Neural-network quantum states for exotic hadrons and nuclei

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Peking University

Based on: PRL136 (2026) 071901; arXiv: 2606.09254

Coauthors: Lu Meng (孟璐), Liang-Zhen Wen (温梁臻), Shi-Lin Zhu (朱世琳)

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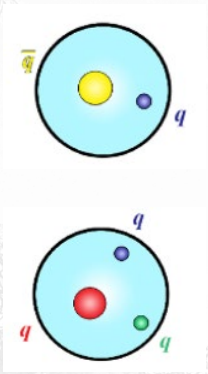
01 Introduction to Neural-network Quantum State

02 Meson-Nucleus System arXiv: 2606.09254

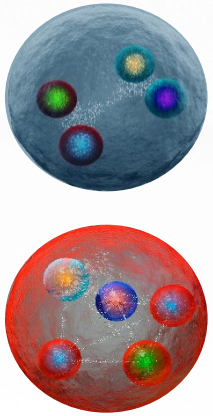
03 Multiquark System PRL 136 (2026) 071901

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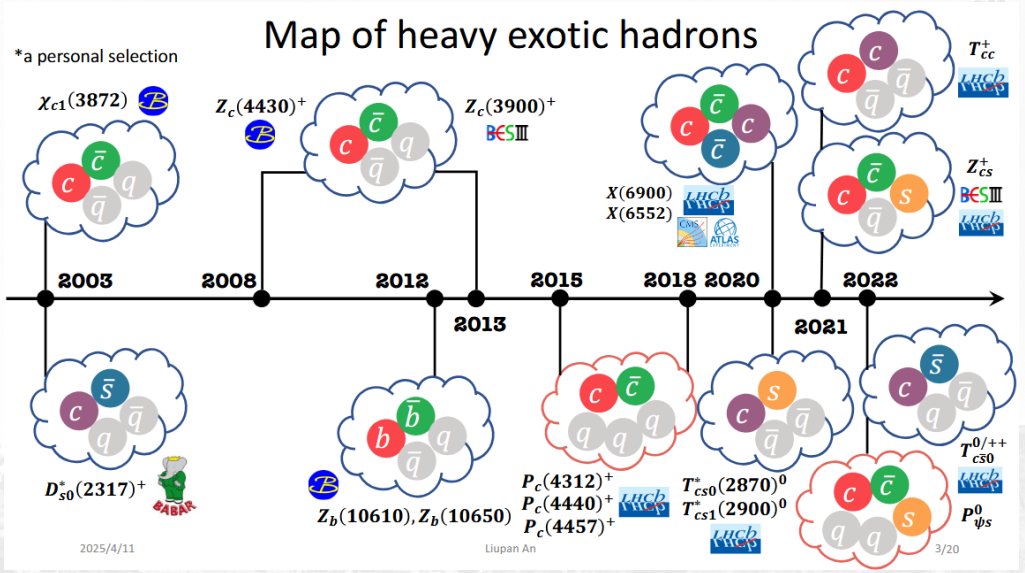
Exotic hadrons and nuclei



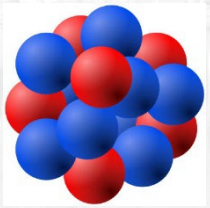
Conventional Hadron



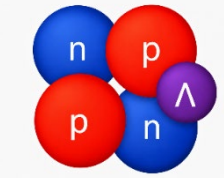
Multiquark



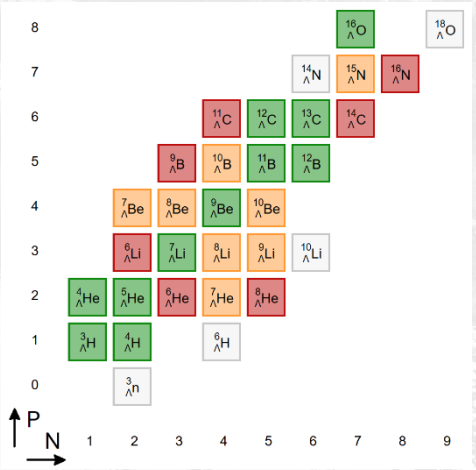
Credit: Liu-Pan An



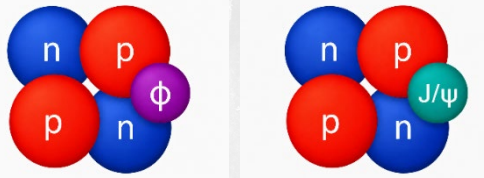
Nuclei



Hypernuclei:
rich spectroscopy



hypernuclei.kph.uni-mainz.de

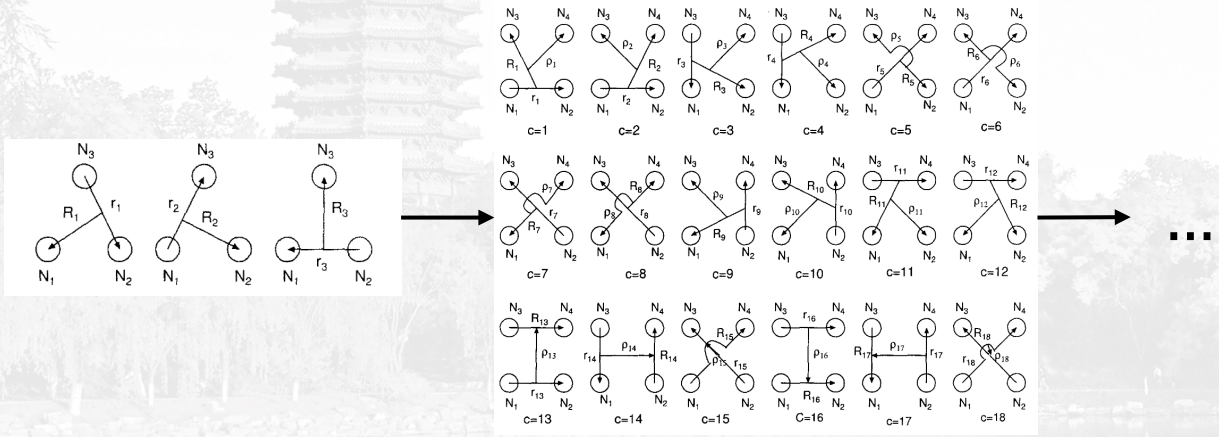


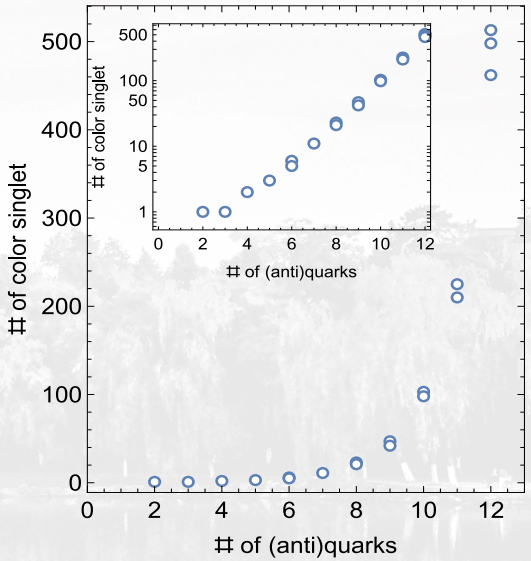
Mesic nuclei:
yet to be found

Quantum few-body problem

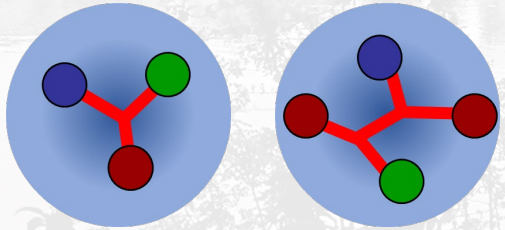
We need to deal with multiple-quark/hadron systems:

- Dimensionality curse: exponential growth of basis





- Many-body force
  - 3NF, 4NF, …
  - Possible flux-tube confinement interaction



T.T. Takahashi, PRL 86, 18 ;
F. Okiharu, PRD 72, 014505 ;
P. Bicudo, PRD 96, 074508

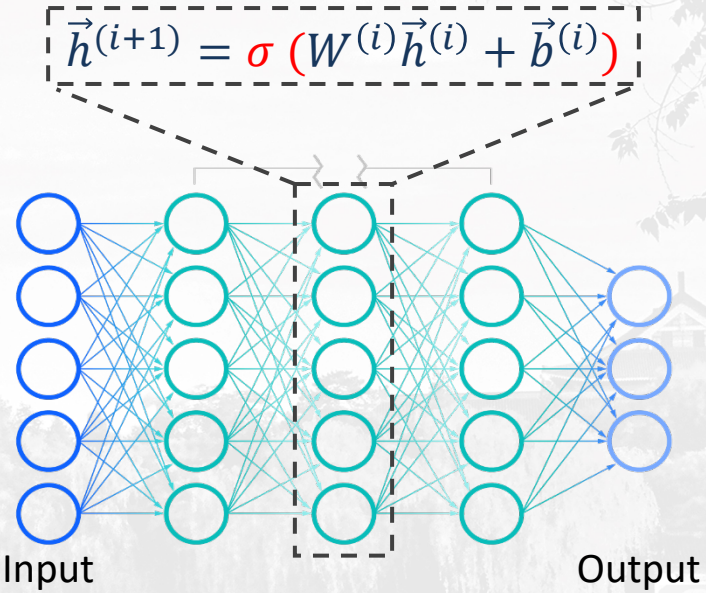
Neural-network Quantum State (NQS)

A new framework for solving quantum many-body problems:

Deep neural network (DNN)

- Representation of high-dimensional functions
- Trainable parameters $W^{(i)}, \vec{b}^{(i)}$
- Nonlinear activation function $\sigma(x)$



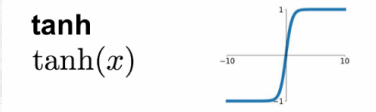


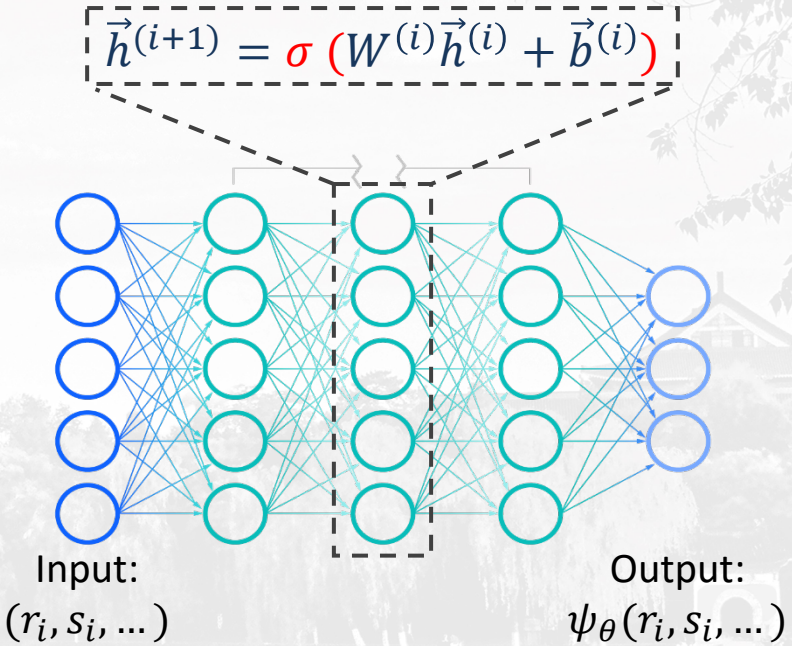
Neural-network Quantum State (NQS)

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NQS: many-body wave function $\psi_\theta(r_i, s_i, \dots)$ represented by DNN

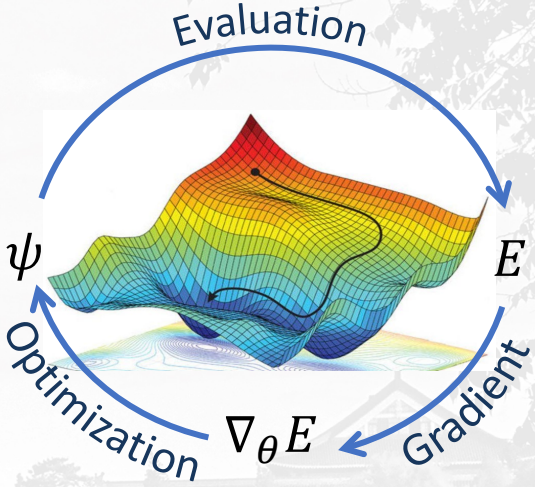
G. Carleo, Science 355, 602

- Polynomial scaling with N
- Capable of describing different configurations
- Architecture enforces physical symmetries
- Unsupervised learning: no additional data, trained by physical equation

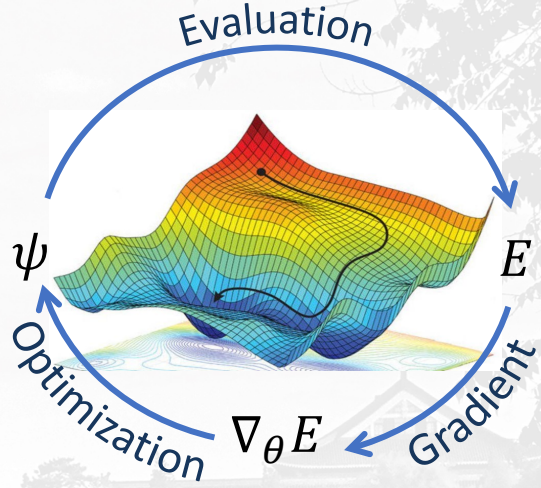
Variational Monte Carlo (VMC)

Variational Principle. $E[\theta] = \langle\psi_\theta|H|\psi_\theta\rangle / \langle\psi_\theta|\psi_\theta\rangle \geq E_0$

- Minimizing $E[\theta]$ approaches ground state
- Traditional ansatz: based on *a priori* knowledge, limited expressiveness
- **NQS: an expressive, flexible ansatz**



Variational Monte Carlo (VMC)



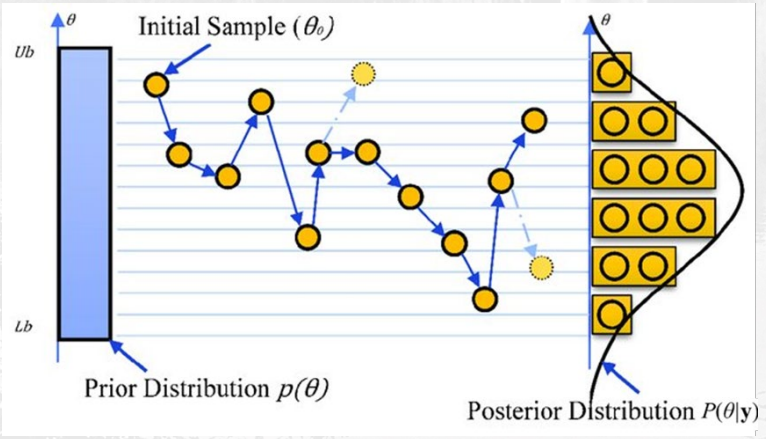
Variational Principle. $E[\theta] = \langle\psi_\theta|H|\psi_\theta\rangle/\langle\psi_\theta|\psi_\theta\rangle \geq E_0$

- Minimizing $E[\theta]$ approaches ground state
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- **NQS: an expressive, flexible ansatz**

Monte Carlo Estimation. High-dim integrals via importance sampling:

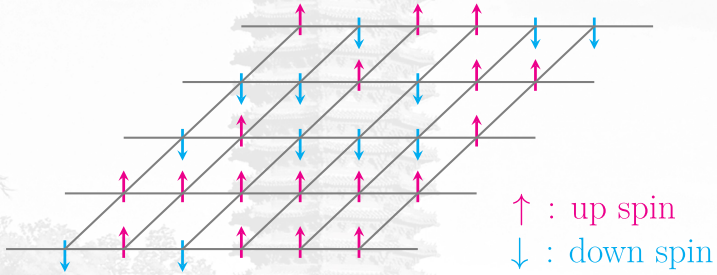
$$E[\theta] = \frac{\int |\psi_\theta(r)|^2 \frac{H\psi(r)}{\psi(r)} dr}{\int |\psi_\theta(r)|^2 dr} \approx \frac{1}{N}\sum_i E_L(r_i) \qquad E_L(r) = \frac{H\psi(r)}{\psi(r)} = \frac{T\psi(r)}{\psi(r)} + V(r)$$

- Samples $r_i \sim |\psi_\theta|^2$ generated by Metropolis-Hastings algorithm
- Efficient evaluation with statistical error $\sim 1/\sqrt{N}$
- easily handles complicated potentials, only needs $V(r_i)$



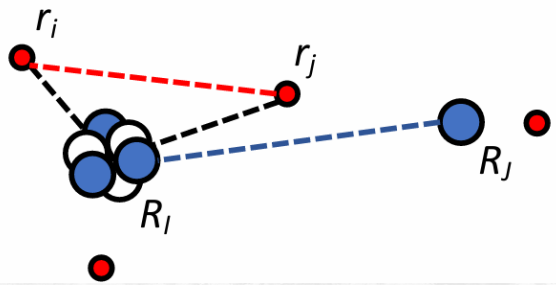
Applications in many-body physics

Spin Models



Science 355, 602 (2017), …

Atomic & Molecular Systems

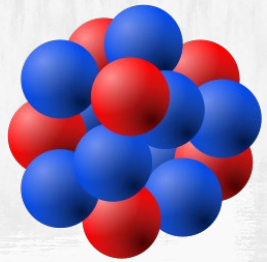


FermiNet: Phys. Rev. Res. 2, 033429 (2020).

PauliNet: Nat. Chem. 12, 891 (2020).

…

Nuclear Physics



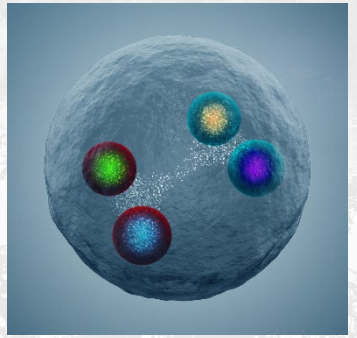
$A \leq 4$**:** PRL 127, 022502 (2021).

FeynmanNet: Phys.Rev.C 107, 034320 (2023).

Hypernuclei: PLB 874, 140285 (2026), …

Mesic nuclei: 2606.09254

Hadron Physics



DeepQuark: PRL 136, 071901 (2026)

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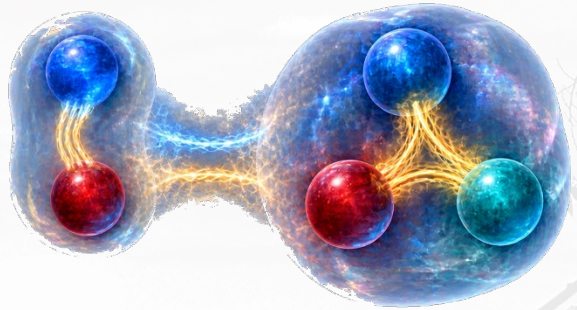
02 Meson-Nucleus System arXiv: 2606.09254

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Meson-Nucleon Interaction

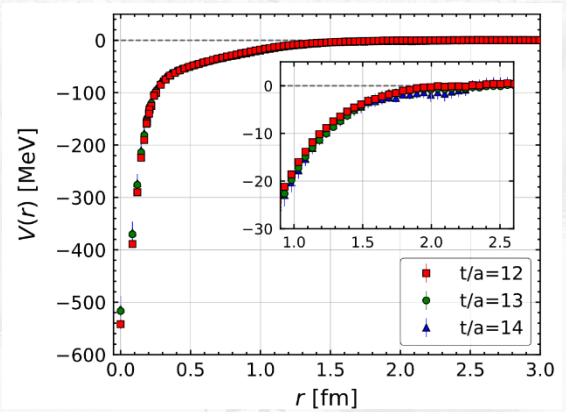
- Meson $(s\bar{s}, c\bar{c})$ - Nucleon (qqq)
  - No common valence quarks, meson exchange suppressed
  - Probe of gluonic dynamics



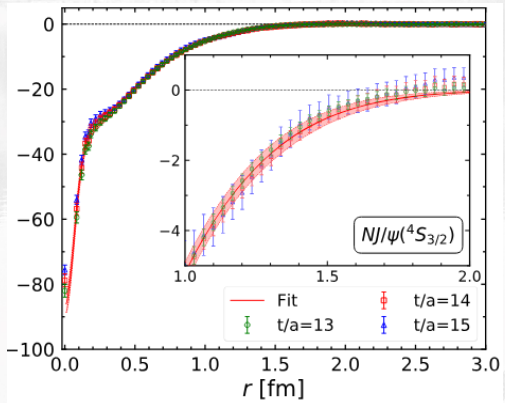
- Experiments: photoproduction, correlation function
  PRL 95, 182001 (2005); PRL 123,072001 (2019); PRL 127, 172301 (2021); …
- Lattice HAL QCD potential

} Attractive, but insufficient for MN bound state

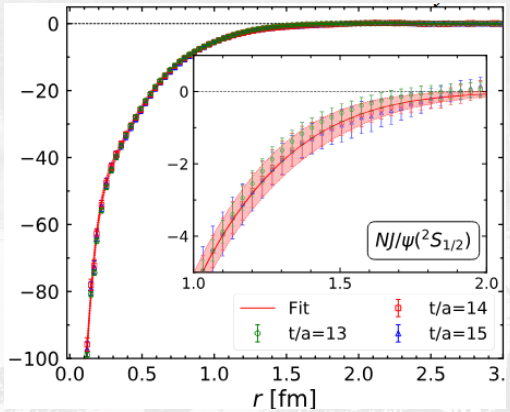
Y. Lyu, PRD 106, 074507; Y. Lyu, PLB 860, 139178



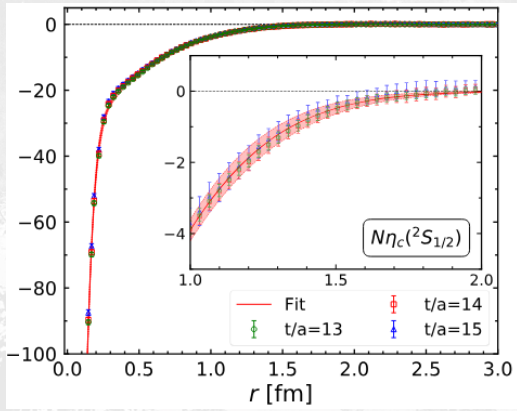
$N\phi(4S_{3/2})$



$NJ/\psi(4S_{3/2})$



$NJ/\psi(2S_{1/2})$



$N\eta_c(^2S_{1/2})$

Meson-Nucleon Interaction

- Lattice HAL QCD potential
  - ✓ $NJ/\psi(2S_{1/2})$, $NJ/\psi(4S_{3/2})$
  - ✓ $N\phi(4S_{3/2})$
  - ✓ $N\eta_c(2S_{1/2})$
- $N\phi(2S_{1/2})$ is missing
  - ➢ $\phi N \leftrightarrow K\Lambda \leftrightarrow K\Sigma$ coupled-channel effects
  - ➢ Model A: fitting $p\phi$ correlation function

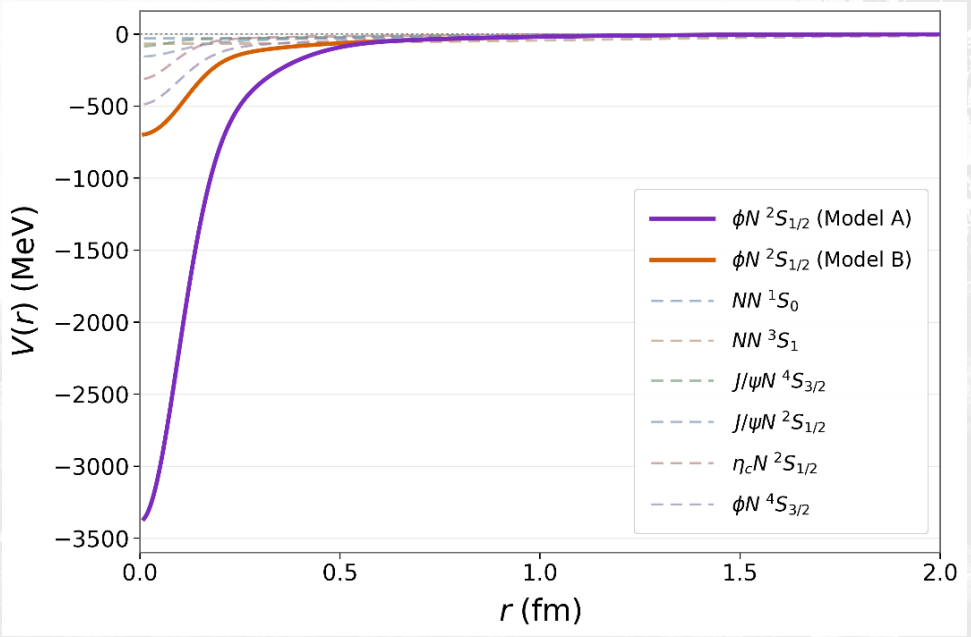
E. Chizzali, PLB 848, 138358

  - ➢ Model B: assuming mass dependence of spin interactions ✓

L.Z. Wen, PRD 111, 114004

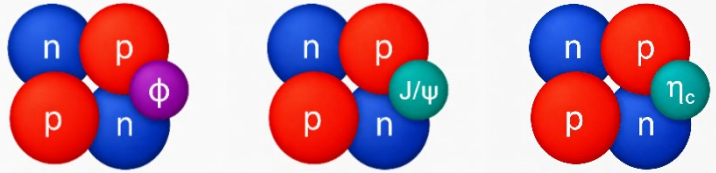
$$V^s_{\phi N}(r) = \frac{m_{J/\psi}}{m_\phi} V^s_{J/\psi N}(r) = \frac{m_{J/\psi}}{m_\phi} \frac{2(V^{^4S_{3/2}}_{J/\psi N} - V^{^2S_{1/2}}_{J/\psi N})}{3},$$

$$V^{^2S_{1/2}}_{\phi N}(r) = V^{^4S_{3/2}}_{\phi N}(r) - \frac{3}{2} V^s_{\phi N}(r).$$



Meson-Nucleus Systems

- Attractive MN interaction may give rise to M-A bound states



  - Prediction of M-A bound states S.J. Brodsky, PRL 64, 1011 (1990) ; H. Gao, PRC 63, 022201
  - Now: state-of-the-art calculations + first-principles potential

- $(A+1)$-body quantum mechanical problem

- Hamiltonian

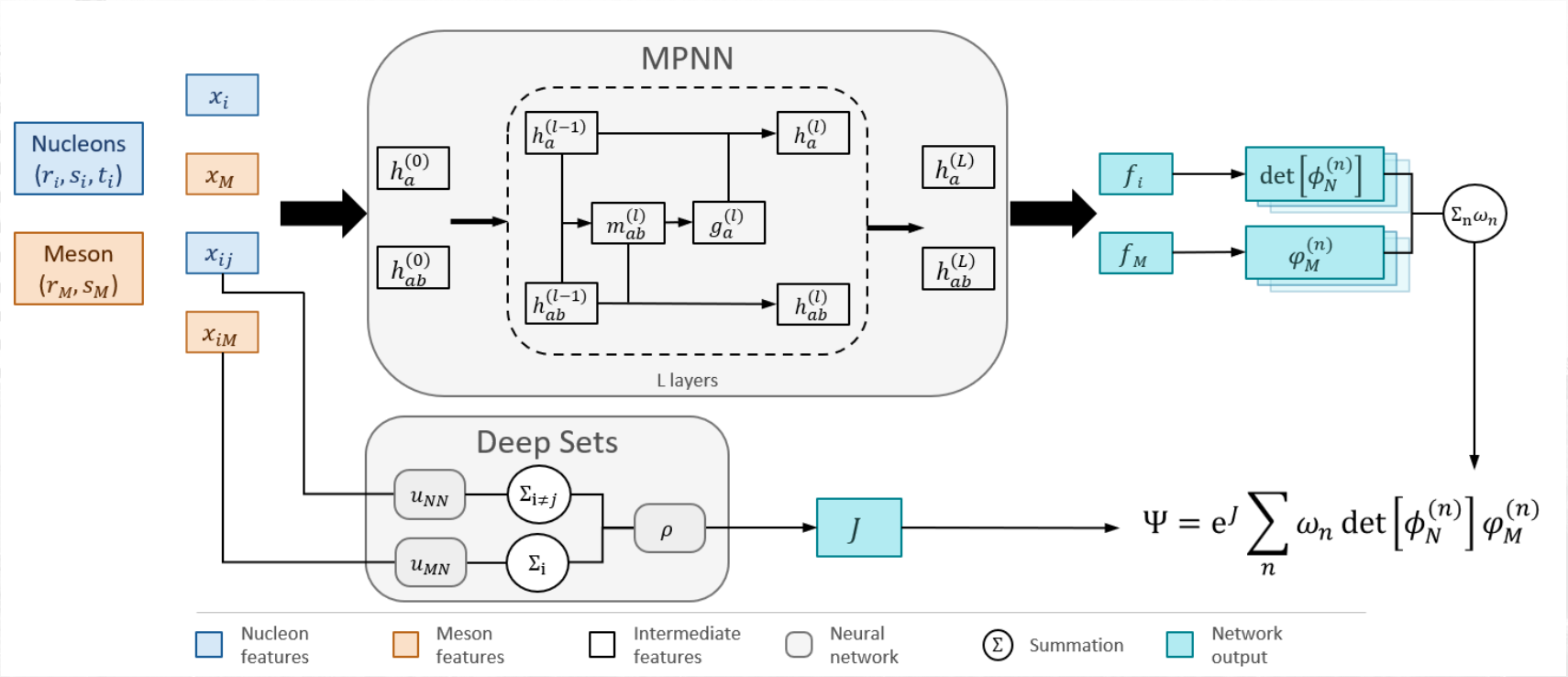
$$H = \sum_i \frac{p_i^2}{2m_i} + V_{NN} + V_{3N} + V_{MN}$$

  - Nucleon-nucleon interaction: LO pionless EFT R. Schiavilla, PRC 103, 054003
  - Meson-nucleon interaction: HAL QCD potential at $m_\pi = 146$ MeV

Y. Lyu, PRD 106, 074507; Y. Lyu, PLB 860, 139178

Meson-Nucleus NQS

W.L. Wu, L.Z. Wen, S.L. Zhu, arXiv: 2606.09254



Slater determinant

- Enforces fermionic antisymmetry
- Message-passing neural network
  - $\phi_i(x_j) \rightarrow \phi_i(x_j; x_1, \ldots, x_N)$

Jastrow factor

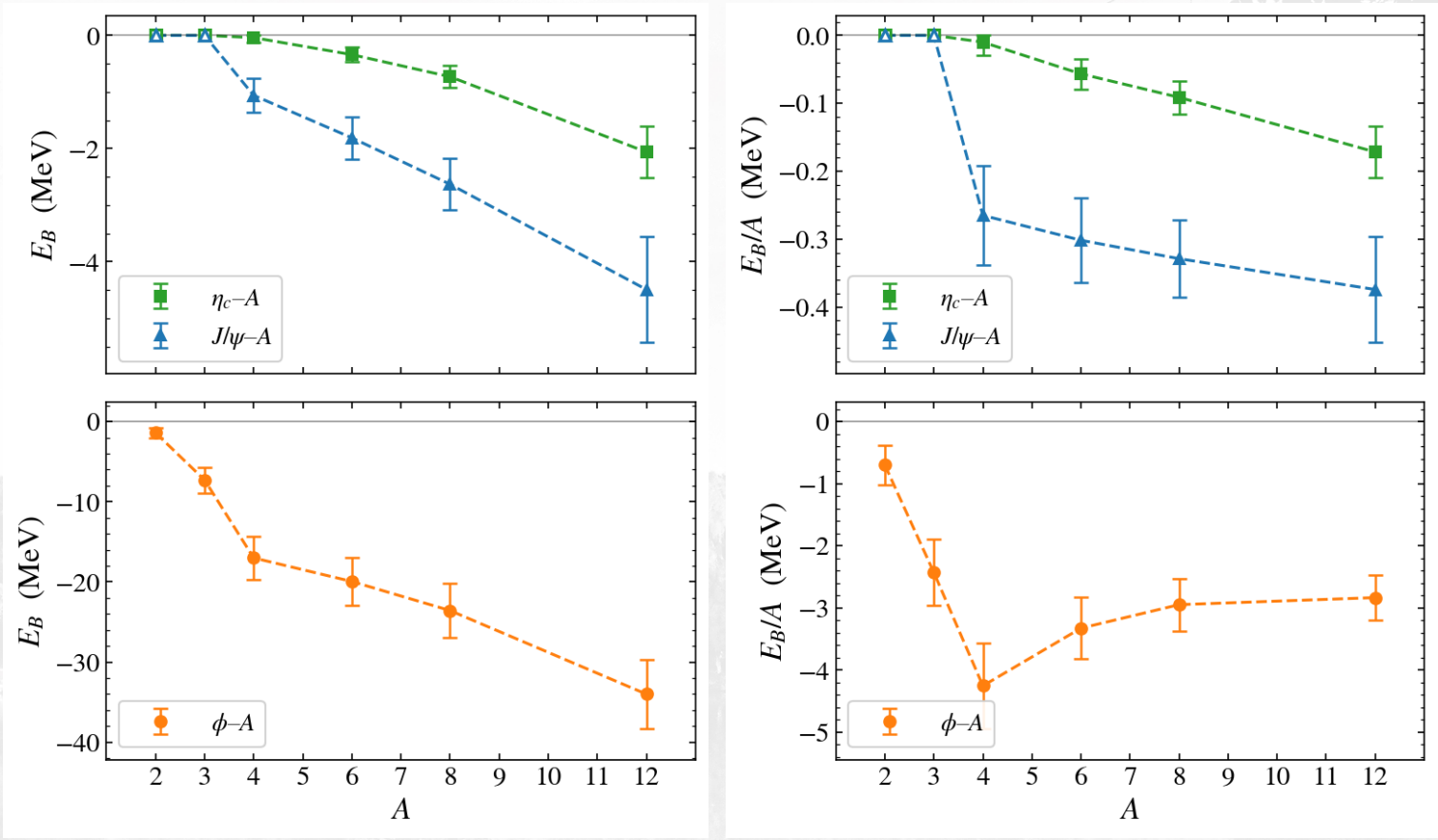
- Permutation invariant
- Deep Sets neural network
  - $J = \sum_{i<j} u(x_i, x_j) \rightarrow J(x_1, \ldots, x_N)$

Beyond single-particle orbitals and pairwise correlations

Binding energy

- Binding mass number:
  - $A \geq 2$ for ϕ
  - $A \geq 4$ for J/ψ
  - $A \geq 6$ for η_c
- E_B/A - A:
  - Nearly linear for charmonium
  - Peak at $A = 4$ for ϕ

→ Feature of short-range attractive interaction



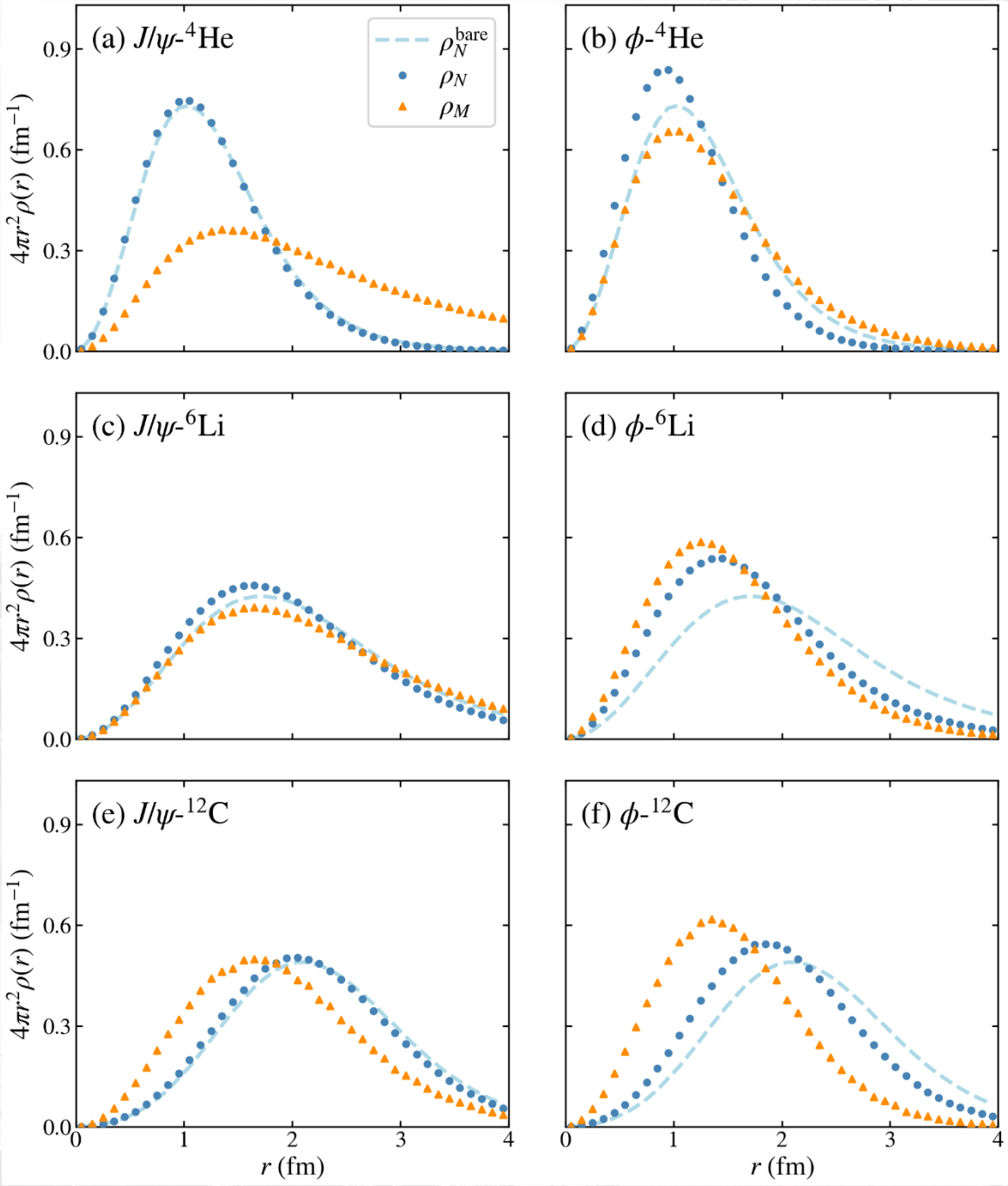
Error bar: uncertainty from HAL QCD potential

Spatial Structure

- Normalized radial density:

$$\rho_N(r) = \frac{1}{4\pi r^2} \frac{\langle\Psi|\sum_i \delta(|\boldsymbol{r}_i - \boldsymbol{R}_{\rm n}| - r)|\Psi\rangle}{A\langle\Psi|\Psi\rangle}, \qquad \rho_M(r) = \frac{1}{4\pi r^2} \frac{\langle\Psi|\delta(|\boldsymbol{r}_M - \boldsymbol{R}_{\rm n}| - r)|\Psi\rangle}{\langle\Psi|\Psi\rangle},$$

- Charmonium-Nucleus:
  - Meson halo → nuclear interior
  - Small modification of nuclear density
- ϕ-Nucleus:
  - Meson embedded inside nucleus
  - "Glue" effect: nuclear distribution is compressed
  - Short-range interaction strength ~ density overlap: maximum at ^{4}He



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Quark Potential Model

- Quark potential model
  - Constituent quarks
  - Minimal potential model

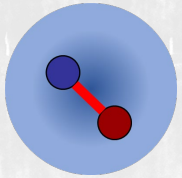
Cornell model: PRD 17,3090 (1978); BGS model: PRD 72,054026 (2005) , …

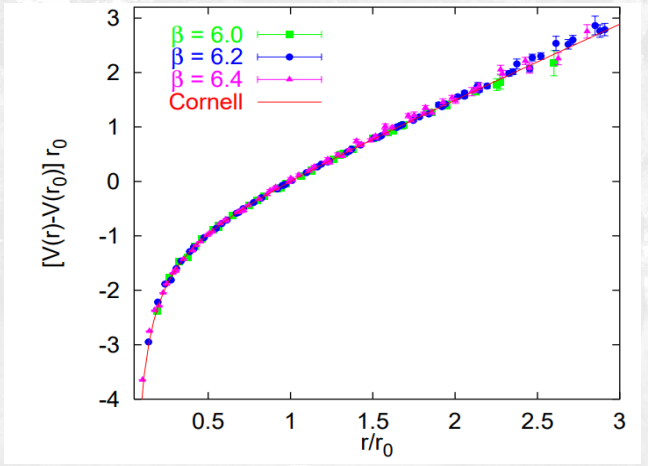
$$V_{ij}(r) = V_{ij}^{OGE} + V_{ij}^{conf.} = \left[\left(\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_Q^2}\,\tilde{\delta}(\sigma;r)\boldsymbol{S_i}\cdot\boldsymbol{S_j}\right) + \left(-\frac{3}{4}br\right)\right]\frac{\lambda_i\cdot\lambda_j}{4}$$

one-gluon-exchange



linear confinement





G.S. Bali, Phys. Rept. 343, 1-136

- Great success in conventional hadrons
- Extension to multiquarks?

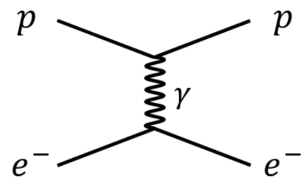
✓ $T_{cc}(3875)^+$ Q. Meng, PLB 814, 136095

✓ $T_{cc\bar{c}\bar{c}}(6900), T_{cc\bar{c}\bar{c}}(7200)$ **W.L. Wu**, PRD 109, 054034

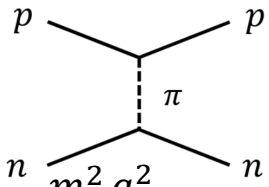
✓ $T_{cs0}^*(2870)^0$ Y.K. Chen, PRD 109, 014010

Challenges in multiquark systems

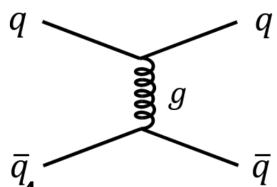
- Stronger interactions among quarks



$e^2 = 0.1$



$\frac{m_\pi^2 g_A^2}{16\pi f_\pi^2} \sim 0.1$



$\frac{4}{3}\alpha_s(m_c) \sim 0.4$

  - significant correlation effects
  - no evidence of shell structure in multiquarks
  - Slater wave function is not a natural starting point

$$\sum_{\mathcal{P}} \mathrm{sign}(\mathcal{P}) \prod_i \phi_i^k(\mathbf{x}_{\mathcal{P}_i}) = \begin{vmatrix} \phi_1^k(\mathbf{x}_1) & \dots & \phi_1^k(\mathbf{x}_n) \\ \vdots & & \vdots \\ \phi_n^k(\mathbf{x}_1) & \dots & \phi_n^k(\mathbf{x}_n) \end{vmatrix}$$

$$= \det\left[\phi_i^k(\mathbf{x}_j)\right] = \det[\mathbf{\Phi}^k],$$

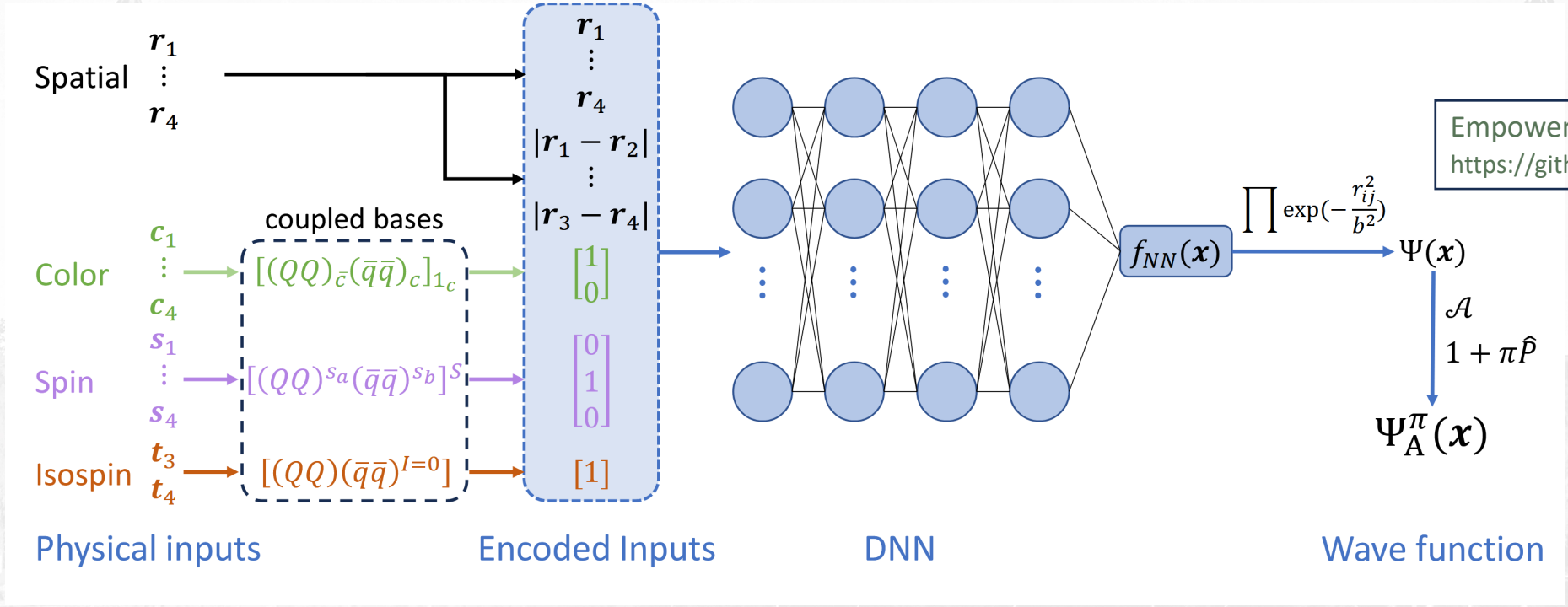
$$\psi(\mathbf{x}_1, \dots, \mathbf{x}_n) = \sum_k \omega_k \det[\mathbf{\Phi}^k],$$

- Symmetry constraints
  - color projection to ensure SU(3) singlet
  - spin projection to target states of interest

    $[cc\bar{q}\bar{q}]_{S=0}$: DD threshold, $[cc\bar{q}\bar{q}]_{S=1}$: T_{cc} state

DeepQuark Architecture

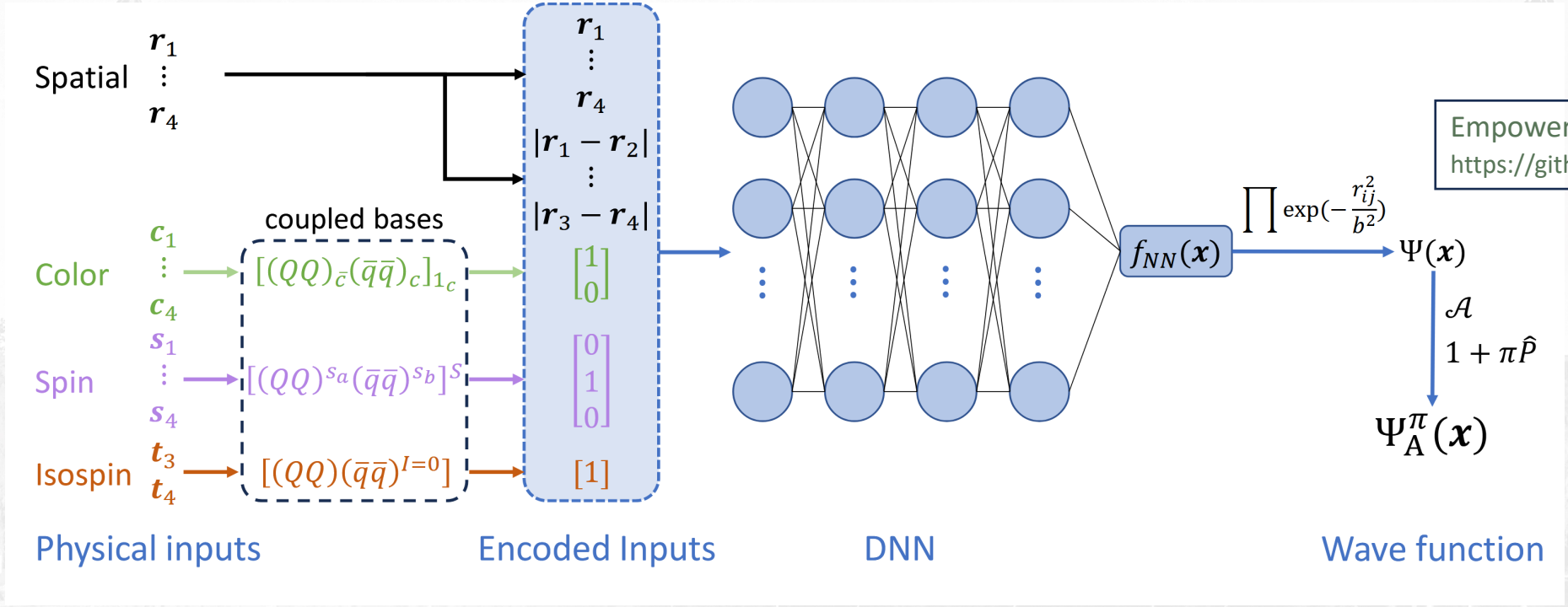
W.L. Wu, L. Meng, S.L. Zhu, PRL 136, 071901 (2026)



- Coupled bases instead of single-particle (s_i, t_i, c_i)
  - satisfy discrete symmetry
  - describe multichannel coupling

DeepQuark Architecture

W.L. Wu, L. Meng, S.L. Zhu, PRL 136, 071901 (2026)

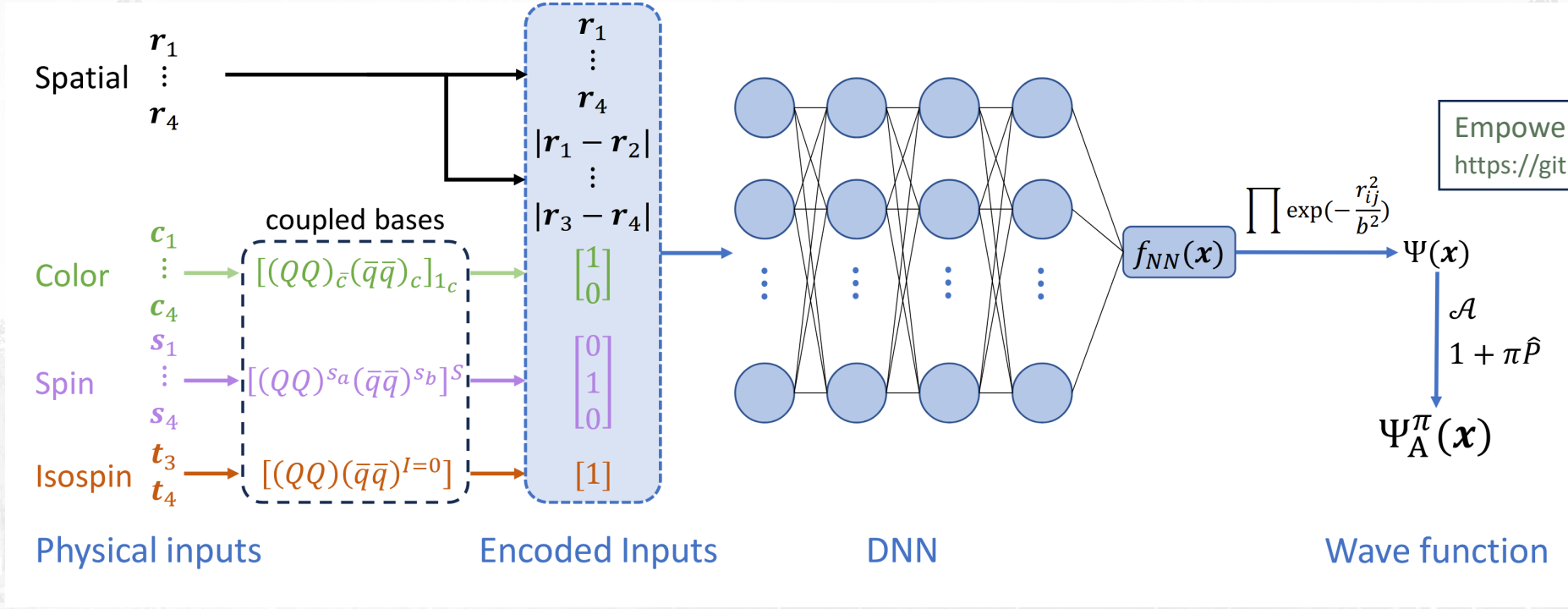


- Coupled bases instead of single-particle (s_i, t_i, c_i)
  - satisfy discrete symmetry
  - describe multichannel coupling
- Fully connected network with only $O(10^3)$ parameters

  vs. GEM for tetraquark:~10^4 bases; $A \leq 4$ nuclei: ~10^4 paras

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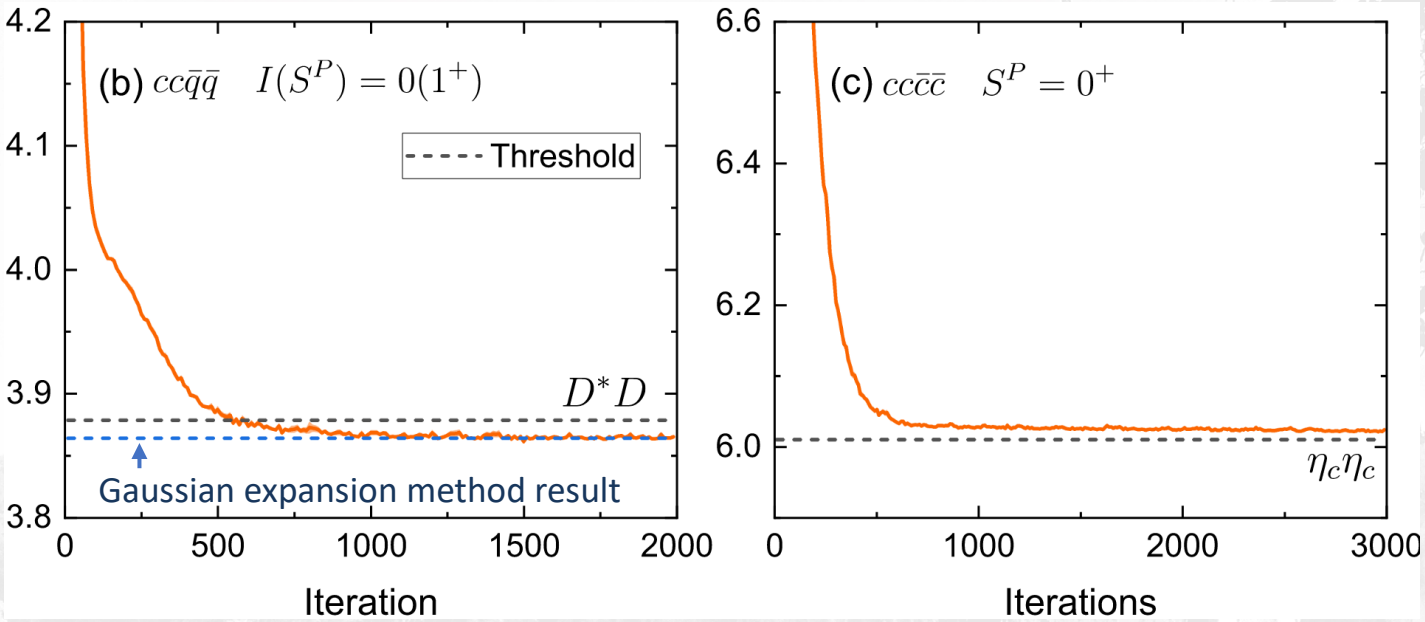
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  vs. GEM for tetraquark:~10^4 bases; $A \leq 4$ nuclei: ~10^4 paras

- Antisymmetrization $\mathcal{A} = \sum_{\mathcal{P}} (-1)^{\mathcal{P}} \mathcal{P}$
  - $O(n!)$ for identical particles
  - manageable up to hexaquark
  - the price for enforcing symmetry

Benchmark: Tetraquarks

- Convergence within 2000 iterations
- Agreement with known results



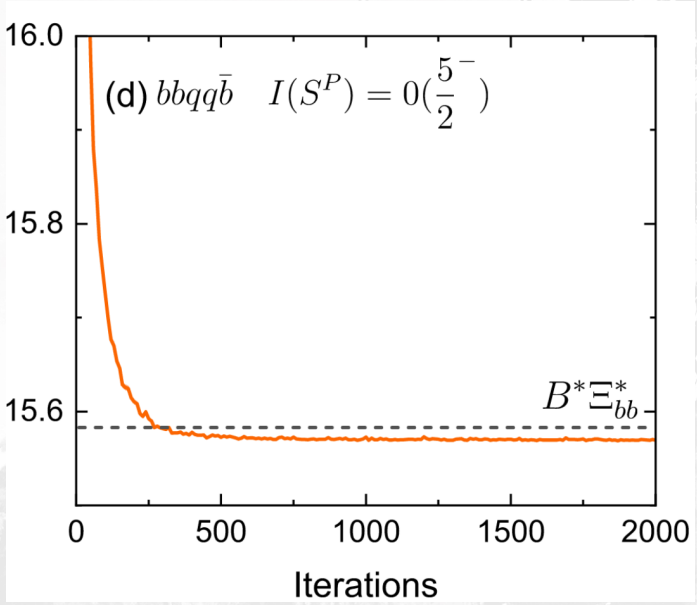
- $cc\bar{q}\bar{q}$: molecular state, agree with exp.
- $bb\bar{q}\bar{q}$: compact tetraquark
- $QQ\bar{Q}\bar{Q}$: no bound states

	$I(S^P)$	Thresh.	ΔE	$P_{\bar{3}_c\otimes 3_c}$	$P_{6\otimes 6_c}$	r_{QQ}	$r_{\bar{q}\bar{q}}$	$r_{Q\bar{q}}$
$cc\bar{q}\bar{q}$	$0(1^+)$	DD^*	-15	55%	45%	1.24	1.41	1.06
$bb\bar{q}\bar{q}$	$0(1^+)$	$\bar{B}\bar{B}^*$	-153	97%	3%	0.33	0.78	0.69

Extension to Pentaquarks

- Five-body calculations without few-body approximations using DeepQuark
- Moderate increase in computational cost compared to tetraquark

  (2000 iters: ~40/60 mins on a A100 GPU for tetra/penta-quark)



- Pentaquark bound states $QQ\bar{Q}qq$

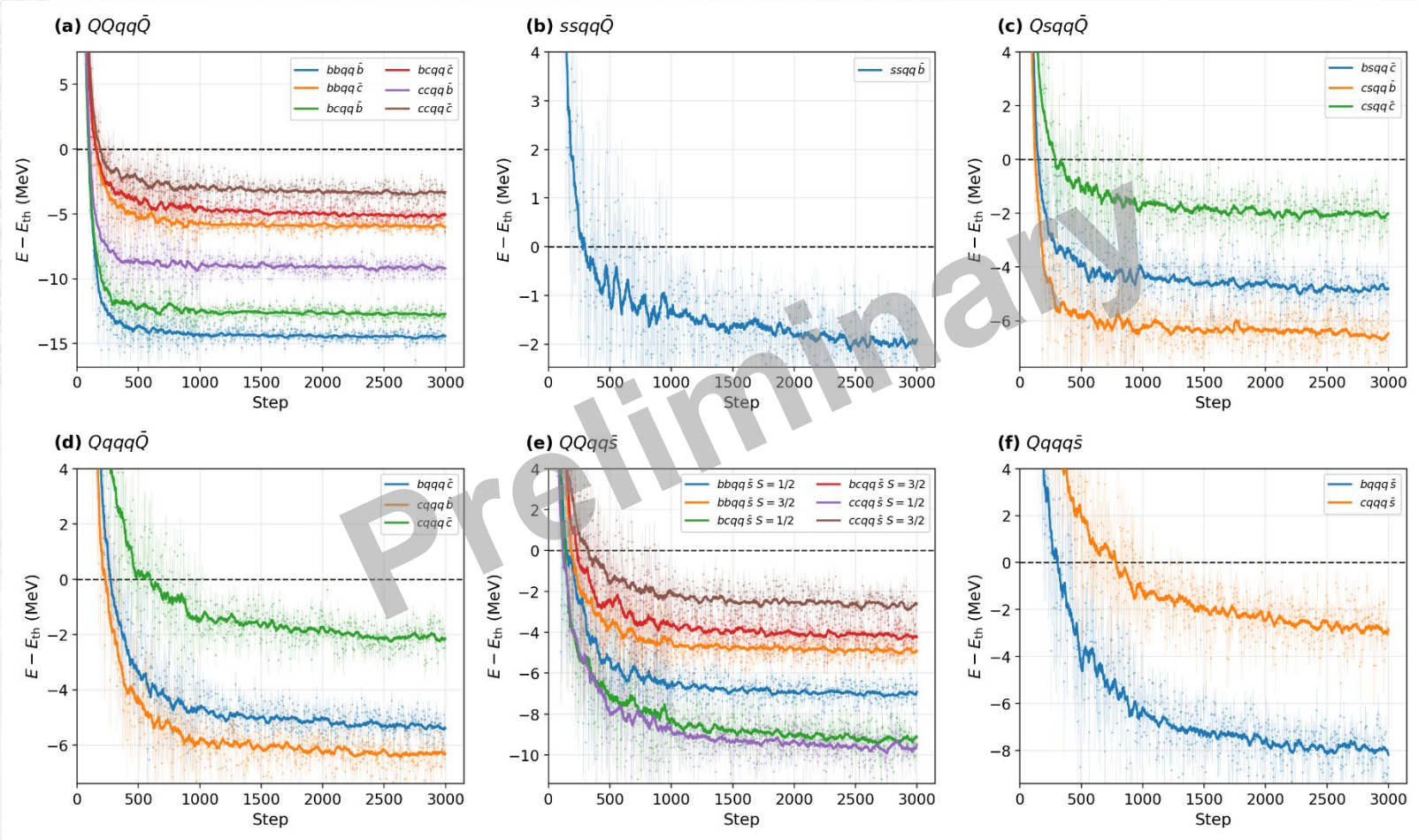
  F.K. Guo, PRD 88, 054014
  Z.Y. Wang, PRD 110, 076014

  - T_{cc} analogue under heavy diquark-antiquark symmetry: $(QQ)_{\bar{3}_c} \to \bar{Q}$
  - Possible search in D-wave $J/\psi\Lambda_c$

	$I(S^P)$	Thresh.	ΔE	r_{QQ}	r_{Qq}	r_{qq}	$r_{Q\bar{Q}}$	$r_{q\bar{Q}}$
$cc\bar{c}qq$	$0(1/2^-, 3/2^-)$	$\eta_c\Lambda_c, J/\psi\Lambda_c$	Unbound					
	$0(5/2^-)$	$\bar{D}^*\Xi^*_{cc}$	-3	0.50	1.32	1.82	1.64	1.32
$bb\bar{b}qq$	$0(1/2^-, 3/2^-)$	$\eta_b\Lambda_b, \Upsilon\Lambda_b$	Unbound					
	$0(5/2^-)$	$B^*\Xi^*_{bb}$	-14	0.31	0.96	1.32	1.02	0.95

$\to J/\psi\Lambda_c$ (D-wave)

Extension to Pentaquarks



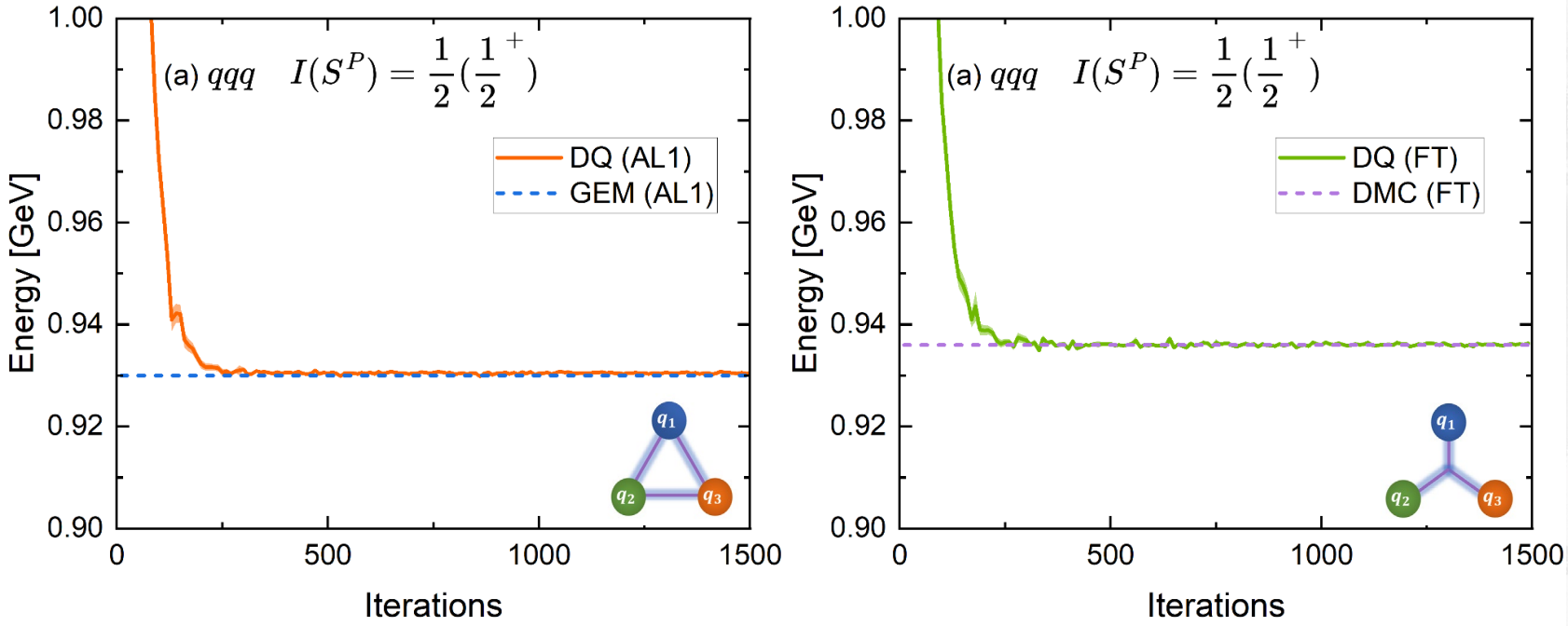
- Many baryon-meson molecular states are realized in constituent quark model
- No deeply bound pentaquark state exists (in contrast to T_{bb})

Beyond 2-body force: flux tube in baryons

- Lattice QCD favors flux-tube confinement interactions for baryons T.T. Takahashi, PRL 86, 18-21

$$V_{\text{conf}} = \sum_{i<j} \lambda r_{ij} \rightarrow \sigma L_{\text{min}} \qquad L_{\text{min}} = \begin{cases} \left[\frac{1}{2}(a^2+b^2+c^2) + \frac{\sqrt{3}}{2}\sqrt{d(d-2a)(d-2b)(d-2c)}\right]^{1/2} & \max(\theta_a,\theta_b,\theta_c) < \frac{2\pi}{3}, \\ d - \max(a,b,c) & \max(\theta_a,\theta_b,\theta_c) > \frac{2\pi}{3}, \end{cases}$$

- DeepQuark efficiently incorporates flux-tube type interactions using VMC
- Ready for exploring different confinement mechanisms in tetraquarks



Summary

- NQS + VMC framework
  - DNN as many-body wave function: efficient, general, unbiased
  - VMC: solves ground state using variational principle; handles complicated interactions
- Mesic-Nuclei bound states:
  - Full $(A+1)$-body calculations up to $A = 12$
  - Bound states: $A \geq 2$ for ϕ, $A \geq 4$ for J/ψ, $A \geq 6$ for η_c
  - E_B/A peaks at ϕ-^{4}He
- DeepQuark:
  - Architecture for strongly correlated multiquark systems with symmetry constraints
  - Outperforms traditional methods starting from pentaquarks
  - Capable of flux-tube interactions

Thanks for your attention!

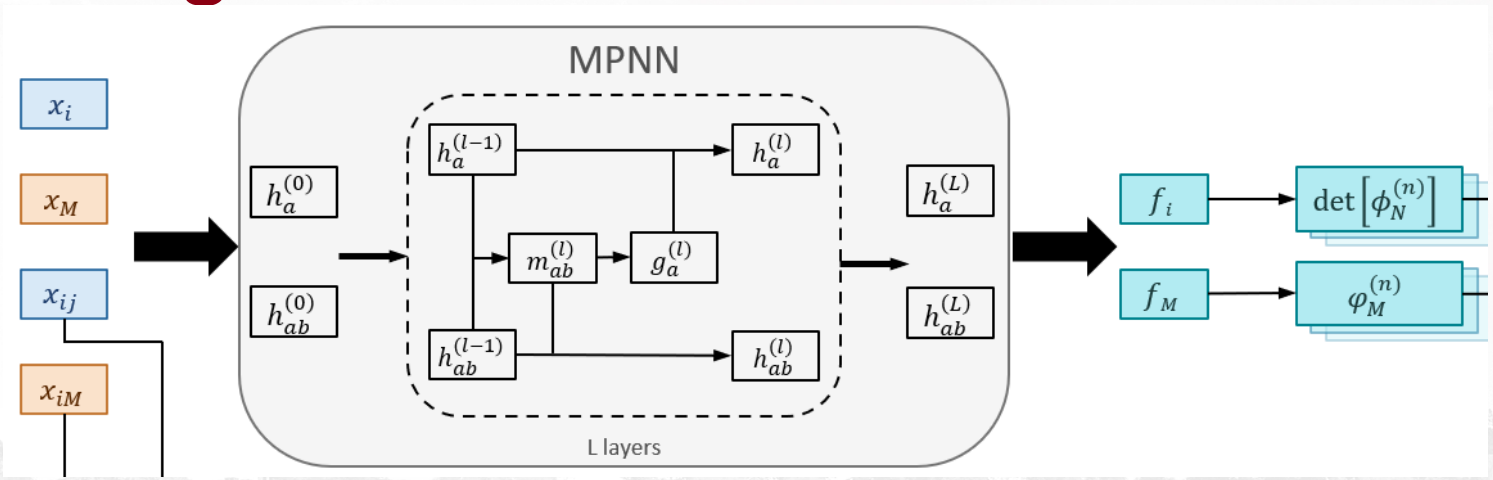
Backup

Nuclear ground states

TABLE I. Nuclear ground-state energies and S^2 expectation values. E uses the lattice nucleon mass $m_N = 954$ MeV, while E_{phys} uses the physical $m_N = 938.9$ MeV. Statistical errors are shown in parentheses. Experimental values from [61] are listed for comparison.

Nucleus	E (MeV)	E_{phys} (MeV)	$\langle S^2 \rangle$	Exp. (MeV)
^{2}H	$-2.416(0)$	$-2.240(1)$	2.00	-2.224
^{3}H	$-8.921(1)$	$-8.470(3)$	0.75	-8.482
^{3}He	$-8.250(1)$	$-7.808(2)$	0.75	-7.718
^{4}He	$-29.072(1)$	$-28.161(3)$	0.00	-28.30
^{6}Li	$-32.478(4)$	$-31.134(12)$	2.01	-31.99
^{8}Be	$-57.873(7)$	$-55.689(26)$	0.02	—
^{12}C	$-91.621(16)$	$-87.724(28)$	0.03	-92.16

Message-passing neural network



Backflow-transformed orbitals $\phi_i(x_j; x_1, \ldots, x_N)$:

- Antisymmetry: nucleon exchange $i \leftrightarrow j$ swaps orbital rows $\phi_{i\mu} \leftrightarrow \phi_{j\mu}$
- Input $h_i^{(0)}: (r_i, s_i, t_i)$, $h_{ij}^{(0)}: (r_{ij}, s_i, s_j, t_i, t_j)$
- Message-passing iteration

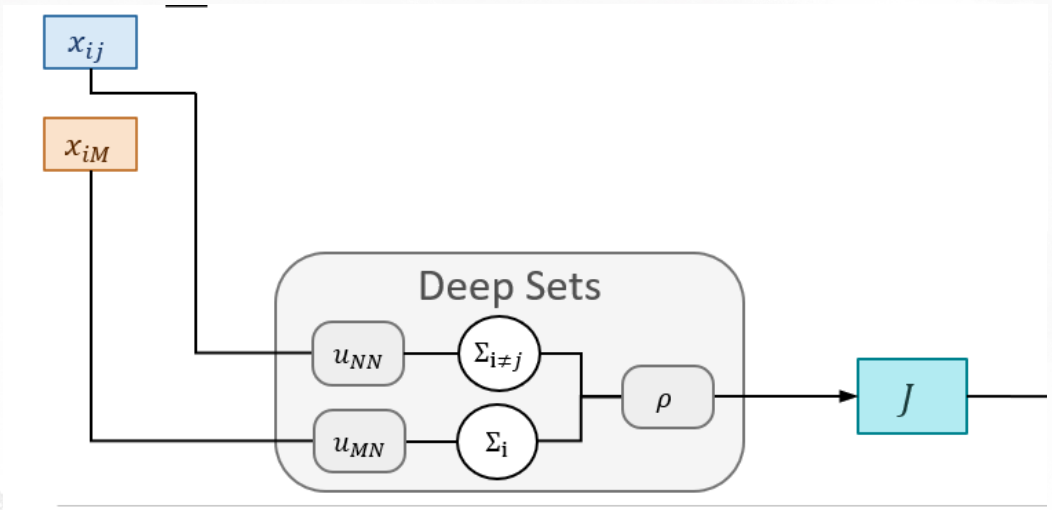
$$\begin{aligned}
\boldsymbol{m}_{ab}^{(l)} &= M^{(l)}\left(\boldsymbol{h}_a^{(l-1)}, \boldsymbol{h}_b^{(l-1)}, \boldsymbol{h}_{ab}^{(l-1)}\right), \\
\boldsymbol{m}_a^{(l)} &= \sum_{b\neq a} \boldsymbol{h}_b^{(l-1)} \odot \boldsymbol{m}_{ab}^{(l)}, \\
\boldsymbol{h}_a^{(l)} &= \boldsymbol{h}_a^{(l-1)} + U^{(l)}\left(\boldsymbol{h}_a^{(l-1)}, \boldsymbol{m}_a^{(l)}\right), \\
\boldsymbol{h}_{ab}^{(l)} &= \boldsymbol{h}_{ab}^{(l-1)} + V^{(l)}\left(\boldsymbol{h}_{ab}^{(l-1)}, \boldsymbol{m}_{ab}^{(l)}\right).
\end{aligned}$$

- Output

$$\boldsymbol{f}_a = \boldsymbol{h}_a^{(L)} + \sum_{b=1}^{A+1} \lambda\left(\boldsymbol{h}_b^{(L)}\right),$$

$$\begin{aligned}
\phi_{i\mu}^{(n)} &= O_N^{(n)}(\boldsymbol{f}_i)_\mu \exp\left(-\xi^{(n)2} \bar{r}_i^2\right), \\
\varphi_M^{(n)} &= O_M^{(n)}(\boldsymbol{f}_M) \exp\left(-\sum_{\alpha=1}^{3} \eta_\alpha^{(n)2} \bar{r}_{M,\alpha}^2\right),
\end{aligned}$$

Deep Sets



Deep Sets： a permutation-invariant form by summing over all index

$$J = \rho\left(\sum_{i\neq j} u_{NN}(\boldsymbol{x}_{ij}),\ \sum_{i} u_{MN}(\boldsymbol{x}_{iM})\right)$$

- u_{NN}, u_{MN}: maps x_{ij}, x_{iM} into latent space R^F
- $\rho: R^{2F} \rightarrow R$