Factorized Approach to QED Radiations in Semi-inclusive Deep Inelastic Scatterings

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Tianbo Liu (刘天博)

Key Laboratory of Particle Physics and Particle Irradiation (MOE)
Institute of Frontier and Interdisciplinary Science, Shandong University

In collaboration with: W. Melnitchouk, J.W. Qiu, N. Sato

山东大学(青岛)
SHANDONG UNIVERSITY, QINGDAO

粒子物理与粒子辐照教育部重点实验室
Key Laboratory of Particle Physics and Particle Irradiation (MOE)
粒子科学技术研究中心
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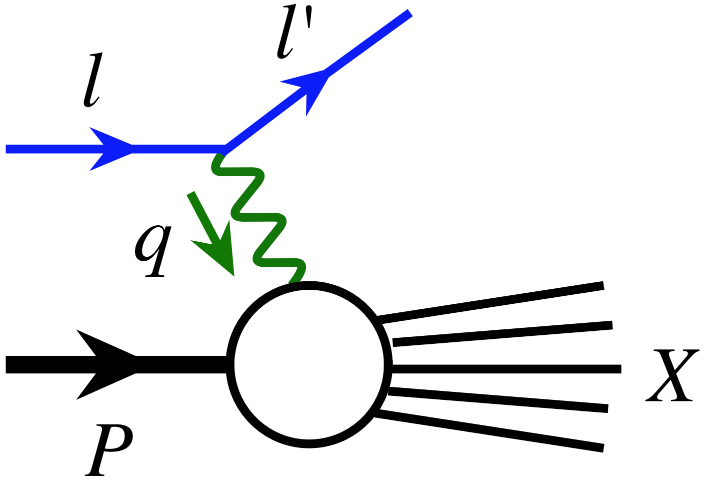
Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer $Q \gg \Lambda_{\rm QCD}$

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization: $\sigma \propto H(Q) \otimes \phi_{a/P}(x, \mu^2)$
- overall corrections suppressed by $1/Q^n$

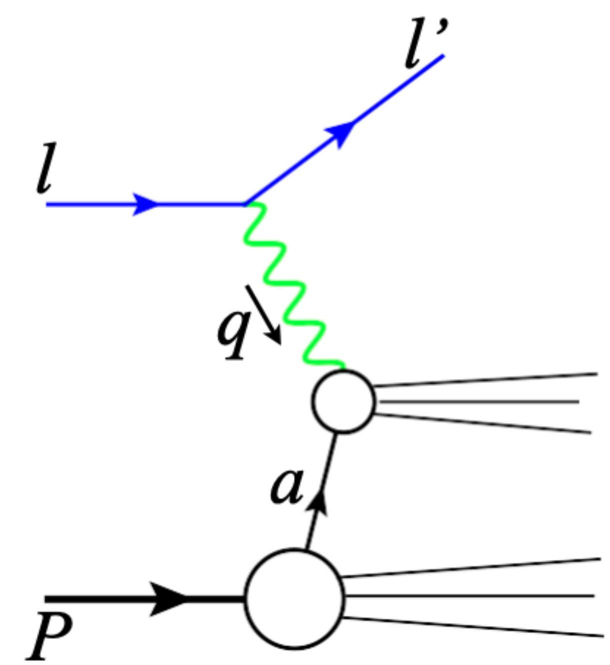


QCD factorization

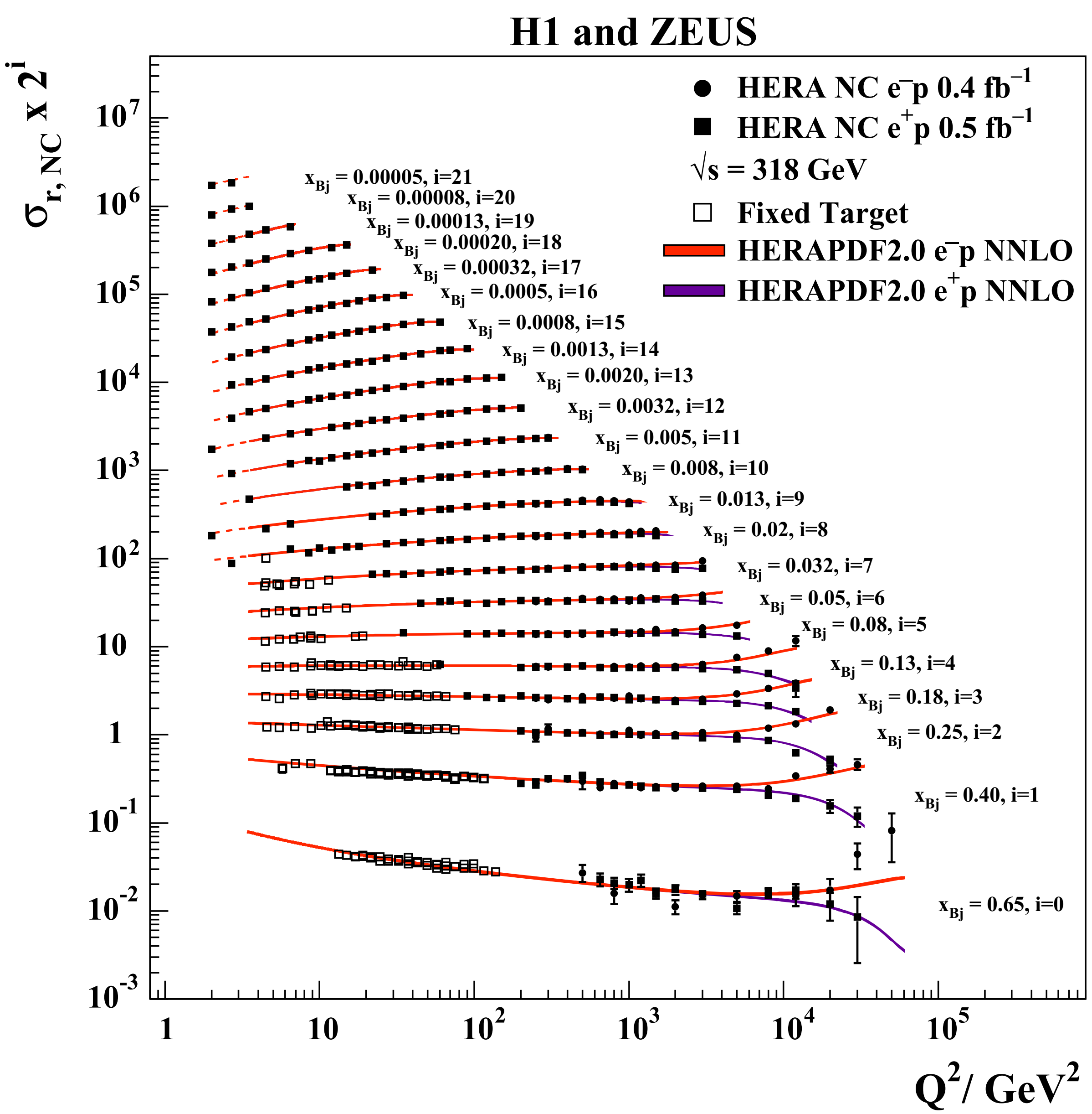
- provides the probe to "see" quarks, gluons and their dynamics indirectly
- predictive power relies on

— precision of the probe

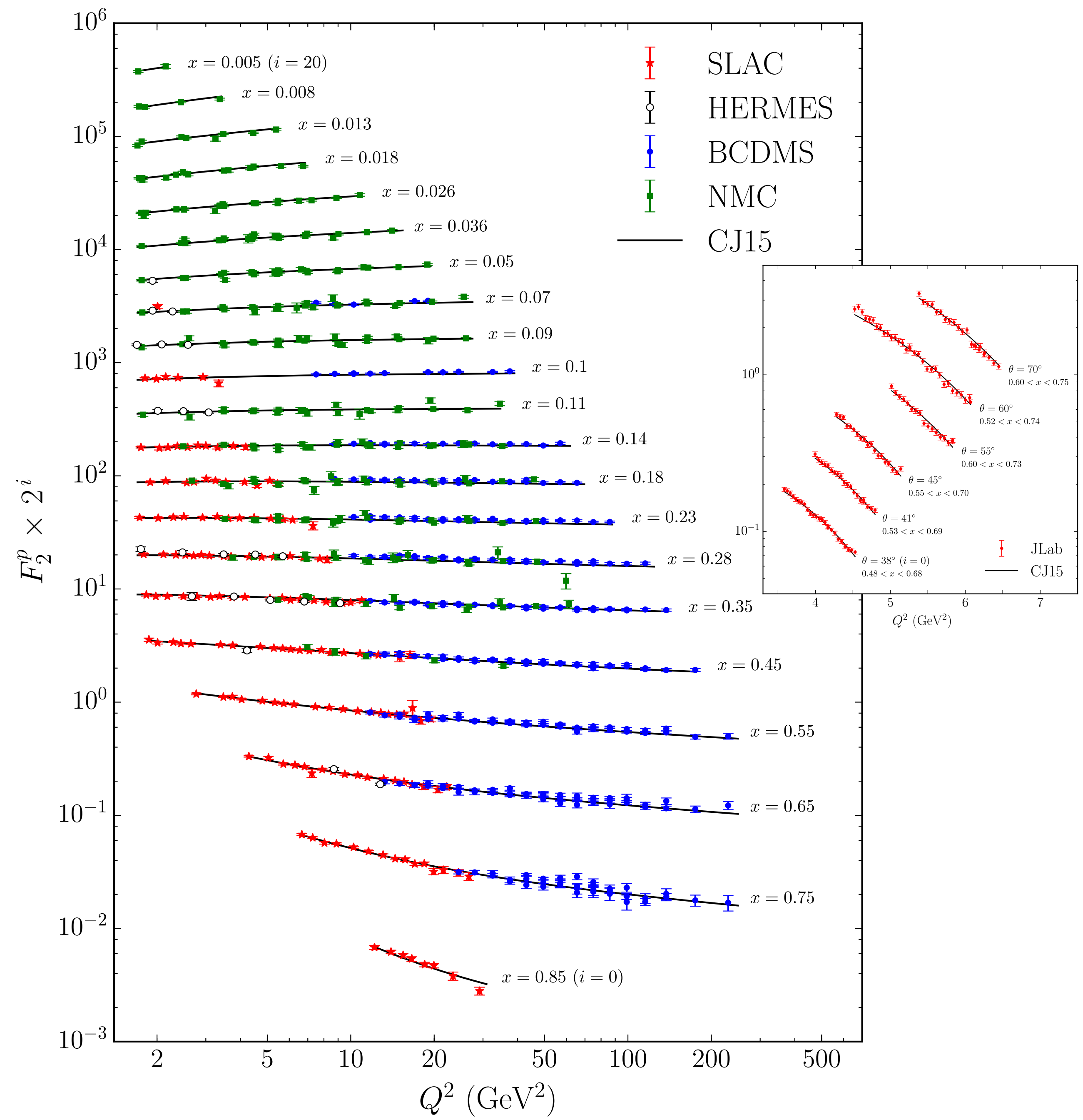
— universality of $\phi_{a/P}(x, \mu^2)$



Lepton-Hadron Deep Inelastic Scattering



H. Abramowicz *et al.*, EPJC 78, 580 (2015).

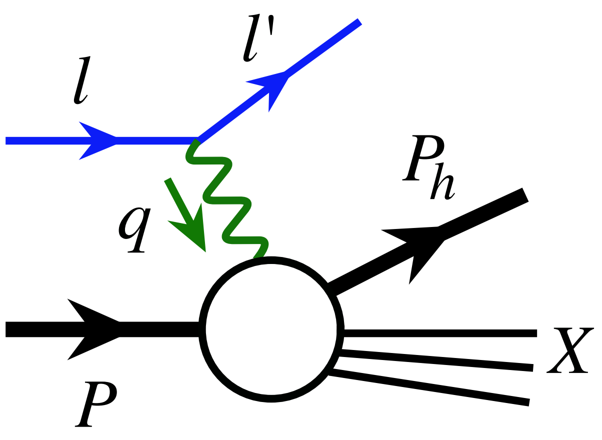


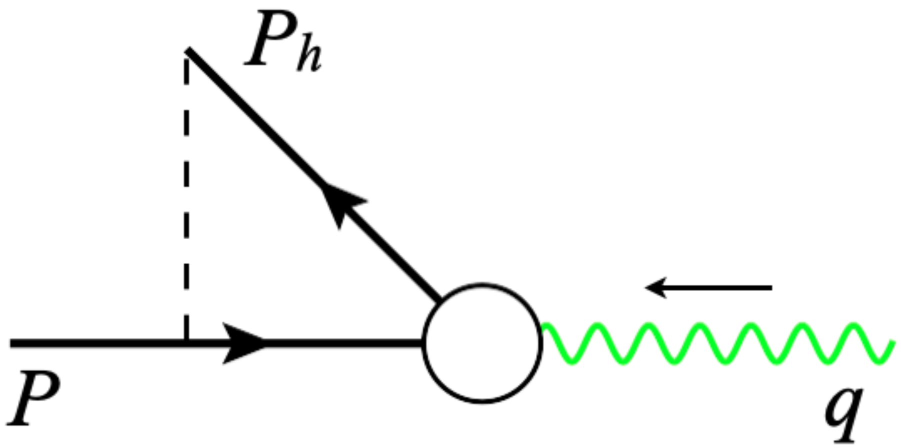
A. Accardi *et al.*, PRD 93, 114017 (2016).

Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, …
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{h_T}



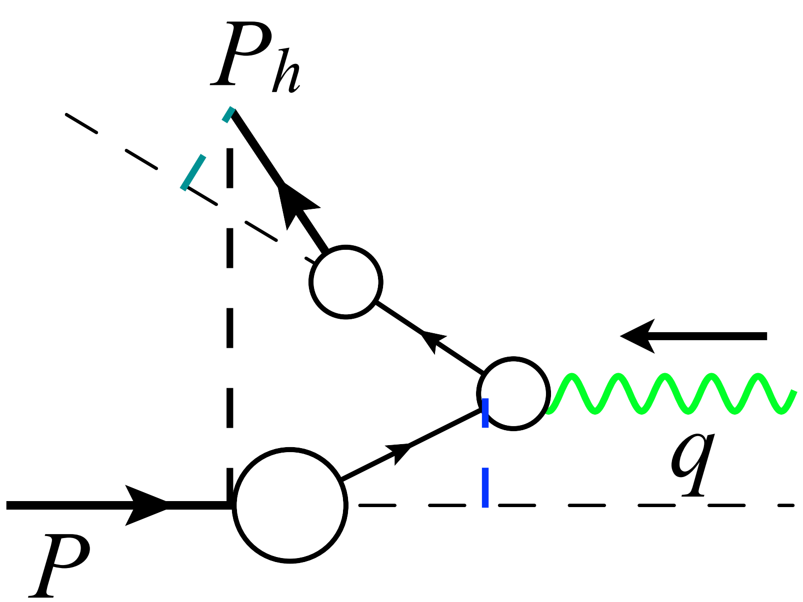


Small and Large Transverse Momentum

Small transverse momentum: $P_{hT} \ll Q$

- the hard scale Q localizes the probe to "see" quarks and gluons
- the soft scale P_{hT} is sensitive to the confined motion of quarks and gluons
- TMD factorization

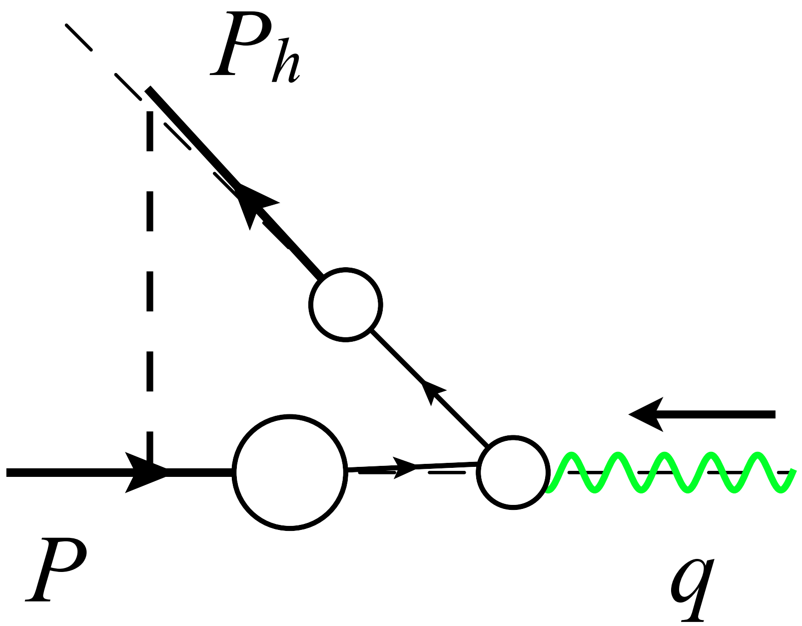
$$\sigma \propto H(Q) \otimes \phi_{a/P}(x, k_T, \mu^2) \otimes D_{f\to h}(z, p_T, \mu^2)$$



Large transverse momentum: $P_{hT} \sim Q$

- dominated by a single scale
- not sensitive to the active parton's transverse momentum
- collinear factorization

$$\sigma \propto H(Q, P_{h_T}) \otimes \phi_{a/P}(x, \mu^2) \otimes D_{f\to h}(z, \mu^2)$$



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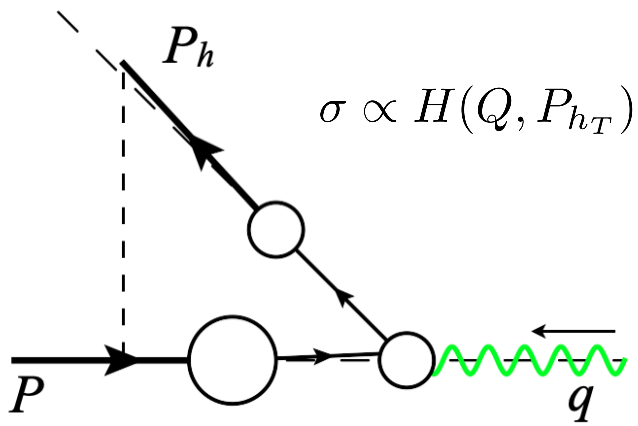
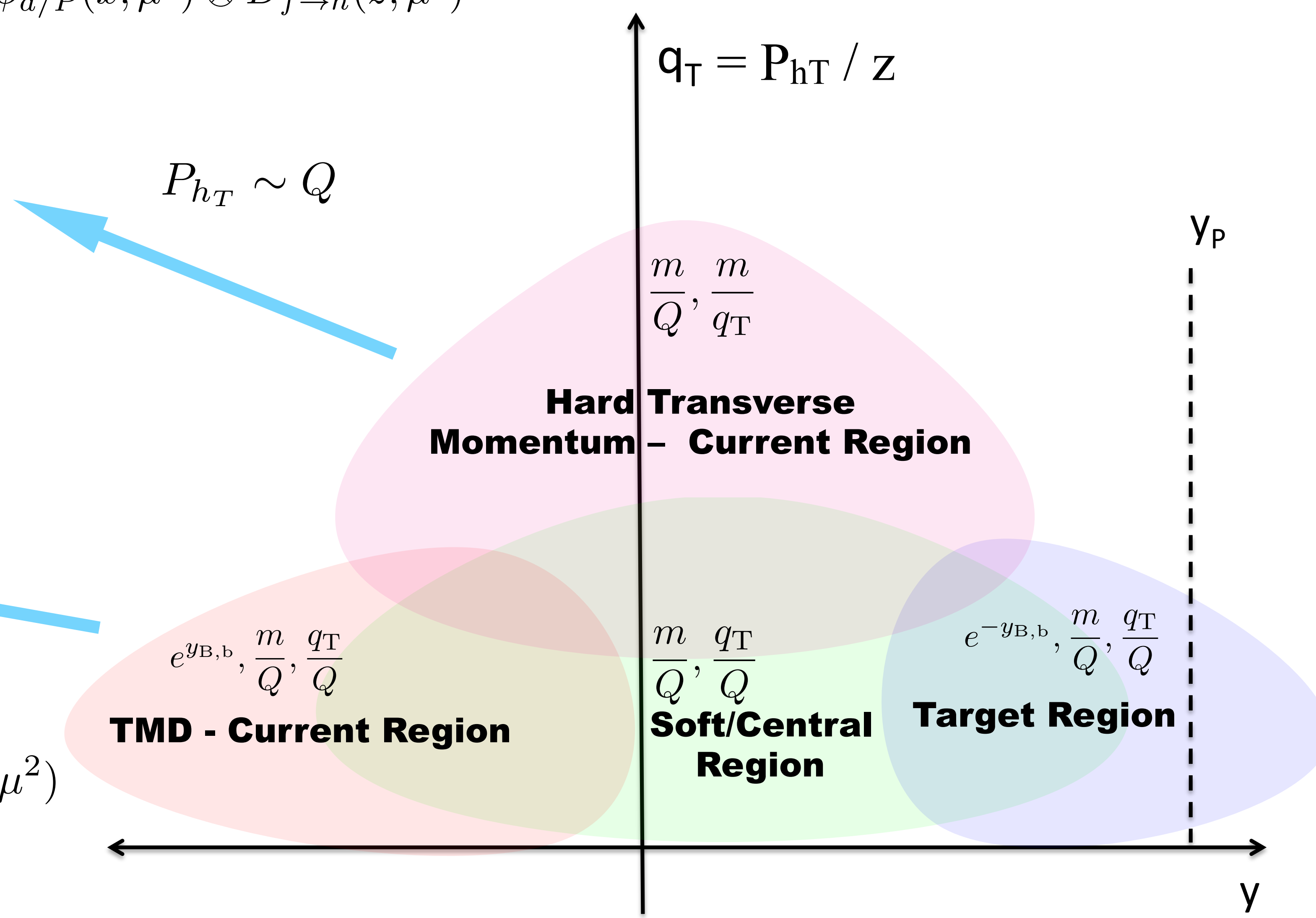
SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron

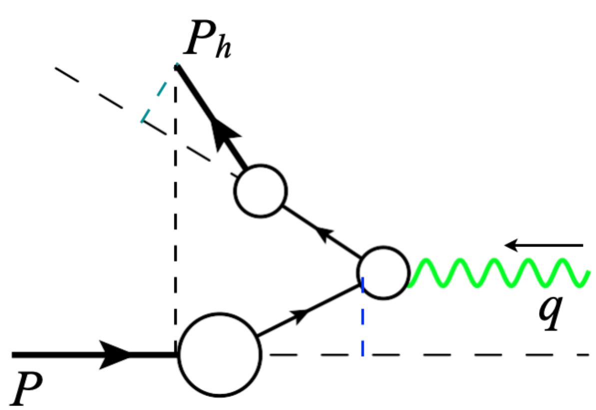


$$\sigma \propto H(Q, P_{h_T}) \otimes \phi_{a/P}(x, \mu^2) \otimes D_{f\to h}(z, \mu^2)$$

$P_{h_T} \sim Q$



$P_{h_T} \ll Q$



$$\sigma \propto H(Q) \otimes \phi_{a/P}(x, k_T, \mu^2) \otimes D_{f\to h}(z, p_T, \mu^2)$$

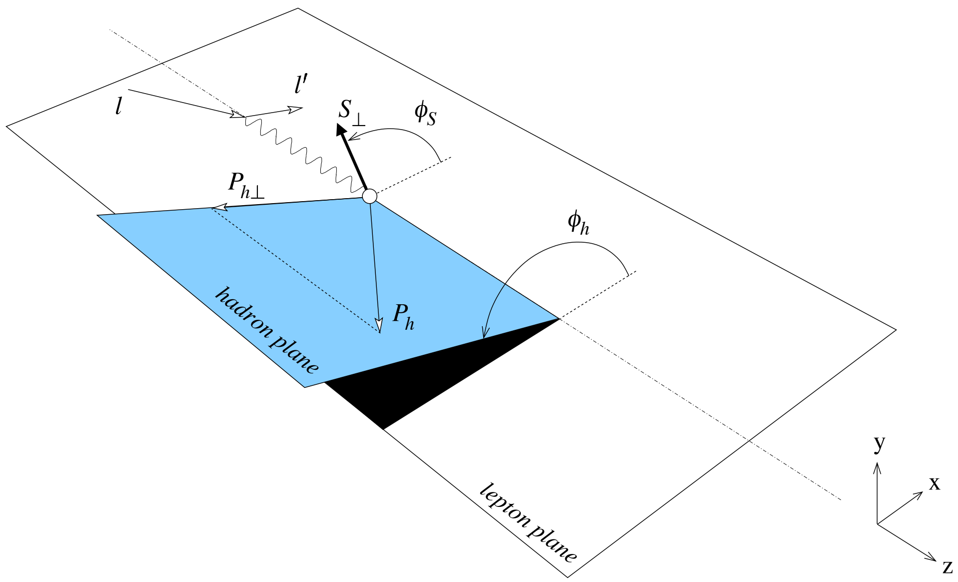
P_{hT} is defined in the photon-hadron frame

[Figure from JHEP10(2019)122]

SIDIS in Trento Convention

SIDIS differential cross section

18 structure functions $F(x_B, z, Q^2, P_{hT})$,
(one photon exchange approximation)



[Trento conventions, PRD70,117504 (2004)]

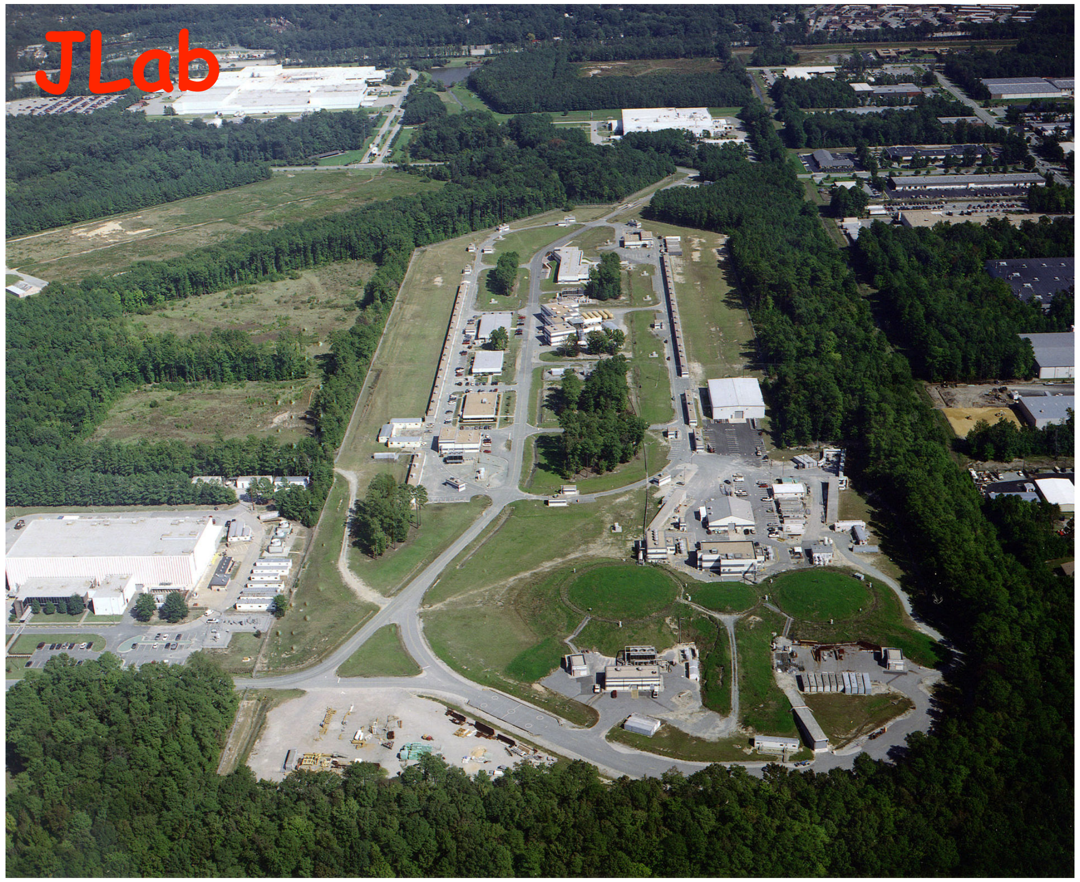
$$
\begin{aligned}
\frac{d^6\sigma}{dx_B\, dy\, dz\, dP_{hT}^2\, d\phi_h\, d\phi_S}
&= \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\\
&\times \Big\{F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h + \lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h\\
&+ S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h + \epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right] + \lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL} + \sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right]\\
&+ S_T\Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin(\phi_h-\phi_S) + \epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)\\
&+ \epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S) + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big]\\
&+ \lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\\
&+ \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S + \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}
\end{aligned}
$$

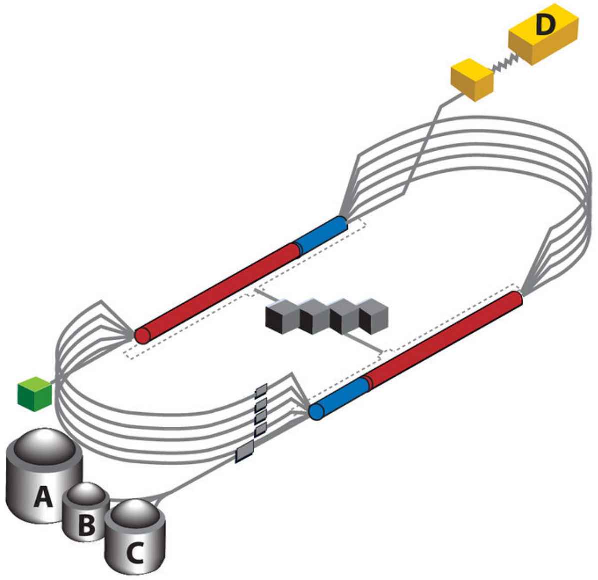
Need to know the photon-hadron frame.

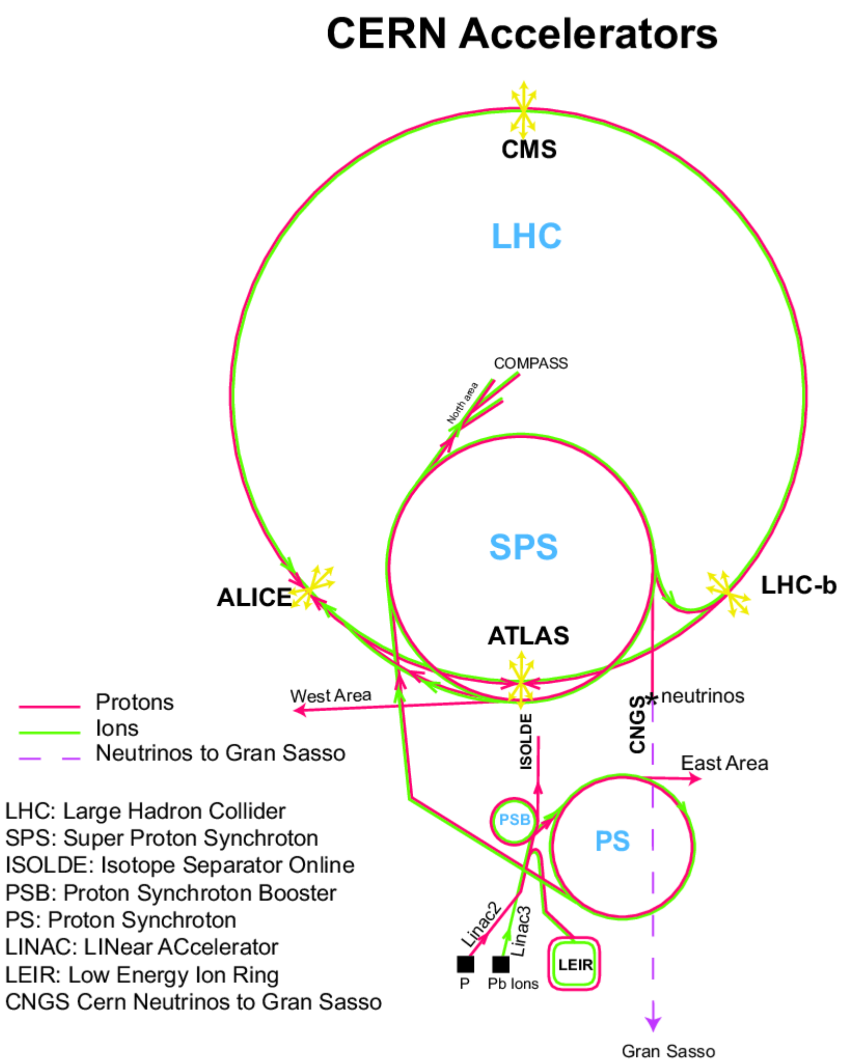
Leading Twist TMDs

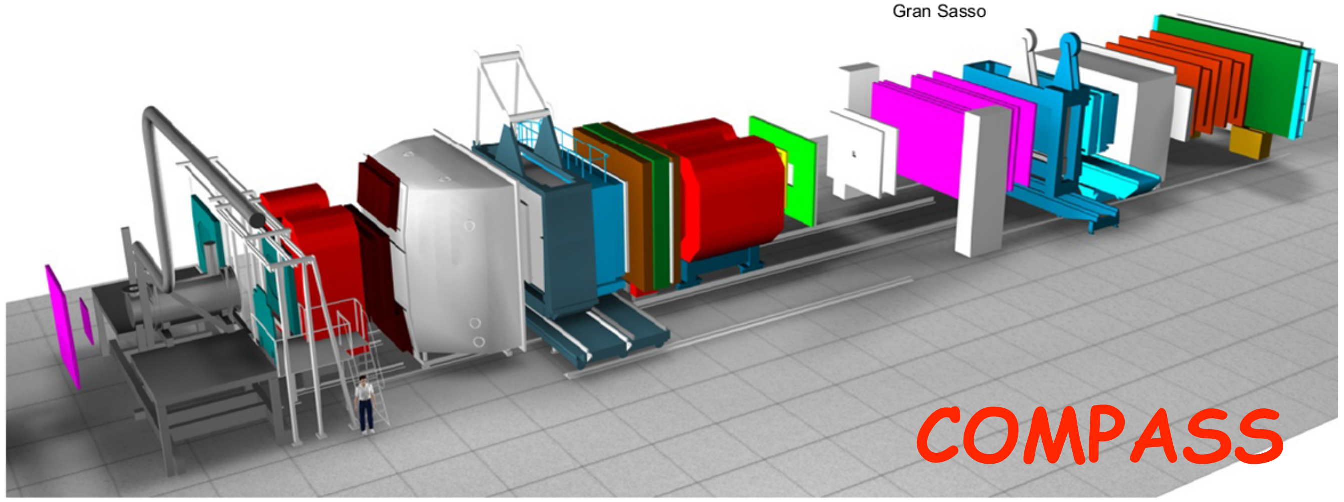
		Quark Polarization		
		U	L	T
Nucleon Polarization	U	f_1 unpolarized		$h_1^{\perp}$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^{\perp}$ longi-transversity (worm-gear)
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} trans-helicity (worm-gear)	h_1 transversity $h_{1T}^{\perp}$ pretzelosity

Fixed Target Experiments (Existing)

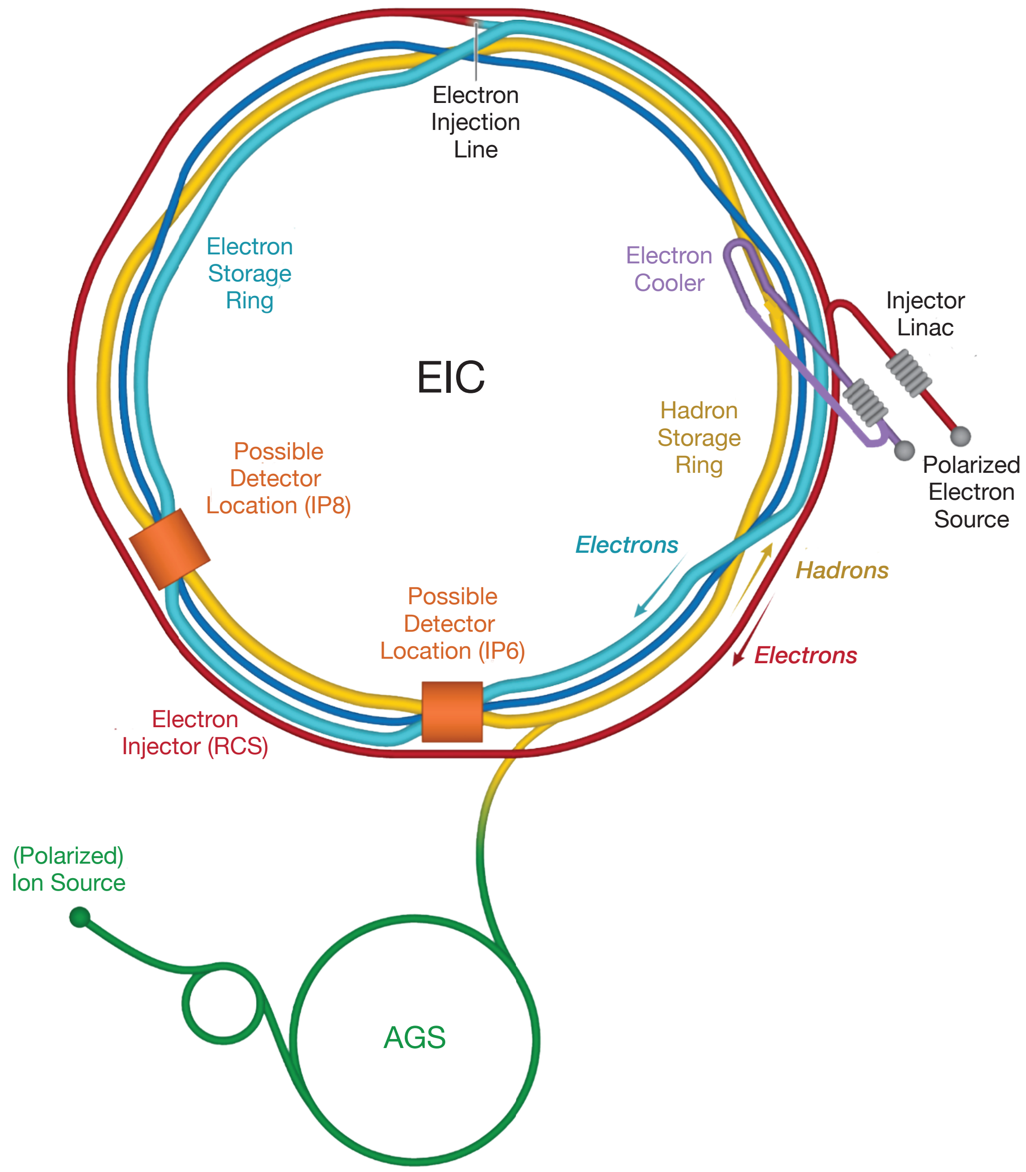




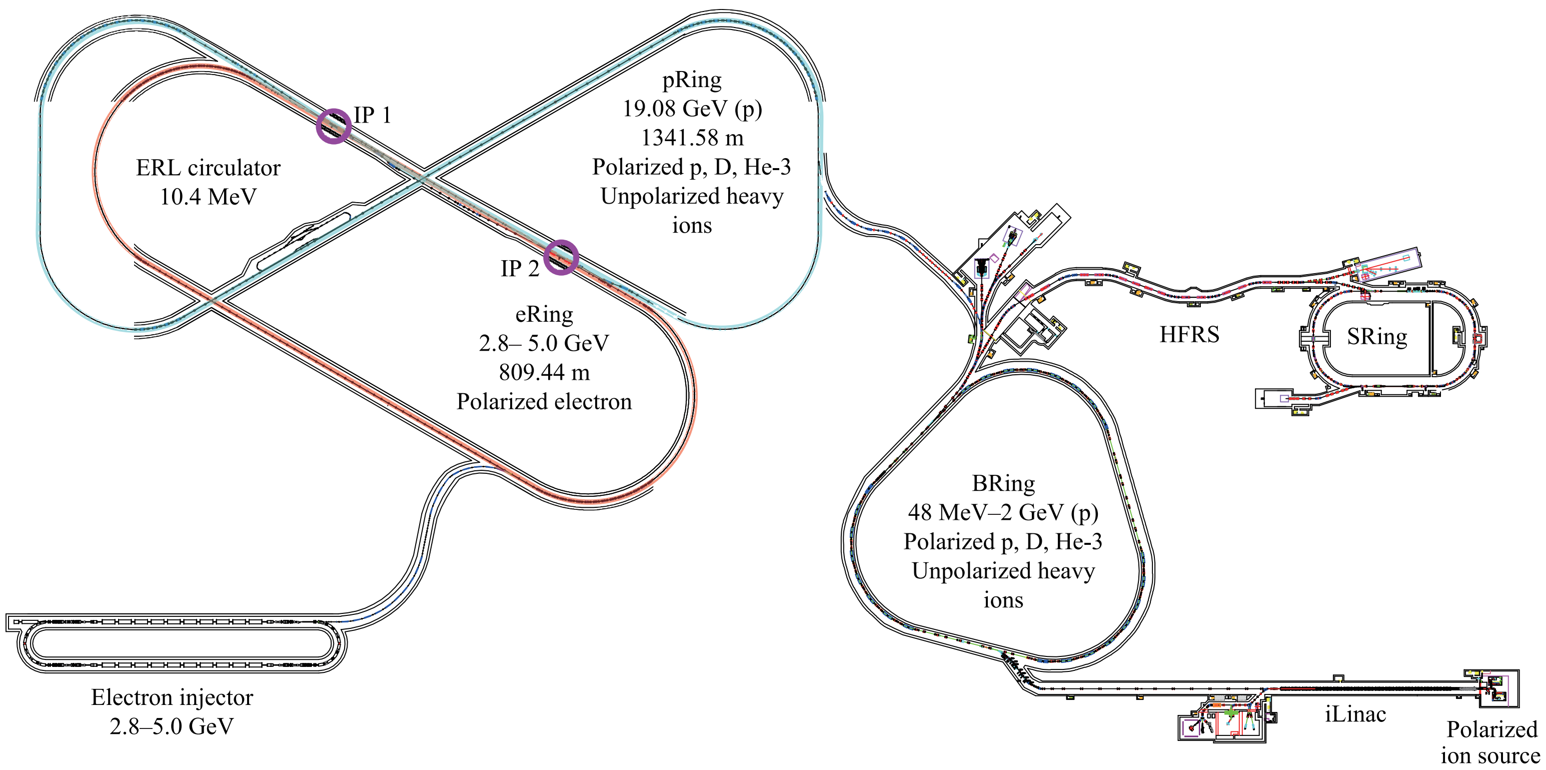




Electron-Ion Colliders (Future)

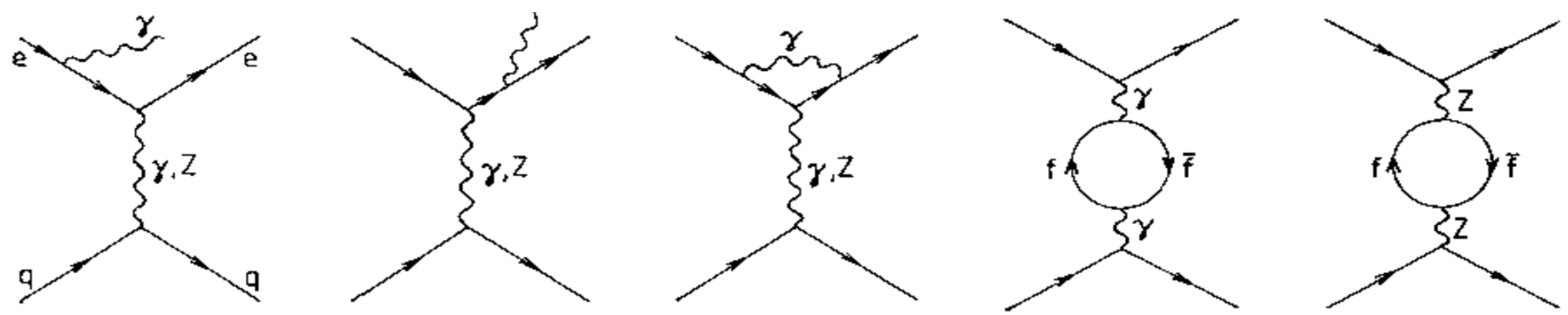


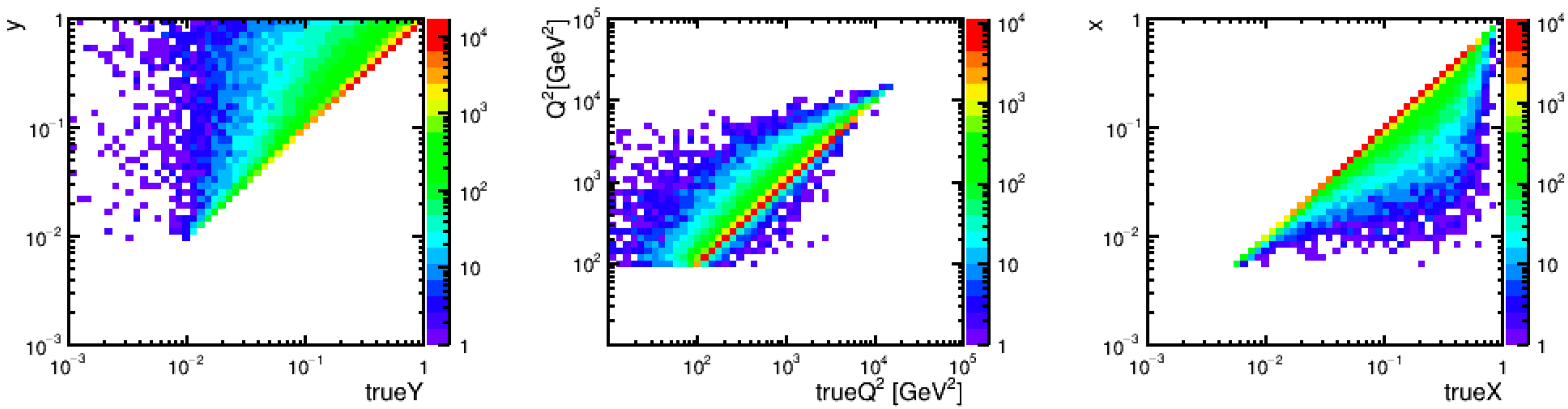
[Figure from EIC Yellow Report]



[Figure from EicC Whitepaper]

Kinematics with Radiative Effects





[Figures from X. Chu at 2nd EIC YR workshop]

Kinematic experience by the parton ≠ *Kinematic reconstructed from observed momenta*

QED radiation will have significant impact due to kinematic shift, although α is small.

Traditional Method to Handle QED Radiation

Radiative correction (RC) to Born kinematics:

$$\sigma_{\text{measured}} = \sigma_{\text{No QED radiation}} \otimes \eta_{\text{RC}}$$

→ RC factor

"In many nuclear physics experiments, radiative corrections quickly become a dominant source of systematics. In fact, the uncertainty on the corrections might be the dominant source for high-statistics experiment"

—— EIC Yellow Report

Problems or challenges:

The determination of RC factor relies on Monte Carlo simulation.

Usually depends on the physics we want to extract, hence introducing bias.

Also depends on experimental acceptance.

increasingly difficult for reactions beyond inclusive DIS, e.g. SIDIS ...

Multidimensional kinematic shift, challenge to decouple 18 structure functions.

Almost impossible to determine the virtual photon event by event, and thus the *true photon-hadron frame*.

Problematic to define P_{hT} and azimuthal angles, essential for TMD physics.

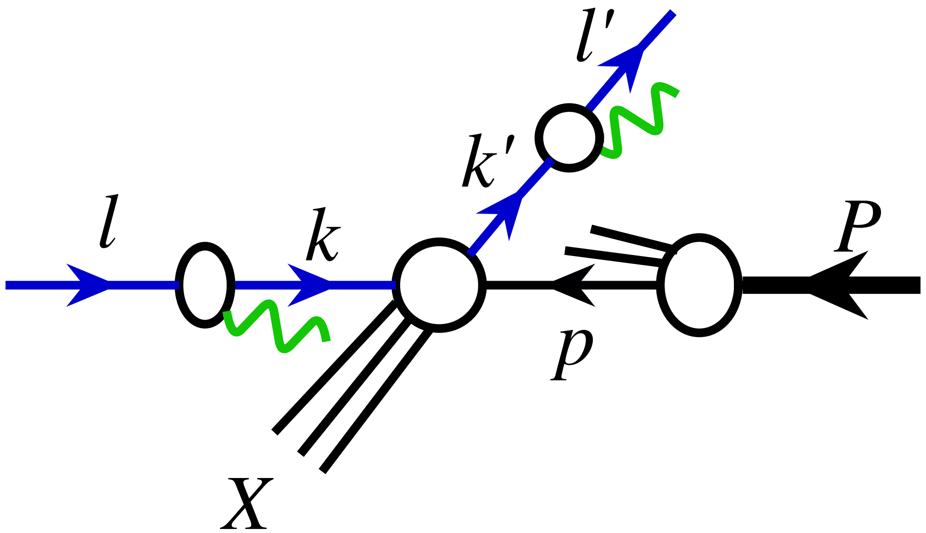
Basic Ideas of Our Approach

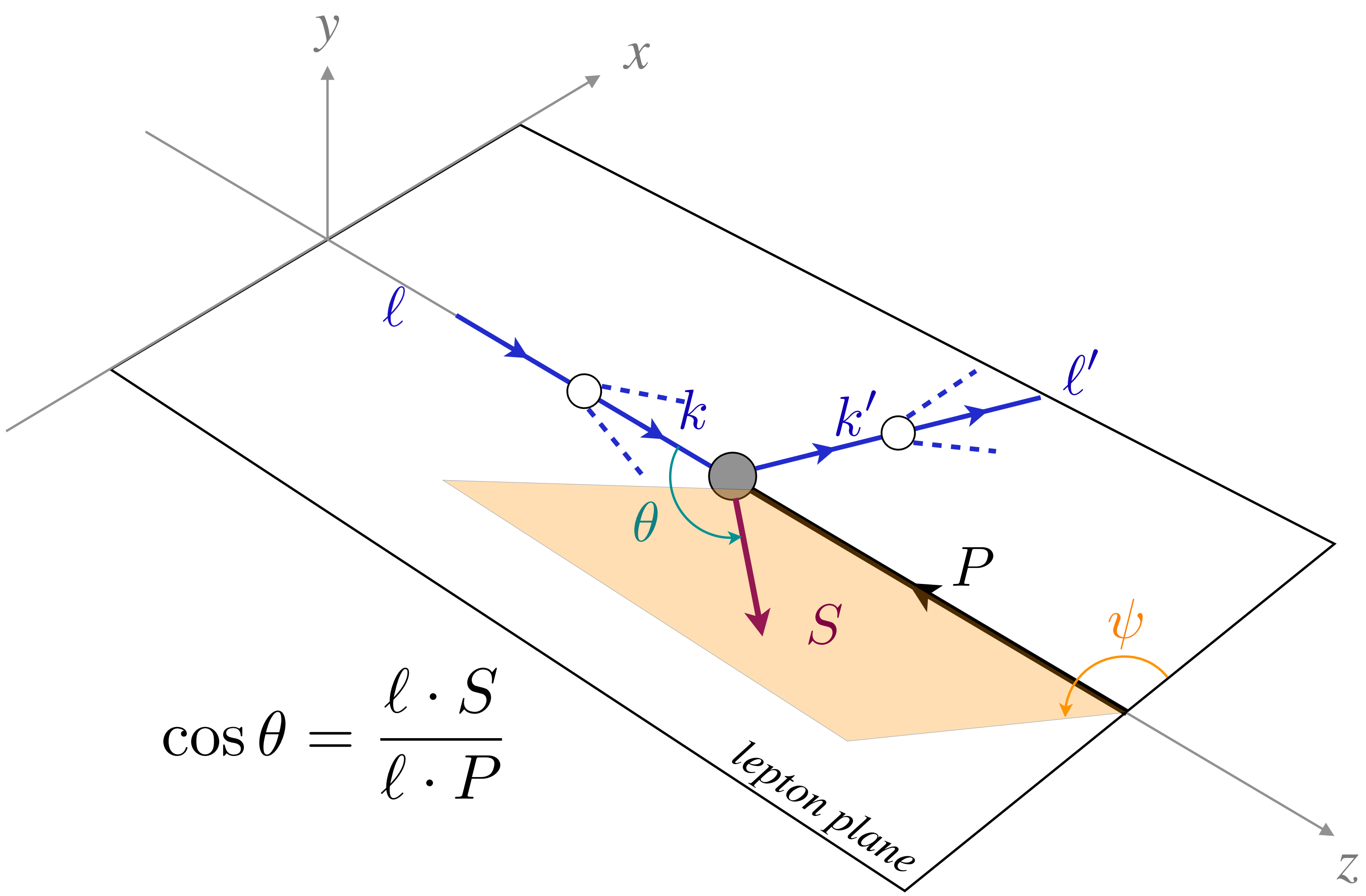
- Do not try to invent any scheme to treat QED radiation to match Born kinematics. — No radiative correction!

- Generalize the QCD factorization to include Electroweak theory, resum the logarithmic enhanced QED contributions.
  — QED radiation is part of the production cross sections.
  — treat QED radiation in the same way as QCD radiation is treated.

- Same systematically improvable treatment of QED contributions for both inclusive DIS and SIDIS.

Inclusive DIS with QED





$$\cos\theta = \frac{\ell \cdot S}{\ell \cdot P}$$

Define inclusive DIS as inclusive lepton scattering with large ℓ'_T

in lepton-hadron frame

Factorized Approach to inclusive DIS

Unpolarized inclusive DIS cross section:

lepton fragmentation function (LFF)

lepton distribution function (LDF)

$$E' \frac{\mathrm{d}\sigma_{\ell P \to \ell' X}}{\mathrm{d}^3 \ell'} = \frac{1}{2s} \sum_{i,j,a} \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} \frac{\mathrm{d}\xi}{\xi} D_{e/j}\left(\zeta, \mu^2\right) f_{i/e}\left(\xi, \mu^2\right)$$

$$\times \int_{x_{\min}}^{1} \frac{\mathrm{d}x}{x} f_{a/N}\left(x, \mu^2\right) \widehat{H}_{ia \to jX}\left(\xi\ell, xP, \ell'/\zeta, \mu^2\right) + \cdots$$

$$\zeta_{\min} = -\frac{t+u}{s}, \quad \xi_{\min} = -\frac{u}{\zeta s + t}, \quad x_{\min} = -\frac{\xi t}{\zeta \xi s + u}$$

one-photon exchange approximation:

$$\frac{\mathrm{d}\sigma_{\ell P \to \ell' X}}{\mathrm{d}x_B\, \mathrm{d}y} \approx \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} \mathrm{d}\xi D_{e/e}\left(\zeta, \mu^2\right) f_{e/e}\left(\xi, \mu^2\right)$$

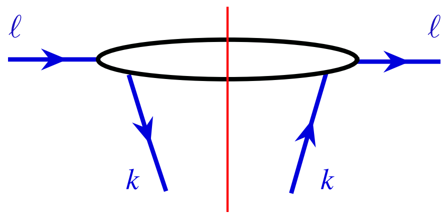
$$\times \frac{4\pi\alpha^2}{\hat{x}_B \hat{y} \widehat{Q}^2} \left[\hat{x}_B \hat{y}^2 F_1\left(\hat{x}_B, \widehat{Q}^2\right) + \left(1 - \hat{y} - \frac{1}{4}\hat{y}^2\hat{\gamma}^2\right) F_2\left(\hat{x}_B, \widehat{Q}^2\right)\right]$$

$$\widehat{Q}^2 = -\hat{q}^2 = \frac{\xi}{\zeta} Q^2, \quad \hat{x}_B = \frac{\widehat{Q}^2}{2P \cdot \hat{q}}, \quad \hat{y} = \frac{P \cdot \hat{q}}{P \cdot k}, \quad \hat{\gamma} = \frac{2M\hat{x}_B}{\widehat{Q}}$$

LDF and LFF

Lepton distribution function:

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi\ell^+ z^-} \langle e| \overline{\psi}_i(0)\gamma^+ \Phi_{[0,z^-]}\, \psi_i(z^-)|e\rangle$$



LO: $f^{(0)}_{i/e}(\xi) = \delta_{ie}\delta(1-\xi)$ NLO($\overline{\rm MS}$): $f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$

Lepton fragmentation function:

$$D_{e/j}(\zeta) = \frac{\zeta}{2}\sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^-/\zeta}\, {\rm Tr}\left[\gamma^+ \langle 0| \overline{\psi}_j(0)\Phi_{[0,\infty]}\, |e,X\rangle\langle e,X|\psi_j(z^-)\Phi_{[z^-,\infty]}|0\rangle\right]$$

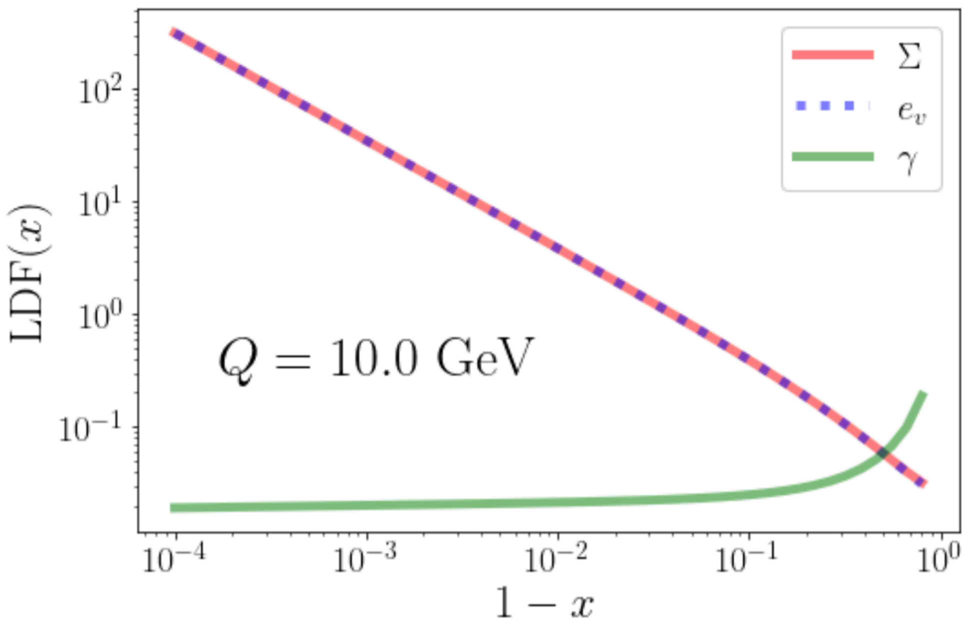
LO: $D^{(0)}_{e/j}(\zeta) = \delta_{ej}\delta(1-\zeta)$ NLO($\overline{\rm MS}$): $D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$

Resum:

$$\frac{\partial}{\partial \ln\mu^2}\begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\gamma} \\ P_{\gamma e} & P_{\gamma\gamma} \end{pmatrix} \otimes \begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix}$$

QED DGLAP evolution

Similar for LFF



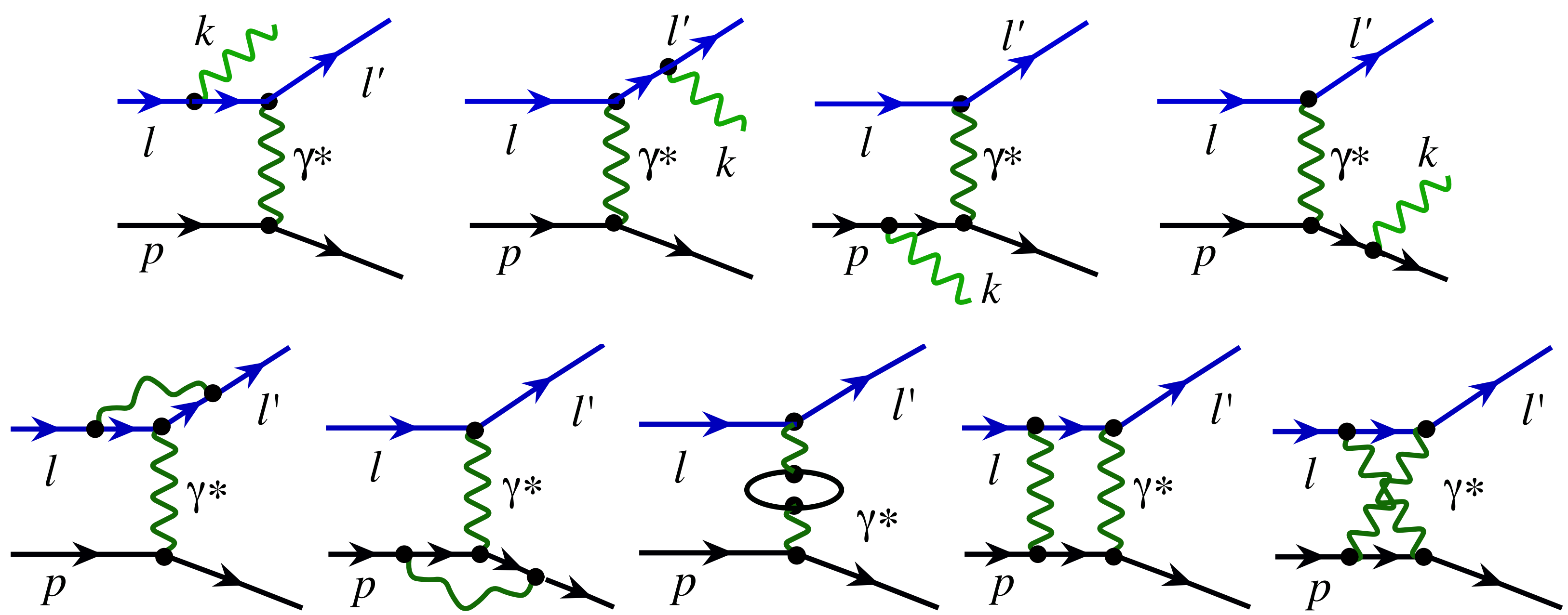
Hard Part of Inclusive DIS

LO:

$$\sigma_{eq}^{(2,0)} = D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{q/q}^{(0)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} = \widehat{H}_{eq\to eX}^{(2,0)}$$

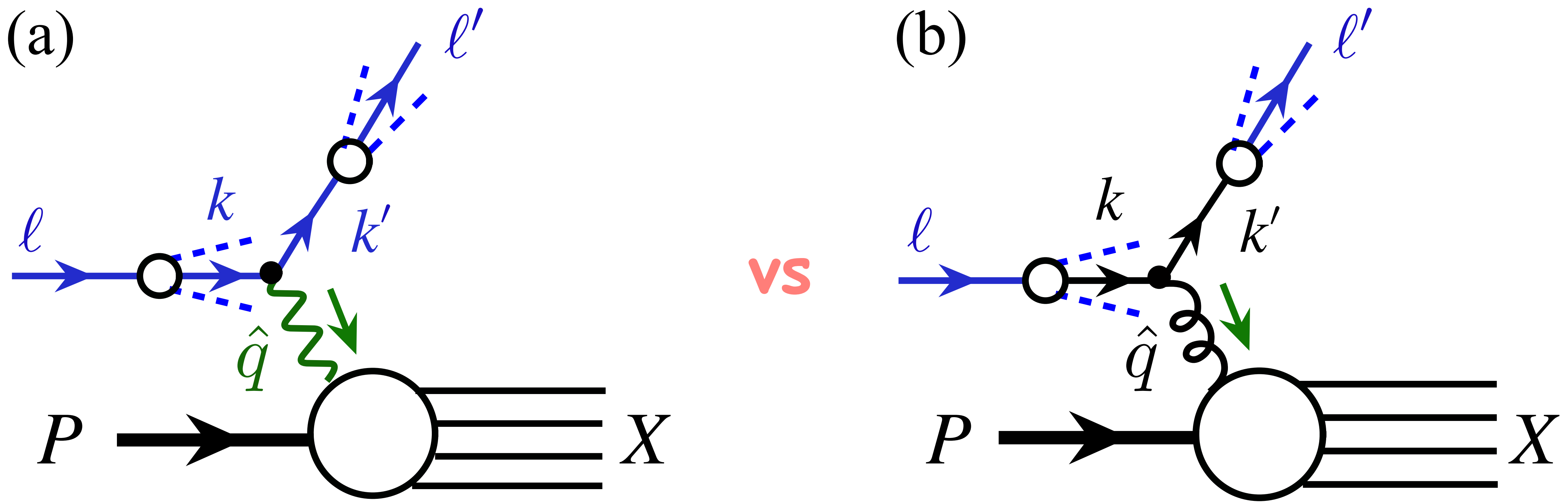
$$\widehat{H}_{eq\to eX}^{(2,0)} = \frac{4\alpha^2 e_q^2}{\zeta}\left[\frac{(\zeta\xi x s)^2 + (xu)^2}{(\xi t)^2}\right]\delta(\zeta\xi x s + xu + \xi t)$$

NLO:



$$\widehat{H}_{eq\to eX}^{(3,0)} = \sigma_{eq}^{(3,0)} - D_{e/e}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} - f_{e/e}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} - f_{q/q}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)}$$

Quark in Lepton at Higher Order

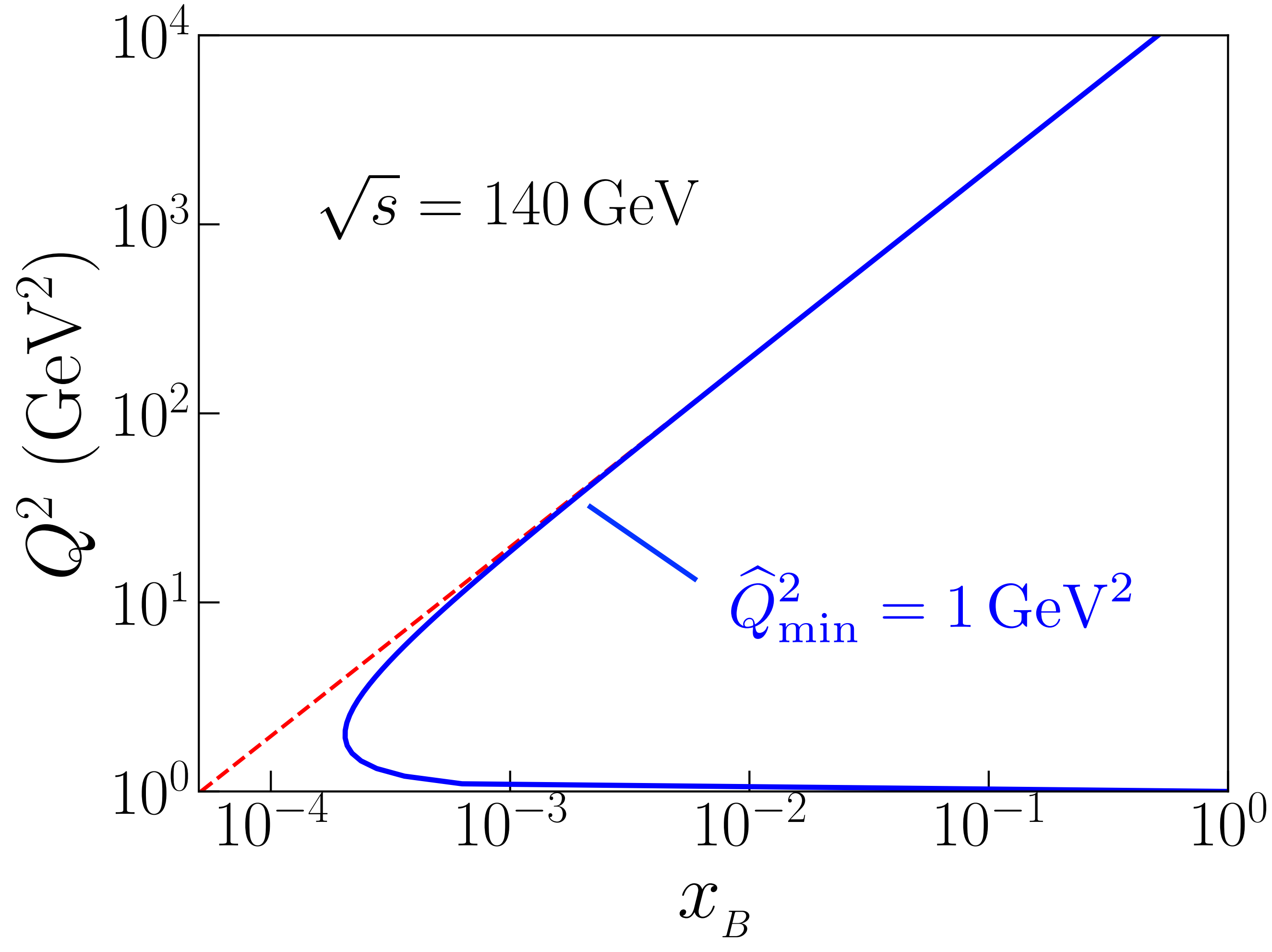


At higher order one can find quark/gluon distribution in LDF and LFF.

(b) is suppressed by selecting events in which the lepton does not have much hadronic energy around it.

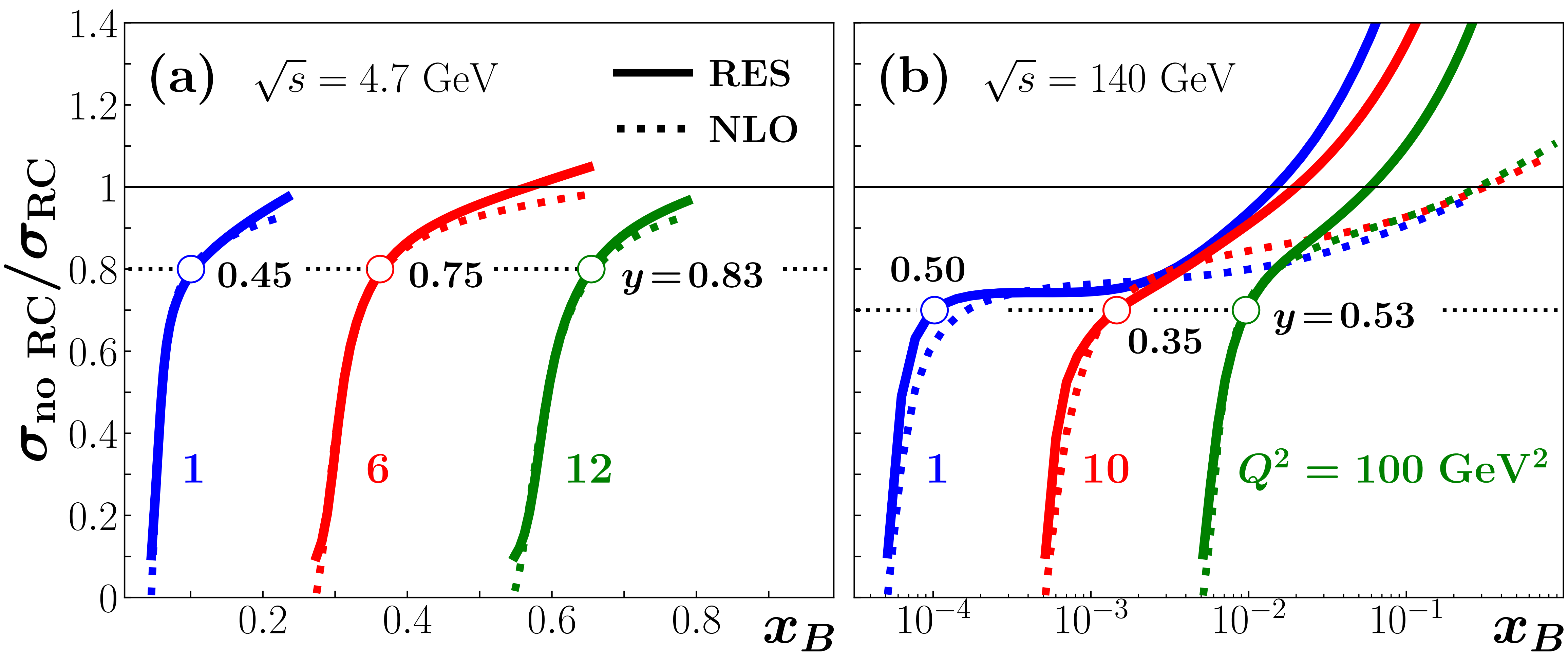
The Hard Scale

Collision induced QED radiation changes the hard scale from Q^2 to $\widehat{Q}^2$

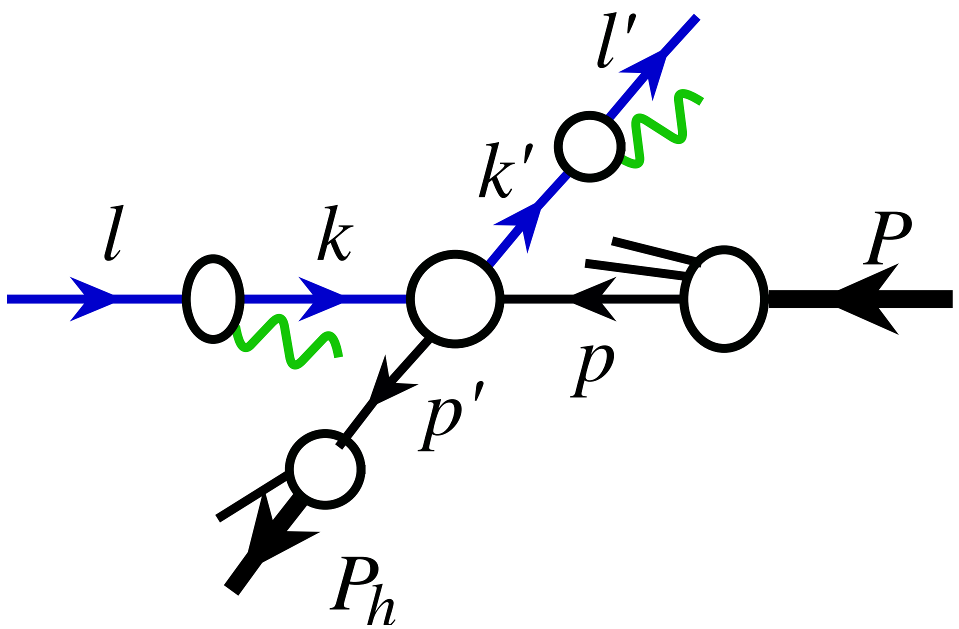


$\widehat{Q}^2$ has a minimum value $\widehat{Q}^2_{\min} = \dfrac{1-y}{1-yx_B} Q^2 < Q^2$

Impact on Inclusive DIS



Semi-inclusive DIS with QED



Define SIDIS as inclusive production of large ℓ'_T lepton plus large P_{hT} hadron.

in lepton-hadron frame

$$\overline{\mathrm{P}}_T \equiv \left|\ell'_T - \boldsymbol{P}_{hT}\right|/2 \qquad\qquad \overline{\mathrm{p}}_T \equiv \left|\ell'_T + \boldsymbol{P}_{hT}\right|$$

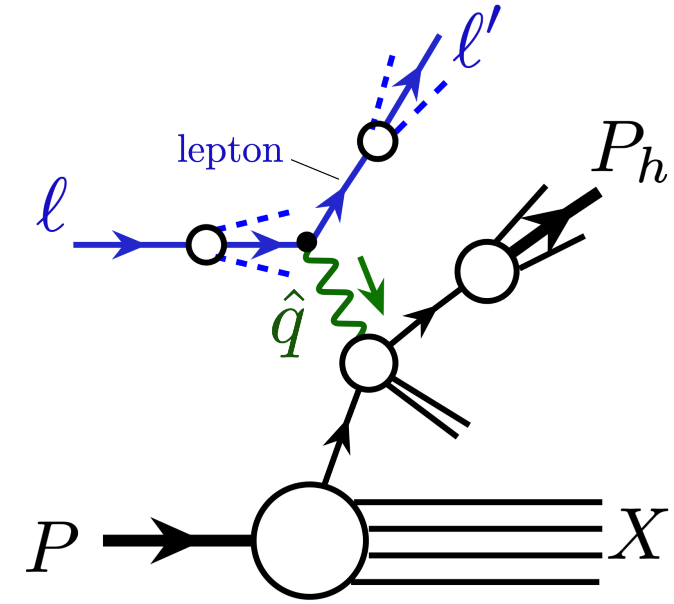
$\overline{\mathrm{P}}_T \gg \overline{\mathrm{p}}_T$ TMD factorization

$\overline{\mathrm{P}}_T \sim \overline{\mathrm{p}}_T$ collinear factorization

SIDIS Cross Section with QED Radiations

Differential cross section

$$d\sigma_{\ell P\to\ell' P_h X} = \frac{1}{2s}\left|M_{\ell P\to\ell' P_h X}\right|^2 \mathrm{dPS}$$



one photon exchange approximation:

$$E_{\ell'}E_{P_h}\frac{d^6\sigma_{\ell P\to\ell' P_h X}}{d^3\ell' d^3P_h} \approx \frac{\alpha^2}{2s}\int d^4\hat{q}\left(\frac{1}{\hat{q}^2}\right)^2 \widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q})\,\widetilde{W}_{\mu\nu}(\hat{q},P,P_h,S)$$

Hadronic tensor:

$$\widetilde{W}_{\mu\nu}(\hat{q},P,P_h,S) = \sum_{X_h}\int\prod_{i\in X_h}\frac{d^3p_i}{(2\pi)^3 2E_i}\delta^{(4)}\left(\hat{q}+P-P_h-\sum_{i\in X_h}p_i\right) \times \langle P,S\,|J_\mu(0)|\,P_hX_h\rangle\langle P_hX_h\,|J_\nu(0)|\,P,S\rangle$$

Leptonic tensor:

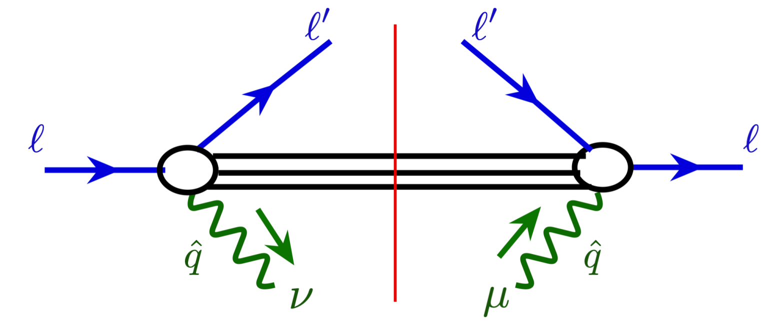
$$\widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q}) \equiv \sum_{X_L}\int\prod_{i\in X_L}\frac{d^3k_i}{(2\pi)^3 2E_i}\delta^{(4)}\left(\ell-\ell'-\hat{q}-\sum_{i\in X_L}k_i\right) \times \langle \ell\,|j^\mu(0)|\,\ell' X_L\rangle\langle \ell' X_L\,|j^\nu(0)|\,\ell\rangle$$

The lowest order recovers no QED radiation expression:

$$\widetilde{L}^{\mu\nu(0)}(\ell,\ell',\hat{q}) = 2\left(\ell^\mu\ell'^\nu+\ell'^\mu\ell^\nu-\ell\cdot\ell' g^{\mu\nu}\right)\delta^{(4)}(\ell-\ell'-\hat{q})$$

Lepton Structure Functions

Current conserved decomposition of leptonic tensor



$$\widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q}) = -\widetilde{g}^{\mu\nu}L_1 + \frac{\widetilde{\ell}^{\mu}\widetilde{\ell}^{\nu}}{\ell\cdot\ell'}L_2 + \frac{\widetilde{\ell}'^{\mu}\widetilde{\ell}'^{\nu}}{\ell\cdot\ell'}L_3 + \frac{\widetilde{\ell}^{\mu}\widetilde{\ell}'^{\nu}+\widetilde{\ell}'^{\mu}\widetilde{\ell}^{\nu}}{2\ell\cdot\ell'}L_4$$

$$\widetilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{\hat{q}^{\mu}\hat{q}^{\nu}}{\hat{q}^2}, \quad \widetilde{\ell}^{\mu} = \widetilde{g}^{\mu\nu}\ell_{\nu} = \ell^{\mu} - \frac{\ell\cdot\hat{q}}{\hat{q}^2}\hat{q}^{\mu}, \quad \widetilde{\ell}'^{\mu} = \widetilde{g}^{\mu\nu}\ell'_{\nu} = \ell'^{\mu} - \frac{\ell'\cdot\hat{q}}{\hat{q}^2}\hat{q}^{\mu}$$

Lepton structure functions: $L_i(\xi_B,\zeta_B,\hat{\mathbf{q}}_T^2,Q^2), \quad i=1,2,3,4$

$$\xi_B = \frac{\hat{q}\cdot\ell'}{\ell\cdot\ell'}, \quad \frac{1}{\zeta_B} = -\frac{\hat{q}\cdot\ell}{\ell\cdot\ell'} \quad \hat{\boldsymbol{q}}_T^2 = \widehat{Q}^2 - \frac{\xi_B}{\zeta_B}Q^2$$

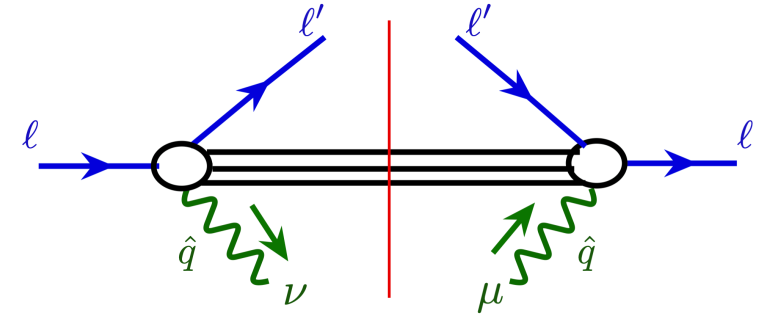
In lepton back-to-back frame:

$$\ell^{\mu} = (\ell^{+},0,\mathbf{0}_T), \quad \ell'^{\mu} = (0,\ell'^{-},\mathbf{0}_T) \qquad \ell^{+} = \ell'^{-} = Q/\sqrt{2}$$

$$\hat{q}^{\mu} = (\hat{q}^{+},\hat{q}^{-},\hat{\boldsymbol{q}}_T) = \left(\xi_B\ell^{+}, -\frac{1}{\zeta_B}\ell'^{-}, \hat{\boldsymbol{q}}_T\right)$$

Lepton SFs in Helicity Basis

Basis vectors and polarization vectors:



$$T^\mu = \frac{\sqrt{\xi_B\zeta_B}}{Q}\widetilde{\ell}^\mu + \frac{1}{\sqrt{\xi_B\zeta_B}Q}\widetilde{\ell}'^\mu,$$

$$X^\mu = -\frac{\widehat{Q}\sqrt{\xi_B\zeta_B}}{Q\sqrt{\hat{\boldsymbol{q}}_T^2}}\widetilde{\ell}^\mu + \frac{\widehat{Q}}{Q\sqrt{\xi_B\zeta_B}\sqrt{\hat{\boldsymbol{q}}_T^2}}\widetilde{\ell}'^\mu,$$

$$Y^\mu = \varepsilon^{\mu\nu\rho\sigma}Z_\nu T_\rho X_\sigma,$$

$$Z^\mu = \frac{\hat{q}^\mu}{Q}$$

$$\epsilon_0^\mu(\hat{q}) = T^\mu,$$

$$\epsilon_+^\mu(\hat{q}) = -\frac{1}{\sqrt{2}}X^\mu - \frac{i}{\sqrt{2}}Y^\mu,$$

$$\epsilon_-^\mu(\hat{q}) = \frac{1}{\sqrt{2}}X^\mu - \frac{i}{\sqrt{2}}Y^\mu,$$

Helicity basis lepton structure functions:

$$\begin{aligned}\widetilde{L}^{\mu\nu} &= \epsilon_0^{*\mu}\epsilon_0^\nu L_{00} + (\epsilon_+^{*\mu}\epsilon_+^\nu + \epsilon_-^{*\mu}\epsilon_-^\nu)L_{++} + (\epsilon_+^{*\mu}\epsilon_-^\nu + \epsilon_-^{*\mu}\epsilon_+^\nu)L_{+-} \\ &\quad - \epsilon_0^{*\mu}(\epsilon_+^\nu - \epsilon_-^\nu)L_{0+} - (\epsilon_+^\mu - \epsilon_-^\mu)^*\epsilon_0^\nu L_{+0} \\ &= T^\mu T^\nu L_{00} + (X^\mu X^\nu + Y^\mu Y^\nu)L_{TT} \\ &\quad + (T^\mu X^\nu + T^\nu X^\mu)L_\Delta + (Y^\mu Y^\nu - X^\mu X^\nu)L_{\Delta\Delta},\end{aligned}$$

Expansion in α:

$$L_{\rho\sigma} = e^2\sum_{N=0}^{\infty}\left(\frac{\alpha}{\pi}\right)^N L_{\rho\sigma}^{(N)}$$

Leading order: $L_{TT}^{(0)} = 2\,\delta(\xi-1)\delta(\frac{1}{\zeta}-1)\delta^{(2)}(\hat{\boldsymbol{q}}_T)$ *the other three vanish.*

Factorization of Lepton Structure Function

CSS factorization

"W+Y" formalism:

$$L_{TT}\left(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2\right)=\int\frac{\mathrm{d}^2\boldsymbol{b}}{(2\pi)^2}e^{i\hat{\boldsymbol{q}}_T\cdot\boldsymbol{b}}\widetilde{W}_{TT}\left(\xi_B,\zeta_B,Q^2,b\right)+Y_{TT}\left(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2\right)$$

b-space resummed form:

$$\widetilde{W}_{TT}\left(\xi_B,\zeta_B,Q^2,b\right)=2\int_{\zeta_B}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_B}^{1}\frac{d\xi}{\xi}\left[C_D\left(\frac{\zeta_B}{\zeta},\alpha\right)D\left(\zeta,\mu_b^2\right)\right]\left[C_f\left(\frac{\xi_B}{\xi},\alpha\right)f\left(\xi,\mu_b^2\right)\right]$$
$$\times\exp\left\{-\int_{\mu_b^2}^{\mu_Q^2}\frac{d\mu'^2}{\mu'^2}\left[A\left(\alpha\left(\mu'\right)\right)\ln\frac{\mu_Q^2}{\mu'^2}+B\left(\alpha\left(\mu'\right)\right)\right]\right\}$$

Expansion in α:

$$A=\sum_{N=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^N A^{(N)}$$

$$B=\sum_{N=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^N B^{(N)}$$

$$C_{f,D}=\sum_{N=0}^{\infty}\left(\frac{\alpha}{\pi}\right)^N C_{f,D}^{(N)}$$

$$A^{(1)}=1,\qquad C_f^{(0)}(\lambda)=\delta(1-\lambda)$$

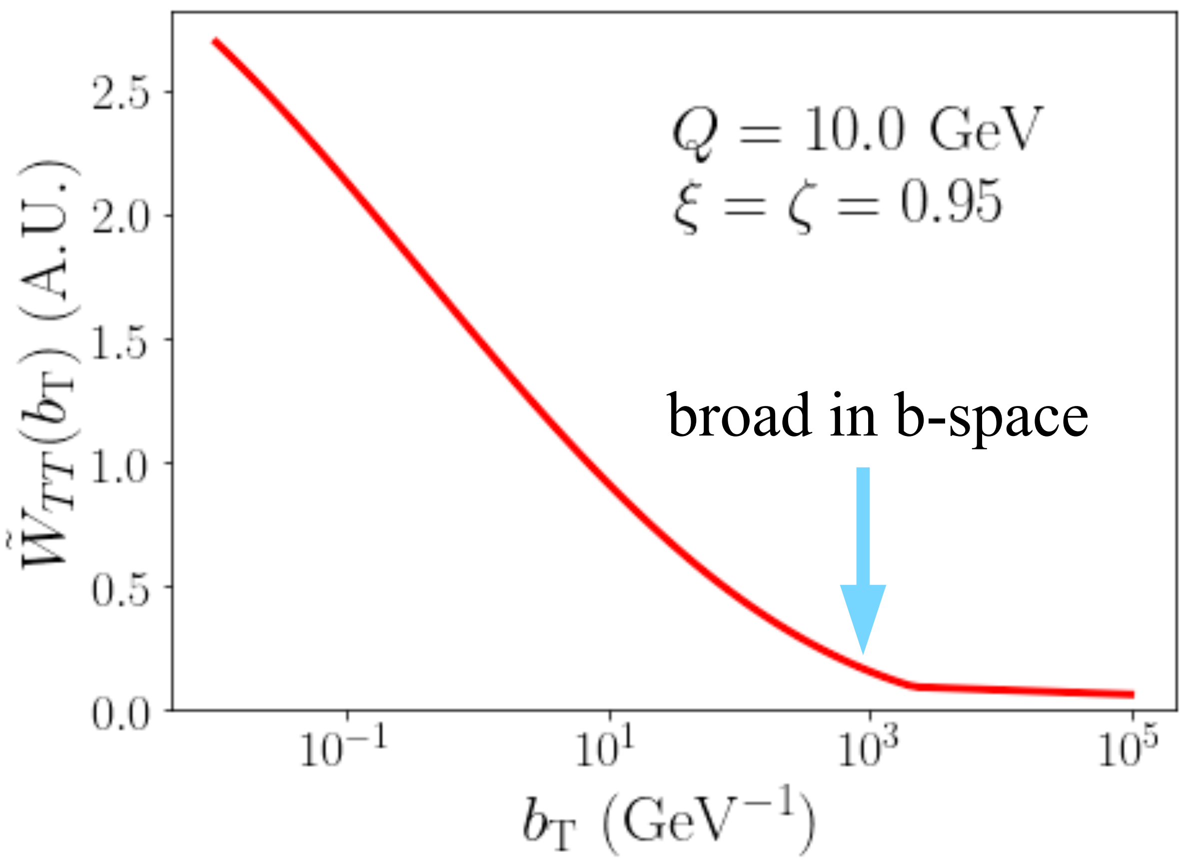
$$B^{(1)}=-\frac{3}{2}\qquad C_D^{(0)}(\eta)=\delta(1-\eta)$$

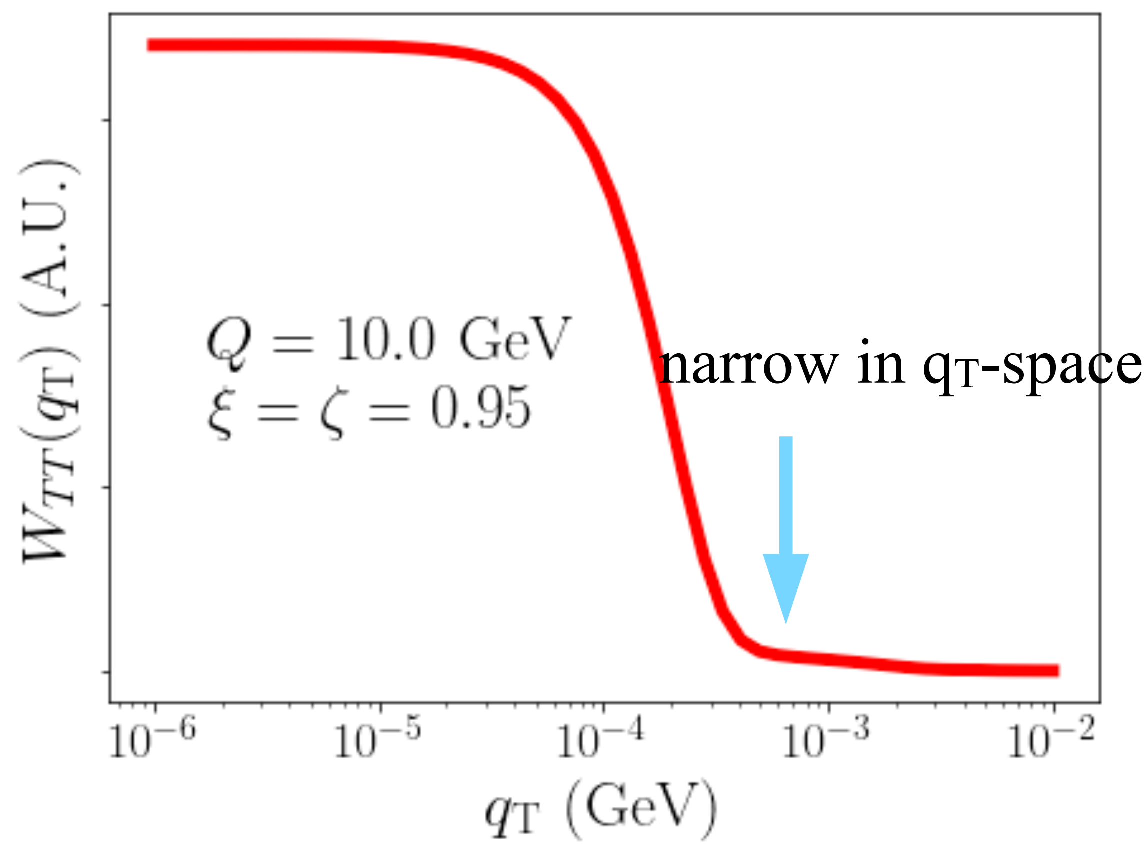
$$C_f^{(1)}(\lambda)=\frac{1}{2}(1-\lambda)-\left(\frac{1+\lambda^2}{1-\lambda}\right)_+\ln\frac{\mu_{\overline{\mathrm{MS}}}}{\mu_b}-2\delta(1-\lambda),$$

$$C_D^{(1)}(\eta)=\frac{1}{2\eta}(1-\eta)-\frac{1}{\eta}\left(\frac{1+\eta^2}{1-\eta}\right)_+\ln\frac{\mu_{\overline{\mathrm{MS}}}}{\mu_b}-2\delta(1-\eta)$$

Lepton TMD

QED shower generates very small transverse momentum





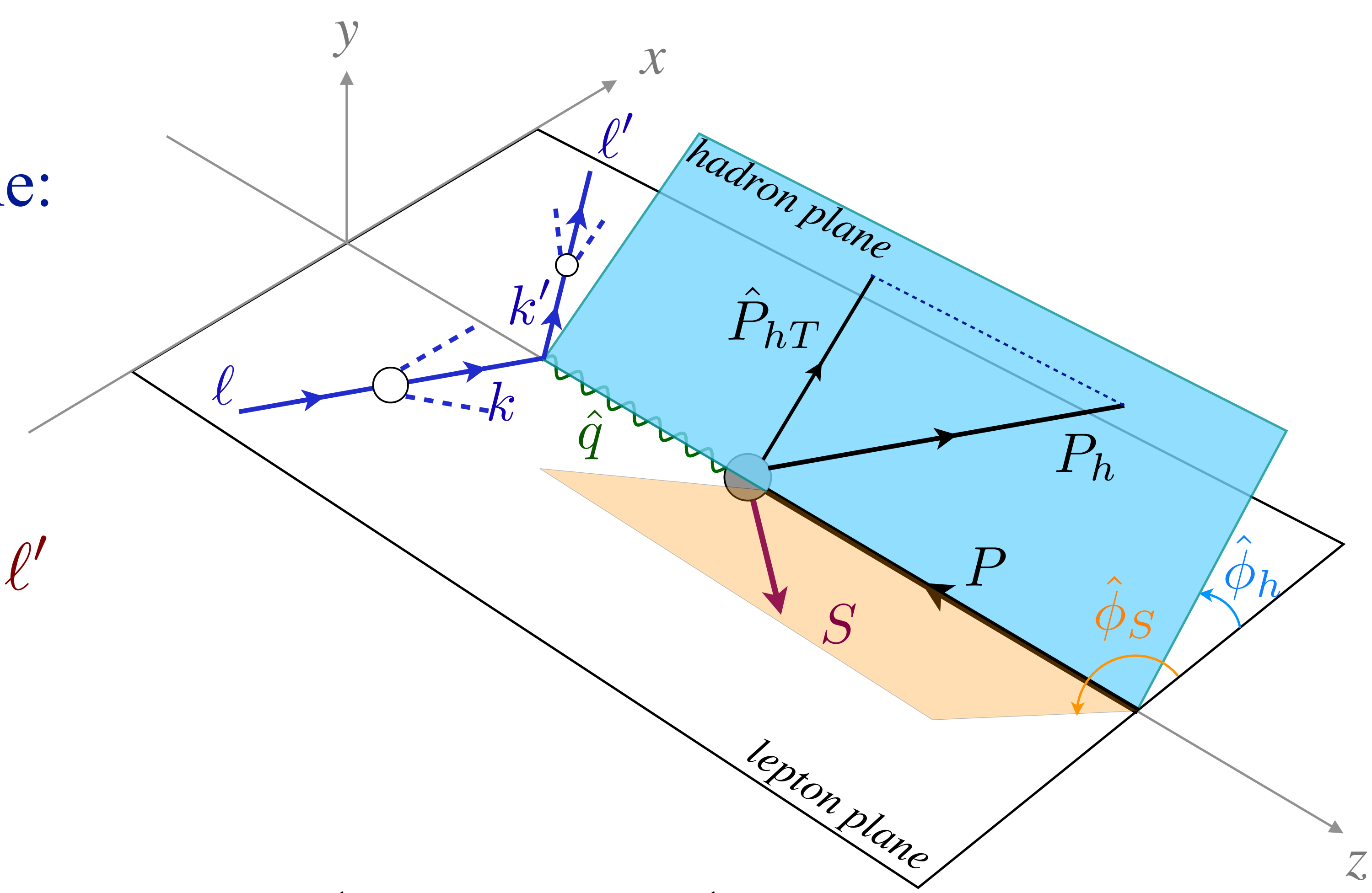
Collinear LDF and LFF are good approximation of lepton TMDs.

Impact on hadron P_{hT} in "photon-hadron frame" is mainly caused by logarithmic enhanced collinear radiation.

SIDIS with Collinear Factorized QED

True photon-hadron frame:

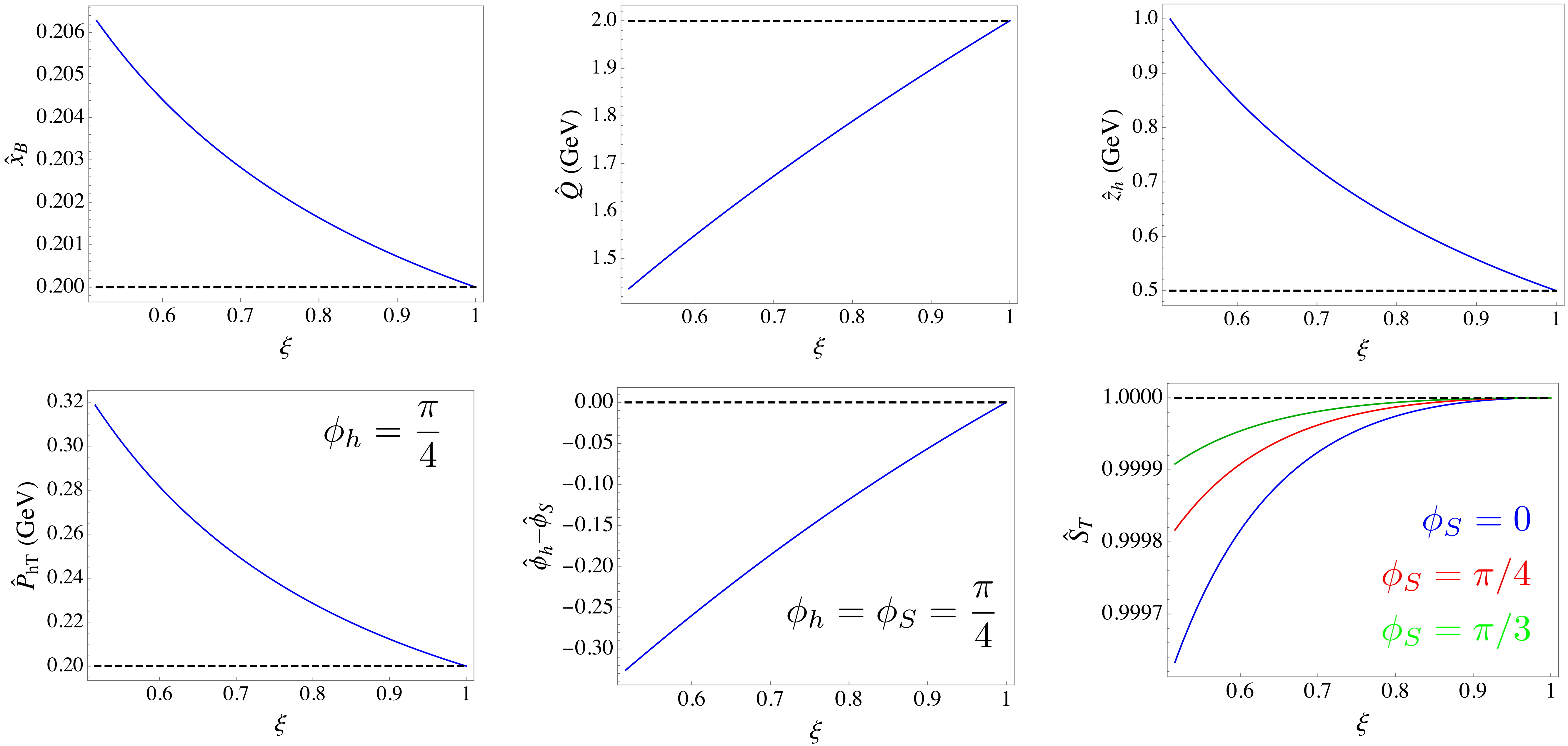
$$\hat{q} = \xi\ell - \frac{1}{\zeta}\ell' \neq \ell - \ell'$$



$$E_{\ell'} E_{P_h} \frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\mathrm{d}^3 P_h} \approx \sum_{i,j,\lambda_k} \int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 \mathrm{d}\xi f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \times \left[E_{k'} E_{P_h} \frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3 k'\mathrm{d}^3 P_h} \right]_{k=\xi\ell, k'=\ell'/\zeta}$$

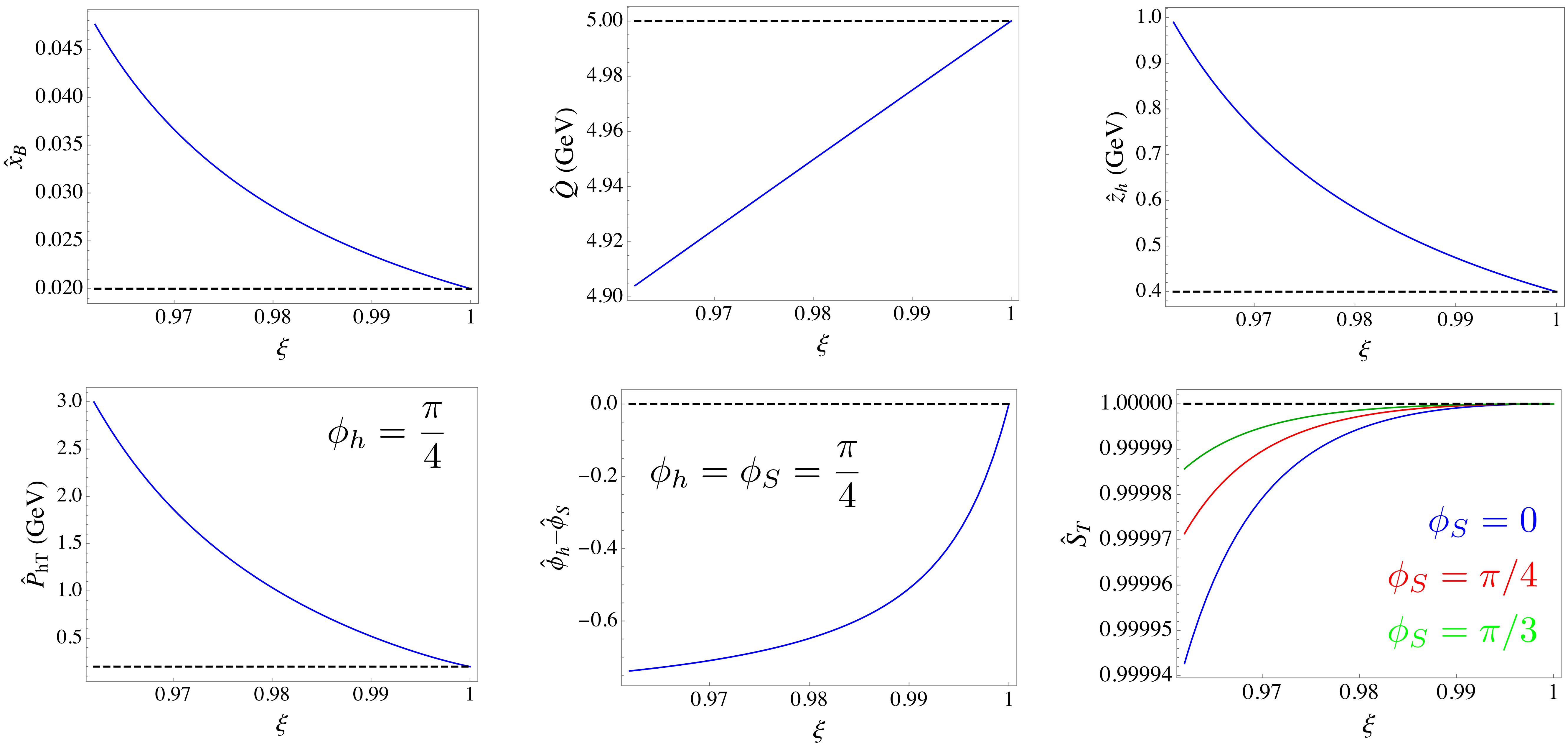
$$E_{k'} E_{P_h} \frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3 k'\mathrm{d}^3 P_h} = \left(\frac{4\hat{x}_B}{\widehat{Q}^2} \sqrt{\hat{z}^2 - \left(\hat{\gamma}\widehat{P}_{hT}/\widehat{Q}\right)^2} \right) \frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}\hat{x}_B\ \mathrm{d}\hat{y}\ \mathrm{d}\hat{\phi}_S \mathrm{d}\hat{z}\ \mathrm{d}\hat{\phi}_h\ \mathrm{d}\widehat{P}_{hT}^2}$$

Shift of Kinematics



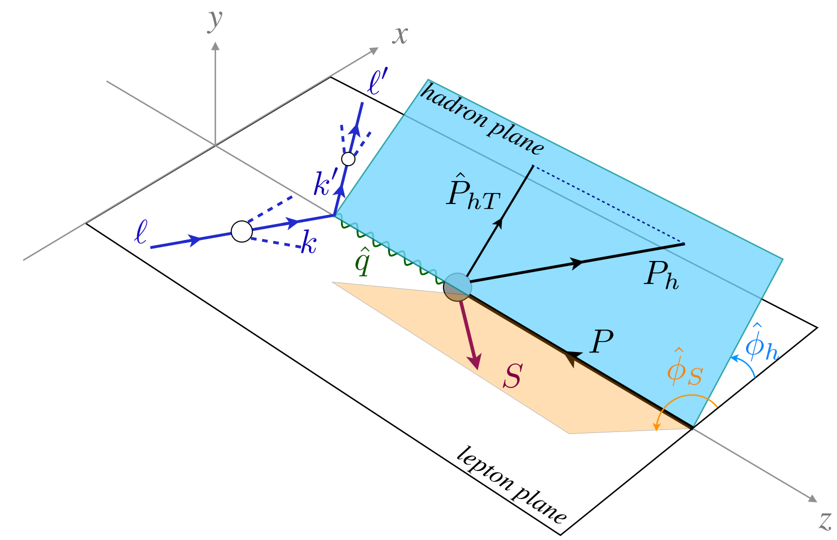
$$\sqrt{s} = 4.64\,\text{GeV}, \quad x_B = 0.2, \quad Q = 2\,\text{GeV},$$
$$z_h = 0.5, \quad P_{hT} = 0.2\,\text{GeV} \quad \zeta = 1$$

Shift of Kinematics



$$\sqrt{s} = 140\,\text{GeV}, \quad x_B = 0.02, \quad Q = 5\,\text{GeV},$$
$$z_h = 0.4, \quad P_{hT} = 0.2\,\text{GeV} \quad \zeta = 1$$

SIDIS with Collinear QED Factorization



$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{hT}^2\,\mathrm{d}\phi_h\,\mathrm{d}\phi_S}$$

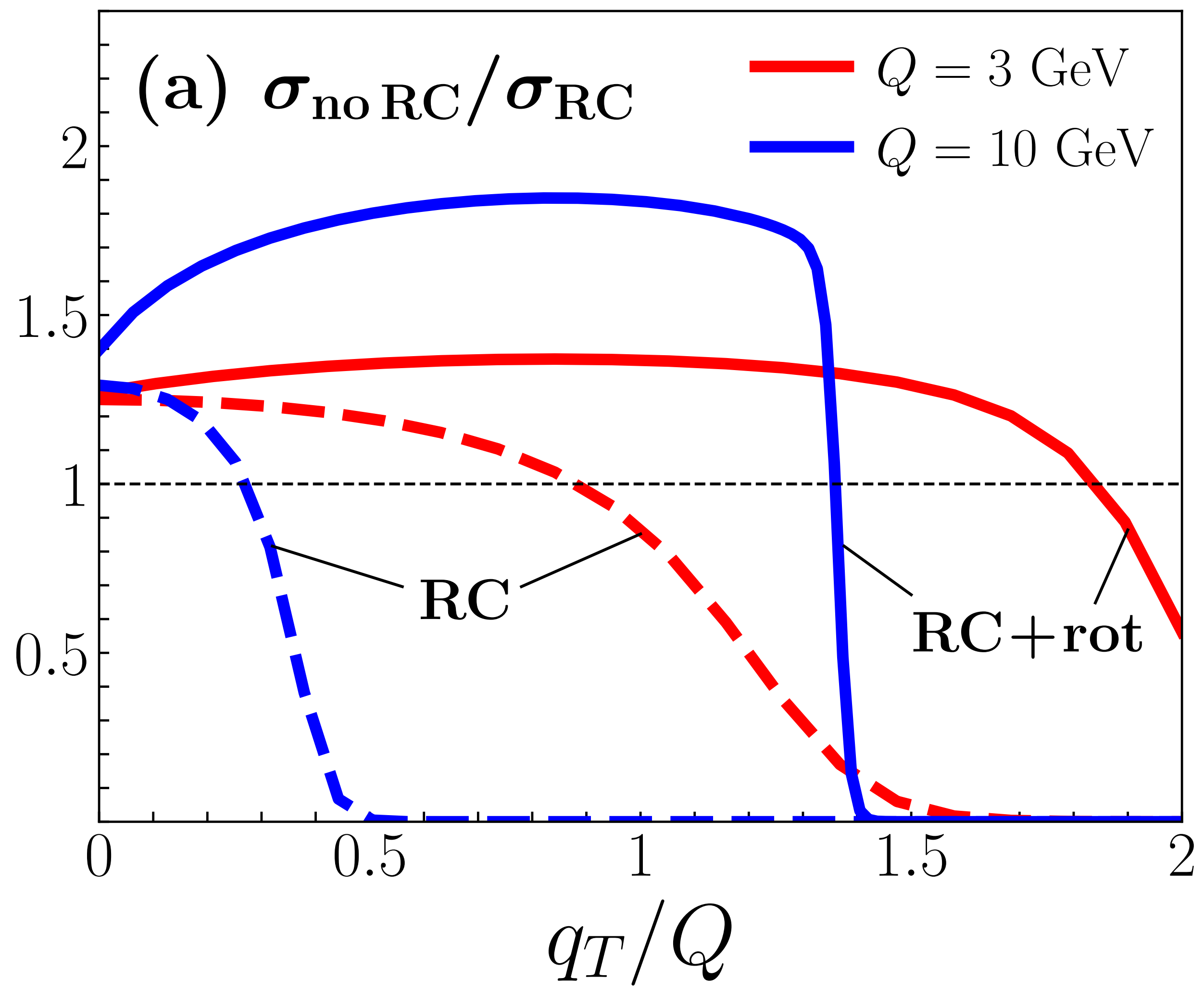
$$= \Bigg[\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$
$$\times \Big\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h + \lambda_e\sqrt{2\epsilon(1-\epsilon)}\,F_{LU}^{\sin\phi_h}\sin\phi_h$$
$$+ S_L\left[\sqrt{2\epsilon(1+\epsilon)}\,F_{UL}^{\sin\phi_h}\sin\phi_h + \epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right] + \lambda_e S_L\left[\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\,F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$
$$+ S_T\Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin(\phi_h-\phi_S) + \epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)$$
$$+ \epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S) + \sqrt{2\epsilon(1+\epsilon)}\,F_{UT}^{\sin\phi_S}\sin\phi_S + \sqrt{2\epsilon(1+\epsilon)}\,F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big]$$
$$+ \lambda_e S_T\Big[\sqrt{1-\epsilon^2}\,F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)$$
$$+ \sqrt{2\epsilon(1-\epsilon)}\,F_{LT}^{\cos\phi_S}\cos\phi_S + \sqrt{2\epsilon(1-\epsilon)}\,F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}$$
$$\{Q^2, x_B, y, \gamma, \epsilon, z, P_{hT}, \phi_h, \phi_S, S_T, S_L\} \to \{\widehat{Q}^2, \hat{x}_B, \hat{y}, \hat{\gamma}, \hat{\epsilon}, \hat{z}, \widehat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \widehat{S}_T, \widehat{S}_L\}\Bigg]$$

$$\otimes f_{e/e}(\xi) \otimes D_{e/e}(\zeta)\left(\frac{\hat{x}_B}{x_B\xi\zeta}\right)$$

→ Jacobian between the two frames

$\widehat{Q}^2, \hat{x}_B, \hat{z}, \widehat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \widehat{S}_T, \widehat{S}_L$ are functions of $\xi, \zeta, Q^2, x_B, z, P_{hT}, \phi_h, \phi_S, S_T, S_L$

Impact of QED Effects: P_{hT} Distribution

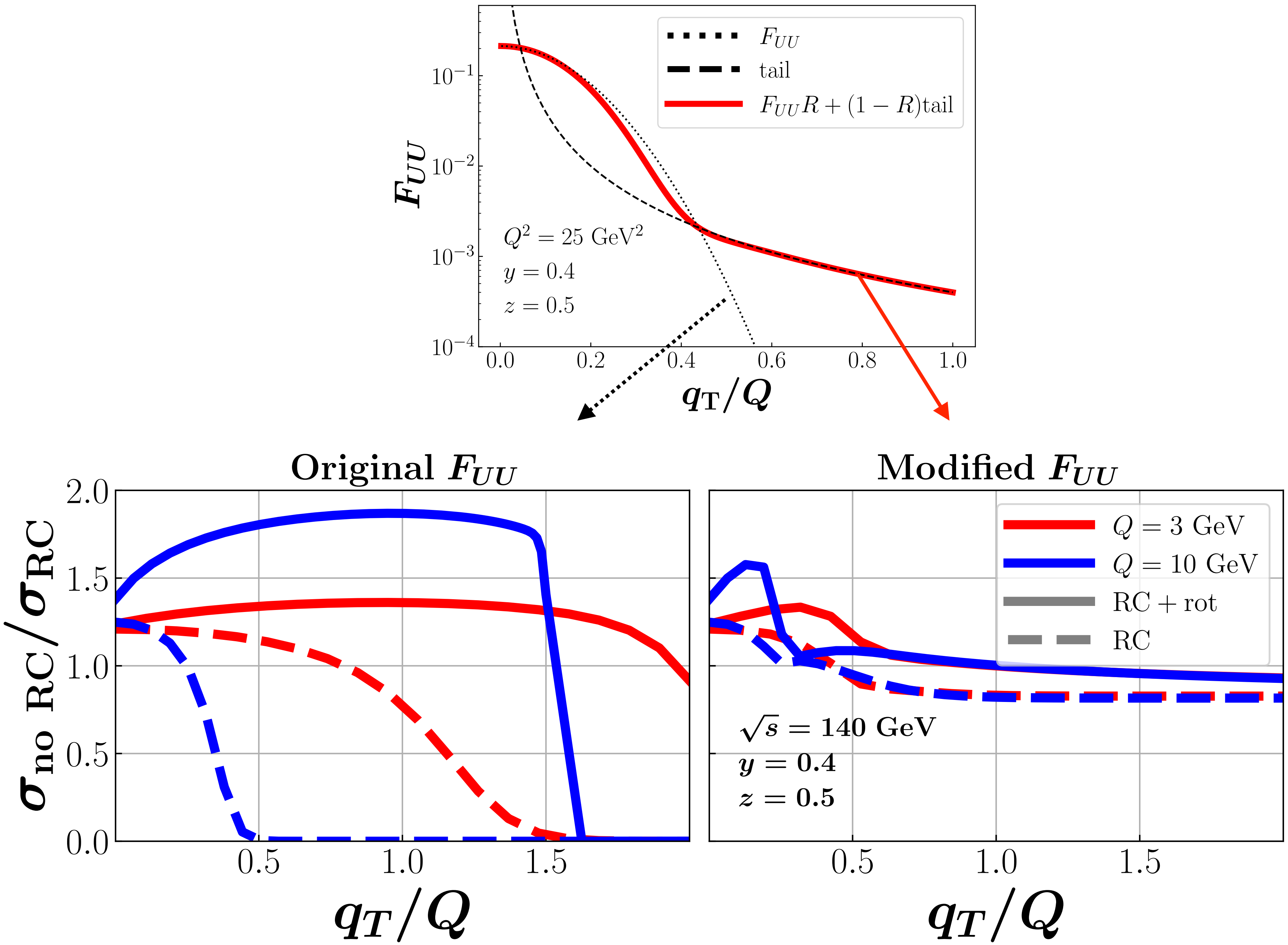


$$\sqrt{s} = 140\,\text{GeV}$$
$$y = 0.4$$
$$z_h = 0.5$$

Lorentz transform from experimental "photon-hadron frame" to the true photon-hadron frame (It is a rotation in target rest frame)

The rotation effect is huge.

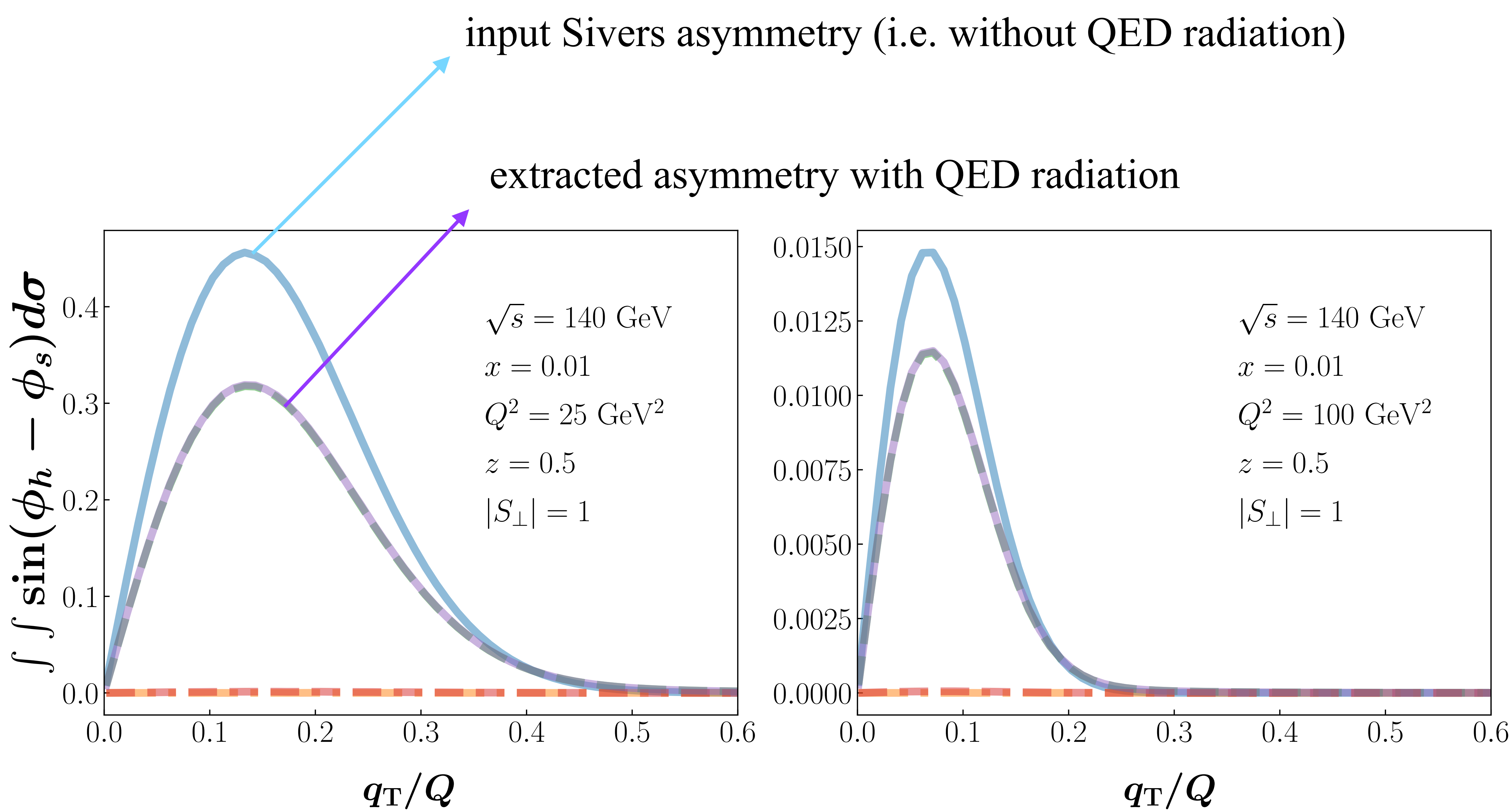
Impact of QED Effects: P_{hT} Distribution



Radiative correction factor depends on the hadronic physics we want to extract.

Impact of QED Effects: Azimuthal Asymmetries

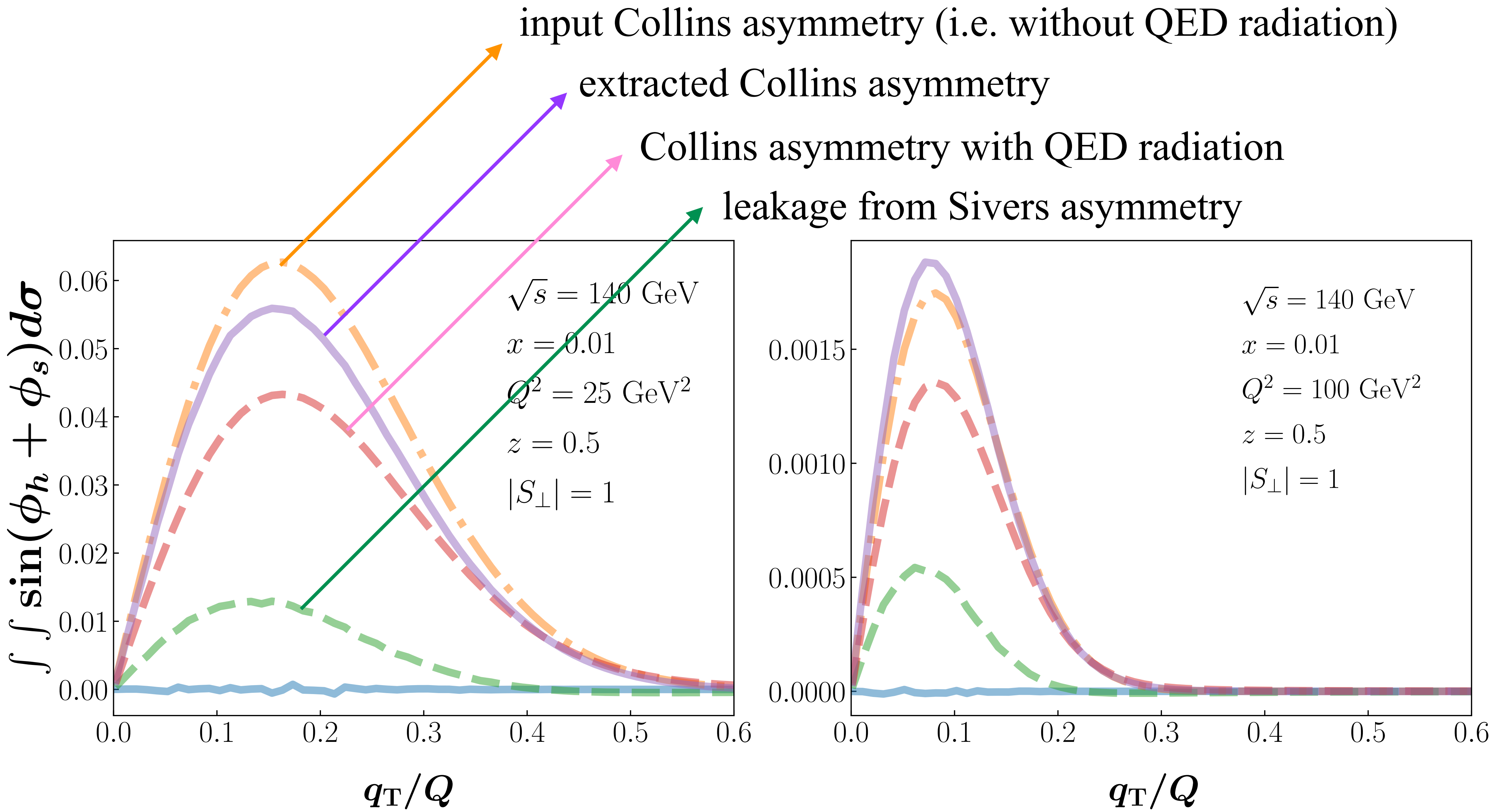
Sivers asymmetry:



Asymmetry amplitude is affected by QED radiation.

Impact of QED Effects: Azimuthal Asymmetries

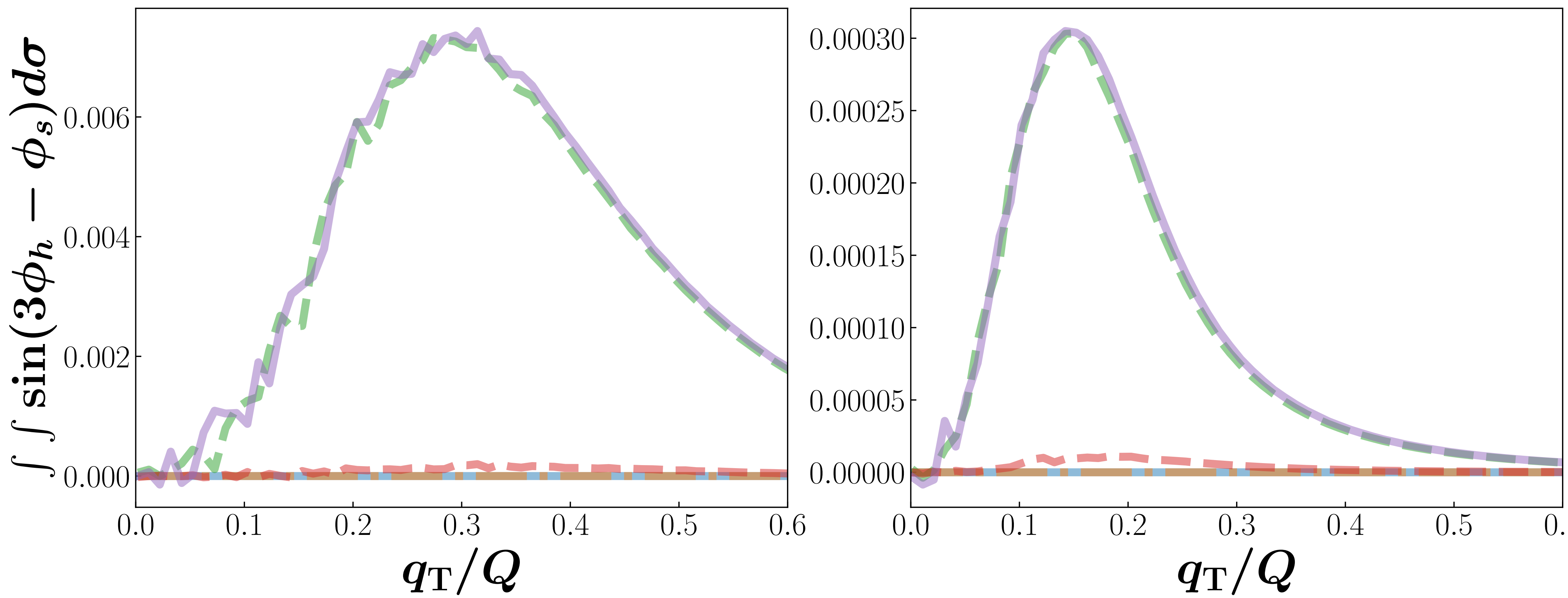
Collins asymmetry:



Azimuthal modulations mix with each other.

Impact of QED Effects: Azimuthal Asymmetries

Pretzelosity asymmetry:



Input pretzelosity is zero. Extracted pretzelosity asymmetry is all from the leakage of other asymmetries (Sivers, Collins).

Mixing effect could be a disaster for pretzelosity measurement.

Summary

- QED radiation effects are important in SIDIS, and hence precise extractions of TMDs.
  - Experimental "photon-hadron frame" does not coincide with the *true photon hadron frame*, where the factorization works.
  - Almost impossible to determine/reconstruct the *true photon hadron frame* event by event.
  - Challenge to match to Born kinematics without introducing model/theory bias.
- We propose a factorized approach to treat QED radiations.
  - Treat QED radiation as a part of the production cross section.
  - Generalize QCD factorization to include QED. All perturbatively calculable hard parts are IR safe.
  - Transverse momentum generated by QED shower is small, and one can apply collinear factorization for the leptonic tensor.
  - Huge and nontrivial effects on P_{hT} dependence and azimuthal modulations.

Thank you!