Quantum Simulation of Nuclear Scattering

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Outline

1. Background and basics

2. Time-Dependent Basis Function on Qubits (TBFQ) method

3. Future efforts and summary
Limitations of world leading supercomputing techniques

Problems:

- Exascale supercomputer extremely power consuming
- Complex architecture and algorithms
- Transistor size ($\sim 8 - 10 \, nm$) decreases; quantum effects manifest
Computation resources at exascale and beyond

**Grand challenge:** to describe nuclear physics at all scales based on the most fundamental *d.o.f.*

**Difficulty:** exponential growth in the number of states as particle number increases
A scaling problem for classical computation

**Question:** what is the dimension of the wave function of a spin system?

<table>
<thead>
<tr>
<th>Particle number</th>
<th>Basis set</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{</td>
<td>↑⟩,</td>
</tr>
<tr>
<td>2</td>
<td>{</td>
<td>↑↑⟩,</td>
</tr>
<tr>
<td>3</td>
<td>{</td>
<td>↑↑↑⟩,</td>
</tr>
<tr>
<td>...</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>N</td>
<td>...</td>
<td>2^N</td>
</tr>
</tbody>
</table>

- Laptop: 40-particle system (~2^{40} bytes)
- 100-particle system **beyond** the capability of largest supercomputer nowadays (< 2^{100} bytes)!
Seminal idea: let’s make the computation fully quantum mechanical

“I’m not happy with all the analyses that go with the classical theory, because nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.” --- R. P. Feynman’s vision in 1982

From classical to quantum mechanical

<table>
<thead>
<tr>
<th>Classical bit</th>
<th>Quantum bit (qubit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>0\rangle$ or $</td>
</tr>
</tbody>
</table>

2 states  
transistor ON or OFF

<table>
<thead>
<tr>
<th>Classical bit</th>
<th>Quantum bit (qubit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>0\rangle$</td>
</tr>
</tbody>
</table>

infinite number of states

![Quantum state vector](image)
The principles of quantum computing

Superposition, e.g.,

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

Entanglement, e.g.,

\[ |\psi\rangle = \frac{|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2}{\sqrt{2}} \]

Fundamental change

- computation methodology and algorithm
- information storage and manipulation
Operating a qubit (illustration)

Quantum operation

\[ |q_0\rangle \xrightarrow{X} X|q_0\rangle \]

Various other operations, e.g., the phase gate, the Hadamard gate etc.

Matrix interpretation

\[ X|0\rangle = |1\rangle, \quad X|1\rangle = |0\rangle \]

\[ X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \]

\[ |q_0\rangle = \alpha|0\rangle + \beta|1\rangle \]

\[ X|q_0\rangle = \alpha|1\rangle + \beta|0\rangle \]
Quantum computers

- Quantum computers make use of quantum systems to perform calculations.
- Such quantum systems need to be **well controlled** and **sufficiently isolated from the environment**.
- Quantum computers can potentially **circumvent** the **roadblock of exponential cost** in computational science.
Quantum leap in computation power

Factorization into two prime numbers

\[ \text{rsa}_250 = 21403246502407449612644230728393335630086147151447550 \\
1779775492088141802344714013664334551909580467961099285187247 \\
0914587687396261921557363047454770520805119056493106687691590 \\
0197594056934574522305893259766974716817380693648946998715784 \\
94975937497937 \]

\[ = p \times q \]

Need 2700 core-years!

\[ p = 64135289477071580278790190170577 \\
3890848250147429434472081168596320 \\
2453234463023862359875266834770873 \\
7661925585694639798853367 \]

\[ q = 3337202759497815655622601060535 \\
5114227940760344767554666678452098 \\
702384172921003708025744867329688 \\
1877565718986258036932062711 \]

Classical: \( T \sim O(e^{3\sqrt{n}}) \)
Quantal: \( T \sim O(n^3) \)

[https://lists.gforge.inria.fr/pipermail/cado-nfs-discuss/2020-February/001166.html]
Natural way of processing quantum information

Expressing a quantum state of $N$-particle system

- Classical computer: intractable as $N$ increases (exponential)
- Quantum computer: $\sim N$ qubits

$$|\Psi\rangle = (\alpha_1 |\uparrow\rangle_1 + \beta_1 |\downarrow\rangle_1) \otimes (\alpha_2 |\uparrow\rangle_2 + \beta_2 |\downarrow\rangle_2) \cdots \otimes (\alpha_N |\uparrow\rangle_N + \beta_N |\downarrow\rangle_N)$$

Quantum computer: natural way to express and manipulate quantum states
Hardware development

Hardware improvement/innovations
- joint tasks for scientists from many areas

Challenges: decoherence time, precise control/readout, scalability with error corrections
Circuit-based quantum processors

These QPUs are based on the quantum circuit and quantum logic gate-based model of computing.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Name/Codename/Designation</th>
<th>Architecture</th>
<th>Layout</th>
<th>Socket</th>
<th>Fidelity</th>
<th>Qubits</th>
<th>Release date</th>
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<tbody>
<tr>
<td>Google</td>
<td>N/A</td>
<td>Superconducting</td>
<td>N/A</td>
<td>N/A</td>
<td>99.5%[1]</td>
<td>20 qb</td>
<td>2017</td>
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<td>99.7%[1]</td>
<td>49 qb[2]</td>
<td>Q4 2017 (planned)</td>
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<tr>
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<td>6×12 lattice</td>
<td>N/A</td>
<td>99% (readout) 99.9% (1 qubit) 99.4% (2 qubits)</td>
<td>72 qb[3][4]</td>
<td>5 March 2018</td>
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<td>Sycamore</td>
<td>Nonlinear superconducting resonator</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>54 transmon qb 53 qb effective</td>
<td>2019</td>
</tr>
<tr>
<td>IBM</td>
<td>IBM Q 5 Tenerife</td>
<td>Superconducting</td>
<td>bow tie</td>
<td>N/A</td>
<td>99.897% (average gate) 98.64% (readout)</td>
<td>6 qb</td>
<td>2016[1]</td>
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<td>bow tie</td>
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<td>20 qb[7]</td>
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<td>Superconducting</td>
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<td>N/A</td>
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<td>T</td>
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<td>N/A</td>
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<td>5 qb</td>
<td>13 September 2019</td>
</tr>
</tbody>
</table>

The Noisy Intermediate-Scale Quantum era is arriving!
Requests for quantum algorithms in nuclear physics

Rapidly developing quantum hardware boosts/requests inventions/discoveries of new quantum algorithms.

Interesting problem set:
- Many-particle problems
- Strongly coupled systems
- Real-time evolution and dynamics

[J. Preskill, arXiv: 1801.00862]
Potential applications of quantum computing

Quantum Field Theories and Fundamental Symmetries
- indefinite particle number
- gauge symmetries and constraints
- entangled ground states

Real-Time Dynamics
- nuclear reactions
- neutrino-nucleus interactions
- neutrinos in matter
- early universe
- non-equilibrium - heavy-ions
- parton showers fragmentation

Dense Matter
- neutron stars
- gravity waves?
- heavy nuclei
- chemical potentials

[Adapted from M. J. Savage’s talk in NUCLEI - SciDAC, June 2020]
Ground-breaking activities in nuclear physics

**RESEARCH ARTICLE**

**Quantum Algorithms for Quantum Field Theories**

Stephen P. Jordan¹,*, Keith S. M. Lee², John Preskill³

[Science vol. 336, pp. 1130-1133 (2012)]

**Quantum computing for neutrino-nucleus scattering**

Alessandro Roggero,¹,* Andy C. Y. Li¹,²,† Joseph Carlson,³,‡ Rajan Gupta,⁴,§ and Gabriel N. Perdue²,‖

[PRD 101, 074038 (2020)]

**Cloud Quantum Computing of an Atomic Nucleus**

E. F. Dumitrescu,¹ A. J. McCaskey,² G. Hagen,³,⁴ G. R. Jansen,⁵,³ T. D. Morris,⁴,³ T. Papenbrock,⁴,³,* R. C. Pooser,¹,⁴ D. J. Dean,³ and P. Lougovski¹,†

[PRL 120, 210501 (2018)]

**Neutrino oscillations in a quantum processor**

C. A. Argüelles¹ and B. J. P. Jones²

[Phys. Rev. Research 1, 033176 (2019)]

**Real-time dynamics of lattice gauge theories with a few-qubit quantum computer**

Esteban A. Martinez¹, Christine A. Muschik²,³, Philipp Schindler¹, Daniel Nigg¹, Alexander Erhard¹, Markus Heyl²,⁴, Philipp Hauke²,³, Marcello Dalmonte²,³, Thomas Monz², Peter Zoller²,³ & Rainer Blatt¹,²

[Nature vol. 538, pp.517 (2016)]
Elements of quantum computing: qubit system

Quantum bit (qubit)

- the basic element in quantum computation
- a two-level quantum system (e.g., spin-$\frac{1}{2}$ system)

Basis states of two-level quantum system

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
Elements of quantum computing: qubit system

1-qubit case

- a general **superposition** of basis states (infinite state vectors)

\[
|\psi\rangle = \alpha|0\rangle + \beta|1\rangle
\]

\[
|\psi\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}
\]

- normalization

\[
|\alpha|^2 + |\beta|^2 = 1
\]

**Bloch representation** of a general state vector
Elements of quantum computing: qubit system

2-qubit case

- **Superposition of basis states**

\[ |\psi\rangle = \alpha|0\rangle_1|0\rangle_2 + \beta|0\rangle_1|1\rangle_2 + \gamma|1\rangle_1|0\rangle_2 + \delta|1\rangle_1|1\rangle_2 \]

- **$2^2$ basis states**

\[
|0\rangle_1|0\rangle_2 \equiv |0\rangle_1 \otimes |0\rangle_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad |0\rangle_1|1\rangle_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad |1\rangle_1|0\rangle_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad |1\rangle_1|1\rangle_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

- **Normalization**

\[ |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1 \]
Elements of quantum computing: qubit system

N-qubit case

- **Superposition of basis states**

\[
|\psi\rangle = \sum_{i_1 \in \{0,1\}} \sum_{i_2 \in \{0,1\}} \cdots \sum_{i_N \in \{0,1\}} \alpha_{i_1 i_2 \cdots i_N} |i_1\rangle_1 |i_2\rangle_2 \cdots |i_N\rangle_N
\]

- **\(2^N\) basis states**

\[
|0\rangle_1 |0\rangle_2 \cdots |0\rangle_N, \ |0\rangle_1 |0\rangle_2 \cdots |1\rangle_N, \ \cdots, \ |1\rangle_1 |1\rangle_2 \cdots |1\rangle_N
\]

- **Normalization**

\[
\sum_{i_1 \in \{0,1\}} \sum_{i_2 \in \{0,1\}} \cdots \sum_{i_N \in \{0,1\}} |\alpha_{i_1 i_2 \cdots i_N}|^2 = 1
\]
Elements of quantum computing: quantum gates

Quantum gates: controlled local unitary operations applied on qubit(s)

- **Identity gate**
  \[
  I |0\rangle = |0\rangle, \quad I |1\rangle = |1\rangle
  \]
  \[
  I = \begin{bmatrix}
  1 & 0 \\
  0 & 1
  \end{bmatrix}
  \]

- **X gate (NOT gate)**
  \[
  X |0\rangle = |1\rangle, \quad X |1\rangle = |0\rangle
  \]
  \[
  X = \begin{bmatrix}
  0 & 1 \\
  1 & 0
  \end{bmatrix}
  \]
Elements of quantum computing: quantum gates

- **Hadamard gate**

\[ H \ket{0} = \frac{1}{\sqrt{2}} \ket{0} + \frac{1}{\sqrt{2}} \ket{1}, \quad H \ket{1} = \frac{1}{\sqrt{2}} \ket{0} - \frac{1}{\sqrt{2}} \ket{1} \]

\[ H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \]

- **Phase gate**

\[ R_\phi \ket{0} = \ket{0}, \quad R_\phi \ket{1} = e^{i\phi} \ket{1} \]

\[ R_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} \]
Elements of quantum computing: quantum gates

- Rotational gates

\[ R_x(\theta) \equiv e^{-i\theta X/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} X = \begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix} \]

\[ R_y(\theta) \equiv e^{-i\theta Y/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Y = \begin{bmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix} \]

\[ R_z(\theta) \equiv e^{-i\theta Z/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \]

with the Pauli gates

\[ X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ; \quad Y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} ; \quad Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]
Elements of quantum computing: quantum gates

- **Controlled-NOT (CNOT) gate**

  \[ |x\rangle \quad \text{black}\quad |x\rangle \quad \text{XOR} \]

  \[ |y\rangle \quad \text{white}\quad |x \oplus y\rangle \]

  \[
  \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & 1 & 0 \\
  \end{bmatrix}
  \]

  \[|00\rangle \rightarrow |00\rangle \quad |01\rangle \rightarrow |01\rangle \quad |10\rangle \rightarrow |11\rangle \quad |11\rangle \rightarrow |10\rangle\]

- **Controlled phase gate**

  \[ R_\phi \]

  \[
  \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & e^{i\phi} \\
  \end{bmatrix}
  \]

  \[|00\rangle \rightarrow |00\rangle \quad |01\rangle \rightarrow |01\rangle \quad |10\rangle \rightarrow |10\rangle \quad |11\rangle \rightarrow e^{i\phi} |11\rangle\]
Elements of quantum computing: quantum gates

- **SWAP gate**

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
|00\rangle \rightarrow |00\rangle \quad |01\rangle \rightarrow |10\rangle \quad |10\rangle \rightarrow |01\rangle \quad |11\rangle \rightarrow |11\rangle
\]

- **More gates**

Available at:  
Basic concepts of quantum computing

Quantum circuit

- a sequence of quantum gates output the solution to a problem

\[
|0\rangle \rightarrow H(\frac{|0\rangle + |1\rangle}{\sqrt{2}}) \rightarrow |0\rangle
\]

\[
|0\rangle \otimes |0\rangle \equiv |00\rangle \quad \frac{|00\rangle + |10\rangle}{\sqrt{2}} \quad \frac{|00\rangle + |11\rangle}{\sqrt{2}}
\]

Quantum algorithm

- designing a quantum circuit for a particular problem
- performance of a quantum algorithm characterized by
  - the number of gates
  - the run time as a function of the problem size (dimension)
Algorithm illustration: quantum Fourier transformation

\[ |j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i jk/N} |k\rangle \]

\[ = \frac{1}{2^{n/2}} \left( |0\rangle + e^{2\pi i 0.j_n} |1\rangle \right) \left( |0\rangle + e^{2\pi i 0.j_{n-1}j_n} |1\rangle \right) \cdots \left( |0\rangle + e^{2\pi i 0.j_1j_2\cdots j_n} |1\rangle \right) \]

[Figure credit: Dean Lee]

Exponential speedup

Quantum Fourier transform \( \sim O(n^2) \) gates
Classical Fourier transform \( \sim O(n2^n) \) gates
Outline

1. Background and basics

2. Time-Dependent Basis Function on Qubits (TBFQ) method

3. Future efforts and summary
Motivations

- Construct a unified *ab initio* theory for nuclear structure and reaction
- Study complex nuclear processes (e.g., rare isotopes)
- Investigate nuclear interaction (both on- and off- shell properties)
Requests for theories with predictive power for nuclear structures and reactions
Background

Existing methods, e.g.,

- No-core Shell Model with Continuum
- No-core Shell Model/Resonating Group Method
- Gamow Shell Model
- Harmonic Oscillator Representation of Scattering Equation
- Green’s Function Approaches
- Nuclear Lattice Effective Field Theory

Challenges

- Retaining full quantal coherence
- Tracking all potentially possible nuclear processes (rare isotopes)
- Large and complex reaction systems (exponential scaling)

Solutions

- Hamiltonian dynamics
- Real-time evolution; information on the amplitude level
- Quantum computation
Time-dependent Basis Function on Qubits (TBFQ) algorithm (Hamiltonian simulation)

- Unified structure and reaction theory
- Based on successful nuclear structure theory
- *Ab initio* approach
- Non-perturbative method
- Retaining full quantal coherence & entanglement
- Circumventing the exponential cost in computation resource in simulating real-time many-body dynamics
Demonstration problem: Coulomb excitation of deuterium system by peripheral scattering with heavy ion

- $H_0$: Target (deuteron in trap) Hamiltonian
- $\varphi$: Coulomb field from heavy ion ($U^{92+}$) sensed by target
- $\rho$: Charge density distribution of target

\[
H(t) = H_0 + V(t) \\
H_0 = T_{rel} + V_{NN} + U_{trap} \\
V(t) = \int \rho(\vec{R}, t) \varphi(\vec{R}, t) \, d\vec{R}
\]

[Weijie Du et al., 2017]
Elements of TBFQ

Construct the basis representation from *ab initio* nuclear structure calculation

\[
H_0 |\beta_i\rangle = E |\beta_i\rangle \\
H_0 = T_{rel} + V_{QCD}
\]

Basis representation
\[
\{ |\beta_1\rangle, |\beta_2\rangle, \cdots, |\beta_n\rangle \}
\]

Diagonalization

Free Hamiltonian $H_0$

Eigenenergies and eigenbases

**Game plan for TBFQ**

1. Prepare the initial state – can be entangled state
2. Time-evolve the state – Trotterized evolution operator & qubitization
3. Measurement
The algorithm (Hamiltonian simulation)

State vector evolution

\[ |\psi; t\rangle_I = U_I(t; t_0) |\psi; t_0\rangle_I = \hat{T}\left\{ \exp \left[ -i \int_{t_0}^{t} V_{\text{int}}^I(t') dt' \right] \right\} |\psi; t_0\rangle_I \]

Time discretization

\[ U_I(t; t_0) \approx \hat{T}\left\{ \exp \left[ -i \left[ V_{\text{int}}^I(t) \delta t + V_{\text{int}}^I(t_{n-1}) \delta t + \cdots + V_{\text{int}}^I(t_1) \delta t \right] \right] \right\} \]

Trotterization (1\textsuperscript{st} order)

\[ U_I(t; t_0) = \underbrace{e^{-i V_{\text{int}}^I(t) \delta t}}_{U(t; t_{n-1})} \cdots \underbrace{e^{-i V_{\text{int}}^I(t_k) \delta t}}_{U(t_k; t_{k-1})} \cdots \underbrace{e^{-i V_{\text{int}}^I(t_1) \delta t}}_{U(t_1; t_0)} + \mathcal{O}(\delta t^2) \]

Qubitization

\[ \{ |\beta_0\rangle, |\beta_1\rangle, \cdots, |\beta_N\rangle \} \]

\[ |000\cdots\rangle_n, |100\cdots\rangle_n, |111\cdots\rangle_n \]

\[ n \sim \lfloor \log N_{\text{basis}} \rfloor \]

Quantum circuit
Basis set of the inelastic scattering problem

1. 7 basis states of the target solved via \textit{ab initio} structure calculation
2. Initial state set to be antiparallel to z-axis

\[
\begin{pmatrix}
(3S_1, 3\ D_1) & M = -1 & -0.65289 \text{ MeV} \\
(3S_1, 3\ D_1) & M = 0 & -0.65289 \text{ MeV} \\
(3S_1, 3\ D_1) & M = +1 & -0.65289 \text{ MeV} \\
3P_0 & M = 0 & 12.0733 \text{ MeV} \\
3P_1 & M = -1 & 12.7585 \text{ MeV} \\
3P_1 & M = 0 & 12.7585 \text{ MeV} \\
3P_1 & M = +1 & 12.7585 \text{ MeV}
\end{pmatrix}
\]
1. 7 basis states of the target solved via \textit{ab initio} structure calculation
2. Initial state set to be antiparallel to z-axis
3. E1 radiative transitions retained in dynamics (time-evolution operator)
1. 7 basis states of the target solved via ab initio structure calculation
2. Initial state set to be antiparallel to z-axis
3. E1 radiative transitions retained in dynamics (time-evolution operator)
4. Trotterization; 7 basis states \textit{mapped} to 3 qubits

\[ n \sim \log N_{\text{basis}} \]

\[ \begin{array}{c|c|c}
|000> & (^{3}S_{1}, ^{3}D_{1}) & M = -1 & -0.65289 \text{ MeV} \\
|100> & (^{3}S_{1}, ^{3}D_{1}) & M = 0 & -0.65289 \text{ MeV} \\
|010> & (^{3}S_{1}, ^{3}D_{1}) & M = +1 & -0.65289 \text{ MeV} \\
|110> & ^{3}P_{0} & M = 0 & 12.0733 \text{ MeV} \\
|001> & ^{3}P_{1} & M = -1 & 12.7585 \text{ MeV} \\
|101> & ^{3}P_{1} & M = 0 & 12.7585 \text{ MeV} \\
|011> & ^{3}P_{1} & M = +1 & 12.7585 \text{ MeV} \\
\end{array} \]
The initial state in the qubit representation is $|000\rangle$

The quantum circuit is constructed by Quantum Shannon Decomposition

By measurement, we obtain the final state in terms of probability distribution.
Transition probabilities and observables
Outline

1. Background and basics

2. Time-Dependent Basis Function on Qubits (TBFQ) method

3. Future efforts and summary
Future efforts

Generalization to real scattering

- Channels and states increase dramatically

Real-time dynamics in gauge fields

- Electron scattering in intense E&M fields (e.g., laser facilities)
- Quark scattering in color field
- Particle production and evolution in the glasma field

[X. Zhao et al., PRD 88 065064 (2013)]
[G. Chen et al., PRD 95 096012 (2017)]
[M. Li et al., PRD 101 076016 (2020)]
[M. Li et al., private communication]

[P. Yin, WD et al., arXiv: 1910.10586]
Summary

- Quantum computing techniques make use quantum mechanics principles to outperform classical computers

- The rapid development of quantum computing techniques shed light on many challenging problems in nuclear physics

- We develop the TBFQ algorithm to simulate low-energy nuclear scattering on quantum computers

- This work can be further generalized to study complicated problems in low-energy nuclear, QED and QCD scattering processes

- Stay tuned...
Thanks!