Molecular interpretation of the P_c states

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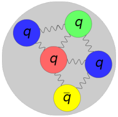
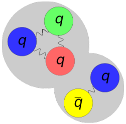
Penta-Quark States



Pentaquark

↪ Compact object formed from q and $\bar{q}$



Hadronic-Molecule

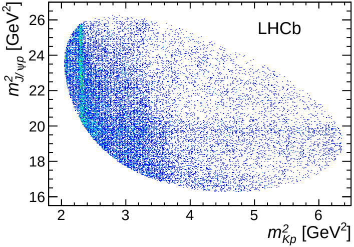
↪ Extended object made of Baryon and Meson

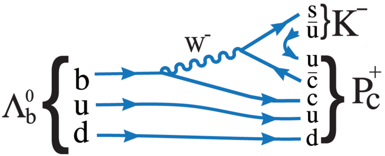
- $\Lambda(1405)$
  ↪ $\bar{K}N$ predicted by Dalitz and Tuan, 1959 PRL2,425
  ↪ $\Lambda(1405) \to \Sigma\pi$ observed by Alston et al., PRL6,698
- "$\theta(1540)$"
  predicted by Diakonov et al., 1997 ($Z(1530)^+$) ZPA359, 305
  ↪ NOT supported by many high statistics experiments

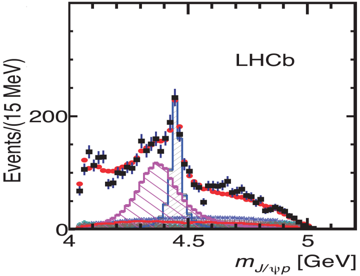
Charmonium-pentaquark states (I)

Observation of exotic structures (P_c) in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL 115, 072001 (2015)







$P_c(4380)^+$: $M = 4380 \pm 8 \pm 29$ MeV

$\Gamma = 205 \pm 18 \pm 86$ MeV

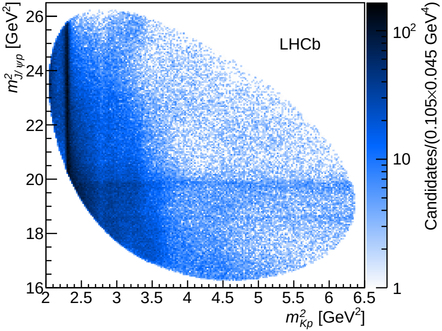
$P_c(4450)^+$: $M = 4449.8 \pm 1.7 \pm 2.5$ MeV

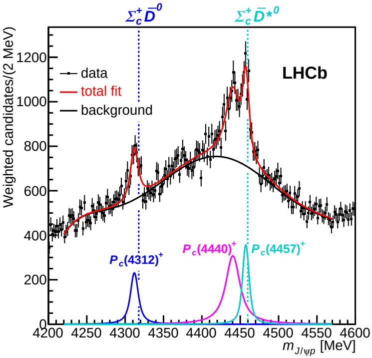
$\Gamma = 39 \pm 5 \pm 19$ MeV

Preferred Parity: Opposite

Charmonium-pentaquark states (II)

LHCb, PRL 122, 222001 (2019)

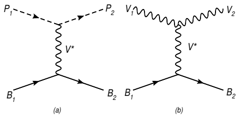




State	M [MeV]	Γ [MeV]	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	$0.53 \pm 0.16^{+0.15}_{-0.13}$

Charmonium-pentaquark states (predictions)

- Molecular states ($\bar{D}^{(*)}\Sigma_c^{(*)}$)
  - local hidden gauge — Wu et al., PRL 105, 232001 (2010)



(I,S)	M	Γ			Γ_i		
$(1/2,0)$			πN	ηN	$\eta' N$	$K\Sigma$	$\eta_c N$
$\bar{D}\Sigma_c$	4261	56.9	3.8	8.1	3.9	17.0	23.4
$(1/2,0)$			ρN	ωN	$K^*\Sigma$		$J/\psi N$
$\bar{D}^*\Sigma_c$	4412	47.3	3.2	10.4	13.7		19.2

  - vector exchange + coupled-channel — Wu et al., PRC85, 044002(2012)
    $\hookrightarrow$ $\bar{D}^{(*)}\Sigma_c$, binding $0 \sim 24$ MeV, width $0 \sim 15$ MeV.
  - HQSS + local hidden gauge — Xiao et al., PRD88, 056012(2013)
    $\hookrightarrow$ 7 $\bar{D}^{(*)}\Sigma_c^{(*)}$, binding ~ 50 MeV, width < 60 MeV
  - One-boson-exchange model — Yang et al., CPC36(2012)6
    $\hookrightarrow$ $\bar{D}\Sigma_c(1/2^-)$, $\bar{D}^*\Sigma_c(1/2^-, 3/2^-)$: $0 \sim 50$ MeV
  - Chiral quark model — Wang et al., PRC84, 015203(2011)
    $\hookrightarrow$ $\bar{D}\Sigma_c$: binding energy $5 \sim 42$ MeV

and many more including also quark model ...

Charmonium-pentaquark (theoretical)

- Compact pentaquark — Cheng et al., PRD100(2019)054002

  $P_c(4312)$, $P_c(4440)$, $P_c(4457)$: $J^P = 3/2^-, 1/2^-, 3/2^-$

- Compact diquark model — Ali et al., JHEP1910(2019)256

$3/2^-$	4240 ± 29
$3/2^+$	4440 ± 35
$5/2^+$	4457 ± 35

- $P_c(4312)$: virtual state — Fernández-Ramírez et al., PRL123(2019)092001
- K-matrix: $J/\psi p - \Sigma_c \bar{D} - \Sigma_c \bar{D}^*$ — Kuang et al., EPJC80(2020)433

  $\hookrightarrow$ $P_c(4312)$: $\Sigma_c \bar{D}$, $P_c(4457)$: ? cusp effect
- Molecule (HQSS) — Liu et al., PRL122,242001 (2019)

	Molecule	J^P	M (MeV)		Molecule	J^P	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4311.8 − 4313.0	B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4306.3 − 4307.7
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4376.1 − 4377.0	B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4370.5 − 4371.7
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4440.3*	B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4457.3*
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4457.3*	B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4440.3*
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4500.2 − 4501.0	B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4523.2 − 4523.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4510.6 − 4510.8	B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4516.5 − 4516.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4523.3 − 4523.6	B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4500.2 − 4501.0

- quantum numbers? line shape? the existence of $P_c(4380)$?

Heavy-quark spin symmetry (HQSS)

- For a heavy-quark Q ($c$harm, $b$ottom) with $m_Q \gg \Lambda_{\text{QCD}}$

  ☞ chromomag. interaction $\propto \dfrac{\sigma \cdot \mathbf{B}}{m_Q}$

  $\hookrightarrow$ HQSS: $\dfrac{\Lambda_{\text{QCD}}}{m_Q} \to 0$: independent of heavy-quark spin

- For a hadron containing a heavy-quark Q: $J = s_Q + j_\ell$,

  s_Q: heavy quark spin

  j_ℓ: total angular momentum of light degrees of freedom

  ☞ HQSS: s_Q and j_ℓ conserved separately in heavy quark limit

  ☞ spin multiplets:

	$c\bar{q}$	$c(qq)_1$	$c(qq)_0$
$s_Q \otimes j_\ell$	$\frac{1}{2} \otimes \frac{1}{2}$	$\frac{1}{2} \otimes 1$	$\frac{1}{2} \otimes 0$
doublets $\Psi(J^P)$	$D(0^-)$, $D^*(1^-)$	$\Sigma_c(\frac{1}{2}^+)$, $\Sigma_c(\frac{3}{2}^+)$	$\Lambda_c^+(\frac{1}{2}^+)$

- For heavy quarkonia ($c\bar{c}$),

  $\hookrightarrow$ S-wave: $s_Q = \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$: $\{\eta_c, J/\psi\}$;

  $\hookrightarrow$ P-wave: $s_Q = \frac{1}{2} \otimes \frac{1}{2} \otimes 1 = 1 \oplus 0 \oplus 1 \oplus 2$: $\{h_c, \chi_{c0}, \chi_{c1}, \chi_{c2}\}$;

Heavy-Light decomposition

- A S-wave two-hadron system can be transformed into basis of Heavy-Light degrees of freedom $|s_Q \otimes j_\ell\rangle$

$$\left|[s_{Q_1} j_{\ell_1}]_{j_1} [s_{Q_2} j_{\ell_2}]_{j_2}\right\rangle_J = \sum_{s_Q, j_\ell} \sqrt{(2j_1+1)(2j_2+2)(2s_Q+1)(2j_\ell+1)} \times \begin{Bmatrix} s_{Q_1} & j_{\ell_1} & j_1 \\ s_{Q_2} & j_{\ell_1} & j_2 \\ s_Q & j_\ell & J \end{Bmatrix} \left|[s_{Q_1} s_{Q_2}]_{s_Q} \otimes [j_{\ell_1} j_{\ell_2}]_{j_\ell}\right\rangle_J .$$

☞ S-wave $\bar{D}^{(*)}\Sigma_c^{(*)}$ Heavy-Light decomposition $|s_Q \otimes j_\ell\rangle$

$$\begin{pmatrix} |\Sigma_c \bar{D}\rangle \\ |\Sigma_c \bar{D}^*\rangle \\ |\Sigma_c^* \bar{D}^*\rangle \end{pmatrix}_{\frac{1}{2}} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{-1}{2\sqrt{3}} & \frac{5}{6} & \frac{\sqrt{2}}{3} \\ \sqrt{\frac{2}{3}} & \frac{\sqrt{2}}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \end{pmatrix},$$

$$\begin{pmatrix} |\Sigma_c \bar{D}^*\rangle \\ |\Sigma_c^* \bar{D}\rangle \\ |\Sigma_c^* \bar{D}^*\rangle \end{pmatrix}_{\frac{3}{2}} = \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{3} & \frac{\sqrt{5}}{3} \\ \frac{1}{2} & \frac{-1}{\sqrt{3}} & \frac{1}{2}\sqrt{\frac{5}{3}} \\ \frac{1}{2}\sqrt{\frac{5}{3}} & \frac{\sqrt{5}}{3} & \frac{1}{6} \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{3}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \end{pmatrix},$$

$$|\Sigma_c^* \bar{D}^*\rangle_{\frac{5}{2}} = \mathbb{I}|1 \otimes \frac{3}{2}\rangle$$

$\bar{D}^{(*)}\Sigma_c^{(*)}$ interaction: contact interactions

- Contact interaction: short-range interaction
- Heavy-quark spin symmetry

  $\hookrightarrow$ strong interaction only depends on the spin of light degrees of freedom

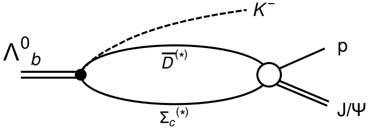
  $\hookrightarrow$
$$C_{\frac{1}{2}} \equiv \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle, \quad C_{\frac{3}{2}} \equiv \langle s_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{3}{2}\rangle,$$
- Contact potentials:

$$V^C_{\frac{1}{2}^-} = \begin{pmatrix} \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} \\ \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{7}{9}C_{\frac{1}{2}} + \frac{2}{9}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} \\ \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{8}{9}C_{\frac{1}{2}} + \frac{1}{9}C_{\frac{3}{2}} \end{pmatrix},$$

$$V^C_{\frac{3}{2}^-} = \begin{pmatrix} \frac{1}{9}C_{\frac{1}{2}} + \frac{8}{9}C_{\frac{3}{2}} & -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} \\ -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & +\frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} \\ -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{5}{9}C_{\frac{1}{2}} + \frac{4}{9}C_{\frac{3}{2}} \end{pmatrix},$$

$$V^C_{\frac{5}{2}^-} = C_{\frac{3}{2}}.$$

$\Lambda_b^0 \to K^- J/\psi p$



☞ $m_{J/\psi p} \sim 4440$ MeV
$\hookrightarrow |\mathbf{p}| \sim 810$ MeV
$\hookrightarrow J/\psi p(S),\ J/\psi p(D)$

☞

$$|J/\psi p\rangle \begin{cases} S\text{-wave}: |1 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |1 \otimes \frac{3}{2}\rangle \end{cases}, \qquad |\eta_c p\rangle \begin{cases} S\text{-wave}: |0 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |0 \otimes \frac{3}{2}\rangle \end{cases}.$$

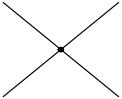
☞ $\mathcal{V}_{\alpha i}$: $\left(\Sigma_c^{(*)}\bar{D}^{(*)}\right)_\alpha \to (J/\psi p)_i$, $\mathcal{V}'_{\alpha i}$: $\left(\Sigma_c^{(*)}\bar{D}^{(*)}\right)_\alpha \to (\eta_c p)_i$

$$g_S \equiv \langle 1_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_S = \langle 0_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_S,$$
$$g_D k^2 \equiv \langle 1_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_D = \langle 0_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_D.$$

☞ Bare production vertices

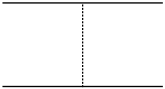
$$P_{s_Q \otimes j_\ell} \equiv \langle \Lambda_b^0|H_W|K^-(s_Q \otimes j_\ell)\rangle.$$

Effective Lagraingian $\Sigma_c^{(*)}\bar{D}^{(*)}$, $J/\psi p$, $\eta_c p$, $\Lambda_c\bar{D}^{(*)}$



- Contact Lagrangian

$$\begin{aligned}\mathcal{L} &= -C_a \vec{S}_c^\dagger \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] \\ &\quad -C_b i\epsilon_{ijk}(S_c^\dagger)_j (S_c)_k \mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c]. \\ &\quad +C_c\Big(S_{ab}^{i\dagger} T_{ca}\langle \bar{H}_c^\dagger \sigma^i \bar{H}_b\rangle - T_{ca}^\dagger S_{ab}^i \langle \bar{H}_b^\dagger \sigma^i \bar{H}_c\rangle\Big) \\ &\quad +C_d T_{ab}^\dagger T_{ba}\langle \bar{H}_c^\dagger \bar{H}_c\rangle\end{aligned}$$



- One-pion exchange (OPE)

$$\begin{aligned}\mathcal{L}_{DD\pi} &= \frac{g}{4}\langle \sigma\cdot u_{ab}\bar{H}_b \bar{H}_a^\dagger\rangle, \\ \mathcal{L}_{\Sigma_c\Sigma_c\pi} &= i\frac{3}{2} g_1 \epsilon_{ijk}\langle \bar{S}_i u_j S_k\rangle. \\ \mathcal{L}_{\Sigma_c\Lambda_c\pi} &= -\frac{1}{\sqrt{2}} g_3 (S_{ab}^{i\dagger} u_{bc}^i T_{ca} + T_{ab}^\dagger u_{bc}^i S_{ca}^i),\end{aligned}$$

☞ Effective Lagrangian for $\bar{D}^{(*)}\Sigma_c^{(*)} \to J/\psi p$ $(\eta_c p)$

$$\mathcal{L} = \frac{g_S}{\sqrt{3}} N^\dagger \sigma^i \bar{H} J^\dagger S^i - \sqrt{3} g_D N^\dagger \sigma^i \bar{H}(\partial^i\partial^j - \frac{1}{3}\delta^{ij}\partial^2) J^\dagger S^j,$$

$$\vec{S}_c = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*, \qquad \bar{H}_c = \frac{1}{2}\Big(-\bar{D} + \vec{\sigma}\cdot\vec{\bar{D}}^*\Big), \qquad J = -\eta_c + \sigma\cdot\psi.$$

Lippman-Schwinger equaitons

$$U^J_\alpha(E,p) = P^J_\alpha(E,p) - \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} V^J_{\alpha\beta}(E,p,q) G_\beta(E,q) U^J_\beta(q),$$

$$U^J_i(E,p) = -\sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} \mathcal{V}_{\beta i} G_\beta(E,q) U^J_\beta(q).$$

☞ The effective potentials

$$V^J_{\alpha\beta} = V^J_{\mathrm{CT},\alpha\beta} + V^J_{\mathrm{OPE},\alpha\beta} + \mathcal{G}^J_{\alpha\beta},$$

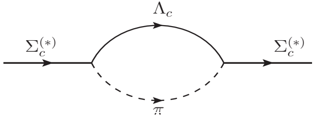
The effective contributions from the $J/\psi p$ and $\eta_c p$ bubble loop ($k \sim 0.9$ GeV)

$$\mathcal{G}^J_{\alpha\beta} = \sum_i$$

$$= R_{\mathcal{G}} - \sum_i i \frac{1}{2\pi E} m_{\psi(\eta_c)} m_p \mathcal{V}^{(\prime)J}_{\alpha i} \mathcal{V}^{(\prime)J}_{\beta i} k.$$

Lippman-Schwinger equaitons: Two-body propagator

☞ The self-energy function $\tilde{\Sigma}_R^{(*)}(s) \sim ig^2 \frac{p^3}{\sqrt{s}}$



☞ Two-body propagator:

$$G_\beta(E, \mathbf{q}) = \frac{m_{\Sigma_c^{(*)}} m_{D^{(*)}}}{E_{\Sigma_c^{(*)}}(\mathbf{q}) E_{D^{(*)}}(\mathbf{q})} \frac{1}{E_{\Sigma_c^{(*)}}(\mathbf{q}) + E_{D^{(*)}}(\mathbf{q}) - E - \frac{\tilde{\Sigma}_R^{(*)}(s)}{2E_{\Sigma_c^{(*)}}(\mathbf{q})}},$$

$s = \left(E - E_{D^{(*)}}(\mathbf{q})\right)^2 - \mathbf{q}^2$ is the off-shellness of $\Sigma_c^{(*)}$.

☞ Nonrelativistic limit

$$G_\beta(E, \mathbf{q}) = \frac{1}{\frac{\mathbf{q}^2}{2\mu} + m_{D^{(*)}} + m_{\Sigma_c^{(*)}} - E}. \tag{1}$$

$\hookrightarrow m_{\Sigma_c^{(*)}} \to m_{\Sigma_c^{(*)}} - i\Gamma_{\Sigma_c^{(*)}}/2.$

Fit Schemes

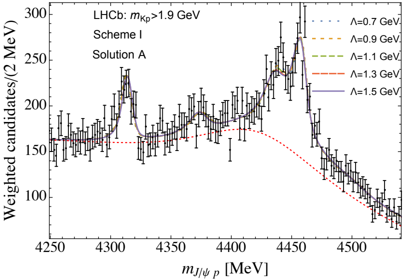
☞ Fit schemes:

- Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$
- Scheme II: Scheme I + elastic OPE w/o $\Lambda_c\bar{D}^{(*)}$ (+ CT for $\Lambda_c\bar{D}^{(*)}$)
- Scheme III: Scheme II + S-D counter term w/o $\Lambda_c\bar{D}^{(*)}$

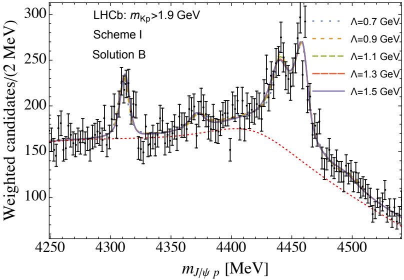
  ↪ coupled channel
- Scheme IV: contact + OPE + S-D counter terms w/ $\Lambda_c\bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



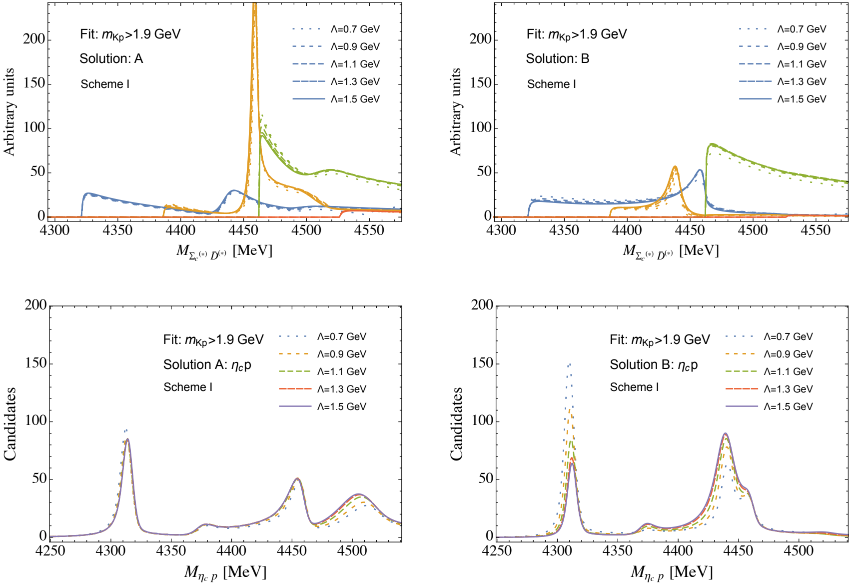
Solution B



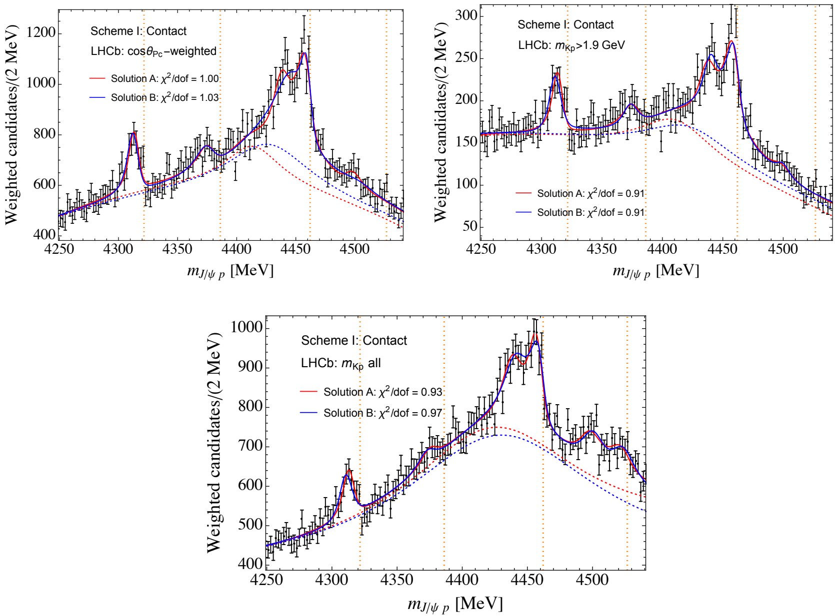
- ☞ $\Lambda_{\rm soft} \sim 0.7$ GeV
- ☞ Cutoff-independent for both solution A and B
- ☞ No need for $\Lambda_c\bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c \bar{D}^{(*)}$

Solution A | Solution B



Scheme I: pure contact potentials w/o $\Lambda_c \bar{D}^{(*)}$



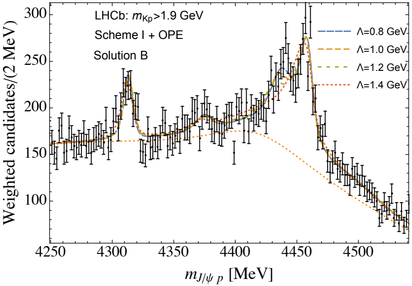
Scheme I: pole positions

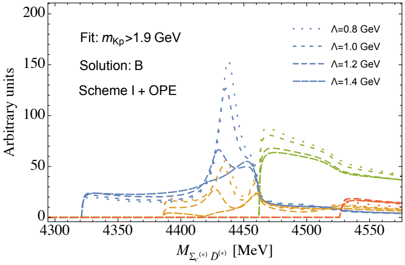
			Solution A		Solution B
	DC ([MeV])	J^P	Pole [MeV]	J^P	Pole [MeV]
$P_c(4312)$	$\Sigma_c\bar{D}$ (4321.6)	$\frac{1}{2}^-$	$4314(1)-4(1)i$	$\frac{1}{2}^-$	$4312(2)-4(2)i$
$\mathbf{P_c(4380)}^*$	$\Sigma_c^*\bar{D}$ (4386.2)	$\frac{3}{2}^-$	$4377(1)-7(1)i$	$\frac{3}{2}^-$	$4375(2)-6(1)i$
$P_c(4440)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{1}{2}^-$	$4440(1)-9(2)i$	$\frac{3}{2}^-$	$4441(3)-5(2)i$
$P_c(4457)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{3}{2}^-$	$4458(2)-3(1)i$	$\frac{1}{2}^-$	$4462(4)-5(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{1}{2}^-$	$4498(2)-9(3)i$	$\frac{1}{2}^-$	$4526(3)-9(2)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{3}{2}^-$	$4510(2)-14(3)i$	$\frac{3}{2}^-$	$4521(2)-12(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{5}{2}^-$	$4525(2)-9(3)i$	$\frac{5}{2}^-$	$4501(3)-6(4)i$

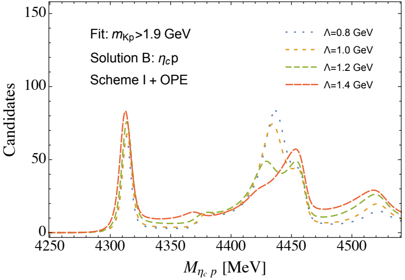
- ☞ * NOT the broad $P_c(4380)$ reported by LHCb in 2015
- ☞ Bound states with respect to the dominant channel (DC)

Scheme II: scheme I + OPE w/o $\Lambda_c\bar{D}^{(*)}$

- ☞ No solution A
- ☞ Solution B:
  Cut-off dependent
- ☞ $\Lambda_{\rm soft} \sim 700$ MeV

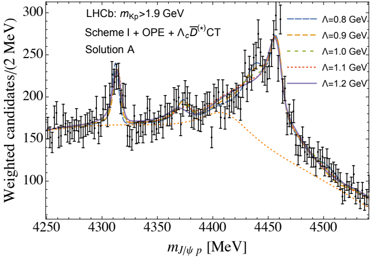




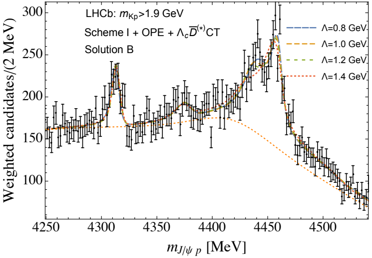


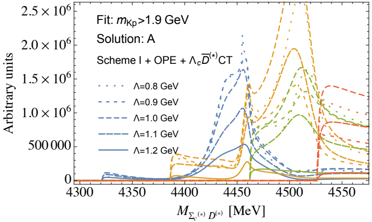
Scheme I + OPE + CT for $\Lambda_c \bar{D}^{(*)}$

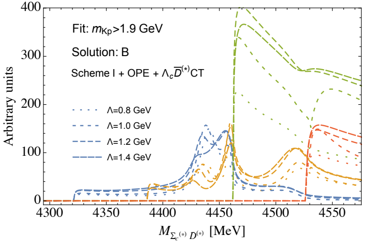
Solution A



Solution B







- Cut-off dependent
- $\Lambda_{\rm soft} \sim 900$ MeV $\Lambda_c \bar{D}^{(*)}$

Scheme III: contact + OPE + S-D counter-term

☞ Next-leading order (NLO) effective Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{\rm NLO} = \\
&-D_a^{SS}\left(\partial^i \vec{S}^\dagger_{ab}\cdot\vec{S}_{ba}\langle\partial^i\bar{H}^\dagger_c\bar{H}_c\rangle + \vec{S}^\dagger_{ab}\cdot\partial^i\vec{S}_{ba}\langle\bar{H}^\dagger_c\partial^i\bar{H}_c\rangle\right) \\
&-D_b^{SS} i\epsilon_{jik}\left(\partial^\ell S^{j\dagger}_{ab}S^k_{ba}\langle\partial^\ell\bar{H}^\dagger_c\sigma^i\bar{H}_c\rangle + S^{j\dagger}_{ab}\partial^\ell S^k_{ba}\langle\bar{H}^\dagger_c\sigma^i\partial^\ell\bar{H}_c\rangle\right) \\
&-D_b^{SD} i\epsilon_{jik}\Big[\partial_i S^\dagger_j S_k\langle\partial_\ell\bar{H}^\dagger\sigma_\ell\bar{H}\rangle + \partial_\ell S^\dagger_j S_k\langle\partial_i\bar{H}^\dagger\sigma_\ell\bar{H}\rangle - \frac{2}{3}\partial_\ell S^\dagger_j S_k\langle\partial_\ell\bar{H}^\dagger\sigma_i\bar{H}\rangle \\
&+S^\dagger_j\partial_i S_k\langle\bar{H}^\dagger\sigma_\ell\partial_\ell\bar{H}\rangle + S^\dagger_j\partial_\ell S_k\langle\bar{H}^\dagger\sigma_\ell\partial_i\bar{H}\rangle - \frac{2}{3}S^\dagger_j\partial_\ell S_k\langle\bar{H}^\dagger\partial_\ell\sigma_i\bar{H}\rangle\Big] \\
&+D_c^{SD}\frac{4}{3}\sqrt{2}\Big[\partial^i S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}^\dagger_c\sigma^j\bar{H}_b\rangle + \partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^i\bar{H}^\dagger_c\sigma^j\bar{H}_b\rangle - \frac{2}{3}\partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}^\dagger_c\sigma^i\bar{H}_b\rangle \\
&-\partial^i T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}^\dagger_b\sigma^j\bar{H}_c\rangle - \partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^i\bar{H}^\dagger_b\sigma^j\bar{H}_c\rangle + \frac{2}{3}\partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}^\dagger_b\sigma^i\bar{H}_c\rangle \\
&+S^{i\dagger}_{ab}\partial^i T_{ca}\langle\bar{H}^\dagger_c\sigma^j\partial^j\bar{H}_b\rangle + S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}^\dagger_c\sigma^j\partial^i\bar{H}_b\rangle - \frac{2}{3}S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}^\dagger_c\sigma^i\partial^j\bar{H}_b\rangle \\
&-T^\dagger_{ca}\partial^i S^i_{ab}\langle\bar{H}^\dagger_b\sigma^j\partial^j\bar{H}_c\rangle - T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}^\dagger_b\sigma^j\partial^i\bar{H}_c\rangle + \frac{2}{3}T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}^\dagger_b\sigma^i\partial^j\bar{H}_c\rangle\Big] \\
&+D_d^{SS}\left(\partial^i T^\dagger_{ab}T_{ba}\langle\partial^i\bar{H}^\dagger_c\bar{H}_c\rangle + T^\dagger_{ab}\partial^i T_{ba}\langle\bar{H}^\dagger_c\partial^i\bar{H}_c\rangle\right)
\end{aligned}
$$

NLO contact potential

☞ Channels

J^P	S-wave	D-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}^*$

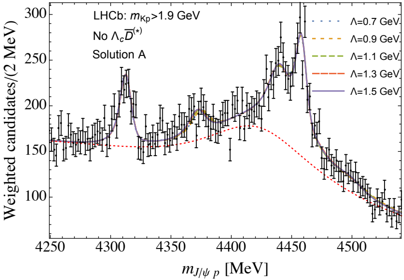
☞ NLO potentials

$$V^J_{\mathrm{NLO}}(p,p') = \begin{pmatrix} (p^2+p'^2)V^J_{SS} & p'^2 V^J_{SD} \\ p^2\left(V^J_{SD}\right)^T & 0 \end{pmatrix}.$$

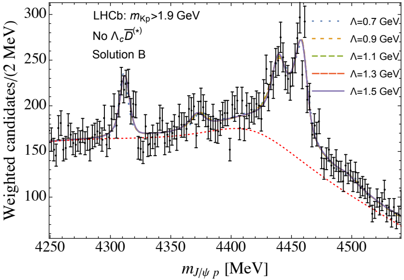
$$V^{\frac{1}{2}}_{SD} = -\frac{128}{3}\begin{pmatrix} \frac{D_b^{SD}}{4\sqrt{6}} & 0 & -\frac{D_b^{SD}}{8\sqrt{30}} & -\frac{1}{8}\sqrt{\frac{3}{10}}D_b^{SD} & \frac{D_c^{SD}}{8\sqrt{3}} \\ -\frac{D_b^{SD}}{12\sqrt{2}} & -\frac{D_b^{SD}}{8\sqrt{6}} & \frac{D_b^{SD}}{6\sqrt{10}} & -\frac{D_b^{SD}}{8\sqrt{10}} & -\frac{D_c^{SD}}{24} \\ \frac{D_b^{SD}}{48} & -\frac{D_b^{SD}}{16\sqrt{3}} & \frac{7D_b^{SD}}{48\sqrt{5}} & \frac{D_b^{SD}}{8\sqrt{5}} & -\frac{D_c^{SD}}{24\sqrt{2}} \\ \frac{D_c^{SD}}{8\sqrt{3}} & 0 & \frac{D_c^{SD}}{8\sqrt{15}} & \frac{1}{8}\sqrt{\frac{3}{5}}D_c^{SD} & 0 \\ -\frac{D_c^{SD}}{24} & \frac{D_c^{SD}}{8\sqrt{3}} & -\frac{D_c^{SD}}{6\sqrt{5}} & \frac{D_c^{SD}}{8\sqrt{5}} & 0 \end{pmatrix}$$

Scheme III: contact + OPE + S-D w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



Solution B

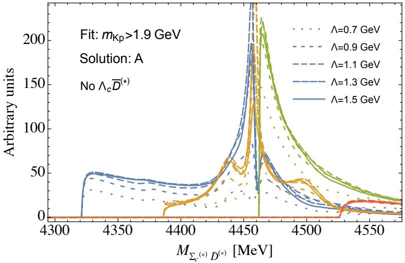


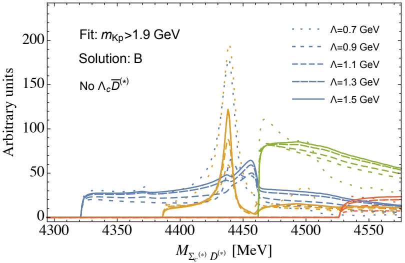
☞ $\Lambda_{\rm soft} \sim 0.7$ GeV

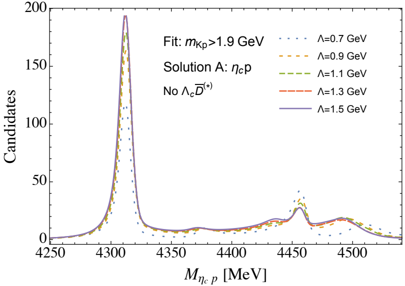
☞ Cutoff-independent for both solution A and B

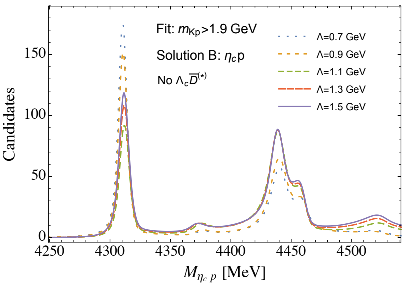
Scheme III: contact + OPE + S-D w/o $\Lambda_c \bar{D}^{(*)}$

Solution A $\Lambda_{\rm soft} \sim 0.7$ GeV Solution B



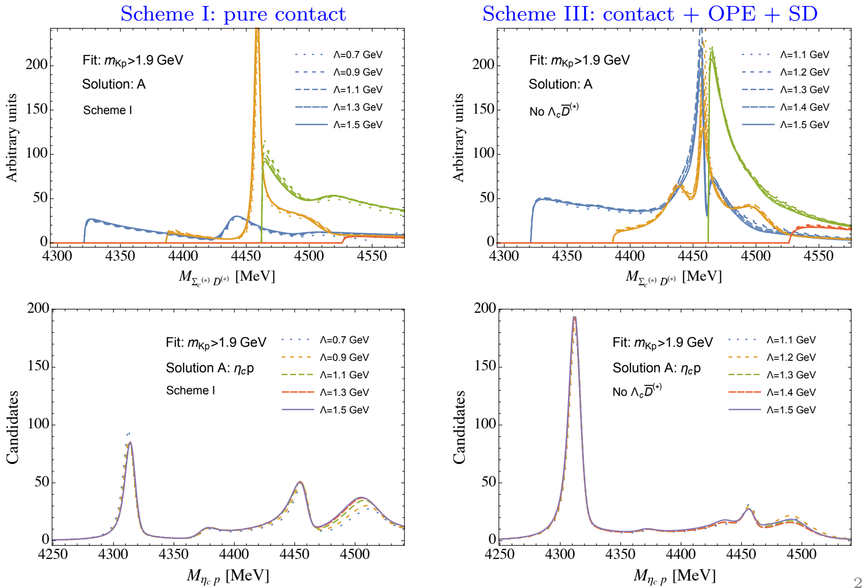






Scheme I vs Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.7$ GeV

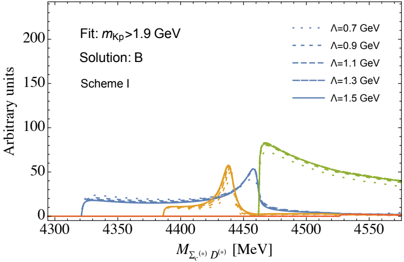
Solution A



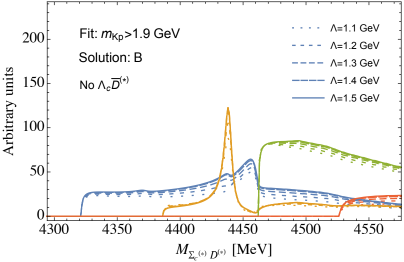
Scheme I vs Scheme III w/o $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.7$ GeV

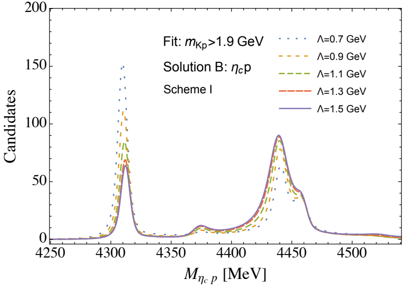
Solution B

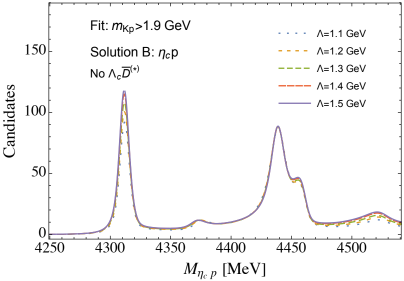
Scheme I: pure contact



Scheme III: contact + OPE + SD

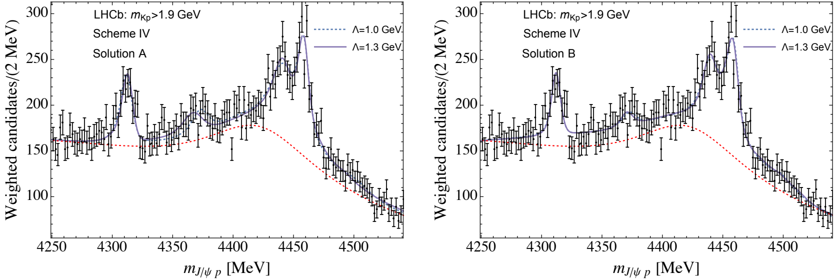






Scheme IV: CT + OPE + SD w/ $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.9$ GeV

Solution A   Solution B



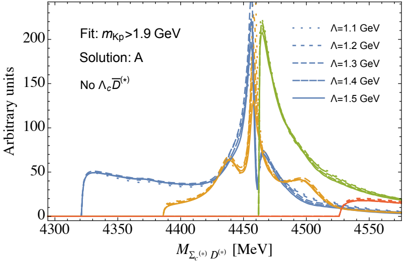
☞ Cutoff-independent for both solution A and B

☞ ? uncertainty

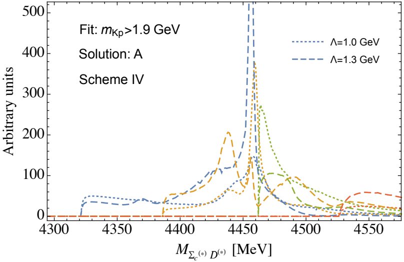
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

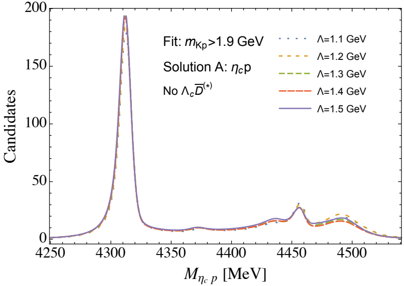
Solution A

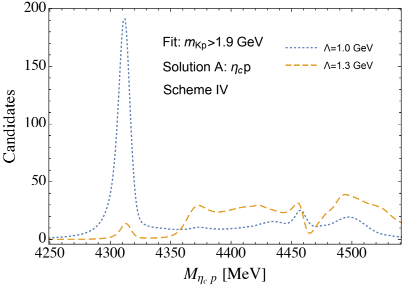
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



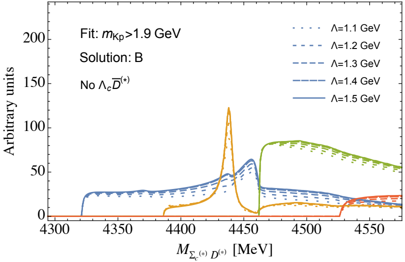




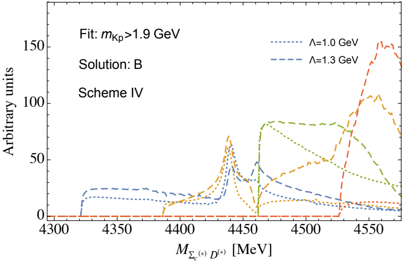
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

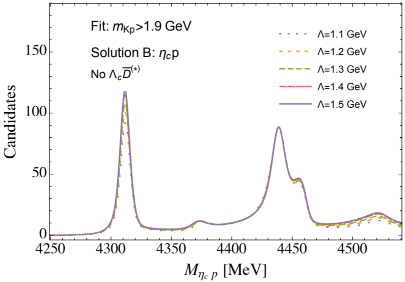
Solution B

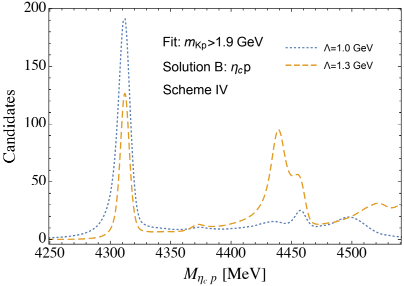
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



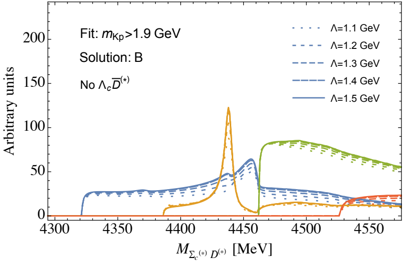




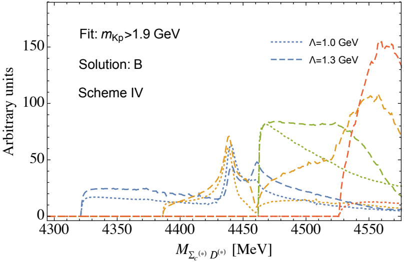
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

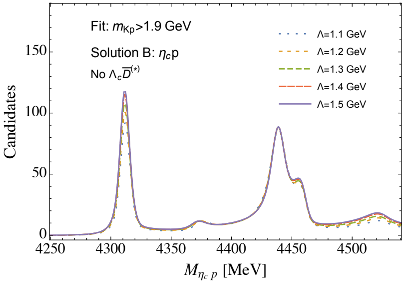
Solution B :)

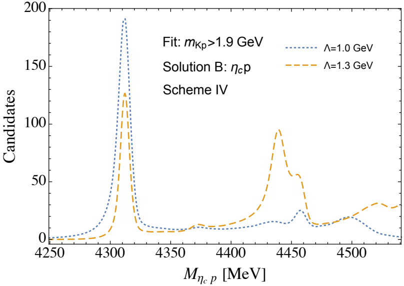
Scheme III: $\Lambda_{\text{soft}} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\text{soft}} \sim 0.9$ GeV







Summary

☞ Solving Lippmann-Schwinger equation with respect to

- Unitarity, three-body cut
  $\hookrightarrow$ width of $\Sigma_c^{(*)}$
- Coupled-channels
  $\hookrightarrow$ cut-off independence: OPE $\rightarrow$ SD counter term
- Heavy quark spin symmetry
  $\hookrightarrow$ 7 $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecular states

☞ $\Lambda_{\text{cutoff}} = 1.3$ GeV
$\hookrightarrow \Lambda_{\text{cutoff}} \gg \Lambda_{\text{soft}}$

- Solution A is scheme dependent
- Solution B is consistent for all cut-off independent schemes
  $P_c(4440)$: $J^P = \frac{3}{2}^-$, $P_c(4457)$: $J^P = \frac{1}{2}^-$ preferred ?

☞ Formalism consistent
$\hookrightarrow$ we can not say much about $\Lambda_c\bar{D}^{(*)}$ interaction without data in this channel.

☞ A narrow $P_c(4380)$, different from the broad one reported by LHCb in 2015.

Thank you very much for your attention!

Isospin structures

The isospin wave function of $P_c^{(*)}$

$$P_c^{+(*)} = -\sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Sigma_c^{(*)+} + \sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Sigma_c^{(*)++}.$$

The corresponding potential

$$\begin{aligned}
V_{\Sigma_c^{(*)}\bar{D}^{(*)}} &= \frac{1}{3}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} + \frac{2}{3}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&\quad -\frac{\sqrt{2}}{3}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} - \frac{\sqrt{2}}{3}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} \\
&= 2V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} = -\sqrt{2}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&= -\sqrt{2}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}},
\end{aligned}$$

$$\begin{aligned}
V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} &= 0 \\
V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} &= -\frac{1}{\sqrt{2}}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&= -\frac{1}{\sqrt{2}}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}}
\end{aligned}$$

Heavy-quark spin symmetry (HQSS)

- In the limit $\Lambda_{\mathrm{QCD}}/m_Q \to 0$
  $\hookrightarrow$ strong interactions are independent on heavy-quark spin
- S-wave $\bar{D}^{(*)}\Sigma_c^{(*)}$ $\Lambda_c\bar{D}^{(*)}$ spin decomposition $|s_Q \otimes j_\ell\rangle$

$$\begin{pmatrix} |\Sigma_c\bar{D}\rangle \\ |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{1}{2}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 & 0 \\ \frac{1}{2\sqrt{3}} & \frac{5}{6} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ \sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \\ |0 \otimes \frac{1}{2}\rangle' \\ |1 \otimes \frac{1}{2}\rangle' \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{3}{2}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{3} & \frac{\sqrt{5}}{3} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & \frac{1}{2}\sqrt{\frac{5}{3}} & 0 \\ \frac{1}{2}\sqrt{\frac{5}{3}} & \frac{\sqrt{5}}{3} & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{3}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle' \end{pmatrix}, \tag{3}$$

$$|\Sigma_c^*\bar{D}^*\rangle_{\frac{5}{2}} = |1 \otimes \frac{3}{2}\rangle. \tag{4}$$

Contact interactions

- Contact interaction: short-range interaction
- strong interaction: spin of light degrees of freedom

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Sigma_c^{(*)}\bar{D}^{(*)}$:

$$C_{\frac{1}{2}} \equiv \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle, \quad C_{\frac{3}{2}} \equiv \langle s_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{3}{2}\rangle,$$

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C'_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle = \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle'.$$

$\hookrightarrow$ $\Lambda_c\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C''_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle',$$

- $J/\psi p$ ($\eta_c p$) Heavy-Light spin decomposition:

$$|J/\psi p\rangle \begin{cases} S\text{-wave}: |1 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |1 \otimes \frac{3}{2}\rangle \end{cases}, \qquad |\eta_c p\rangle \begin{cases} S\text{-wave}: |0 \otimes \frac{1}{2}\rangle \\ D\text{-wave}: |0 \otimes \frac{3}{2}\rangle \end{cases}.$$

- $\Sigma_c^{(*)}\bar{D}^{(*)} \to J/\psi p$ ($\eta_c p$):

$$\begin{aligned} g_S &\equiv \langle 1 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_S = \langle 0 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_S, \\ g_D k^2 &\equiv \langle 1 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_D = \langle 0 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_D \end{aligned}$$

Contact potentials

$$V_C^{\frac{1}{2}^-} = \begin{pmatrix} \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{7}{9}C_{\frac{1}{2}} + \frac{2}{9}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{8}{9}C_{\frac{1}{2}} + \frac{1}{9}C_{\frac{3}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} \\ 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} & 0 \\ \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} & 0 & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{3}{2}^-} = \begin{pmatrix} \frac{1}{9}C_{\frac{1}{2}} + \frac{8}{9}C_{\frac{3}{2}} & -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & -\frac{1}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & +\frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{5}{9}C_{\frac{1}{2}} + \frac{4}{9}C_{\frac{3}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3}C'_{\frac{1}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{5}{2}^-} = C_{\frac{3}{2}}.$$

channels

J^P	S-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$,$\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}^*$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c^*\bar{D}^*$

J^P	D-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$

Next-leading order (NLO) effective Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{NLO}} = \\
&-D_a^{SS}\left(\partial^i \vec{S}^\dagger_{ab}\cdot\vec{S}_{ba}\langle\partial^i \bar{H}_c^\dagger \bar{H}_c\rangle + \vec{S}^\dagger_{ab}\cdot\partial^i\vec{S}_{ba}\langle\bar{H}_c^\dagger\partial^i\bar{H}_c\rangle\right)\\
&-D_b^{SS} i\epsilon_{jik}\left(\partial^\ell S^{j\dagger}_{ab} S^k_{ba}\langle\partial^\ell\bar{H}^\dagger_c\sigma^i\bar{H}_c\rangle + S^{j\dagger}_{ab}\partial^\ell S^k_{ba}\langle\bar{H}^\dagger_c\sigma^i\partial^\ell\bar{H}_c\rangle\right)\\
&-D_b^{SD} i\epsilon_{jik}\Big[\partial_i S^\dagger_j S_k\langle\partial_\ell\bar{H}^\dagger\sigma_\ell\bar{H}\rangle + \partial_\ell S^\dagger_j S_k\langle\partial_i\bar{H}^\dagger\sigma_\ell\bar{H}\rangle - \frac{2}{3}\partial_\ell S^\dagger_j S_k\langle\partial_\ell\bar{H}^\dagger\sigma_i\bar{H}\rangle\\
&+S^\dagger_j\partial_i S_k\langle\bar{H}^\dagger\sigma_\ell\partial_\ell\bar{H}\rangle + S^\dagger_j\partial_\ell S_k\langle\bar{H}^\dagger\sigma_\ell\partial_i\bar{H}\rangle - \frac{2}{3}S^\dagger_j\partial_\ell S_k\langle\bar{H}^\dagger\partial_\ell\sigma_i\bar{H}\rangle\Big]\\
&+\frac{4}{3}\sqrt{2}D_c^{SD}\Big[\partial^i S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}^\dagger_c\sigma^j\bar{H}_b\rangle + \partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^i\bar{H}^\dagger_c\sigma^j\bar{H}_b\rangle - \frac{2}{3}\partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}^\dagger_c\sigma^i\bar{H}_b\rangle\\
&-\partial^i T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}^\dagger_b\sigma^j\bar{H}_c\rangle - \partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^i\bar{H}^\dagger_b\sigma^j\bar{H}_c\rangle + \frac{2}{3}\partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}^\dagger_b\sigma^i\bar{H}_c\rangle\\
&+S^{i\dagger}_{ab}\partial^i T_{ca}\langle\bar{H}^\dagger_c\sigma^j\partial^j\bar{H}_b\rangle + S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}^\dagger_c\sigma^j\partial^i\bar{H}_b\rangle - \frac{2}{3}S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}^\dagger_c\sigma^i\partial^j\bar{H}_b\rangle\\
&-T^\dagger_{ca}\partial^i S^i_{ab}\langle\bar{H}^\dagger_b\sigma^j\partial^j\bar{H}_c\rangle - T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}^\dagger_b\sigma^j\partial^i\bar{H}_c\rangle + \frac{2}{3}T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}^\dagger_b\sigma^i\partial^j\bar{H}_c\rangle\Big]\\
&+D_d^{SS}\left(\partial^i T^\dagger_{ab}T_{ba}\langle\partial^i\bar{H}^\dagger_c\bar{H}_c\rangle + T^\dagger_{ab}\partial^i T_{ba}\langle\bar{H}^\dagger_c\partial^i\bar{H}_c\rangle\right)
\end{aligned}
$$

NLO contact potential

$$V^J_{\mathrm{NLO}}(p,p') = \begin{pmatrix} (p^2+p'^2)V^J_{SS} & p'^2 V^J_{SD} \\ p^2\left(V^J_{SD}\right)^T & 0 \end{pmatrix}.$$

$$V^{\frac{1}{2}}_{SD} = -\frac{128}{3}\begin{pmatrix} \frac{D_b^{SD}}{4\sqrt{6}} & 0 & -\frac{D_b^{SD}}{8\sqrt{30}} & -\frac{1}{8}\sqrt{\frac{3}{10}}D_b^{SD} & \frac{D_c^{SD}}{8\sqrt{3}} \\ -\frac{D_b^{SD}}{12\sqrt{2}} & -\frac{D_b^{SD}}{8\sqrt{6}} & \frac{D_b^{SD}}{6\sqrt{10}} & -\frac{D_b^{SD}}{8\sqrt{10}} & -\frac{D_c^{SD}}{24} \\ \frac{D_b^{SD}}{48} & -\frac{D_b^{SD}}{16\sqrt{3}} & \frac{7D_b^{SD}}{48\sqrt{5}} & \frac{D_b^{SD}}{8\sqrt{5}} & -\frac{D_c^{SD}}{24\sqrt{2}} \\ \frac{D_c^{SD}}{8\sqrt{3}} & 0 & \frac{D_c^{SD}}{8\sqrt{15}} & \frac{1}{8}\sqrt{\frac{3}{5}}D_c^{SD} & 0 \\ -\frac{D_c^{SD}}{24} & \frac{D_c^{SD}}{8\sqrt{3}} & -\frac{D_c^{SD}}{6\sqrt{5}} & \frac{D_c^{SD}}{8\sqrt{5}} & 0 \end{pmatrix}$$