A NONTRIVIAL FLUCTUAION THEOREM FOR ADIABATIC PUMPING PROCESSES

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See Y. Hino & HH. arXiv:1908.10597,
K. Fujii, HH, Y. Hino & K. Takahashi, arXiv:1909.02202

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Contents

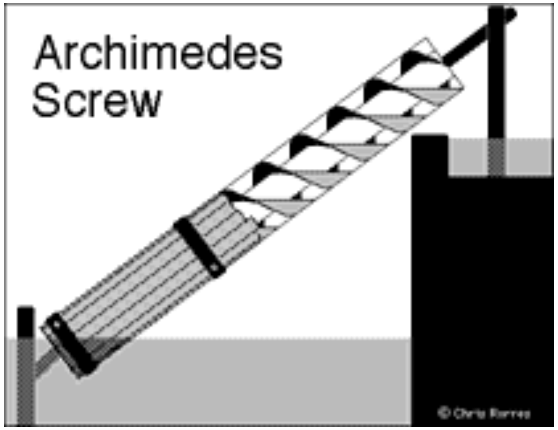
Introduction

- I am talking on geometric pumping proposed by D. Thouless (1983) who got the Nobel prize in 2016 and passed away recently.
- The essence of Thouless pumping is Berry's phase proposed by M. Berry (1984).
- The idea by two big shots can be applied to non-equilibrium driven systems.
- Non-Gaussian fluctuations play important roles.

Photos taken from wikipedia

Pumping process

- Pump=>We need a bias.



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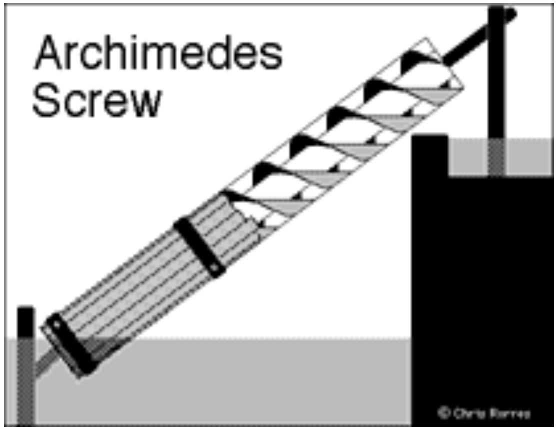

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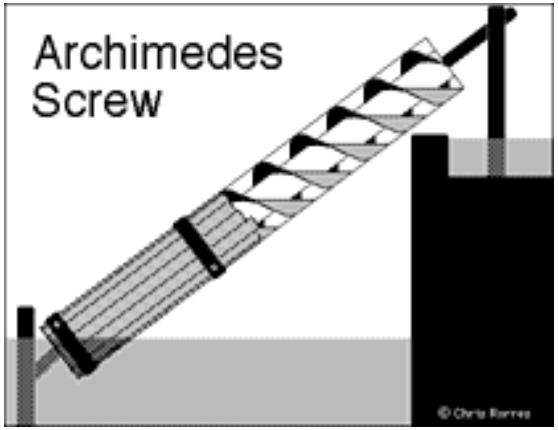
  The current can flow in a mesoscopic system without dc bias =>*Geometric (Thouless) pumping*.



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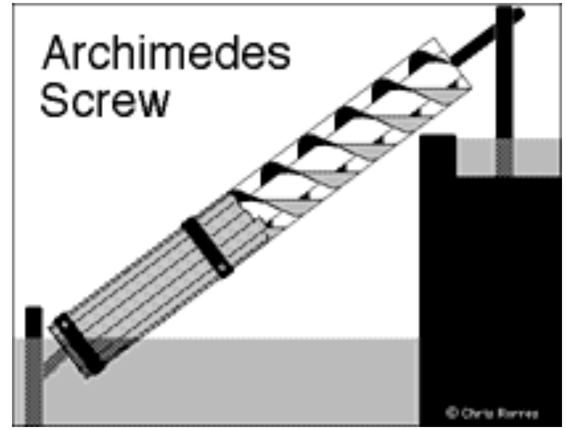

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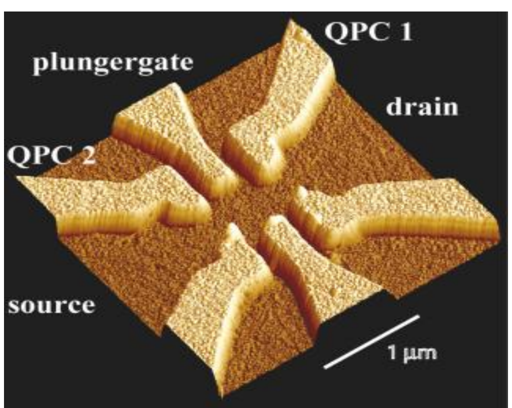

A nano-machine to extract a work

Pumping process

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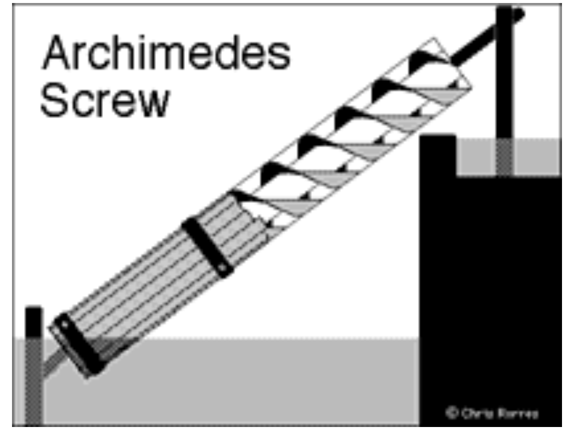
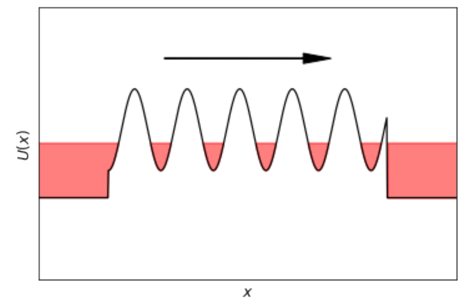
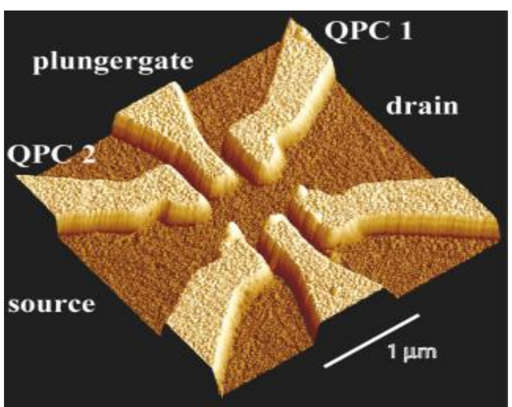
The current can flow in a mesoscopic system without dc bias =>*Geometric (Thouless) pumping*.



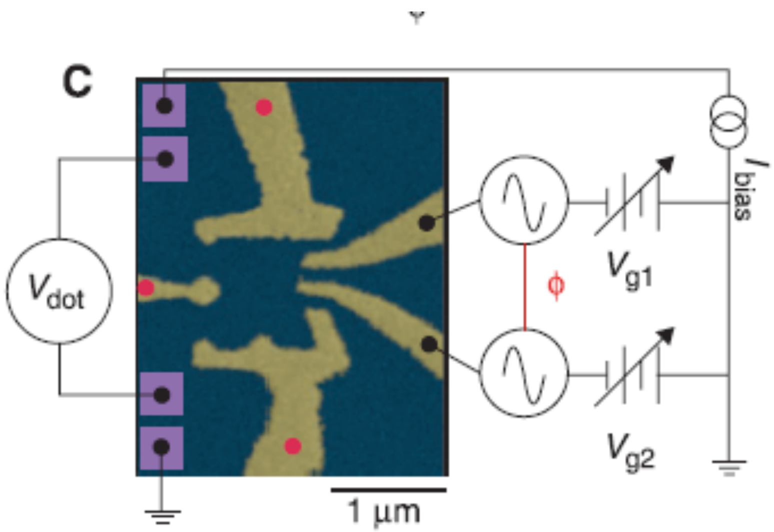


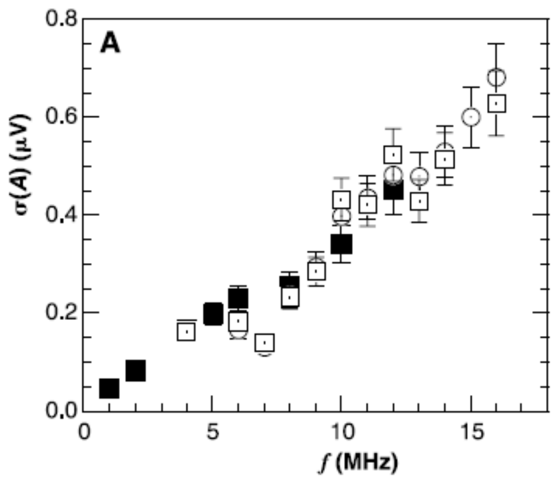


A nano-machine to extract a work

Previous studies

- Experiments
  - Pothier et al. (1992) get a classical pumping for a mesoscopic system.
  - Switkes et al. (1999) control the gate voltage and get 20 electrons current per a cycle in a quantum dot system.





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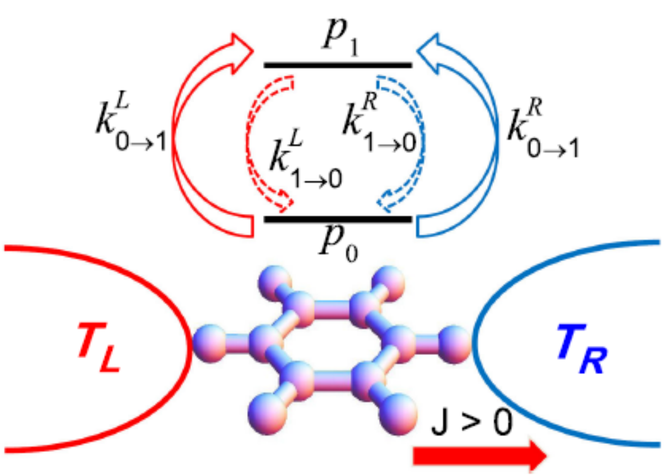
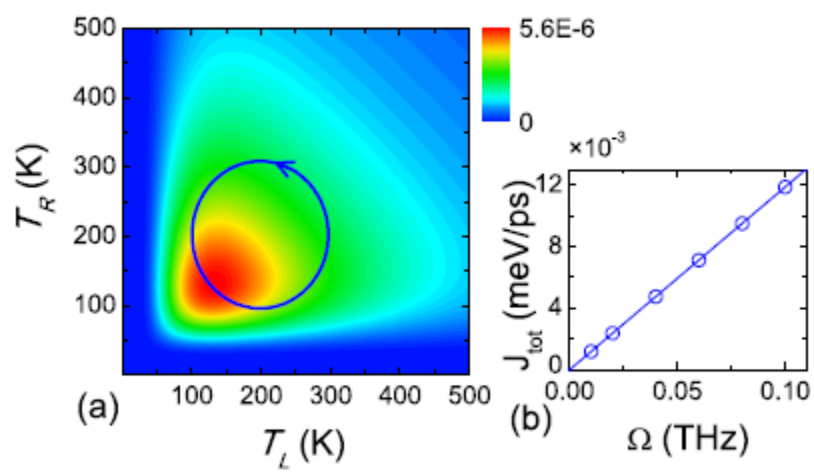
Previous theoretical studies

- Adiabatic geometric pumping (theories)
  - Thouless (1983) for a closed system
  - Open quantum system (P. W. Brower, (1998)).
- Sintsyin & Nemenman (2007) indicated that Berry's phase can be used to nonequilibrium stochastic processes.
- Ren-Hänggi-Li (2010) analyzed a spin-boson system and to clarify the role of Berry's phase.





Fluctuation theorem (FT)

- Fluctuation theorem gives the basis of non-equilibrium processes.
  - It was proposed by Evans & Morriss (1993), Galavotti & Cohen (1995) et al, and is related to Jarzynski equality (1997).
- We can derive the fluctuation-dissipation theorem, Onsager-Casimir relation, and Green-Kubo formula as well as 2nd law of thermodynamics.
- How can we apply systems far from equilibrium such as geometric pumping?

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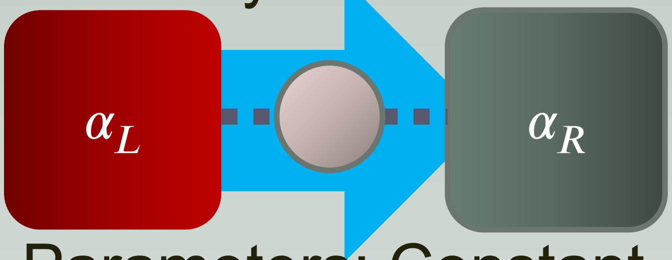
Steady Fluctuation Theorem

$$\lim_{\tau\to\infty}\frac{1}{\tau}\ln\frac{P(J)}{P(-J)} = \mathcal{A}J$$

Steady Current $\boldsymbol{J}$



Parameters: Constant

$\mathcal{A} = \alpha_R - \alpha_L$:affinity

Integral FT

$$\left\langle e^{\mathcal{A}J\tau}\right\rangle = 1$$

Fluctuation Dissipation Theorem

$$\left.\frac{\partial\langle J\rangle}{\partial\mathcal{A}}\right|_{\mathcal{A}=0} = 2\left.\left\langle (J-\langle J\rangle)^2\right\rangle\right|_{\mathcal{A}=0}$$

Reciprocal relation

$$\frac{\partial^2\langle J\rangle}{\partial\mathcal{A}^2} = \frac{\partial\langle J^2\rangle}{\partial\mathcal{A}}$$

Other nonlinear relations

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Motivation of our study

- To demonstrate adiabatic pumping by controlling the bias such as chemical potentials.
- To get the extended fluctuation theorem in geometrical pumping
  - This is an example of the fluctuation theorem for systems with non-Gaussian fluctuations.
- How can we extend adiabatic pumping to non-adiabatic processes?

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Contents

See Y. Hino & HH. arXiv:1908.10597.

Counting Statistics & Master Eq.

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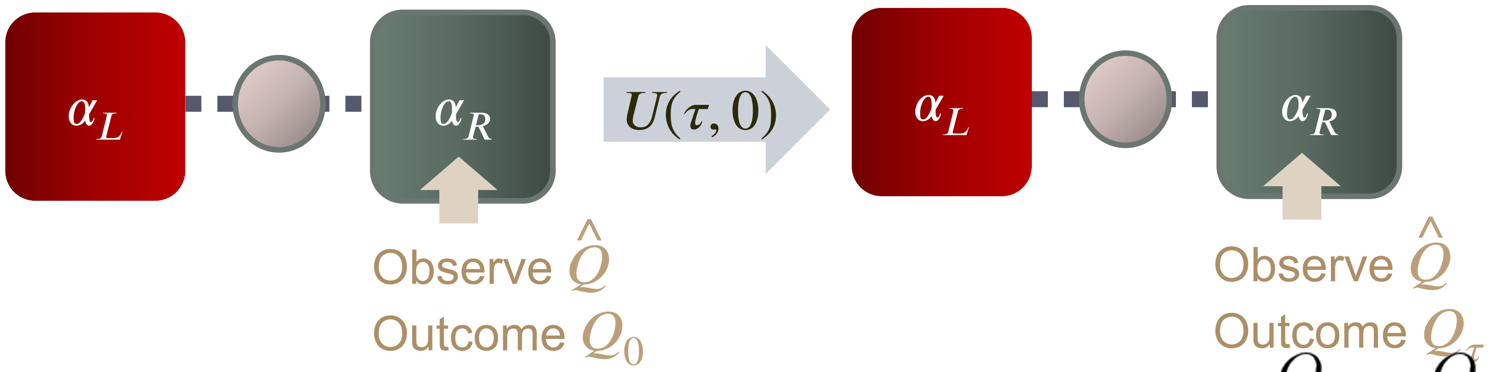
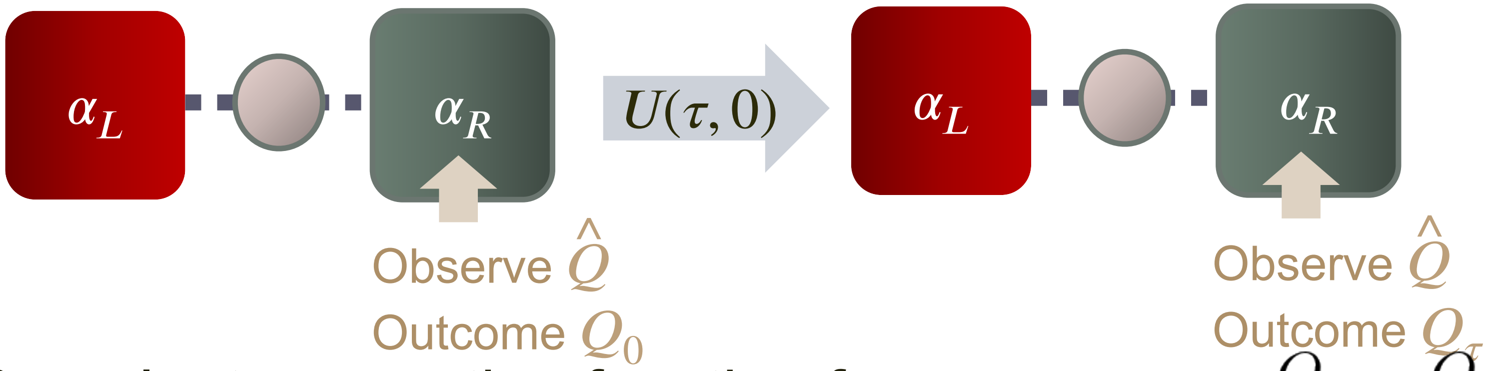
Counting Statistics & Master Eq.



$$J = \frac{Q_\tau - Q_0}{\tau}$$

Counting Statistics & Master Eq.



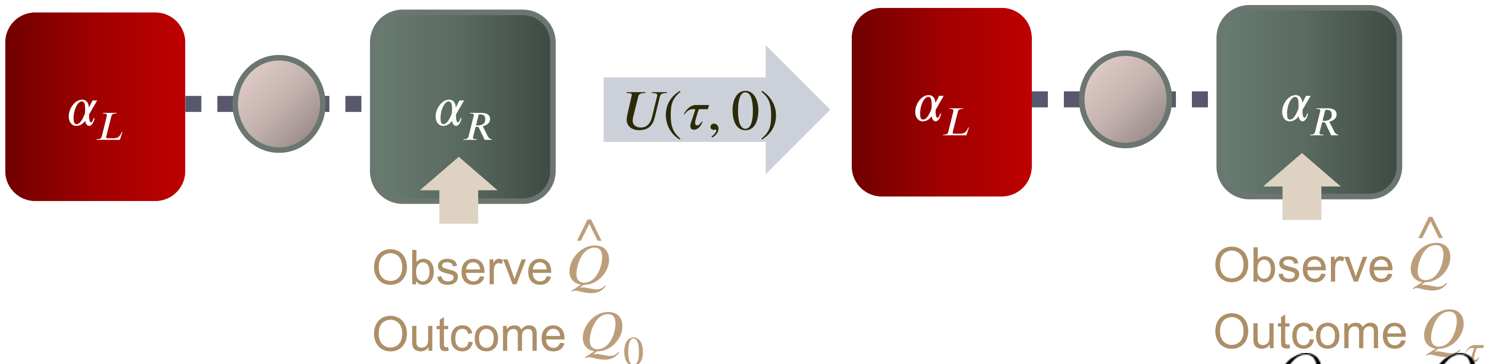
$$J = \frac{Q_\tau - Q_0}{\tau}$$

Cumulant-generating function for $P(J)$:

$$S(\chi) = \frac{1}{\tau} \mathrm{Tr}_B [\rho(\chi, \tau)]$$

Generalized density op.

Counting Statistics & Master Eq.



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Generalized Lindblad type master eq.

$$\frac{d}{dt}|\rho(\chi, t)\rangle = \mathcal{K}(\chi, \alpha)|\rho(\chi, t)\rangle$$

Assumption: weak coupling, Markovian process

Counting Statistics & Master Eq.

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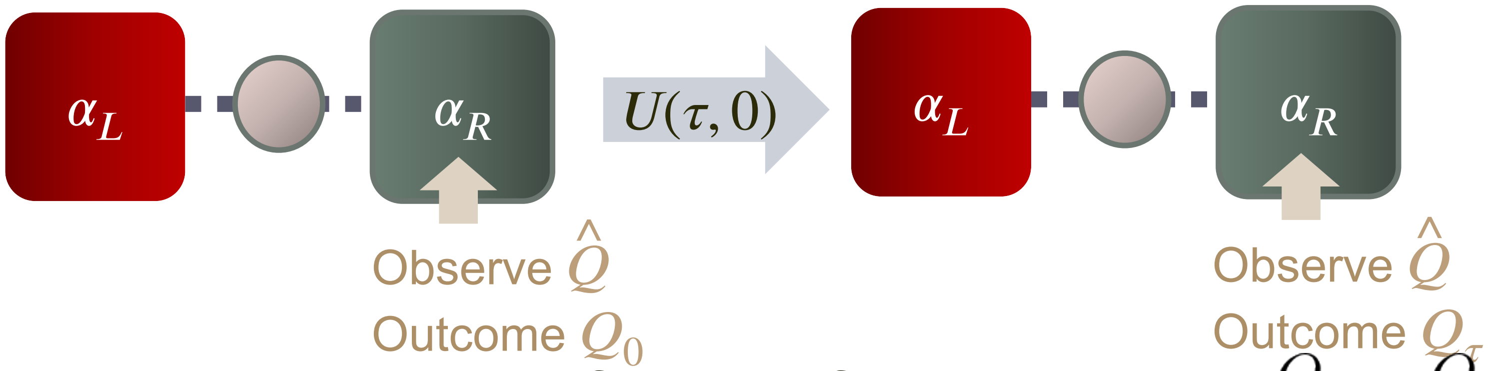

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Lindblad operator

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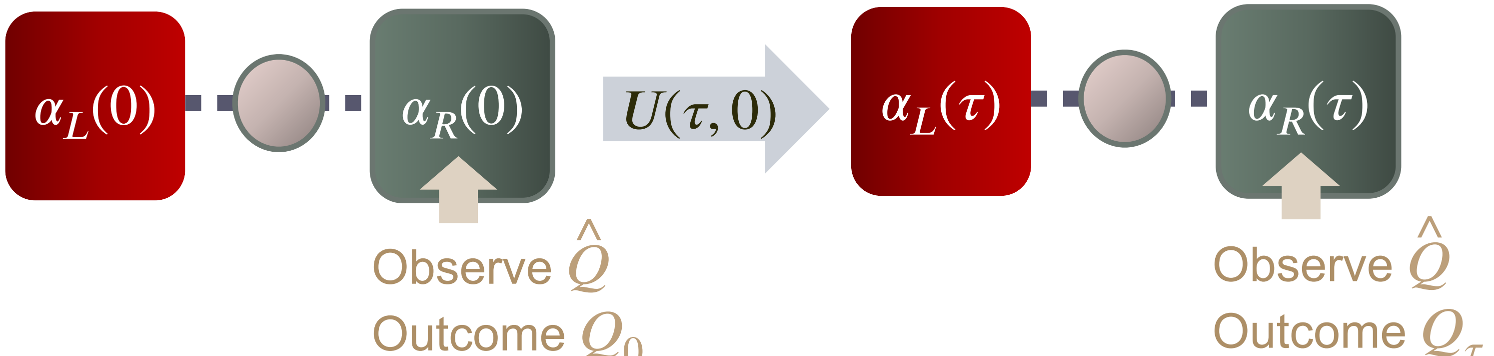

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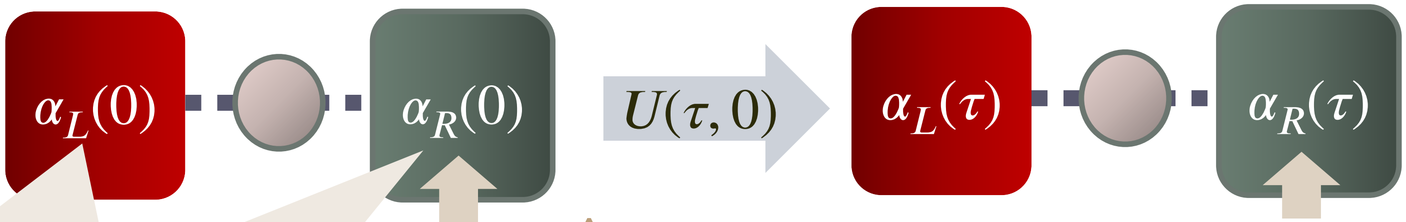

Time dependent

…serve $\hat{Q}$

…come Q_0

Observe $\hat{Q}$

Outcome Q_τ

$$J = \frac{Q_\tau - Q_0}{\tau}$$

Cumulant-generating function for $P(J)$:

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Time dependent

Assumption: w… …vian process

Periodic modulation

$$\partial_\theta |\rho(\theta,\chi)\rangle = \epsilon^{-1}\hat{K}(\boldsymbol{\alpha}(\theta),\chi)|\rho(\theta,\chi)\rangle,$$

$$\Omega = 2\pi/\tau \qquad \theta := \Omega(t-t_0)$$

t_0 : time for loss of the initial information

$$\epsilon = \Omega/\Gamma,\ \hat{K}(\boldsymbol{\alpha}(t),\chi) = \Gamma^{-1}K(\boldsymbol{\alpha}(t),\chi)$$

Γ : e.g. the hopping rate

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$$|\rho(\theta,\chi)\rangle \simeq e^{\epsilon^{-1}\Lambda(\theta,\chi)-V(\theta,\chi)} \times |r(\boldsymbol{\alpha}(\theta),\chi)\rangle\langle 1|r(\boldsymbol{\alpha}(0),\chi)\rangle^{-1},$$

Periodic modulation

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Eigenvalue for the dynamic part of Lindblad operator

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The right eigenvector

$$|\rho(\theta,\chi)\rangle \simeq e^{\epsilon^{-1}\Lambda(\theta,\chi)-V(\theta,\chi)} \times |r(\boldsymbol{\alpha}(\theta),\chi)\rangle\langle 1|r(\boldsymbol{\alpha}(0),\chi)\rangle^{-1},$$

Eigenvalue for the dynamic part of Lindblad operator

$$\Lambda(\theta,\chi) := \int_0^\theta d\theta' \lambda(\boldsymbol{\alpha}(\theta'),\chi),$$

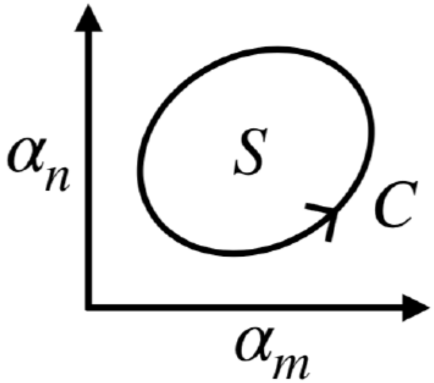
Geometric contribution

$$\overline{V}(\chi) := V(2\pi, \chi)$$

$$\overline{V}(\chi) = \int_0^{2\pi} d\theta \langle l(\boldsymbol{\alpha}(\theta), \chi) | \partial_\theta r(\boldsymbol{\alpha}(\theta), \chi) \rangle$$

$$= \iint_S d\alpha_m d\alpha_n \mathcal{F}_{mn}(\boldsymbol{\alpha}, \chi),$$

$$\mathcal{F}_{mn}(\boldsymbol{\alpha}, \chi) := \partial_{\alpha_m} \langle l_0(\boldsymbol{\alpha}, \chi) | \wedge \partial_{\alpha_n} | r_0(\boldsymbol{\alpha}, \chi) \rangle$$



Berry-Sintsyin-Nemenmann (BSN) curvature

Large deviation theory

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Current probability distribution

$$P_\epsilon(J) \simeq \frac{1}{\sqrt{2\pi\epsilon\overline{\Lambda}^{(2)}(\chi_c(J))}} e^{-\epsilon^{-1}I(J)-\overline{V}(\chi_c(J))},$$

$$I(J) := i \max_\chi [\chi J - \hat{\Lambda}(\chi)] = i\chi_c(J)J - \Lambda(\chi_c(J)),$$

$$\Lambda^{(2)}(\chi_c(J)) := \partial_{i\chi}^2 \Lambda(\chi)\Big|_{\chi=\chi_c(J)}$$

$$I(J) - I(-J) = -\mathcal{A}J$$

Cyclic fluctuation theorem

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$$\epsilon \ln \frac{P_\epsilon(J)}{P_\epsilon(-J)} = \mathcal{A}J - \epsilon[\overline{V}(\chi_c(J)) - \overline{V}(\chi_c(-J))] - \frac{1}{2}\ln \frac{\overline{\Lambda}^{(2)}(\chi_c(J))}{\overline{\Lambda}^{(2)}(\chi_c(-J))}.$$

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$$- \frac{1}{2} \ln \frac{\overline{\Lambda}^{(2)}(\chi_c(J))}{\overline{\Lambda}^{(2)}(\chi_c(-J))}.$$

If the average bias is zero, we have

$$\mathcal{A} = 0 \quad \overline{V}(-\chi) = -\overline{V}(\chi), \quad \overline{\Lambda}(\chi) = \overline{\Lambda}(-\chi)$$

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The cyclic FT is reduced to

$$\ln \frac{P_\epsilon(J)}{P_\epsilon(-J)} = -2\overline{V}(\chi_c(J)),$$

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Contents

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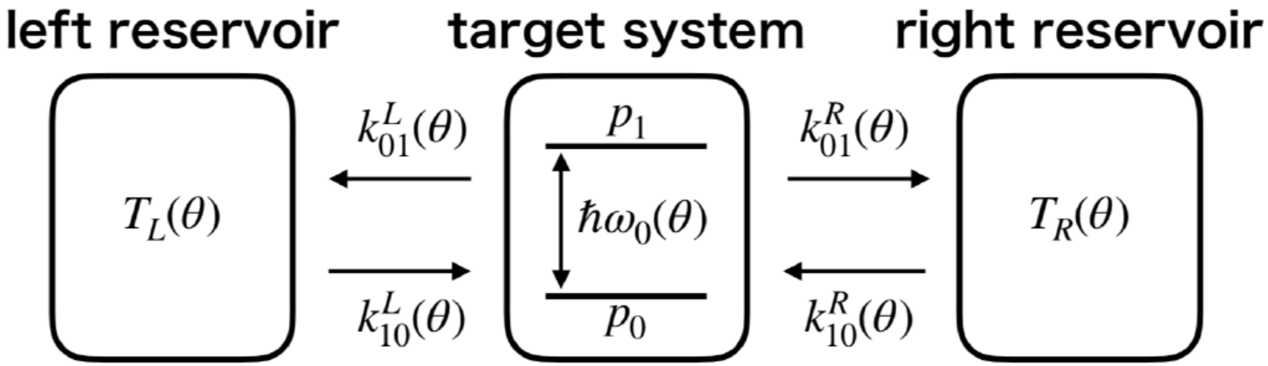
Spin-boson system



$$H_S(\alpha_S) = \frac{\hbar\omega_0}{2}\sigma_z,$$

$$H_\nu = \sum_k \hbar\omega_{k,\nu} b_{k,\nu}^\dagger b_{k,\nu},$$

$$H_{S\nu}(\alpha_{S\nu}) = \hbar\sigma_x \otimes \sum_k g_{k,\nu}(b_{k,\nu} + b_{k,\nu}^\dagger),$$

Control parameters

- Case A

$$\hat{T}_L(\theta) := (\hbar\omega_0\beta_L(\theta))^{-1} = \hat{T}_0 + \hat{T}_A \cos(\theta + \pi/4),$$
$$\hat{T}_R(\theta) := (\hbar\omega_0\beta_R(\theta))^{-1} = \hat{T}_0 + \hat{T}_A \sin(\theta + \pi/4).$$

- Case B

$$\gamma_L(\theta) = \gamma_R + \gamma_A \cos(\theta),$$
$$\hat{\omega}_0(\theta) := \beta\hbar\omega_0(\theta) = \hat{\omega}_C + \hat{\omega}_A \sin(\theta)$$

- Dynamic affinity

$$\mathcal{A} = \ln \frac{\int_0^{2\pi} d\theta n_L(\theta)(1 + n_R(\theta))}{\int_0^{2\pi} d\theta n_R(\theta)(1 + n_L(\theta))}$$

Control parameters

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$$\hat{T}_R(\theta) := (\hbar\omega_0\beta_R(\theta))^{-1} = \hat{T}_0 + \hat{T}_A \sin(\theta + \pi/4).$$

- Case B $\gamma_\nu := \Gamma_\nu/\Gamma, \quad \Gamma := (\Gamma_L + \Gamma_R)/2$

$$\gamma_L(\theta) = \gamma_R + \gamma_A \cos(\theta),$$
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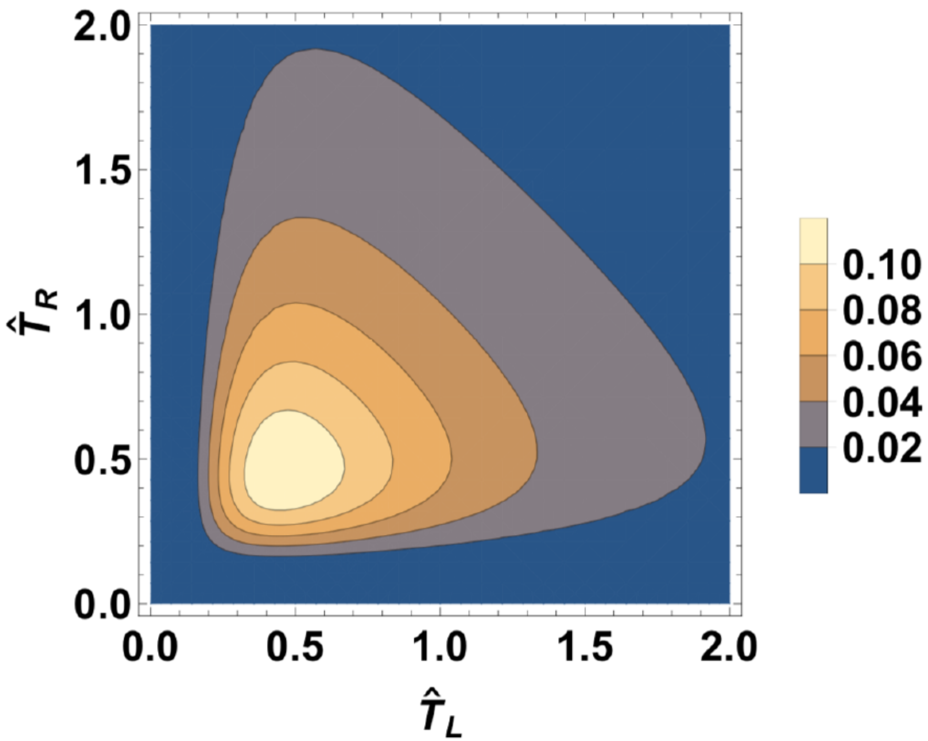
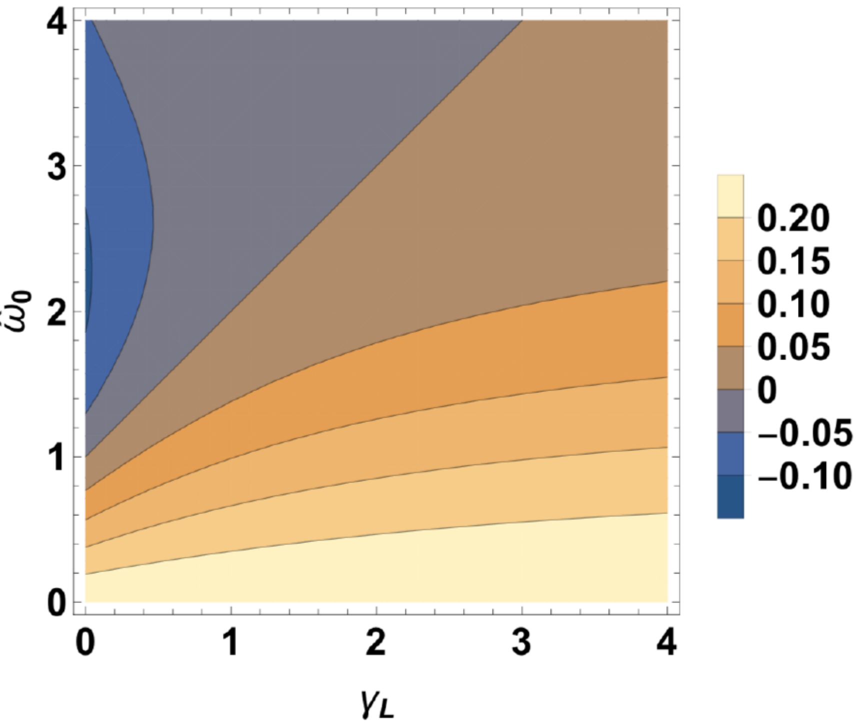
BSN curvatures



Case A



Case B

Cyclic fluctuation theorem

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- FT is given by

$$\ln \frac{P_\epsilon(J)}{P_\epsilon(-J)} \simeq -2V_1(\alpha)[J + \frac{1}{3}\hat{V}_3(\alpha)J^3 + O(J^5)],$$

- where

Cyclic fluctuation theorem

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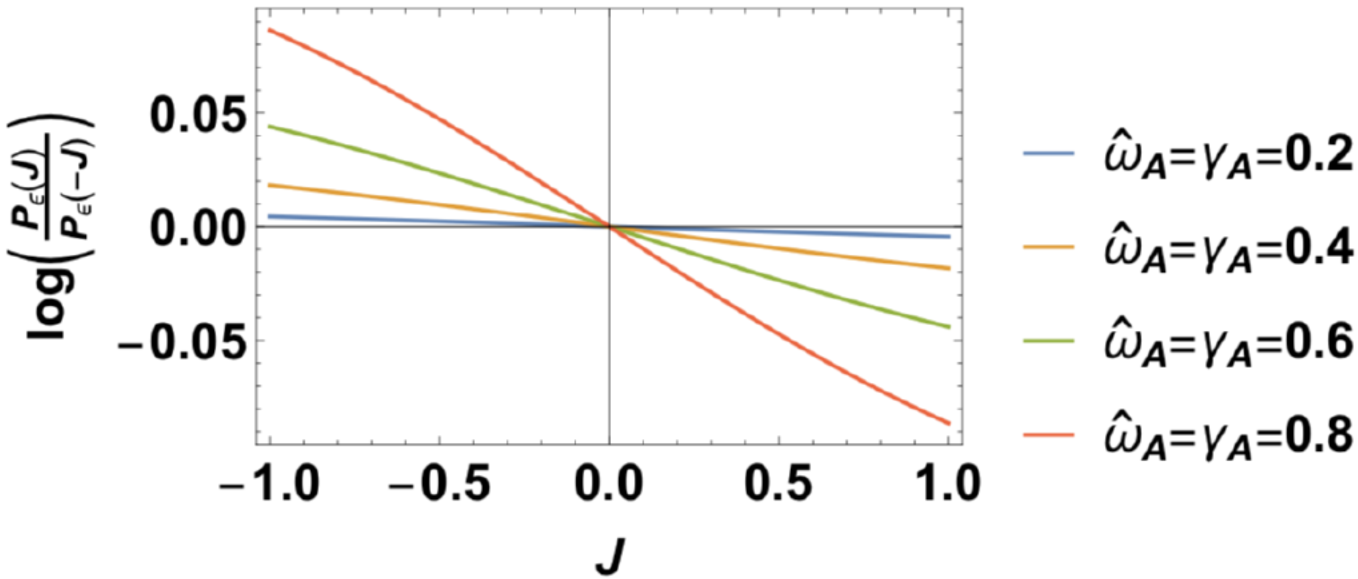
- where

$$V_n(\alpha) \quad := \quad \partial_J^n V(\chi_c(J))|_{J=0},$$

$$\hat{V}_n(\alpha) \quad := V_n(\alpha)/V_1(\alpha)$$

$$\alpha = (\hat{T}_0, \hat{T}_A) \text{ or } (\hat{\omega}_C, \hat{\omega}_A, \gamma_R, \gamma_A)$$

Cyclic FT for case B



Cyclic FT for case B

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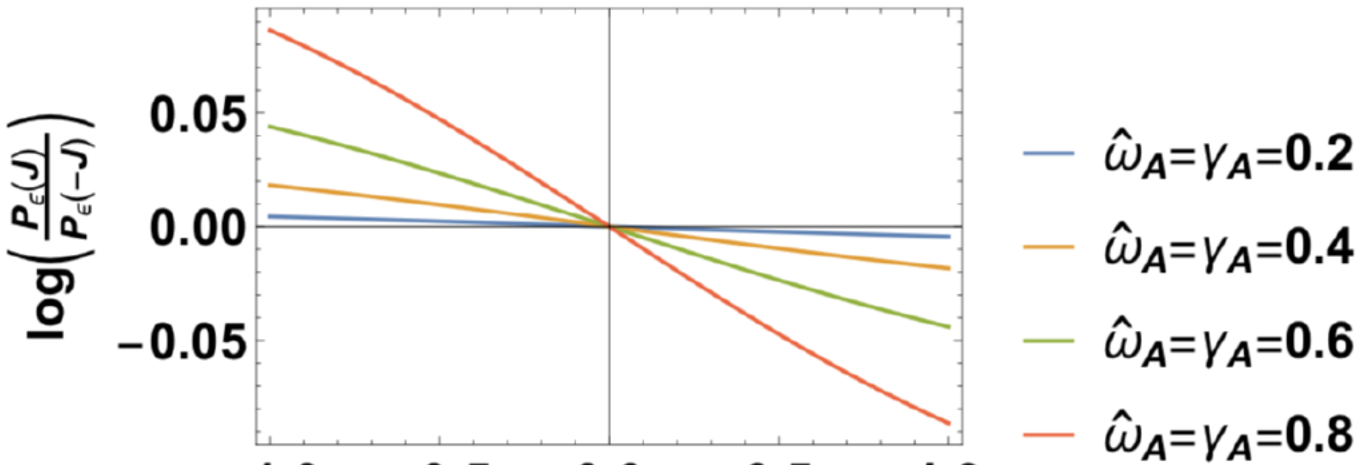

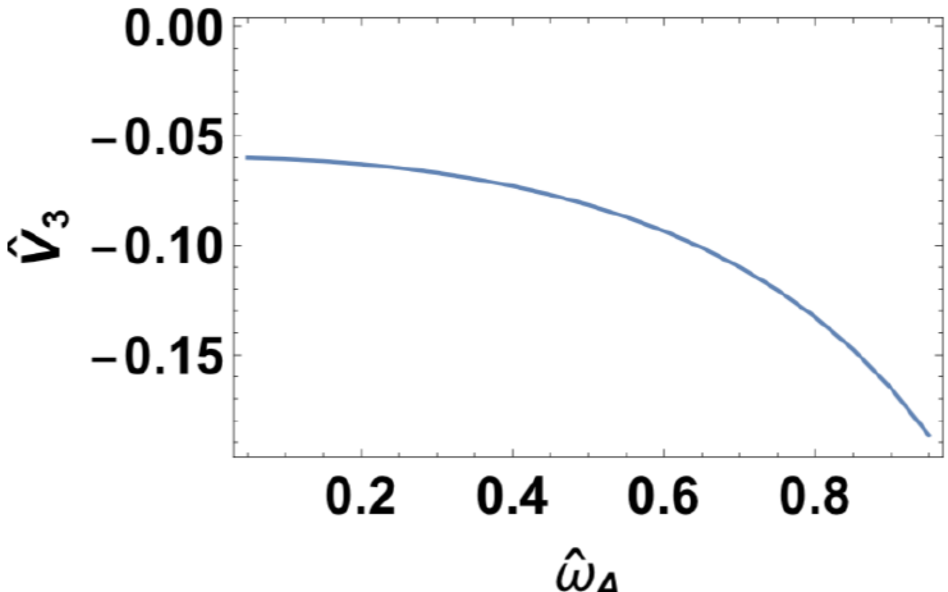

Violations of conventional relations

- Integral FT
- N-th Current cumulant
- Violation of FDR
- Violation of reciprocal relation

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- Integral FT

$$\left\langle e^{-A(J+BJ^3)} \right\rangle \simeq 1$$

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- Violation of reciprocal relation

$$L_{12} - L_{21} + B(L_{32} + 3L_{12}L_{20} + 6L_{11}L_{21} - 2L_{41}) = 0$$

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- N-th Current cumulant

$$\langle J^n \rangle_c = \sum_m L_{nm} A^m / m!$$

- Violation of FDR

$$2L_{11} - L_{20} + \boxed{2B(L_{31} + 3L_{11}L_{20} + L_{40})} = 0$$

- Violation of reciprocal relation

$$L_{12} - L_{21} + \boxed{B(L_{32} + 3L_{12}L_{20} + 6L_{11}L_{21} - 2L_{41})} = 0$$

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Contents

See K. Fujii, HH, Y. Hino & K. Takahashi, arXiv:1909.02202

Nonadiabatic control of geometric current

- Let us consider the nonadiabatic control of geometric current in terms of the master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}|p(t)\rangle = W(t)|p(t)\rangle$$

$$W(t) = \begin{pmatrix} -k_{\mathrm{in}}(t) & k_{\mathrm{out}}(t) \\ k_{\mathrm{in}}(t) & -k_{\mathrm{out}}(t) \end{pmatrix},$$

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$$W(t) = \begin{pmatrix} -k_{\mathrm{in}}(t) & k_{\mathrm{out}}(t) \\ k_{\mathrm{in}}(t) & -k_{\mathrm{out}}(t) \end{pmatrix},$$

$$k_{\mathrm{in}}^{(\mathrm{L})}(t) = k_0\left(1 + \frac{1}{2}\cos\omega t\right),$$

$$k_{\mathrm{in}}^{(\mathrm{R})}(t) = k_0\left(1 + \frac{1}{2}\sin\omega t\right),$$

$$k_{\mathrm{out}}^{(\mathrm{L})}(t) = k_0,$$

$$k_{\mathrm{out}}^{(\mathrm{R})}(t) = k_0.$$

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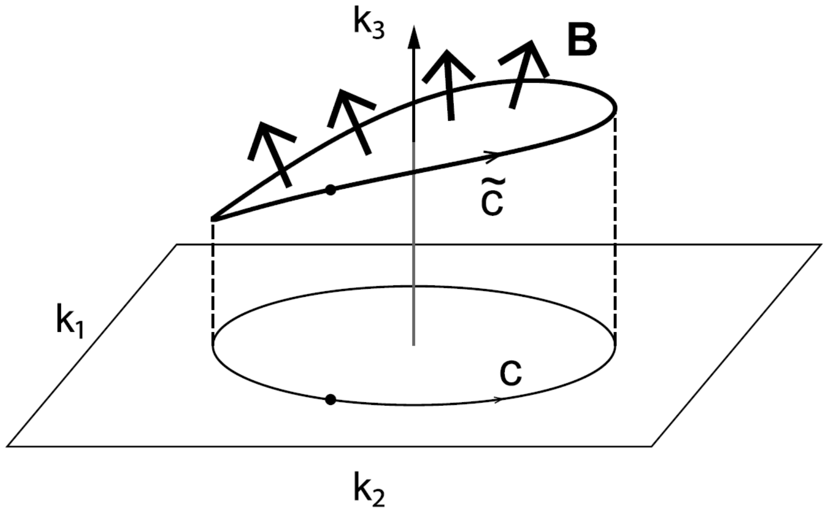
Vector potential & geometric current

$$\tilde{\boldsymbol{k}} = (\boldsymbol{k}, k_3) \qquad \boldsymbol{k}(t) = (k_1(t), k_2(t))$$

$$k_1 = k_{\mathrm{in}}^{(\mathrm{L})}(t) \qquad k_2 = k_{\mathrm{in}}^{(\mathrm{R})}(t) \qquad k_3 = \delta_2$$

$$J_{\mathrm{g}} = \oint_{\tilde{C}} \mathrm{d}\tilde{\boldsymbol{k}} \cdot \boldsymbol{A}(\boldsymbol{k}),$$



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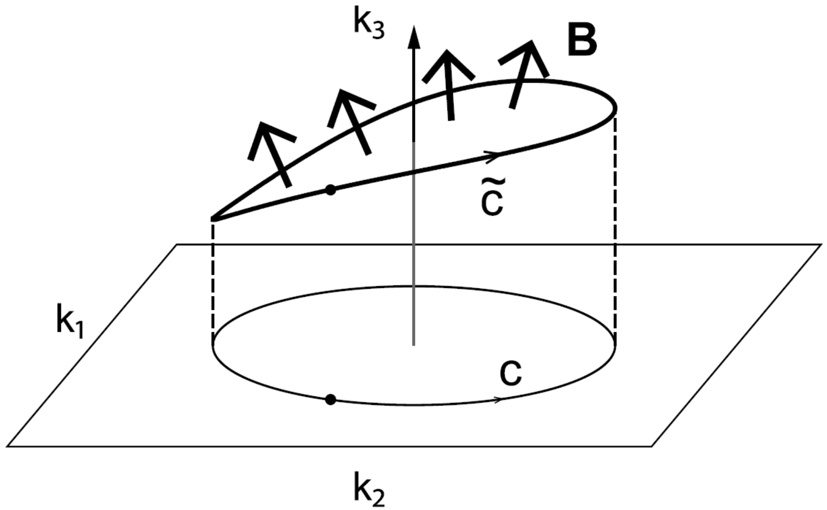
$$J_{\rm g} = \oint_{\tilde{C}} {\rm d}\tilde{\boldsymbol{k}} \cdot \boldsymbol{A}(\boldsymbol{k}),$$

$$\boldsymbol{A}(\boldsymbol{k}) = \frac{\omega}{2\pi} \begin{pmatrix} p^{(\rm R)} \partial_1 p_{\rm out} \\ p^{(\rm R)} \partial_2 p_{\rm out} \\ p^{(\rm R)} \end{pmatrix}$$



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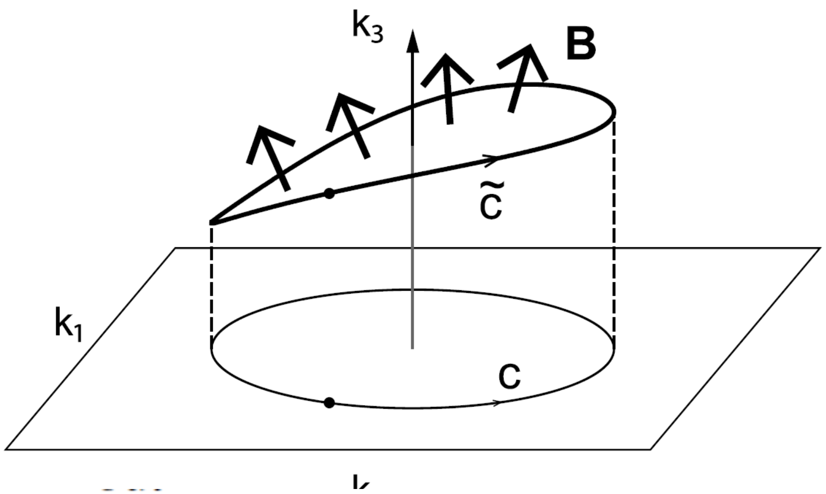
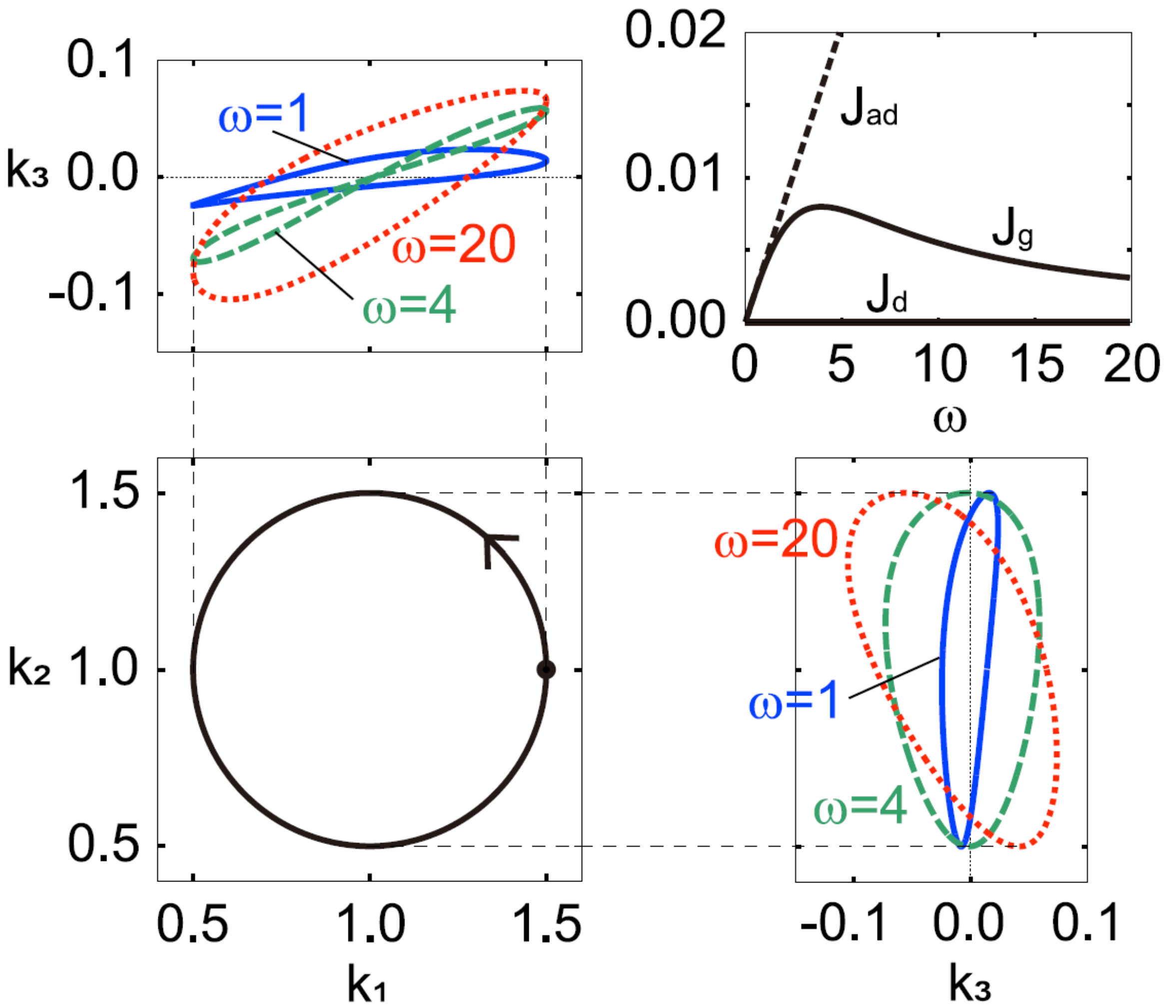


$$p^{(\rm R)}(t) = (k_{\rm in}^{(\rm R)}(t) + k_{\rm out}^{(\rm R)}(t))/(k_{\rm in}(t) + k_{\rm out}(t)),$$

$$p_{\rm out}(t) = k_{\rm out}(t)/(k_{\rm in}(t) + k_{\rm out}(t))$$

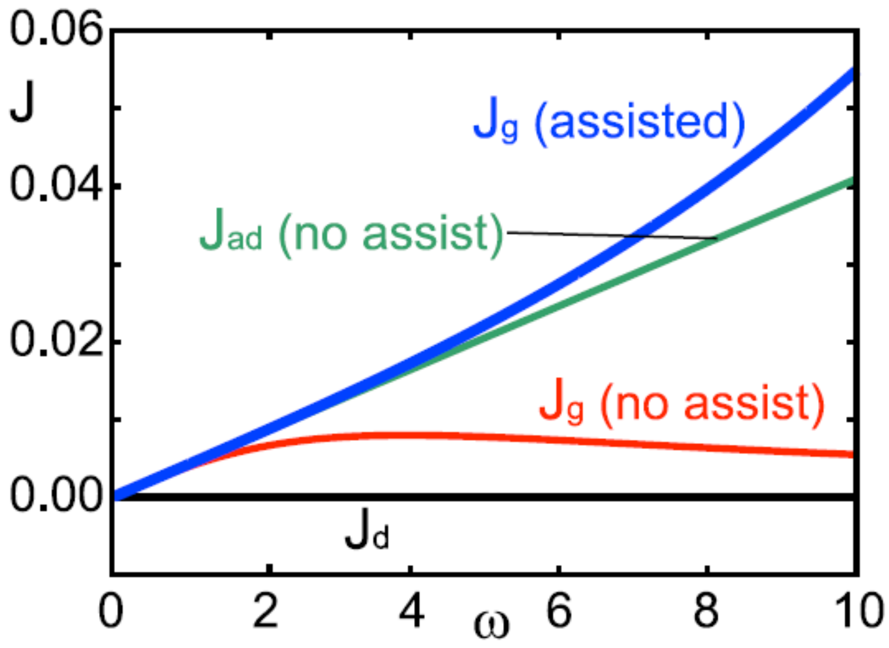
Nonadiabatic current



Shortcuts to adiabacity

- The counterdiabatic term helps us to get the large current in non-adiabatic region.

$$W_{\mathrm{CD}}(t) = \frac{\mathrm{d}p_{\mathrm{out}}(t)}{\mathrm{d}t}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$



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Contents

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Summary

- We have formulated the description of geometrical pumping processes in terms of master equation.
- We formulate the non-trivial fluctuation theorem including the effects of geometric current.
- The non-trivial FT is governed by non-Gaussian fluctuations.
- FDR and reciprocal relation are violated in this system.
- Non-adiabatic current can be obtained, at least, for two-level systems.
- We obtain large currents with the aid of shortcuts to adiabaticity.

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Collaborators

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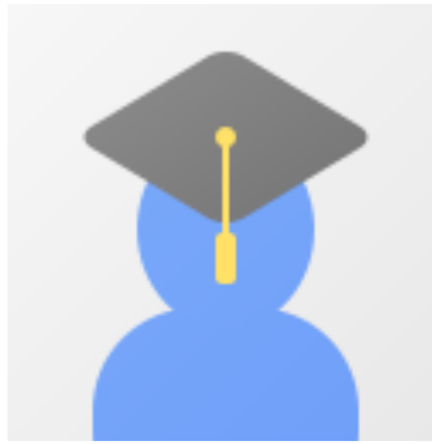

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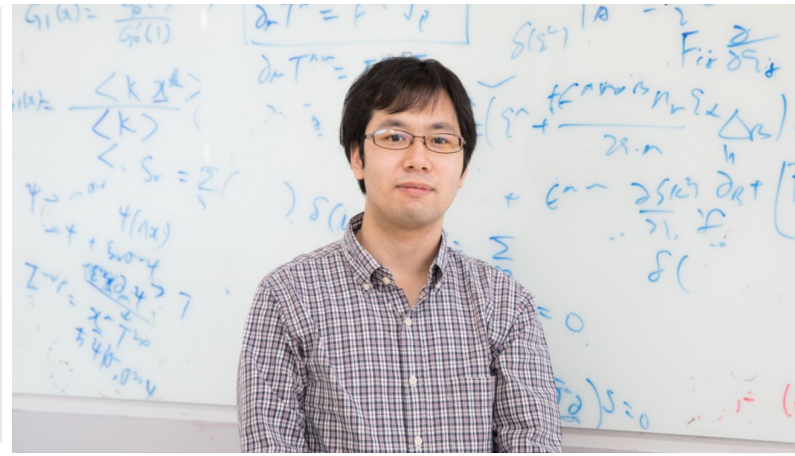

Keisuke Fujii
(Tokyo Tech)

See Y. Hino & HH. arXiv:1908.10597,
K. Fujii, HH, Y. Hino & K. Takahashi, arXiv:1909.02202

FRONTIERS IN NON-EQUILIBRIUM PHYSICS: STATISTICAL MECHANICS OF ATHERMAL SYSTEMS

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JUNE 3RD — JULY 3RD, 2020

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