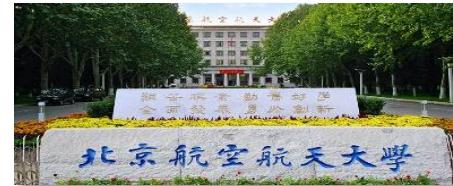




北京航空航天大學  
BEIHANG UNIVERSITY



# Is there new matter of nuclei like in Nature?

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# Contents

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- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$  and  $Ds1(2460)$  as  $DK/D^*K$  molecules: theory & lattice
- $DDK$  molecule:  $R^{++}(4140)$
- $D\bar{D}^*K$  and  $D\bar{D}K$  molecules:  $K^*(4307)$  and  $Kc(4180)$
- Where to search for these 3-body molecules
- Summary and outlook

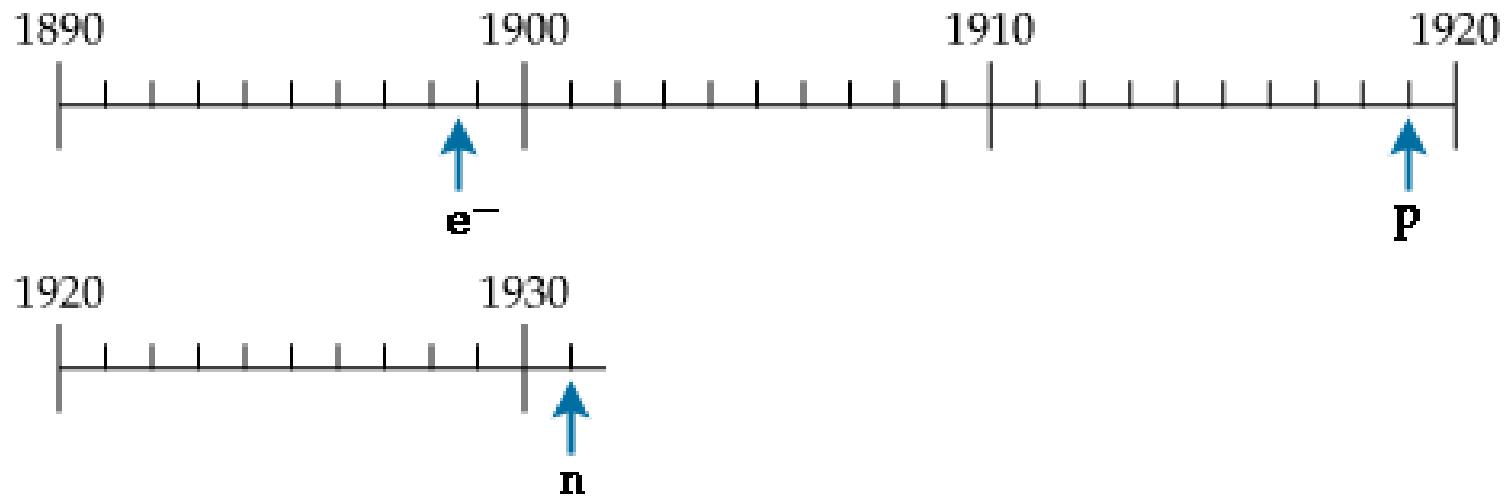
# Contents

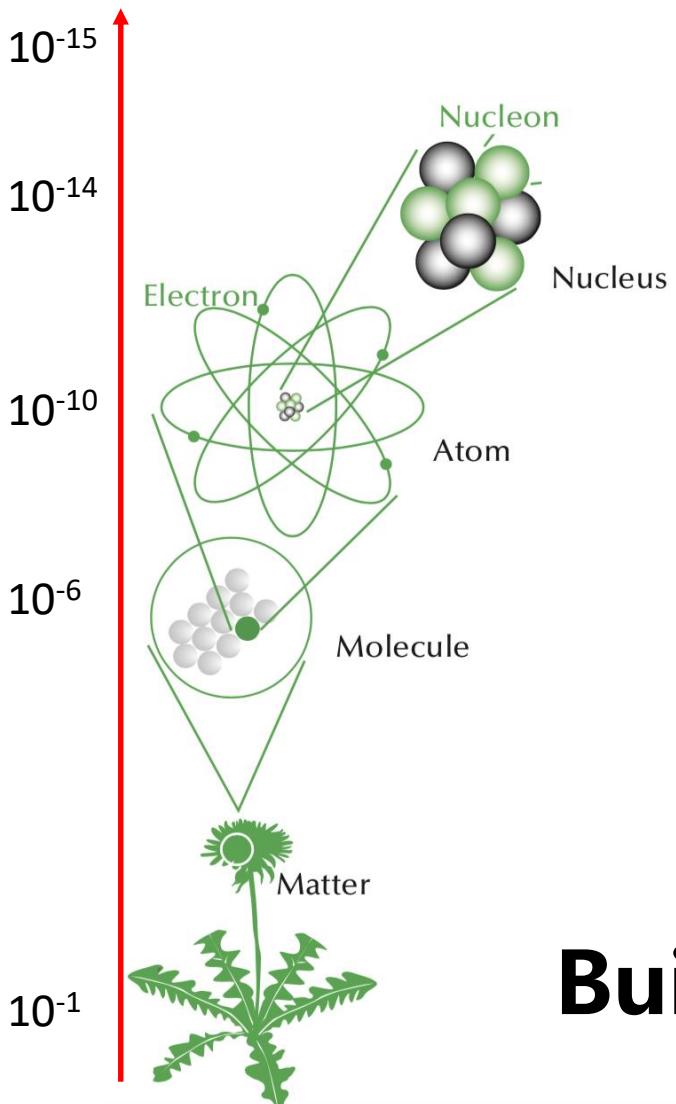
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# The world was once very simple

Particles discovered before 1932





IUPAC Periodic Table of the Elements

<b>1</b>	<b>H</b>	<b>hydrogen</b>	<b>1.008</b>	<b>2</b>
<b>3</b>	<b>Li</b>	<b>lithium</b>	<b>(6.938, 6.997)</b>	<b>4</b>
				<b>Be</b>
				<b>beryllium</b>
			<b>9.0122</b>	
<b>11</b>	<b>Na</b>	<b>sodium</b>	<b>(22.990, 22.997)</b>	<b>12</b>
				<b>Mg</b>
				<b>magnesium</b>
			<b>(24.304, 24.307)</b>	
<b>19</b>	<b>K</b>	<b>potassium</b>	<b>(39.098, 40.0784)</b>	<b>20</b>
	<b>Ca</b>	<b>calcium</b>		<b>Sc</b>
				<b>scandium</b>
			<b>44.956</b>	<b>21</b>
				<b>Ti</b>
				<b>titanium</b>
			<b>47.867</b>	<b>22</b>
				<b>V</b>
				<b>vanadium</b>
			<b>50.942</b>	<b>23</b>
				<b>Cr</b>
				<b>chromium</b>
			<b>51.996</b>	<b>24</b>
				<b>Mn</b>
				<b>manganese</b>
			<b>54.938</b>	<b>25</b>
				<b>Fe</b>
				<b>iron</b>
			<b>55.845(2)</b>	<b>26</b>
				<b>Co</b>
				<b>cobalt</b>
			<b>58.933</b>	<b>27</b>
				<b>Ni</b>
				<b>nickel</b>
			<b>58.693</b>	<b>28</b>
				<b>Cu</b>
				<b>copper</b>
			<b>63.546(3)</b>	<b>29</b>
				<b>Zn</b>
				<b>zinc</b>
			<b>65.38(2)</b>	<b>30</b>
				<b>Ga</b>
				<b>gallium</b>
			<b>69.723</b>	<b>31</b>
				<b>Ge</b>
				<b>germanium</b>
			<b>72.63(8)</b>	<b>32</b>
				<b>As</b>
				<b>arsenic</b>
			<b>74.922</b>	<b>33</b>
				<b>Se</b>
				<b>selenium</b>
			<b>78.91(8)</b>	<b>34</b>
				<b>Cl</b>
				<b>chlorine</b>
			<b>39.46(5), 35.45(7)</b>	<b>35</b>
				<b>Ar</b>
				<b>argon</b>
			<b>39.982</b>	<b>36</b>
				<b>Kr</b>
				<b>krypton</b>
			<b>83.78(2)</b>	
<b>37</b>	<b>Rb</b>	<b>rubidium</b>	<b>85.468</b>	<b>38</b>
	<b>Sr</b>	<b>strontium</b>	<b>87.62</b>	<b>39</b>
			<b>84.906</b>	<b>Y</b>
				<b>yttrium</b>
			<b>91.224(2)</b>	<b>40</b>
				<b>Zr</b>
				<b>zirconium</b>
			<b>91.992</b>	<b>41</b>
				<b>Nb</b>
				<b>niobium</b>
			<b>92.906</b>	<b>42</b>
				<b>Mo</b>
				<b>molybdenum</b>
			<b>95.95</b>	<b>43</b>
				<b>Tc</b>
				<b>technetium</b>
			<b>101.97(2)</b>	<b>44</b>
				<b>Ru</b>
				<b>rhuthenium</b>
			<b>101.97(1)</b>	<b>45</b>
				<b>Rh</b>
				<b>rhodium</b>
			<b>102.91</b>	<b>46</b>
				<b>Pd</b>
				<b>paladium</b>
			<b>106.42</b>	<b>47</b>
				<b>Ag</b>
				<b>silver</b>
			<b>107.67</b>	<b>48</b>
				<b>Cd</b>
				<b>cadmium</b>
			<b>112.41</b>	<b>49</b>
				<b>In</b>
				<b>indium</b>
			<b>114.82</b>	<b>50</b>
				<b>Sn</b>
				<b>tin</b>
			<b>116.71</b>	<b>51</b>
				<b>Sb</b>
				<b>antimony</b>
			<b>121.76</b>	<b>52</b>
				<b>Te</b>
				<b>tellurium</b>
			<b>127.65(3)</b>	<b>53</b>
				<b>I</b>
				<b>iodine</b>
			<b>131.29</b>	<b>54</b>
				<b>Xe</b>
				<b>xenon</b>
			<b>131.97</b>	
<b>55</b>	<b>Cs</b>	<b>caesium</b>	<b>132.61</b>	<b>56</b>
	<b>Ba</b>	<b>barium</b>	<b>137.33</b>	<b>57</b>
				<b>Hf</b>
				<b>hafnium</b>
			<b>178.49(2)</b>	<b>58</b>
				<b>Ta</b>
				<b>tautonium</b>
			<b>180.95</b>	<b>59</b>
				<b>W</b>
				<b> tungsten</b>
			<b>183.84</b>	<b>60</b>
				<b>Os</b>
				<b>osmium</b>
			<b>186.21</b>	<b>61</b>
				<b>Pm</b>
				<b>promethium</b>
			<b>191.23(3)</b>	<b>62</b>
				<b>Sm</b>
				<b>samarium</b>
			<b>195.96</b>	<b>63</b>
				<b>Eu</b>
				<b>europium</b>
			<b>197.2(3)</b>	<b>64</b>
				<b>Gd</b>
				<b>gadolinium</b>
			<b>197.93</b>	<b>65</b>
				<b>Tb</b>
				<b>terbium</b>
			<b>202.50</b>	<b>66</b>
				<b>Dy</b>
				<b>dysprosium</b>
			<b>202.50</b>	<b>67</b>
				<b>Ho</b>
				<b>holmium</b>
			<b>164.93</b>	<b>68</b>
				<b>Er</b>
				<b>erbium</b>
			<b>168.93</b>	<b>69</b>
				<b>Tm</b>
				<b>thulium</b>
			<b>173.05</b>	<b>70</b>
				<b>Yb</b>
				<b>ytterbium</b>
			<b>174.97</b>	<b>71</b>
				<b>Lu</b>
				<b>lutetium</b>
<b>89</b>	<b>Ac</b>	<b>actinium</b>	<b>232.04</b>	<b>57</b>
	<b>Th</b>	<b>thorium</b>	<b>232.04</b>	<b>58</b>
				<b>Pr</b>
				<b>praseodymium</b>
			<b>140.91</b>	<b>59</b>
				<b>Nd</b>
				<b>neodymium</b>
			<b>144.24</b>	<b>60</b>
				<b>Pm</b>
				<b>promethium</b>
			<b>150.36(2)</b>	<b>61</b>
				<b>Sm</b>
				<b>samarium</b>
			<b>151.96</b>	<b>63</b>
				<b>Eu</b>
				<b>europium</b>
			<b>157.2(3)</b>	<b>64</b>
				<b>Gd</b>
				<b>gadolinium</b>
			<b>158.93</b>	<b>65</b>
				<b>Tb</b>
				<b>terbium</b>
			<b>162.50</b>	<b>66</b>
				<b>Dy</b>
				<b>dysprosium</b>
			<b>164.93</b>	<b>67</b>
				<b>Ho</b>
				<b>holmium</b>
			<b>164.93</b>	<b>68</b>
				<b>Er</b>
				<b>erbium</b>
			<b>168.93</b>	<b>69</b>
				<b>Tm</b>
				<b>thulium</b>
			<b>173.05</b>	<b>70</b>
				<b>Yb</b>
				<b>ytterbium</b>
			<b>174.97</b>	<b>71</b>
				<b>Lu</b>
				<b>lutetium</b>

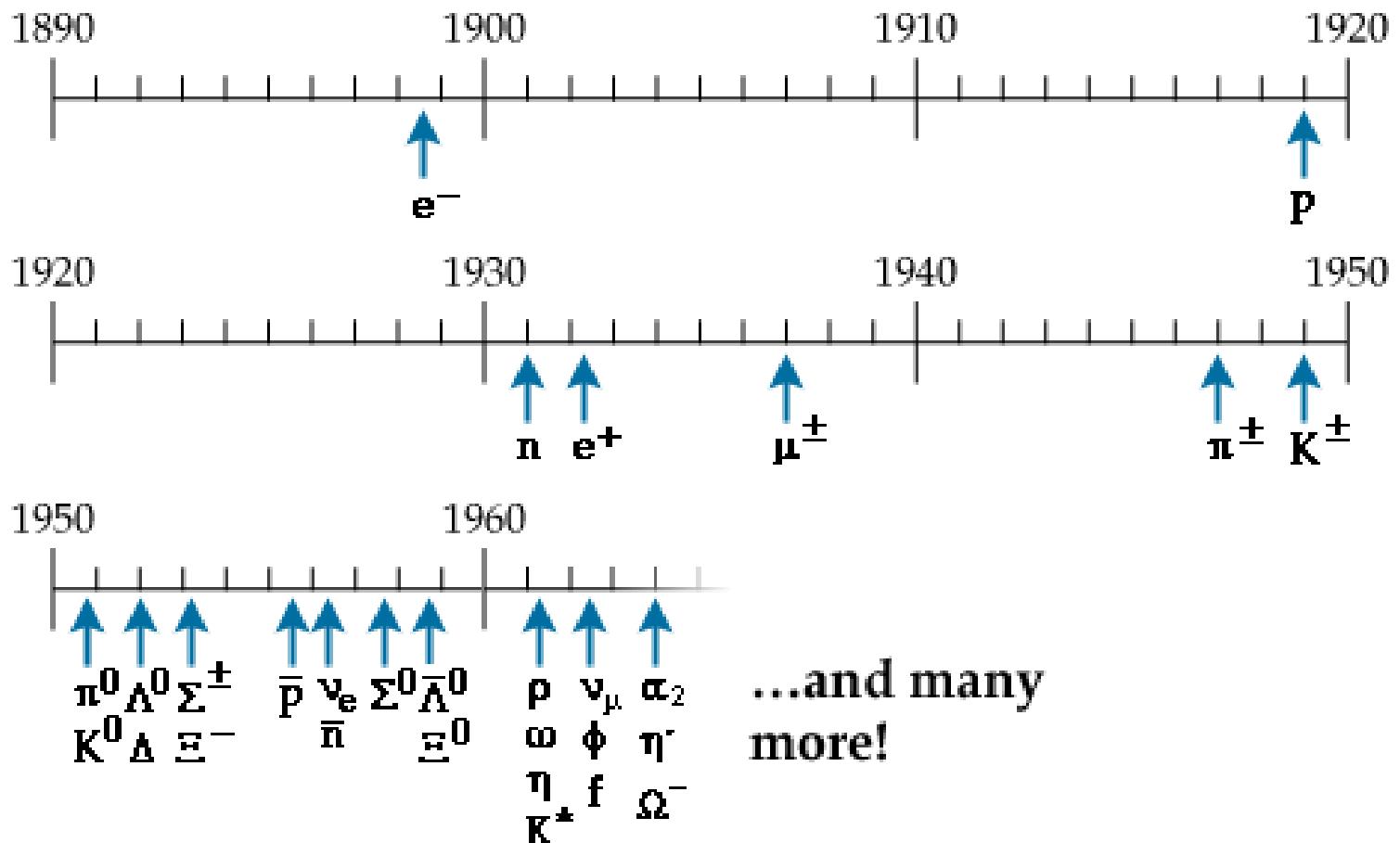
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For notes and updates to this table, see [www.iupac.org](http://www.iupac.org). This version is dated 1 December 2018.  
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# Building up the atomic world

# Many particles observed in the 1960' s

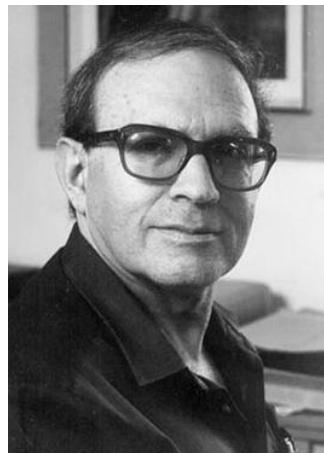


# Hadron spectroscopy—QM—QCD

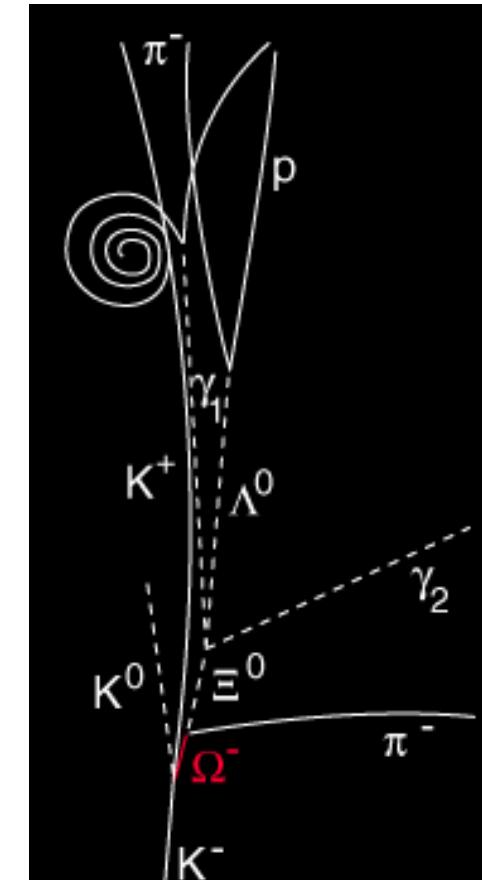
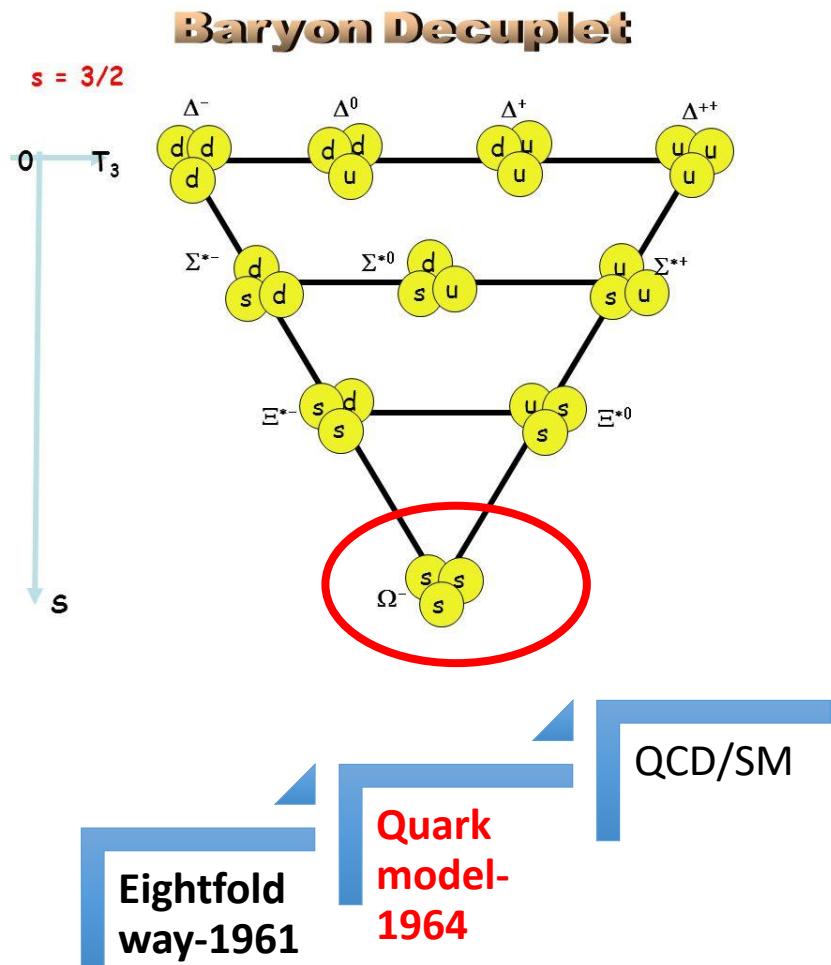
*Put an end to the then chaotic situation*



Murray Gell-Mann



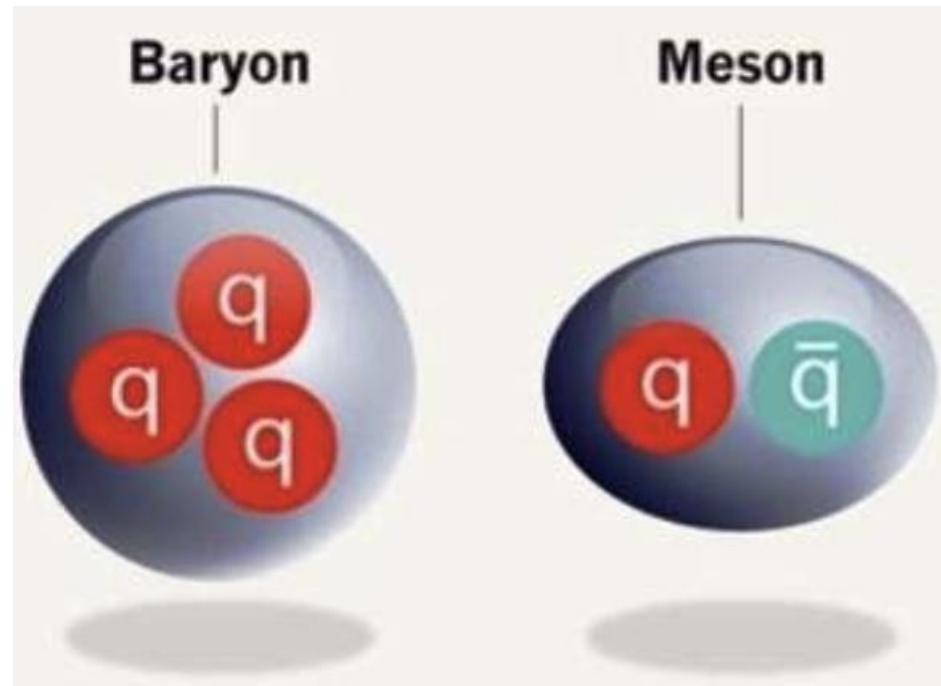
Yuval Ne'eman.



V. E. Barnes et al., Phys.  
Rev. Lett. 12, 204 (1964)

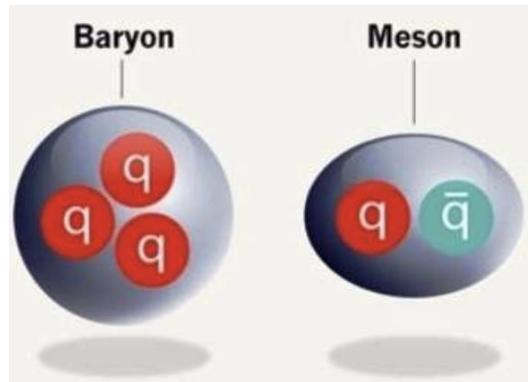
# Naive QM: hadron structure

---



# Beyond Naïve QM, more complicated structures allowed

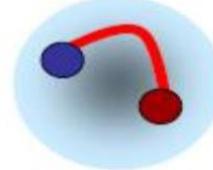
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In the naïve quark model

In principle,  
QCD allows

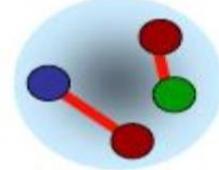
Hybrid



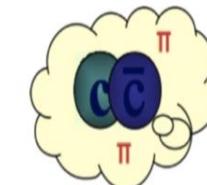
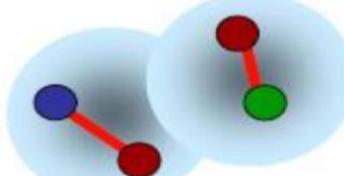
Glueball



Tetraquark

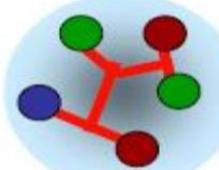


Hadronic molecule



Hadro-  
quarkonium

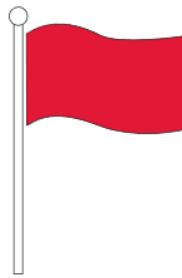
Pentaquark



# Naïve quark models more or less fine until 2003

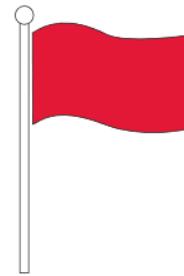
$\Lambda(1405)$ ,  $N^*(1535), \dots$   
 $f_0(500)$ ,  $f_0(980)$ ,  $a_0(980)$ , ...

# Beginning of a new era: 2003



LEPS, 0301020  
 $> 1100$

$\Theta^+ (1540)$



BaBar, 0304021  
 $> 900$

$D_{s0}^*(2317)$

**BABAR**

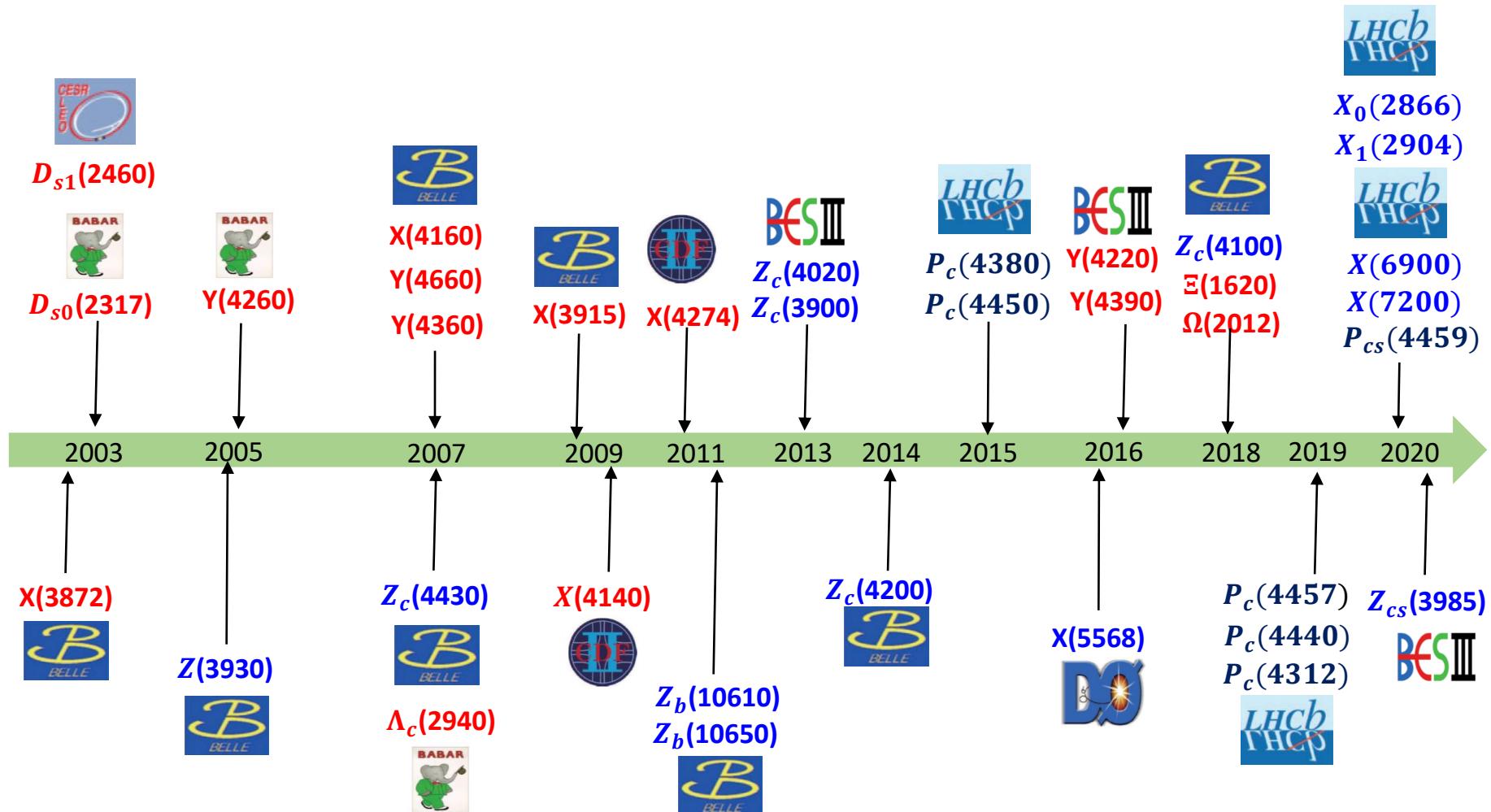


Belle, 0309032  
 $> 1800$

**X(3872)**



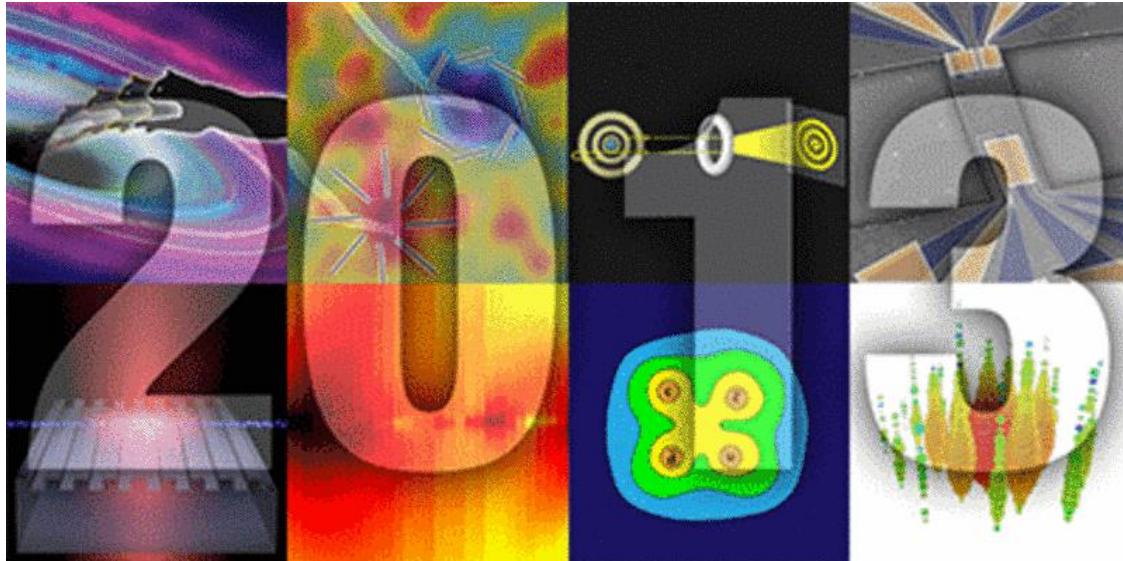
# Exotic mesons or baryons Tetraquark states Pentaquark states



# Highlights of the year

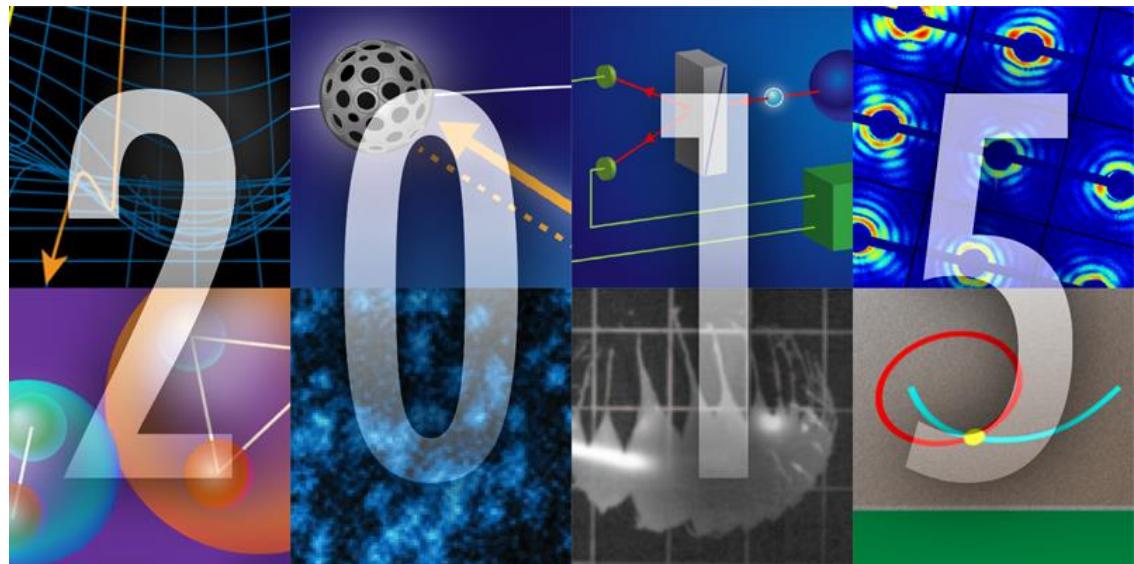
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the research covered in Physics that **really made waves in and beyond the physics community.**

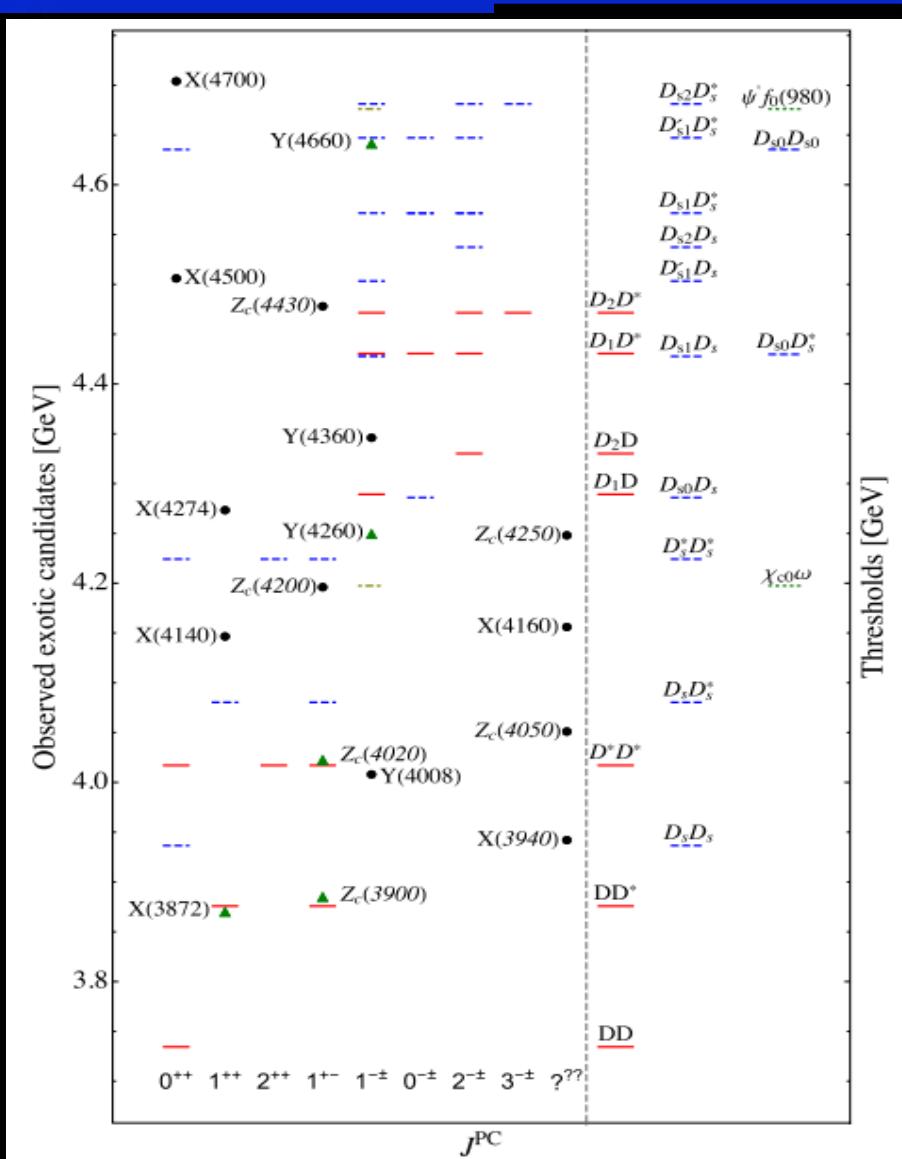


Four-Quark Matter/BESIII

Particle High Five/LHCb



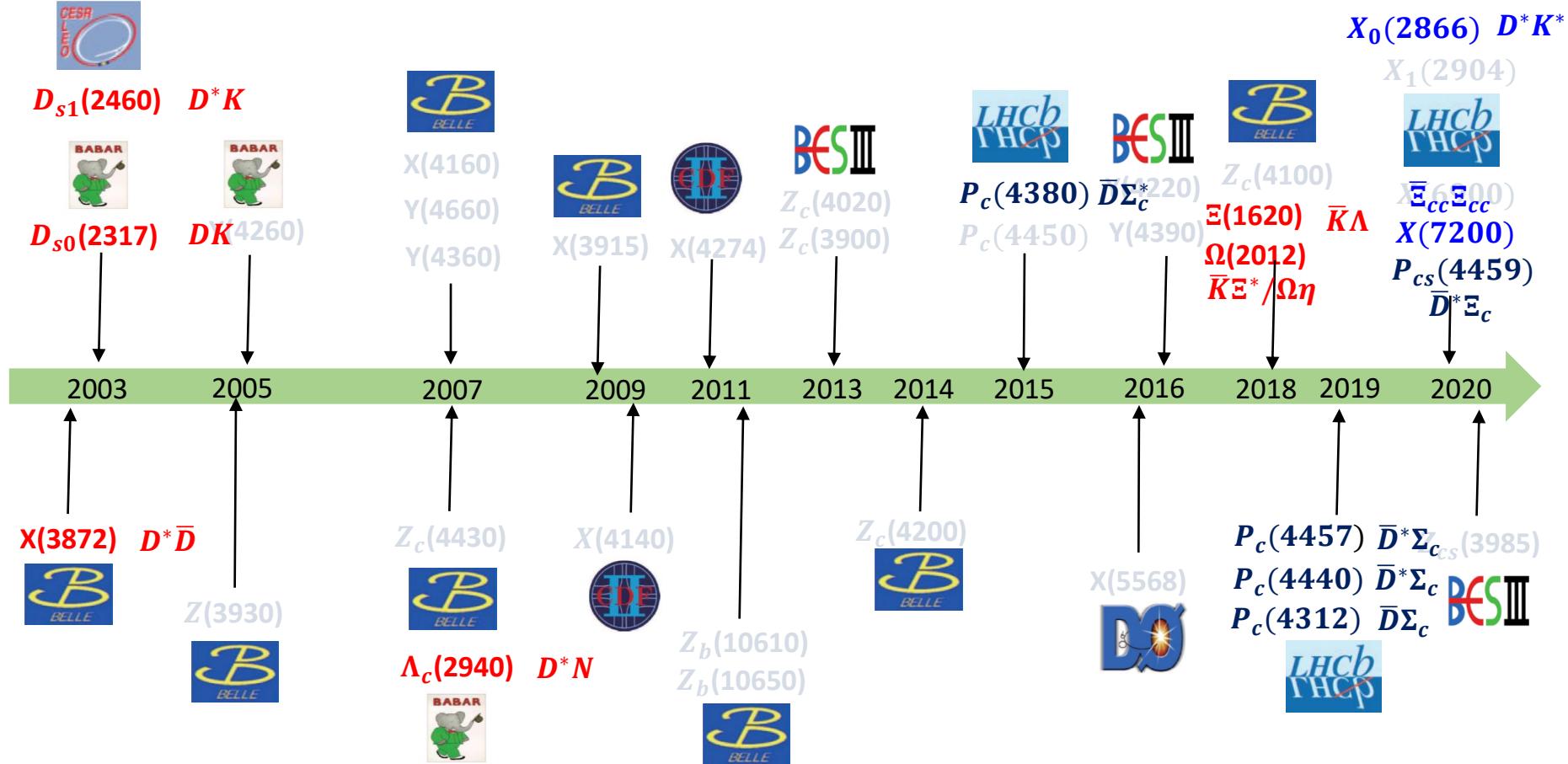
# Many (if not all) of them close to thresholds



Feng-Kun Guo, Christoph Hanhart,  
 Ulf-G. Meißner, Qian Wang,  
 Qiang Zhao, Bing-Song Zou.  
*Rev.Mod.Phys.* 90 (2018) 015004.

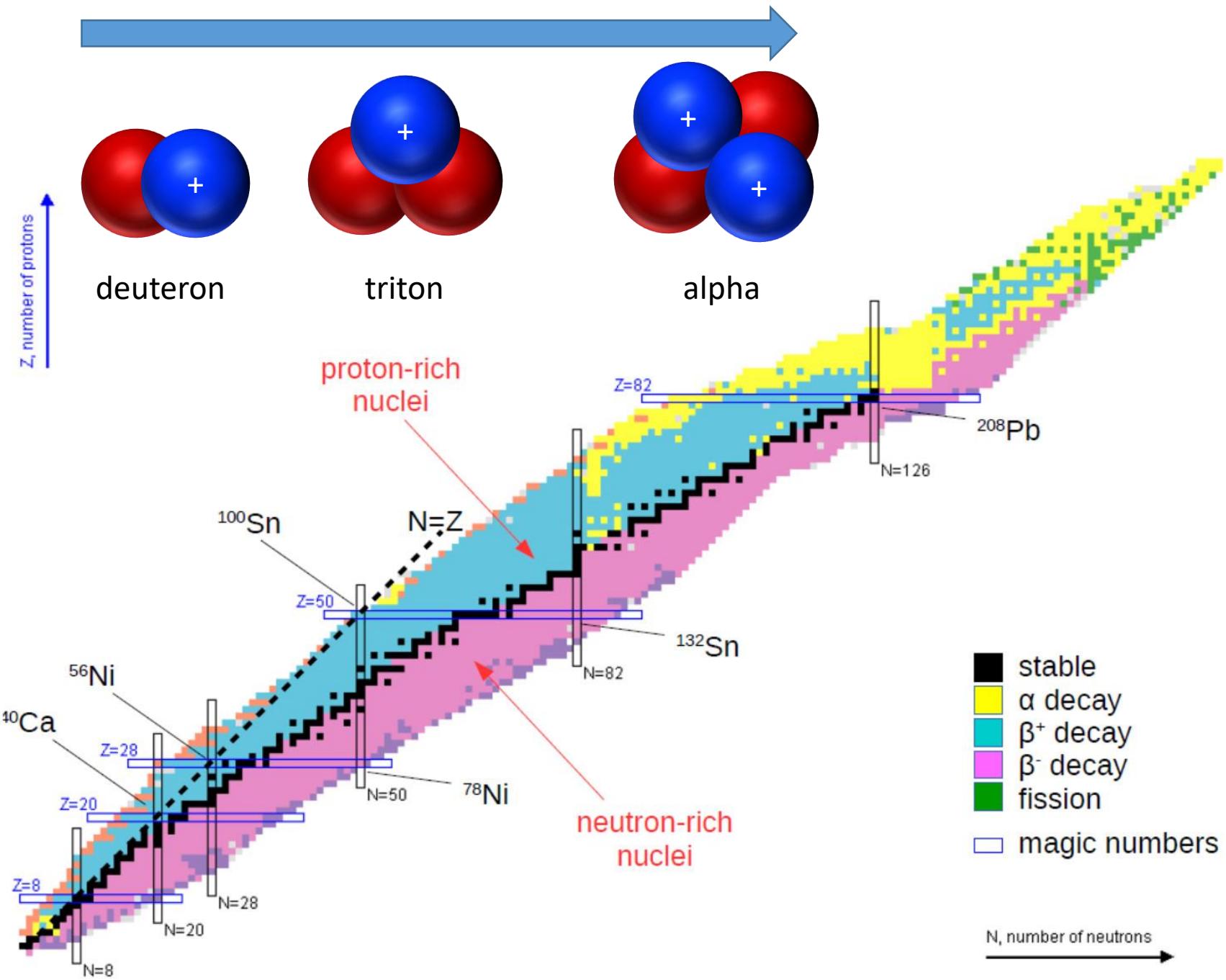
# Exotic mesons or baryons Tetraquark states Pentaquark states

## Molecular candidates

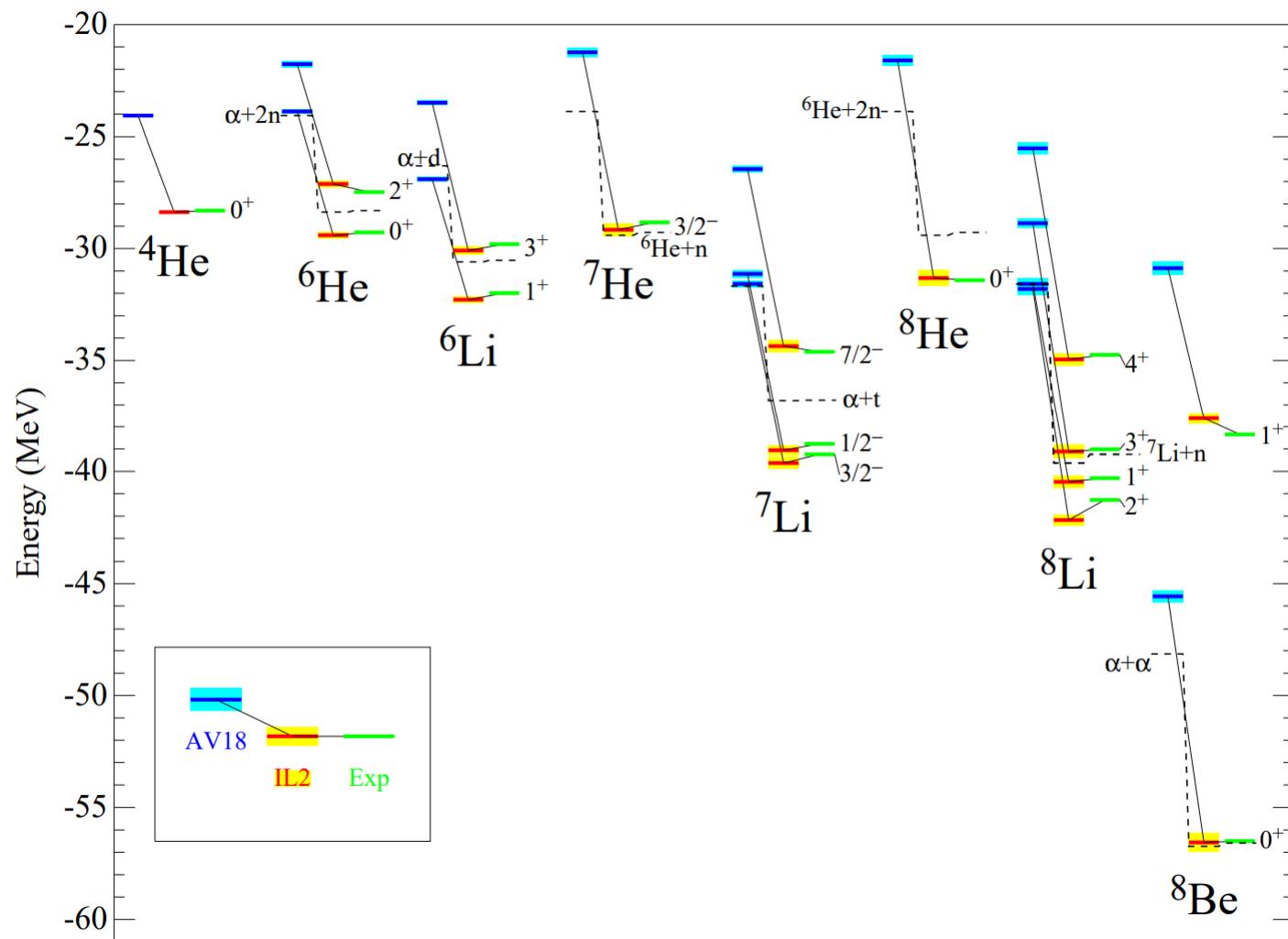


How to check the **molecular** picture?  
our naïve answer—go to many body





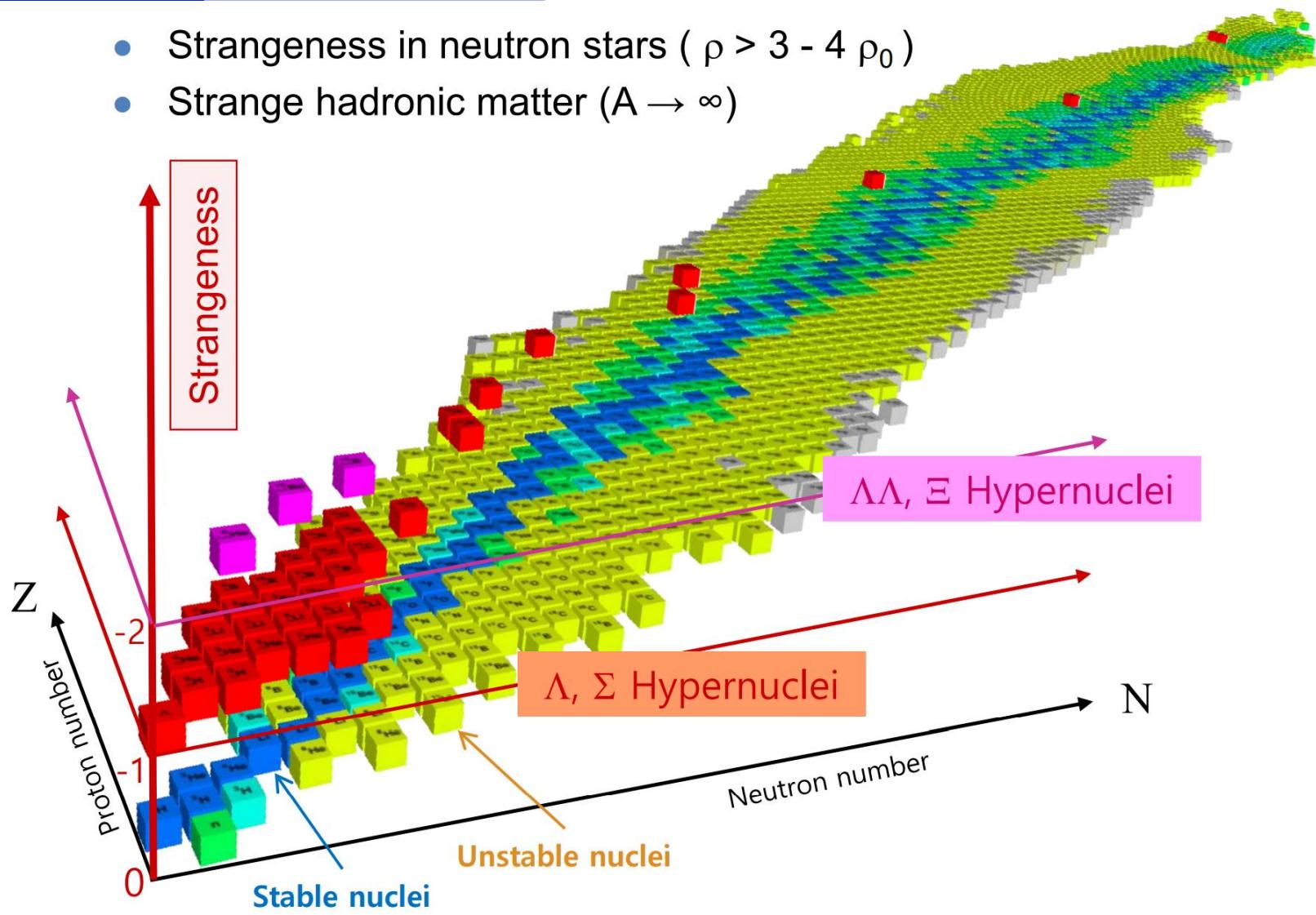
# Quantum Monte Carlo Calculations of light Nuclei



# Adding hyperons → the 3-D nuclear chart

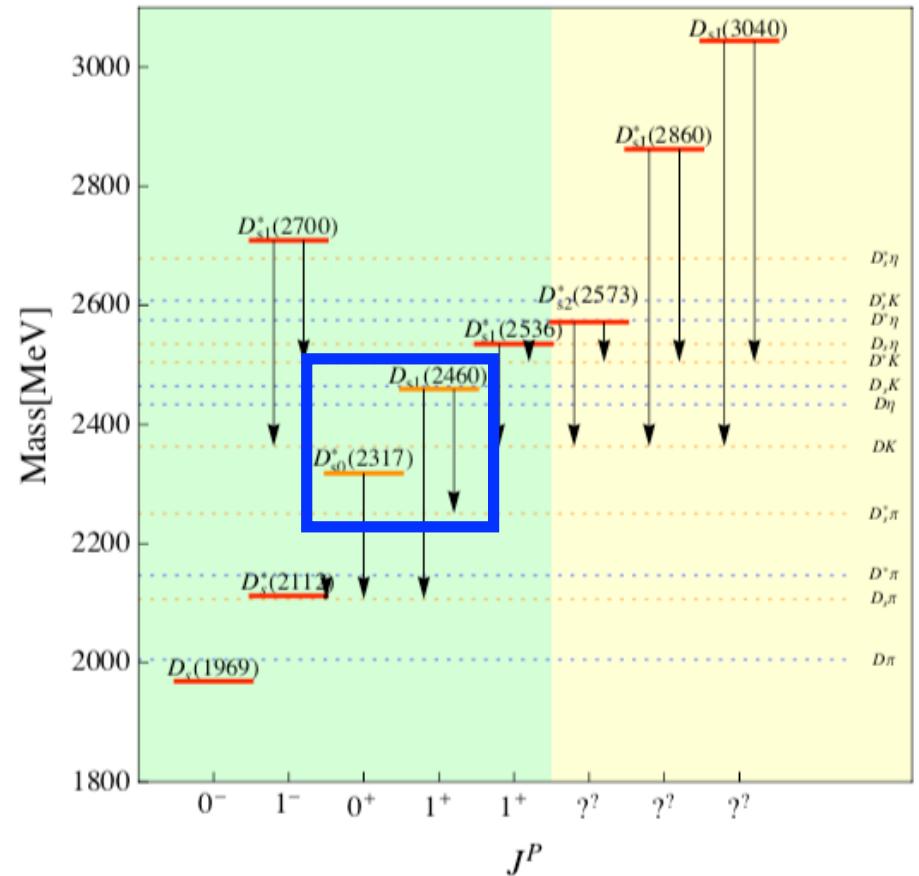
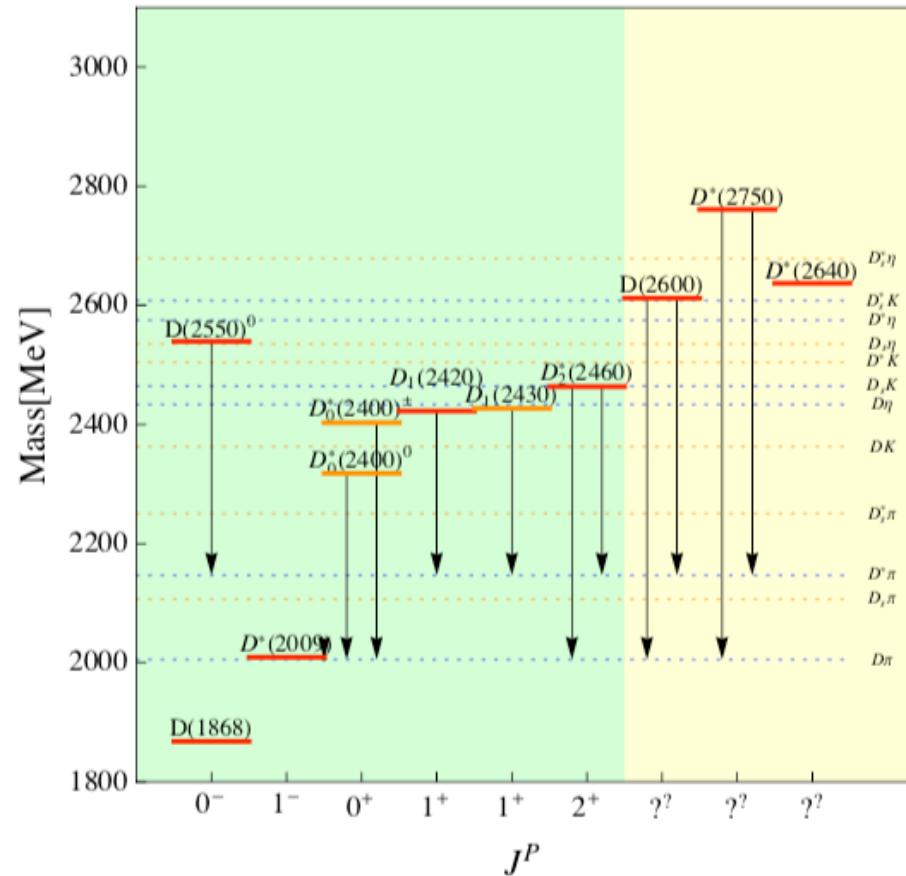
- Strangeness in neutron stars ( $\rho > 3 - 4 \rho_0$ )
  - Strange hadronic matter ( $A \rightarrow \infty$ )

## Higher density



# Next best two-body molecule candidates

adding charm

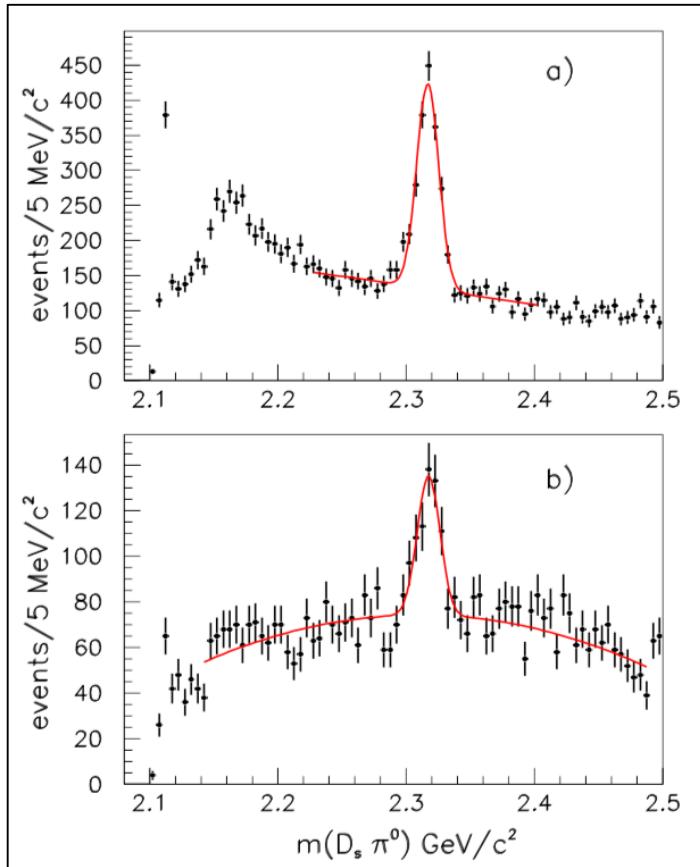


**Ds0<sup>\*</sup>(2317)**

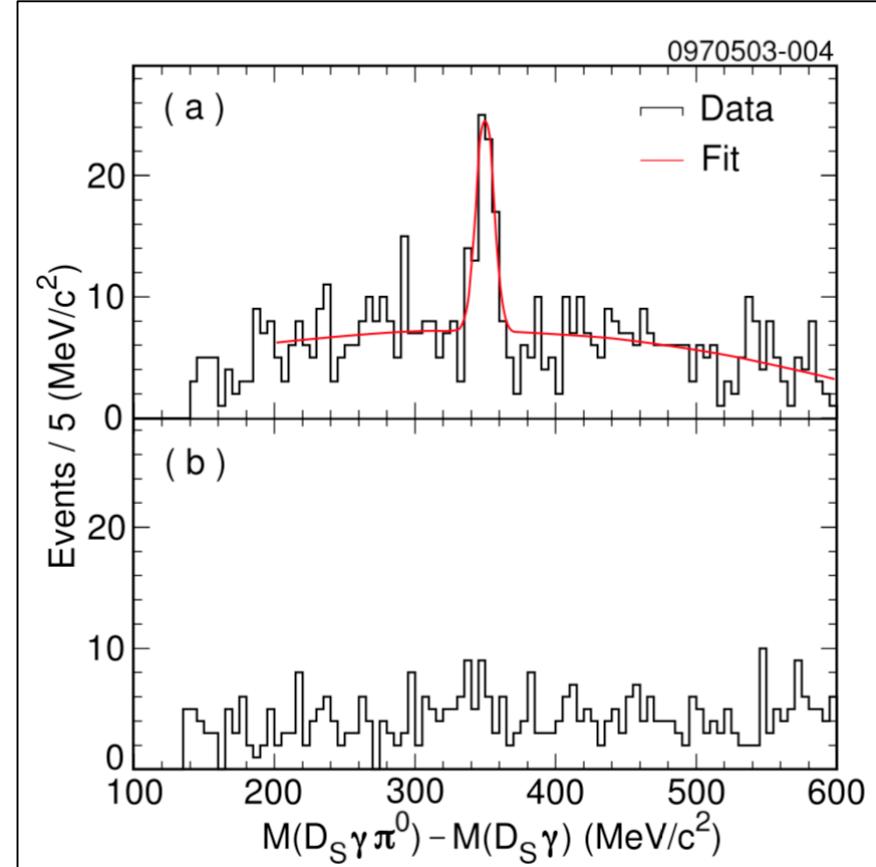
**Ds1(2460)**

# Discovery channels

**Ds<sub>0</sub>\*<sup>(2317)</sup>**



**Ds<sub>1</sub>(2460)**



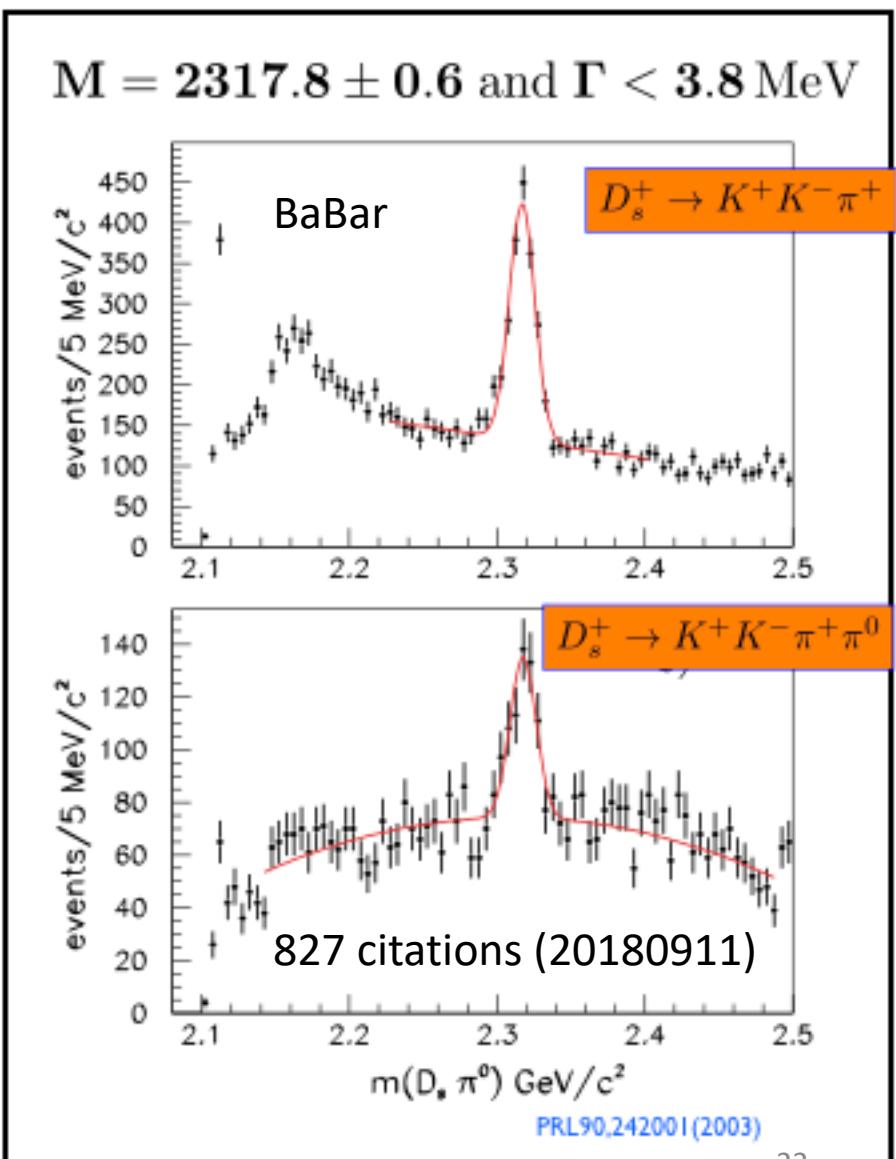
BaBar PRL90,242001(2003)

CLEO PRD68,032002(2003)

# What are special about these two states

- $D_{s0}^*(2317)$ ,  $D_{s1}(2460)$
- 160/70 MeV lower than the GI quark model predictions--difficult to be understood as conventional csbar states.
- “Dynamically generated” from strong DK interaction
  - ✓ E. E. Kolomeitsev 2004, [\[SEP\]](#)
  - ✓ F. K. Guo 2006,
  - ✓ D. Gamermann 2007

$$m_{D_{s1}(2460)} - m_{D_{s0}^*(2317)} \approx m_{D^*} - m_D$$

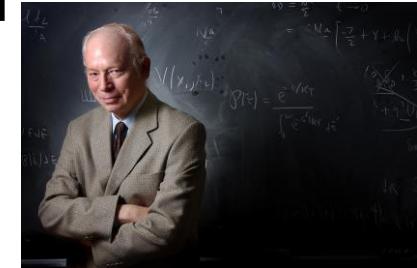


# Contents

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- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- **Ds0\*(2317) and Ds1(2460) as DK/D\*K molecules: theory & lattice**
- **DDK molecule: R++(4140)**
- **$D\bar{D}^*K$  and  $D\bar{D}K$  molecules: K\*(4307) and Kc(4180)**
- **(Future) experimental searches**
- **Summary and outlook**

# UChPT in Bethe-Salpeter equation



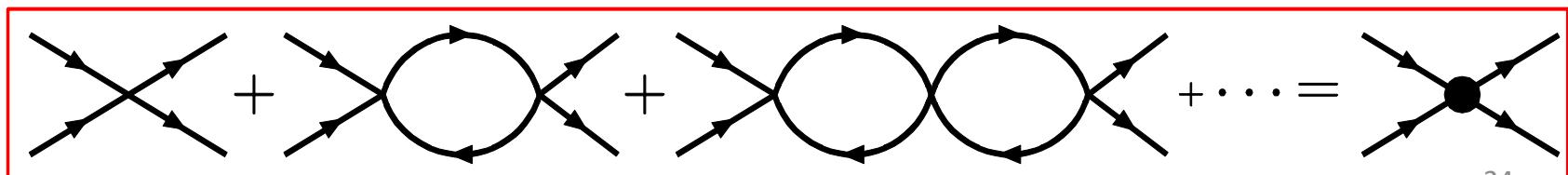
- Model independent DK interaction from ChPT

$$\mathcal{V}_{\text{WT}}(P(p_1)\phi(p_2) \rightarrow P(p_3)\phi(p_4)) = \frac{1}{4f_0^2} \mathcal{C}_{\text{LO}} (s - u)$$

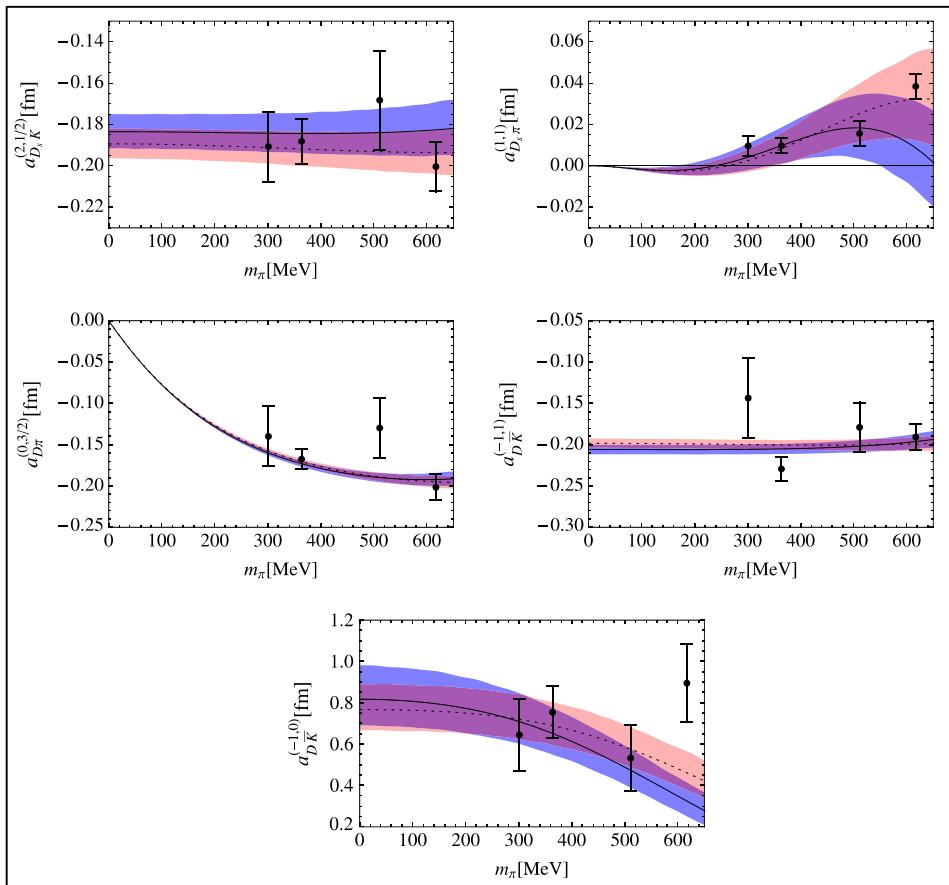
Weinberg-Tomazawa

$$\begin{aligned} \mathcal{V}_{\text{NLO}}(P(p_1)\phi(p_2) \rightarrow P(p_3)\phi(p_4)) = & -\frac{8}{f_0^2} C_{24} \left( c_2 p_2 \cdot p_4 - \frac{c_4}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & -\frac{4}{f_0^2} C_{35} \left( c_3 p_2 \cdot p_4 - \frac{c_5}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & -\frac{4}{f_0^2} C_6 \frac{c_6}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 - p_1 \cdot p_2 p_3 \cdot p_4) \\ & -\frac{8}{f_0^2} C_0 c_0 + \frac{4}{f_0^2} C_1 c_1 , \end{aligned} \quad (11)$$

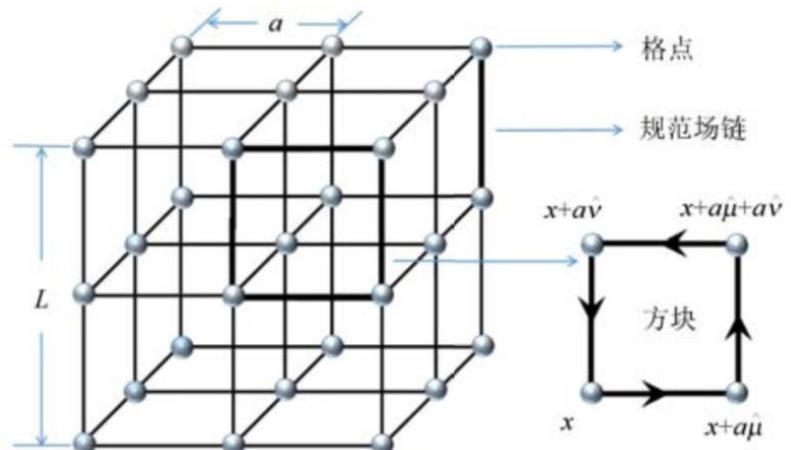
- Resumed in the Bethe-Salpeter equation (two-body elastic unitarity)



# Fixing the LECs using latest LQCD\* data



- NLO ChPT kernel: 5 LECs
- A quite good description of the 20 Lattice **scattering lengths of pseudoscalar mesons and D mesons (I=0 DK excluded)** can be achieved.



# D<sub>s0</sub> and D<sub>s1</sub> dynamically generated

- Charm sector

“Post-diction”

$$D_{s0}^*(2317), D_{s1}(2460)$$

TABLE V. Pole positions  $\sqrt{s} = M - i\frac{\Gamma}{2}$  (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

$(S, I)$	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	$2317 \pm 10$	$2457 \pm 17$
(0, 1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$

- Bottom Sector

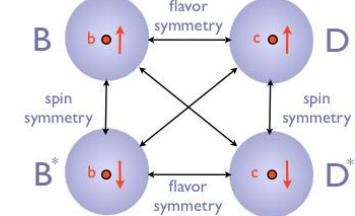


TABLE VI. Pole positions  $\sqrt{s} = M - i\frac{\Gamma}{2}$  (in units of MeV) of bottom mesons dynamically generated in the HQS UChPT.

$(S, I)$	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	$5726 \pm 28$	$5778 \pm 26$
(0, 1/2)	$(5537 \pm 14) - i(118 \pm 22)$	$(5586 \pm 16) - i(124 \pm 25)$

# Predicted $B_s0$ and $B_s1$ states

Physics Letters B 750 (2015) 17–21



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## Predicting positive parity $B_s$ mesons from lattice QCD



C.B. Lang<sup>a</sup>, Daniel Mohler<sup>b,\*</sup>, Sasa Prelovsek<sup>c,d</sup>, R.M. Woloshyn<sup>e</sup>

<sup>a</sup> Institute of Physics, University of Graz, A-8010 Graz, Austria

<sup>b</sup> Fermi National Accelerator Laboratory, Batavia, IL 60510-5011, USA

<sup>c</sup> Department of Physics, University of Ljubljana, 1000 Ljubljana, Slovenia

<sup>d</sup> Jozef Stefan Institute, 1000 Ljubljana, Slovenia

<sup>e</sup> TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

**Table 5**

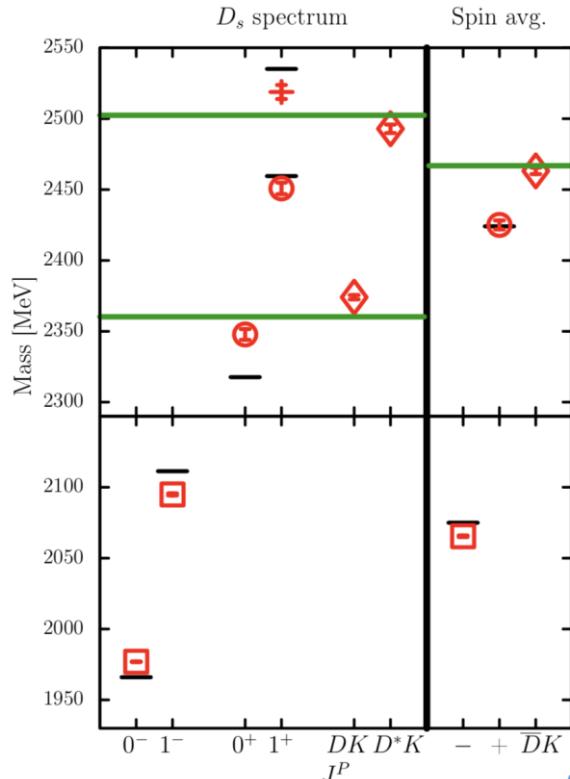
Comparison of masses from this work to results from various model based calculations; all masses in MeV.

$J^P$	$0^+$	$1^+$
Covariant (U)ChPT [24]	5726(28)	5778(26)
NLO UHMChPT [19]	5696(20)(30)	5742(20)(30)
LO UChPT [17,18]	5725(39)	5778(7)
LO $\chi$ -SU(3) [16]	5643	5690
HQET + ChPT [20]	5706.6(1.2)	5765.6(1.2)
Bardeen, Eichten, Hill [15]	5718(35)	5765(35)
rel. quark model [5]	5804	5842
rel. quark model [22]	5833	5865
rel. quark model [23]	5830	5858
HPQCD [30]	5752(16)(5)(25)	5806(15)(5)(25)
this work	5713(11)(19)	5750(17)(19)

In agreement with lQCD

# More support from recent IQCD studies

- G.K.C. Cheung et al., arXiv:2008.06432[hep-lat].
- G. S. Bali et al., arXiv:1706.01247 [hep-lat].
- C. B. Lang et al., arXiv:1403.8103 [hep-lat].
- D. Mohler et al., arXiv:1308.3175 [hep-lat].



**“DK components substantial”**

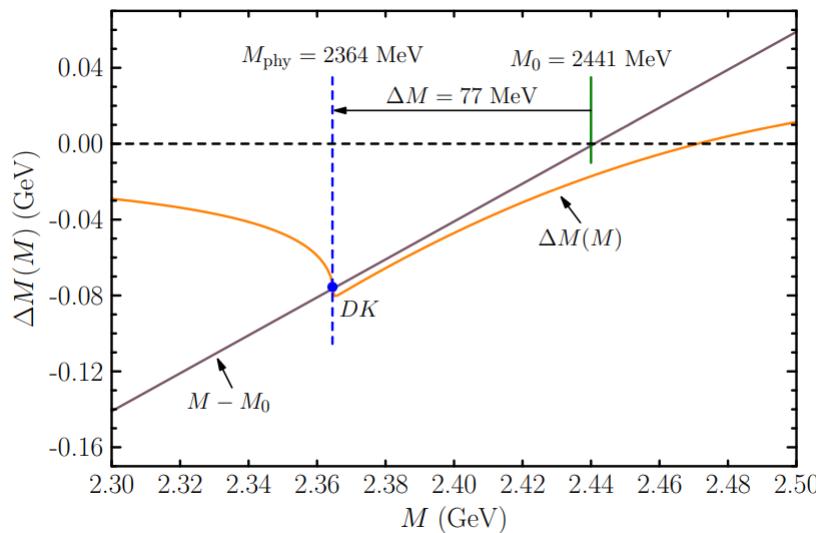
FIG. 12. On the left, our final results for the lower lying  $D_s$  spectrum as detailed in Table VII. The short horizontal black lines indicate the corrected experimental values (see Section II) while the green horizontal lines give the positions of the  $DK$  and  $D^*K$  non-interacting thresholds. Our lattice results for the finite volume thresholds are labelled  $DK$  and  $D^*K$ , respectively. The errors indicated are statistical only. On the right, the negative parity spin-averaged  $1S$  mass  $m_- = \frac{1}{4}(m_{0-} + 3m_{1-})$  is shown and denoted  $-$ , while the same spin-average of the positive parity  $0^+$  and  $1^+$  states is labelled with  $+$  and the weighted average of the threshold is labelled as  $\overline{DK}$ .

# Support from unquenched quark model

Predicting a new resonance as charmed-strange baryonic analogue of  $D_{s0}^*(2317)$

#

Si-Qiang Luo (Lanzhou U. and Lanzhou, Inst. Modern Phys.), Bing Chen (AHSTU, Fengyang and Gansu Lianhe U., Lanzhou), Xiang Liu (Lanzhou U. and Lanzhou, Inst. Modern Phys. and Gansu Lianhe U., Lanzhou), Takayuki Matsuki (Tokyo Kasei U.) (Feb 1, 2021)  
e-Print: 2102.00679 [hep-ph]



$$|D_{s0}^*(2317)\rangle = c_{c\bar{s}}|c\bar{s}(1^3P_0)\rangle + \int d^3\mathbf{p} c_{DK}(\mathbf{p})|DK, \mathbf{p}\rangle.$$

$$\begin{pmatrix} \hat{H}_0 & \hat{H}_I \\ \hat{H}_I & \hat{H}_{DK} \end{pmatrix} \begin{pmatrix} c_{c\bar{s}}|c\bar{s}(1^3P_0)\rangle \\ c_{DK}|DK\rangle \end{pmatrix} = M \begin{pmatrix} c_{c\bar{s}}|c\bar{s}(1^3P_0)\rangle \\ c_{DK}|DK\rangle \end{pmatrix}$$

# Further **tests** of the DK interaction

---

- Experiments, theory, and lattice QCD all show that  $DK$  or  $D^*K$  interaction is strong enough to form  $Ds0^*(2317)$  or  $Ds1(2460)$
- A natural question is: if we add one more  $D(\bar{D})$  or  $D^*(\bar{D}^*)$ , can they form molecules of three hadrons?
- This seems to be a rather straightforward and naive question, but remains unexplored until quite recently

# Contents

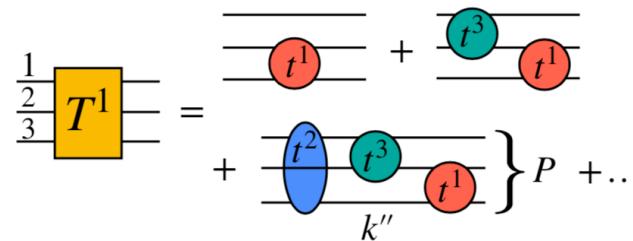
---

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$  and  $Ds1(2460)$  as  $DK/D^*K$  molecules: theory & lattice
- ***DDK molecule: R<sub>++</sub>(4140)***
- $D\bar{D}^*K$  and  $D\bar{D}K$  molecules:  $K^*(4307)$  and  $Kc(4180)$
- (Future) experimental searches
- Summary and outlook

# An explicit three-body study of DDK

- Coupled-three-channel problem:  $D(\text{DK} - D_s\pi - D_s\eta)$
- Three-body scattering matrix (Faddeev)

$$T = \sum_{i=1}^3 T^i$$



$$T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + \sum_{j \neq i=1}^3 T_R^{ij}, \quad i = 1, 2, 3,$$

$$T_R^{ij} = t^i g^{ij} t^j + t^i \left[ G^{iji} T_R^{ji} + G^{ijk} T_R^{jk} \right],$$

*A. Martínez Torres, K. P. Khemchandani, and E. Oset PRC 77, 042203(R)*

*A. Martinez Torres, K.P. Khemchandani, LSG, M. Napsuciale, E. Oset, PRD78 (2008) 074031*

# Two-body inputs

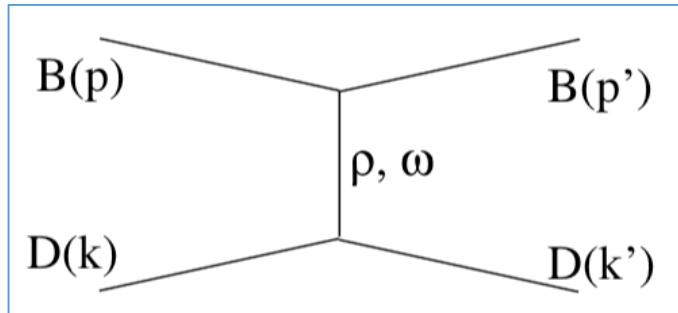
- DK: leading order UChPT  $DK, D_s\eta$  and  $D_s\pi$

$$V_{ij} = -\frac{C_{ij}}{4f^2}(s - u)$$

$a(\mu) = -1.846, \mu = 1000 \text{ MeV} \Rightarrow$   
Pole=2318 MeV

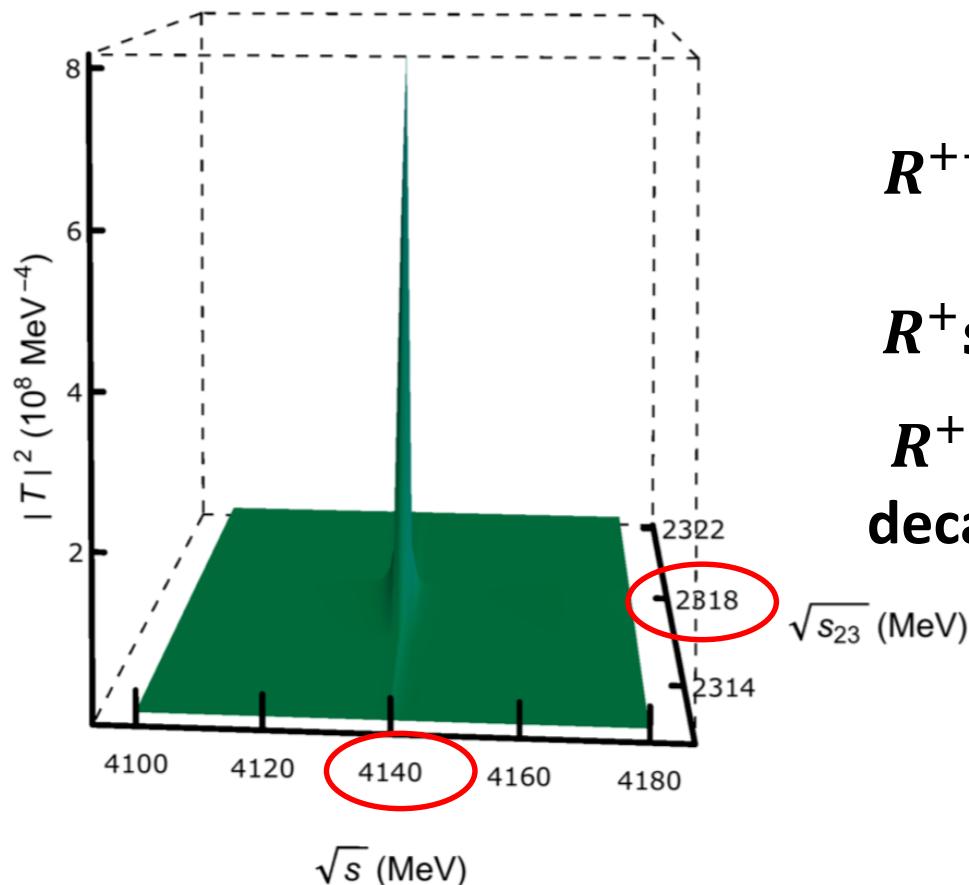
F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, PL B641, 278 (2006).

- DD(Ds): local hidden gauge theory



$a(\mu) = -1.3 \sim -1.5, \mu = 1500 \text{ MeV} \Leftarrow \text{fixed}$   
from  $D\bar{D}/D\bar{D}^*$ --X(3700)  
/X(3872)

# Three-body amplitudes

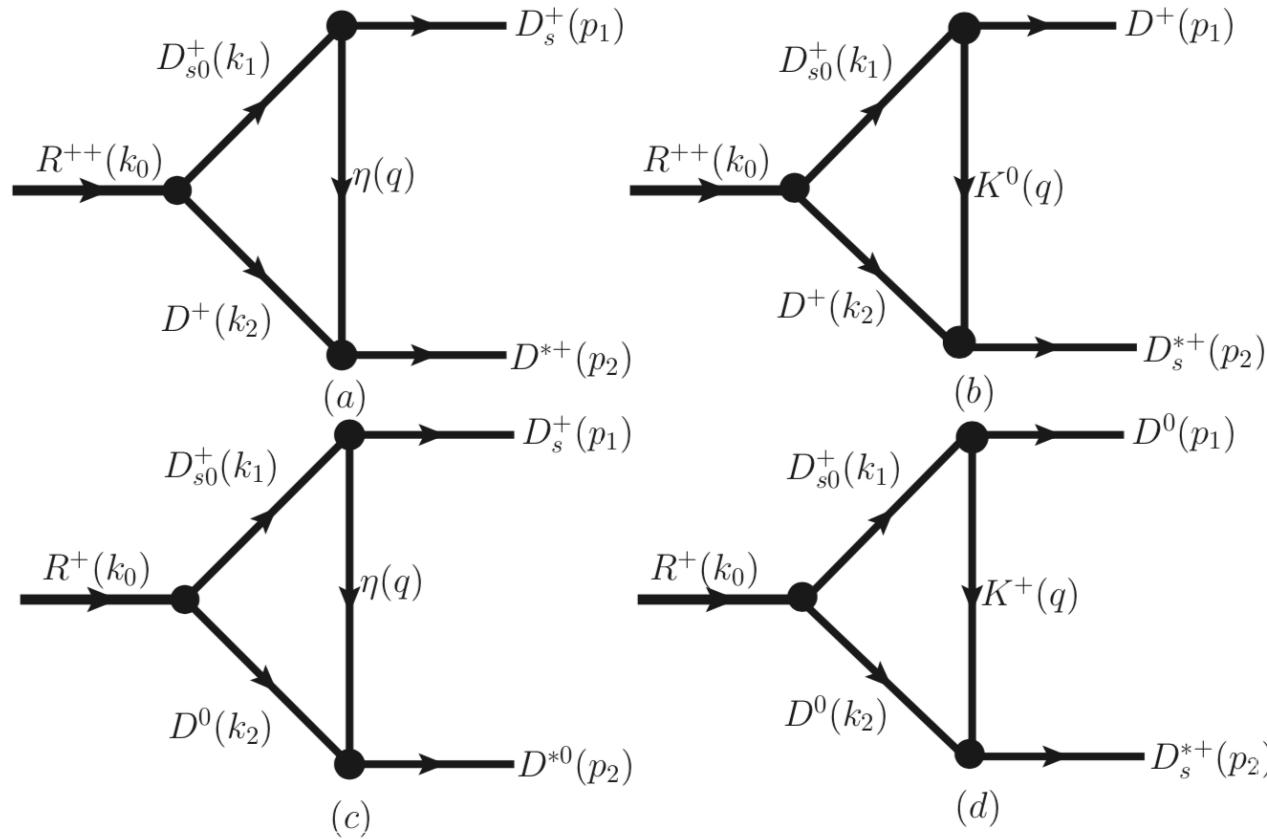
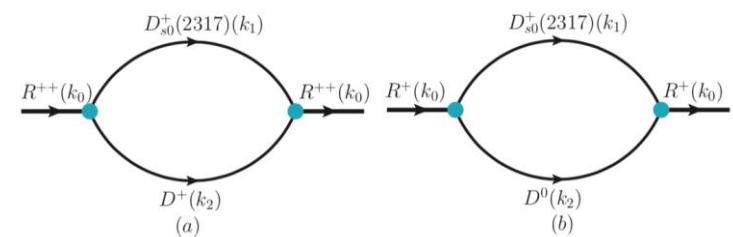


$$R^{++} = (I, I_{23}) = \left(\frac{1}{2}, \mathbf{0}\right)$$

$R^+$  should also exist

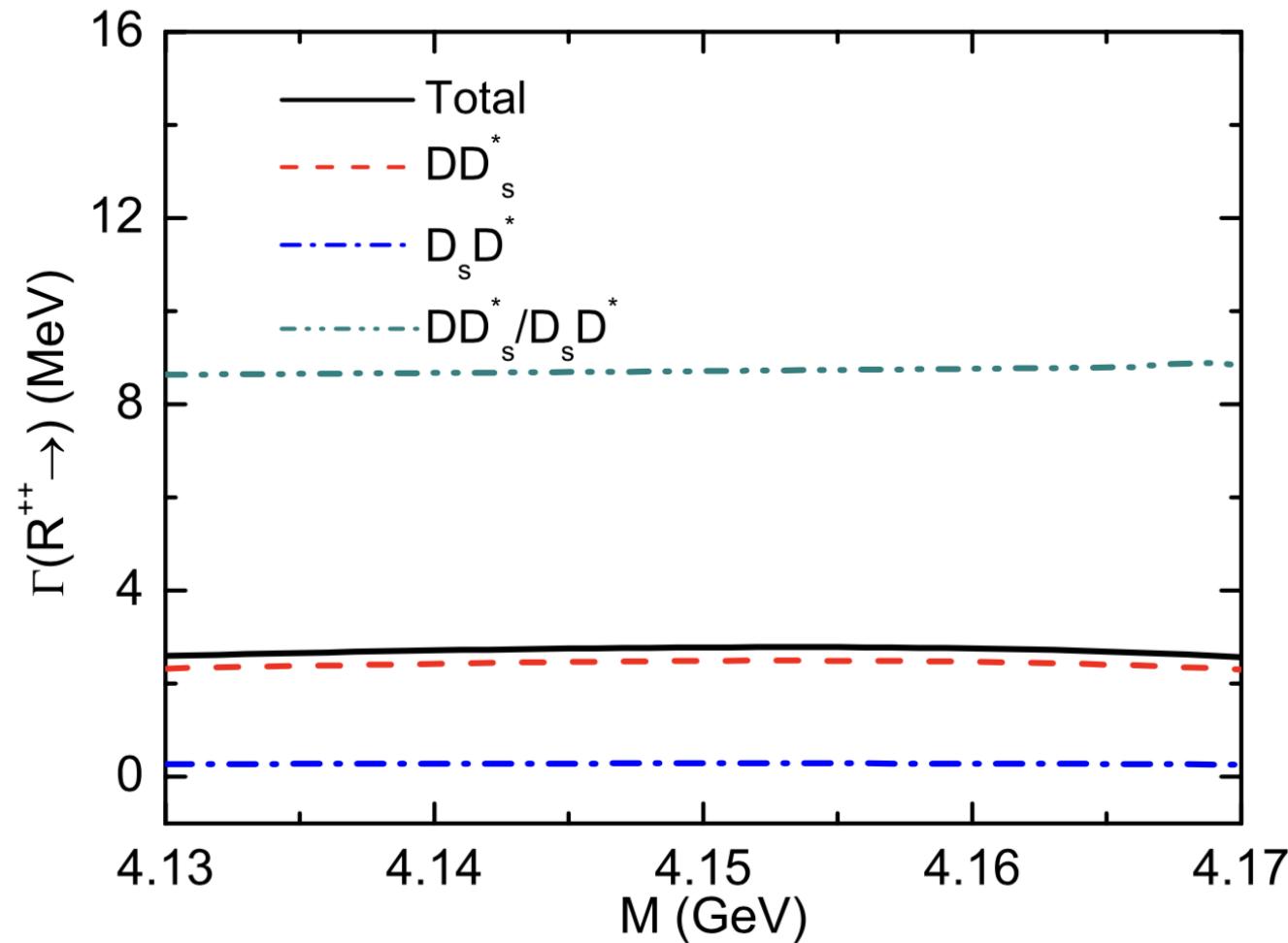
$R^{++}$  is a bound state, but can decay strongly

# Two-body decay width



# Two-body decay width

Kaon-Exchange Dominant

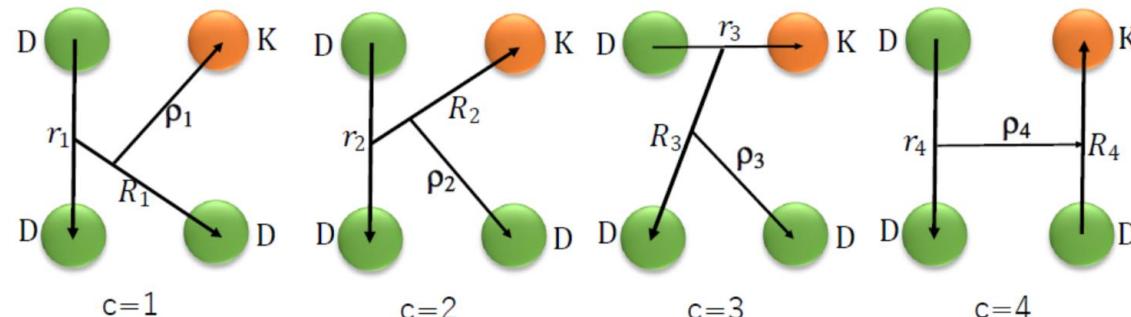
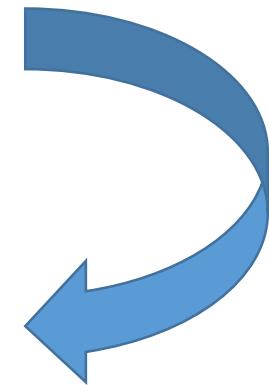
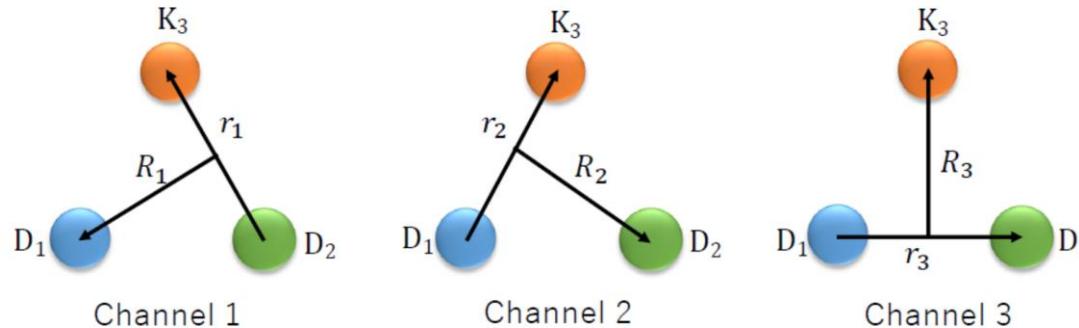


# A DDDK state

$1(0^+)$

Gaussian Expansion Method

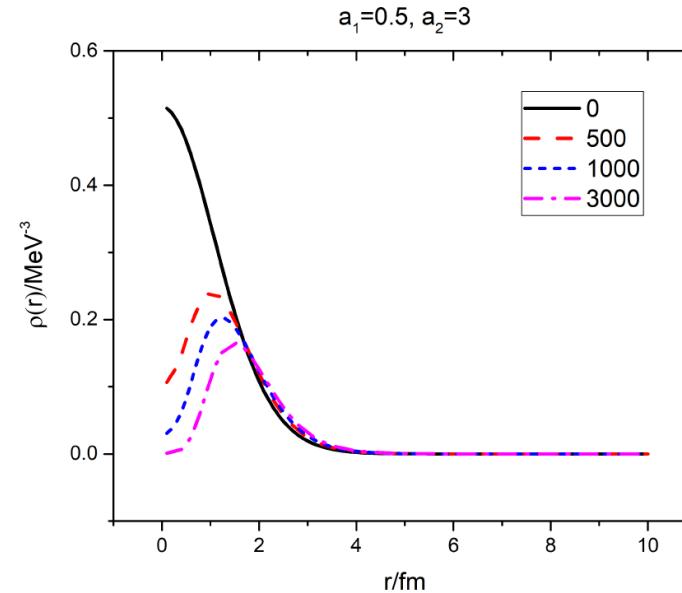
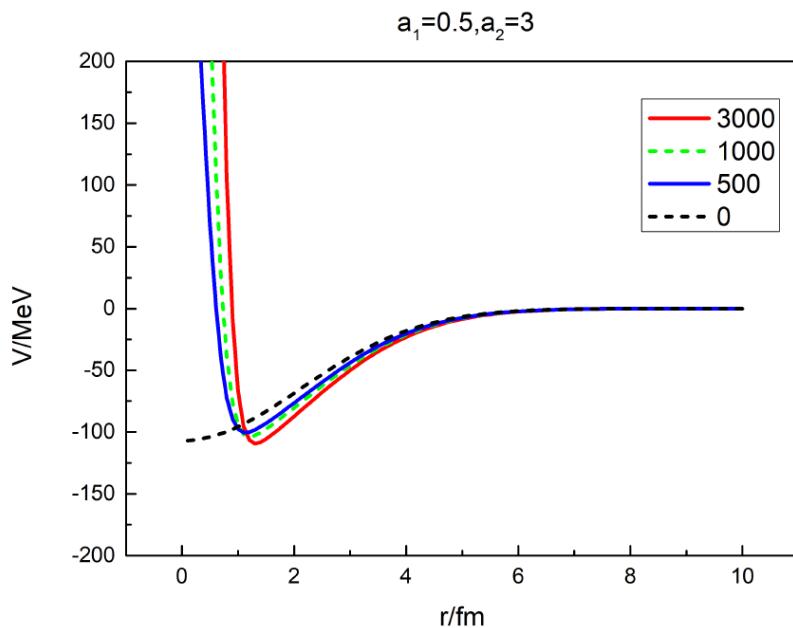
What if we add one more D?



# A DDDK state

$1(0^+)$

What if we add one more D? Our study shows that such a state exists as well



Uncertainties are at the order of 10-20 MeV

$$V_{DK}(r) = C_1 e^{-r^2/a_1^2} + C_2 e^{-r^2/a_2^2}$$

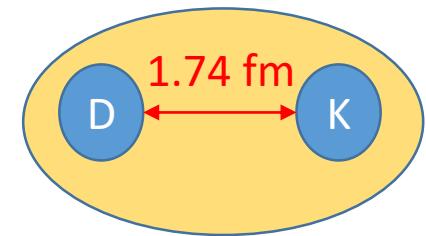
	DK*	DDK	DDDK
Binding	45 MeV	(67-71) MeV	91-107 MeV

# DD interactions play a minor role

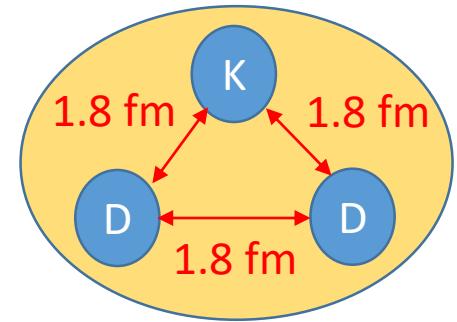
$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$E_2$	$E_3(\text{only } V_{DK})$	$E_3(V_{DK} + V_{DD})$	$E_4(\text{only } V_{DK})$	$E_4(V_{DK} + V_{DD})$
$R_S = 0.5\text{fm}$						
0	-320.1	-45.0	-65.8	-71.2	-89.4	-106.8
500	-455.4	-45.0	-65.8	-70.4	-89.2	-103.5
1000	-562.6	-45.0	-65.7	-69.7	-88.8	-101.4
3000	-838.7	-45.0	-65.0	-68.4	-87.0	-97.3
$R_S = 0.5\text{fm}$						
0	-149.1	-45.0	-66.0	-68.8, -45.1	-88.7, -66.3	-97.6, -70.7
500	-178.4	-45.0	-65.9	-68.2, -45.5	-88.5, -66.7	-95.5, -70.9
1000	-195.0	-45.0	-65.8, -45.2	-67.9, -45.8	-88.2, -66.9	-94.5, -71.2
3000	-225.9	-45.0	-65.3, -45.6	-67.2, -46.6	-87.0, -67.0	-92.6, -71.7
$R_S = 0.5\text{fm}$						
0	-107.0	-45.0	-66.2, -47.3	-68.0, -48.3	-88.8, -70.2	-94.4, -74.3
500	-119.4	-45.0	-66.2, -48.2	-67.7, -49.3	-88.7, -71.0	-93.2, -74.8
1000	-125.6	-45.0	-66.1, -48.7	-67.5, -49.8	-88.4, -71.3	-92.5, -75.2
3000	-136.2	-45.0	-65.8, -49.4	-67.1, -50.7	-87.6, -71.7	-91.4, -75.7

# Spatial distributions

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$\langle T \rangle$	$\langle V_{DK} \rangle$	$\langle V_{DD} \rangle$
$R_S = 0.5\text{fm}$ $R_c = 1\text{fm}$							
0	-320.1	1.28	1.32	1.36	124.37	-189.61	-5.98
500	-455.4	1.39	1.44	1.47	99.51	-164.83	-5.03
1000	-562.6	1.46	1.53	1.54	91.43	-156.67	-4.51
3000	-838.7	1.61	1.69	1.68	93.24	-157.80	-3.82
$R_S = 0.5\text{fm}$ $R_c = 2\text{fm}$							
0	-149.1	1.74	1.80	1.80	60.20	-125.74	-3.23
500	-178.4	1.91	1.98	1.96	51.00	-116.59	-2.64
1000	-195.0	1.99	2.07	2.04	50.63	-116.12	-2.43
3000	-225.9	2.13	2.22	2.15	53.61	-118.59	-2.24
$R_S = 0.5\text{fm}$ $R_c = 3\text{fm}$							
0	-107.0	2.13	2.19	2.17	39.49	-105.35	-2.13
500	-119.4	2.31	2.38	2.34	34.80	-100.73	-1.77
1000	-125.6	2.37	2.47	2.42	34.90	-100.77	-1.65
3000	-136.2	2.53	2.61	2.53	36.66	-102.24	-1.54

Ds<sub>0</sub>\*<sup>0</sup>(2317)

R(4140)



# Contents

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- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$  and  $Ds1(2460)$  as  $DK/D^*K$  molecules: theory & lattice
- $DDK$  molecule:  $R^{++}(4140)$
- $D\bar{D}^*K$  and  $D\bar{D}K$  molecules:  $K^*(4307)$  and  $Kc(4180)$
- (Future) experimental searches
- Summary and outlook

# Instead of a $D$ , adding a $\bar{D}^*$ to the DK pair

- Fixed center approximation (FCA):

$$K(D\bar{D}^* + \bar{D}D^*) \sim KX(3872)/Zc(3900)$$

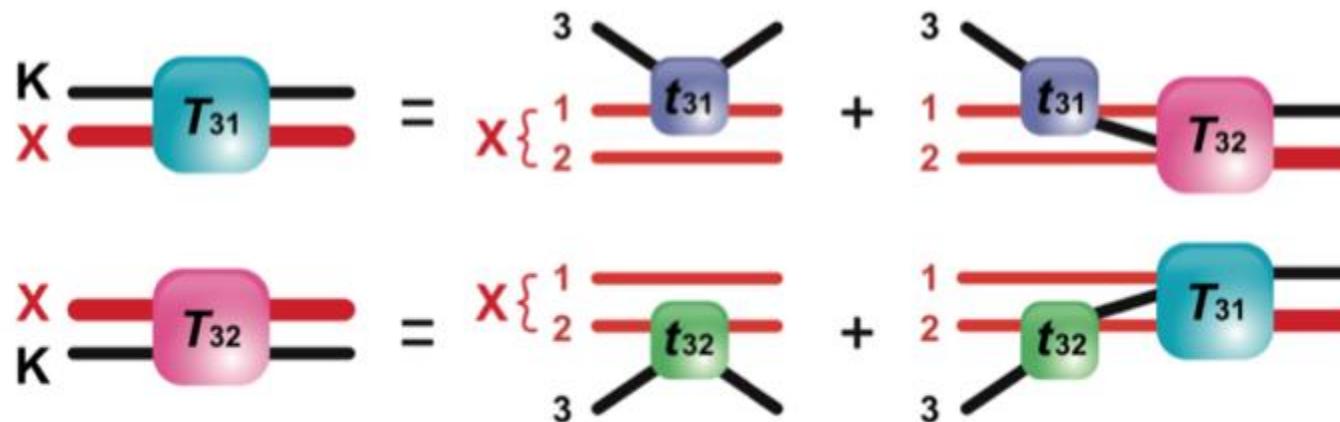
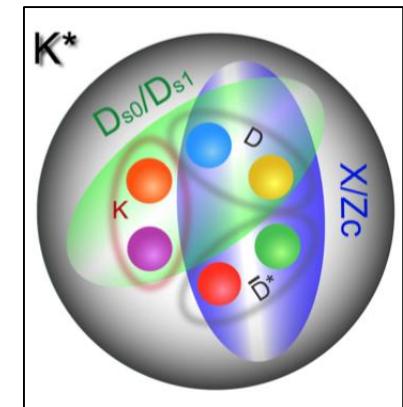
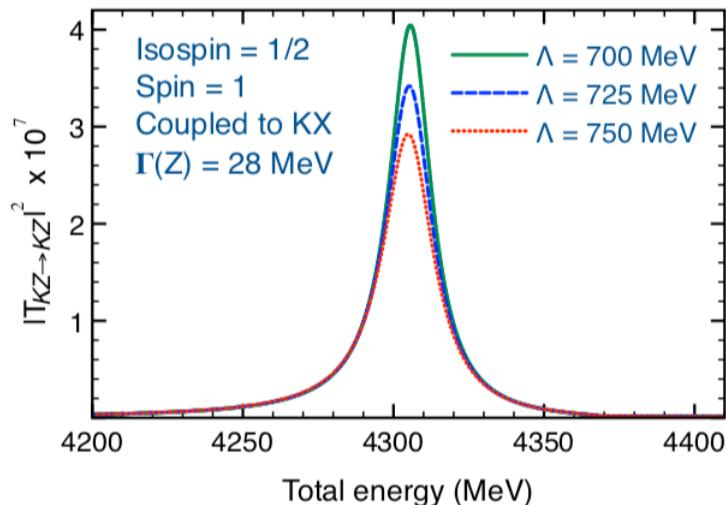
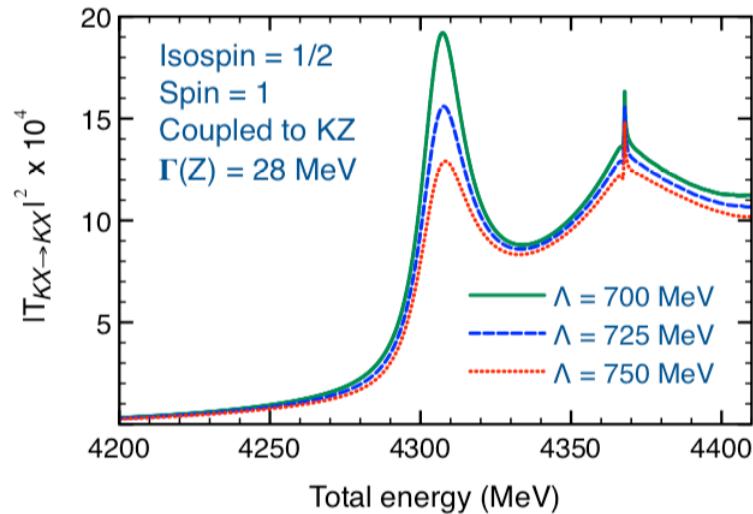


Figure 2: Diagrams showing the scattering of the particle labeled “3” ( $K$ ) on a cluster ( $X$ ) made of particles 1 ( $D$ ) and 2 ( $\bar{D}^*$ ).

1805.08330

# K<sup>\*</sup>(4307)



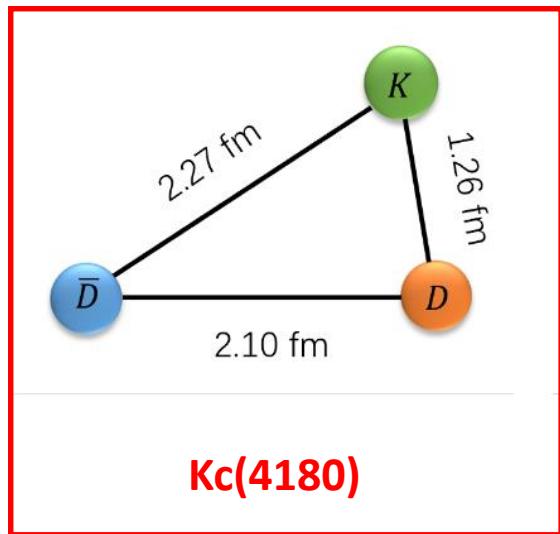
- Treating KX and KZ as coupled channel systems
- A resonance with  $M=(4307 \pm 2) - i(9 \pm 2)$  MeV with  $I(J^P) = 1/2(1^-)$

In agreement with Li Ma, Qian Wang, Ulf-G. Meißner, 1711.06143, but with completely different dynamics

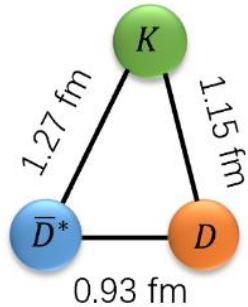
# Instead of a $D$ , adding a $\bar{D}$ to the DK pair

[2012.01134](#)

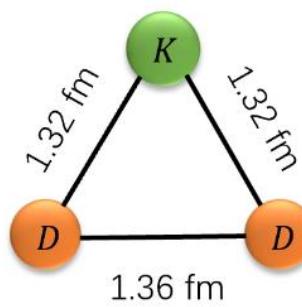
## The Three Musketeers



**Kc(4180)**



**$K^*(4307)$**

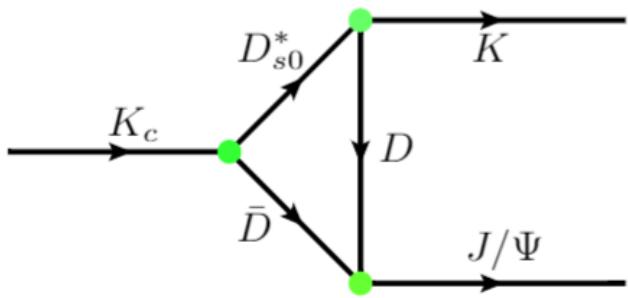


**R(4140)**

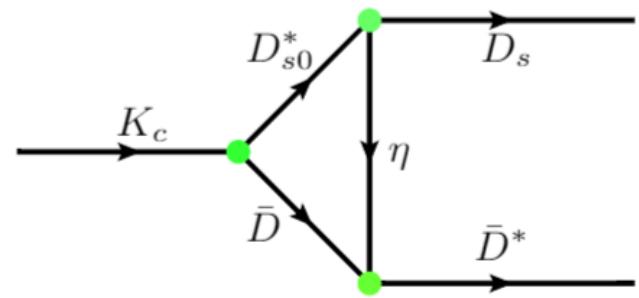
	This work	Ref [28]	Ref [29]
Method	GEM(SE)	BOA(SE)	FCA(FE)
Interaction Models	$\chi$ EFT+OBE	delocalized $\pi$ bond	$\chi$ EFT+OBE
$\frac{1}{2}(0^-) D\bar{D}K$	$4181.2^{+2.4}_{-1.4} (B_3 \simeq 48.9^{+1.4}_{-2.4})$	-	-
$\frac{1}{2}(1^-) D\bar{D}^*K$	$4294.1^{+6.6}_{-3.1} (B_3 \simeq 77.3^{+3.1}_{-6.6})$	$4317.92^{+6.13}_{-6.55} (B_3 \simeq 53.52^{+6.55}_{-6.13})$	$4307 \pm 2 (B_3 \simeq 64 \pm 2)$

# $K_c(4180)$ decay

[2012.01134](#)

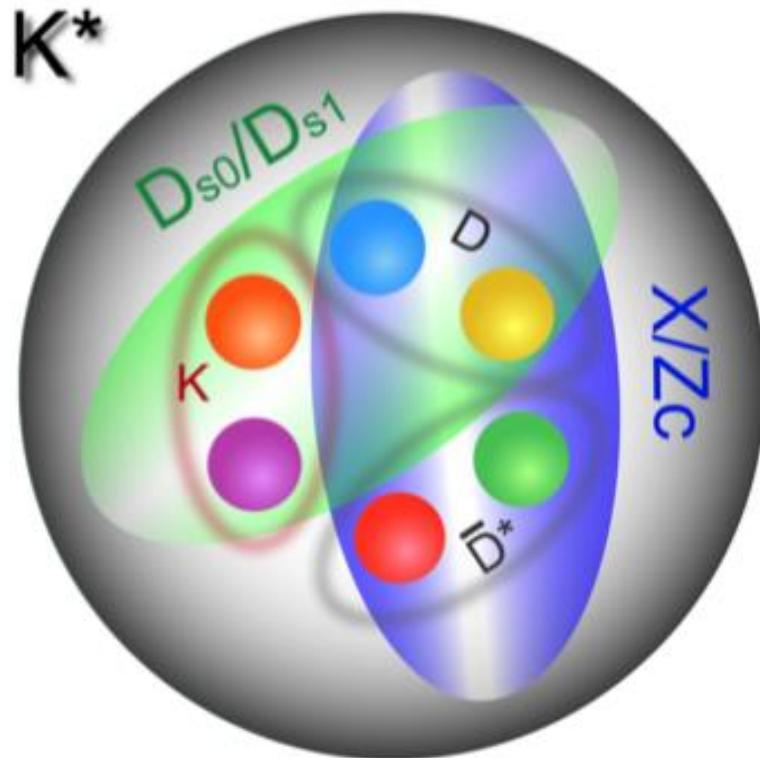


$\sim 1$  MeV

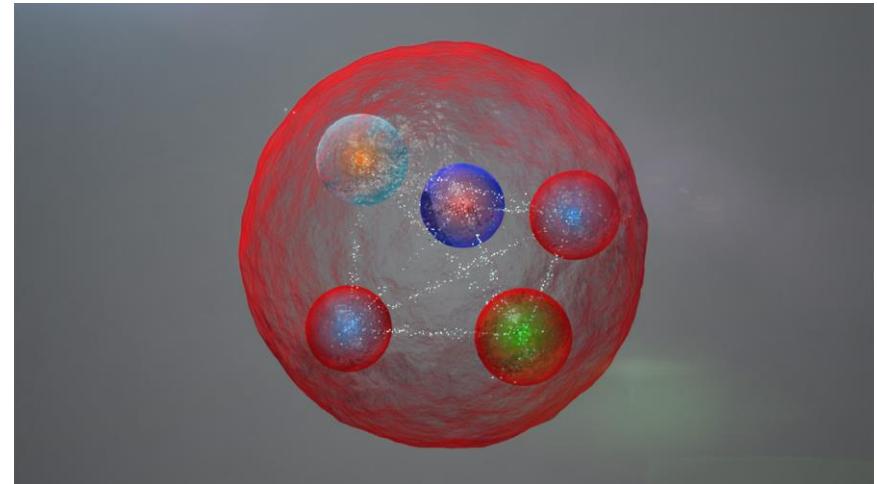


$\sim 1$  MeV

# $K^*(4307)/K_c(4180)$ —bosonic counterpart of $P_c$



but with 3 constituents



Pentaquark ( $N^*$ ) by LHCb

*Phys.Rev.Lett.* 115 (2015) 072001

Prediction of narrow  $N^*$  and  $\Lambda^*$  resonances with hidden charm above 4 GeV,  
Jia-Jun Wu, R. Molina, E. Oset, B.S. Zou, 1007.0573

# Analogy between $KD$ and $\bar{K}N$

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$D_{s0}^*(2317)$

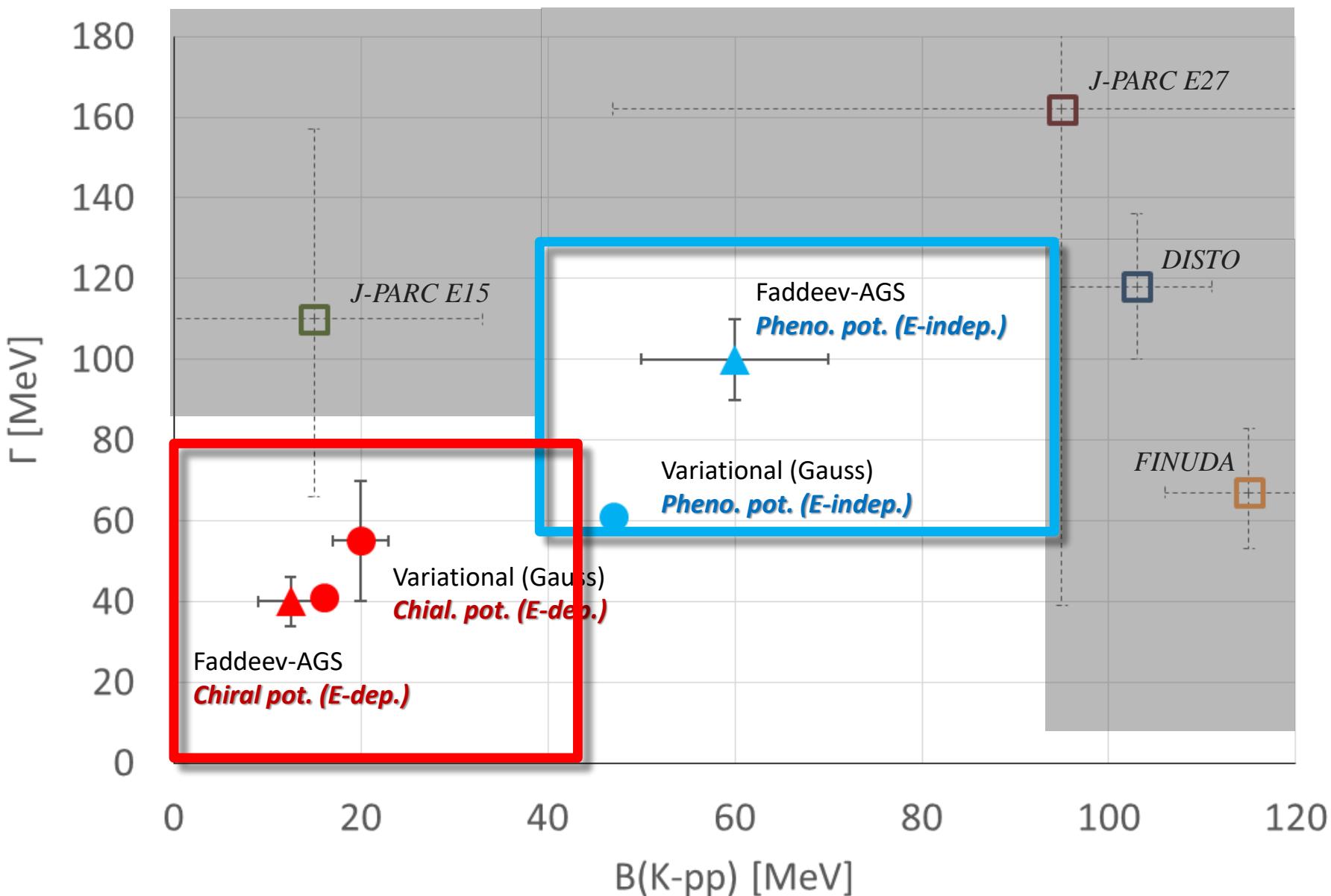
$\Lambda(1405)$

- DK bound state
- Dynamically generated--  
Unitary heavy hadron  
chiral perturbation theory
- N-Kbar bound state
- Dynamically generated--  
Unitary baryon chiral  
perturbation theory

The interaction between a kaon and a heavy particle seems to play an important role

# "Current" status on " $K^-pp$ "

A. Dote, Menu2019



# $\bar{K}N$ in our framework vs. more refined study

$NN$ : OPE fixed by reproducing the deuteron

$\bar{K}N$ : Weinberg-Tomozawa fixed by reproducing  $\Lambda(1405)$

$C(R_c)$	$R_c$	$B_2(N\bar{K})$	$r_2(N\bar{K})$	$B_3(NN\bar{K})$	$r_3(NN)$	$r_3(N\bar{K})$
$C_S = 0$				$R_s = 0.1$		
-925.9	0.5	29.4	1.28	35.2	2.07	2.64
-316.4	1.0	29.4		1.55 39.3	1.99	2.39
-132.6	2.0	29.4	2.05	41.8	2.34	2.69
$C_S = 1000$				$R_s = 0.1$		
-946.6	0.5	29.4	1.28	35.4	2.06	2.63
-319.8	1.0	29.4	1.55	39.4	1.99	2.39
-133.2	2.0	29.4	2.05	41.8	2.34	2.69

Variational method

20-40    2.09-2.26    1.85-2.01

Akinobu Dote, Tetsuo Hyodo , Wolfram Weise, [0806.4917 \[nucl-th\]](#), PRC 79 (2009)014003, 224 citations

# Contents

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- Where to search for these 3-body molecules
- Summary and outlook

# Search for the DDK bound state by Belle

2008.13341

Search for a doubly-charged  $DDK$  bound state in  $\Upsilon(1S, 2S)$  inclusive decays and via direct production in  $e^+e^-$  collisions at  $\sqrt{s} = 10.520, 10.580$ , and  $10.867$  GeV

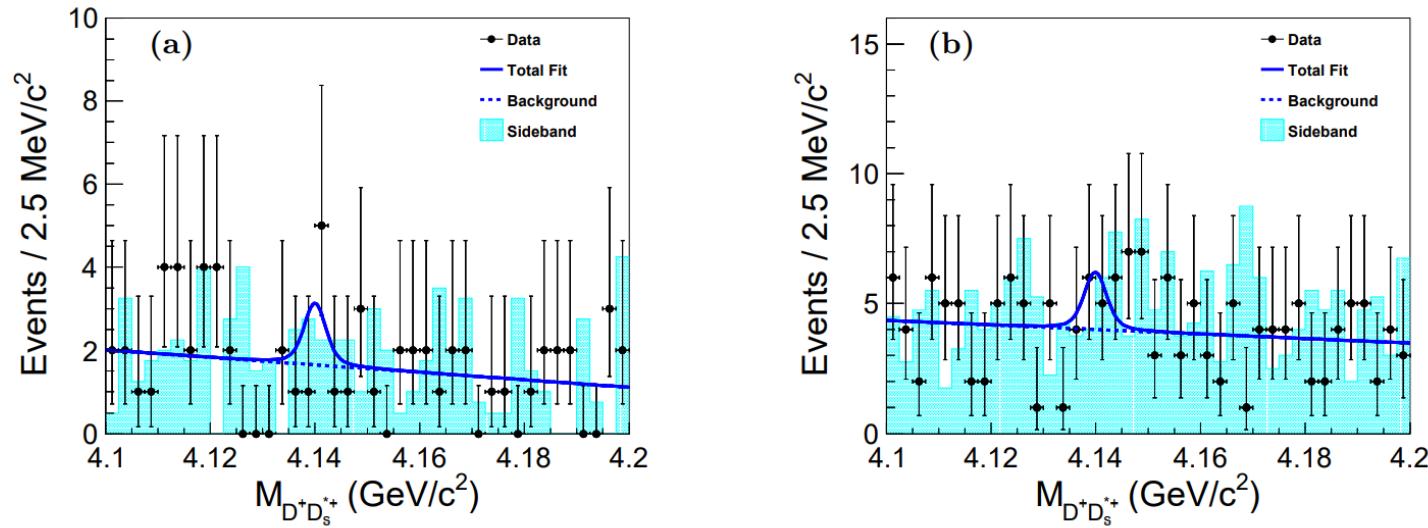


FIG. 4: The invariant-mass spectra of  $D^+ D_s^{*+}$  in the (a)  $\Upsilon(1S)$  and (b)  $\Upsilon(2S)$  data samples. The cyan shaded histograms are from the normalized  $M_{D^+}$  and  $M_{D_s^{*+}}$  sideband events. The blue solid curves show the fitted results with the  $R^{++}$  mass fixed at 4.14 GeV/ $c^2$  and width fixed at 2 MeV, and the blue dashed curves are the fitted backgrounds.

# Search for the DDK bound state by Belle

2008.13341

Search for a doubly-charged  $DDK$  bound state in  $\Upsilon(1S, 2S)$  inclusive decays and via direct production in  $e^+e^-$  collisions at  $\sqrt{s} = 10.520, 10.580$ , and  $10.867$  GeV

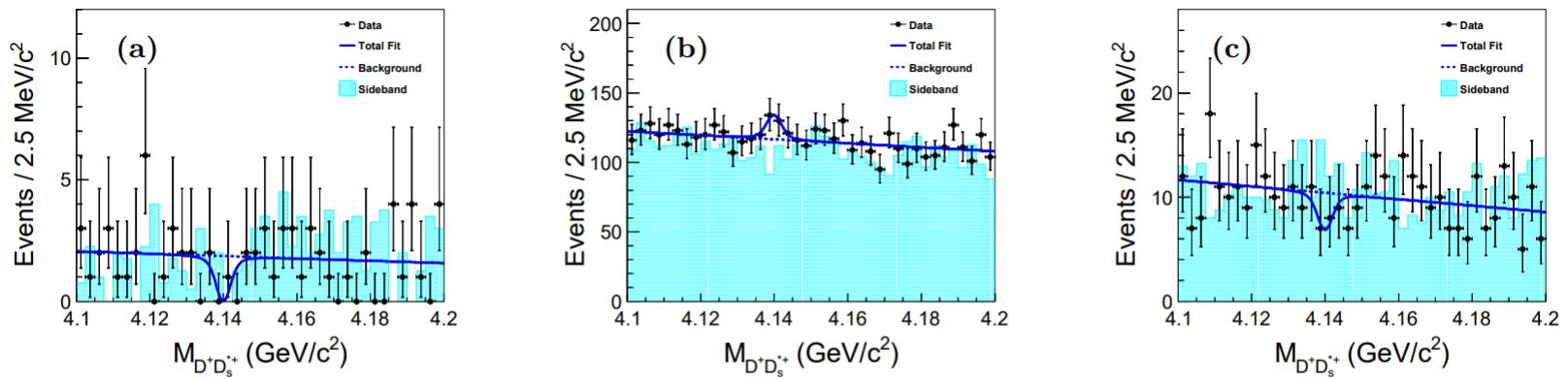


FIG. 10: The invariant-mass spectra of the  $D^+D_s^{*+}$  from  $e^+e^-$  annihilations at (a)  $\sqrt{s} = 10.520$  GeV, (b)  $\sqrt{s} = 10.580$  GeV, and (c)  $\sqrt{s} = 10.867$  GeV data samples. The cyan shaded histograms are from the normalized  $M_{D^+}$  and  $M_{D_s^{*+}}$  sideband events. The blue solid curves show the fitted results with the  $R^{++}$  mass fixed at  $4.14 \text{ GeV}/c^2$  and width fixed at  $2 \text{ MeV}$ , and the blue dashed curves are the fitted backgrounds.

# Where to search for $K_c(4180)$

$$\Lambda_b \rightarrow J/\psi K P$$

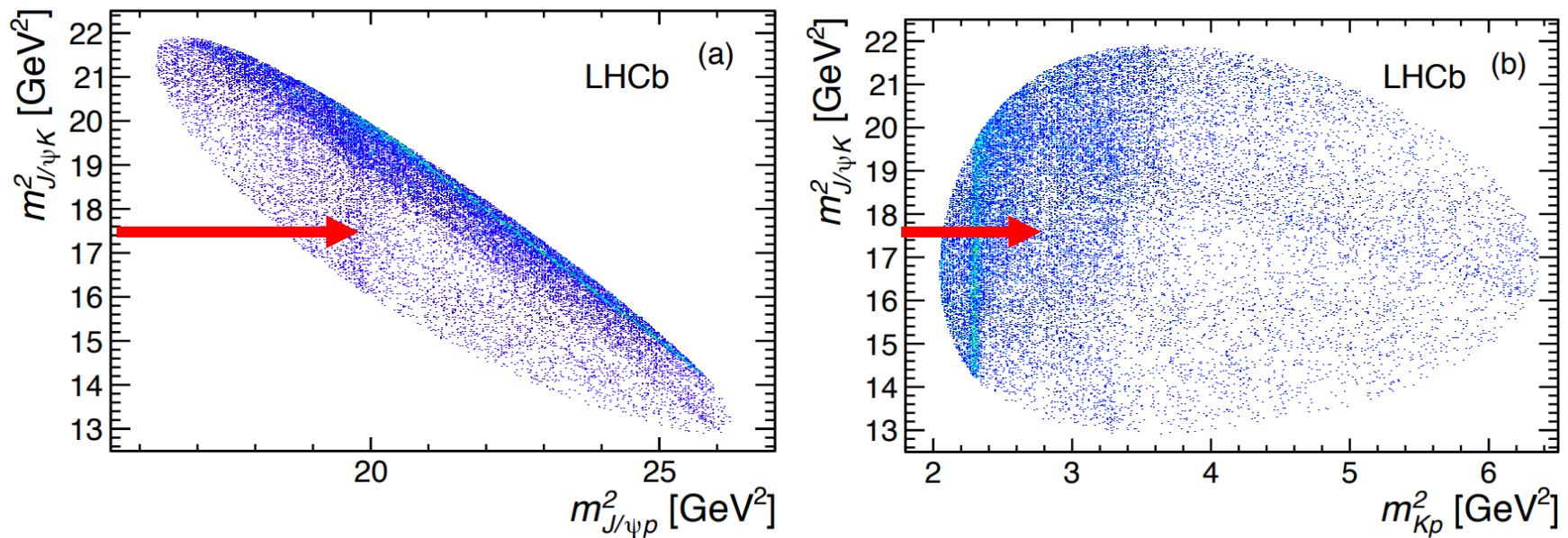


Figure 10: (a) Invariant mass squared of  $J/\psi K^-$  versus  $J/\psi p$  and (b) of  $J/\psi K^-$  versus  $K^- p$  for candidates within  $\pm 15$  MeV of the  $\Lambda_b^0$  mass.

**LHCb:1507.03414:** an integrated luminosity of 3  $\text{fb}^{-1}$

# Where to search for Kc(4180)

$B_s \rightarrow J/\psi KK$

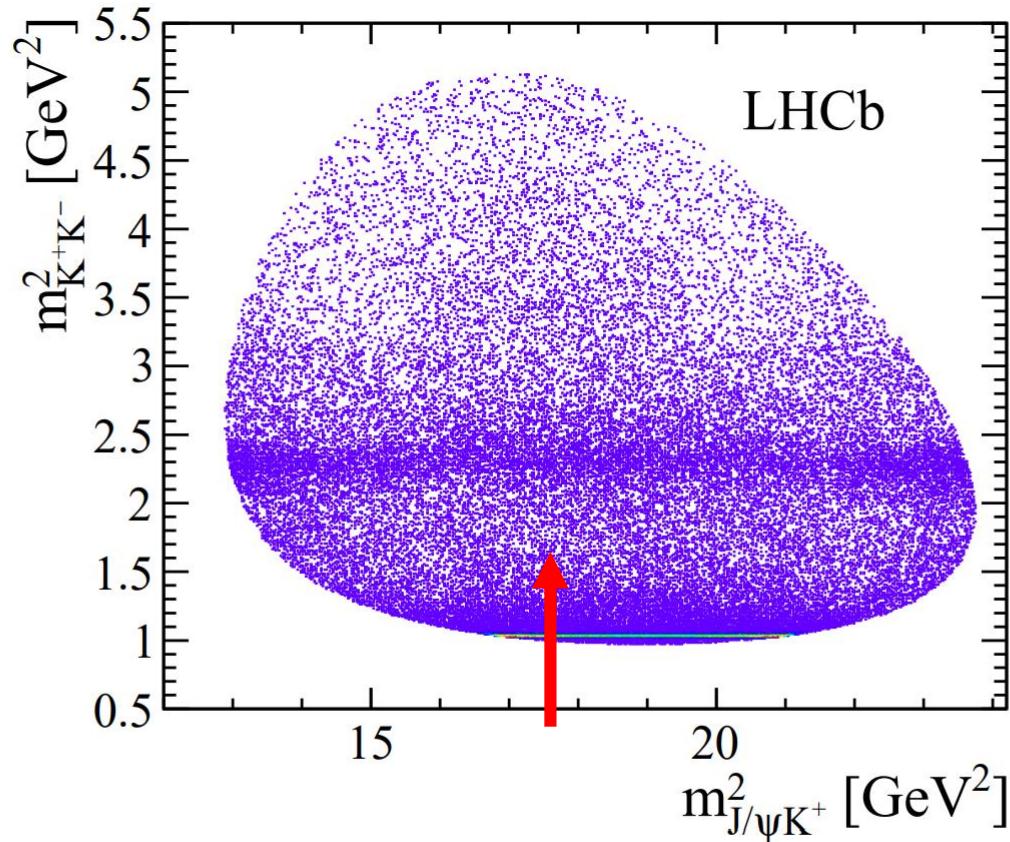


Figure 3: Invariant mass squared of  $K^+K^-$  versus  $J/\psi K^+$  for  $B_s^0 \rightarrow J/\psi K^+K^-$  candidates within  $\pm 15$  MeV of the  $B_s^0$  mass peak. The high intensity  $\phi(1020)$  resonance band is shown with a line (light green).

**LHCb: 1704.08217:** an integrated luminosity of 3  $\text{fb}^{-1}$

# Where to search for Kc(4180)

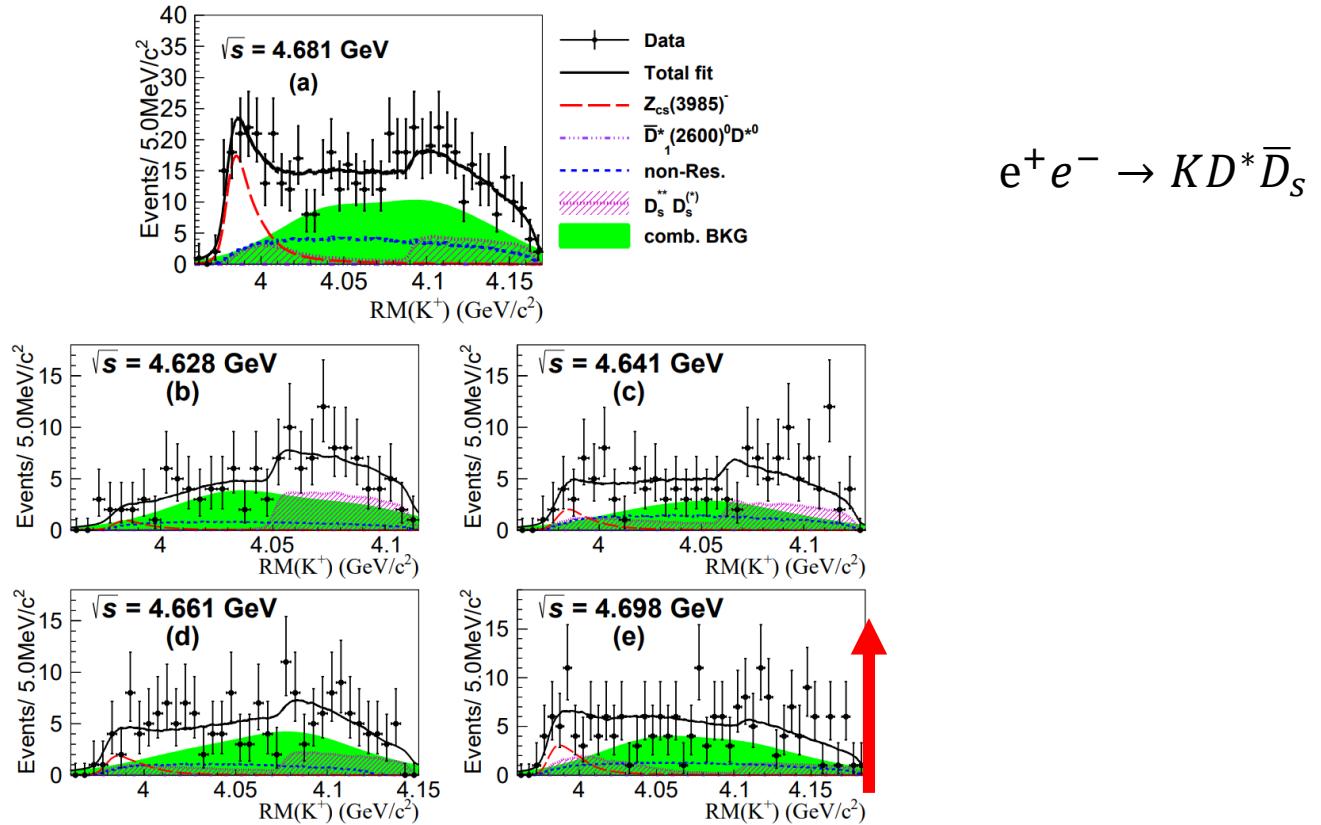


FIG. 3. Simultaneous unbinned maximum likelihood fit to the  $K^+$  recoil-mass spectra in data at  $\sqrt{s}=4.628$ , 4.641, 4.661, 4.681 and 4.698 GeV. Note that the size of the  $D^{*0} \bar{D}_1^*(2600)^0 (\rightarrow D_s^- K^+)$  component is consistent with zero.

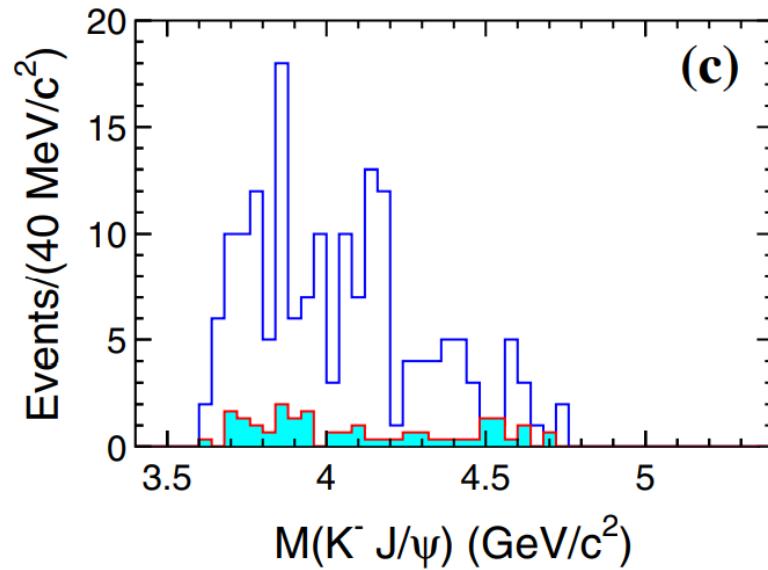
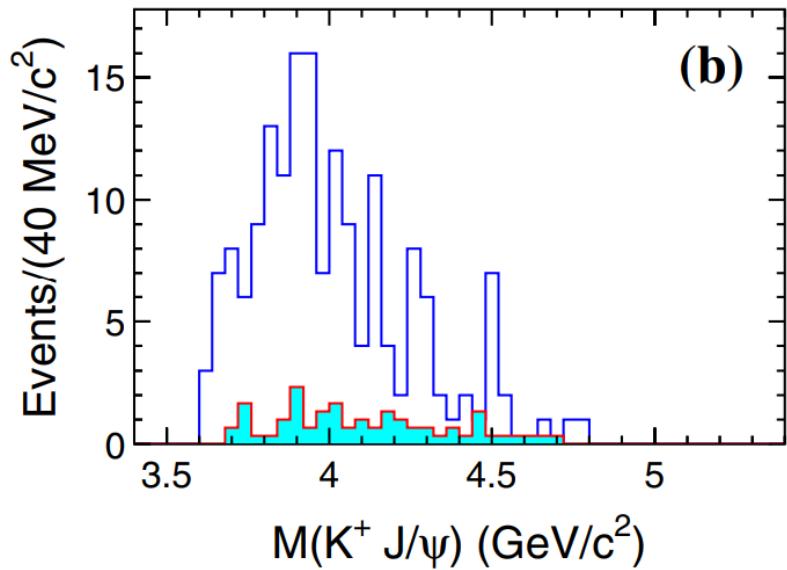
**BESIII: 2011.07855:** an integrated luminosity of  $3.7 \text{ fb}^{-1}$

# Where to search for Kc(4180)

a few pb

(ISR)  $e^+e^- \rightarrow K^+K^-J/\psi$

$4.4 < M(K^+K^-J/\psi) < 5.5 \text{ GeV}/c^2$



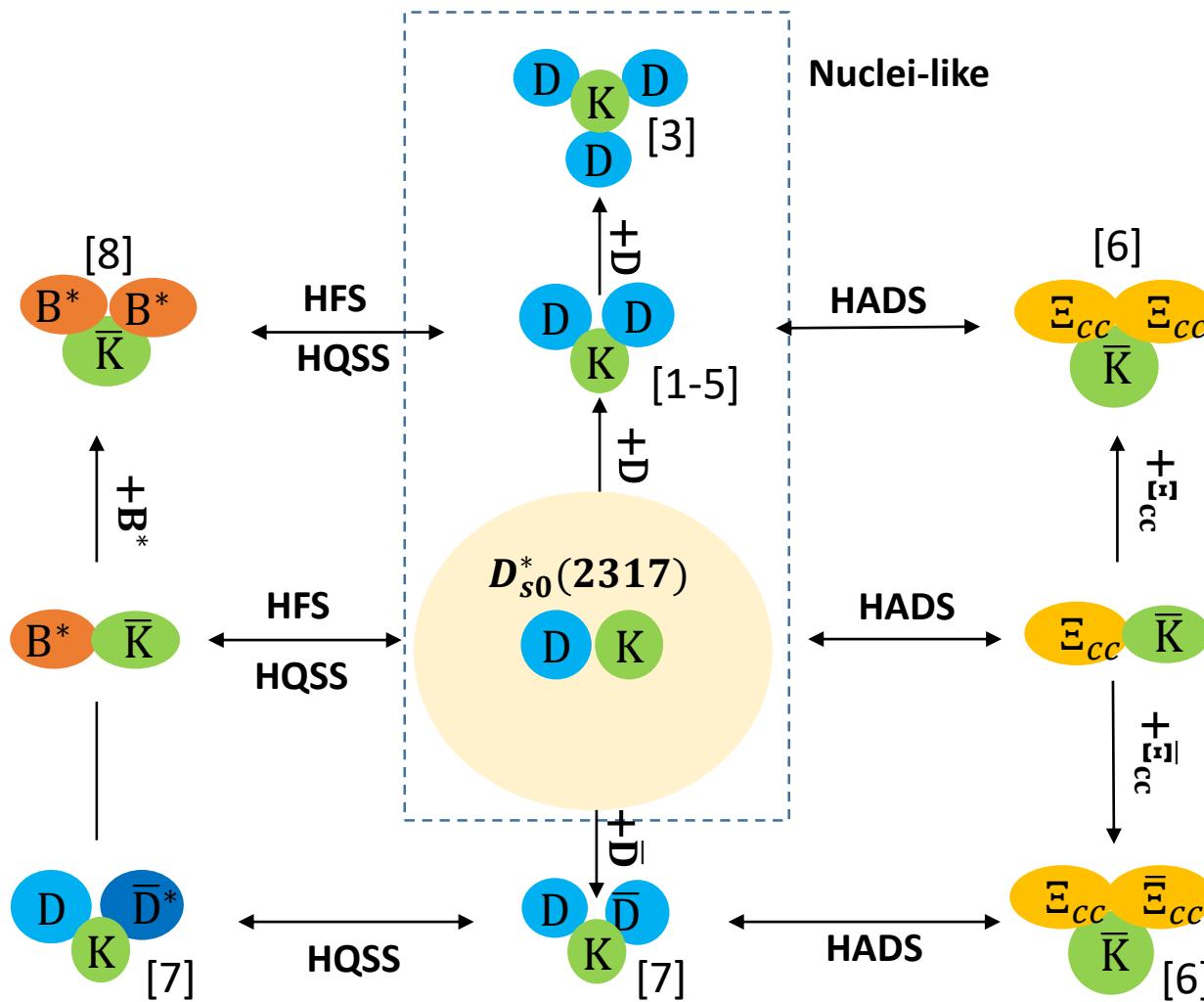
Belle: 1402.6578 : 980 fb<sup>-1</sup>

# Summary and outlook

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- Many interpretations exist for newly discovered exotic hadrons and are difficult to be distinguished from each other
- We proposed a **novel way to validate the molecule picture**.  
Taking the  $Ds0^*(2317)$  as an example, if it **is indeed a molecule of DK**, then new forms of matter may be built upon them, similar to the build up of the nuclear chart.
- We have performed **explicit few-body studies**—demonstrating that indeed both  $DDK$ ,  $D\bar{D}^*K$ ,  $D\bar{D}K$  and  $DDD\bar{K}$  states **bind**
- Now we need experimental or lattice QCD confirmations and further theoretical studies on their production and decay mechanisms

# Mutli-hadron bound states based on the $D_{s0}^*$ state as a DK molecule



[1] Sanchez Sanchez, LSG, Lu, PRD98 (2018) 054001

[2] Torres, Khemchanda, LSG, PRD99 (2019) 076017.

[3] Wu, Liu, LSG, et al., PRD100 (2019) 034029.

[4] Huang, Liu, Pan, LSG, et al., PRD101 (2020) 014022

[5] Pang, Wu, LSG, et al., PRD102 (2020) 114515.

[6] Wu, Liu, LSG, et al., EPJC80 (2020) 901.

[7] Wu, Liu, LSG, PRD103(2021)L031501

[8] Wu, Liu, LSG, et al., in preparation

道生一，一生二，二生三

# 三生万物

Thanks for your attention!

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2017 IF

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2016 IF

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# DDK system in finite volume

Jin-Yi Pang, Jia-Jun Wu , and Li-Sheng Geng, 2008.13014

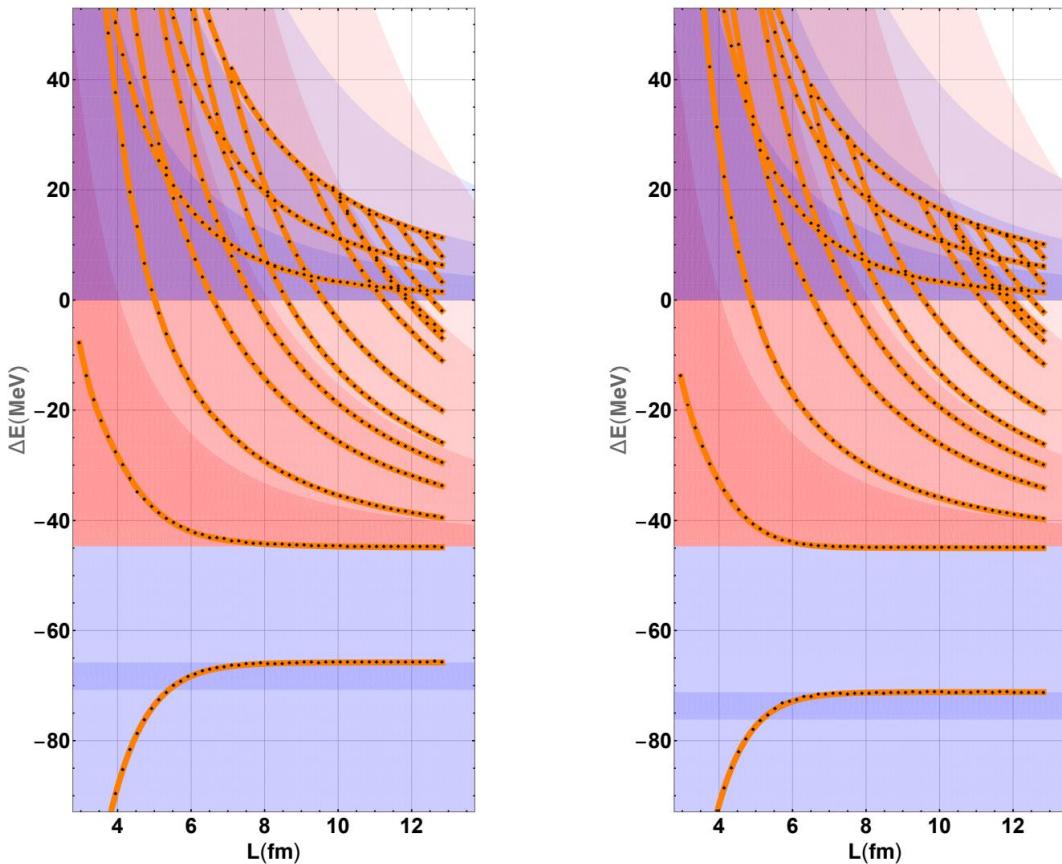


Figure 5:  $DDK$  states in finite volume. Left: only the  $DK$  interaction is considered. Right: both  $DK$  and  $DD$  interactions are taken into account. The upper blue regions indicate the case of 3 free particles in finite volume. The red regions indicate the case of free  $D_{s0}^*(2317)$  and  $D$ . The lower blue regions indicate the  $DDK$  bound state below the  $DD_{s0}^*(2317)$  threshold.