DOUBLY HEAVY HADRONIC MOLECULES

IN COLLABORATION WITH X.-K. DONG, T. JI, V. BARU, L. V. DETTEN., F.-K. GUO, U.-G. MEISSNER, A. NEFEDIEV

August 3, 2026 | Christoph Hanhart | IAS Forschungszentrum Jülich

color meets flavor

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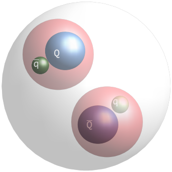
Hadronic Molecules



Guo et al., Rev.Mod.Phys.90(2018)015004

- are few-hadron states, bound by the strong force
- do exist: light nuclei.
  e.g. deuteron as pn & hypertriton as Λd bound state
- are located typically close to relevant continuum threshold;
  e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$
  - $E_B^{\mathrm{deuteron}} = 2.22$ MeV ($\gamma = 45$ MeV)
  - $E_B^{\mathrm{hypertriton}} = (0.11 \pm 0.03)$ MeV (to Λd) ($\gamma = 13$ MeV)

$\boxed{\text{Are there hadronic molecules in the double heavy sector?}}$

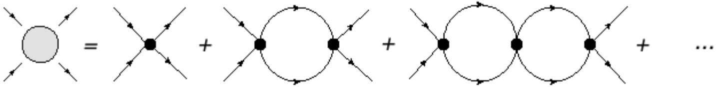
To address this we need to understand, how

- to describe two-hadron states theoretically
- the picture would change for more compact structures
- to exploit QCD symmetries to get a more global view

Two Particle Scattering

following C.H., arxiv: 2504.06043 [hep-ph]
Encyclopedia of Particle Physics

EFT for near threshold poles:



$$T \;=\; C \;+\; C\Pi(E)C \;+\; C\Pi(E)C\Pi(E)C \;+\; \ldots$$

$$= C\sum_{n=0}^{\infty}(C\Pi(E))^n \quad \Longrightarrow \quad \boxed{T(E) = \frac{1}{C^{-1} - \Pi(E)}}$$

with $\quad \Pi(E) = \int \frac{d^3q}{(2\pi)^3}\frac{F_\Lambda^2(q)}{E - q^2/(2\mu) + i\epsilon} = -\frac{\mu}{2\pi}\left(\Lambda + i\sqrt{2\mu E + i\epsilon}\right) + \mathcal{O}(\Lambda^{-1})$

$T(E)$ develops pole at $E = -E_B$ for $C^{-1} = \Pi(-E_B) = -\mu/(2\pi)\,(\Lambda - \gamma)$

... with binding momentum $\gamma = -i\sqrt{2\mu(-E_B) + i\epsilon} = \sqrt{2\mu E_B}$ (first sheet!)

Thus we have for the renormalised T matrix:

$$\boxed{T(E) = \left(\frac{2\pi}{\mu}\right)\frac{1}{\gamma + i\sqrt{2\mu E + i\epsilon}}} = \frac{g_{\rm eff}^2}{E + E_B} + \text{reg. terms;} \qquad \boxed{g_{\rm eff}^2 = \frac{2\pi\gamma}{\mu^2}}$$

Introducing compact component

Weinberg, Phys.Rev.137(1965)B672

One then gets for the scattering amplitude (without range corrections):

$$T = \frac{g_0^2}{E + E_B + g_0^2 \Pi^{\text{reg.}}(E)} = \frac{g_0^2}{E + E_B + g_0^2 \mu/(2\pi)(ip + \gamma)} \quad \left[E = \frac{p^2}{2\mu} \right]$$

where $g_0^2 = \frac{2\pi\gamma}{\mu^2} \left(\frac{1}{\lambda^2} - 1 + \mathcal{O}\left(\frac{\gamma}{\beta}\right) \right)$ with

full wave function ↓

$$\boxed{\lambda^2 = \text{Probability to find } \psi_0 \text{ in } \Psi}$$

compact component of wf. ↑

where β (e.g.$=1/R$) lightest energy scale not considered

$$\boxed{g_{\text{eff}}(\lambda^2)^2 = Z\, g_0^2 = \frac{2\pi\gamma}{\mu^2}\left(1 - \lambda^2\right) + \mathcal{O}\left(\frac{\gamma}{\beta}\right)}$$

$\lambda^2 = 0$ (pure mol.) $\to$ earlier results. Matching to effective range expansion:

$$T(E) = -\frac{2\pi}{\mu} \frac{1}{1/a + (r/2)p^2 - ip} \Longrightarrow \begin{cases} a = -2\frac{1-\lambda^2}{2-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right) \\ r = -\frac{\lambda^2}{1-\lambda^2}\left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right) \end{cases}$$

$\lambda^2 = 0 \Longrightarrow a = -1/\gamma + \mathcal{O}(1/\beta)$; $\boxed{r = \mathcal{O}(1/\beta) > 0}$ (for single channel case)

Admixture of compact component reduces a and gives sizeable neg. r

Summary: Properties of Molecules

- Are located in the “vicinity” of thresholds of the constituents
- For bound states the coupling to the constituents gets maximal
- The preferred decay channel is into the constituents, regardless phase space suppression
  $\Longrightarrow$ explains why Γ small, regardless open channels
- Scattering length is large
  $\Longrightarrow$ Near threshold enhancements!
- Positive, natural effective range
  $\Longrightarrow$ Important exception: Large coupled channel effects
- Dominant isospin violation: Mass differences of constituents
- Virtual states are always molecular

Matuschek et al., EPJA57(2021)101

Disclaimers and Outline

The method presented is 'diagnostic' — especially,

- it does not allow for conclusions on the binding force;
- it allows one only to study individual states;
- quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim R\gamma$)

To go beyond tailor made models/effective field theories needed

$\Longrightarrow$ Allow one to exploit approximate QCD symmetries

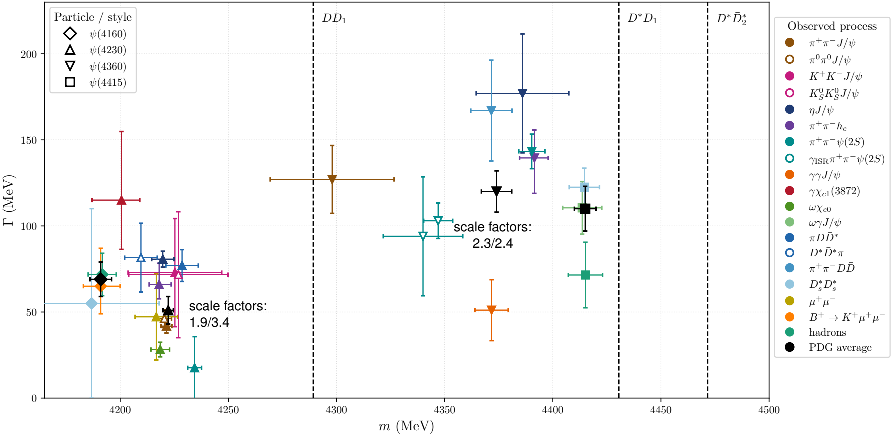
Outline

- Example I: Implications of Heavy Quark Spin Symmetry
  - Exploratory study of 1^{--} system and spin partners
- Example II: Isospin Symmetry and its breaking
  - Chiral EFT for $X(3872)$ and isovector partners

Example I: 1^{--} molecular states (and friends)

Quark model: 2 $\bar{c}c$ states between 4.1 and 4.5 GeV

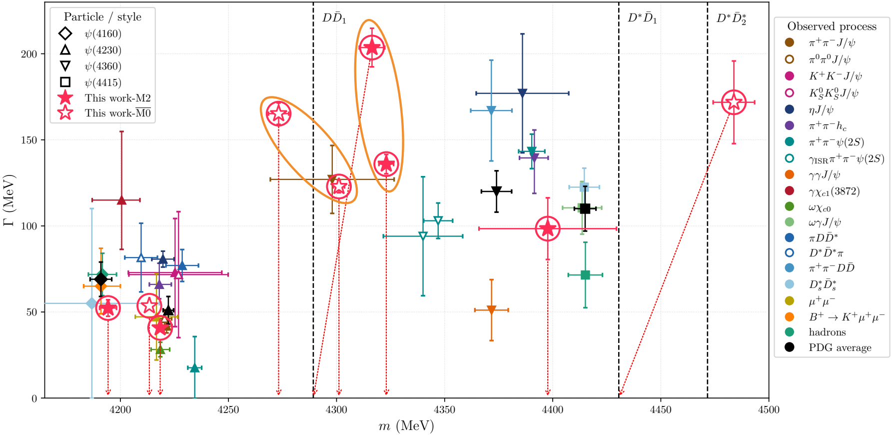
Experimental situation rather unclear: How many states are there?



Example I: 1^{--} molecular states (and friends)

Our study: 2-3 additional states — in studied channels $\bar{c}c$ states not crucial

Very strong effects from coupled channels!



Dong et al., [arXiv:2606.06180 [hep-ph]]; see also Nakamura et al., PRD112(2025)054027

Imposing Heavy Quark Spin Symmetry

Guo et al., Rev.Mod.Phys. 90 (2018) 1

following Bondar et al., PRD84(2011)054010, Mehen and Powell, PRD84(2011)114013 , Voloshin, Phys.Rev.D 84 (2011) 031502

Spins of heavy Quarks decouple! (for $M_Q \to \infty$)

$\Longrightarrow$ conserved light quark total angular momentum controls dynamics

$\{D, D^*\}$: $j_\ell = 1/2$; $\{D_1, D_2^*\}$: $j_\ell = 3/2 \Longrightarrow$ combine to $\boxed{j_\ell^{[\text{tot}]} = 1 \text{ or } 2}$

changing basis: $|D^{(*)}\bar{D}_{1,2}^{(*)}\rangle \Leftrightarrow |s_Q, j_\ell^{[\text{tot}]}, J\rangle$ and $|D_{1,2}^{(*)}\bar{D}^{(*)}\rangle \Leftrightarrow |\overline{s_Q, j_\ell^{[\text{tot}]}, J}\rangle$ gives

$$\boxed{\langle s_Q, j_\ell^{[\text{tot}]}, J|\mathcal{H}_{\text{int}}|s'_Q, j_\ell^{[\text{tot}]\prime}, J\rangle = \langle\overline{s_Q, j_\ell^{[\text{tot}]}, J}|\mathcal{H}_{\text{int}}|\overline{s'_Q, j_\ell^{[\text{tot}]\prime}, J}\rangle = \delta_{s_Q s'_Q}\delta_{j_\ell^{[\text{tot}]} j_\ell^{[\text{tot}]\prime}} F^d_{j_\ell^{[\text{tot}]}}}$$

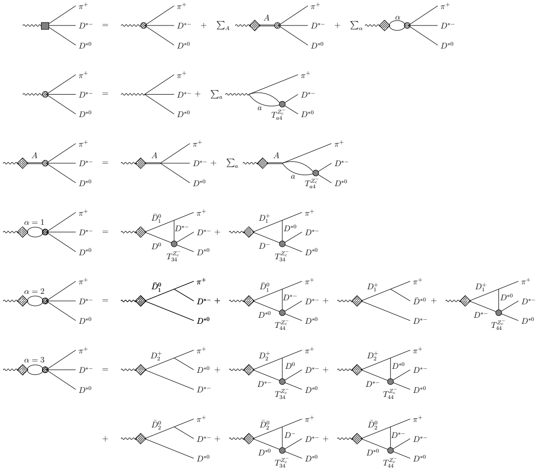
$$\boxed{\langle s_Q, j_\ell^{[\text{tot}]}, J|\mathcal{H}_{\text{int}}|\overline{s'_Q, j_\ell^{[\text{tot}]\prime}, J}\rangle = \langle\overline{s_Q, j_\ell^{[\text{tot}]}, J}|\mathcal{H}_{\text{int}}|s'_Q, j_\ell^{[\text{tot}]\prime}, J\rangle = \delta_{s_Q s'_Q}\delta_{j_\ell^{[\text{tot}]} j_\ell^{[\text{tot}]\prime}} F^c_{j_\ell^{[\text{tot}]}}}$$

- In total 4 complex parameters (3 relevant for $J^{PC}=1^{--}$: $F_1^c, F_1^d, F_2^d - F_2^c$)
- Spin Symmetry automatically provides strong channel coupling

Disclaimer: Too large energy range and too deep binding for systematic power counting $\Longrightarrow$ exploratory study!

Remarks on the parameters

Example: Diagrams for $D^* \bar{D}^* \pi$



Number of parameters

Model $\overline{\text{M0}}$:

- Production vertex: 12
- Scattering: 6
- Transition to 2 body channels: 2
- Parametrisation of Z_c/Z_c' sector: 10
- Parametrisation of $\pi\pi$ interaction: 7

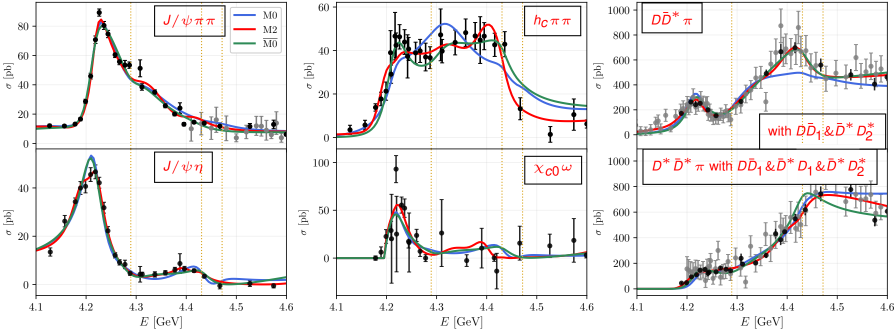
Total: 37

To fit 1240 data points

Results

Model M0 ($\overline{\text{M0}}$) $D^{(*)}\bar{D}^{(*)}_{1/2}$ int. only (reweighted data) [37 para.]

Model M2 with $\psi(4160)$ and $\psi(4415)$ added in [61 para.]



State	M [$\overline{\text{M0}}$]	Γ [$\overline{\text{M0}}$]	M [M2]	Γ [M2]	M [Nak]	Γ [Nak]	M [PDG]	Γ [PDG]
$\psi(4160)$	—	—	4195(4)[111]	52(4)	4192(2)	129(4)	4191(5)	69(10)
$\psi(4230)$	4214(1)[111]	52(2)	4219(1)[111]	40(2)	4230(1)	46(3)	4222(2)	51(8)
$\psi(4320)$	4274(2)[111]	154(4)	4314(6)[111]	202(12)	4308(2)	138(4)	—	—
	4301(1)[211]	116(2)	4321(3)[211]	132(6)				
$\psi(4360)$	4478(7)[211]	168(18)	—	—	4346(4)	123(7)	4374(7)	120(12)
$\psi(4415)$	—	—	4399(2)[211]	92(10)	4390(2)	107(4)	4415(5)	110(13)

blue: believed to be $\bar{c}c$ states
red: believed to be non-conventional states (not all on same sheets)

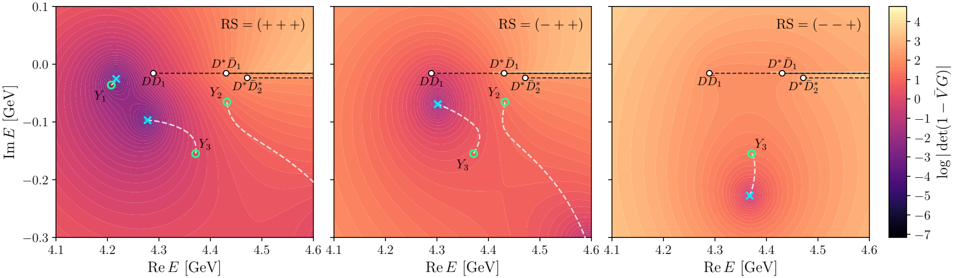
[Nak] = Nakamura et al., PRD112(2025)054027 (quote add. state at 4216 MeV from $D_s^*\bar{D}_s^*$)
all Data: from BESIII, e^+e^- induced

Pole Trajectories *M0*

To understand the pole map: study transition from de-coupled case

o: (near) Decoupling limit

x: Full result



Infinitesimal channel coupling duplicates poles

on unphysical sheets of lower lying thresholds

Spin Partner States & Summary Part I

The same 4 contact interactions can generate in addition:

$$\boxed{J^{PC} = 0^{-\pm}, 1^{-+}, 2^{-\pm}, 3^{-\pm}}$$

e.g. 0^{--} predicted based on meson-exchange model Ji et al., PRL129 (2022) 102002

We have 3 out of 4 constrained. With those we access:

- $0^{--}[D^*\bar{D}_1]$: Pole badly determined — mostly virtual state
- $0^{-+}[D^*\bar{D}_1]$: Bound, $E_b = 50...80$ MeV
- $3^{--}[D^*\bar{D}_2^*]$: Bound, $E_b = 90...100$ MeV

Disclaimer: Imaginary parts dropped; uncertainties incomplete

Summary Part I:

$\Longrightarrow$ 1^{--} channel very rich; number & nature of states still unclear

$\Longrightarrow$ Studies of other quantum numbers can raise the curtain

EFT for doubly heavy states

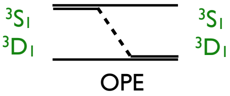
Schemes currently proposed (only initial work cited)—analogous to *NN*

- Contact EFT: no pions — AlFiky et al., PLB640(2006)238
- X-EFT: pert. pions — Fleming et al., PRD76(2007)034006
- chiral EFT: non.-pert. pions — Baru et al., PRD84(2011)074029
  - broadest range of applicability
  - Pion dynamics fully under control (3-body unitarity) $\Longrightarrow$ we use this

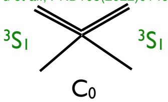
Thus we solve LS-equation

for X: Baru et al., PRD84(2011)074029; for T_{cc}: Du et al., PRD105(2022)014024

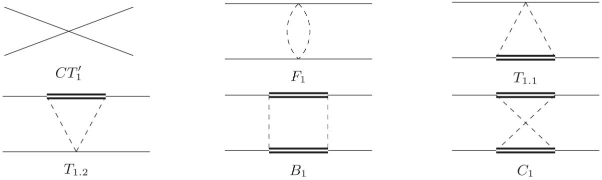
$$T = V + VGT \quad \text{with} \quad V_{LO} =$$



+



$V_{NLO} =$



Note:
reducible box enhanced by $\pi\mu/p$

Observation: Loops can be absorbed into counter terms — Chacko et al. PRD 111 (2025)034042

Example II: $\chi_{c1}(3872)$ aka $X(3872)$

Allow for $I = 0$ and $I = 1$ interaction; inelastic channels added

Ji et al., [arXiv:2502.04458 [hep-ph]]

- Isospin violation
  - $M_{D^{(*)0}} \neq M_{D^{(*)+}}$
  - ρ-ω mixing
- X as bound state

  $E_X = \left(-160^{+57}_{-74} - i\,125^{+23}_{-38}\right)$ keV

  with 2.7 σ below $D^0 \bar{D}^{*0}$ threshold
- Additional state

  W_{c1} with

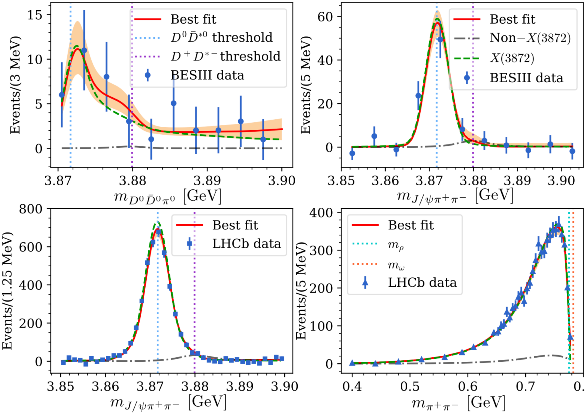
  $E_W = \left(3.1 \pm 0.7 + i\,1.3^{+1.9}_{-0.6}\right)$ MeV

  relative to $D^+ D^{*-}$ threshold

  on hidden sheet (R_{12})

  pred.: Zhang et al. JHEP08(2024)130

  lattice: Sadl et al., PRD111(2025)054513



analysis gives compositeness of $X(3872)$=0.97(2)

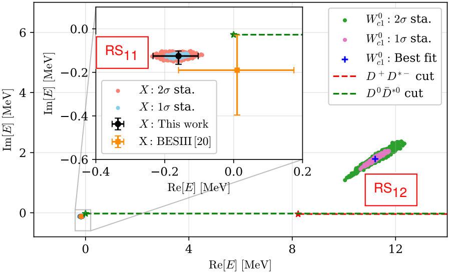
using Li et al. PRD105(2022)L071502

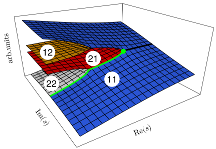
W_{c1} effect amplified through interference with X

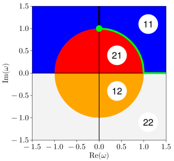
For the experts

Zhang, Guo, PLB 863 (2025) 139387, C.H. [arXiv:2508.20694 [hep-ph]]

see also: Peláez, Rabán and de Elvira, [arXiv:2606.14634 [hep-ph]]







conformal map: $k_{1/2} = \sqrt{\frac{\mu_{1/2}\Delta}{2}}\left(\omega \pm \frac{1}{\omega}\right)$

$$E = \frac{k_1^2}{2\mu_1} = \frac{k_2^2}{2\mu_2} + \Delta = \frac{\Delta}{4}\left(\omega^2 + \frac{1}{\omega^2} + 2\right)$$

Pole trajectories X vs. W
(pure contact int.; particle basis)

$$V^{\rm ct} = \frac{1}{2}\begin{pmatrix} C_{0X}+C_{1X} & C_{0X}-C_{1X} \\ C_{0X}-C_{1X} & C_{0X}+C_{1X} \end{pmatrix}$$

- start: Isospin limit & $C_{0X}=C_{1X}$
  $\rightarrow$ channels decouple
  $\rightarrow$ Poles on RS$_{21}$ and RS$_{12}$
- $M_{D^{(*)+}} \neq M_{D^{(*)0}}$ & $C_{0X} \neq C_{1X}$:
  $\rightarrow$ thresholds move apart
  $\rightarrow$ states couple
  and repel each other
  - one near neutral thres.
    gets bound ($X(3872)$)
  - one near charged thres.
    gets width
    & moves up on RS$_{12}$

$\Longrightarrow$ threshold cusp (W_{c1})

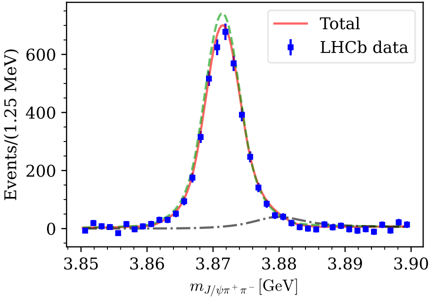
Possible Future measurements

W_{c1} signal could be enhanced by switching production source:

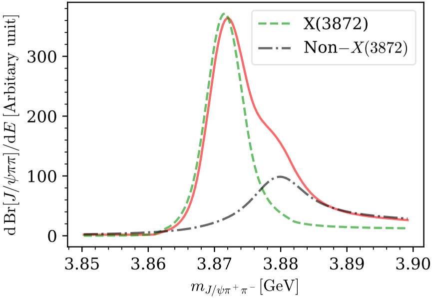
For example $\Gamma(B^+ \to K^+ D^0 \bar{D}^{*0}) > \Gamma(B^+ \to K^+ D^+ D^{*-})$ $(P_\pm / P_0 = 0.5)$

but $\Gamma(B^0 \to K^0 D^0 \bar{D}^{*0}) < \Gamma(B^0 \to K^0 D^+ D^{*-})$ $(P_\pm / P_0 = 2)$

$B^+ \to K^+[J/\psi\pi^+\pi^-]$



$B^0 \to K^0[J/\psi\pi^+\pi^-]$



... analogous prediction for $B^0 \to K^0 D^0 \bar{D}^{*0}$ [BelleII?]

Summary and Perspectives

Recent and future data have the potential to allow us

to identify the prominent components in exotic states

to-do for experiment

- Continue great performance! Especially needed:
  - data for different quantum numbers and
  - data for line shapes in different systems

to-do for theory

- Provide more predictions for the different scenarios in EFTs as well as lattice QCD
- Go beyond simplest approaches e.g. study interplay of regular quarkonia with exotics (in 1^{--} very weak!!)

Thanks a lot for your attention

BACK-UP SLIDES

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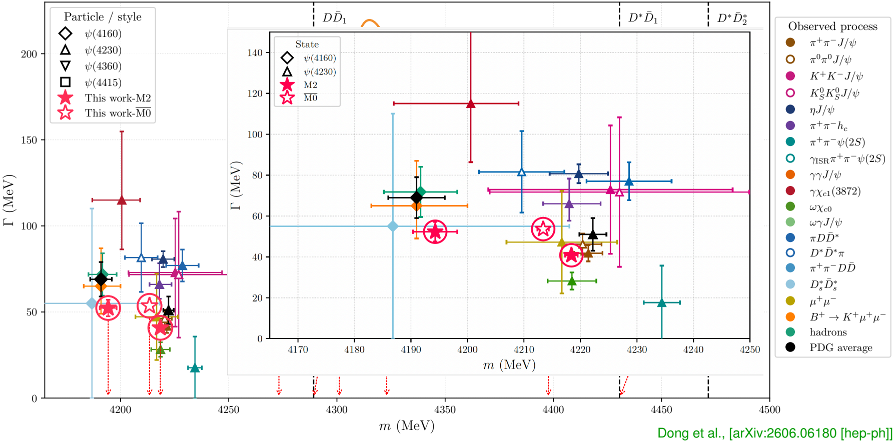

Dong et al., [arXiv:2606.06180 [hep-ph]]

Sheet structure — two channels