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Universal Thermodynamic Signature of Self-dual Quantum Critical Points

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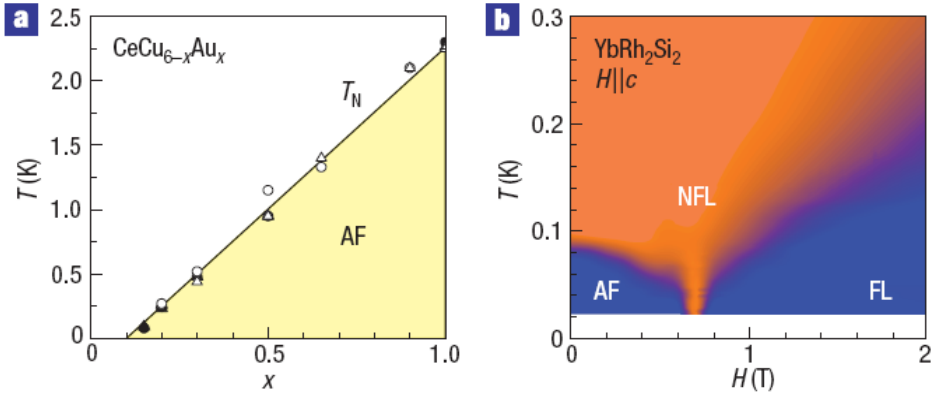
L. Zhang, arXiv:1903.09217 (accepted in PRL)

Outline

- Introduction
  - Critical phenomena and universality
  - Grüneisen ratio (GR)
  - 1D transverse-field Ising model
  - Self-duality
- Non-divergent GR at self-dual QCP
  - QCP with exact self-duality
  - General hyperscaling theory
  - Examples
  - Summary

Critical phenomena

- Universality is the hallmark of critical phenomena.
- Characterized by critical exponents:
  - Correlation length: $\xi \sim |T - T_c|^{-\nu}$
  - Magnetic susceptibility: $\chi \sim |T - T_c|^{-\gamma}$
- Quantum critical points induced by pressure and magnetic field etc. may be responsible for exotic phenomena in cuprates, heavy fermions, etc.



P. Gegenwart et al, Nat. Phys. 4, 186 (2008)

Grüneisen ratio: signature of QCP

- Grüneisen ratio (GR) near a pressure-tuned QCP

$$\Gamma = \frac{\alpha}{c_p} = \frac{(1/V)(\partial V/\partial T)_p}{(T/N)(\partial S/\partial T)_p}.$$

- Near a magnetic field-tuned QCP, replace (p, V) by (B, M). GR is the magnetocaloric coefficient.
- Universally diverging GR as a thermodynamic signature of QCP:

$$\Gamma(T, g = g_c) \sim -G_T T^{-1/z\nu},$$
$$\Gamma(T \to 0, g) \sim \frac{1}{g - g_c},$$

$z\nu$: critical exponents

- It was derived from hyperscaling theory.

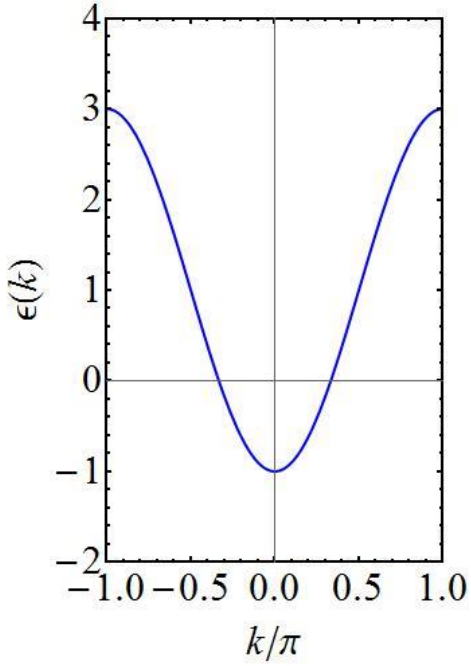
L. Zhu, M. Garst, A. Rosch, and Q. Si, Phys. Rev. Lett. 91, 066404 (2003)

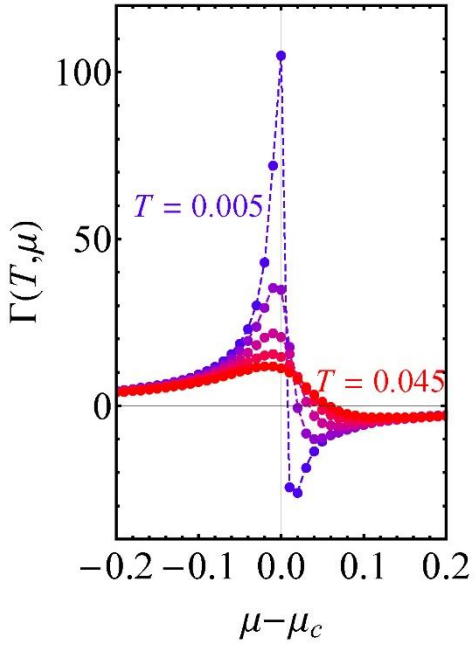
A simple example

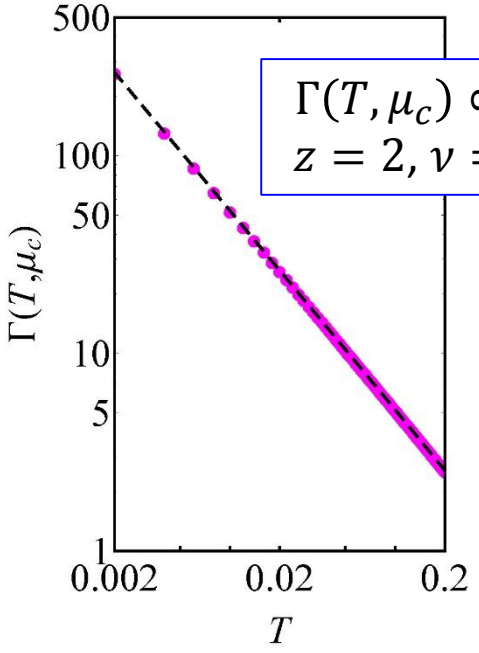
- Chemical potential-tuned MIT in 1D:

$$H = \sum_k \xi_k c_k^\dagger c_k, \quad \xi_k = -2\cos k - \mu.$$

- QCP at $\mu_c = -2$: metal $\mu < \mu_c$, insulator $\mu > \mu_c$.



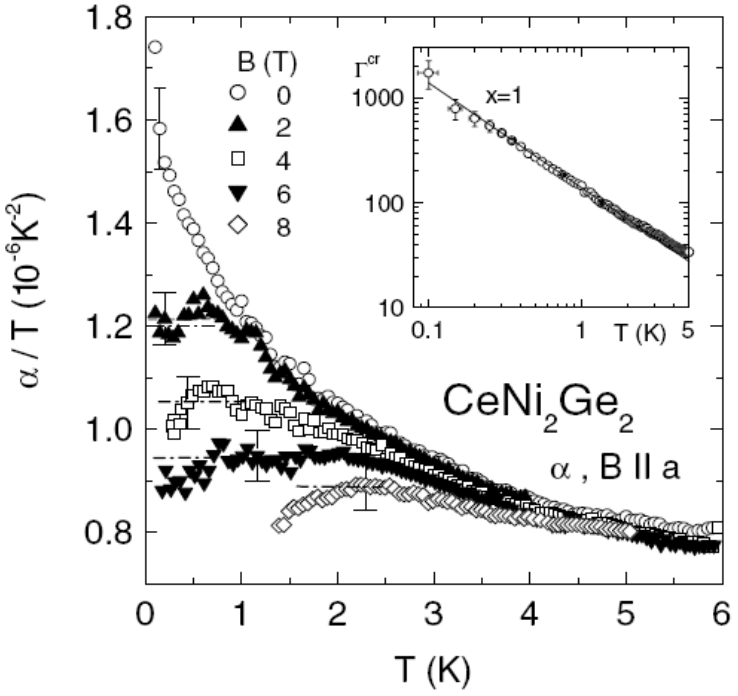


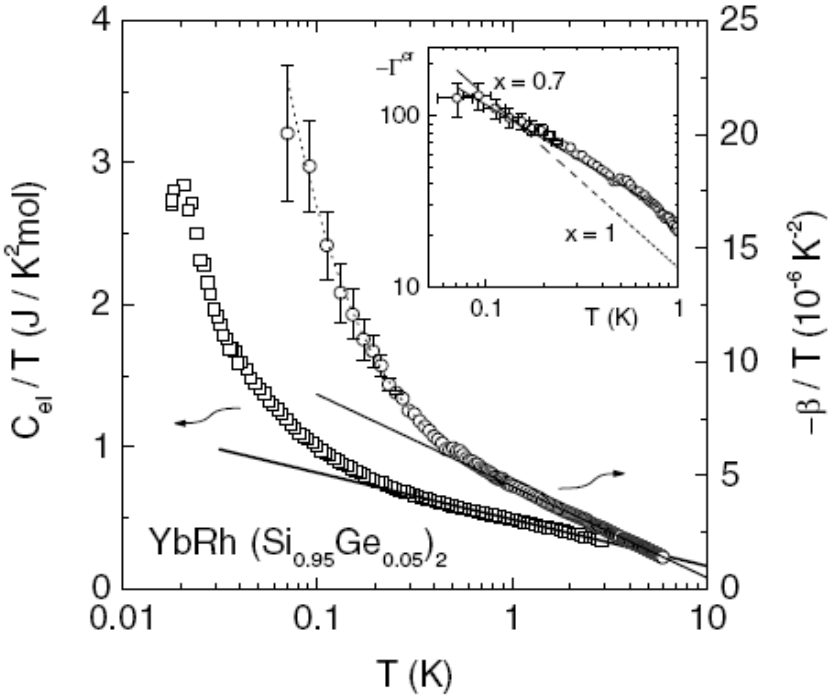


$\Gamma(T,\mu_c) \propto T^{-1}$
$z = 2, \nu = 1/2$

GR in heavy fermion materials

- Diverging GR was soon found in heavy fermion materials, $CeNi_2Ge_2$ and $YbRh(Si_{0.95}Ge_{0.05})_2$, at QCP.
- It has been a standard probe to QCPs.





R. Küchler et al, PRL 91, 066405 (2003); P. Gegenwart, Rep. Prog. Phys. 79, 114502 (2016)

GR of 1D quantum Ising model

- 1D transverse-field Ising model (TFIM)

$$H(g) = -\sum_i \sigma_i^x \sigma_{i+1}^x - g \sum_i \sigma_i^z$$

- Exactly solvable, QCP at $g_c = 1$.

$$\Gamma(T \to 0, g) \sim 1/(g-1), \quad \text{Good!}$$
$$\Gamma(T, g_c) = 1/2. \quad \text{Oops!}$$

- It apparently contradicts the general scaling form:

$$\Gamma(T, g = g_c) \sim -G_T T^{-1/z\nu},$$
$$\Gamma(T \to 0, g) \sim \frac{1}{g - g_c},$$

$z\nu$: critical exponents

- Why?

J. Wu, L. Zhu, and Q. Si, Phys. Rev. B 97, 245127 (2018)

Hypercaling and GR of 1DTFIM

- GR is derived from the hyperscaling ansatz of free energy near a QCP:

$$f_s(T,g) \propto T^{(d+z)/z}\Phi\left(\frac{g-g_c}{T^{1/z\nu}}\right)$$

$$\Gamma(T,g_c) \sim -G_T T^{-1/z\nu}, G_T \propto \Phi'(0).$$

- Special feature in 1DTFIM: $\Phi'(0) = 0.$

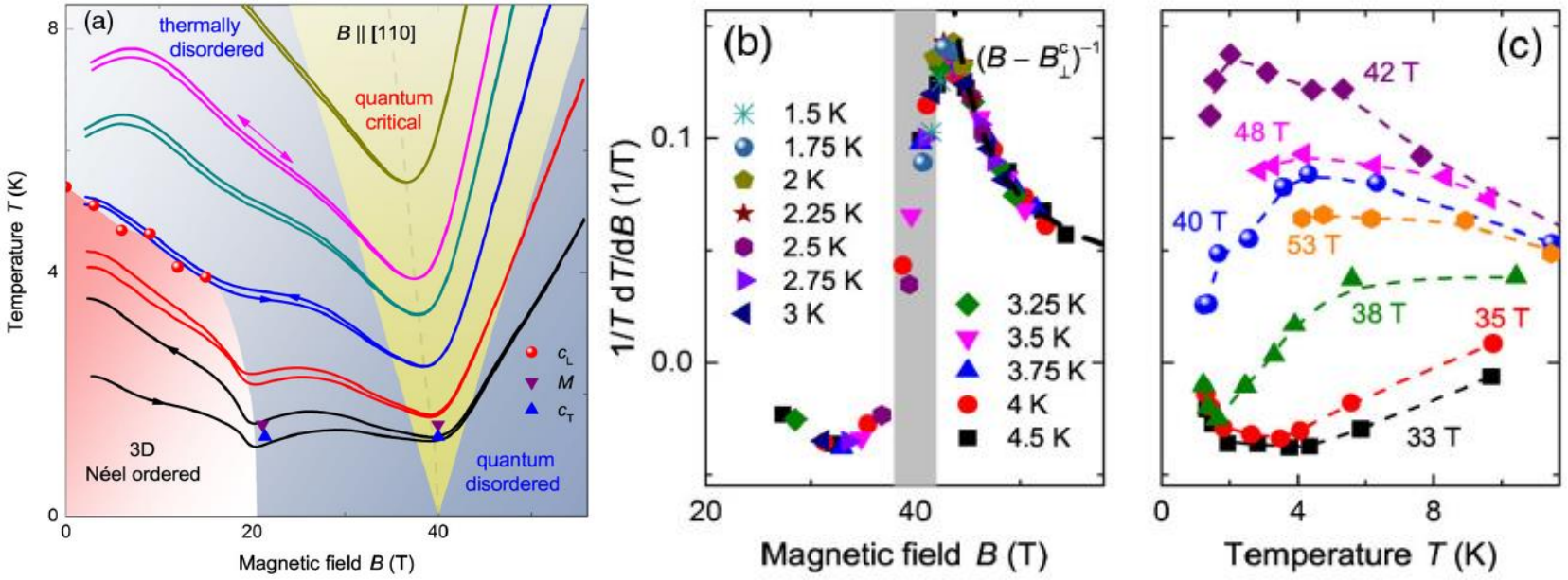
$$f_s(T,g) \propto -T^2(a_g + \frac{(g-1)^2}{2T^2})$$

- This reconciles the finite GR with the hyperscaling.
  - Interpreted as a characteristic feature of 1DTFIM.

L. Zhu et al, PRL 91, 066404 (2003); J. Wu et al, PRB 97, 245127 (2018)

$BaCo_2V_2O_8$: 1D Ising antiferromagnet

- A transverse field-tuned QCP captured by 1DTFIM.
- Magnetocaloric effect: GR changes sign as $(B - B_c)^{-1}$ while saturates close to QCP as $T \to 0$.
  - Taken as evidence of quantum criticality of 1DTFIM (?)



Z. Wang et al, Phys. Rev. Lett. 120, 207205 (2018)

Self-duality of 1DTFIM

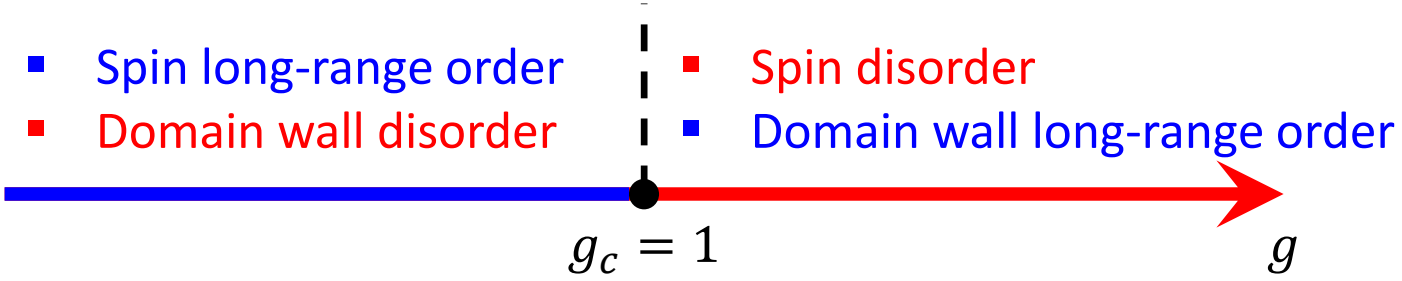
- What is special of 1DTFIM?

$$H(g) = -\sum_i \sigma_i^x \sigma_{i+1}^x - g \sum_i \sigma_i^z$$

- Self-duality!

$$U\sigma_i^x U^{-1} = \prod_{l \le i} \sigma_l^z, \quad U\sigma_i^z U^{-1} = \sigma_i^x \sigma_{i+1}^x.$$

  - $UH(g)U^{-1} = gH(1/g)$, weak-strong duality.
  - Self-dual QCP at $g_c = 1$.
  - Mapping between spin and domain wall operators.



E. Fradkin and L. Susskind, PRD 17, 2637 (1978)

QCP with exact self-duality

- Assume $H(g) = H_1 + gH_2$ with exact self-duality,

$$UH_1U^{-1} = H_2, \quad UH_2U^{-1} = H_1.$$

- At self-dual QCP $g_c = 1$, $UH(g_c)U^{-1} = H(g_c)$. Thermodynamic average,

$$\langle H_1\rangle_T = \langle UH_1U^{-1}\rangle_T = \langle H_2\rangle_T = \frac{1}{2}\langle H(g_c)\rangle_T.$$

- GR at the QCP,

$$\Gamma(T, g_c) = \frac{\partial\langle H_2\rangle_T/\partial T}{\partial\langle H(g_c)\rangle_T/\partial T} = \frac{1}{2}$$

- A generic consequence of exact self-duality without referring to any specific properties of the model.

LZ, arXiv:1903.09217

General hyperscaling theory

- Given a fixed point H^* and a relevant operator R, a self-duality is heuristically defined by

$$UH^*U^{-1} = H^* + \cdots,$$
$$URU^{-1} = -R + \cdots.$$

$\mathbb{Z}_2$ mapping up to irrelevant terms

- The free energy satisfies

$$f_s(T, g_c + \delta g) \simeq f_s(T, g_c - \delta g),$$
$$\Phi\left(\frac{g - g_c}{T^{1/z\nu}}\right) = \Phi\left(-\frac{g - g_c}{T^{1/z\nu}}\right)$$

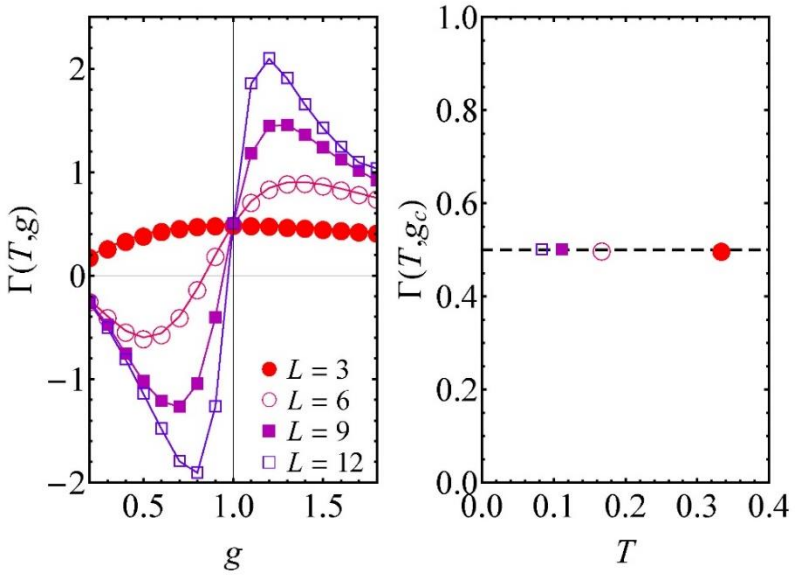
- This implies $\Phi'(0) = 0$ and non-divergent GR at QCP, which is a universal signature of self-dual QCP.

Example: Ising domain wall model

- Ising domain wall model

$$H(g) = -\sum_i \sigma_i^z \sigma_{i+1}^x \sigma_{i+2}^z - g \sum_i \sigma_i^y.$$

- Exact self-duality: $U = \prod_i \sigma_i^x \prod_i e^{-(i\pi/4)(\sigma_i^z \sigma_{i+1}^z - \sigma_i^z - 1)}$
- Self-dual QCP at $g_c = 1$, $SU(2)_1$ WZW



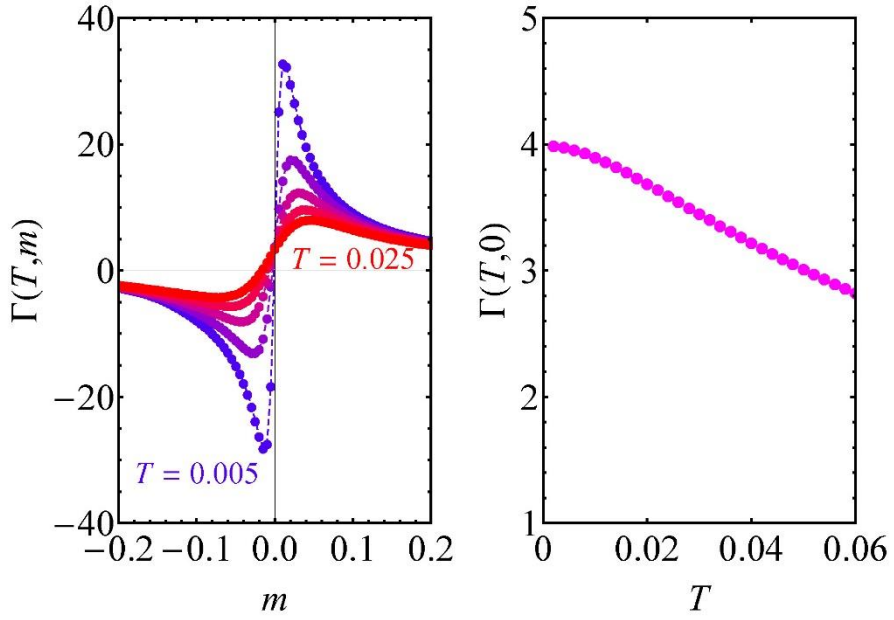
C. Bao et al, arXiv:1801.00719; LZ, arXiv:1903.09217

Example: 2D topological transition

- 2D QAH-normal insulator transition:

$$H = \sum_k \psi_k^\dagger h(k) \psi_k, \quad \psi_k = \begin{pmatrix} c_k \\ v_k \end{pmatrix},$$
$$h(k) \simeq \Delta(k_x \sigma_x + k_y \sigma_y) + m\sigma_z.$$

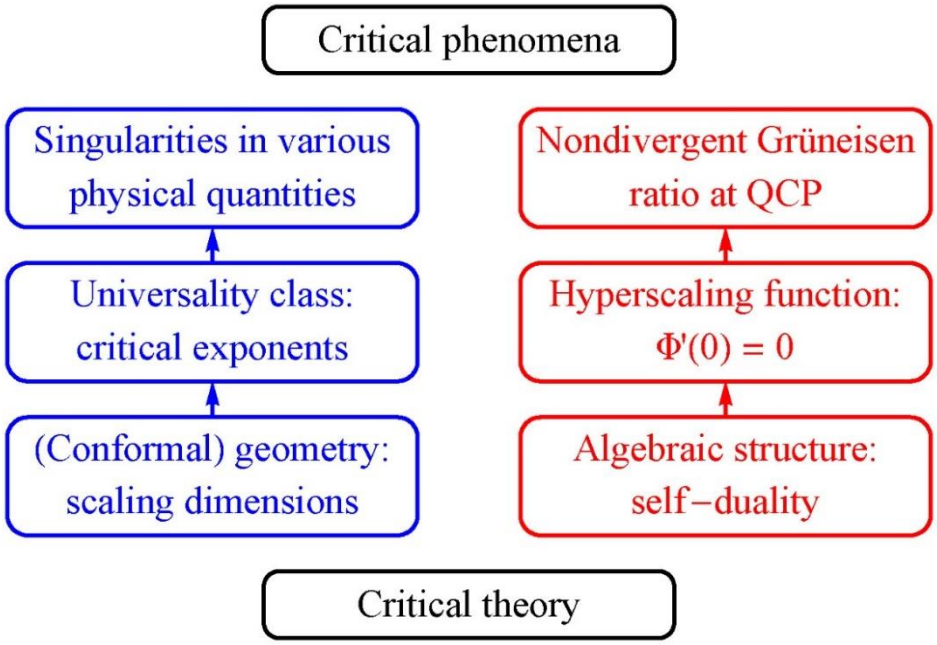
- Anti-unitary self-duality: $U\psi_k U^{-1} = \mathcal{K}\sigma_x \psi_k$



B. A. Bernevig et al, Science 314, 1757 (2006); LZ, arXiv:1903.09217

Summary

- Non-divergent GR is a universal thermodynamic signature of self-dual QCPs.
- Algebraic structure of critical theory, not encoded in critical exponents and scaling dimensions.



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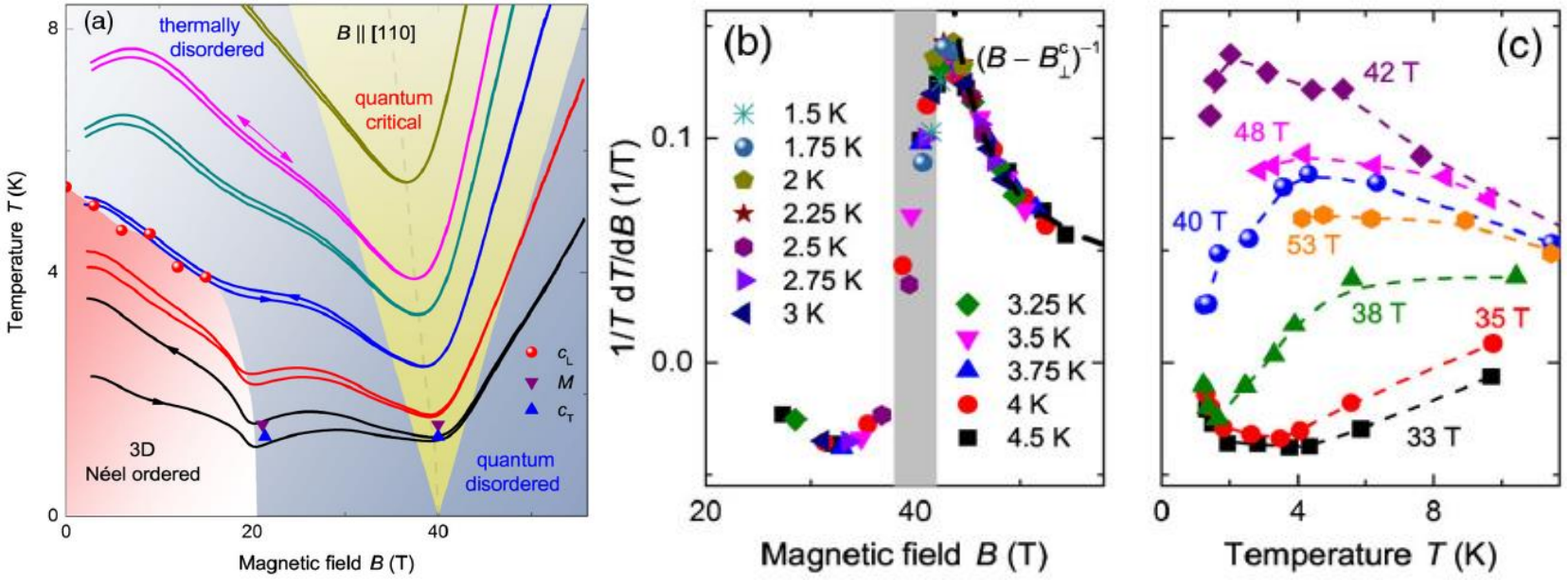

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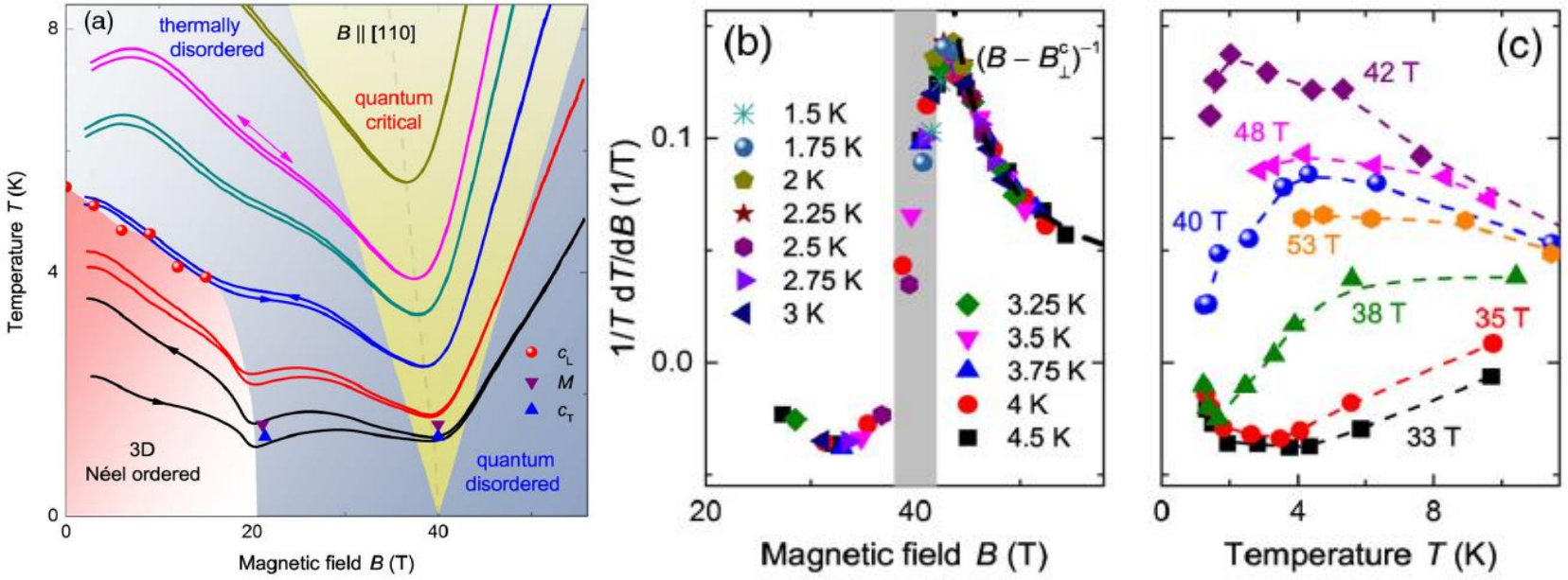
- A transverse field-tuned QCP captured by 1DTFIM.
- Magnetocaloric effect: GR changes sign as $(B - B_c)^{-1}$ while saturates close to QCP as $T \to 0$.
  - First thermodynamic evidence of self-duality!



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