Exactly solvable two-terminal heat engine with asymmetric Onsager coefficients: Origin of the power-efficiency bound

Hyunggyu Park (KIAS)

with Jae Sung Lee and Jong-Min Park

Talk at EAJSSP2019, ITP-CAS, Beijing, China (October 22, 2019)

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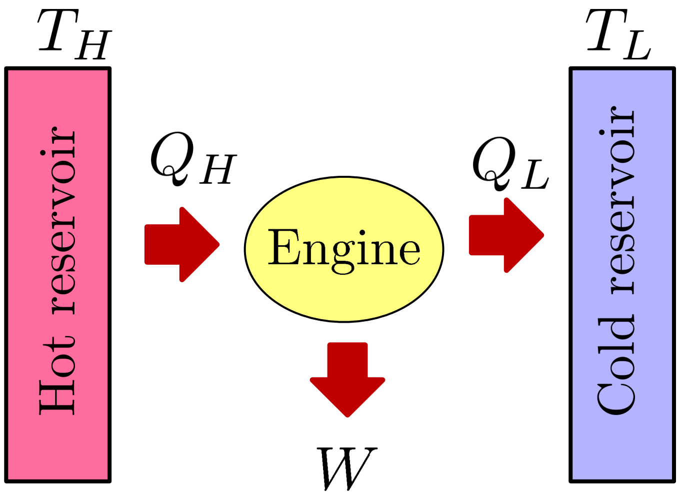

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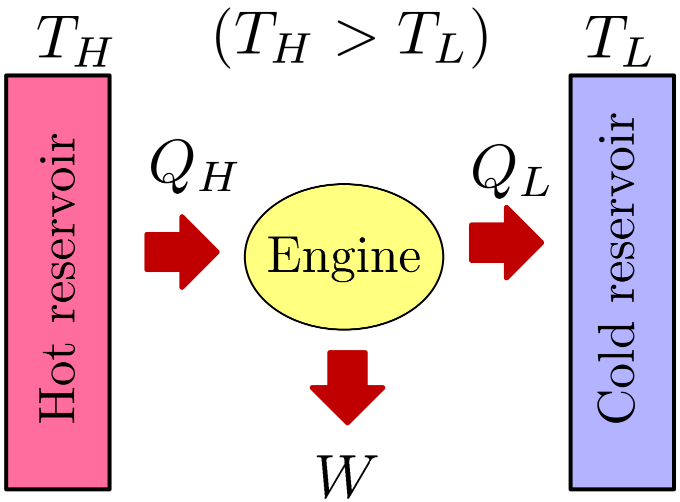

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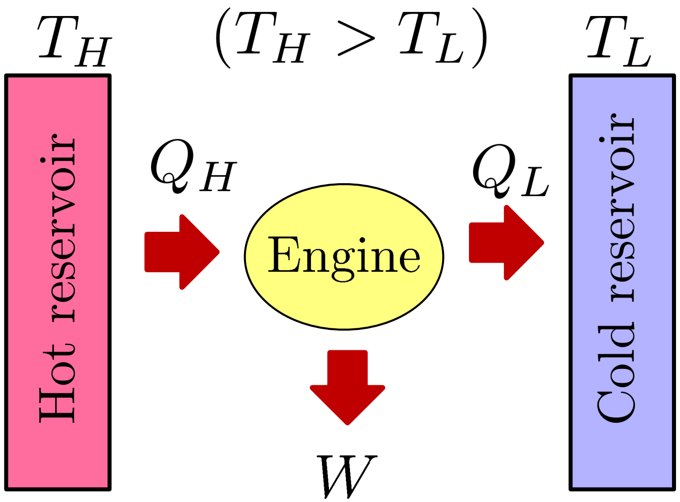

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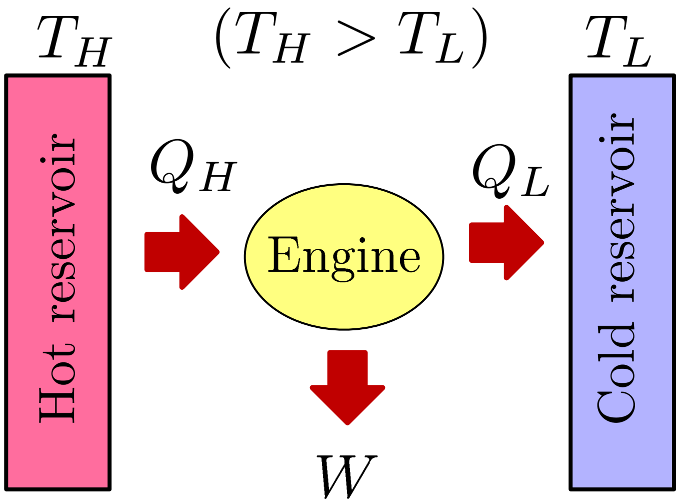

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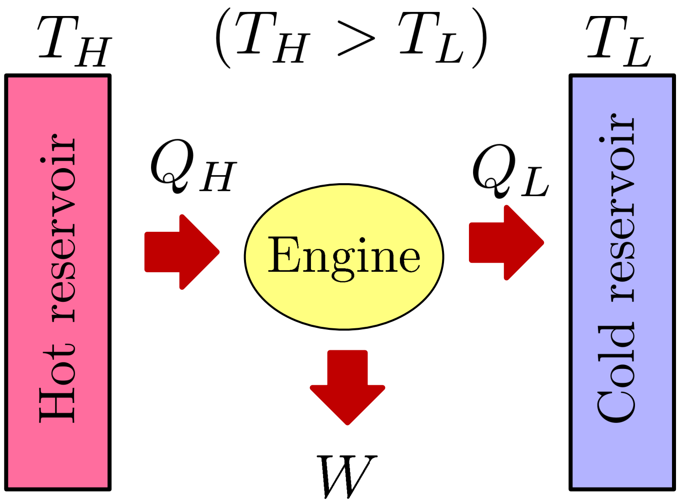

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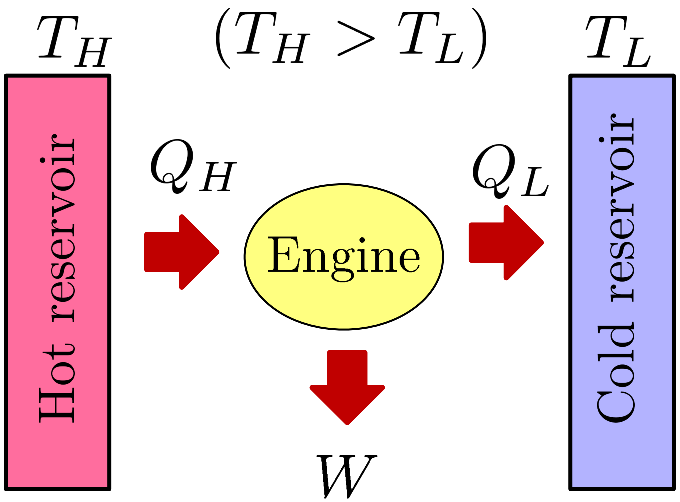

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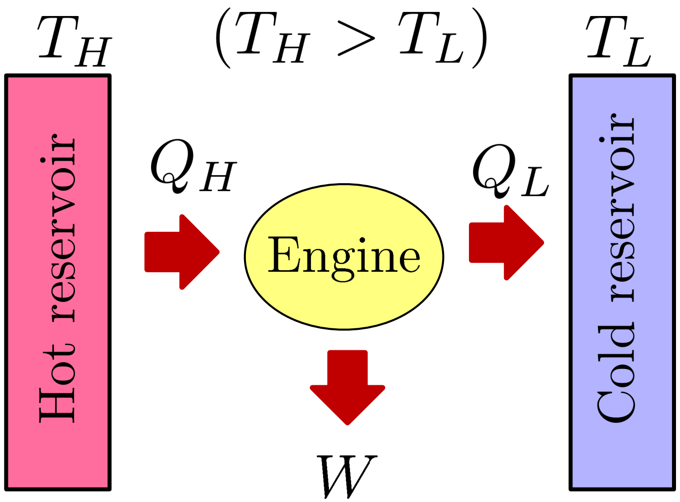

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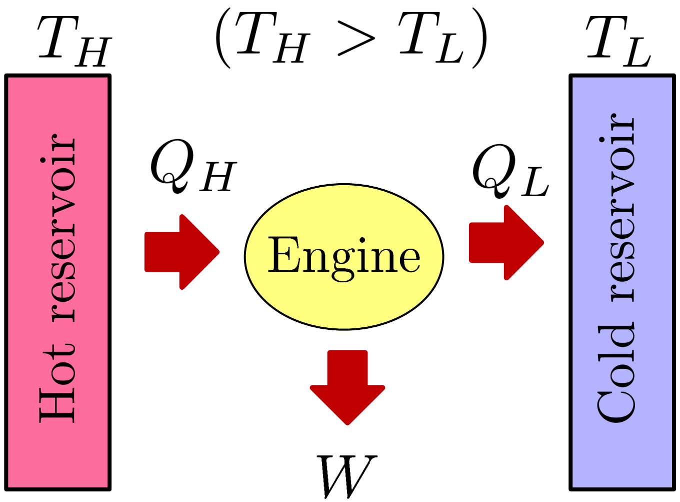

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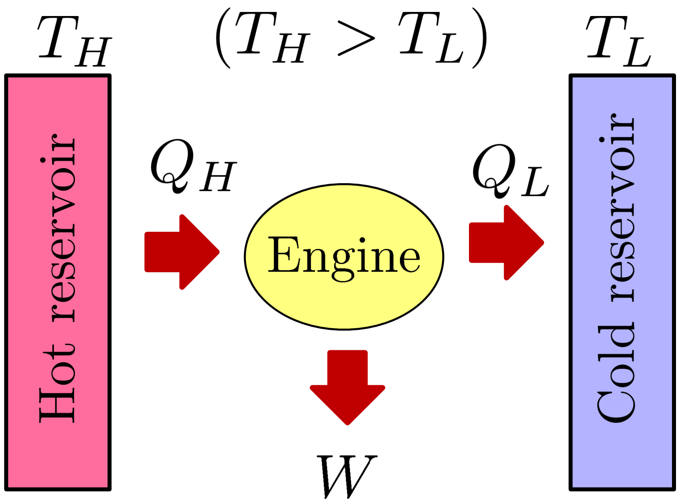

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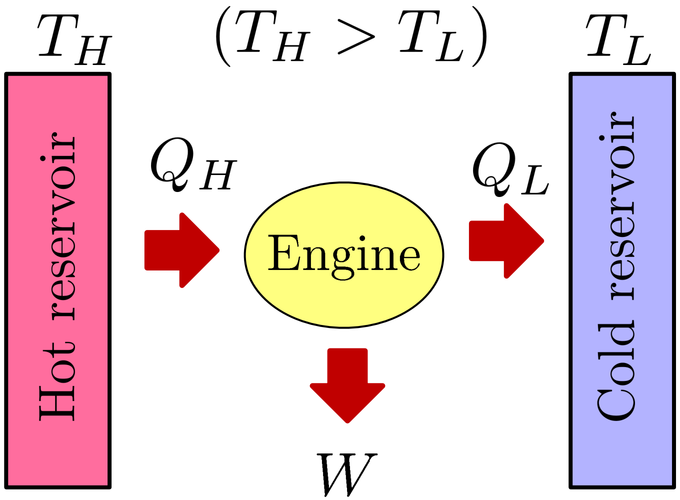

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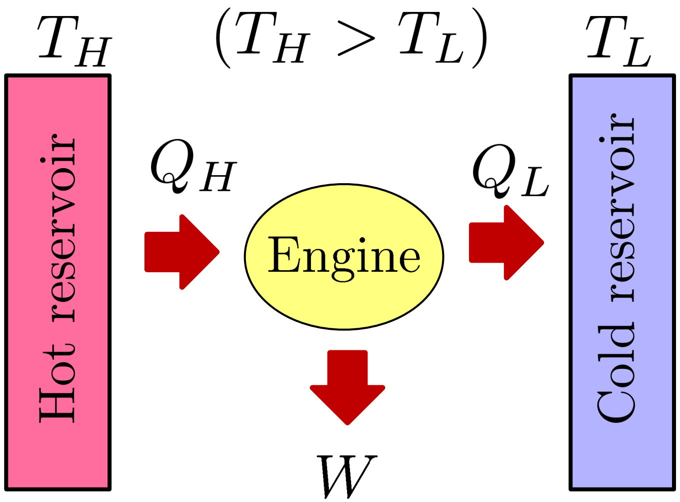

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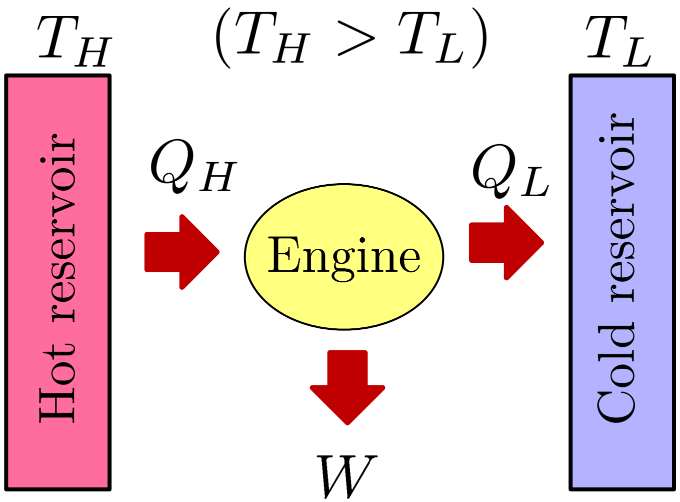

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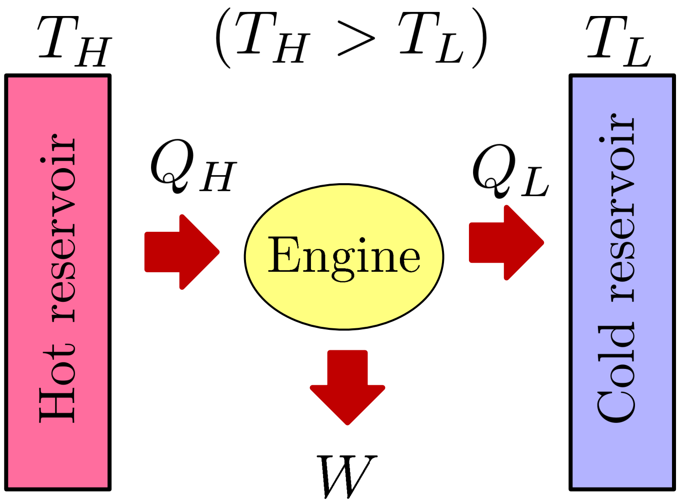

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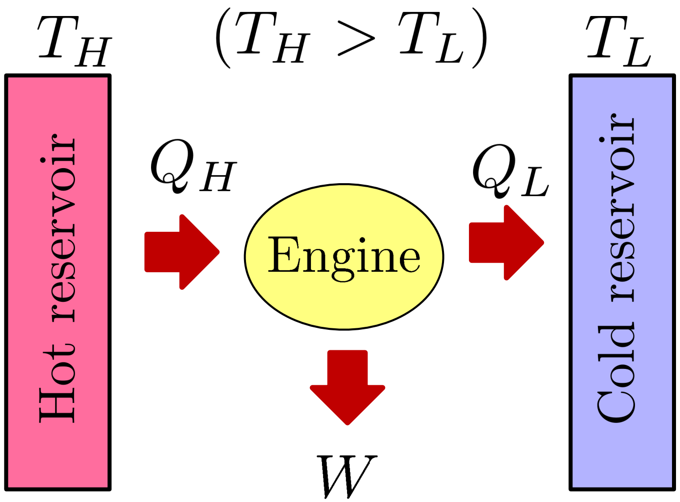

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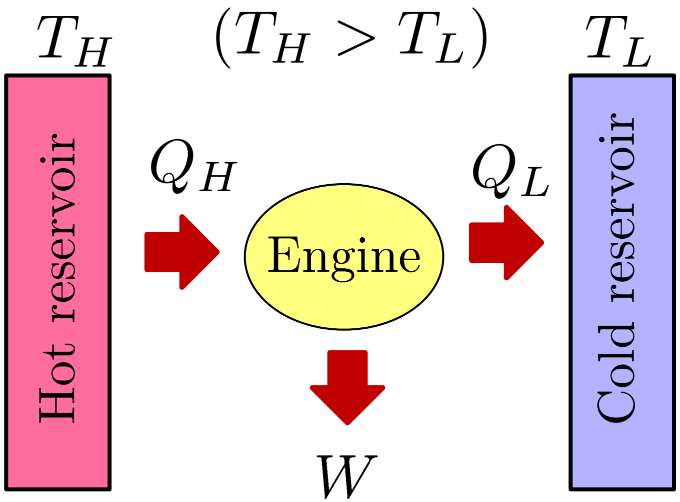

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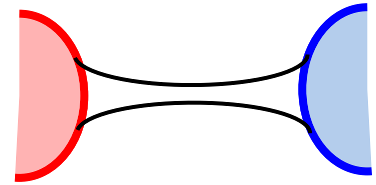

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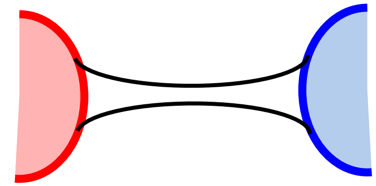

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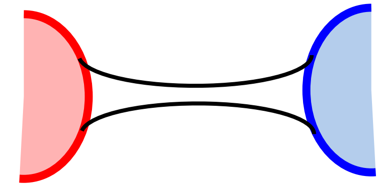

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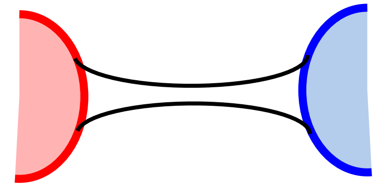

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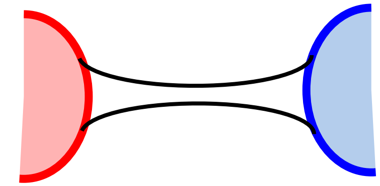

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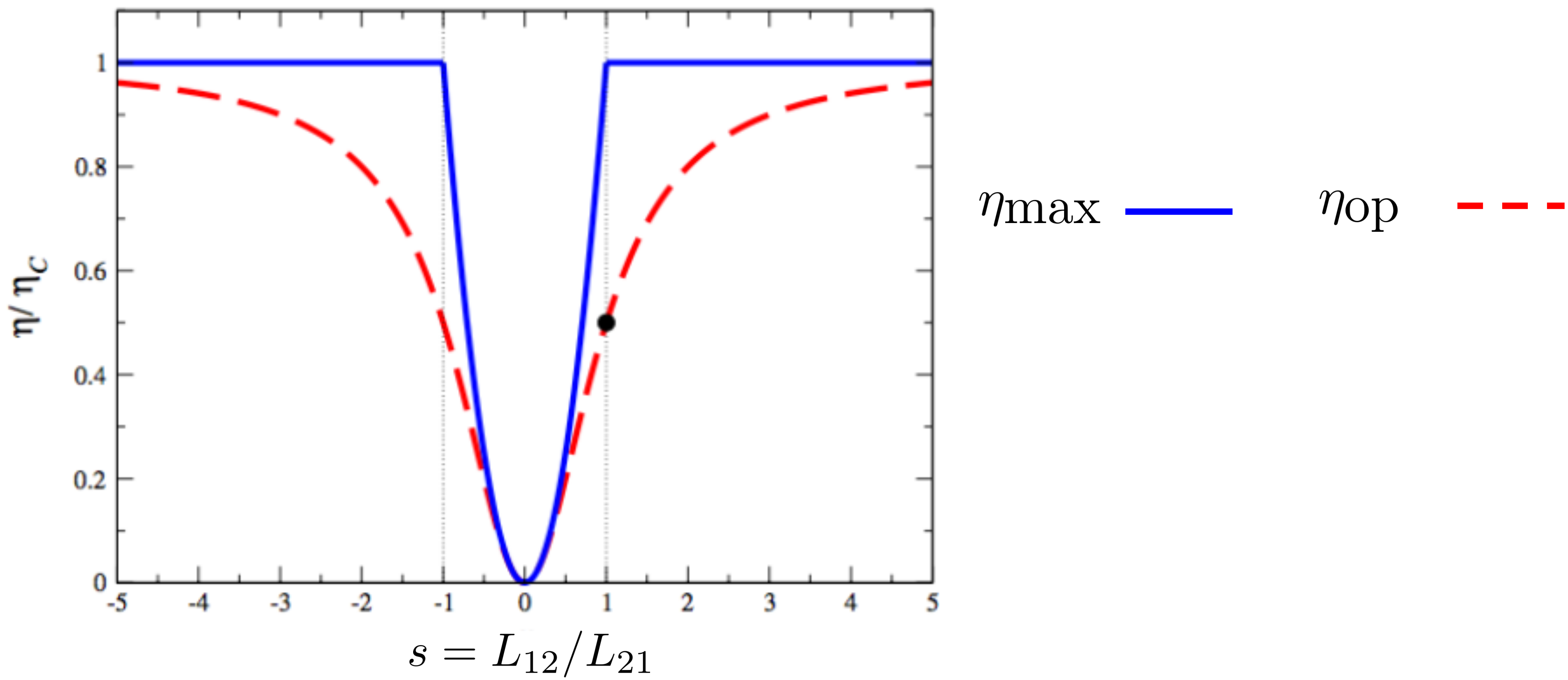

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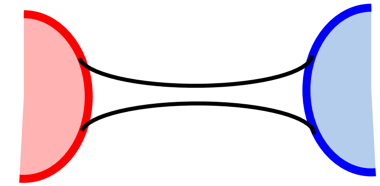

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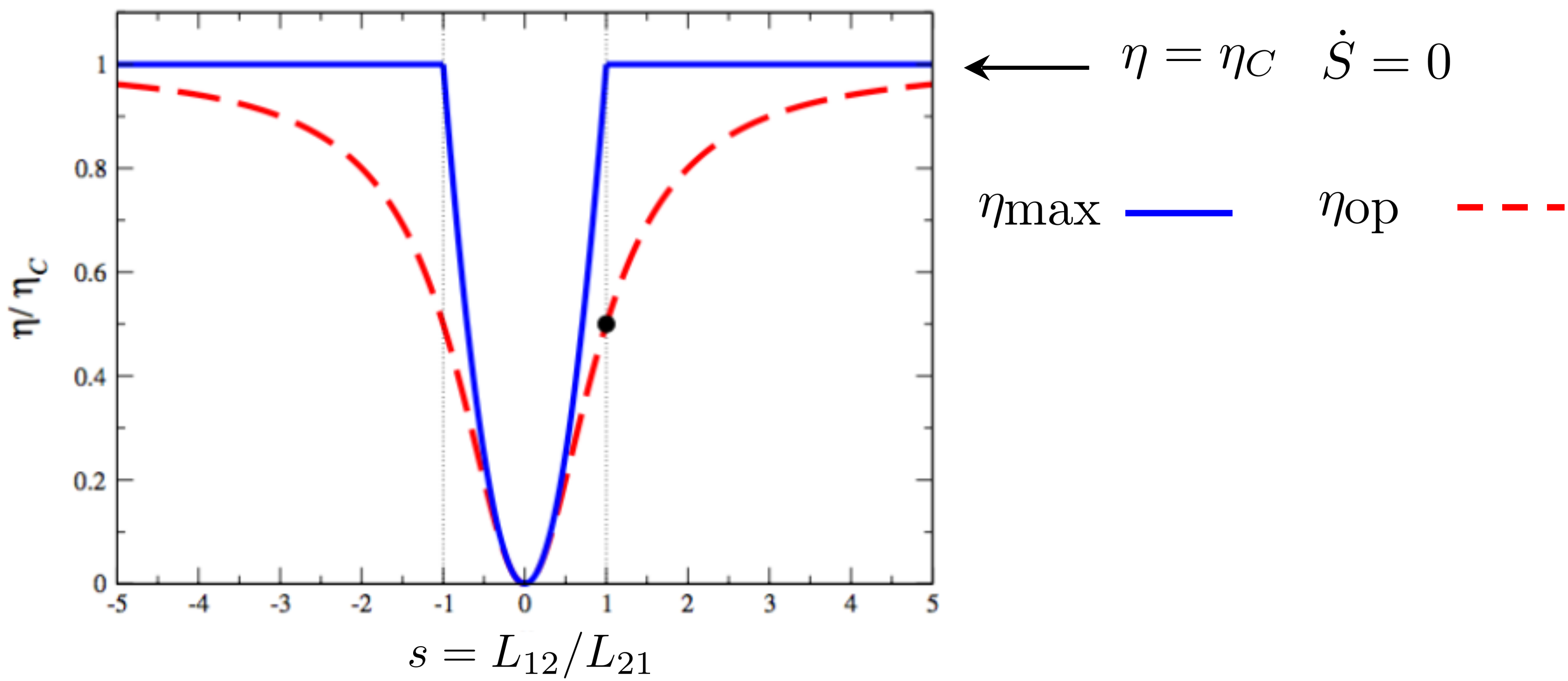

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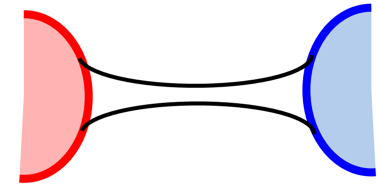

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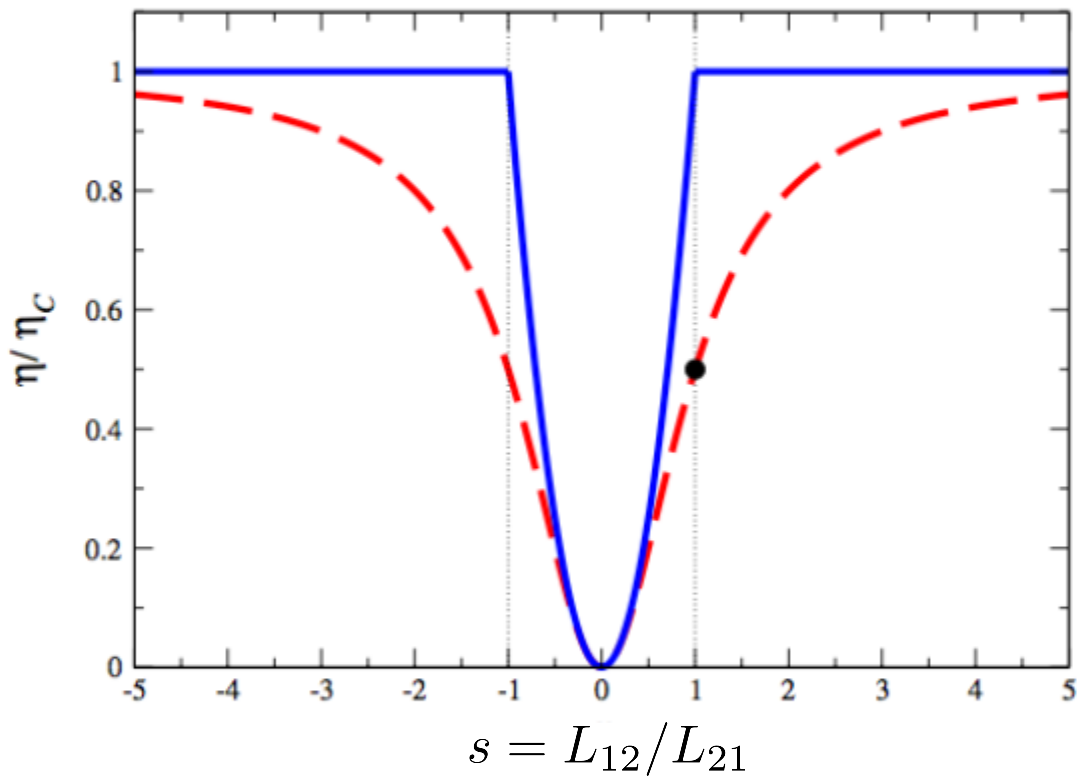

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ηmax ——— ηop - - -

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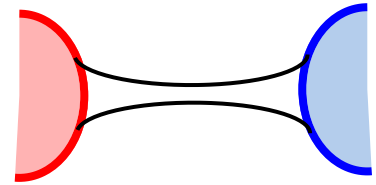

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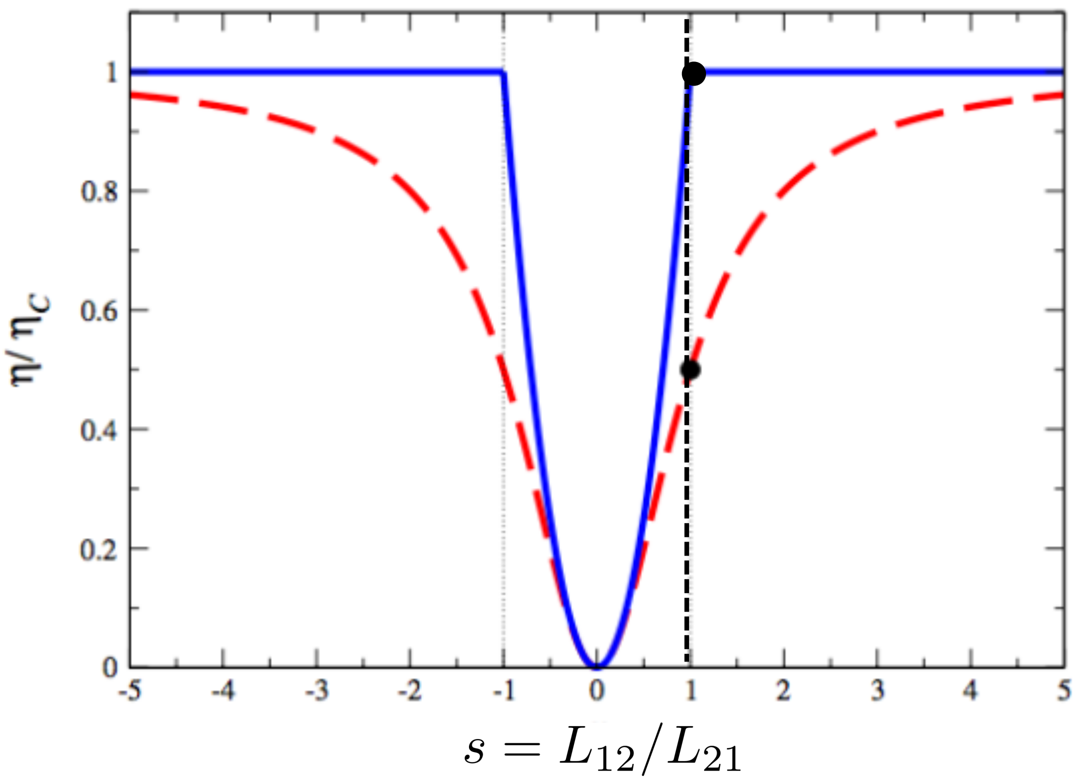

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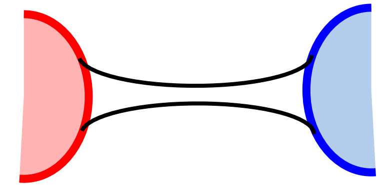

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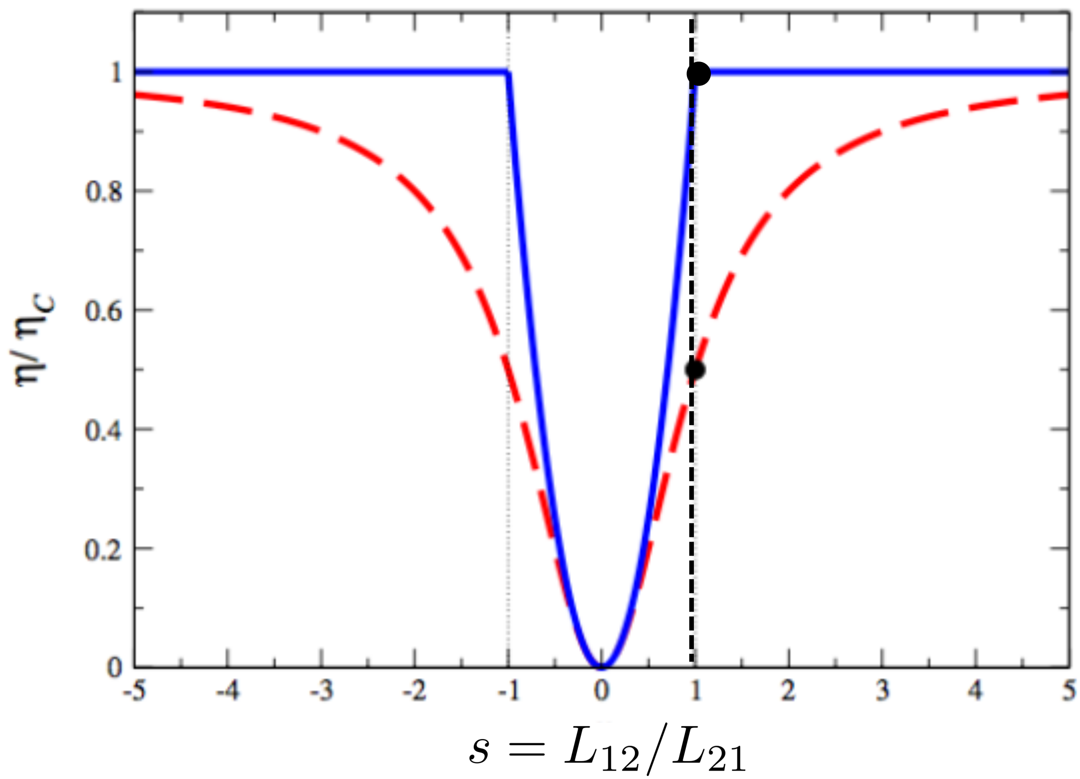

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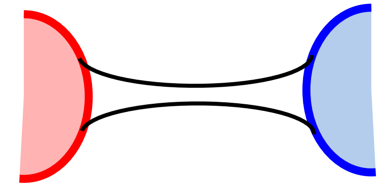

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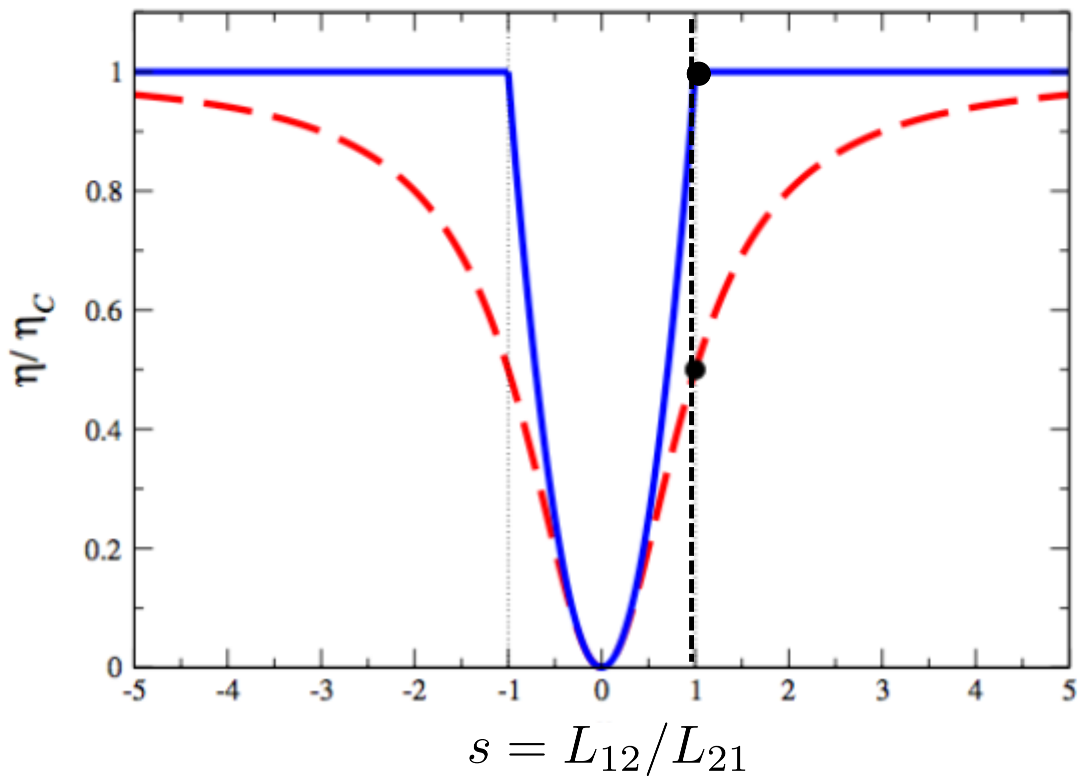

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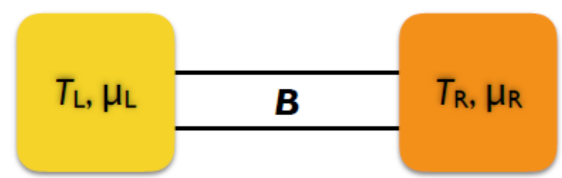

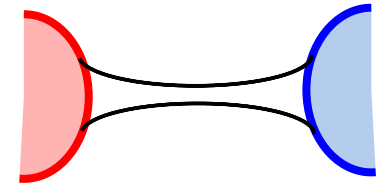

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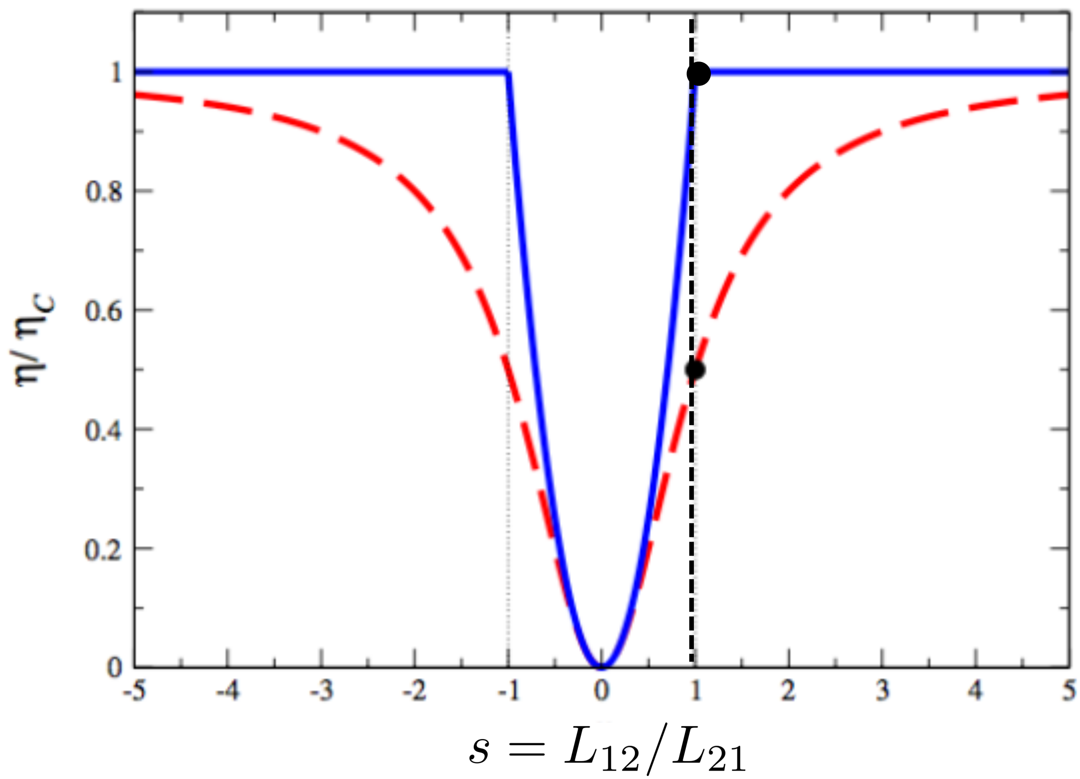

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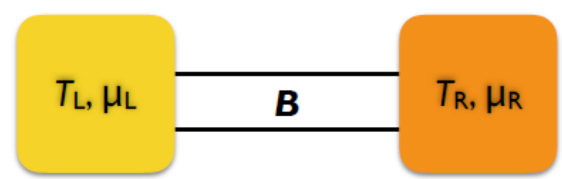

symmetric

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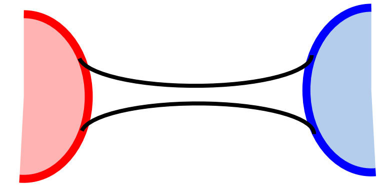

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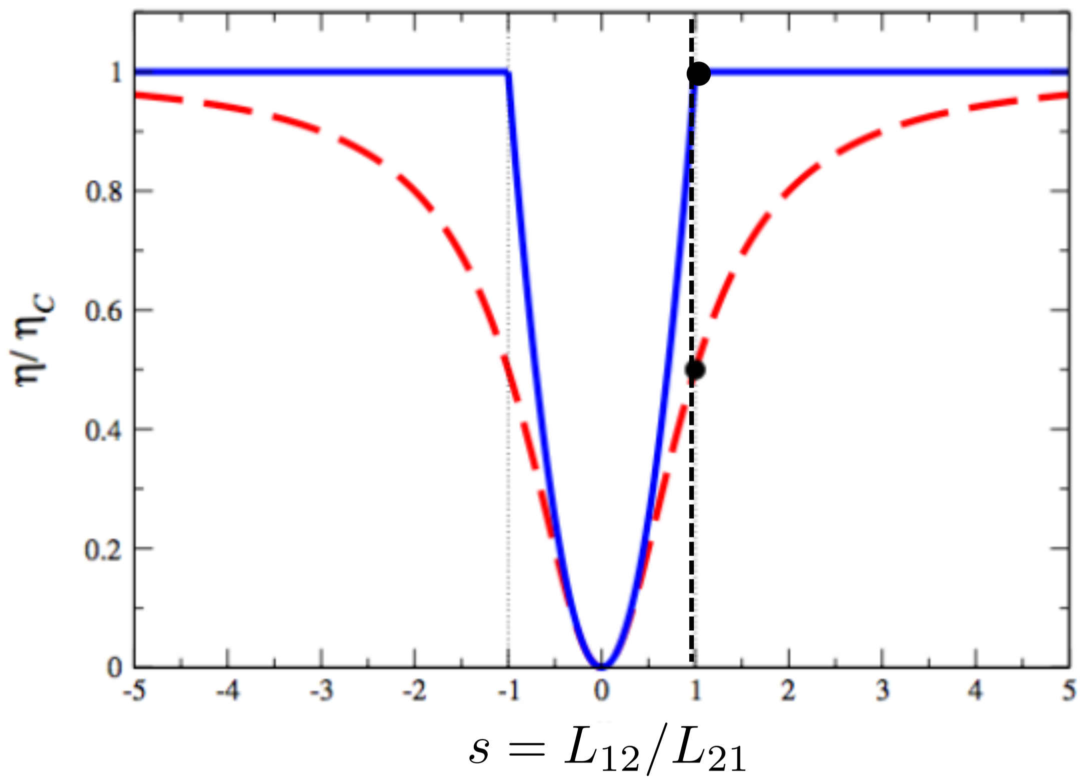
Onsager symmetry: $L_{12} = L_{21}$ $\Leftarrow$ microscopic TR symmetry

Onsager-Casimir: $L_{12}(B) = L_{21}(-B)$ (B: magnetic field) [Onsager, PR(1931)]

$L_{12}(B) \neq L_{21}(B)$

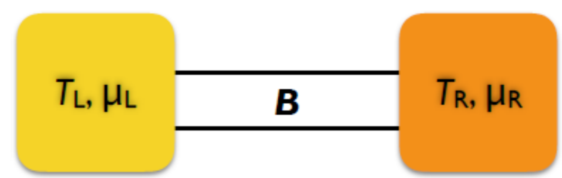


$\eta = \eta_C$ $\dot{S} = 0$

$\dot{W} \sim (s^2 - 1)$

ηmax ηop

- explicit model ?
  - two-terminal transport



symmetric

$L_{12}(B) = L_{21}(B)$

  - "three"-terminal transport

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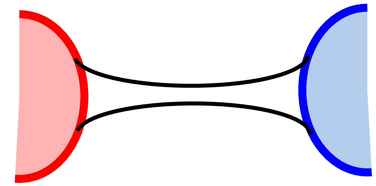

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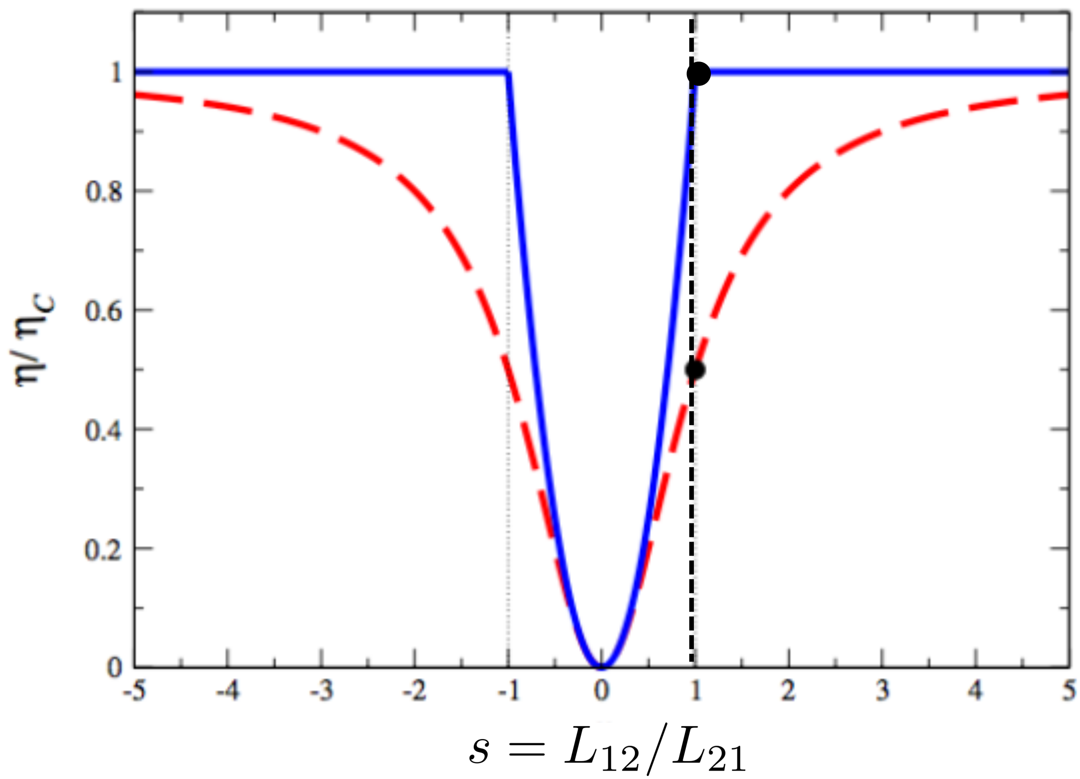

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ηmax —— ηop - - -

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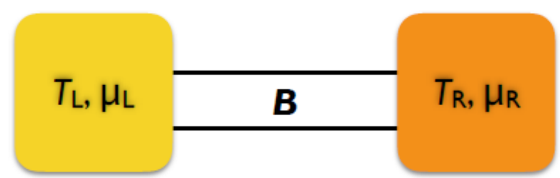

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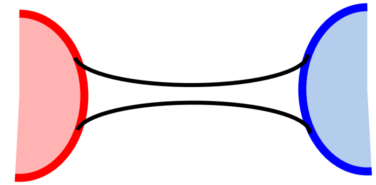

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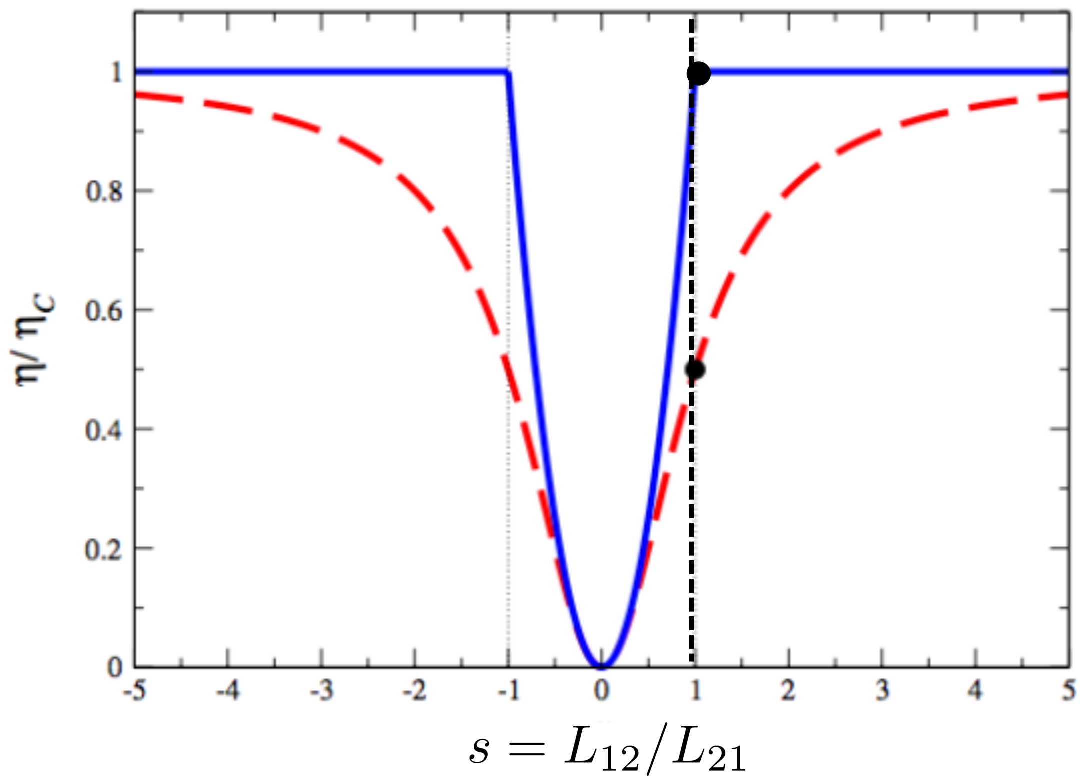

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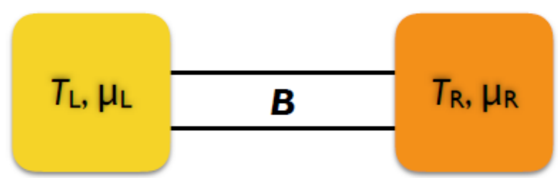

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$\mathbf{L}(B) \neq \mathbf{L}^{\mathrm{T}}(B)$ $\dot{S} > 0$ no dream engine

¶ *universal* trade-off relations between power and efficiency

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- $\dot{W} \leq \frac{\bar{\Theta}\eta}{T_L}(\eta_C - \eta)$ with $\bar{\Theta} = \frac{2\bar{\gamma}\bar{K}}{\bar{T}\bar{m}}$ [Shiraisi,Saito,Tasaki, PRL(2016)]

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Linear Brownian particle engine

Linear Brownian particle engine

¶ underdamped Brownian dynamics

$$m\dot{\mathbf{v}} = -\mathsf{K}\mathbf{x} + \mathsf{F}_{nc}\mathbf{x} + \mathsf{B}\mathbf{v} - \Gamma\mathbf{v} + \boldsymbol{\xi}$$

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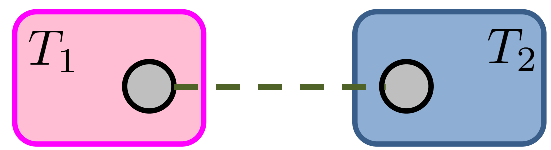

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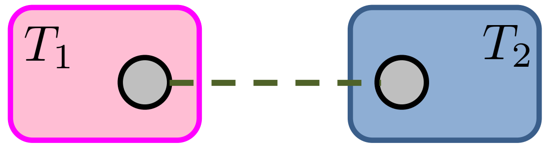

$$\mathsf{K} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

harmonic

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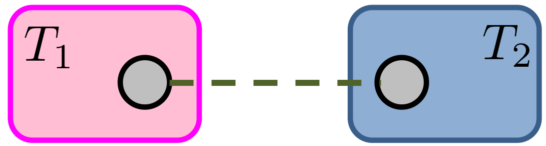

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harmonic   torque

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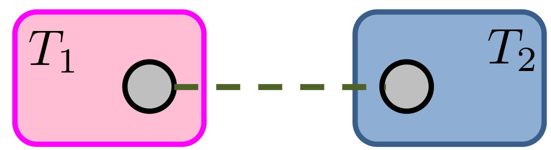

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harmonic torque Lorentz

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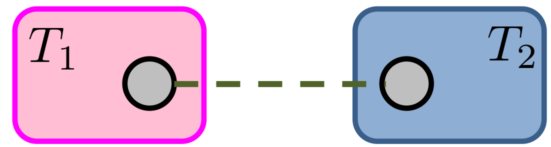

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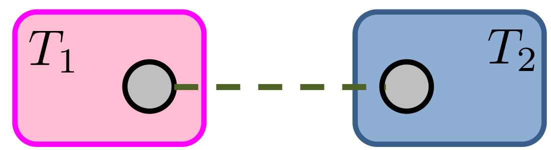

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harmonic torque Lorentz dissipation

- exactly solvable

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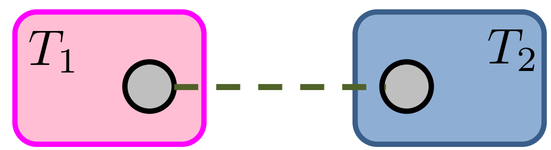

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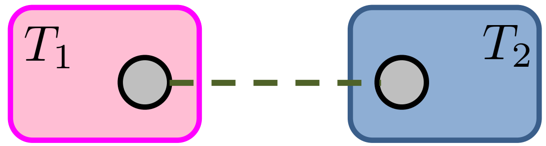

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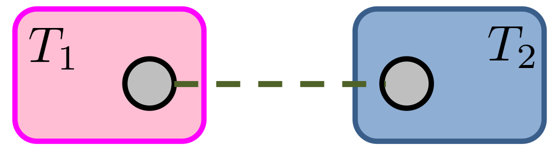

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– No dream engine

[Chun,Um,HP(2018)]

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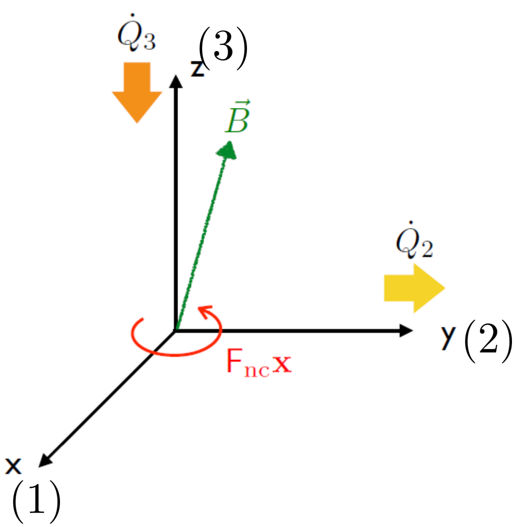

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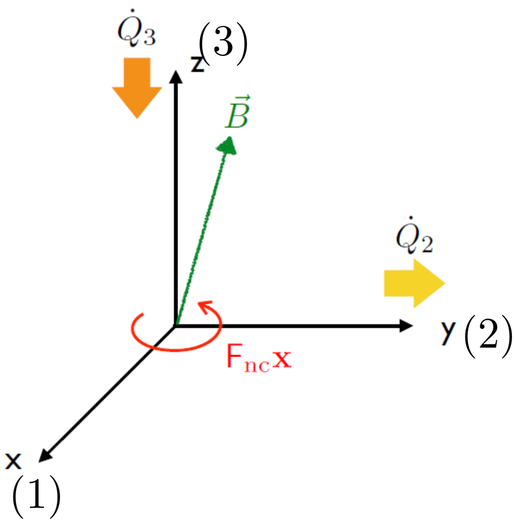
$$\mathsf{\Gamma} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix} \qquad T_1 = 0$$



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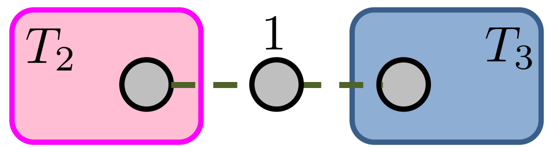
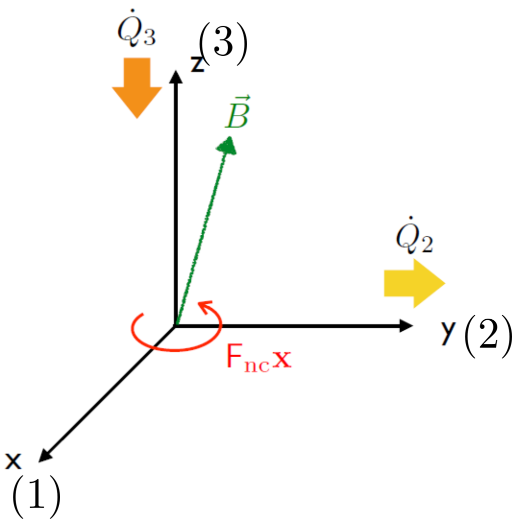
$$\mathsf{K} = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} \quad \mathsf{F}_{nc} = \begin{pmatrix} 0 & \epsilon & 0 \\ \delta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathsf{B} = \begin{pmatrix} 0 & B_3 & -B_2 \\ -B_3 & 0 & B_1 \\ B_2 & -B_1 & 0 \end{pmatrix}$$

$$\mathsf{\Gamma} = \begin{pmatrix} \color{red}{0} & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix} \qquad T_1 = \color{red}{0}$$



⋆ two-terminal engine



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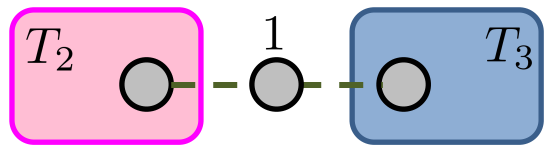

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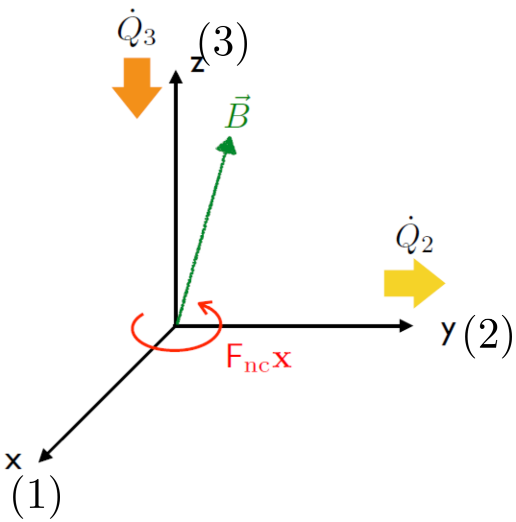
● exactly solvable



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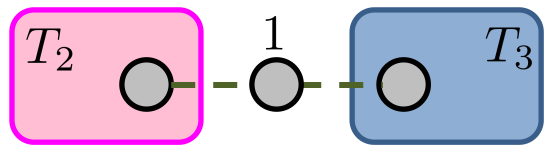
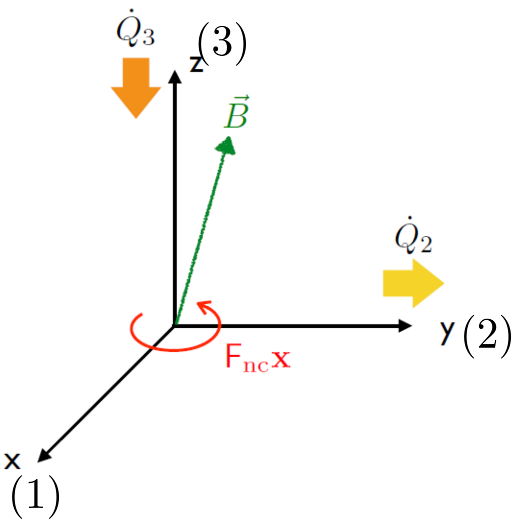


⋆ two-terminal engine

• exactly solvable

$C_0 = B_y^2 + B_z^2$
$C_1 = KC_0 + m\epsilon^2$
$C_2 = B_y^2 + B_z^2 + 2\gamma^2$
$C_3 = KC_2 + m\epsilon^2$
$\rightarrow$ even function in B



$(B_x = 0)$

$$L_{12} = \frac{\epsilon B_y^2 T_2[\gamma(2KC_3 - \epsilon^2 C_2) - \epsilon C_1 B_z]}{C_1(C_3^2 - \gamma^2\epsilon^2 C_2)}$$

$$L_{21} = \frac{\epsilon B_y^2 T_2[\gamma(2KC_3 - \epsilon^2 C_2) + \epsilon C_1 B_z]}{C_1(C_3^2 - \gamma^2\epsilon^2 C_2)}$$

$L_{ij}(\mathbf{B}) \neq L_{ji}(\mathbf{B})$

$L_{ij}(\mathbf{B}) = L_{ji}(-\mathbf{B})$

• Onsager symmetry broken!

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  - dream engine comes true ??   not yet !

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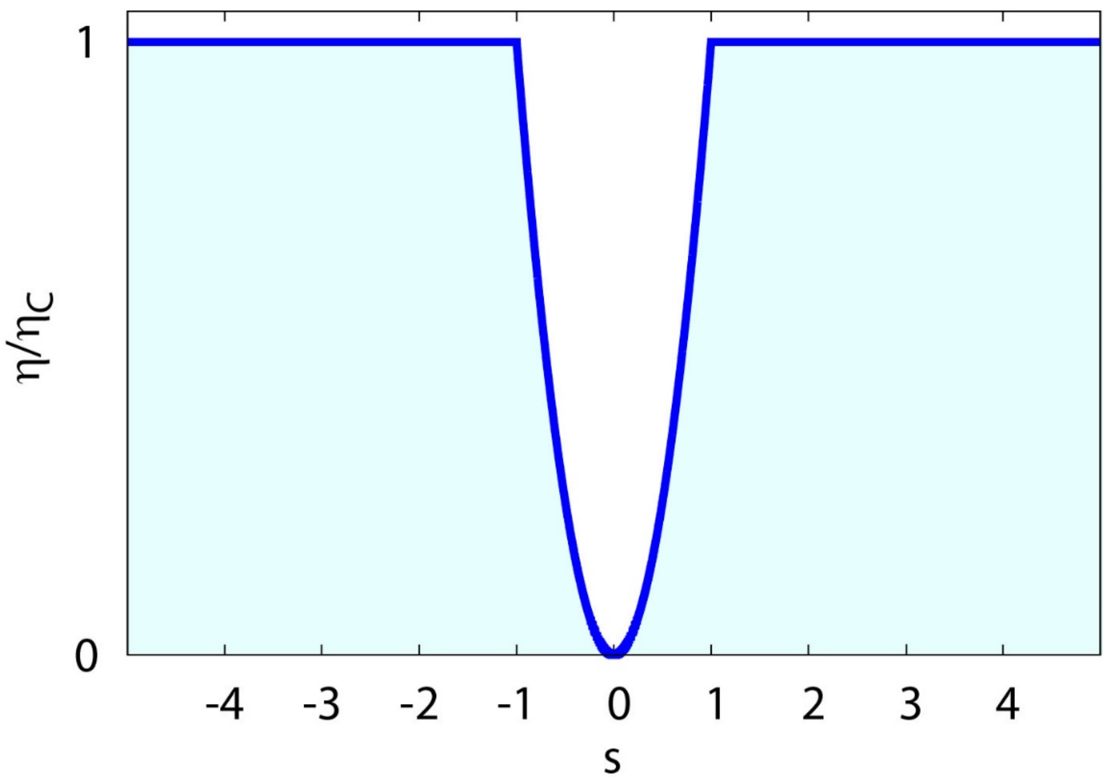

$\eta = \eta_C \quad \dot{W} > 0$

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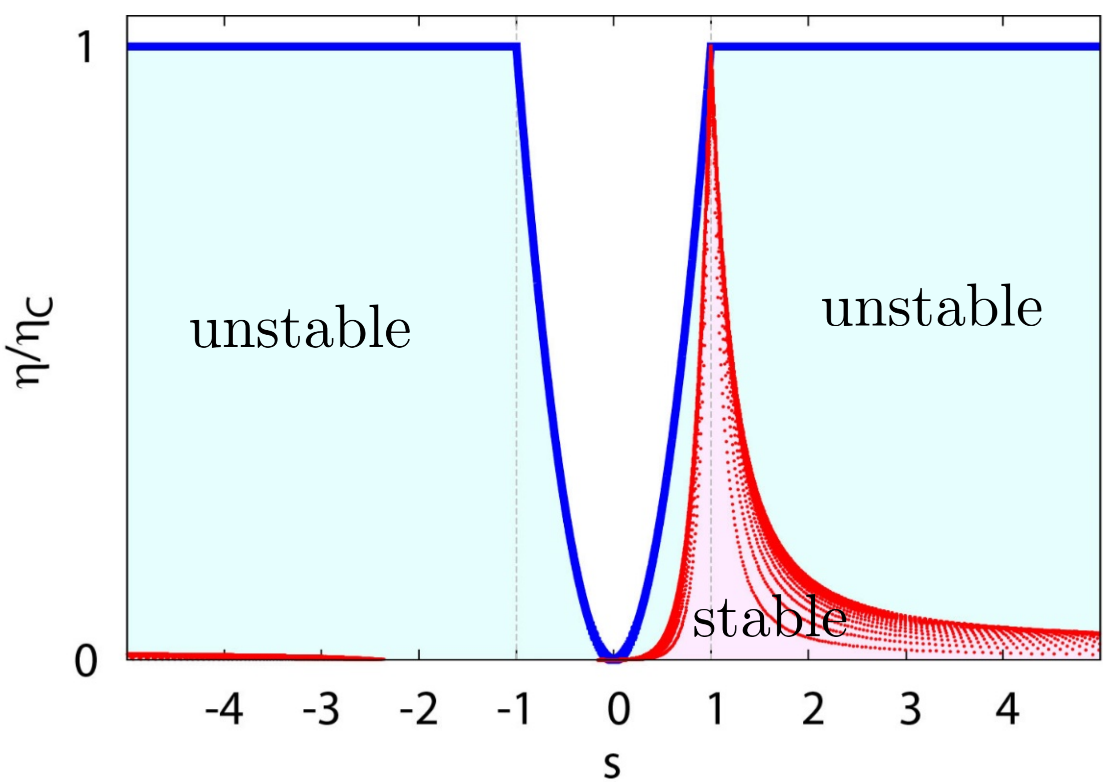

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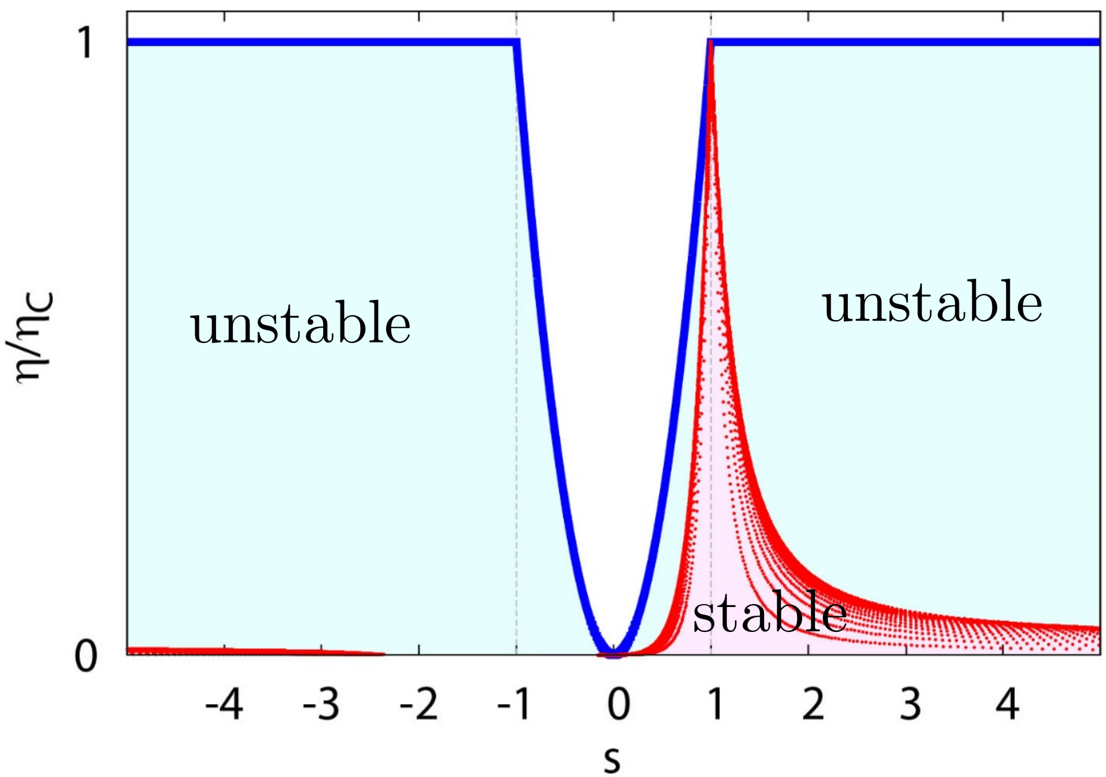

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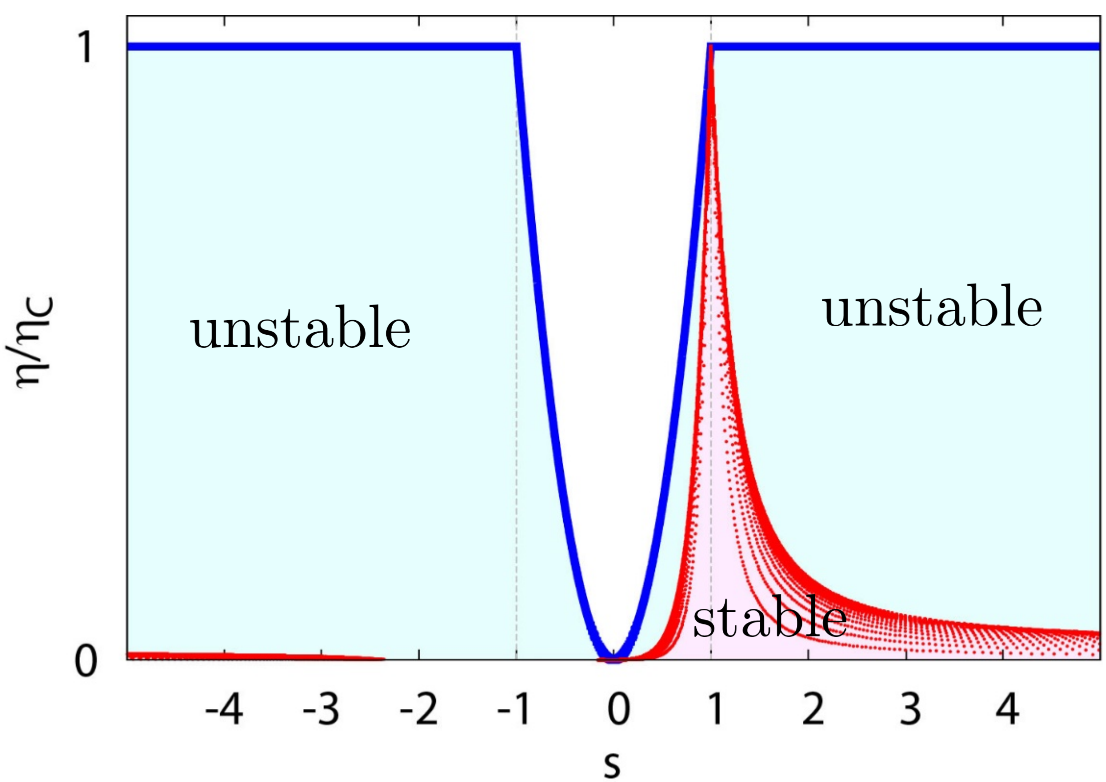

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- No dream engine so far!

Is **dream engine possible ??**

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$\star$ a two-terminal engine with broken Onsager symmetry

$\star$ stability requirement makes it impossible.

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Jae Sung Lee [Thursday 11:00]