Determining system Hamiltonian from eigenstate measurements without correlation functions

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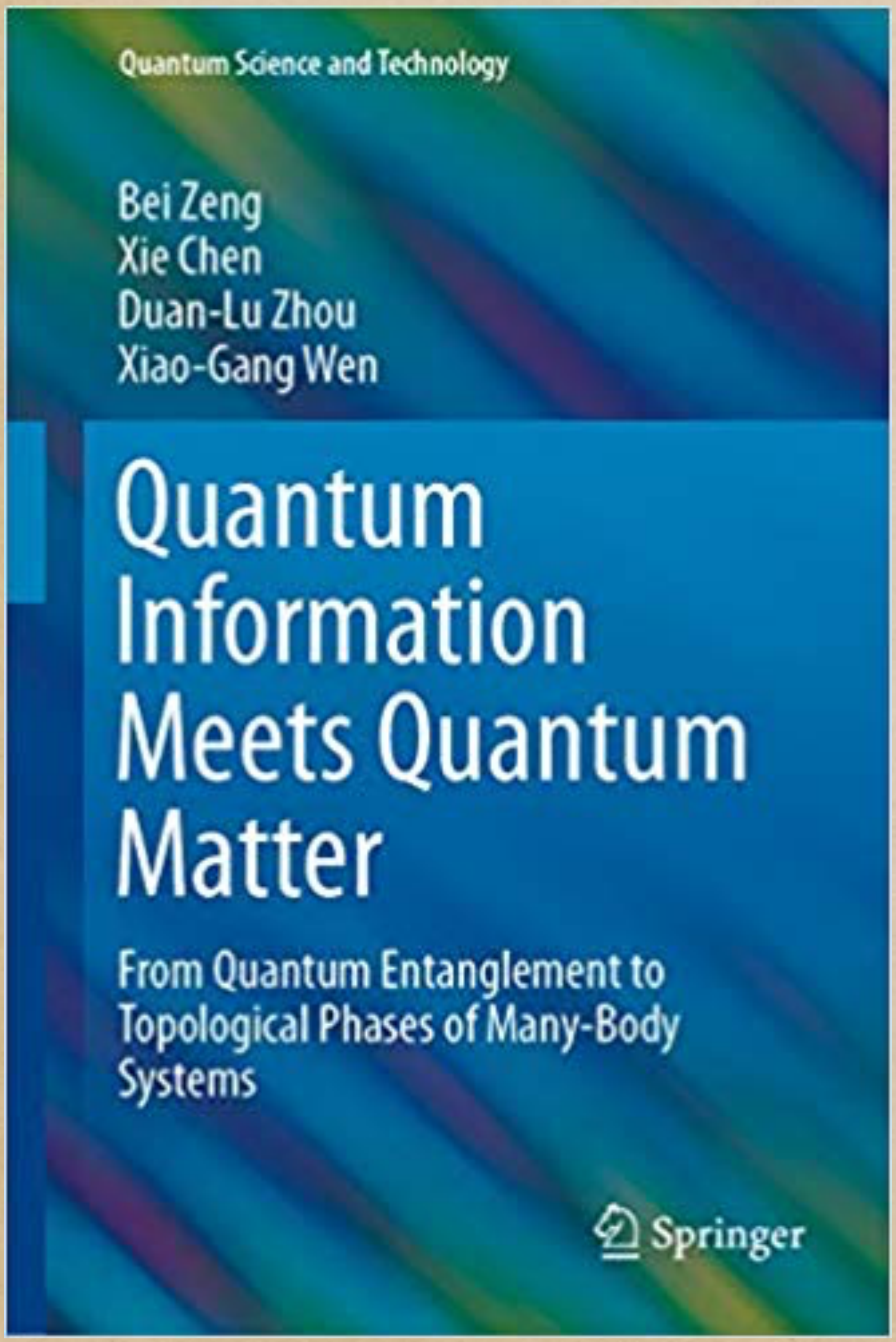
Quantum information meets quantum matter



information = matter

computational complexity
~~~local Hamiltonians

quantum circuits complexity
~~~quantum phase of matter

Local Hamiltonians, Circuit complexity...

$$H = \sum_i H_i$$

Quantum Circuit-depth Lower Bounds For Homological Codes

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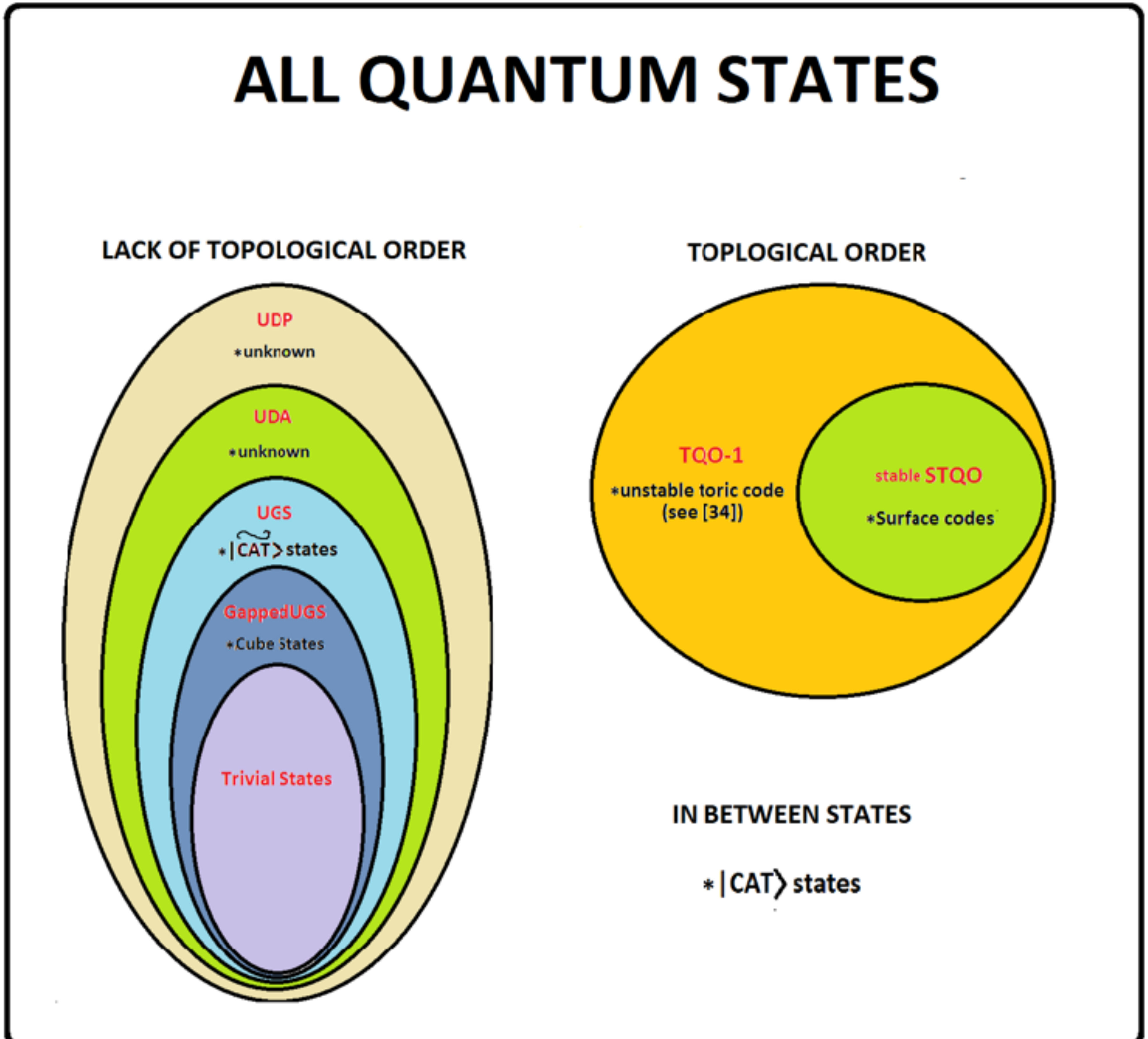
We provide an $\Omega(log(n))$ lower bound for the depth of any quantum circuit generating the unique groundstate of Kitaev's spherical code. No circuit-depth lower bound was known before on this code in the general case where the gates can connect qubits even if they are far away; It is a known hurdle in computional complexity to handle general circuits, and indeed the proof requires introducing new techniques beyond those used to prove the $\Omega(\sqrt{n})$ lower bound which holds in the geometrical case [34]. The lower bound is tight (up to constants) since a MERA circuit of logarithmic depth exists [16]. To the best of our knowledge, this is the first time a quantum circuit-depth lower bound is given for a unique ground state of a *gapped* local Hamiltonian. Providing a lower bound in this case seems more challenging, since such systems exhibit exponential decay of correlations [41] and standard lower bound techniques [31] do not apply. We prove our lower bound by introducing the new notion of γ-separation, and analyzing its behavior using algebraic topology arguments.


ALL QUANTUM STATES
LACK OF TOPOLOGICAL ORDER
UDP
*unknown
UDA
*unknown
UGS
*|CAT⟩ states
GappedUGS
*Cube States
Trivial States
TOPLOGICAL ORDER
TQO-1
*unstable toric code
(see [34])
stable STQO
*Surface codes
IN BETWEEN STATES
*|CAT⟩ states

Eigenstates thermalization

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Does a Single Eigenstate Encode the Full Hamiltonian?

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The eigenstate thermalization hypothesis (ETH) posits that the reduced density matrix for a subsystem corresponding to an excited eigenstate is "thermal." Here we expound on this hypothesis by asking: For which class of operators, local or nonlocal, is ETH satisfied? We show that this question is directly related to a seemingly unrelated question: Is the Hamiltonian of a system encoded within a single eigenstate? We

A single eigenstate

Determining a local Hamiltonian from a single eigen-state

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We ask whether the knowledge of a single eigenstate of a local Hamiltonian is sufficient to uniquely determine the Hamiltonian. We present evidence that the answer is "yes" for generic local Hamiltonians, given either the ground state or an excited eigenstate. In fact, knowing only the two-point equal-time correlation functions of local observables with respect to the eigenstate should generically be sufficient to exactly recover the Hamiltonian for finite-size systems, with numerical algorithms that run in a time that is polynomial in the system size. We also investigate the large-system limit, the sensitivity of the reconstruction to error, and the case when correlation functions are only known for observables on a fixed sub-region. Numerical demonstrations support the results for finite one-dimensional spin chains (though caution must be taken when extrapolating to infinite-size systems in higher dimensions). For the purpose of our analysis, we define the "k-correlation spectrum" of a state, which reveals properties of local correlations in the state and may be of independent interest.

The good old days

PHYSICAL REVIEW A **85**, 040303(R) (2012)

Correlations in excited states of local Hamiltonians

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Physical properties of the ground and excited states of a k-local Hamiltonian are largely determined by the k-particle reduced density matrices (k-RDMs), or simply the k-matrix for fermionic systems—they are at least enough for the calculation of the ground-state and excited-state energies. Moreover, for a nondegenerate ground state of a k-local Hamiltonian, even the state itself is completely determined by its k-RDMs, and therefore contains no genuine $>k$-particle correlations, as they can be inferred from k-particle correlation functions. It is natural to ask whether a similar result holds for nondegenerate excited states. In fact, for fermionic systems, it has been conjectured that any nondegenerate excited state of a 2-local Hamiltonian is simultaneously a unique ground state of another 2-local Hamiltonian, hence is uniquely determined by its 2-matrix. And a weaker version of this conjecture states that any nondegenerate excited state of a 2-local Hamiltonian is uniquely determined by its 2-matrix among all the pure n-particle states. We construct explicit counterexamples to show that both conjectures are false. We further show that any nondegenerate excited state of a k-local Hamiltonian is a unique ground state of another $2k$-local Hamiltonian, hence is uniquely determined by its $2k$-RDMs (or $2k$-matrix). These results set up a solid framework for the study of excited-state properties of many-body systems.

The ground states

$$H|\psi_g\rangle = \lambda_g|\psi_g\rangle \qquad \rho_g = |\psi_g\rangle\langle\psi_g|$$

$$H = \sum_i c_i A_i \qquad H(\vec{c}) \qquad \vec{c} = (c_1, \ldots, c_m)$$

$$\rho_g = \frac{e^{-\beta H}}{\mathrm{Tr} e^{-\beta H}} \qquad \beta = \frac{1}{\kappa T} \to \infty$$

Solve the equations $\mathrm{Tr}(\rho_g(\vec{c})A_i) = a_i$

for $\vec{c} = (c_1, \ldots, c_m)$

Less parameters than $|\psi_g\rangle$

Equation-solving algorithm

$$\mathrm{Tr}(\rho_g(\vec{c})A_i) = a_i \quad \text{for} \quad \vec{c} = (c_1, \ldots, c_m)$$

Or one can define

$$f(\vec{c}) = \sum_i (\mathrm{Tr}(\rho_g(\vec{c})A_i) - a_i)^2$$

And to minimize $f(\vec{c})$

An iterative algorithm:

S. Niekamp, T. Galla, M. Kleinmann, and O. Gühne, Journal of Physics A: Mathematical and Theoretical 46, 125301 (2013).

Idea from information projection. Convergence?

The eigenstates

$$H|\psi\rangle = \lambda|\psi\rangle \qquad \rho = |\psi\rangle\langle\psi|$$

$$H = \sum_i c_i A_i \qquad H(\vec{c}) \qquad \vec{c} = (c_1, \ldots, c_m)$$

$$(H - \lambda)|\psi\rangle = 0|\psi\rangle \qquad (H - \lambda)^2|\psi\rangle = 0|\psi\rangle$$

$$\tilde{H} = (H - \lambda)^2 \qquad \tilde{H}|\psi\rangle = 0|\psi\rangle$$

$|\psi\rangle$ is a ground state of $\tilde{H}$

J. Chen, Z. Ji, Z. Wei, and B. Zeng, Physical Review A 85, 040303 (2012).

Why two-point correlation

$$H|\psi\rangle = \lambda|\psi\rangle \qquad \rho = |\psi\rangle\langle\psi|$$

$$H = \sum_i c_i A_i \qquad \mathrm{Tr}(\rho A_i) = a_i \qquad \lambda = \sum_i c_i a_i$$

$$\tilde{H} = \left(\sum_i c_i (A_i - a_i)\right)^2$$

$$\to \mathrm{Tr}(\rho A_i A_j)$$

If these correlations were known $\to$ ground state algorithm

X.-L. Qi, and D. Ranard, Quantum, 3, 159 (159).

Without two-point correlation?

$$\tilde{H}|\psi\rangle = 0|\psi\rangle \qquad \rho = |\psi\rangle\langle\psi|$$

$$\tilde{H} = \left(\sum_i c_i(A_i - a_i)\right)^2 \qquad \tilde{H}(\vec{c}) \qquad \vec{c} = (c_1, \ldots, c_m)$$

$$\rho = \frac{e^{-\beta\tilde{H}}}{\mathrm{Tr}e^{-\beta\tilde{H}}} \qquad \beta = \frac{1}{\kappa T} \to \infty$$

Solve $\mathrm{Tr}(\rho(\vec{c})A_i) = a_i$ for $\vec{c} = (c_1, \ldots, c_m)$

Or to minimize $f(\vec{c})$ for

$$f(\vec{c}) = \sum_i \left(\mathrm{Tr}(\rho(\vec{c})A_i) - a_i\right)^2$$

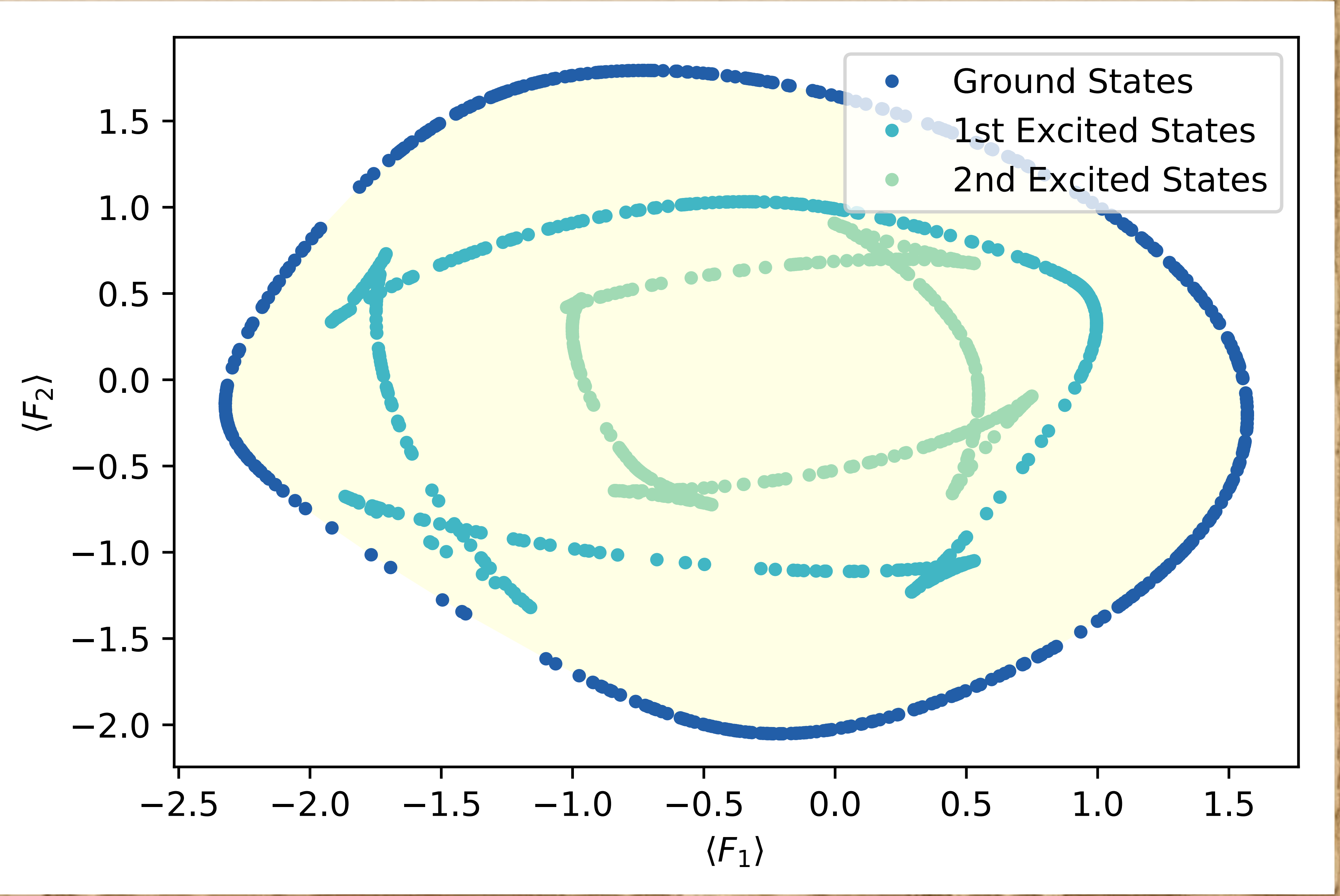
The picture?

$H = h_1 F_1 + h_2 F_2$

(joint) numerical range

$d = 9$

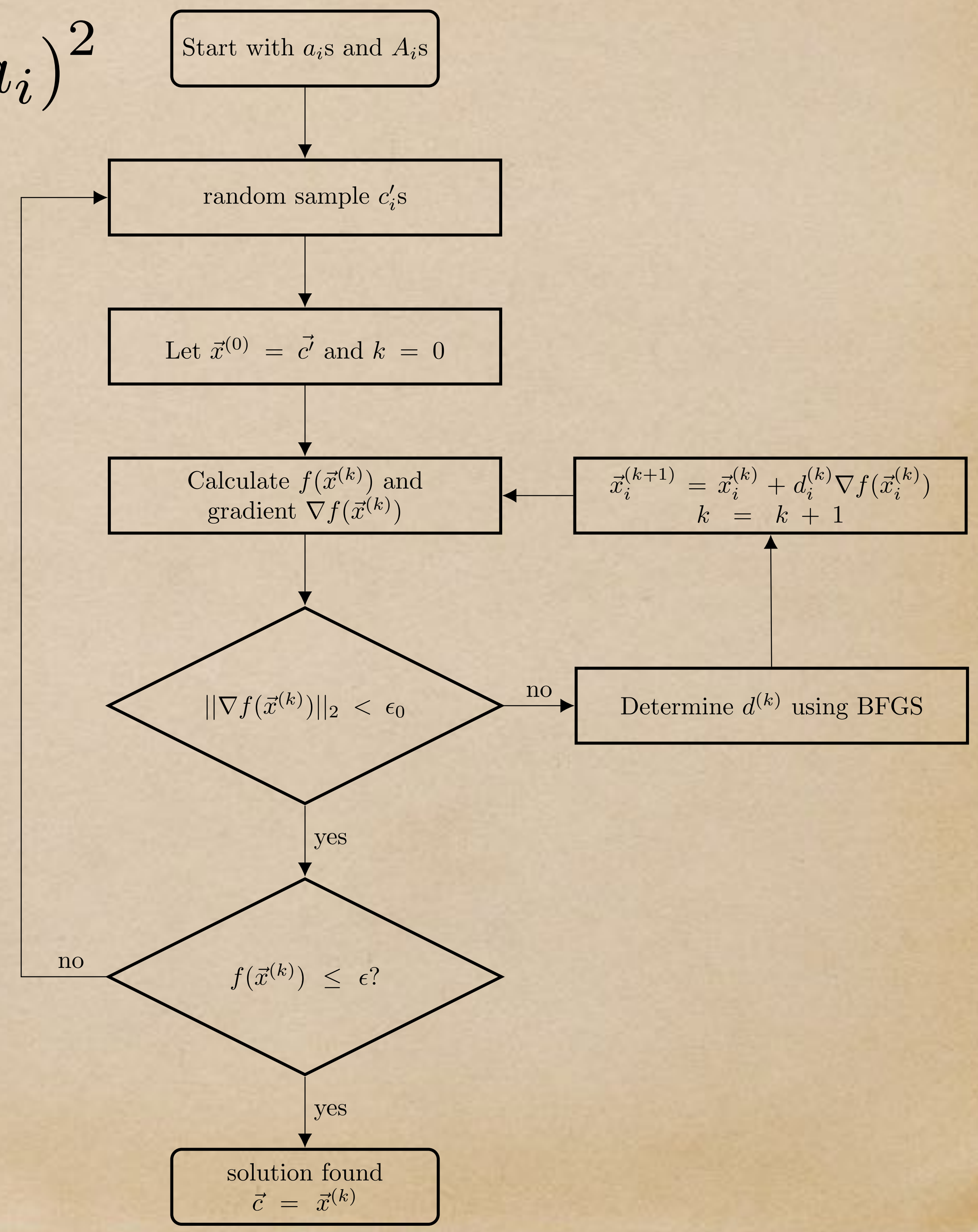
$m = 2$



The algorithm

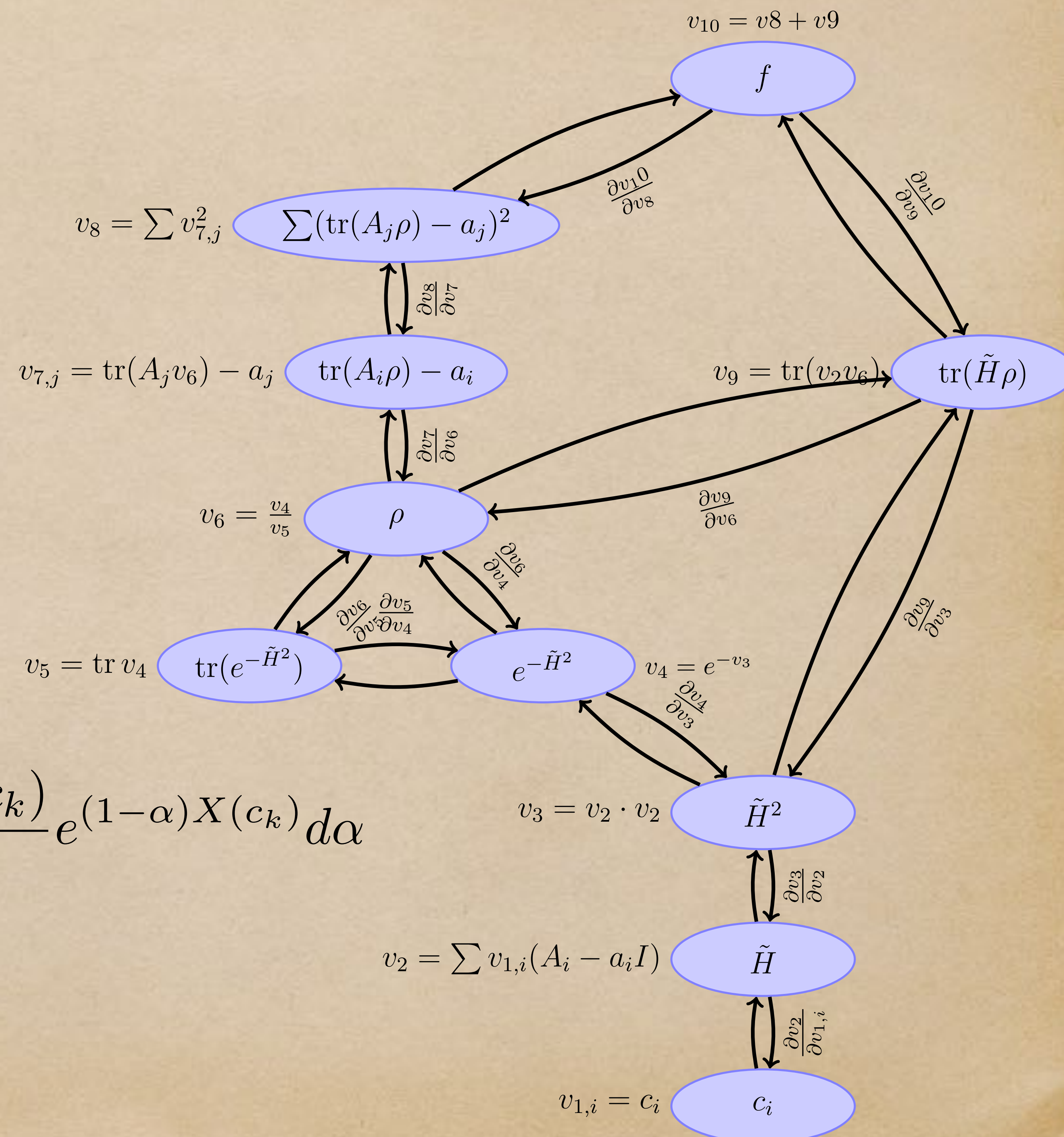
$$f(\vec{c}) = \sum_i \left(\mathrm{Tr}(\rho(\vec{c})A_i) - a_i\right)^2$$

- Random sampling
- quasi-Newton method
- BFGS-formula to approximate Hessian
- Different initial point



The gradient

- Function: chain rule
- Trace
- Operator



The derivative of

$$f(X) = \exp(X(c_k))$$

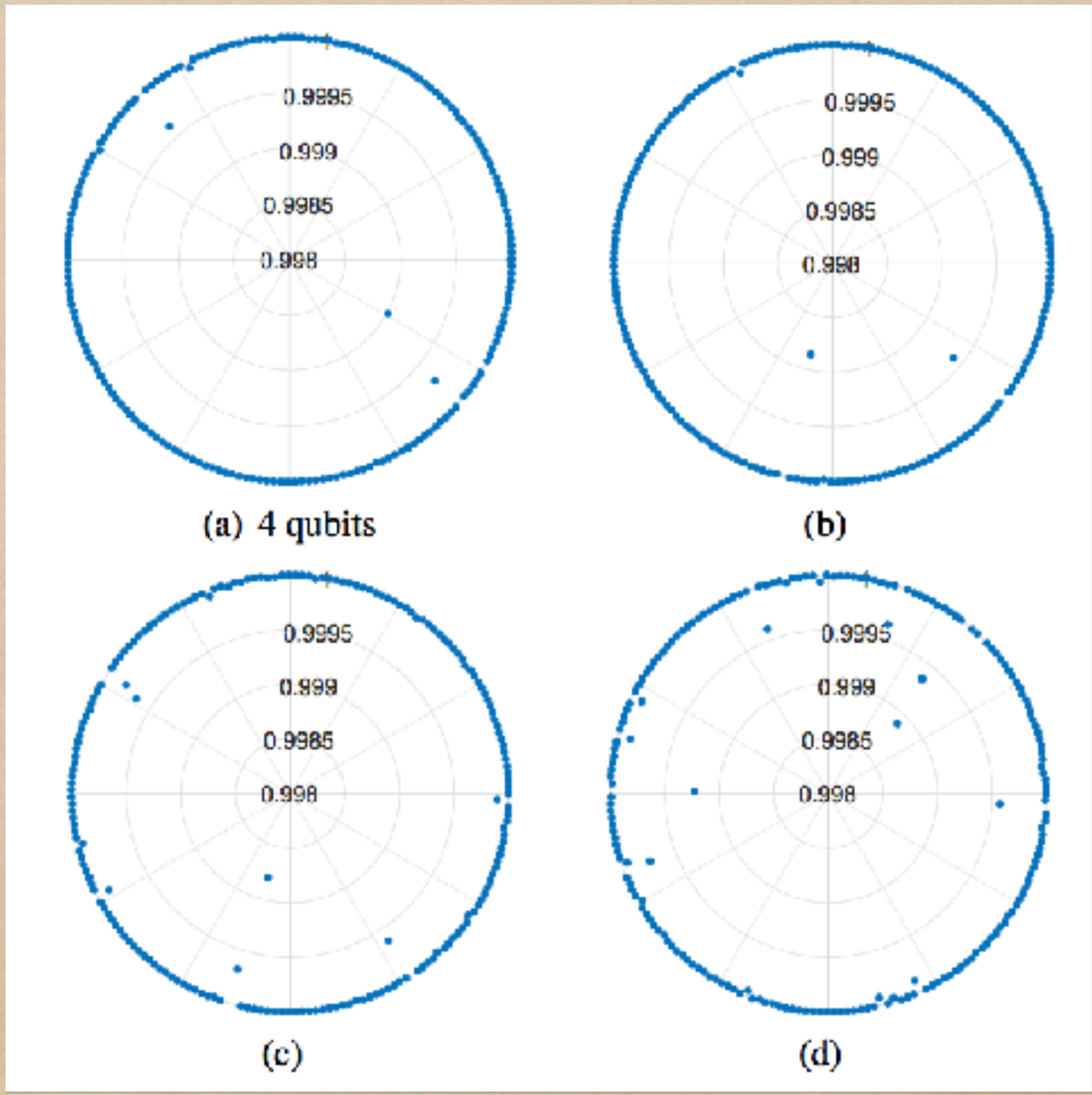
$$\frac{\partial e^{X(c_k)}}{\partial c_k} = \int_0^1 e^{\alpha X(c_k)} \frac{\partial X(c_k)}{\partial c_k} e^{(1-\alpha)X(c_k)} d\alpha$$

$$[X, \frac{\partial X}{\partial c_k}] \neq 0$$

Results: random operators

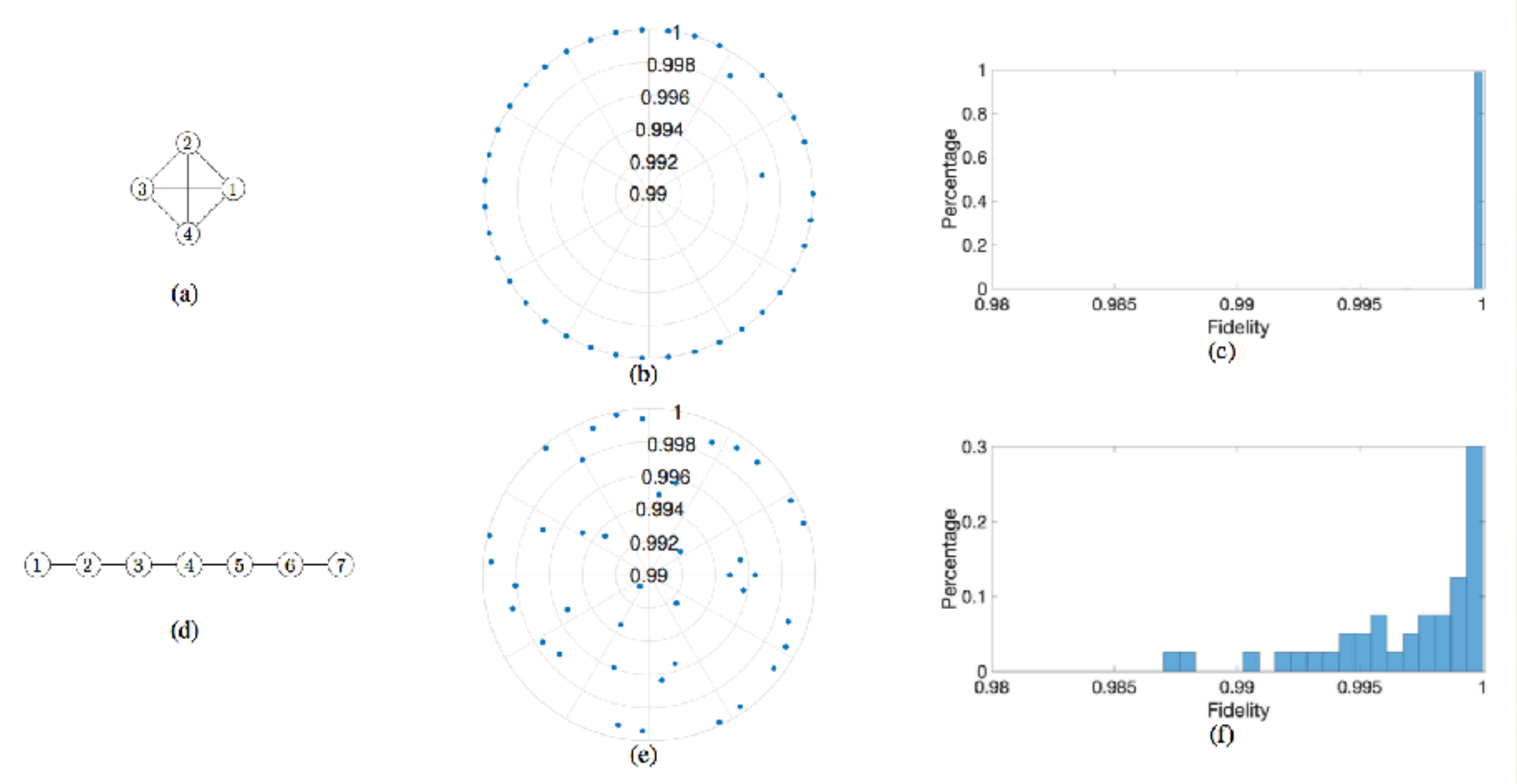
$$f(H_{al}, H_{rd}) = \frac{\mathrm{Tr} H_{al} H_{rd}}{\sqrt{\mathrm{Tr} H_{al}^2}\sqrt{\mathrm{Tr} H_{rd}^2}}.$$

- 3 random operators
- (a) 4-qubit
- (b) 5-qubit
- (c) 6-qubit
- (d) 7-qubit



Results: local operators

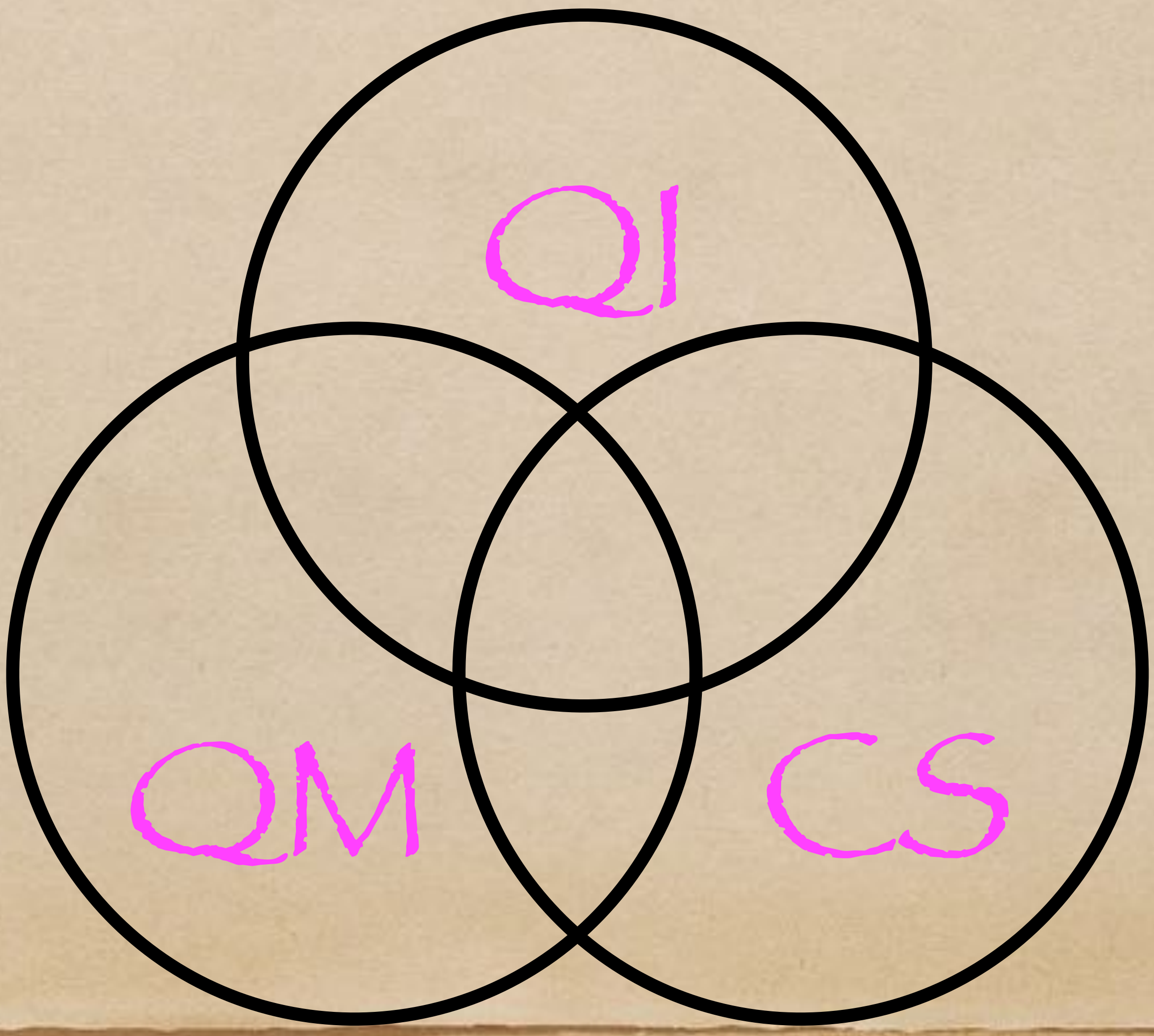
4-qubit; 7-qubit; local operators



Summary

Given $H|\psi\rangle = \lambda|\psi\rangle$ and $H = \sum_i c_i A_i$

Find H $(\lambda, |\psi\rangle)$ by only knowing $\langle\psi|A_i|\psi\rangle = a_i$



Outlook

- More rigorous treatment for uniqueness?
- Information-theoretic meaning for "eigenstates"?
- Complexity analysis of algorithm?
- Other algorithm (e.g. machine learning algorithm)?
- Correlation-locality?
- Eigenstate thermalization?

References

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