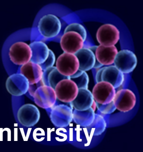
Effective Field Theory for Halo Nuclei



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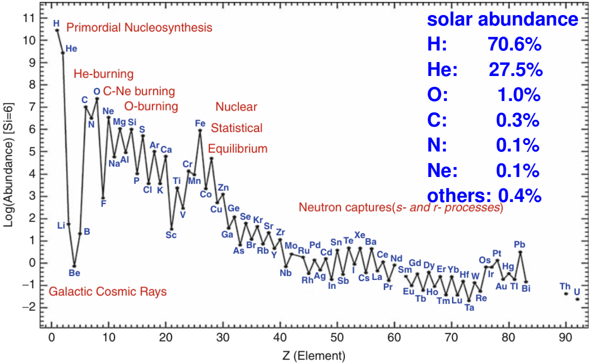
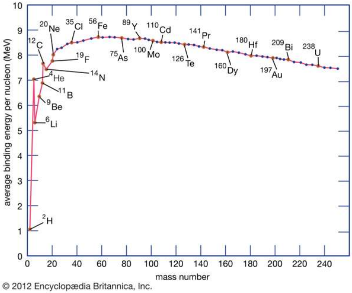
The 17th Hadron Physics Online Forum (HAPOF)
Jan. 15, 2021

Outline

- Halo nuclei and nucleosynthesis
- Introduction to halo effective field theory
- EFT description of structure and reaction in halo nuclei
- Basic Concepts
    - EFT construction
    - Universal correlations in halo nuclei
    - Connect EFT with cluster models, ab initio theories, and experiments

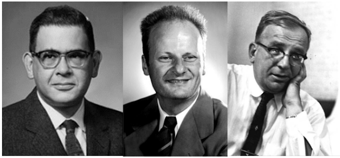
Nuclear abundance and nucleosythesis





- primordial/big-bang nucleosynthesis

Alpher, Bethe, Gamow ($\alpha\beta\gamma$)



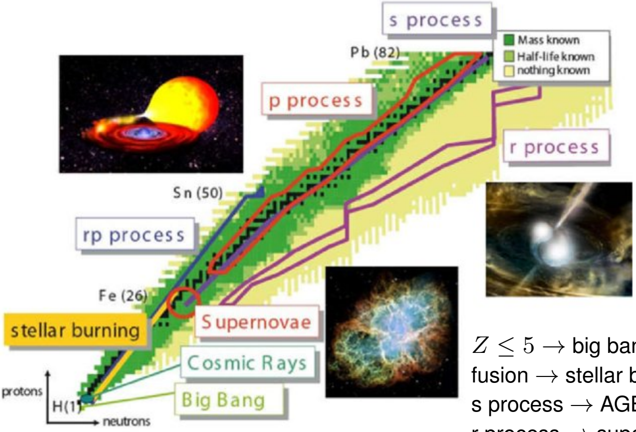
Phys. Rev. 73 (1948) 803

- stellar nucleosynthesis

Burbidge, Burbidge, Fowler, Hoyle (B^2FH)

Rev. Mod. Phys. 29 (1957) 547

Nucleosynthesis & astrophysical processes

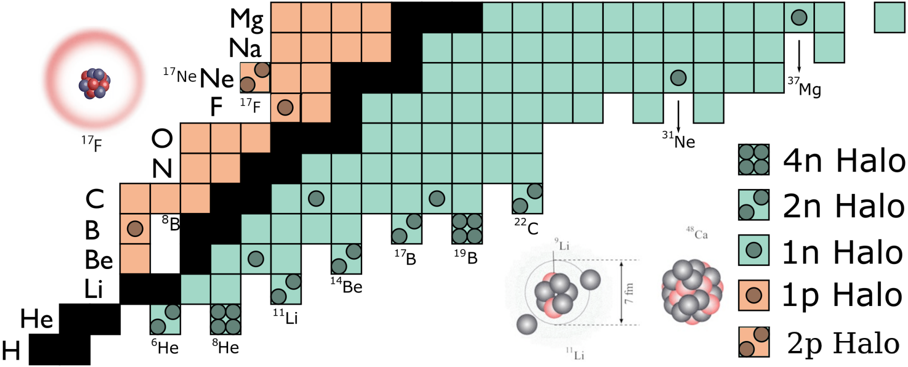


118 known elements
$\sim$3000 known isotopes
$\sim$4000 unknown isotopes

$Z \leq 5 \rightarrow$ big bang / cosmic ray
fusion $\rightarrow$ stellar burning
s process $\rightarrow$ AGB star
r process $\rightarrow$ supernovae & neutron-star merger
p & rp process $\rightarrow$ sun-neutron-star binary

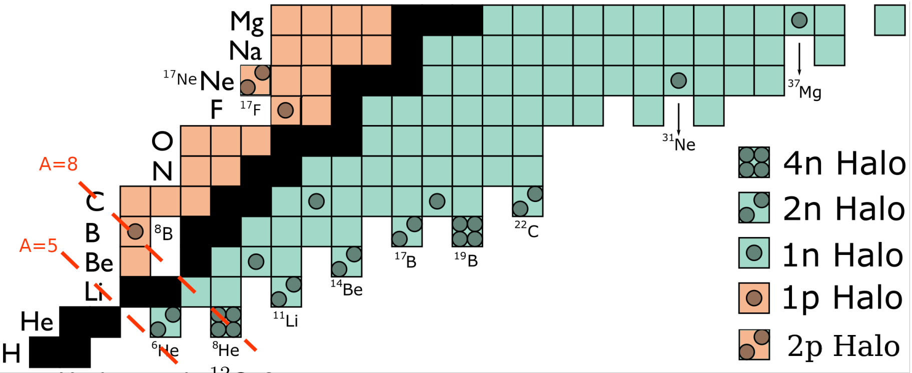
pic: Senger, Particles 3 (2020) 320

Halo nuclei

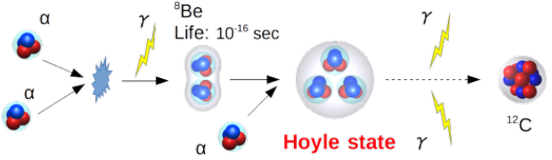


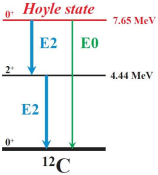
- far from stability (close to drip line)
- exhibit unique quantum features
  - light, p-rich or n-rich
  - bound/resonant states close to breakup threshold
- cluster structures
  - tight core surrounded loosely by valence nucleon(s)
  - large spatial extent
- enhanced cross section in astrophysical reaction at finite temperature

α cluster



- Hoyle state in ^{12}C: 3α resonance
- triple-α reaction is enhanced by Hoyle state
- it bridges the gap between $A = 5$ and $A = 8$ in primordial nucleosynthesis

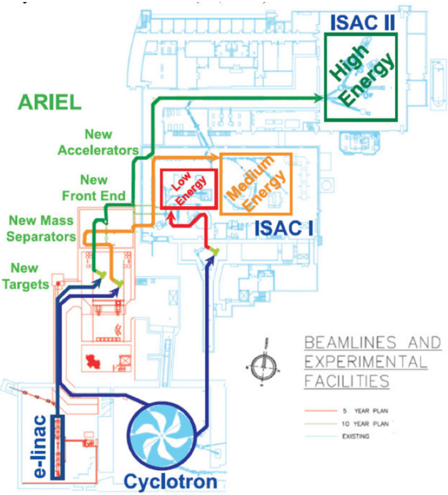




Exotic cluster/halo structures challenge experiments

Rare-Isotope Beam Facilities

- MSU-FRIB (USA)
- GANIL-SPIRAL2 (France)
- CERN-ISOLDE (EU)
- RIKEN-RIBF (Japan)
- TRIUMF-ISAC (Canada)
- HIRFL-RIBLL (China)
- GSI-FAIR (Germany) [construction]
- HIAF (China) [construction]
- RAON (Korea) [construction]
- BISOL (China) [plan]



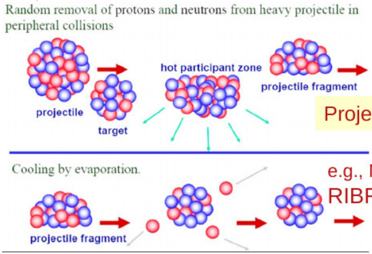
TRIUMF-ISAC

Production of rare isotopes

To produce dripline nuclei:

- In-flight fragmentation
  - heavy projectile
  - projectile fragmentation
  - 2^{nd} beam > 25MeV
  - medium/high-Q reaction
- Isotopic separation on-line (ISOL)
  - light projectile
  - target fragmentation
  - 2^{nd} beam $\sim$ MeV
  - low-Q reaction
  - precision measurements ISOTRAP, spectroscopy,

    $\ldots$



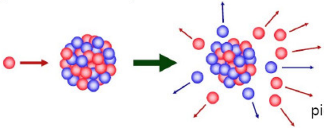
Projectile fragmentation

e.g., NSCL/MSU (US), RIBF/RIKEN (Japan)

Target fragmentation

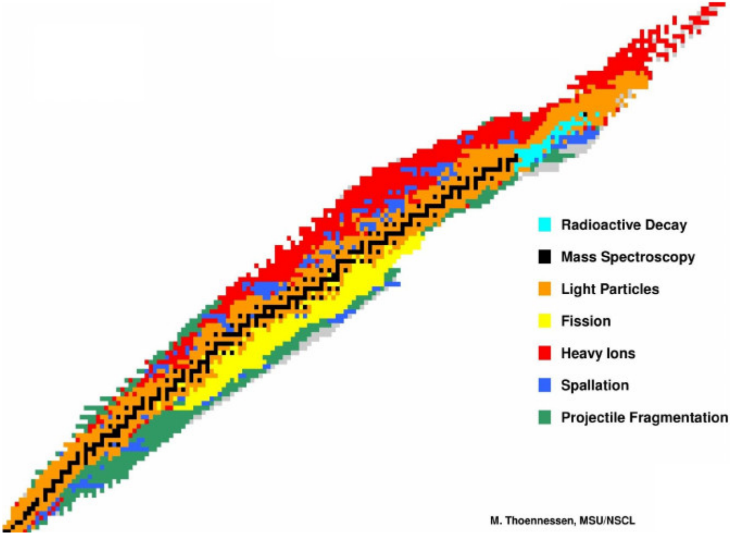
e.g., ISOLDE/CERN (Switzerland) ISAC/TRIUMF (Canada), SPIRAL2 (France)

Random removal of protons and neutrons from heavy target nuclei by energetic light projectiles (pre-equilibrium and equilibrium emissions).



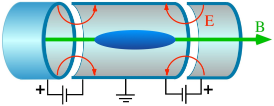
pic credit: A. Gade, MSU/FRIB

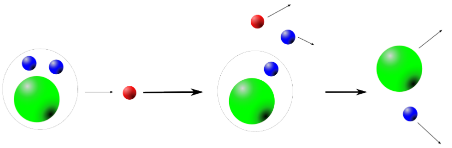
Production of rare isotopes

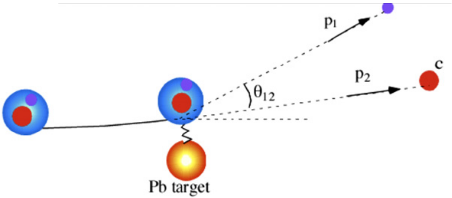


Experimental probes to halo nuclei

- static methods
  - ISOTRAP: atomic mass
  - laser spectroscopy: charge radius
  - β-NMR: μ_M & Q_E

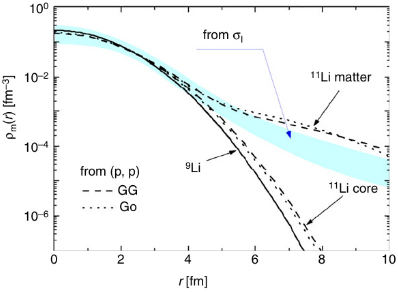






- reaction methods
  - spectroscopy by breakup
    - nuclear breakup $p(^{11}\mathrm{Li}, pn)^{10}\mathrm{Li}$
    - Coulomb breakup $^{11}\mathrm{Be}(\gamma^*, n)^{10}\mathrm{Be}$
  - spatial configuration
    - elastic scattering (p, p)
    - interaction cross section

11Li density



review: Tanihata, Savajols, Kanungo, Prog. Part. Nucl. Phys. 68 (2013) 215

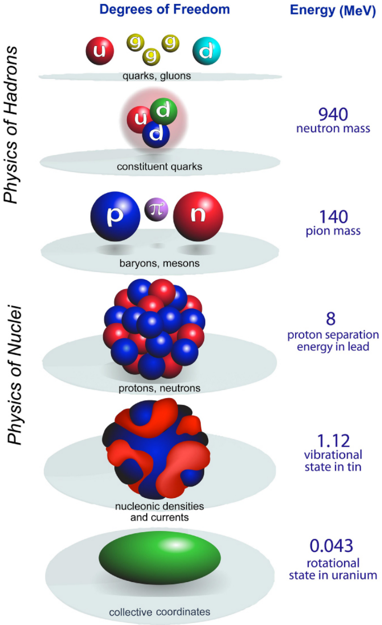
Halo nuclei challenge nuclear theories

- Three phases of halo theories
  - Back-of-the-envelope period (1985–1992)
    - "quick and dirty" estimates of halo properties by reproducing σ_R
    - gaussian spatial distribution $\rightarrow$ reproduce $\sigma_I \rightarrow R_m$ too small!
  - Few-body models period (1992–2000)
    - cluster structure models (core + valence nucleons)
    - few-body reaction models (Glauber, DWBA, CDCC,...)
    - unresolved model dependence
    - limited applicable regimes
  - Microscopic models period (2000–present)
    - ab initio structure theory
    - difficulties in computational power & extension to threshold physics
    - need to develop ab initio reaction theory (e.g. optical potential)

Halo Nuclei, Al-Khalili, Morgan & Claypool Publishers, 2017

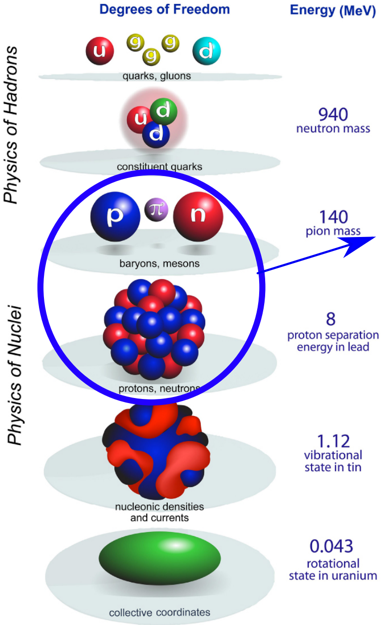
  - Effective field theory (new era)
    - systematically embed microscopic information in cluster model
    - provide guidance to build reaction theory

Introduction to effective field theory



scale hierarchy in nuclear physics

Introduction to effective field theory



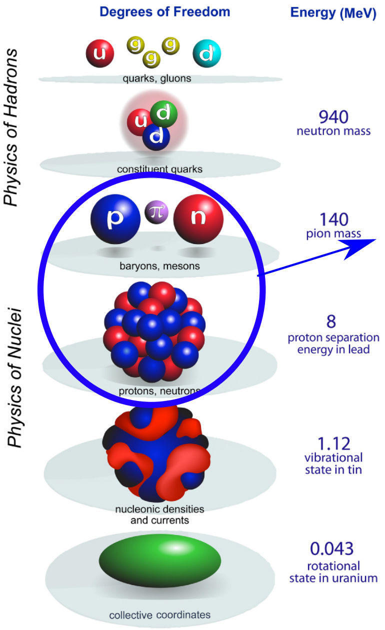
scale hierarchy in nuclear physics

underlying theory

high scale: Λ

low scale: Q

Introduction to effective field theory



scale hierarchy in nuclear physics

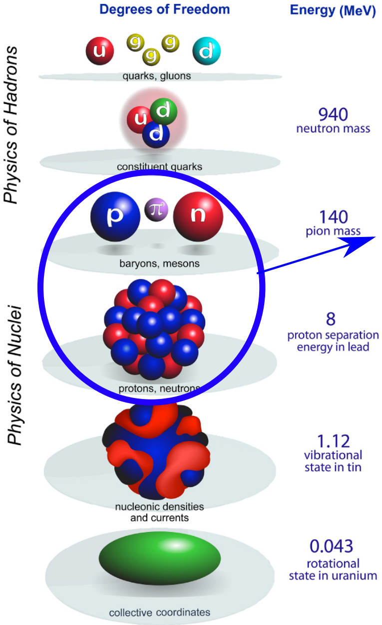
underlying theory

high scale: Λ

low scale: Q

phenomeno-
logical theory

Introduction to effective field theory



scale hierarchy in nuclear physics

underlying theory

high scale: Λ

low scale: Q

phenomeno-logical theory

effective field theory

- EFT emerges from underlying theory
- EFT “inherits” asymptotics from phenomenology

Key elements of an EFT

- Separation of scales $Q \ll \Lambda$:
  - low-energy observables $\rightarrow$ Q
  - short-range interactions $\rightarrow$ Λ

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- Systematic expansion of Lagrangian in Q/Λ:
  - order-by-order construction of effective interactions:

$$V_{\text{eff}} = \sum_n \hat{V}^{(n)}; \qquad \hat{V}^{(n)} \sim (Q/\Lambda)^{n-1}$$

  - prediction uncertainty is controlled by $(Q/\Lambda)^{(n+1)}$

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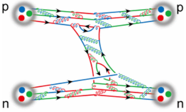
- Predict low-energy physics:
  - low-energy observables (Q) $\xrightarrow{\textbf{insensitive}}$ details of short-range interactions (Λ)
  - EFT unveils universal correlations

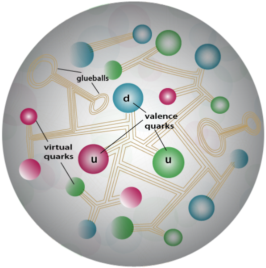
NN interaction in atomic nuclei

QCD

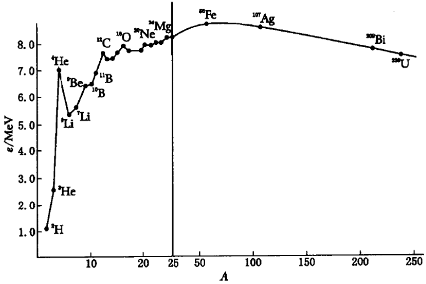


$\Lambda \sim 1$GeV

$Q \sim 100$ MeV



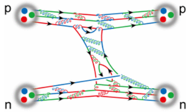
Λ: EFT breakdown scale



$Q \approx \sqrt{2M_N B/A}$: typical scale in EFT

NN interaction in atomic nuclei

QCD



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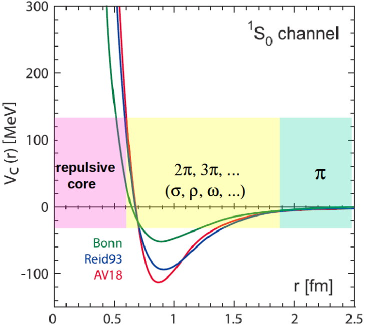
phenomenology



pic: Aoki et al. Comp. Sci. Disc. 2008

NN interaction in atomic nuclei

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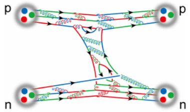

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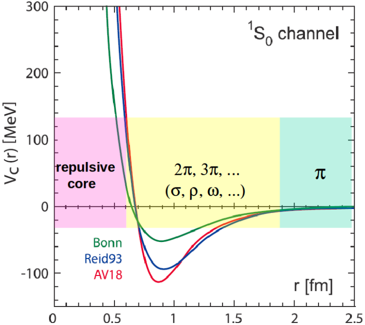
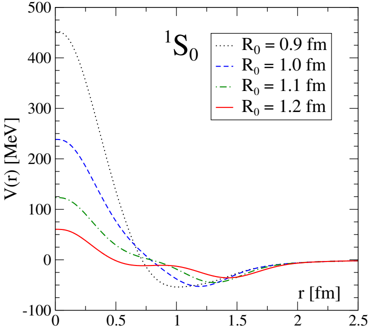


pic: Aoki et al. Comp. Sci. Disc. 2008

chiral EFT



pic: Gezerlis et al. Phys. Rev. C 2014

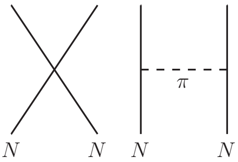
EFT with contact interactions

- Effective field theory with contact interactions originate from pionless EFT

chiral EFT NN force

- short range: $V_s = C_0$
- intermediate/long range:

$$V_{1\pi} \sim \frac{1}{q^2 + m_\pi^2}$$



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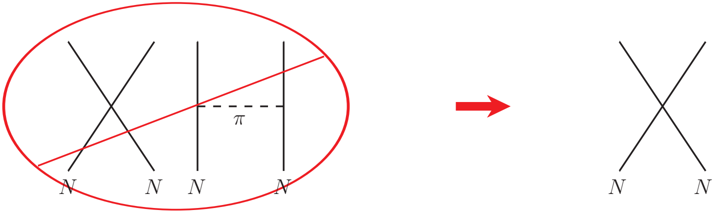
$\not{\pi}$EFT NN force

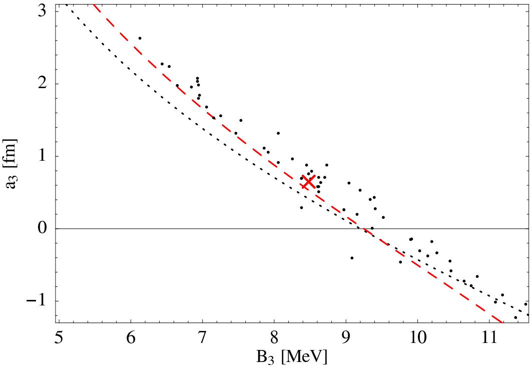
- NN momentum $q^2 \ll m_\pi^2$

$$V_{1\pi} \xrightarrow{q^2 \ll m_\pi^2} C_0 + C_2 q^2 + \cdots$$

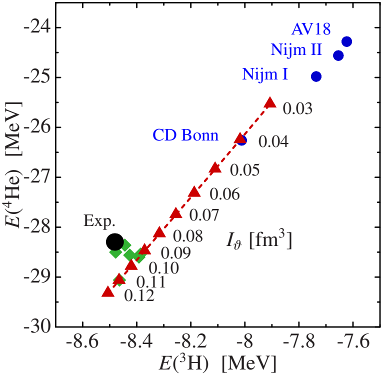


Universality in $\pi\!\!\!/\mathrm{EFT}$

Phillips Line: a_{nd} vs $B(^3\mathrm{H})$



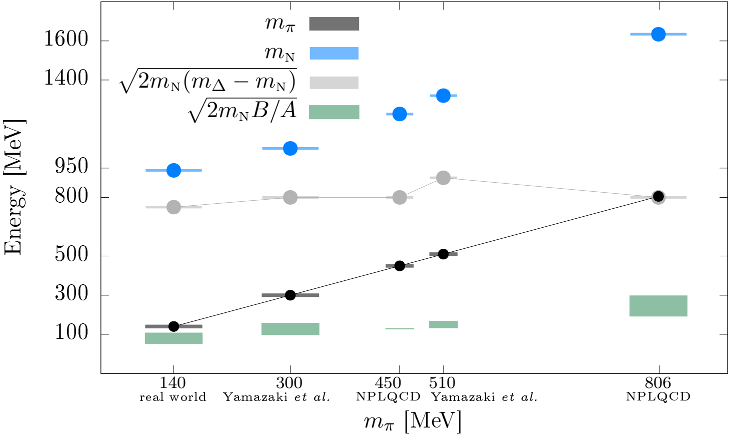
Tjon Line: $B(^3\mathrm{H})$ vs $B(^4\mathrm{He})$



- $\pi\!\!\!/\mathrm{EFT}$ indicates universal correlations among few-body observables
- long-range (low-energy) physics is insensitive to details of short-range interactions

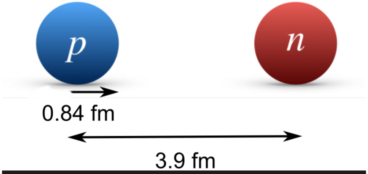
EFT with contact interactions

- Lattice simulated nuclei
  - $M_\pi \gg 140$ MeV; $Q = \sqrt{2M_N B/A} \ll m_\pi$



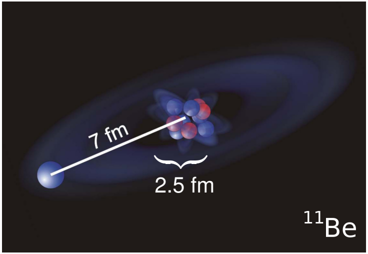
Kirscher, Int. J. Mod. Phys. E 25 (2016) 1641001

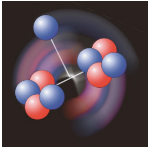
Nuclear halo and cluster

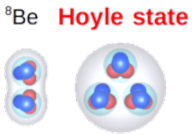


few-body molecular structure

- ^{2}H
  - simplest neutron halo
- neutron halos
  - ^{6}He, ^{11}Be, . . .
- proton halos
  - ^{17}F*, ^{8}B
- α-clustering
  - ^{9}Be: $\alpha + \alpha + n$
  - ^{8}Be, ^{12}C*





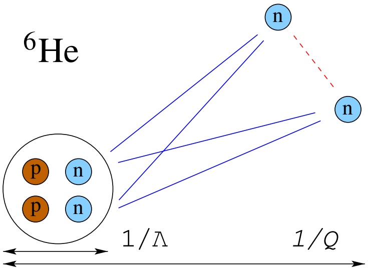


Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

$Q \sim \sqrt{m_N S_N}$



Halo physics near clustering threshold

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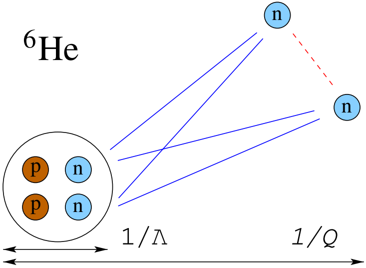
halo physics is difficult for ab initio theories

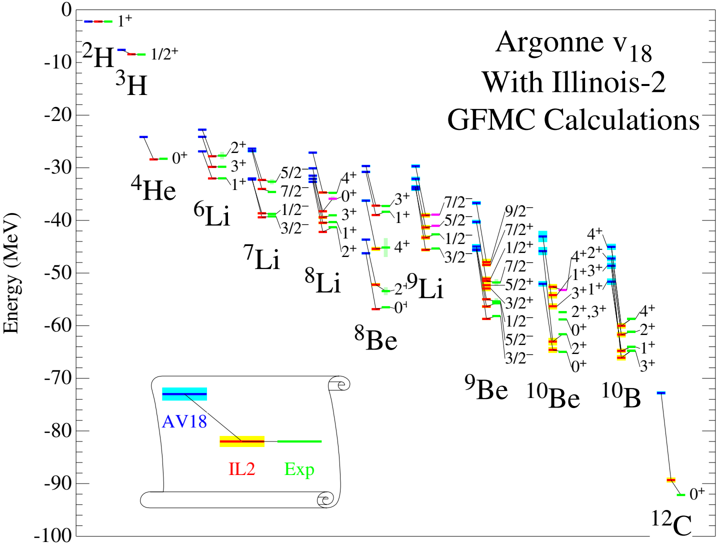
- continuum problem in many-body calculations
  NCSMC, GSM-Bergren, Lattice-EFT, LIT, $\cdots$
- uncertainty control in chiral potentials
  threshold observable converges slower in χEFT

$$\text{halo scale}: Q_{\mathrm{halo}} \ll Q_{\chi\mathrm{EFT}} \approx (2M_N B/A)^{1/2}$$

$$\text{uncertainty}: \ \Delta_{\mathrm{halo}}\% \approx \frac{Q_{\chi\mathrm{EFT}}}{Q_{\mathrm{halo}}}\left(\frac{Q_{\chi\mathrm{EFT}}}{\Lambda_{\chi\mathrm{EFT}}}\right)^{(n+1)}$$



ab initio description of nuclear spectrum



microscopic description of nuclear spectrum is in general accurate

Pieper, Wiringa, Annu. Rev. Nucl. Part. Sci. 51 (2001) 53

ab initio description of halo features

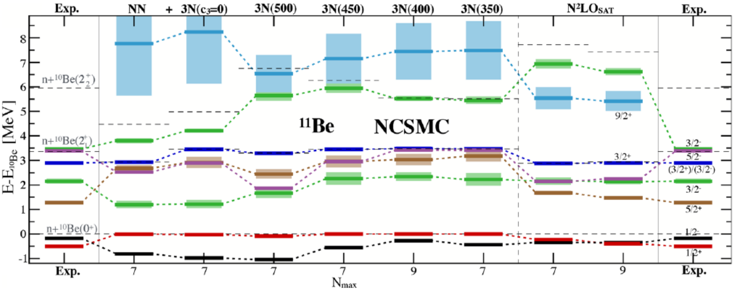


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the $n + {}^{10}$Be threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

- ab initio calculation of ^{11}Be has been done by NCSMC
- predictions of threshold properties rely significantly on the nuclear interactions

Calci et al. Phys. Rev. Lett. 117 (2016) 242501

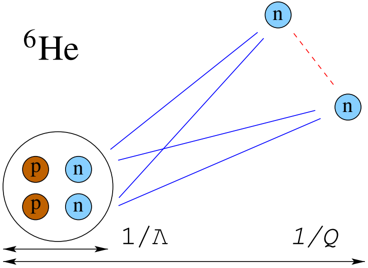
Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

$Q \sim \sqrt{m_N S_N}$

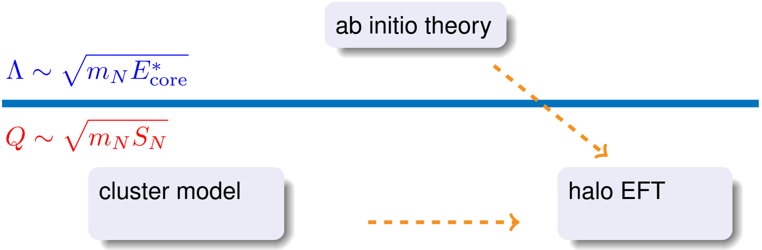
cluster model



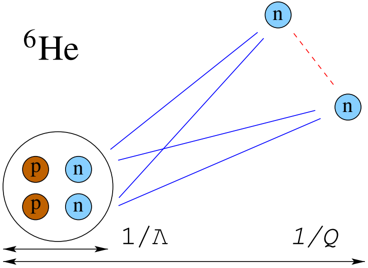
difficulties in cluster models:

- assess model dependence?
- assign theory uncertainty

Halo physics near clustering threshold



- cluster configuration in halo EFT:
  core $+$ valence nucleons d.o.f.
- separation of scales:
  $Q \ll \Lambda \to$ systematic expansion in observables
- short-range physics from underlying theory:
  anti-symmetrization of core nucleons is embedded in contact interactions



Halo Effective Field Theory

- We adopt EFT with contact interactions to describe clustering in halo nuclei
- introduce auxiliary two-body fields for bound/resonance states

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 \\
\mathscr{L}_1 &= n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c \\
\mathscr{L}_2 &= s^\dagger \left[\eta_0 \left(i\partial_0 + \frac{\nabla^2}{4m_n} \right) + \Delta_0 \right] s + \sigma^\dagger \left[\eta_1 \left(i\partial_0 + \frac{\nabla^2}{2(m_n+m_c)} \right) + \Delta_1 \right] \sigma \\
&\quad + g_0 \left[s^\dagger (nn) + \text{h.c.} \right] + g_1 \left[\sigma^\dagger (nc) + \text{h.c.} \right], \\
\mathscr{L}_3 &= h \, (\sigma n)^\dagger (\sigma n)
\end{aligned}
$$

- 2-body contact (LO)

 $= -i\sqrt{2}g$

$g \leftarrow$ 2-body observable

- 3-body contact (LO)

 $= ih$

$h \leftarrow$ 3-body observable

One-neutron s-wave halos

- **scattering amplitude:** $t_0(k) = \frac{2\pi}{\mu}\left(\frac{1}{a_0} - \frac{r_0}{2}k^2 + ik\right)^{-1}$
  - in low-energy bound/virtual state: $a_0 \sim 1/Q$; $r_0 \sim 1/\Lambda$
  - expand t-matrix in r_0/a_0

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One-neutron s-wave halos

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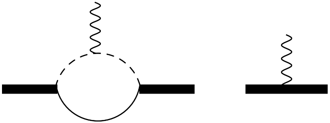
- **tune coupling**
  - LO: $a_0 = \left(\dfrac{2\pi\Delta}{\mu g^2} + \Lambda\right)^{-1}$ NLO: $r_0 = -\eta\dfrac{2\pi}{\mu^2 g^2}$
- **pole expansion**: $t_0(k) \approx \dfrac{2\pi}{\mu}\dfrac{\mathcal{Z}_R}{\gamma_0 + ik}$
  - ANC: $\psi_0(\boldsymbol{r}) = \dfrac{C_\sigma}{\sqrt{4\pi r}}\exp(-\gamma_{0,\sigma} r)$
  - LO: $C_{\sigma,LO} = \sqrt{2\gamma_0}$ NLO: $C_{\sigma,NLO} = \sqrt{\dfrac{2\gamma_0}{1-\gamma_0 r_0}}$
  - renormalization constant $\mathcal{Z}_R = \dfrac{C^2_{\sigma,NLO}}{C^2_{\sigma,LO}} = \dfrac{1}{1-\gamma_0 r_0}$

One-neutron s-wave halos

	^{2}H	^{11}Be	^{15}C	^{19}C
Experiment				
S_{1n} [MeV]	2.224573(2)	0.50164(25)	1.2181(8)	0.58(9)
E_c^* [MeV]	140	3.36803(3)	6.0938(2)	1.62(2)
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.936(12)	6.05(23)	4.15(50)	6.6(5)
	3.95014(156)	5.7(4)	7.2±4.0	6.8(7)
		5.77(16)	4.5(5)	5.8(3)
Halo EFT				
Q/Λ	0.33	0.39	0.45	0.6
r_0/a_0	0.32	0.32	0.43	0.33
$\sqrt{\mathcal{Z}_R}$	1.295	1.3	1.63	1.3
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.954	6.85	4.93	5.72

Electric form factor → radius $\langle r_{nc}^2 \rangle^{1/2}$

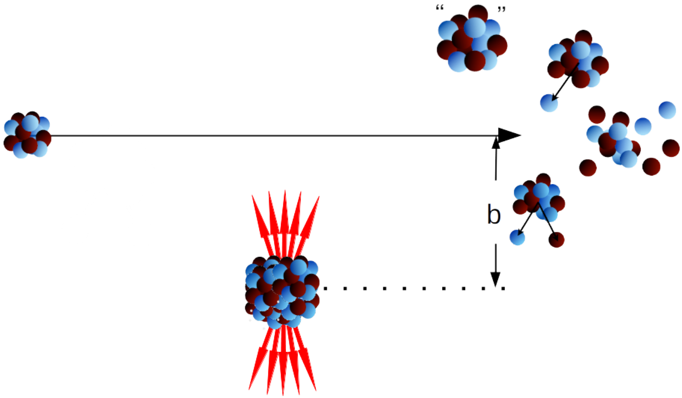


$$F_{nc}(q^2) = \mathcal{Z}_R \frac{2\gamma_0}{q} \arctan\left(\frac{q}{2\gamma_0}\right) + 1 - \mathcal{Z}_R$$

$$F_{nc}(q^2) = 1 - \frac{1}{6}\langle r_{nc}^2 \rangle q^2 + \mathcal{O}(q^4),$$

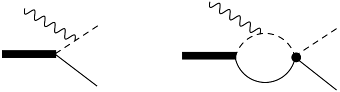
Coulomb dissociation in $1n$ halos

- Coulomb dissociation
  - breakup by colliding a halo nucleus with a high-Z nucleus
  - the halo dynamics dominates when $E_\gamma \sim S_{1n}$

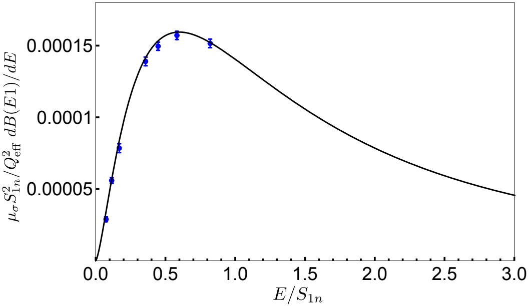


EFT on Coulomb dissociation

- **E1 transition**



$$\frac{dB(E1)}{dE_\gamma} = \frac{1}{(2\pi)^3}\left(|\mathcal{M}_{E1}^{(J=1/2)}|^2 + |\mathcal{M}_{E1}^{(J=3/2)}|^2\right)\frac{d^3p}{dE_\gamma} = \frac{\mathcal{Z}_R Q_{\rm eff}^2}{\mu_{nc} S_{1n}^2}\frac{3\alpha_{em}}{\pi^2}\frac{(E_\gamma/S_{1n})^{3/2}}{(E_\gamma/S_{1n}+1)^4}.$$

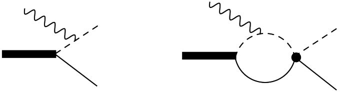


deuteron E1 strength

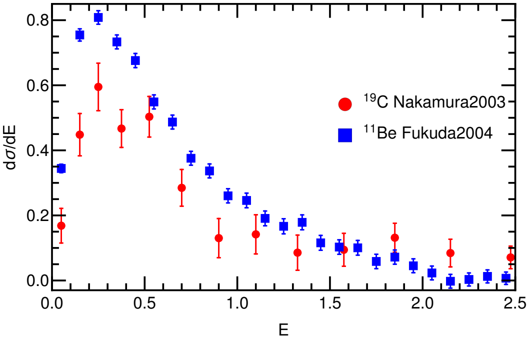
Hammer, CJ, Phillips, JPG 44 (2017) 103002

EFT on Coulomb dissociation

- **E1 transition**



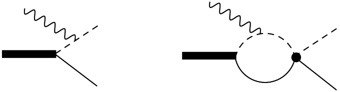
$$\frac{d\sigma}{dE_\gamma} = \frac{16\pi^3}{9} N_{E1}(E_\gamma, R) \qquad \frac{dB(E1)}{dE_\gamma}$$



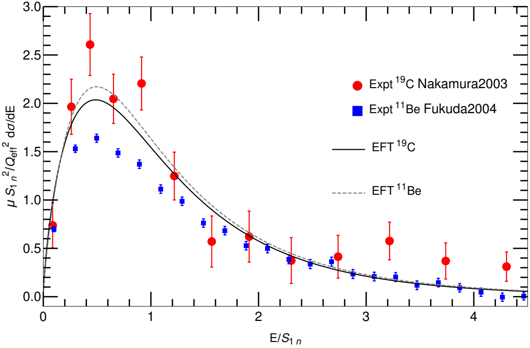
Coulomb dissociation energy spectrum in ^{11}Be and ^{19}C

EFT on Coulomb dissociation

- **E1 transition**



$$\frac{\mu_{nc} S_{1n}^2}{\mathcal{Z}_R Q_{\rm eff}^2}\frac{d\sigma}{dE_\gamma} = \frac{16\pi^3}{9} N_{E1}(E_\gamma, R)\,\frac{\mu_{nc} S_{1n}^2}{\mathcal{Z}_R Q_{\rm eff}^2}\frac{d\mathrm{B(E1)}}{dE_\gamma}$$



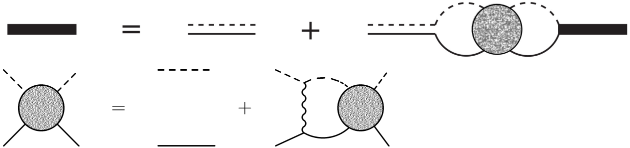
Coulomb dissociation energy spectrum in ^{11}Be and ^{19}C $\rightarrow$ Universality!

^{11}Be: fit ANC in NCSMC [Calci *et al.* '16]

Hammer, Phillips, NPA '11
Acharya, Phillips, NPA '13
Hammer, CJ, Phillips, JPG '17

Halo EFT with Coulomb

- In halo/clustering systems with Coulomb interactions, a new scale $k_c = Q_c \alpha_{em} \mu$ enters
  - $k_c \gtrsim Q$: Coulomb interaction is nonperturbative



  - Coulomb Green's function (non-perturbative)

$$(\boldsymbol{r}|G_C(E)|\boldsymbol{r}') = \int \frac{d^3p}{(2\pi)^3} \frac{\psi_{\boldsymbol{p}}(\boldsymbol{r})\psi^*_{\boldsymbol{p}}(\boldsymbol{r}')}{E - \boldsymbol{p}^2/(2\mu_{nc}) + i\epsilon}$$

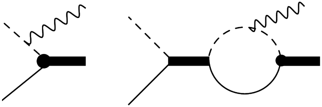
$$\psi_{\boldsymbol{p}}(\boldsymbol{r}) = \sum_{l=0}^{\infty} (2l+1) i^l \exp(i\sigma_l) \frac{F_l(\eta,\rho)}{\rho} P_l(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{r}})$$

p-p scattering [Kong, Ravndal, PLB '99; NPA '10]
p-α and α-α scattering [Higa, Hammer, van Kolck, NPA '08; Higa, FBS '11]
^{17}F* [Ryberg, Forssén, Hammer, Platter, PRC '14; AnnPhys '16]

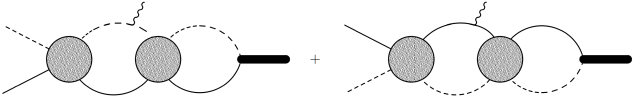
  - $k_c \ll Q$: Coulomb interaction is perturbative
^{3}H and ^{3}He [König, Grießhammer, Hammer, van Kolck, JPG '16]

Radiative Nucleon Captures

neutron captures



proton captures



$$\frac{d\sigma}{d\Omega} = \frac{\mu_{nc} E_\gamma}{8\pi^2 p} \sum_{i=1}^{2} \left| \epsilon_i \cdot \frac{\mathcal{M}}{\sqrt{\Sigma'(-B)}} \right|^2$$

$^7\mathrm{Li} + n \to {}^8\mathrm{Li} + \gamma$

Rupak, Higga, PRL '11; Fernando, Higa, Rupak, EPJA '12; Zhang, Nollett, Phillips, PRC '14

$^{14}\mathrm{C} + n \to {}^{15}\mathrm{C} + \gamma$

Rupak, Fernando, Vaghani, PRC '12

$^7\mathrm{Be} + p \to {}^8\mathrm{B} + \gamma$

Zhang, Nollett, Phillips, PRC '14; Ryberg, *et al.* EPJA '14

$^{16}\mathrm{O} + p \to {}^{17}\mathrm{F}^*(1/2^+) + \gamma$

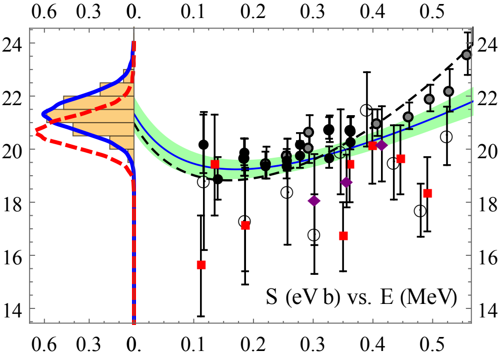
Ryberg, Forssén, Hammer, Platter, PRC '14; AnnPhy '16

Radiative Nucleon Captures

astrophysical S-factor for ^{7}Be$(p, \gamma)^{8}$B

EFT(NLO): VMC ANC + Bayesian error analysis

Zhang, Nollett, Phillips, Phys. Lett. B 751 (2015) 535

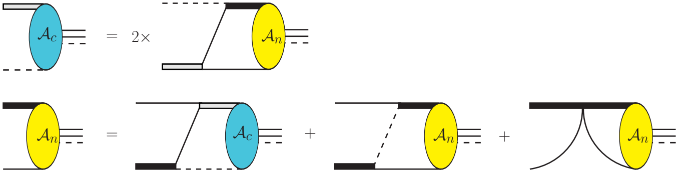


$$S(E) = E e^{2\pi\eta(E)} \sigma(E)$$

$$S(0) = 21.3 \pm 0.7 \text{ eV} \cdot \text{b}$$

$2n$ halos in Faddeev formalism

- **solving transition amplitudes $\mathcal{A}_c$ and $\mathcal{A}_n$**



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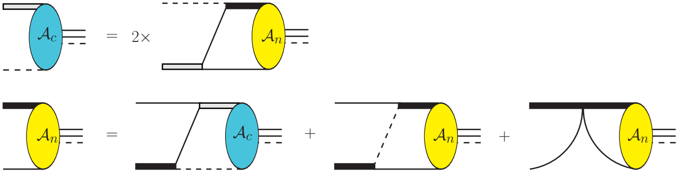
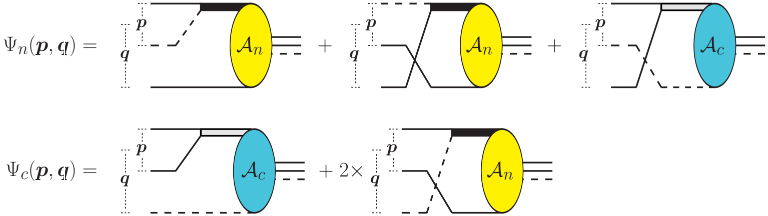


- **three-body wave functions**

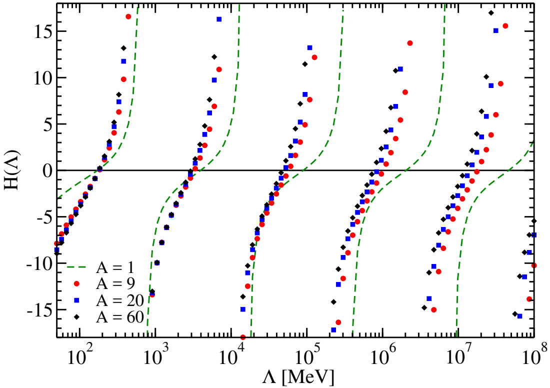


Three-body renormalization

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h / 2mg^2$:
    reproduce one observable in a $2n$-halo

Hammer, CJ, Phillips,

JPG 44 (2017) 103002



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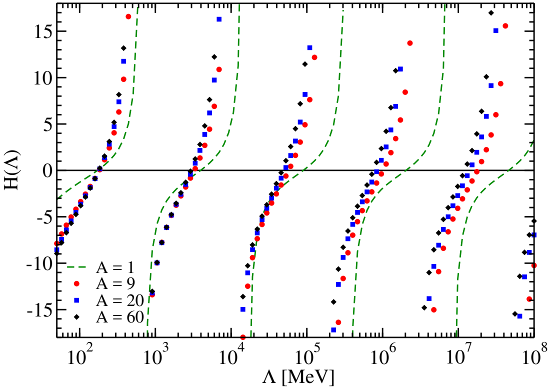
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]



Hammer, CJ, Phillips,

JPG 44 (2017) 103002

Three-body renormalization

- **running of three-body coupling**
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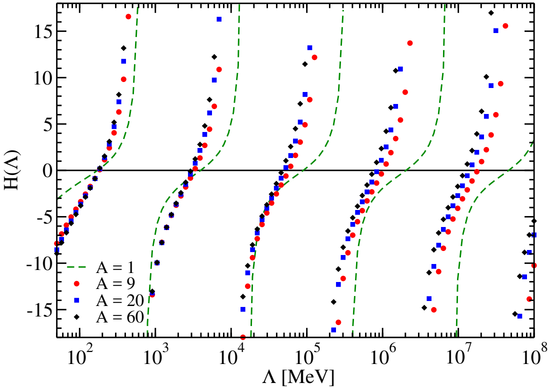
    reproduce one observable in a $2n$-halo
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]
  - $H(\Lambda)$ appears as RG limit cycle [Mohr et al., AnnPhys '06]
  - discrete scale invariance $\to$ Efimov physics



Hammer, CJ, Phillips,

JPG 44 (2017) 103002

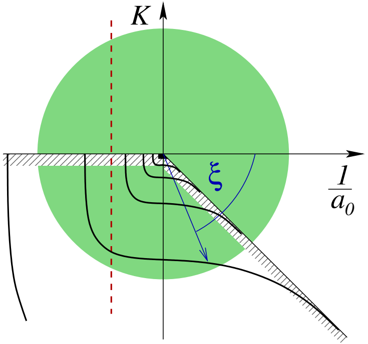
Efimov physics

- a universal spectrum of three-body bound states

$$B_3 = -\frac{1}{ma_0^2} + \left[e^{-2\pi n}\, f(\xi)\right]^{1/s_0}\ \frac{\kappa_*^2}{m}$$

Braaten, Hammer, Phys. Rept. '06

  - atomic physics: vary a_0 through Feshbach resonance
  - nuclear physics: fixed a_0



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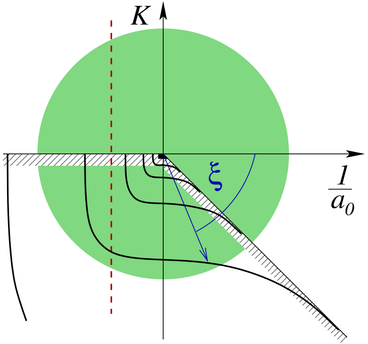
Braaten, Hammer, Phys. Rept. '06

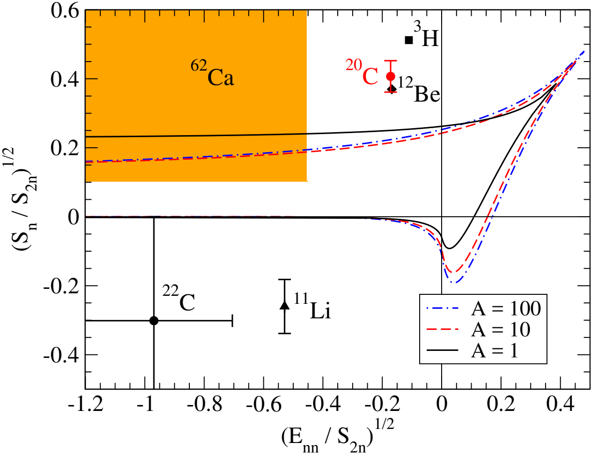
  - atomic physics: vary a_0 through Feshbach resonance
  - nuclear physics: fixed a_0

- unitary limit ($a \to \infty$):
  $B_3 = e^{-2\pi n/s_0}\, \frac{\kappa_*^2}{m}$
- discrete scale invariance:
  $\kappa_* \to \kappa_*, \quad a_0 \to e^{\pi n/s_0} a_0$
- exploring Efimov physics in halo nuclei is an important subject



Efimov universality in $2n$ s-wave halo

- contour constraints on ground-state energy S_{2n} if the excited-state energy $B_3^* = \max\{0, E_{nn}, S_{1n}\}$



Canham, Hammer, EPJA '08; Frederico *et al.* PPNP '12;

Hammer, CJ, Phillips, JPG 44 (2017) 103002

^{22}C: $2n$ Halo

	^{20}C	^{21}C	^{22}C
bound/unbound	bound	unbound	bound
ground state	0^+	$S_{1/2}$	0^+
binding/virtual energy [MeV]	S_{1n}: $2.93(26)$ AME2012	S_{1n}: $-0.01(47)$ AME2012 < -2.9 Mosby et al. '13	S_{2n}: $0.11(6)$ AME2012 S_{2n}: $-0.14(46)$ Gaudefroy et al. '12
matter radius r_m [fm]	$2.97^{+0.03}_{-0.05}$ Togano et al. '16		3.44(8) Togano et al. '16

- Halo EFT

  we fit to ^{22}C matter radius to constrain:

  - S_{1n} in ^{21}C ($a < 0$)
  - S_{2n} in ^{22}C

Correlations in ^{22}C

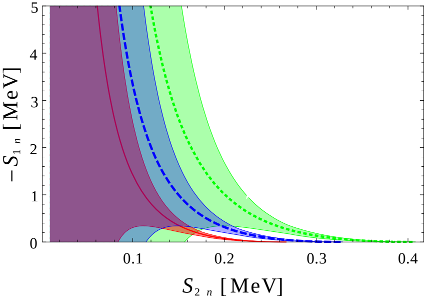
$$\langle r_m^2 \rangle_{2n-\text{halo}} - \frac{A}{A+2} \langle r_m^2 \rangle_{\text{core}} = \mathcal{F}\left(S_{2n}, S_{1n}\right)$$

Acharya, C.J., Phillips, PLB '13

new experimental input:

$$\langle r_m^2 \rangle_{2n-\text{halo}} - \frac{10}{11} \langle r_m^2 \rangle_{\text{core}} = 3.81^{+0.82}_{-0.71} \text{ fm}^2$$

Togano *et al.* PLB '16



Hammer, CJ, Phillips JPG 44 (2017) 103002

bands: uncertainty from NLO EFT

$$\sim \max\left\{ \frac{\sqrt{mE_{nn}}}{\Lambda}, \frac{\sqrt{mS_{1n}}}{\Lambda}, \frac{\sqrt{mS_{2n}}}{\Lambda} \right\}$$

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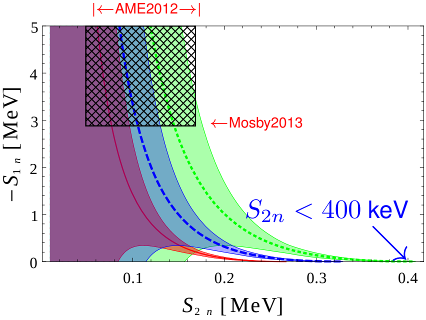


bands: uncertainty from NLO EFT

$$\sim \max\left\{ \frac{\sqrt{mE_{nn}}}{\Lambda}, \frac{\sqrt{mS_{1n}}}{\Lambda}, \frac{\sqrt{mS_{2n}}}{\Lambda} \right\}$$

Other experimental bound:

- AME2012

  $S_{2n} = 110(60)$ keV

- Mosby2013

  $-S_{1n} > 2.9$ MeV

Hammer, CJ, Phillips JPG 44 (2017) 103002

^{6}He: $2n$ Halo with p-wave nc interactions

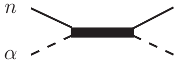
- ***ab intio* calculation**
  - no-core shell model Navrátil *et al.* ’01; Sääf, Forssén ’14
  - NCSM-RGM/Continuum Romero *et al.* ’14 ’16
  - Green’s function Monte Carlo Pieper *et al.* ’01; ’08
  - hyperspherical harmonics (EIHH) Bacca *et al.* ’12

- **Halo EFT in ^{6}He ground state**
  - EFT+Gamow shell model (GSM) Rotureau, van Kolck Few Body Syst. ’13
  - EFT+Faddeev equation C.J., Elster, Phillips, PRC ’14;
    Göbel, Hammer, CJ, Phillips, Few Body Syst. 60 (2019) 61

- $^{6-10}$He effective interaction + GSM Fossez, Rotureau, Nazarewicz, PRC ’18

P-wave neutron halos

- nc interaction in a p-wave bound/resonance state



$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

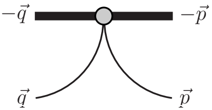
  - $a_1 < 0$: shallow resonance: ^{5}He $(3/2^-)$
  - $a_1 > 0$: shallow bound state: ^{11}Be $(1/2^-)$, ^{8}Li (2^+), ^{8}Li* (1^+)

- p-wave power counting
  - resum ik^3: $1/a_1 \sim Q^3$, $r_1 \sim Q$ [Bertulani, Hammer, van Kolck NPA '02]
  - perturbative ik^3: $1/a_1 \sim Q^2\Lambda$, $r_1 \sim \Lambda$ [Bedaque, Hammer, van Kolck PLB '03]

Running of 3BF Coupling

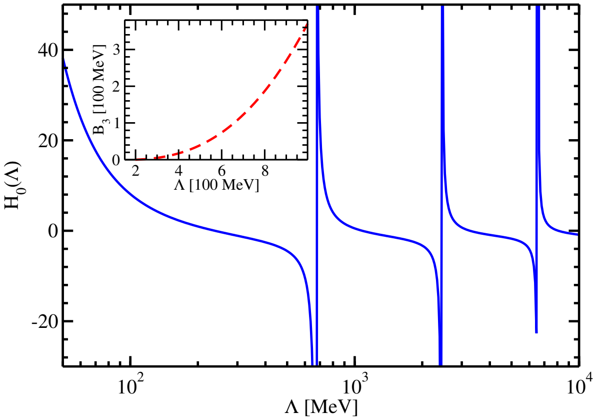
- p-wave 3BF:

  reproduce $S_{2n} = 0.973$ MeV



$$= M_n \, qp \, \frac{H(\Lambda)}{\Lambda^2}$$

- discrete scaling symmetry is broken due to p-wave interactions



Momentum-space probability density

- The probability density:

$$\rho_i(\boldsymbol{p},\boldsymbol{q}) = \langle\Psi|\boldsymbol{p},\boldsymbol{q}\rangle_i\,{}_i\langle\boldsymbol{p},\boldsymbol{q}|\Psi\rangle$$

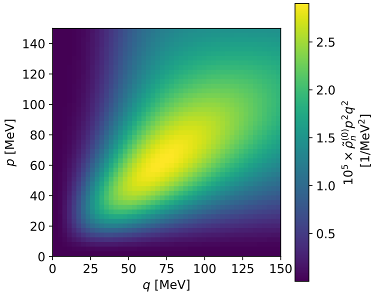
- ℓth moment (partial-wave decomposition)

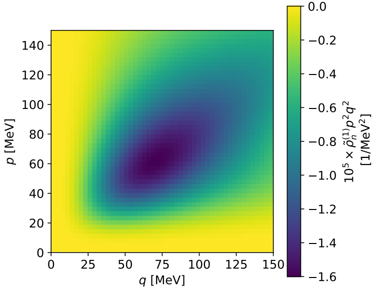
$$\rho_i^{(l)}(p,q) = \int_{-1}^{1} dx P_\ell(x)\rho_i(\boldsymbol{p},\boldsymbol{q})$$

$$x = \frac{\boldsymbol{p}\cdot\boldsymbol{q}}{pq}$$

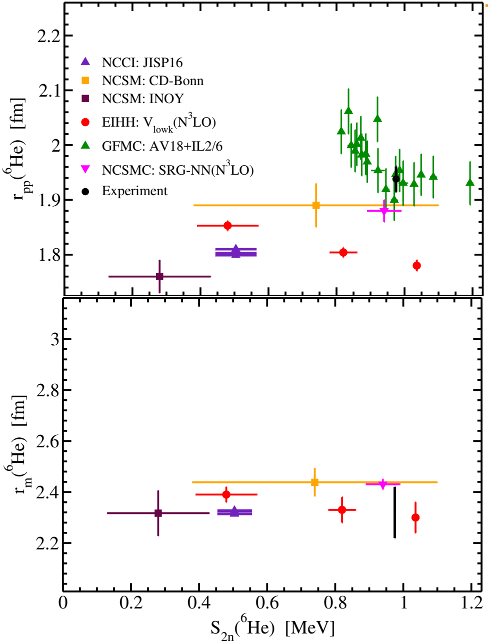
- $\rho_n^{(0)}(p,q)$ and $\rho_n^{(1)}(p,q)$

Göbel, Hammer, CJ, Phillips,
Few Body Syst. 60 (2019) 61





Universal Correlations Among Radii & S_{2n} in ^{6}He



- **He-6 charge radius**
- **He-6 matter radius**

compare with ab initio calculations

NCCI: Caprio, Maris, Vary, PRC ’14

NCSM: Caurier, Navratil, PRC ’06

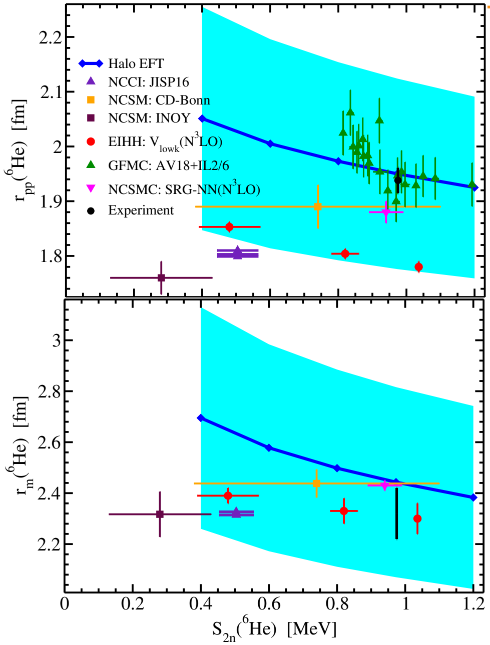
EIHH: Bacca, Barnea, Schwenk, PRC ’12

GFMC: Pieper, RNC ’08

NCSMC: Romero et al., PRL ’16

Halo EFT: preliminary (uncertainty)

Universal Correlations Among Radii & S_{2n} in ^{6}He



- He-6 charge radius
- He-6 matter radius

compare with ab initio calculations

NCCI: Caprio, Maris, Vary, PRC '14

NCSM: Caurier, Navratil, PRC '06

EIHH: Bacca, Barnea, Schwenk, PRC '12

GFMC: Pieper, RNC '08

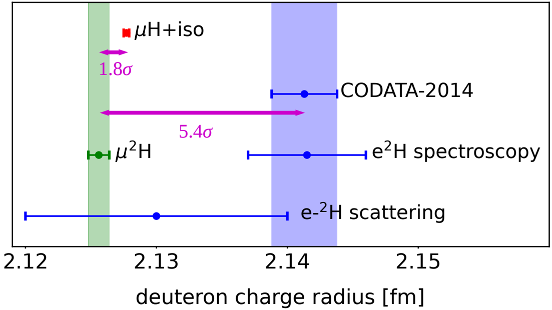
NCSMC: Romero et al., PRL '16

Halo EFT: preliminary (uncertainty)

Nuclear polarization effects in muonic deuterium

- Halo EFT formalism is suitable for study deuteron system ($\not\pi\mathrm{EFT}$)
- Nuclear polarization δ_{pol} plays crucial role in muonic deuterium Lamb shift
- It is important for the experimental determination of deuteron charge radius

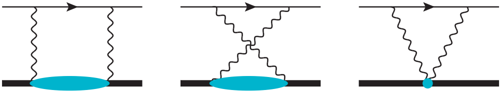
uncertainty dominated by δ_{pol}



deuteron charge radius from Lamb shift in muonic deuterium:
Pohl, *et al.*, Science (2016)

$\delta_{\rm pol}$ from virtual Compton tensor

- $\delta_{\rm pol}$ is originated in two-photon exchange



- $\delta_{\rm pol}$ is dominated by longitudinal response function

$$\delta_{\rm pol} \propto \iint \mathrm{d}q\, \mathrm{d}\omega\, g(\omega, q) S_{\rm L}(\omega, q)$$

$$g(\omega, q) = \frac{1}{2E_q}\left[\frac{1}{(E_q - m_\mu)(E_q - m_\mu + \omega)} - \frac{1}{(E_q + m_\mu)(E_q + m_\mu + \omega)}\right]$$

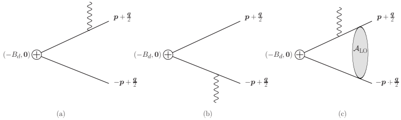
- response function S_L is related to virtual-compton transition matrix elements

$$S_L(\omega, \boldsymbol{q}) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \delta\left(\omega - B_d - \frac{q^2}{4m_N} - \frac{p^2}{m_N}\right) \overline{|\mathcal{M}|^2}$$

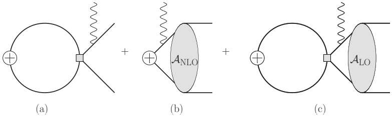
deuteron virtual compton amplitude at NNLO

- compton amplitude is summed to all EM multipolarity

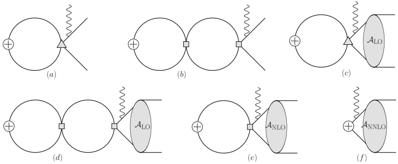
LO



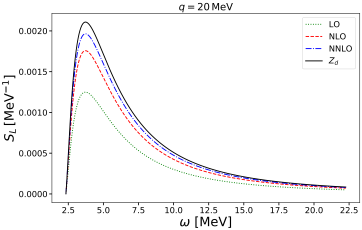
NLO

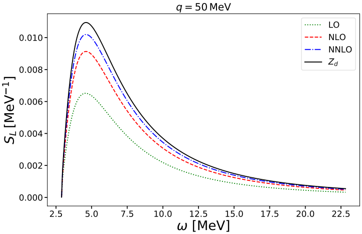


NNLO



- order-by-order convergence
- 3% accuracy at NNLO





Emmons, CJ, Platter, arXiv:2009.08347

Summary

- Halo nuclei play crucial role in nucleosynthesis, and is important for understanding nuclear astrophysical processes

- Effective field theory comes with limited powers determined by Q and Λ, different EFTs may be efficient at different energy regimes

- Halo EFT describes near-threshold physics in halo nuclei in a controlled expansion in Q/Λ

- Halo EFT rejuvenate cluster models with a systematic uncertainty estimates

- Halo EFT can be connected with *ab initio* calculations
    - adopt inputs from *ab initio* results
    - benchmark with *ab initio* calculations
    - explain correlations among observables in *ab initio* work