Three-body coupled-channel approach to vector charmonia

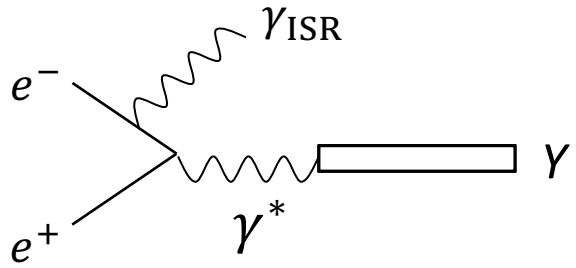
PRD 112, 054027 (2025)

Satoshi Nakamura (Shandong Univ., China)

Collaborators: X.-H. Li, H.-P. Peng, X.-R. Zhou (USTC), Z.-T. Sun (SDU)

Introduction

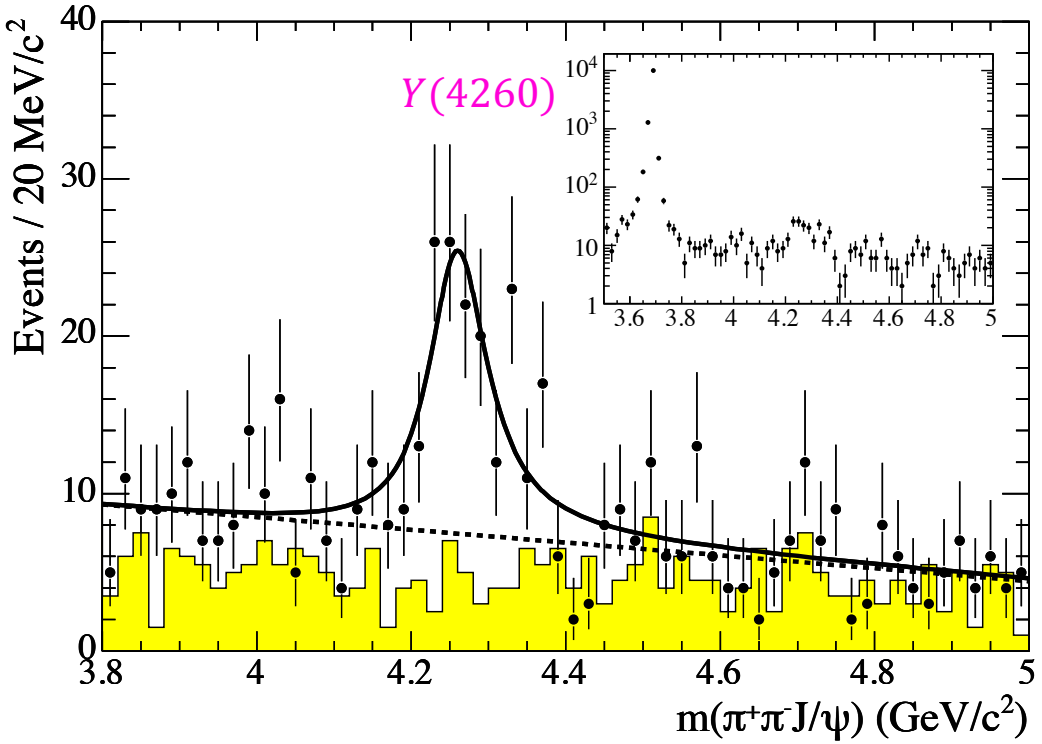
Y states



Exotic hadrons produced in e^+e^- annihilations → $J^{PC} = 1^{--}$

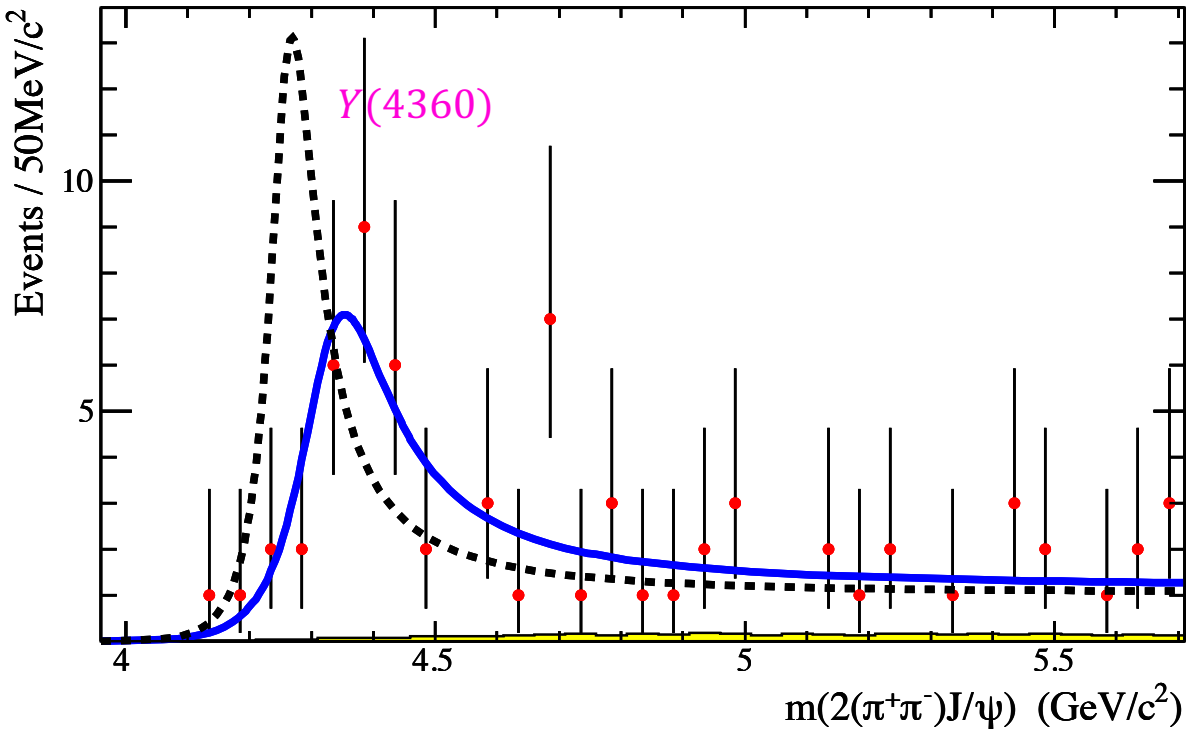
The first discovery: Y(4260) in $e^+e^- \to \gamma_{ISR}\, J/\psi\, \pi^+\pi^-$

BABAR, PRL 95, 142001 (2005)

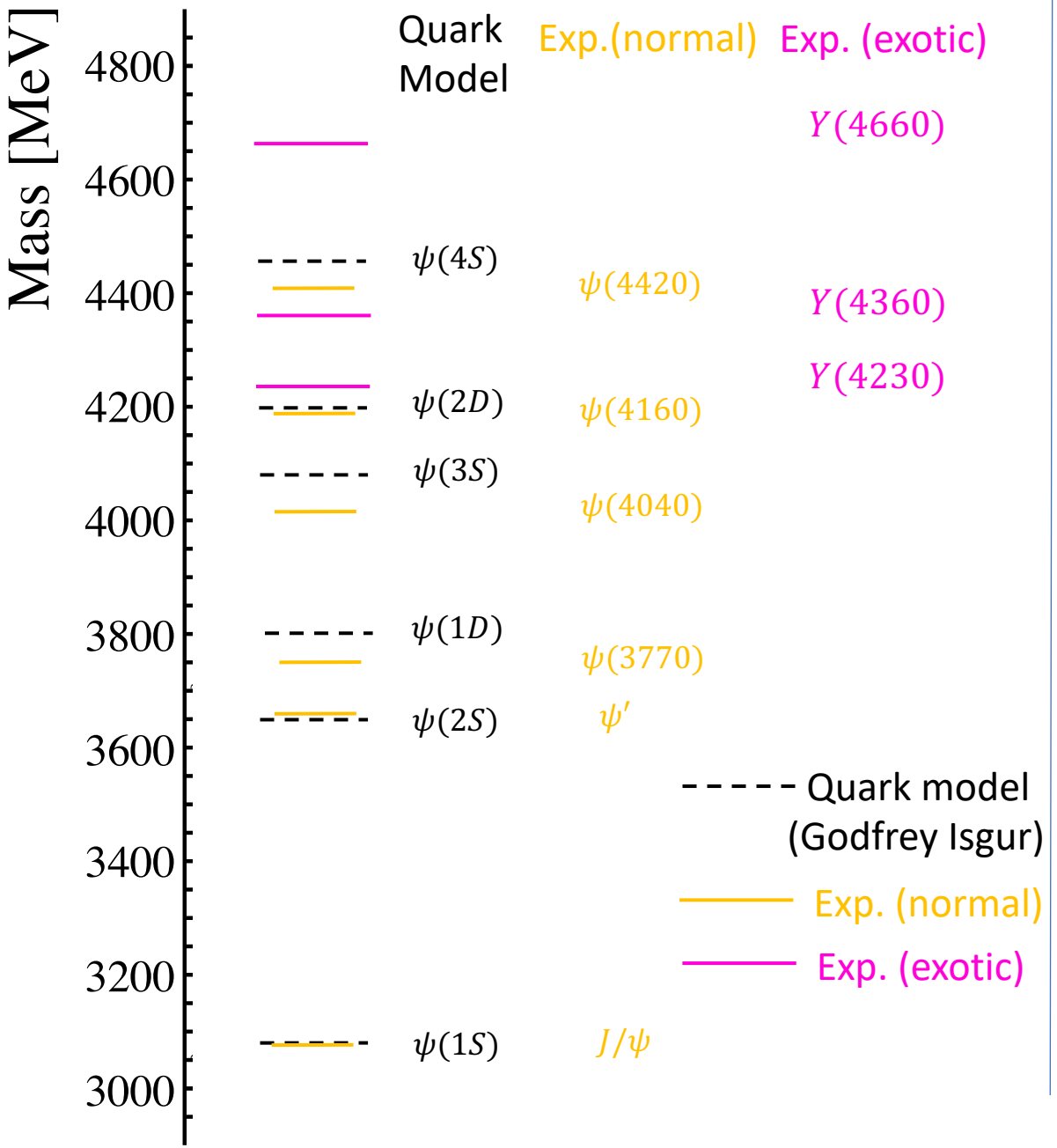


Y(4360) in $e^+e^- \to \gamma_{ISR}\, \psi'\, \pi^+\pi^-$

BABAR, PRL 98, 212001 (2007)



Charmonium spectrum ($J^{PC} = 1^{--}$)



Why Y states considered exotic ?

No quark model counterpart

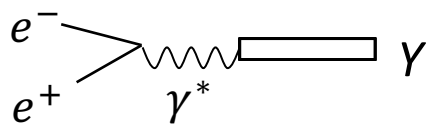
Proposed internal structures

$Y(4230)$: $D_1\bar{D}$ molecule G.-J. Ding, X.-K. Dong et al., T.-J. et al., F.Z. Peng et al., …

hybrid state S.-L. Zhu, F. E. Close et al., E. Kou et al.

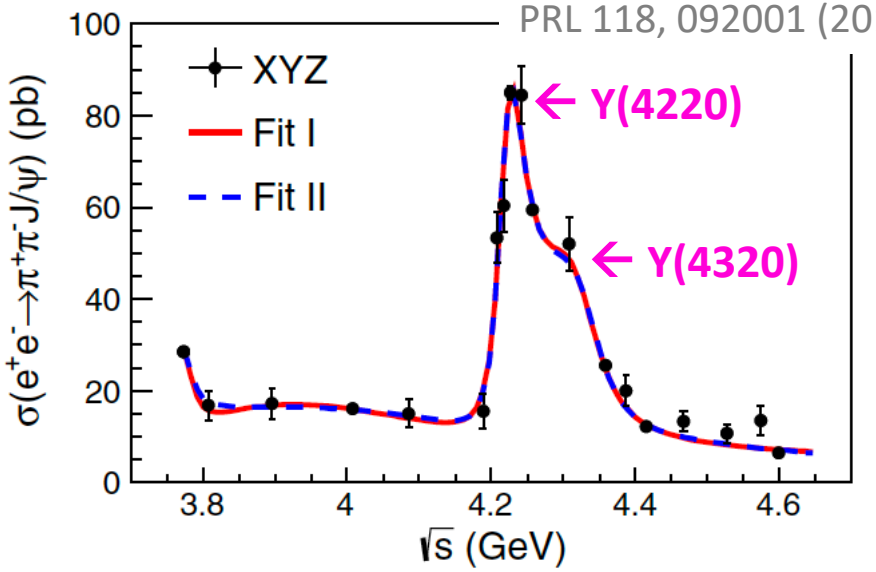
$Y(4360)$: $D_1\bar{D}^*$ molecule X.-K. Dong et al., T.-J. et al.

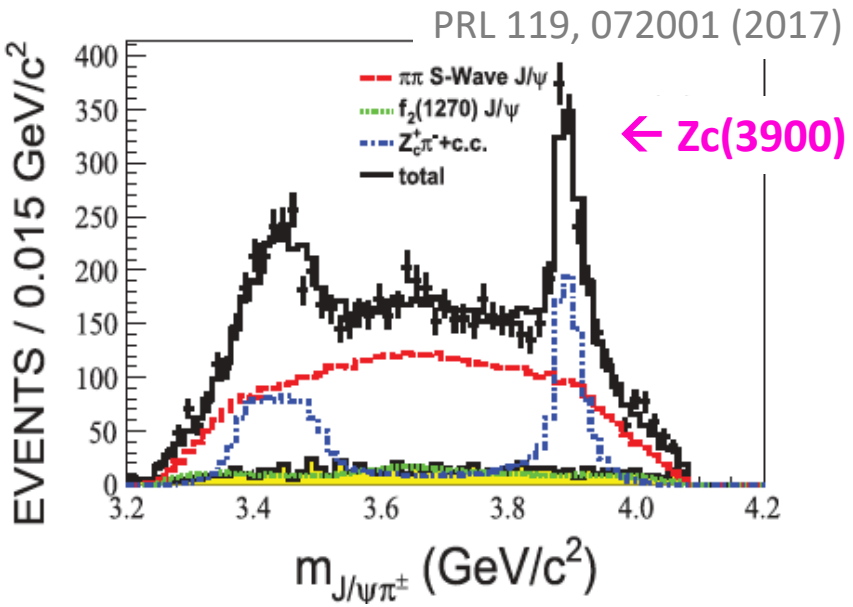
BESIII as Y factory

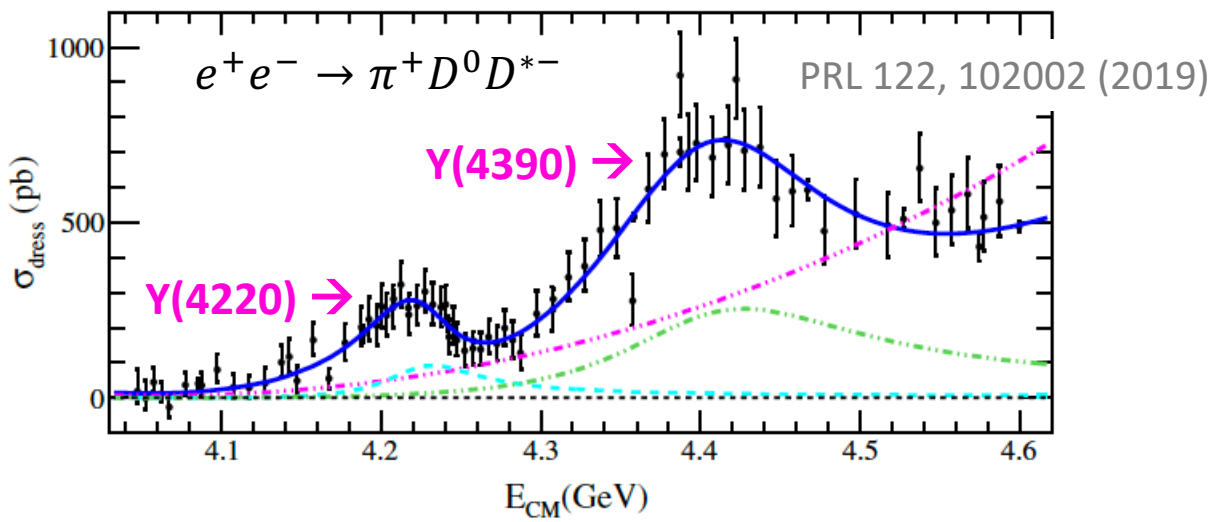


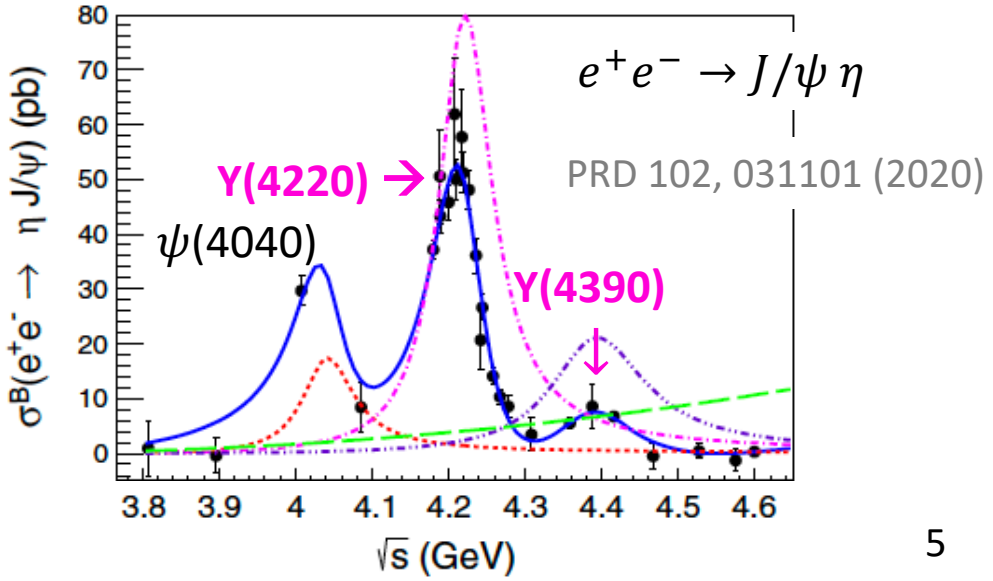
Direct Y productions (No γ_{ISR}) → High precision

$e^+e^- \to J/\psi\,\pi^+\pi^-$ →





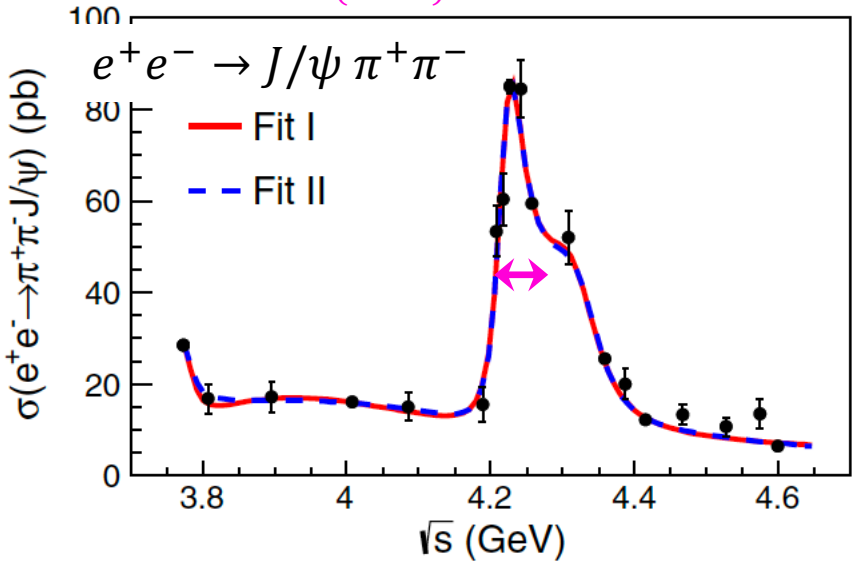




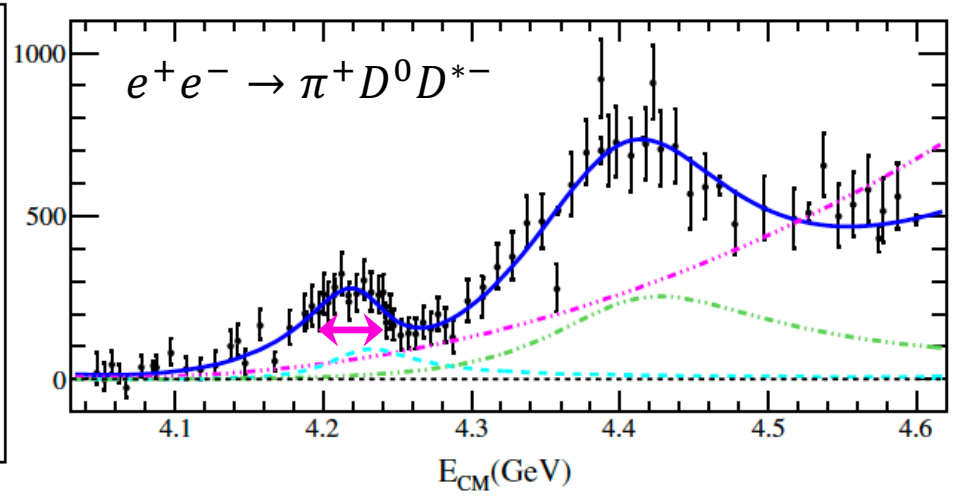
Y width problem

Why Y states seem to have different widths for different final states ? Process-dependent width ?

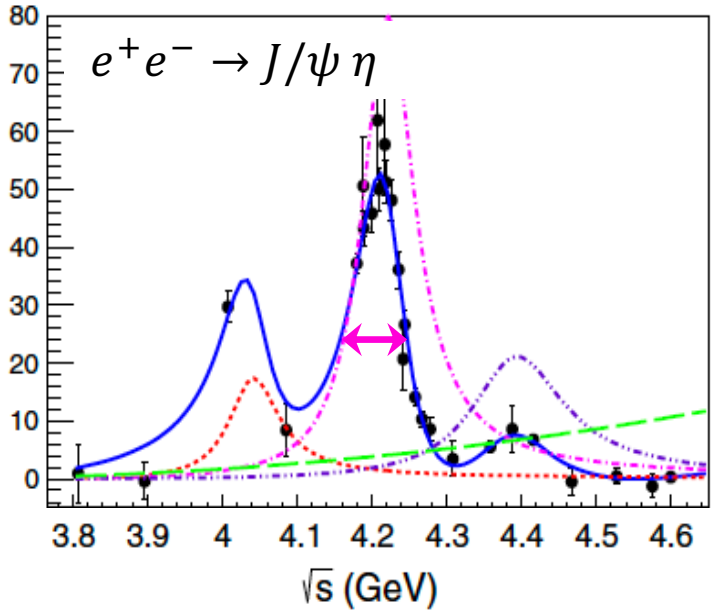
$\Gamma_{Y(4220)}$ ~ 44±4 MeV



$\Gamma_{Y(4220)}$ ~ 77±7 MeV



$\Gamma_{Y(4220)}$ ~ 82±6 MeV



Y width problem revealed limitation of single-channel analysis

Single-channel analysis: analyze different final states with different models (usual experimental analysis)

Interference among overlapping resonances and non-resonant contributions is process dependent

→ Process dependent line-shape observed

Single-channel fits give process-dependent resonance parameters

→ Y width problem created

Y-width problem is artifact of single-channel analysis

Global coupled-channel analysis

Analyze different final states simultaneously with a unified and (semi-)unitary model

Describe different lineshapes in different final states due to:

- Interference between various charmonium states and non-resonant amplitude
- Kinematical effects (threshold opening, triangle singularity)

Coupled-channel analysis clarifies how process-dependent Y lineshapes come about

Prerequisite to studying this

At the same time, the analysis determines:

(i) vector charmonium mass, width (poles)

(ii) couplings of charmonium to decay channels (residues)

BESIII accumulated high-quality data for various $e^+e^- \to c\bar{c}$ processes over wide energy region covering *Y*

$e^+e^- \to D^{(*)}\overline{D}^{(*)}, D_s^{(*)}\overline{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega, \Lambda_c\overline{\Lambda}_c$ (10 two-body final states)

$e^+e^- \to \pi D^{(*)}\overline{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\overline{K}$ (7 three-body final states)

$e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (1 four-body final states)

Good condition for global analysis of $e^+e^- \to c\bar{c}$ data → vector charmonium spectroscopy

Global analysis is important not only for *Y* but also for well-established $\psi(4040)$, $\psi(4160)$, $\psi(4415)$

- Their properties were previously determined by simple Breit-Wigner fit to inclusive (e^+e^- → hadrons) *R* values
- Analyzing precise exclusive data → More detailed and precise information

This work

- Global analysis of BESIII and Belle data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV with a unified coupled-channel model

  $e^+e^- \to D^{(*)}\overline{D}^{(*)}, D_s^{(*)}\overline{D}_s^{(*)}, J/\psi\,\eta^{(\prime)}, \chi_{c0}\omega\,, \Lambda_c\overline{\Lambda}_c$ (10 two-body final states)

  $e^+e^- \to \pi D^{(*)}\overline{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, J/\psi K\overline{K}$ (7 three-body final states)

  $e^+e^- \to \eta_c\rho\pi\ (\rho \to \pi\pi)$ (1 four-body final states)

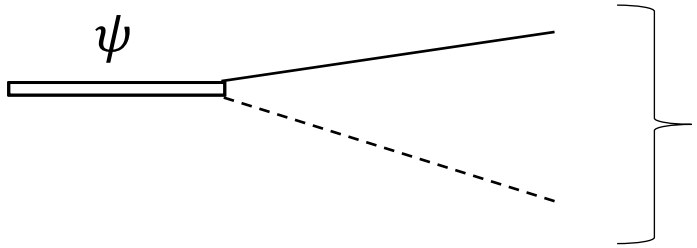
- Approximate three-body unitarity model
- Fit both total cross sections and invariant mass distributions
- Extract vector charmonium poles
- Compositeness → hadron-molecule-dominated states

Recent work towards similar direction: X.-K. Dong, V. Baru, von Detten, F.-K. Guo et al., JHEP 09 (2026) 125

MODEL

Coupled-channels

(quasi) two-body channels included; $J^{PC} = 1^{--}$



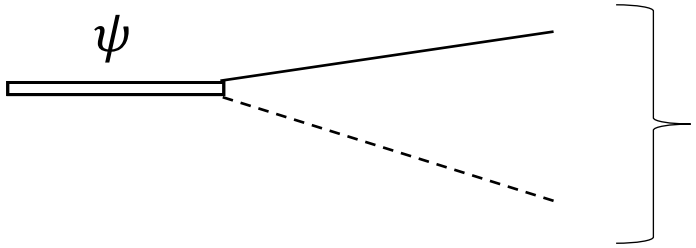
$D_1(2420)\bar{D}^{(*)}$, $D_1(2430)\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}(2536)\bar{D}_s$

$\omega\chi_{c0}$

$D_1(2420)$, $D_1(2430)$, $D_2^*(2460)$, D^*, $D_{s1}(2536)$, ω → Breit-Wigner (BW) propagators; mass and width from PDG

BW partially violate three-body unitarity in our three-body calculation

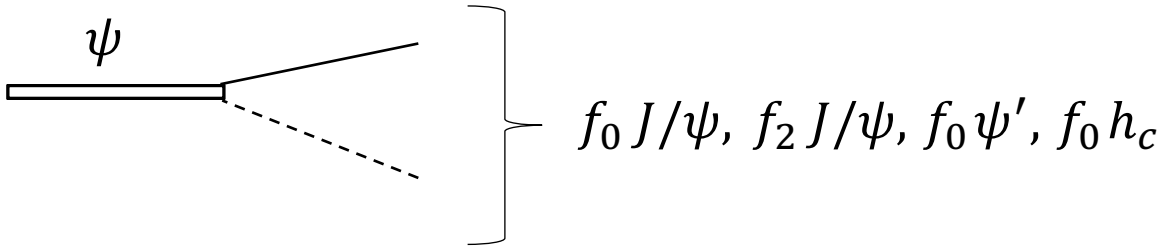
Otherwise the model is manifestly three-body unitary

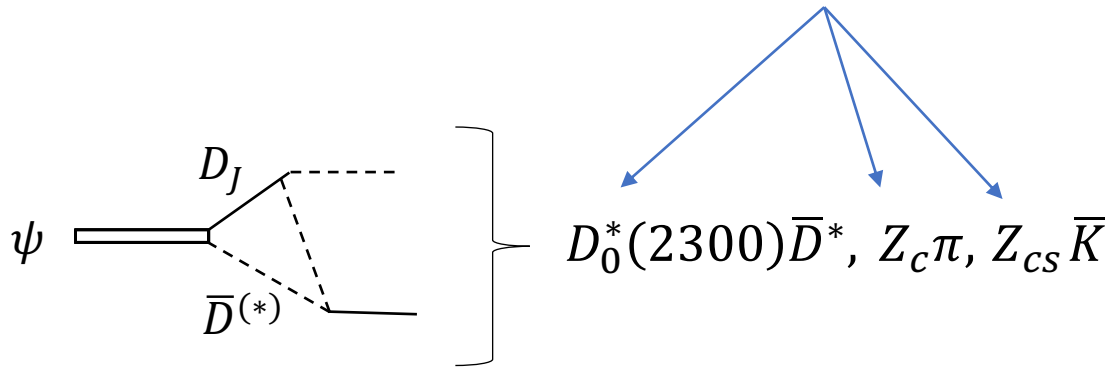


$D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\Lambda_c\overline{\Lambda}_c$ treated as stable particles

Coupled-channels

assumed to be molecule-dominant
→ not directly from ψ decay

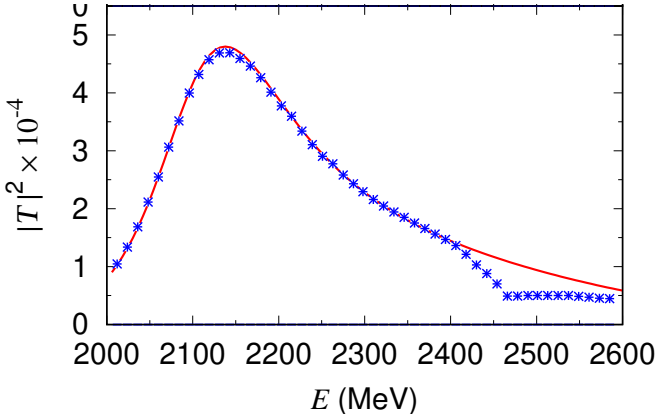




$D_0^*(2300), f_0, f_2, Z_c, Z_{cs}$ as (virtual) poles in two-body amplitudes (bare states + contact interaction models)

$D\pi$ s-wave amplitude fitting LQCD-based amplitude

Albaladejo et al. PLB 767 (2017)

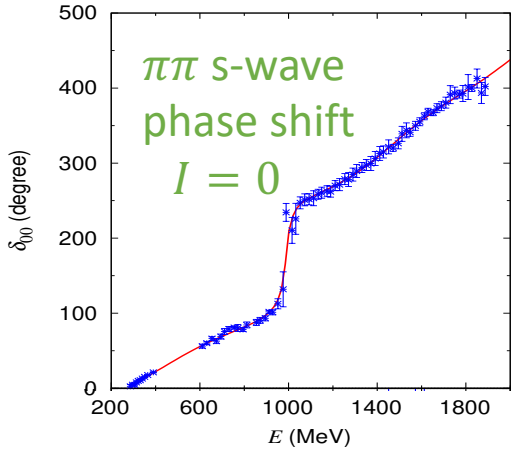


D_0^* pole :

2104 − i 100 MeV (ours)

$2105^{+6}_{-8} - i\ 102^{+10}_{-12}$ MeV (Albaladejo et al.)

$\pi\pi - K\bar{K}$ s[d]-wave amplitude fitting empirical amplitude



$f_0(500), f_0(980),$

$f_0(1370), f_2(1270)$ poles

→ consistent with PDG

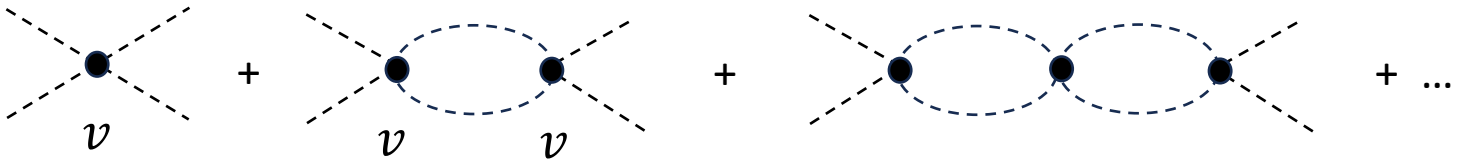
$Z_{c(s)}$ amplitude

Z_c : $I\ J^{PC} = 1\ 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel scattering amplitude

Z_{cs} : $I\ J^P = \frac{1}{2}\ 1^+$ $D_s^*\bar{D} - D_s\bar{D}^* - J/\psi K$

driven by contact interactions; s-wave interactions except $h_c\pi$ p-wave interaction

$Z_{c(s)}$ amplitude



intermediate loops include all possible coupled-channels

Assumptions

$v_{[D^*\bar{D}],[D^*\bar{D}]} = v_{D^*\bar{D}^*,D^*\bar{D}^*} = v_{[D_s^*\bar{D}],[D_s^*\bar{D}]}$ (HQSS, SU(3))

$v_{[D^*\bar{D}],J/\psi\pi} = v_{[D_s^*\bar{D}],J/\psi K}$ (SU(3))

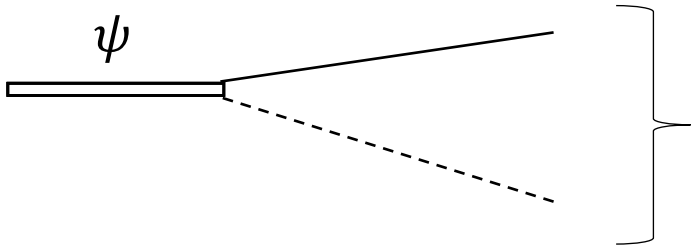
$C = -1$ basis $[D^*\bar{D}] = \frac{1}{\sqrt{2}}(D^*\bar{D} - D\bar{D}^*)$

$[D_s^*\bar{D}] = \frac{1}{\sqrt{2}}(D_s^*\bar{D} - D_s\bar{D}^*)$ SU(3)

no coupling between hidden-charm channels (e.g. $v_{J/\psi\pi,J/\psi\pi} = v_{J/\psi\pi,\psi'\pi} = 0$)

Nonzero couplings are determined by the global fit → poles may be generated if required by data

Coupled-channels

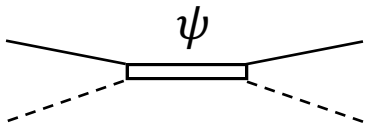


$D_1(2420)\bar{D}^{(*)}$, $D_1(2430)\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}(2536)\bar{D}_s$

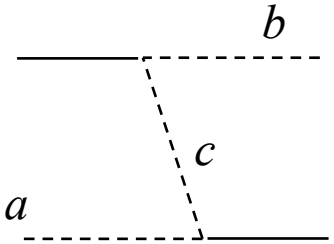
$\omega\chi_{c0}$, $D_s^{(*)}\bar{D}_s^{(*)}$, $\Lambda_c\bar{\Lambda}_c$, $D_0^*(2300)\bar{D}^*$, $f_0\,J/\psi$, $Z_c\pi$, … etc.

These quasi-two-body channels couple with each other through:

- (bare) charmonium excitations

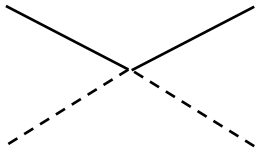


- (potentially on-shell) particle-exchange mechanisms



- Short-range mechanisms

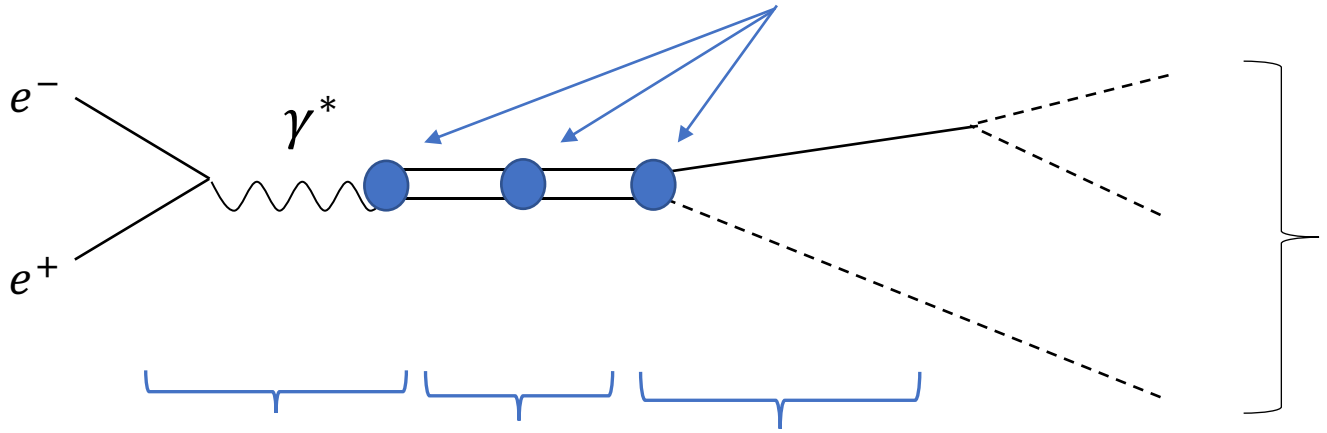
  (between open charm channels)



Full amplitude for $e^+e^- \to$ three-body final states

Dressed vertices (propagator) : bare vertices (propagator) dressed by hadron scattering

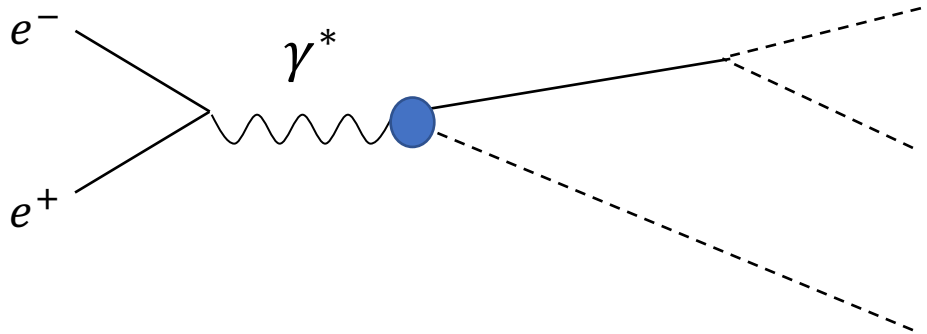
Unitarity requirement



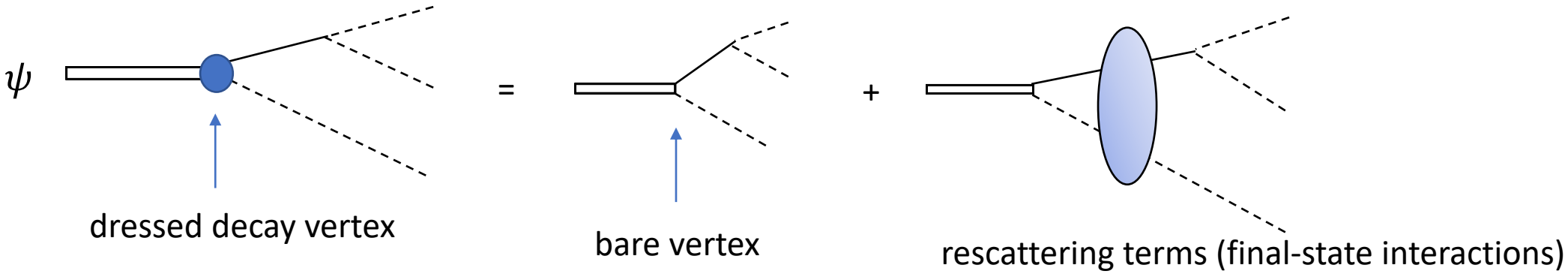
$\pi D^{(*)}\bar{D}^{(*)},\ J/\psi\pi\pi,\ \psi'\pi\pi, h_c\pi\pi, \eta_c\rho\pi, J/\psi K\bar{K}$

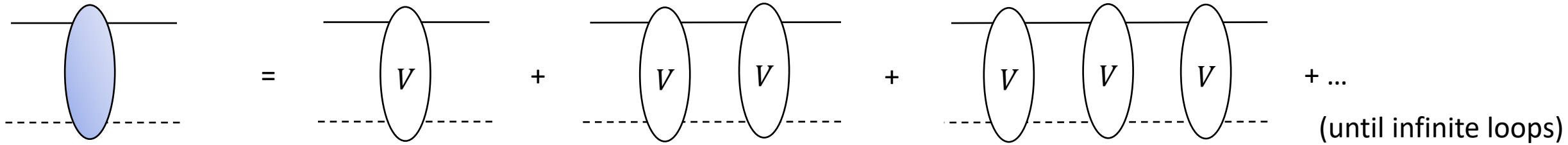
ψ production, propagation, decay

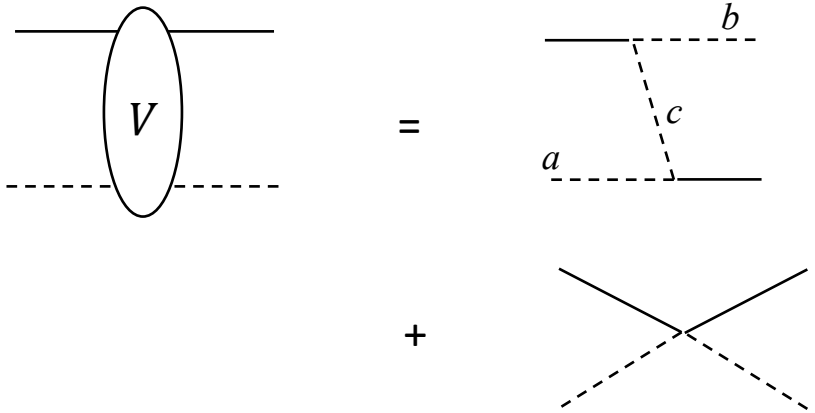
Non-resonant mechanisms are also included (no ψ excitations)



Three-body decays of ψ







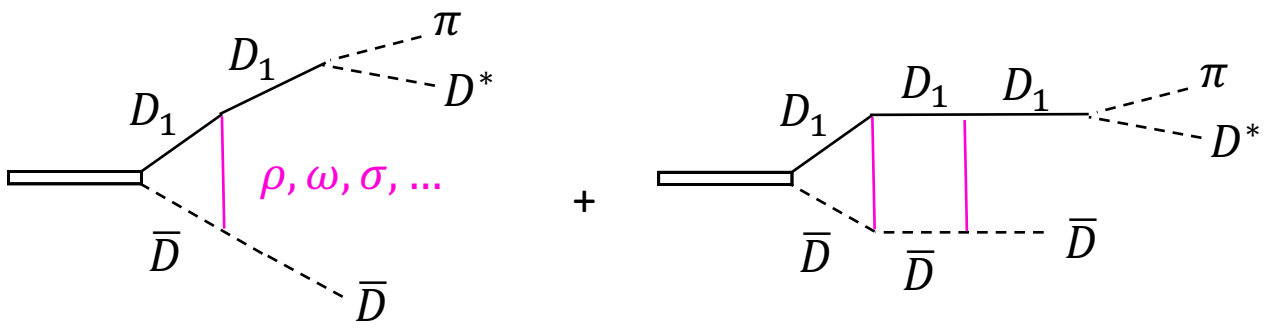
(on-shell) particle-exchange mechanisms → three-body unitarity

$$\propto \frac{1}{E - E_a - E_b - E_c}$$

Short-range mechanisms among open-charm channels (E-independent)

Short-range mechanisms among open-charm channels

example



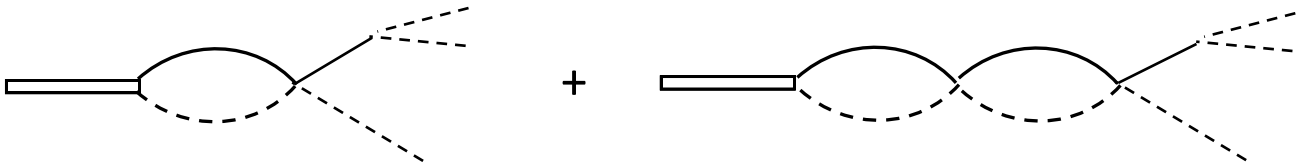
G.-J. Ding, PRD 79, 014001 (2009)
X.-K. Dong et al., PRD 101, 076003 (2020)

→ $D_1\bar{D}$, $D_1\bar{D}^*$ molecule states

Y(4220) Y(4360)

simulate

→



(Our model)

Contact interactions among $D_1\bar{D}^{(*)}$, $D_2^*\bar{D}^*$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}\bar{D}_s$, $D_s^{(*)}\bar{D}_s^{(*)}$, $\Lambda_c\bar{\Lambda}_c$ channels

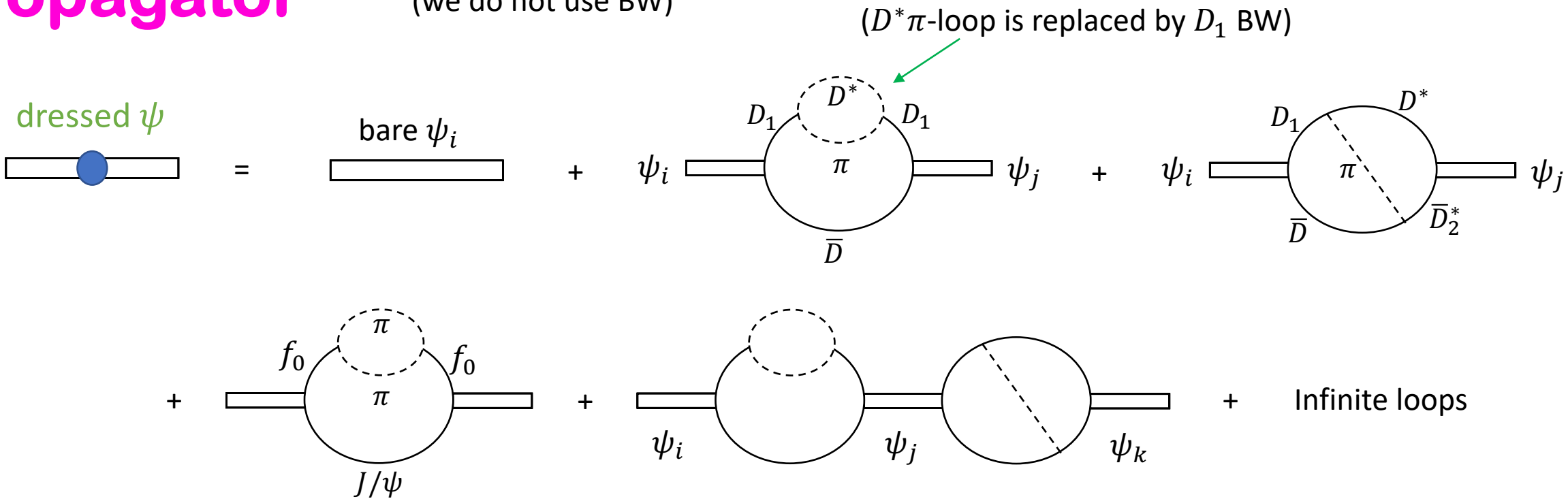
→ fitted to data (advantage of separable interactions)

- High-precision BESIII data require these contributions (enhanced threshold cusps)
- Potentially generate hadron-molecule states

Global analysis can examine if Y(4220) as $D_1\bar{D}$ molecule and Y(4360) as $D_1\bar{D}^*$ molecule

ψ propagator

(we do not use BW)



Charmonium poles are formed by non-perturbative couplings between bare ψ and $D_1\bar{D}$, $f_0\,J/\psi$, …

(= poles of dressed ψ propagator)

Unitary coupled-channel model : resonance pole (mass, width) and decay dynamics are explicitly related.

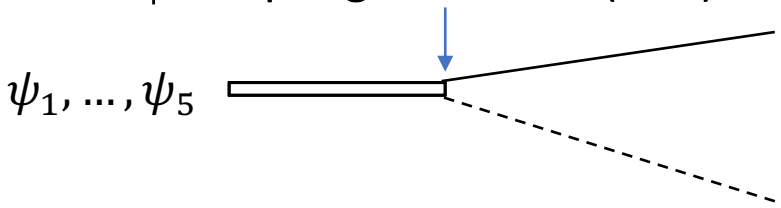
(unitarity requirement)

Breit-Wigner model : decay dynamics are simulated by BW mass and width parameters

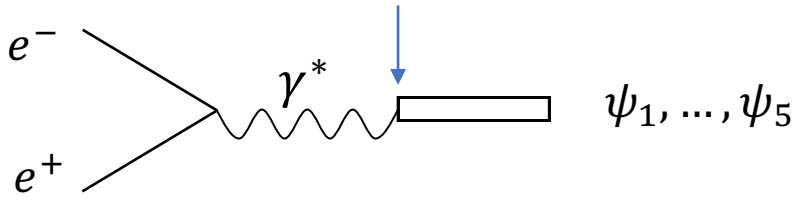
Fitting parameters in global analysis

* bare ψ masses (5 bare states)

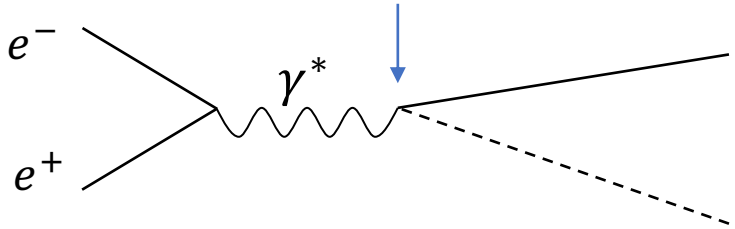
* bare ψ coupling constants (real)



* bare photon-ψ coupling constants (real)



* non-resonant photon coupling constants (real)



* $\psi(4660), \psi(4710)$ Breit-Wigner mass, width, vertices

* coupling constants in Z_c amplitude :

  $v_{D^*\bar{D},D^*\bar{D}}$, $v_{D^*\bar{D},J/\psi\pi}$, $v_{D^*\bar{D},\psi\prime\pi}$ etc.

* Contact-interaction strengths among open-charm channels

* Cutoffs in non-resonant vertices for

  $\gamma^* \to D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, \Lambda_c\bar{\Lambda}_c$

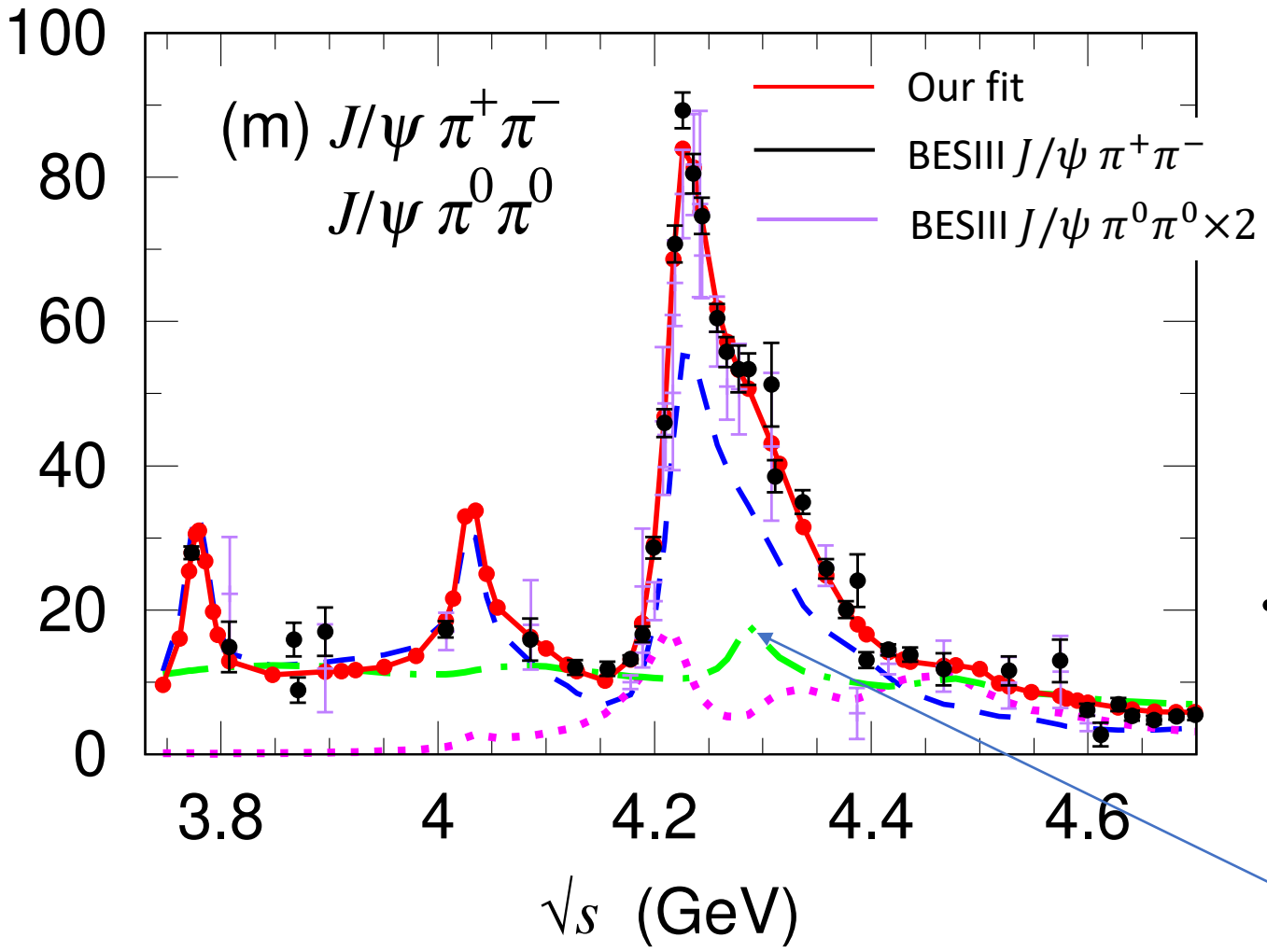
In total, 200 fitting parameters

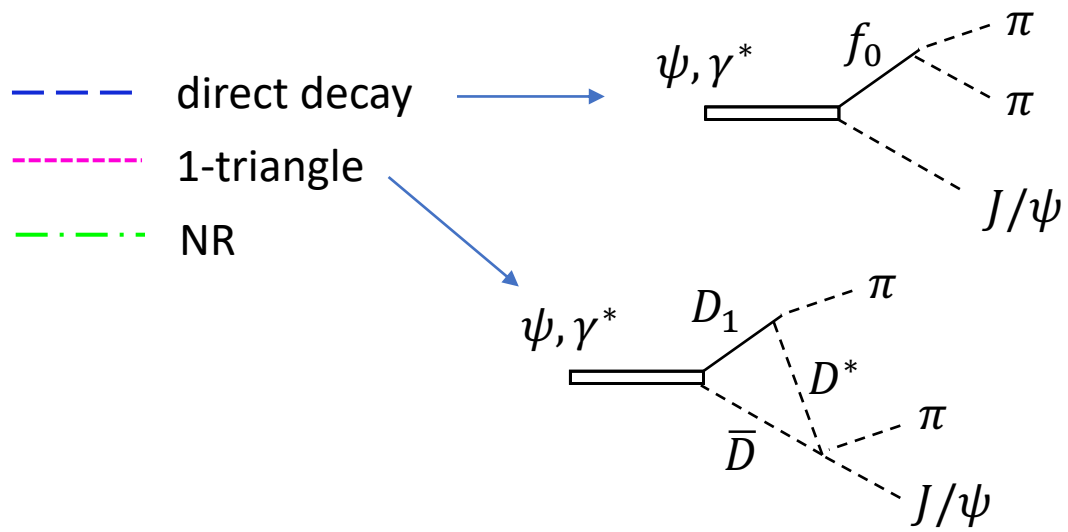
$$\chi^2/\text{ndf} = 2320/(1635-200) \sim 1.6$$

Selected fit results

$e^+e^- \to J/\psi\,\pi^+\pi^-, J/\psi\,\pi^0\pi^0$

Data: BESIII,
PRD 106, 072001 (2022)
PRD 102, 012009 (2020)

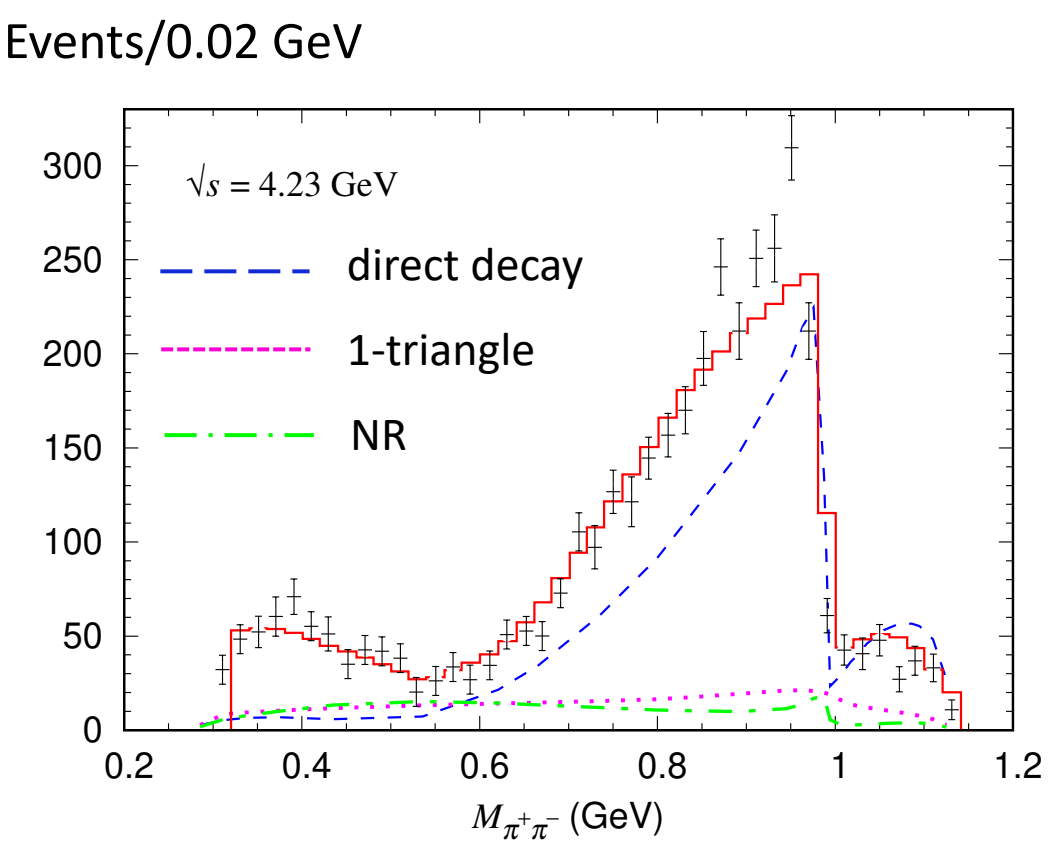


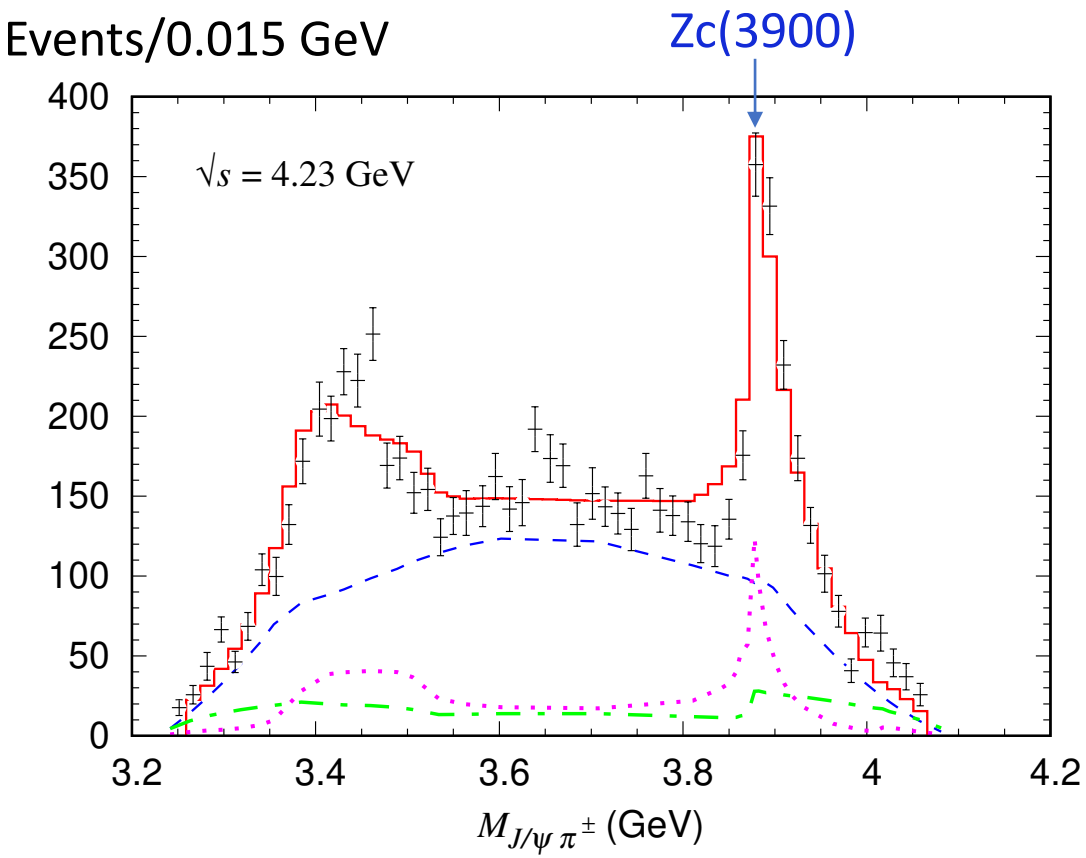


- Peaking structure at $\sqrt{s} \sim 4$ GeV is from $\psi(4040)$
  → consequence of the combined fit
  → supported by Belle-II PRD 113, 112010 (2026)

- Resummation of short-range open-charm interactions → hadron molecule formation

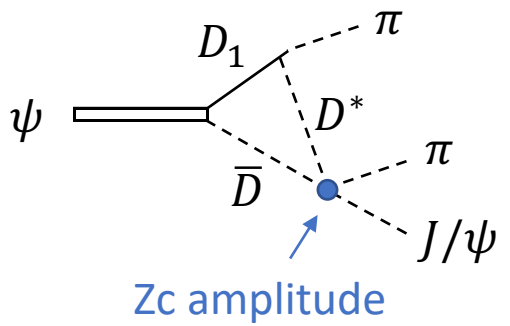
$e^+e^- \to J/\psi\,\pi^+\pi^-$ (invariant mass distributions)



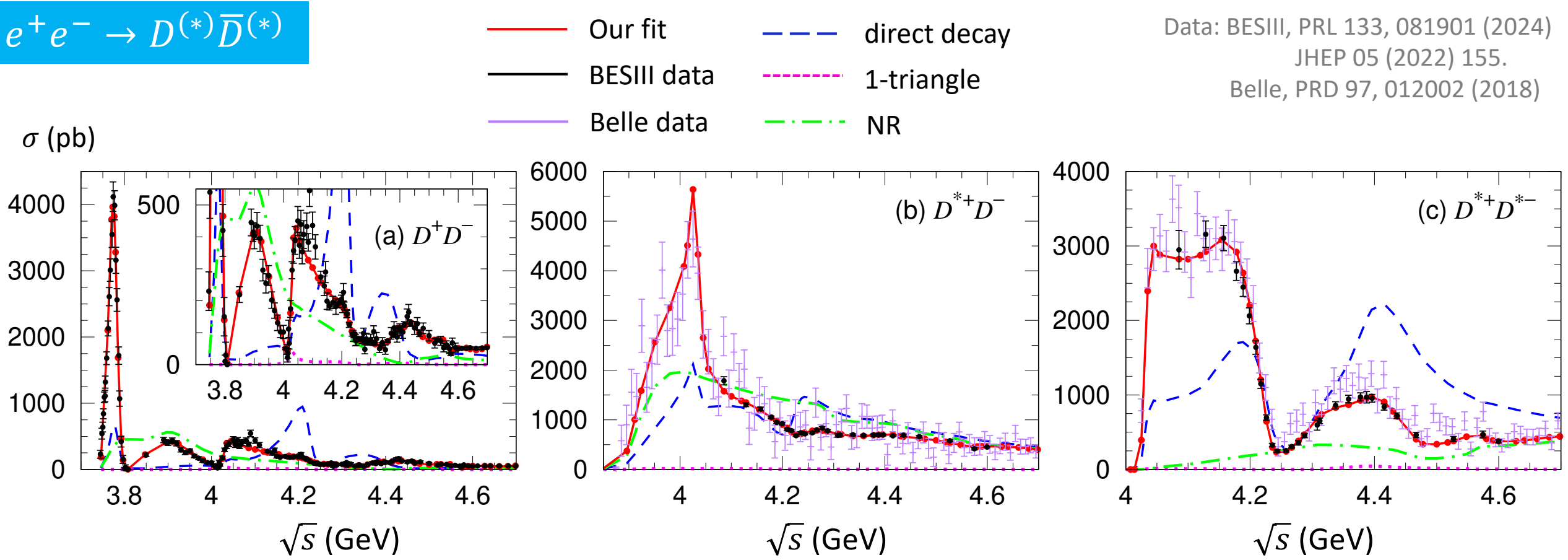


- Zc(3900) peak well fitted
- 1-loop causes $D^*\overline{D}$ threshold cusp enhanced by Zc pole

  a bit off TS at $\sqrt{s} = 4.28$ GeV
- Zc(3900) virtual pole found

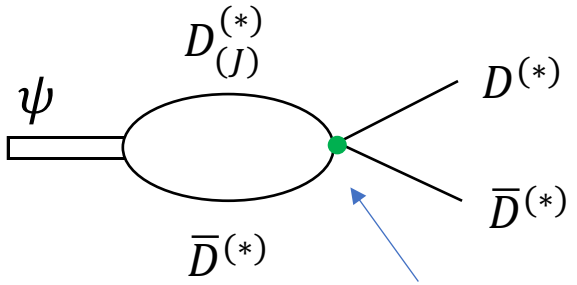


$e^+e^- \to D^{(*)}\overline{D}^{(*)}$



Data: BESIII, PRL 133, 081901 (2024)
JHEP 05 (2022) 155.
Belle, PRD 97, 012002 (2018)

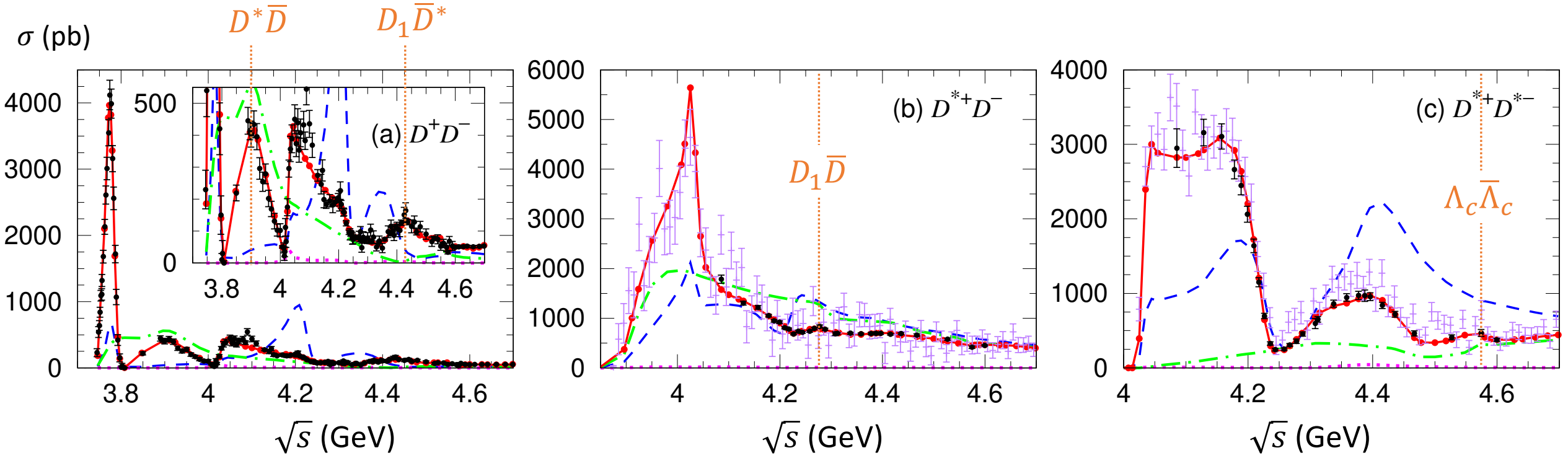
- *Precise* BESIII data are well fitted
- Contact interactions among open-charm channels important (difference between blue and red curves above)
- 1-triangle (particle exchange) is small



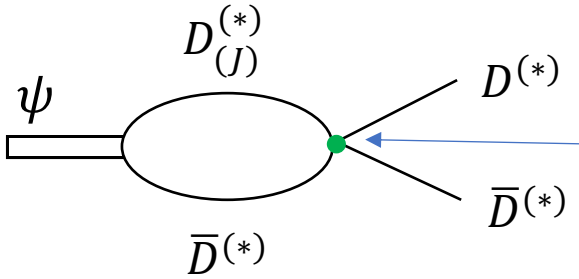
Open-charm rescattering by short-range interactions

→ molecule formation

$e^+e^- \to D^{(*)}\overline{D}^{(*)}$



Fitting precise data, we need threshold cusps from



Open-charm rescattering by short-range interactions

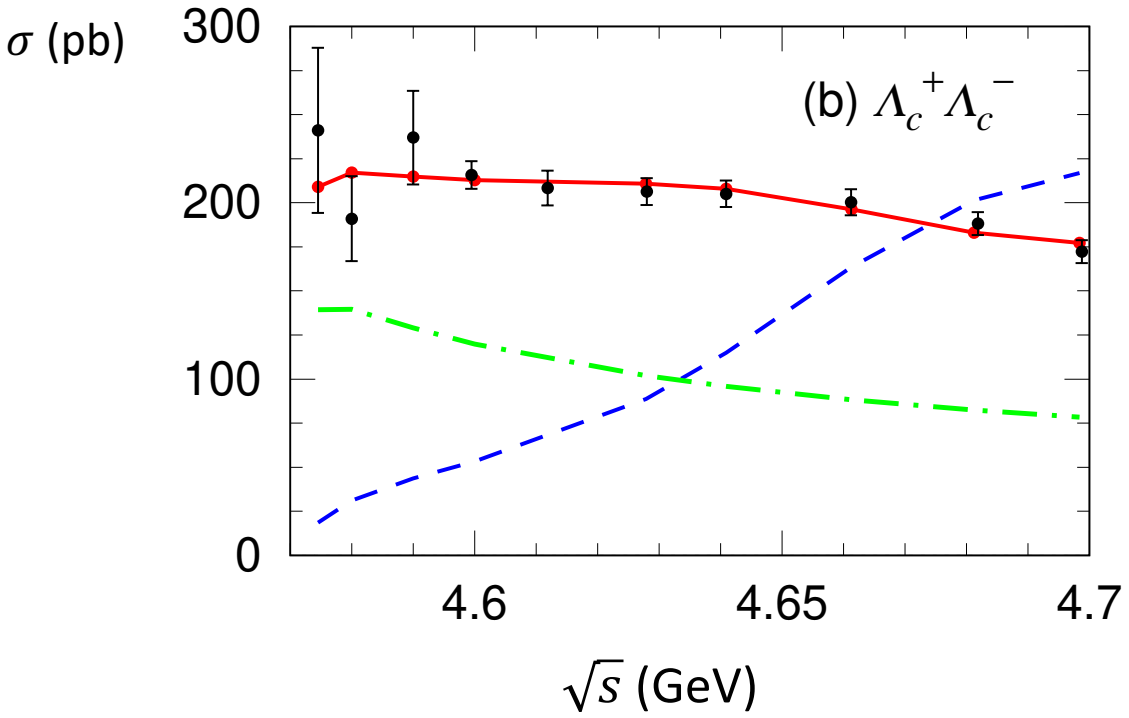
Fitting cusps → good constraints on interactions among open-charm channels

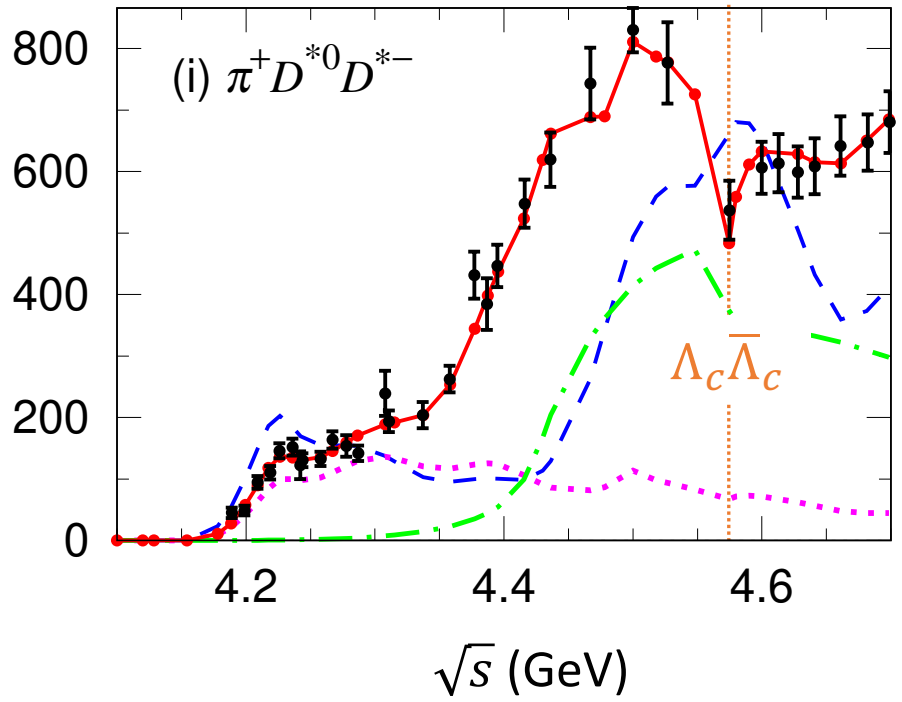
→ good constraints on existence of $D^{(*)}_{(J)}\overline{D}^{(*)}$ molecules

$e^+e^- \to \Lambda_c\overline{\Lambda}_c\,,\ \ \pi D^*\overline{D}^*$

Our fit
direct decay
1-triangle
NR

Data: BESIII,
PRL 120, 132001 (2018)
PRL 131, 191901 (2023)
PRL 130, 121901 (2023)





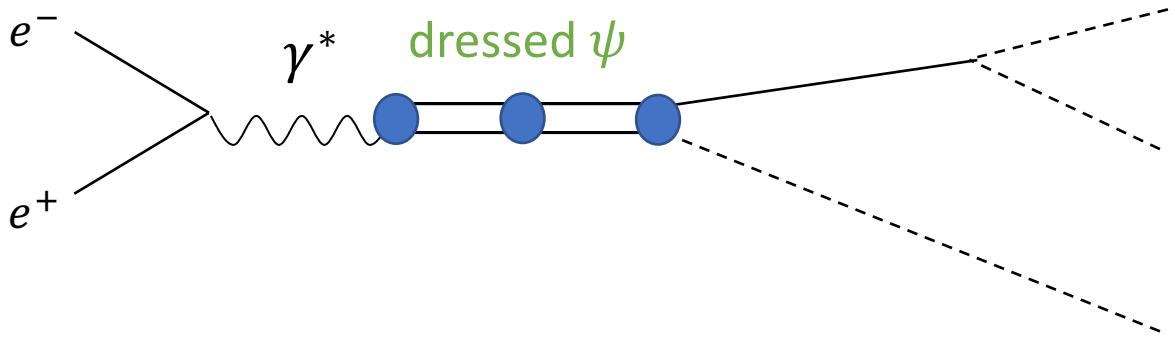
- Non-zero $e^+e^- \to \Lambda_c\overline{\Lambda}_c$ cross section at threshold ← Sommerfeld factor (Coulomb force)
- $\Lambda_c\overline{\Lambda}_c$ threshold enhancement ← attractive $\Lambda_c\overline{\Lambda}_c$ interaction (pole near threshold is likely)
- $\Lambda_c\overline{\Lambda}_c$ threshold cusp is important to fit $e^+e^- \to \pi D^*\overline{D}^*$ data at $\sqrt{s} \sim 4.57$ GeV

Poles and resonance properties

ψ poles from their dressed propagator

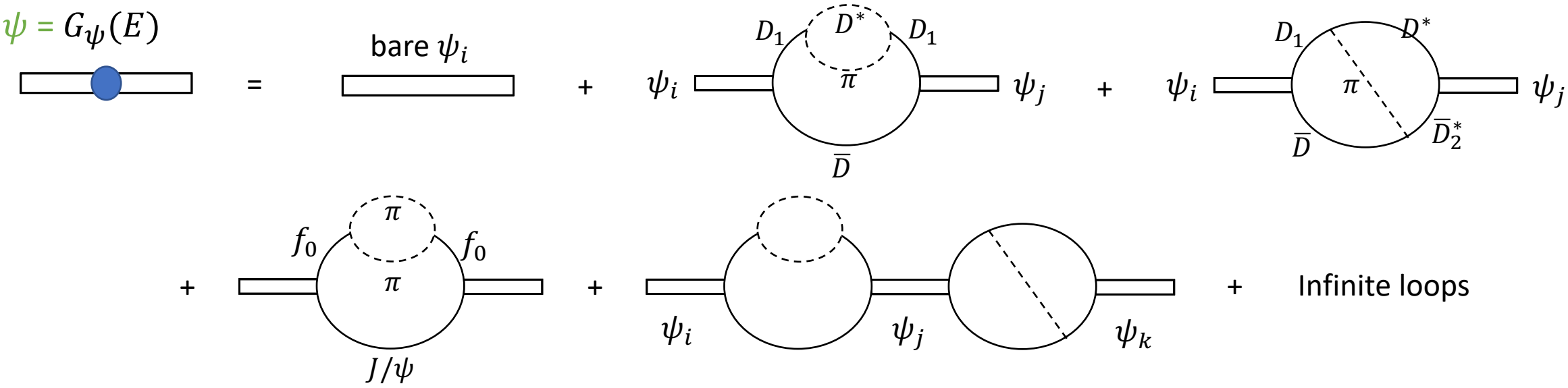
(Not using BW)

Full amplitude



+ non-resonant

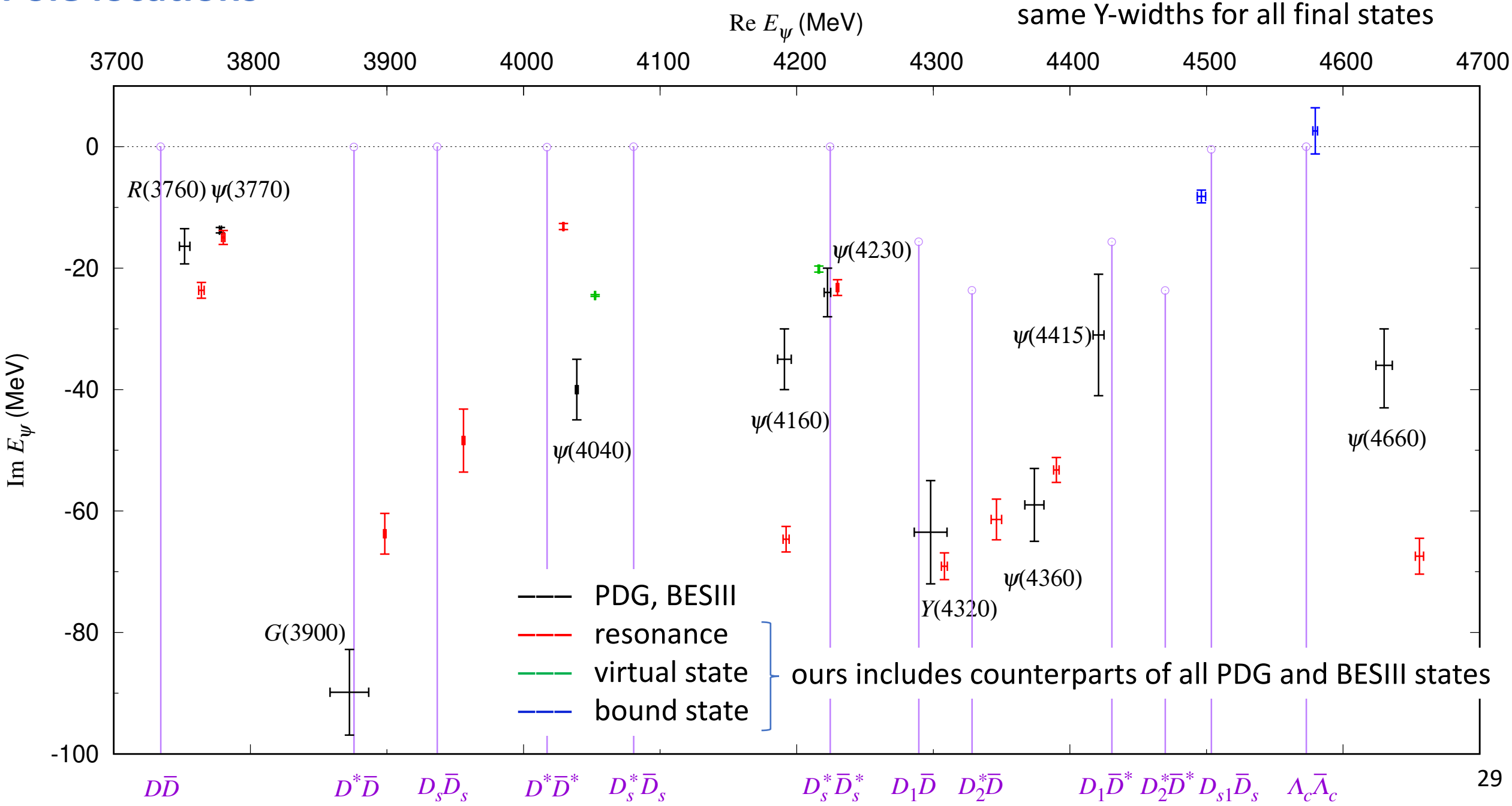
dressed ψ = $G_\psi(E)$



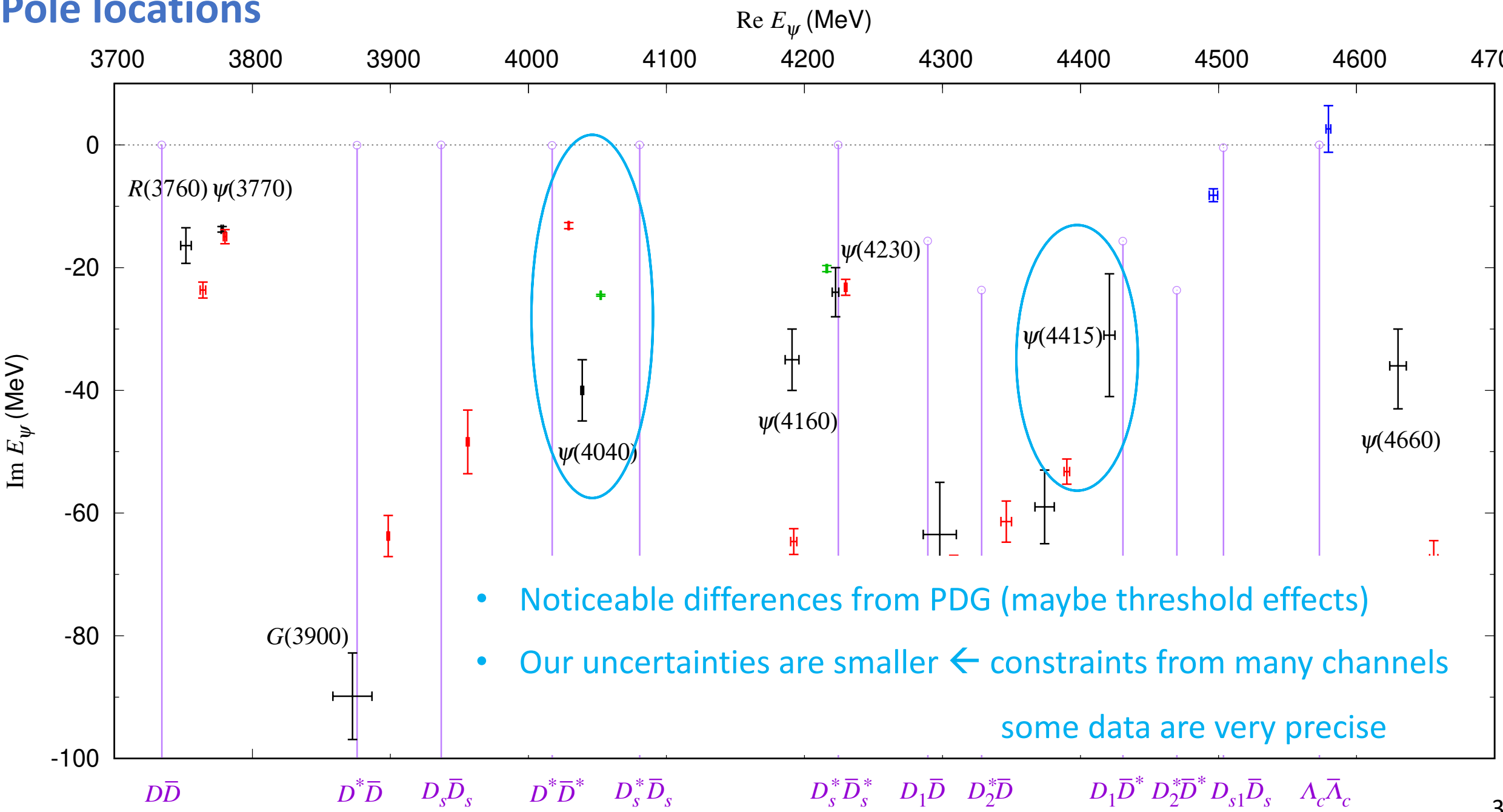
Search complex energy E_ψ where $G_\psi(E_\psi) = \infty$ (E_ψ: pole energy, pole position) by analytical continuation of $G_\psi(E)$

Pole locations

No Y-width puzzle by construction:
same Y-widths for all final states



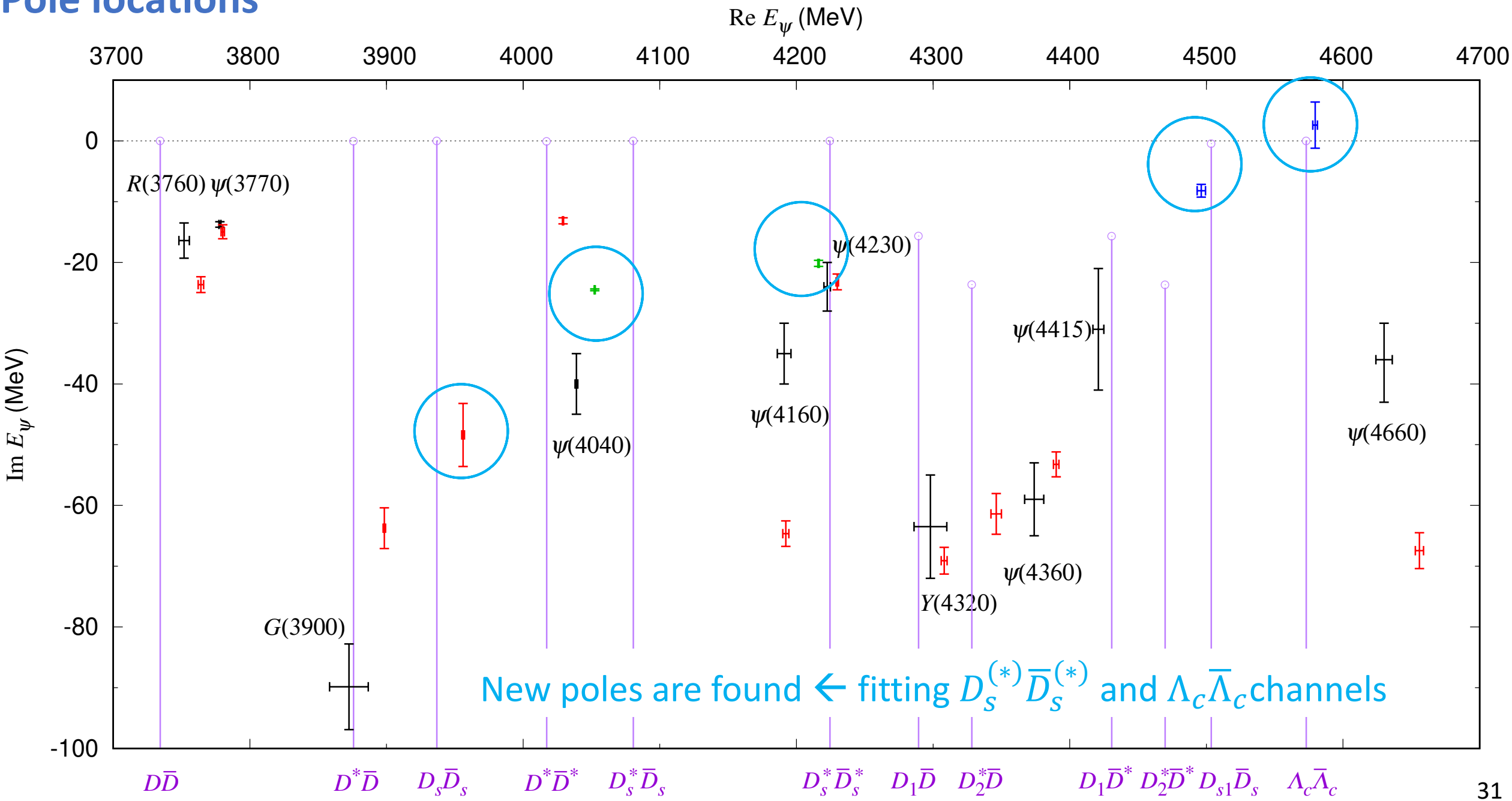
Pole locations



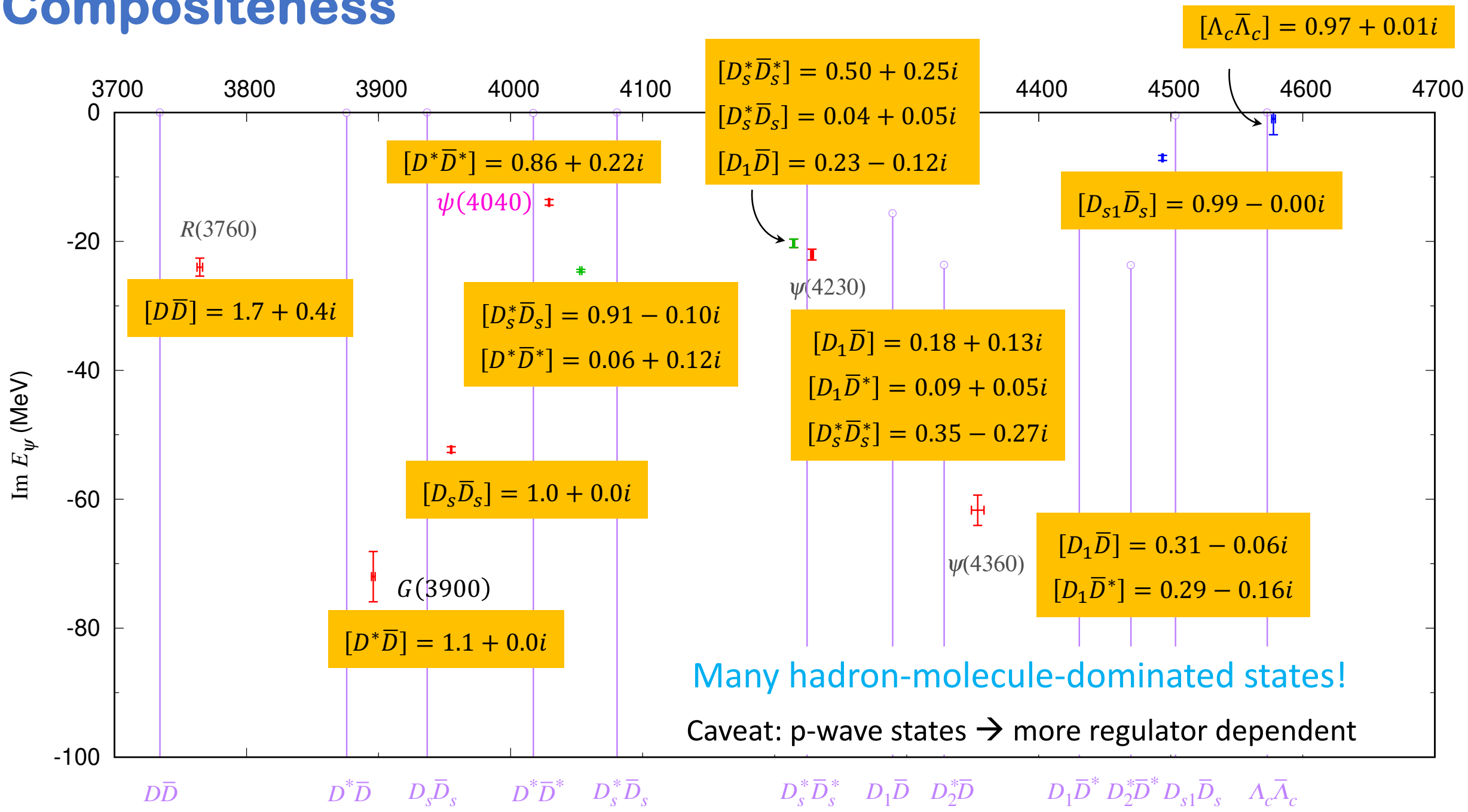
- Noticeable differences from PDG (maybe threshold effects)
- Our uncertainties are smaller ← constraints from many channels

  some data are very precise

Pole locations



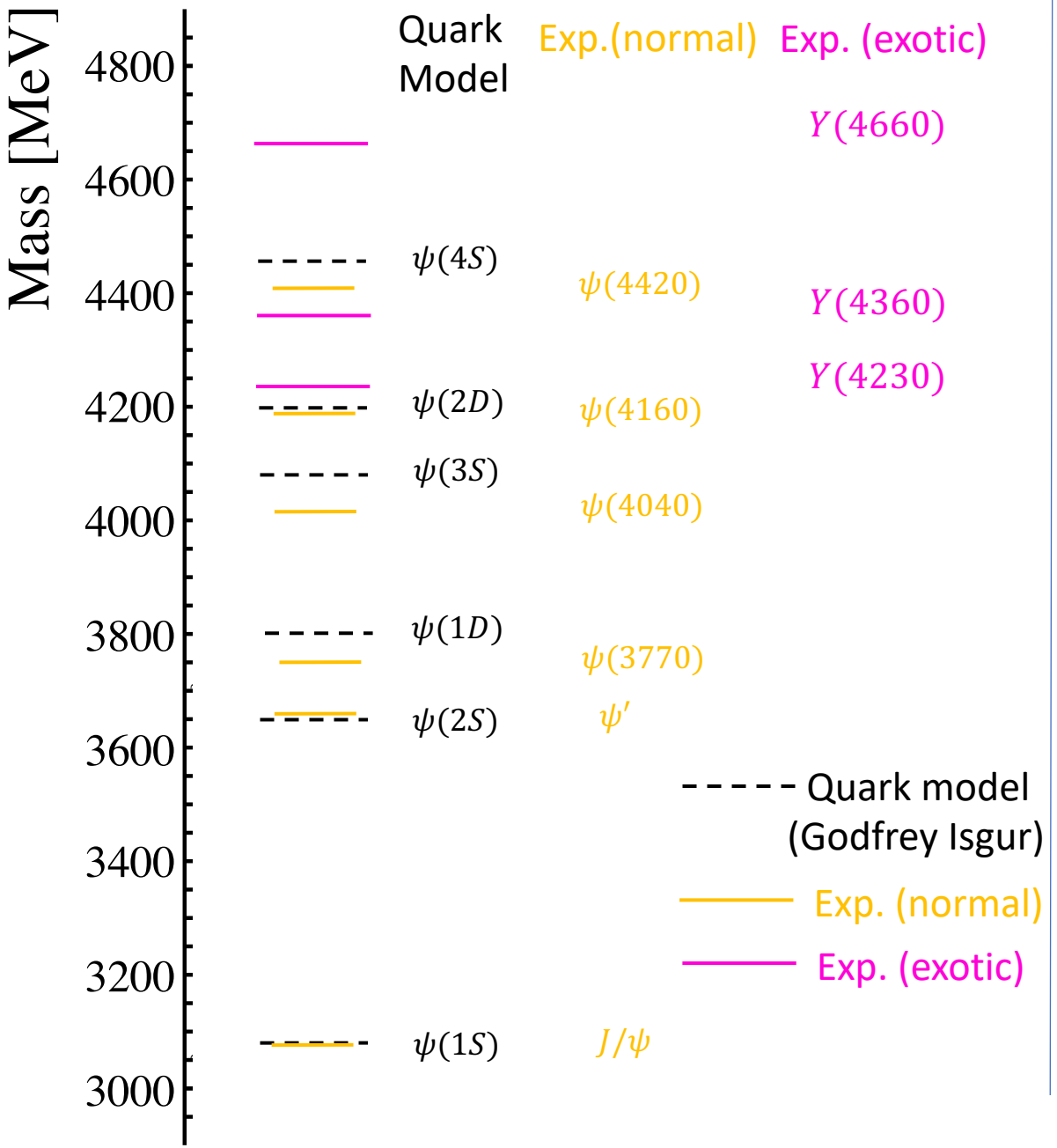
Compositeness



Many hadron-molecule-dominated states!

Caveat: p-wave states → more regulator dependent

Charmonium spectrum ($J^{PC} = 1^{--}$)



Reconsideration of quark model ?

Conventional method

Normal ψ masses → assigned to quark-model states (input)

→ quark-model parameters determined

Our analysis suggests:

- $\psi(4040)$ is mostly $D^*\bar{D}^*$ molecule
- $Y(4230)$ and $Y(4360)$ are mix of molecule (~60%) and $c\bar{c}$ (~40%)

  → resonance mass ≠ quark-model state mass

No simple assignment of quark-model state to resonance

Summary and perspective

Summary

- Global coupled-channel analysis of $e^+e^- \to c\bar{c}$ data in $\sqrt{s} = 3.75 - 4.7$ GeV

  Common analysis method for N*. The $e^+e^- \to c\bar{c}$ analysis gets close to the standard !

- Vector charmonium poles extracted

  -- All PDG states and Y(4320) found

  -- Several more states near open-charm thresholds; some of them are new

- Compositeness calculated → many hadron-molecule-dominated states; $\psi(4040)$ mostly $D^*\overline{D}^*$ molecule

  $\psi(4230)$ is substantial mix of $D_1\overline{D}$, $D_1\overline{D}^*$, $D_s^*\overline{D}_s^*$, $c\bar{c}$

Future

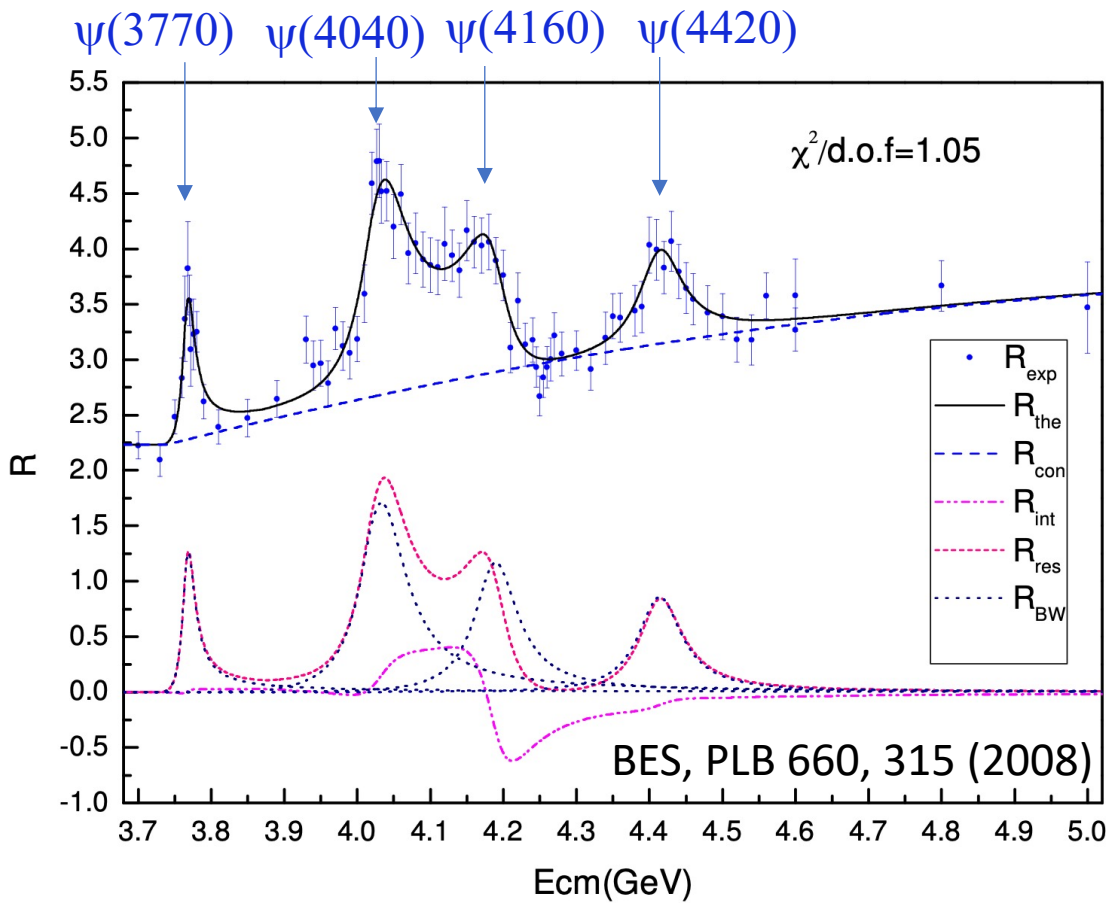
- Address the origin of process-dependent Y lineshapes, calculate charmonium branching ratios
- Update1: Fit with HQSS and SU(3) constraints, more rigorous implementation of three-body unitarity
- Update2: Include data in higher $\sqrt{s}$ and $e^+e^- \to K\overline{D}_s^{(*)}D^{(*)}$ when available → higher charmonium properties

  cross sections + detailed data (Dalitz plot, invariant mass distributions at each $\sqrt{s}$) are important

Back up

Why Y states considered exotic ?

Peculiar decay pattern



R value is dominated by open-charm final states

and hidden-charm final states are minor

Conventional charmonium (ψ) mainly decays to

open charm states ($D\overline{D}$, $D^*\overline{D}$, $D^*\overline{D}^*$, etc.) above $D\overline{D}$ threshold

Y states do not appear in *R* value

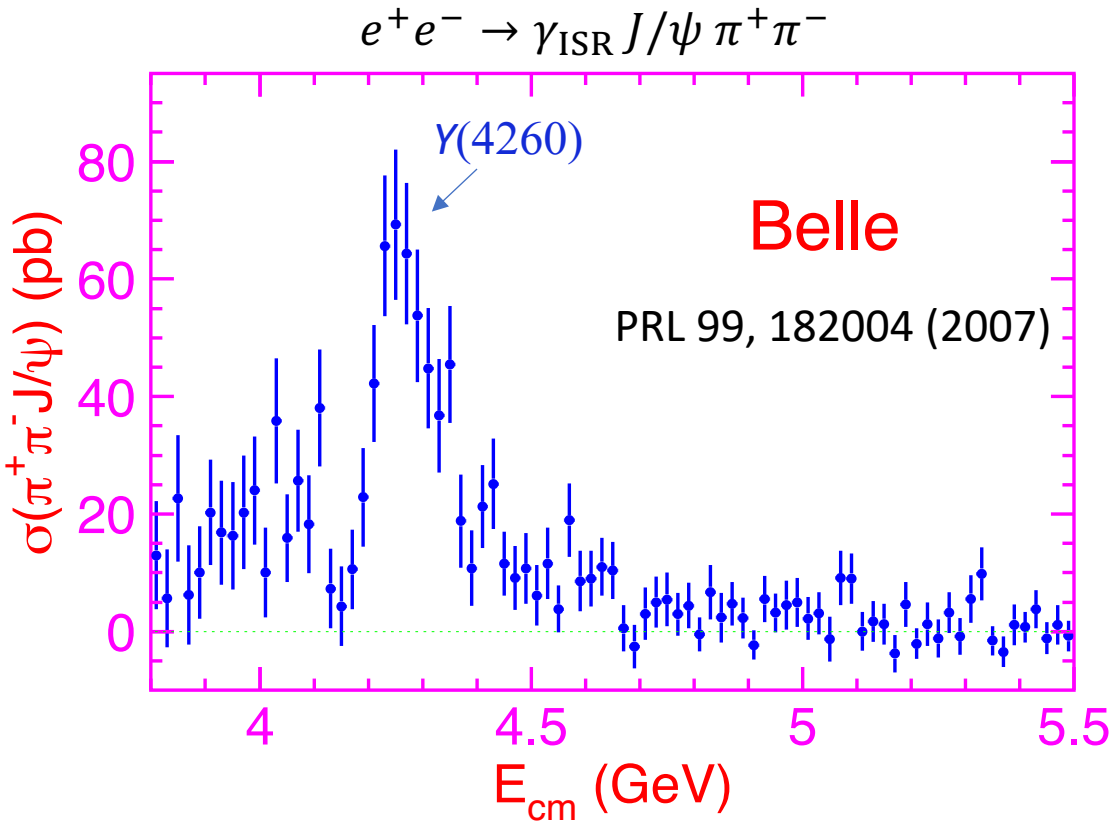
→

Y states are not main contributors to open-charm final states

$$R \equiv (e^+e^- \to \text{hadrons})/(e^+e^- \to \mu^+\mu^-)$$

Why Y states considered exotic ?

Peculiar decay pattern



In hidden-charm final states,

conventional charmonium states don't appear clearly

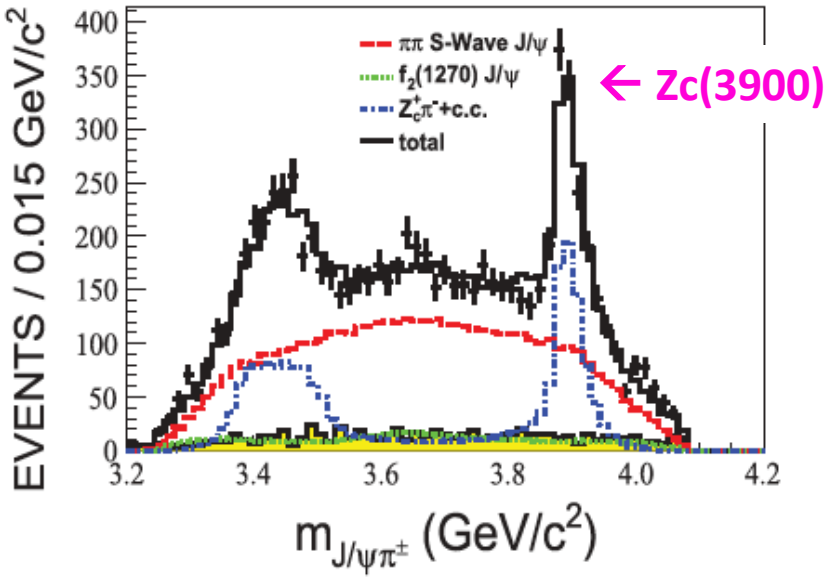
but Y states appear clearly

Conventional $c\bar{c}$ and Y have opposite trends in their decays → Y might have exotic structure

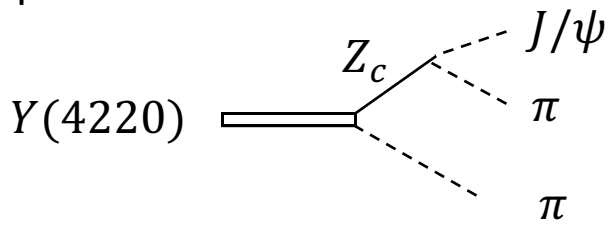
Understanding Y inevitably involves understanding Zc

Zc(3900), Zc(4020) : outstanding exotic candidates including $c\bar{c}u\bar{d}$

$e^+e^- \to J/\psi\,\pi^+\pi^-$ at Y(4220) region →



Zc production appears to be:

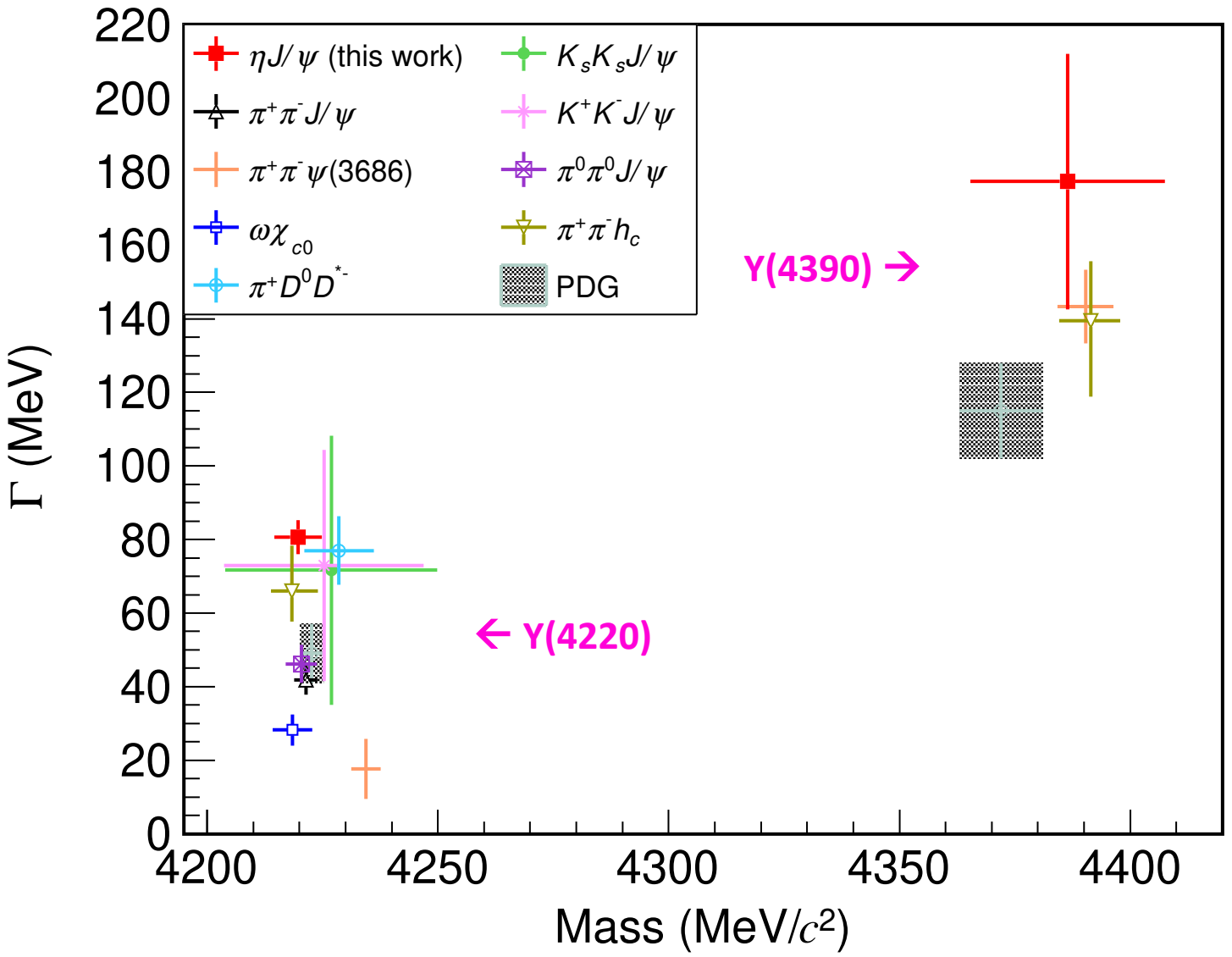


→ Y and Zc properties should be highly correlated

Global $e^+e^- \to c\bar{c}$ analysis consider Zc signals → address Y and Zc properties simultaneously

Y width problem

Process-dependent width ?



BESIII, arXiv:2310.03361

More outcomes from coupled-channel analysis

(i) vector charmonium mass and width (poles)

(ii) couplings of the poles with decay channels (residues)

Input for further calculations

- Compositeness: probability of finding hadron molecule components in resonance

  → helps identify hadron molecule states

  Baru et al., PLB 586, 53 (2004)
  Sekihara et al. PTEP 063D04 (2015)

- Branching ratios: mostly unknown for charmonia of M > 4 GeV

  including well-established $\psi(4040)$, $\psi(4160)$, $\psi(4415)$

  Not straightforward, prescription needed Heuser et al., EPJC 84, 599 (2024)

Related works previously done

Three-body model

* M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Zhao, PRD 90, 074039 (2014)

Analysis of $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross section and invariant mass in $4.1 \lesssim \sqrt{s} \lesssim 4.3$ GeV [Y(4230) region]

Pioneering works, but the data were very limited → limited conclusions on Y(4230) properties

* L. Detten, C. Hanhart, V. Baru, arXiv:2309.11970

Fitting data in Y(4230) region; more final states than the above

Fits to cross section data for 3-4 processes

Our analysis includes significantly more complete dataset

→ More reliable conclusion

* D.-Y. Chen, X. Liu, T. Matsuki, Eur. Phys. J. C 78, 136 (2018)

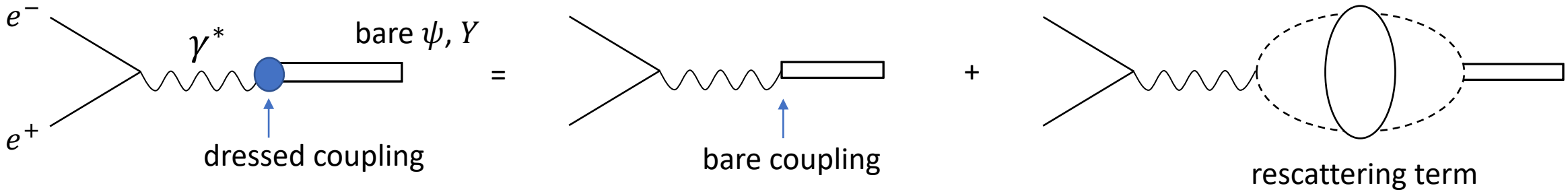
Breit-Wigner fit to $e^+e^- \to \pi D\bar{D}^*$, $J/\psi\pi\pi$, $h_c\pi\pi$ cross sections → Y(4320) and Y(4390) not necessary

* Z.-Y. Zhou, C.-Y. Li, Z. Xiao, arXiv:2304.07052

Two-body unitary model fitted to $e^+e^- \to D^{(*)}\bar{D}^{(*)}$, $\pi D\bar{D}$ cross sections → ψ(4160) is Y(4230)

ψ production mechanisms

$e^+e^- \to c\bar{c}$ data in $3.75 \leq \sqrt{s} \leq 4.7$ GeV region → Charmonium excitations are important mechanism



Data determine how many bare states to be included (5 bare states) and which charmonium states exist

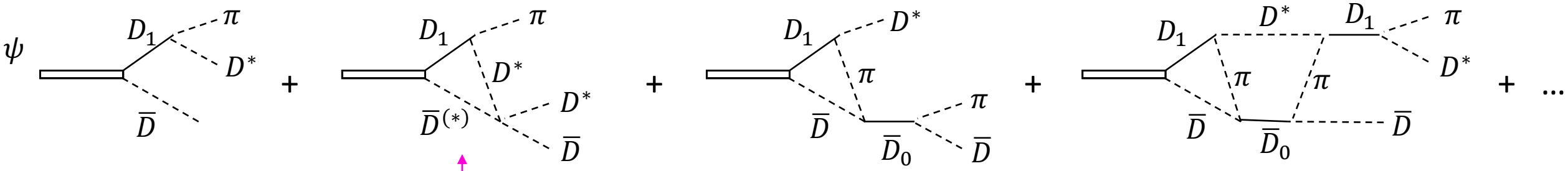
Expected states $\psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415),\ Y(4220),\ Y(4360)$

Data is not sufficient for coupled-channel analysis in $\sqrt{s} > 4.6$ GeV (three-body final states including $c\bar{c}s\bar{s}$)

→ $Y(4660)$ is not included in coupled-channel amplitude → included as a Breit-Wigner amplitude

Three-body decays of ψ

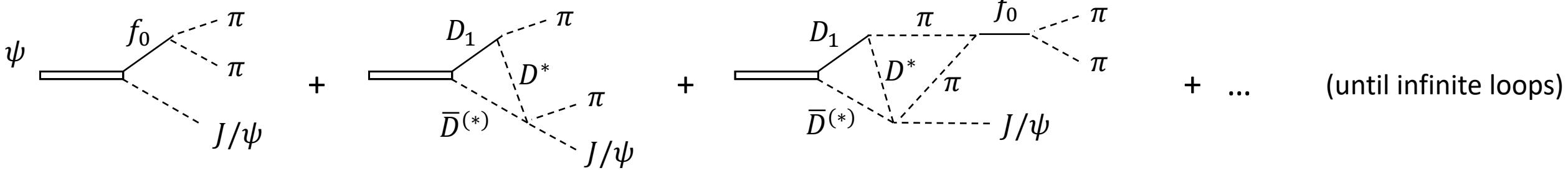
$e^+e^- \to \pi D^* \bar{D}$



(until infinite loops)

Triangle singularity loop

$e^+e^- \to J/\psi\pi\pi$

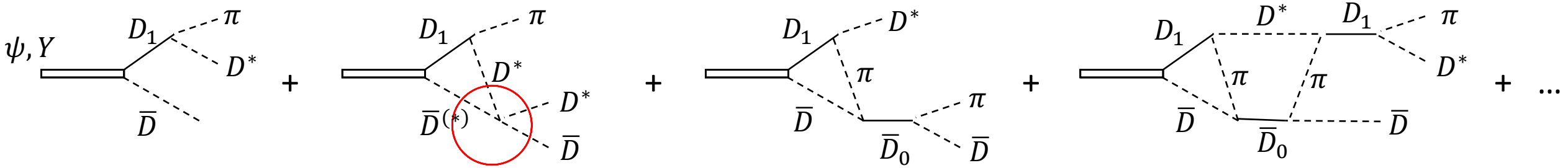


(until infinite loops)

Selected important diagrams; diagrams with more loops are usually more suppressed

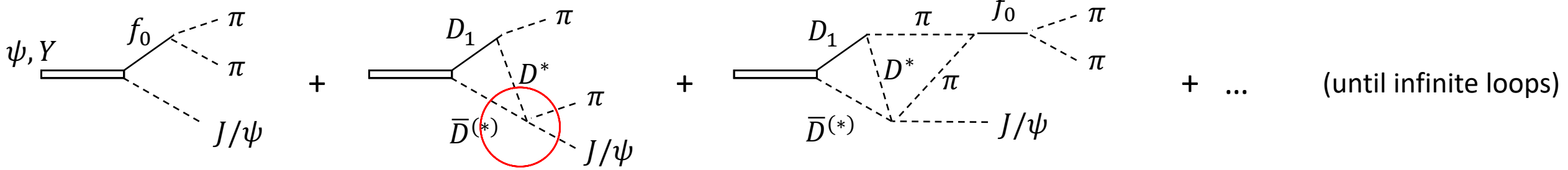
Different processes share the same interactions ← unitarity requirement

$e^+e^- \to \pi D^* \overline{D}$



(until infinite loops)

$e^+e^- \to J/\psi\pi\pi$



(until infinite loops)

Z_c amplitude

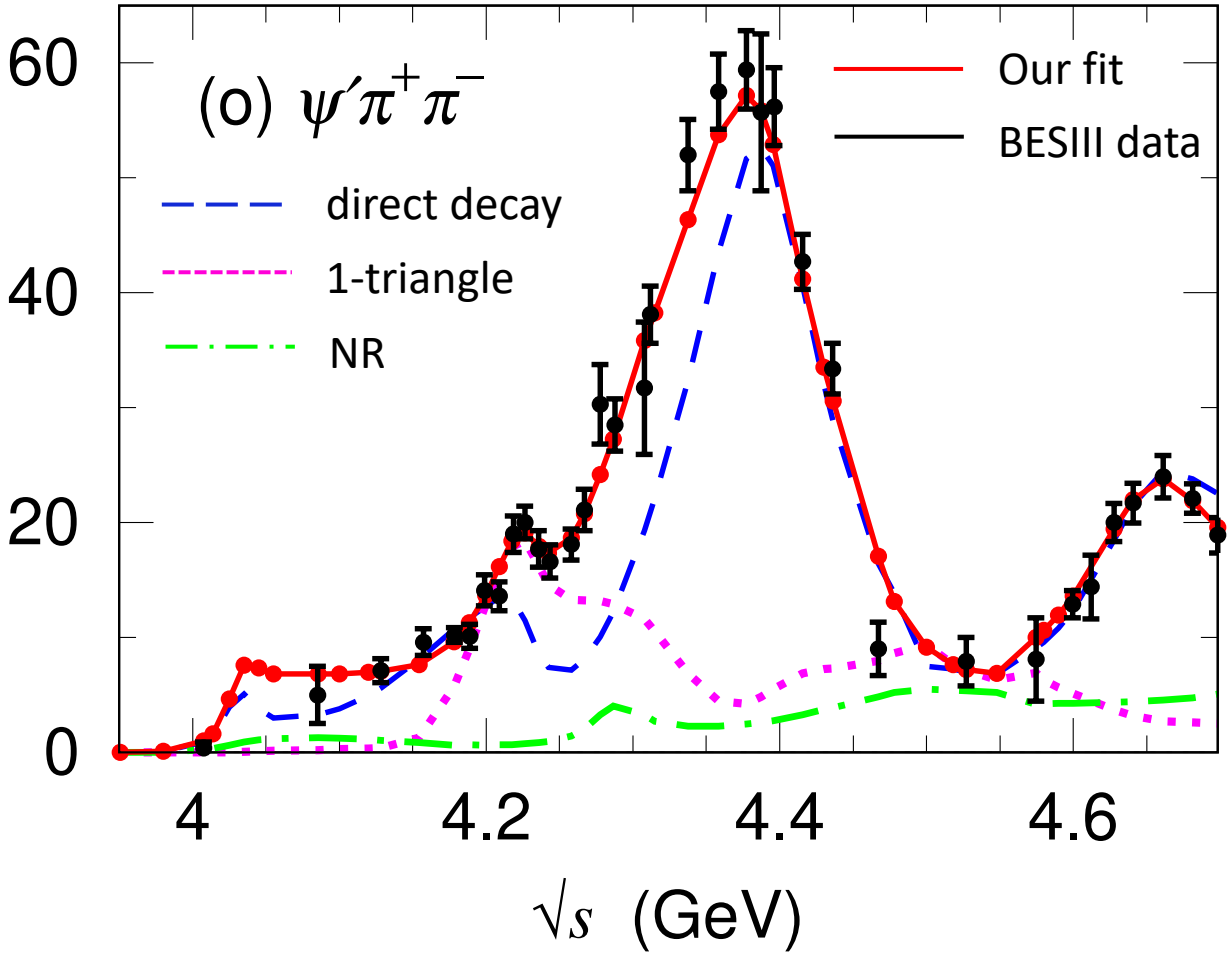
$D^*\overline{D} - D^*\overline{D}^* - J/\psi\,\pi - \psi'\pi - h_c\pi - \eta_c\rho$ coupled-channel scattering amplitude ($J^{PC} = 1^{+-}$)

→ $D^*\overline{D}$ and $D^*\overline{D}^*$ threshold cusps will be created in invariant mass distributions

Zc(3900) and Zc(4020) poles may also be generated (if needed by data) to enhance the cusps

$e^+e^- \to \psi' \pi^+\pi^-$

Data: BESIII, PRD 104, 052012 (2021)



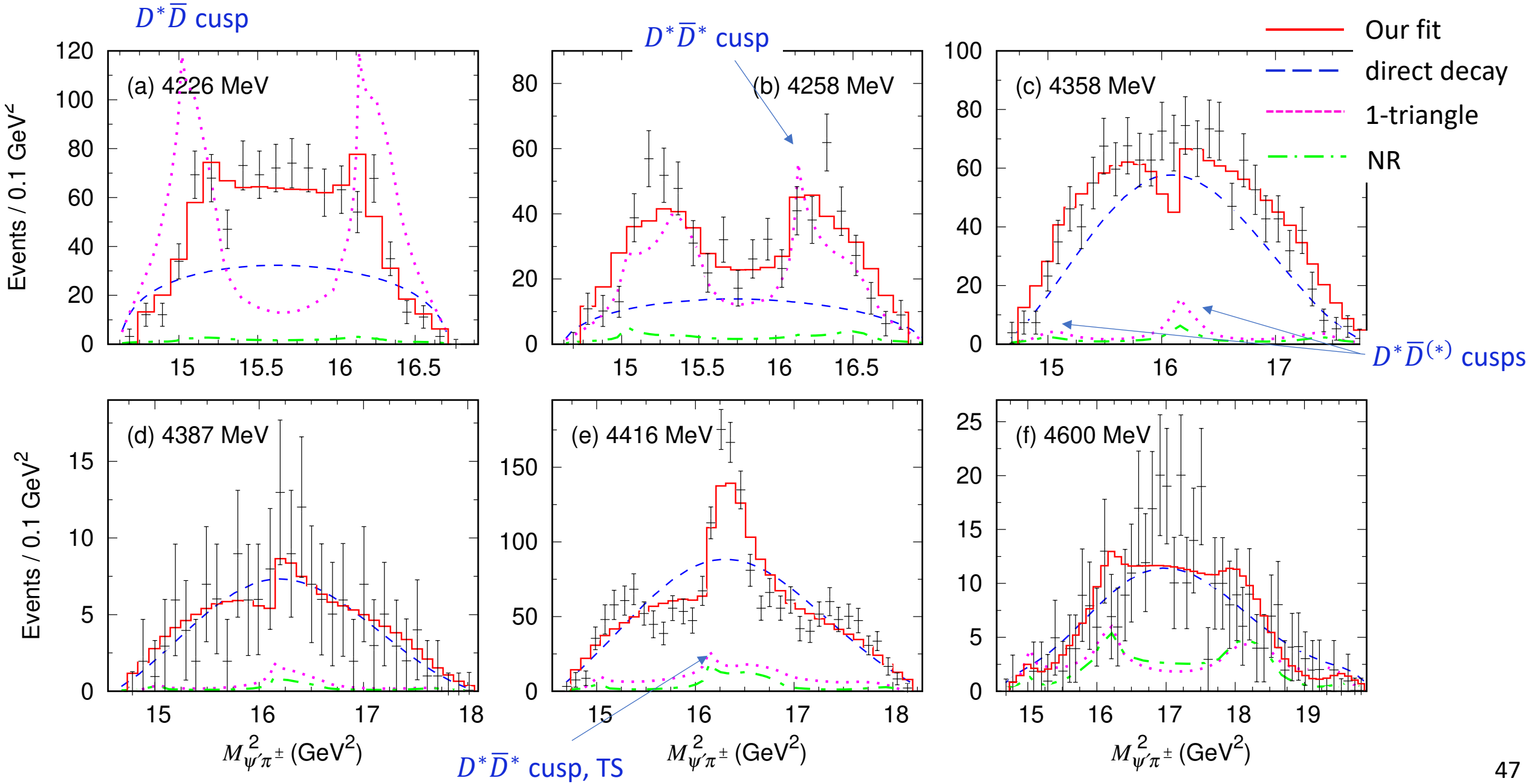
- Overall good fit
- Enhancement at ~ 4.03 GeV is from $\psi(4040)$

  ← consequence of coupled-channel fit
- 1-triangle contribution is large at $\psi(4220)$ peak

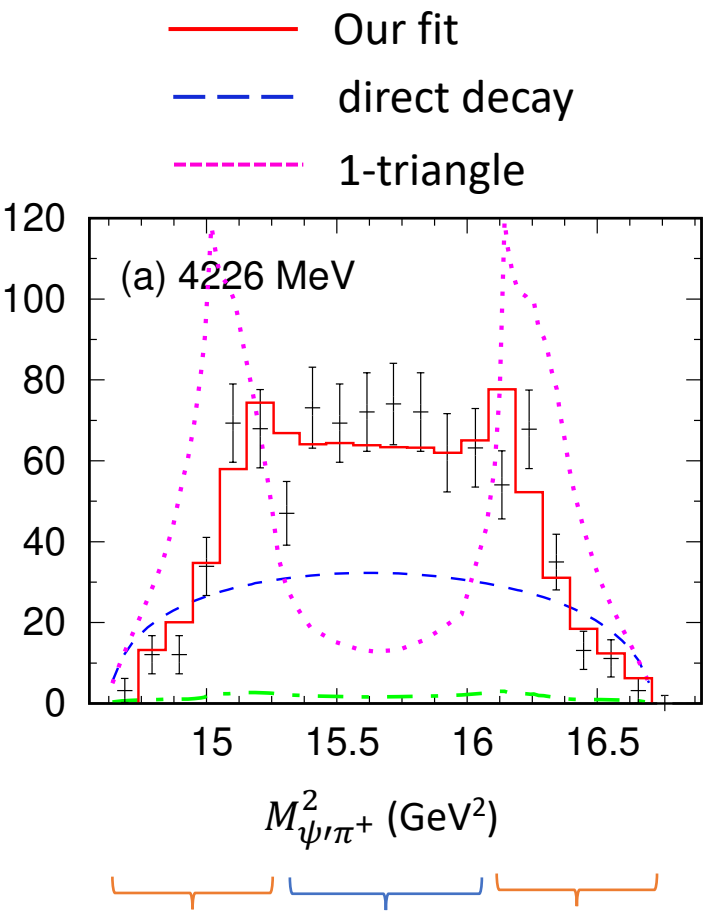
$e^+e^- \to \psi' \pi^+\pi^-$

Fit to $\psi'\pi$ invariant mass distributions

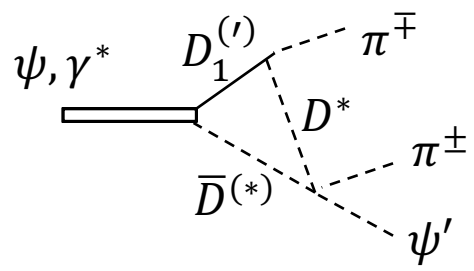
Data: BESIII, PRD 96, 032004 (2017)

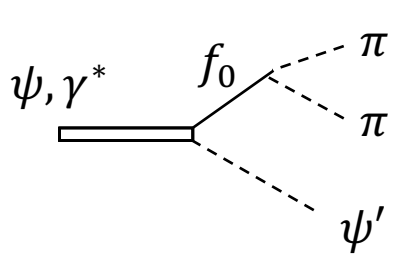


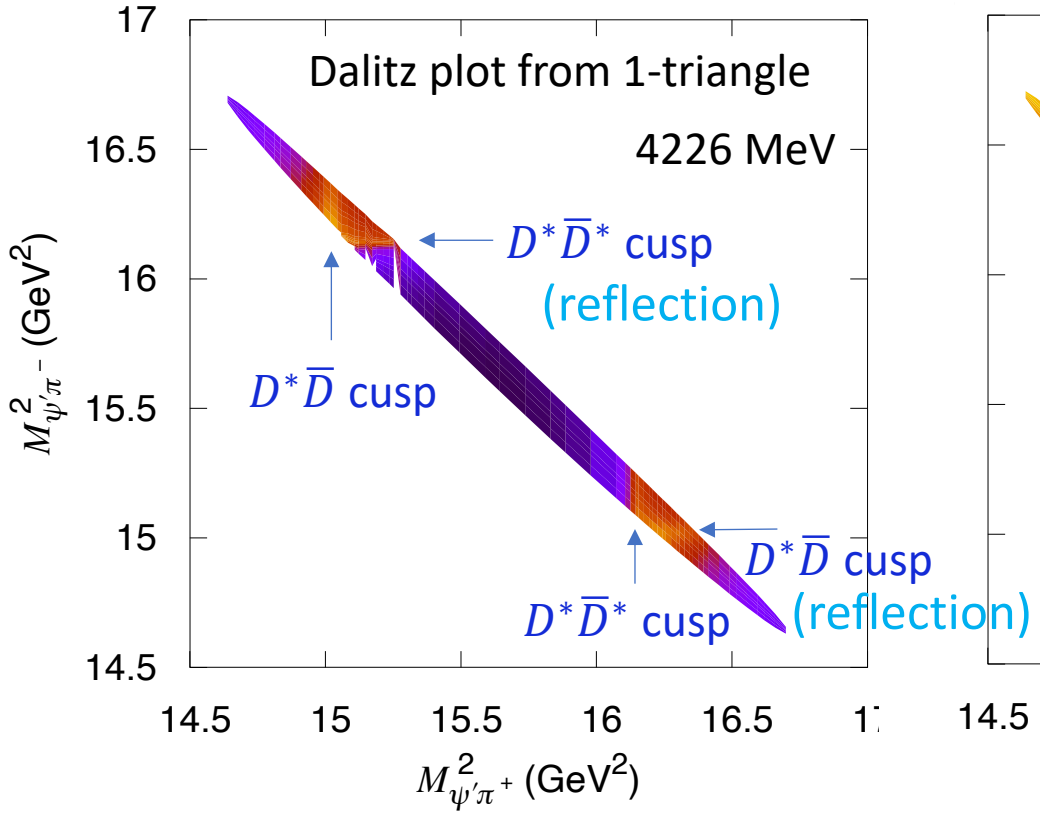
$e^+e^- \to \psi' \pi^+\pi^-$

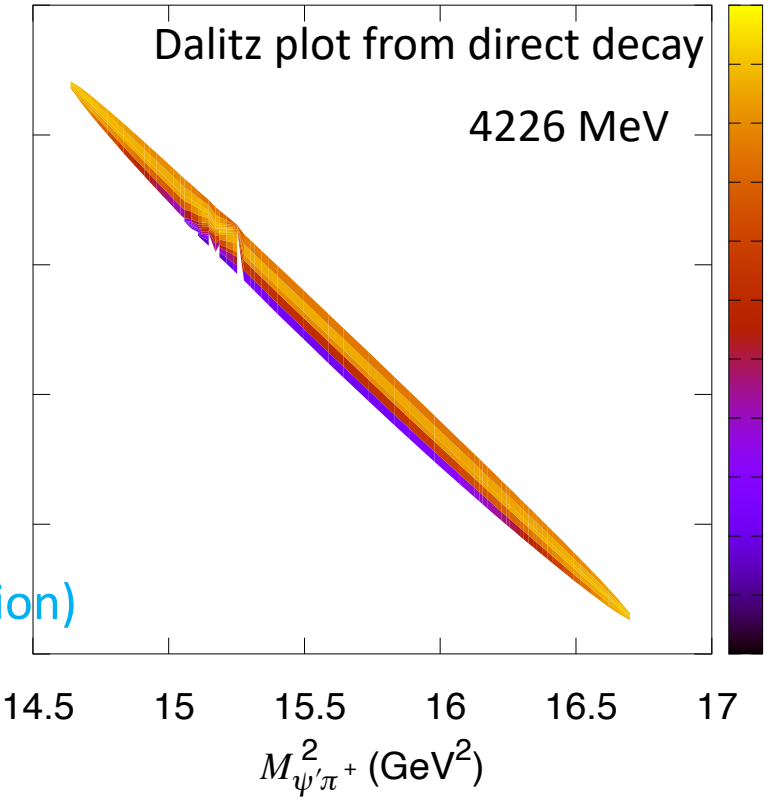


destructive constructive destructive



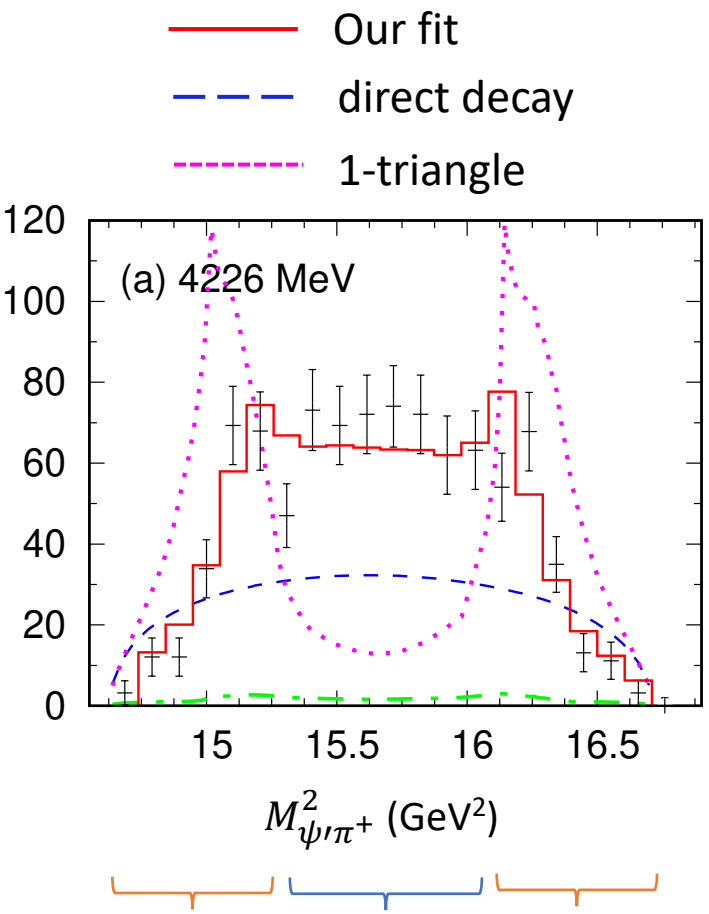




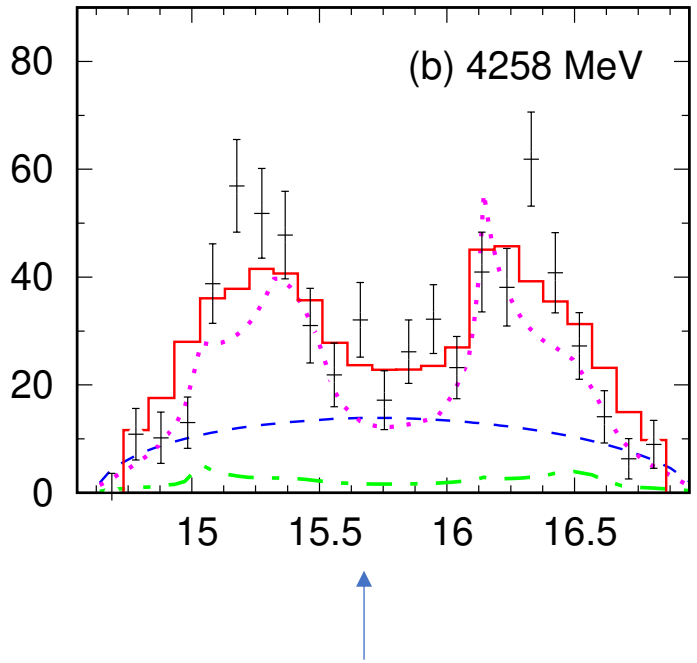


Interference erase cusp structures

$e^+e^- \to \psi' \pi^+\pi^-$



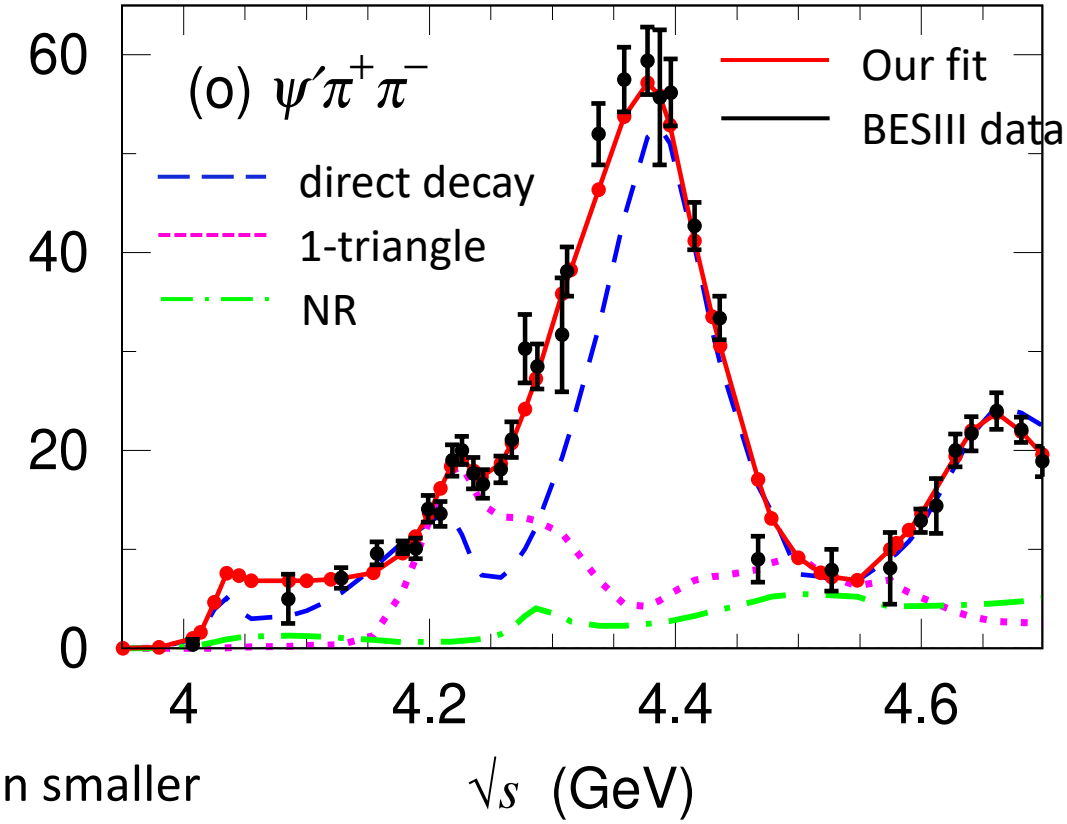
destructive constructive destructive



direct-decay contribution smaller

→ Smaller interference

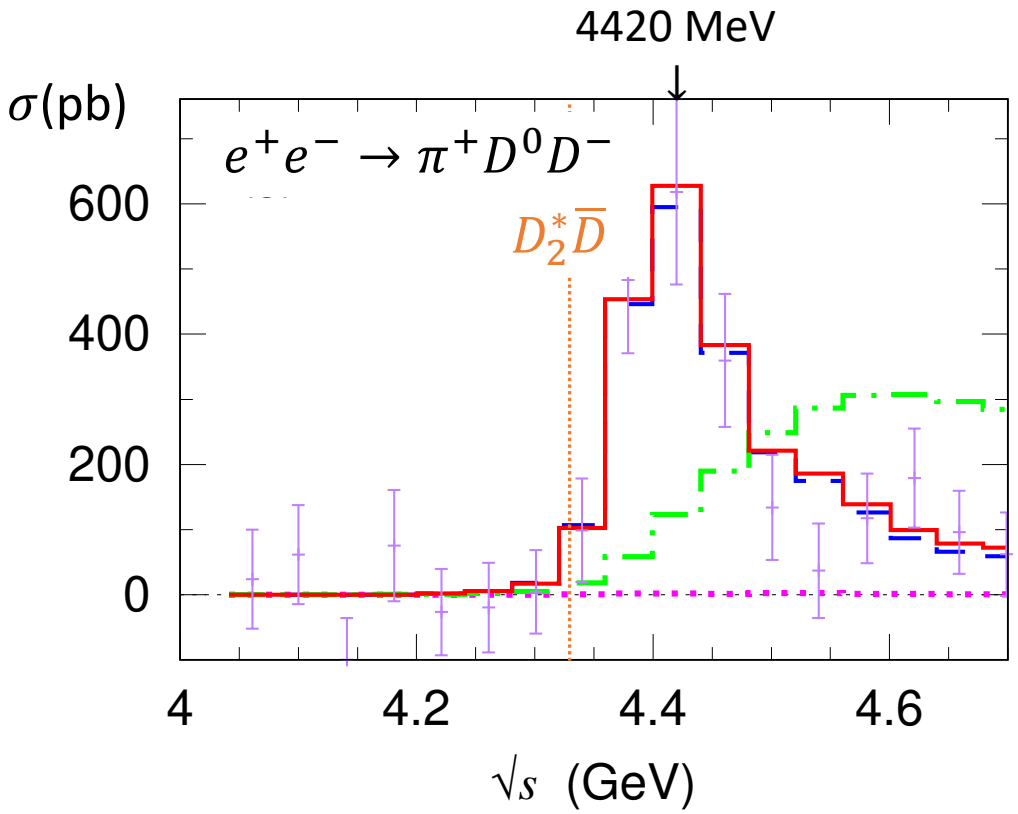
→ Clearer cusp structures

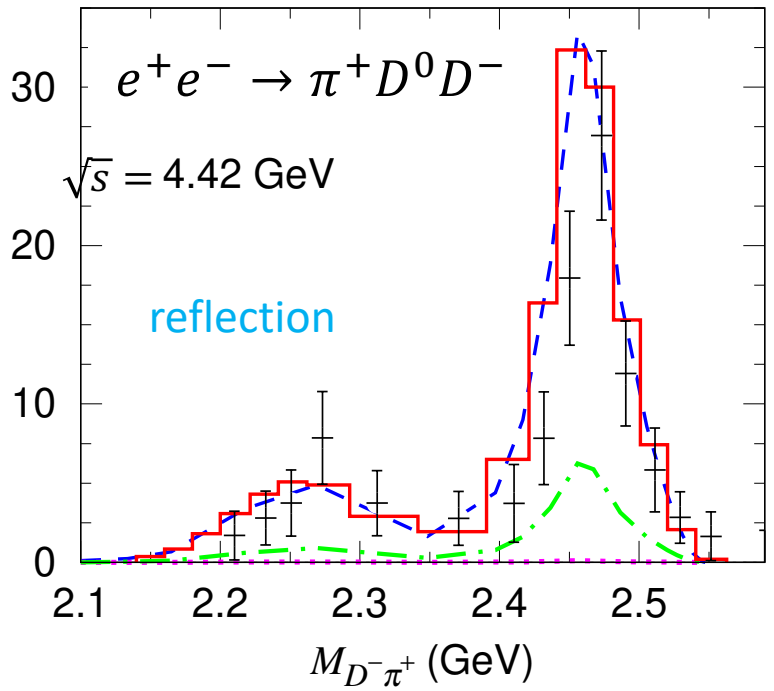


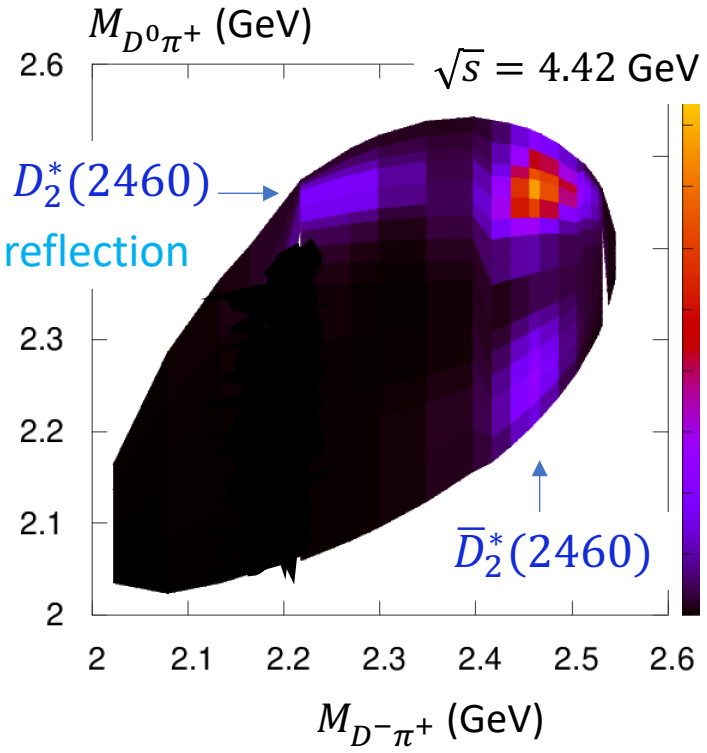
Interference erase cusp structures

First explanation of this rapid change of lineshape

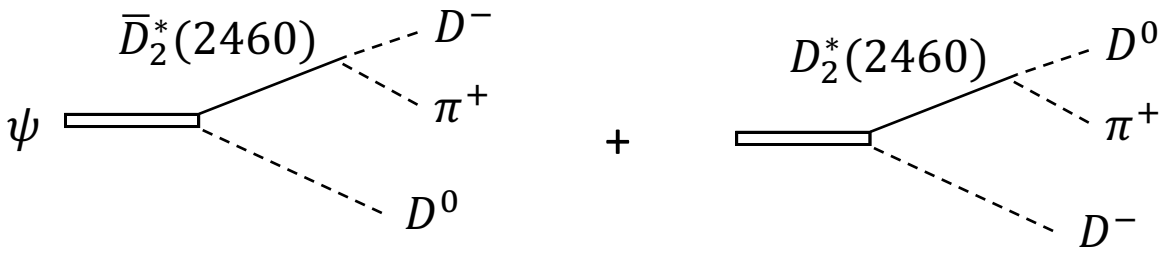
Importance of threshold effects to understand charmonium mass



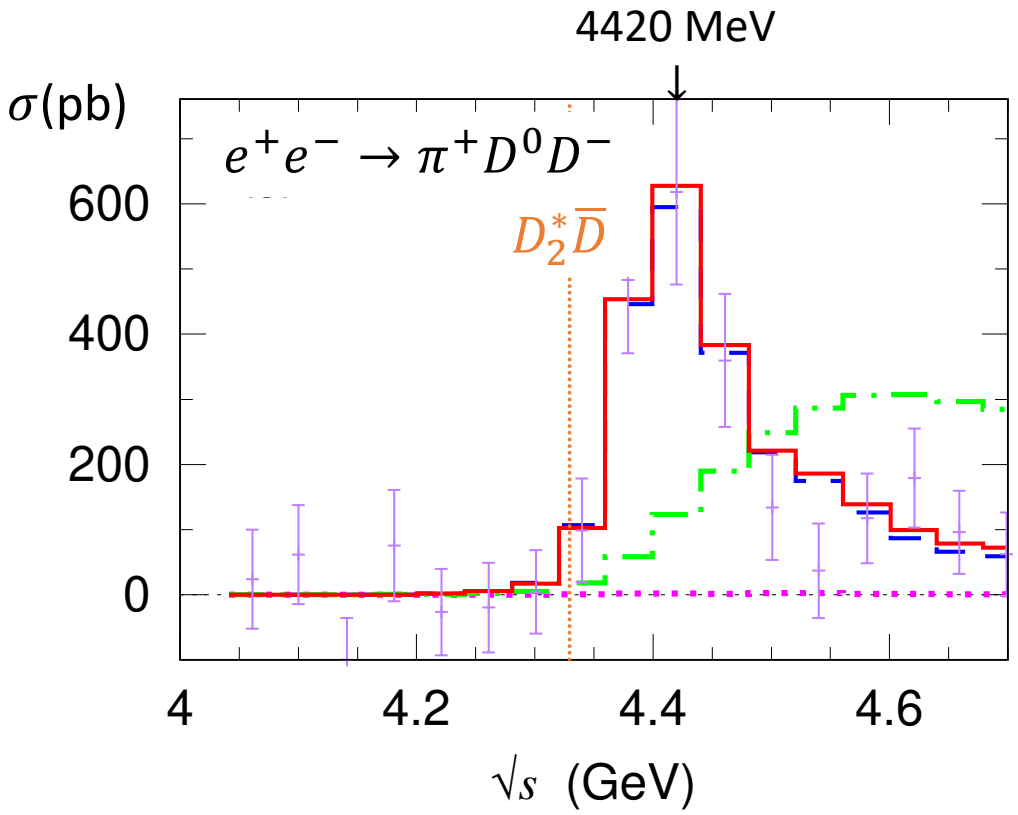


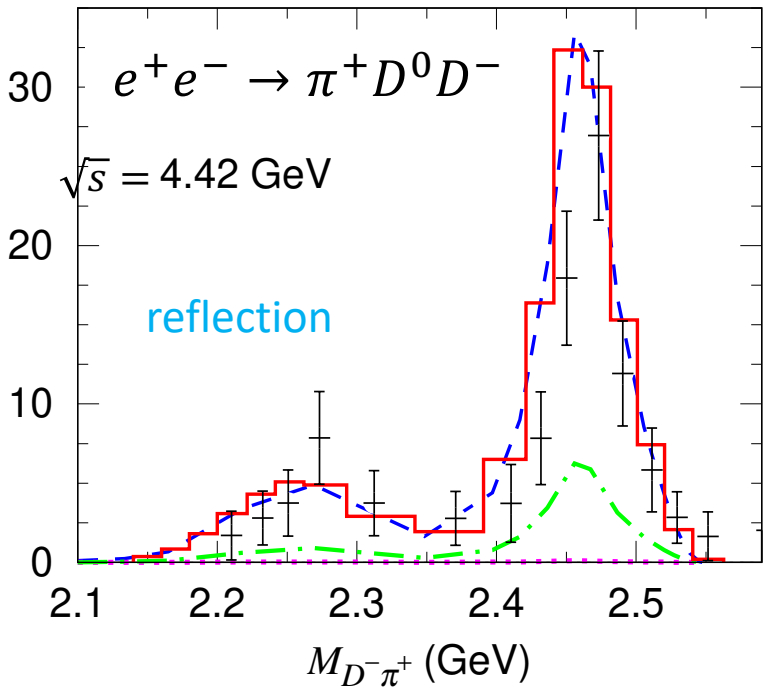


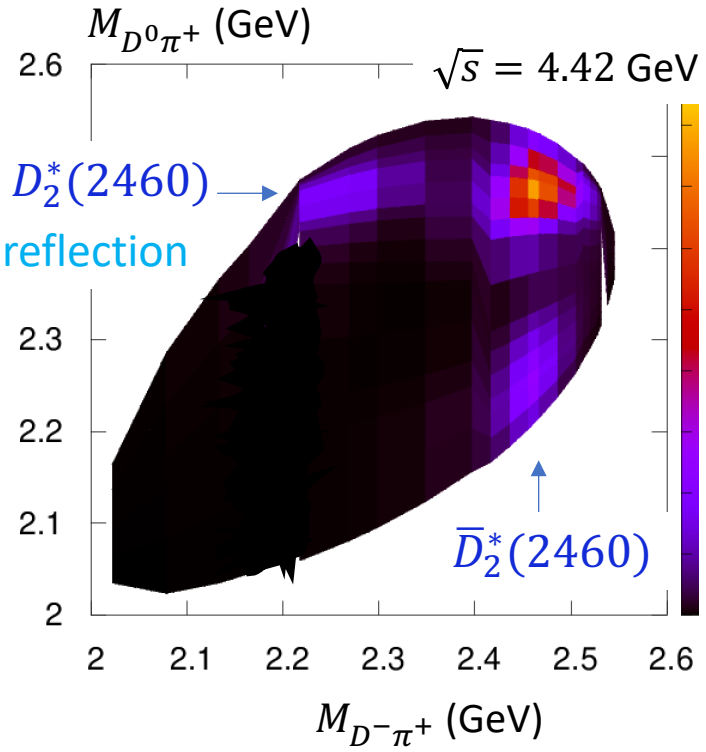
Decay sequence:



Importance of threshold effects to understand charmonium mass

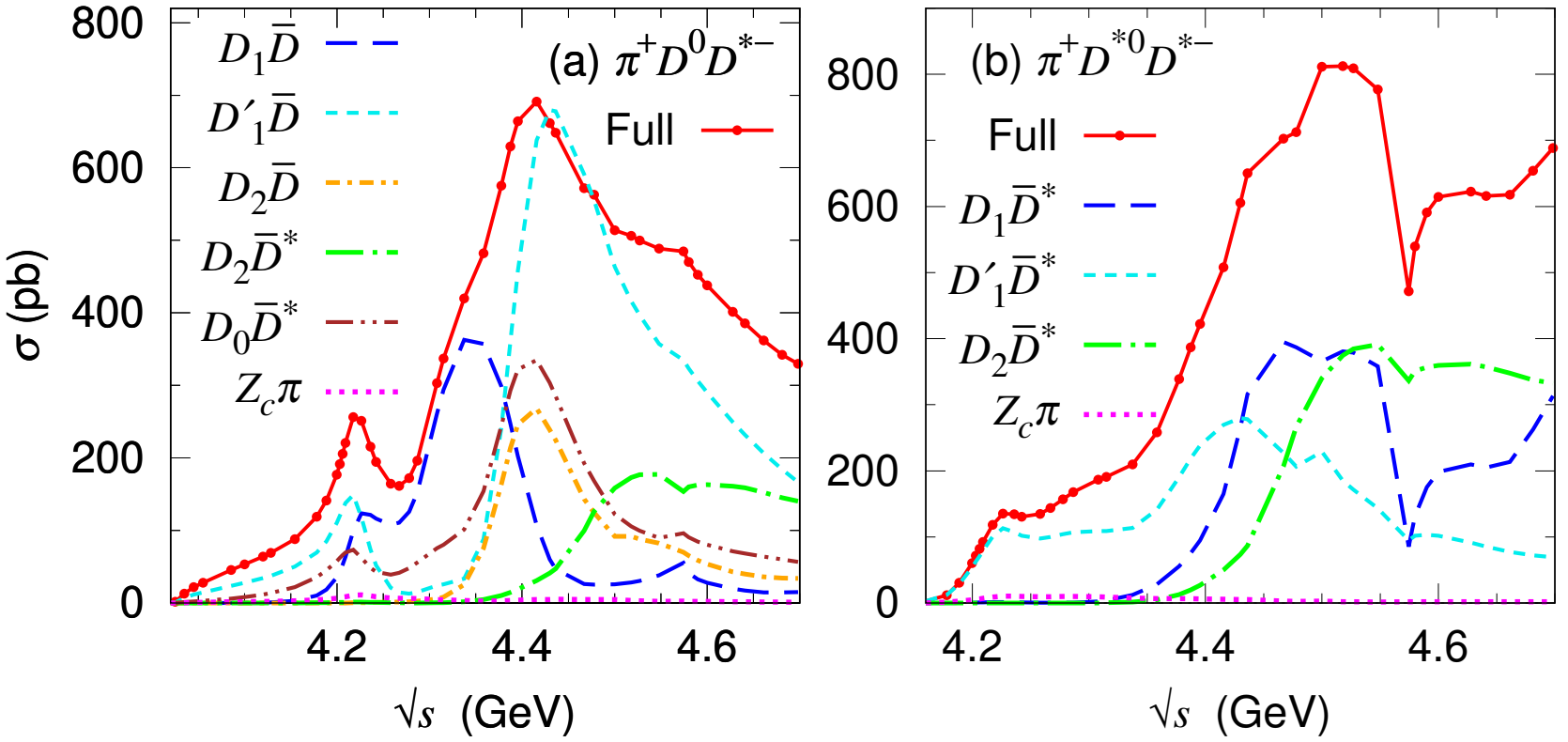


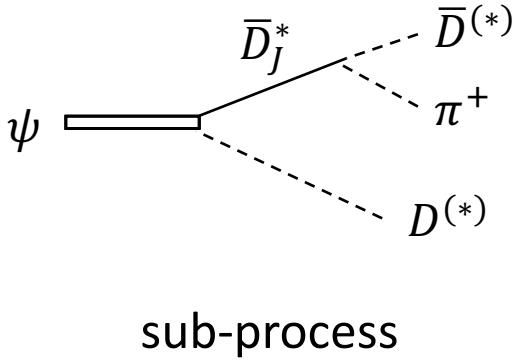




- This process is strongly suppressed below $D_2^*\overline{D}$ threshold (Γ_{D_2}~47 MeV)
- $D_2^*\overline{D}$ channel is d-wave → centrifugal barrier suppression near threshold
- Resonance ($M_{\rm pole}$= 4390 MeV) peak position is shifted by ~30 MeV higher
- Fit cross-section (σ) data with $|A_{BW}|^2\times$ (phase-space) is not justified ($M_{\rm BW}$=4420 MeV)

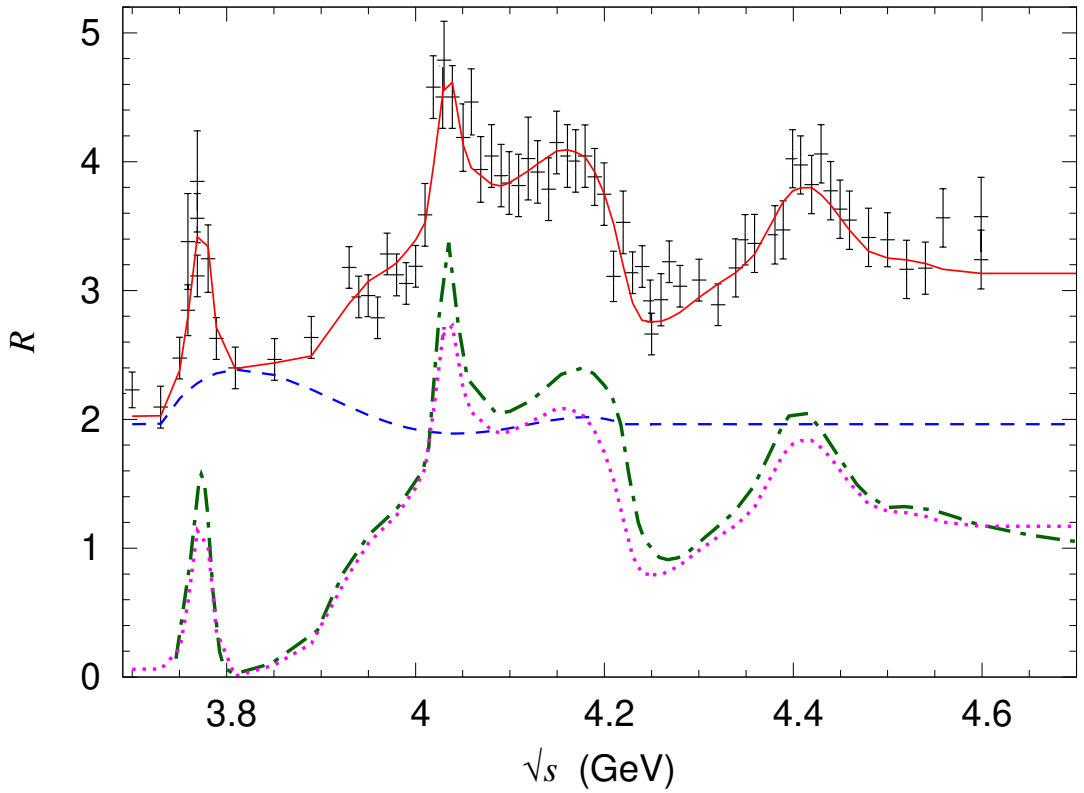
Importance of threshold effects to understand charmonium mass





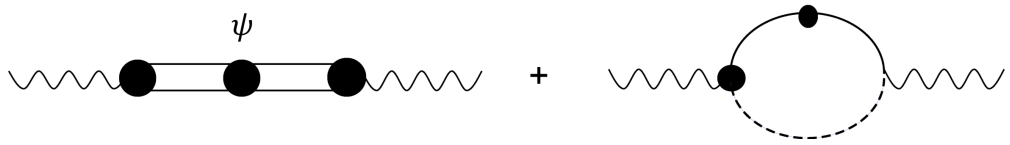
sub-process

- Each sub-process is strongly suppressed below $D_J^*\bar{D}^{(*)}$ threshold (if Γ_{D_J} is narrow)
- Resonance peak position is shifted by threshold effects
- Fit cross-section (σ) data with $|A_{BW}|^2\times$ (phase-space) is not justified
- Dalitz-plot data is important to constrain $D_J^*\bar{D}^{(*)}$ contributions (currently cross-section data only)



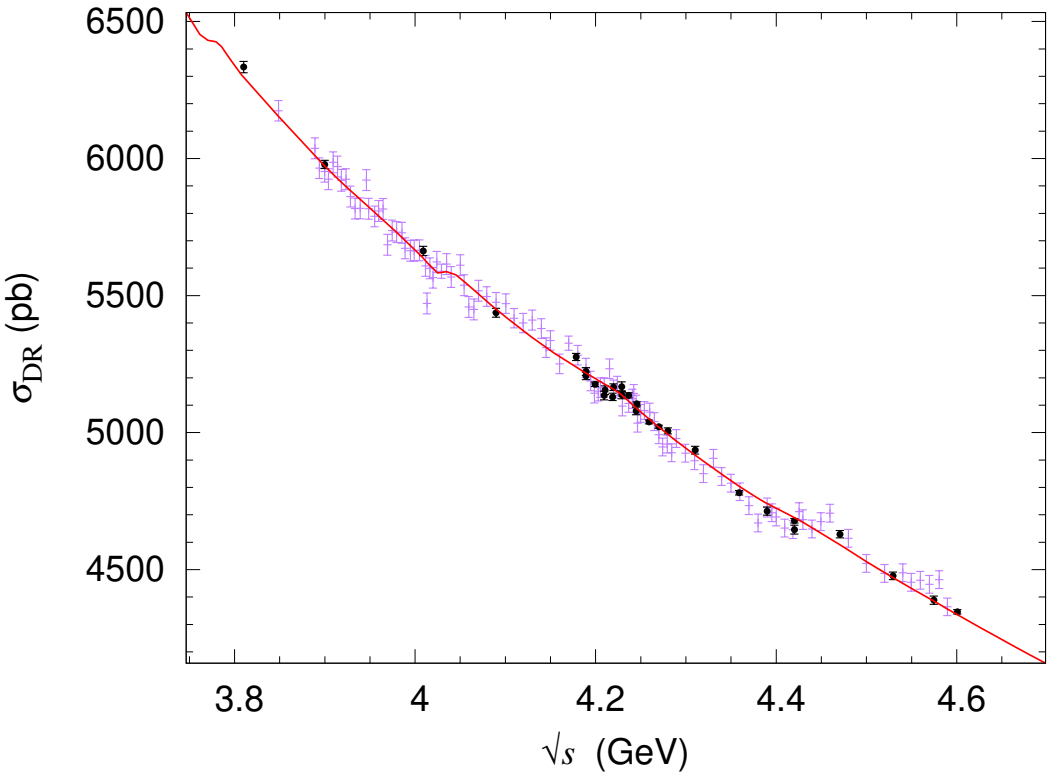
$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma^{\text{tree}}_{e^+e^-\to\mu^+\mu^-}(s)} = R_c(s) + R_{uds}(s),$$

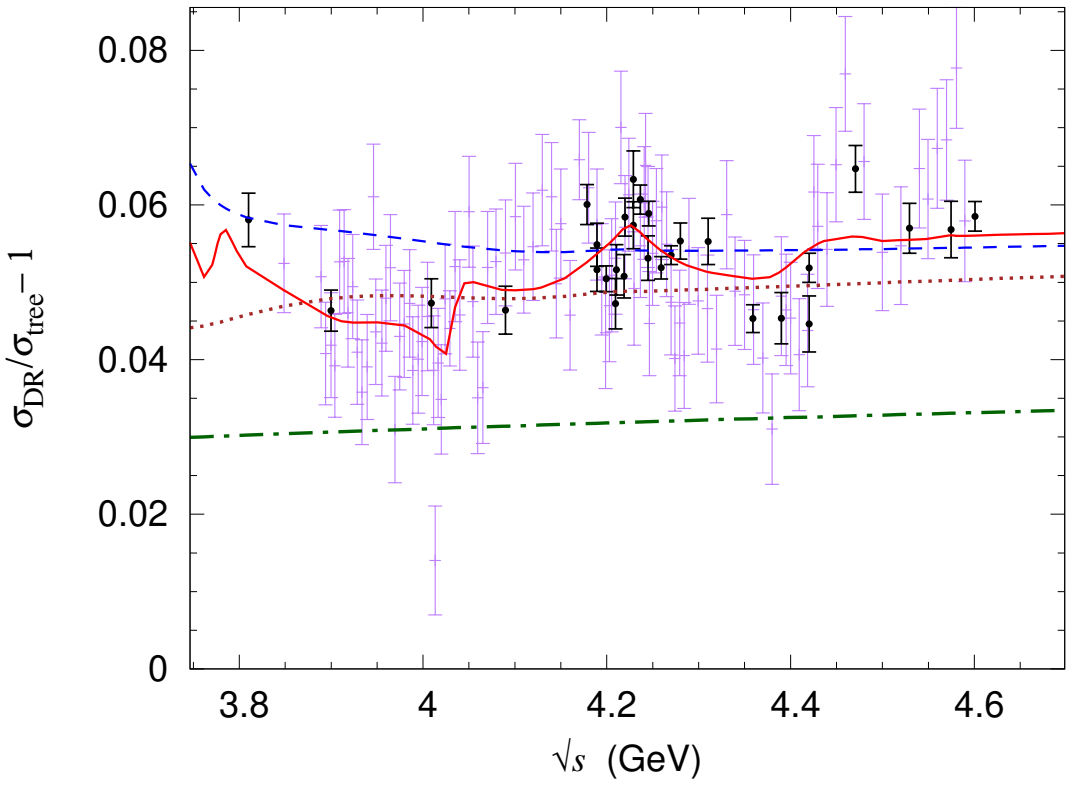
Vacuum polarization $\Pi_c =$



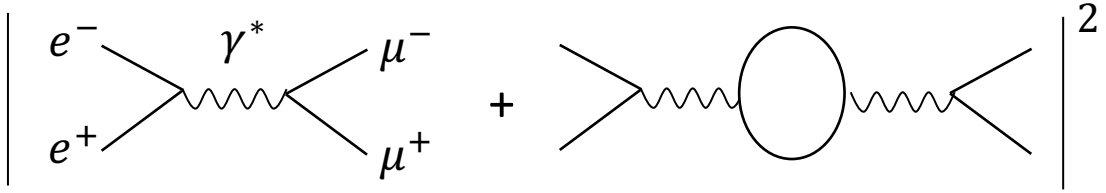
Optical theorem $R_c(s) = -\frac{3}{\alpha}\text{Im}[\Pi_c(s)]$, (green dash-dotted curve)

$e^+e^- \to \mu^+\mu^-$ cross section prediction



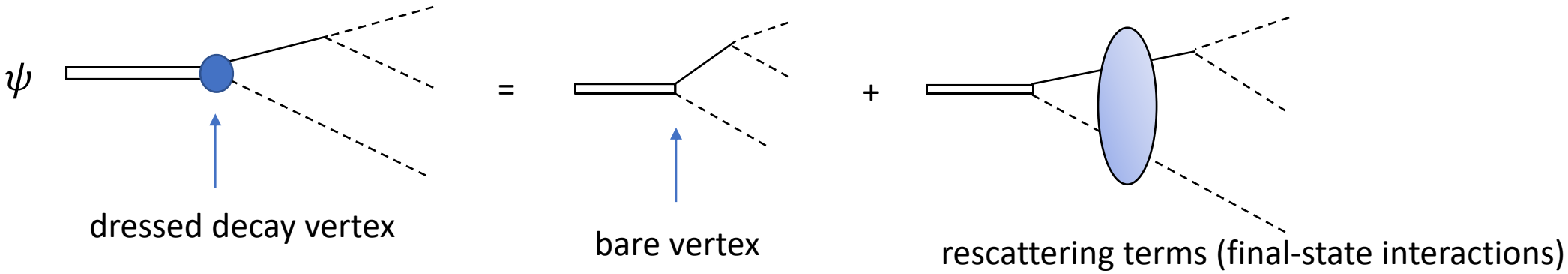


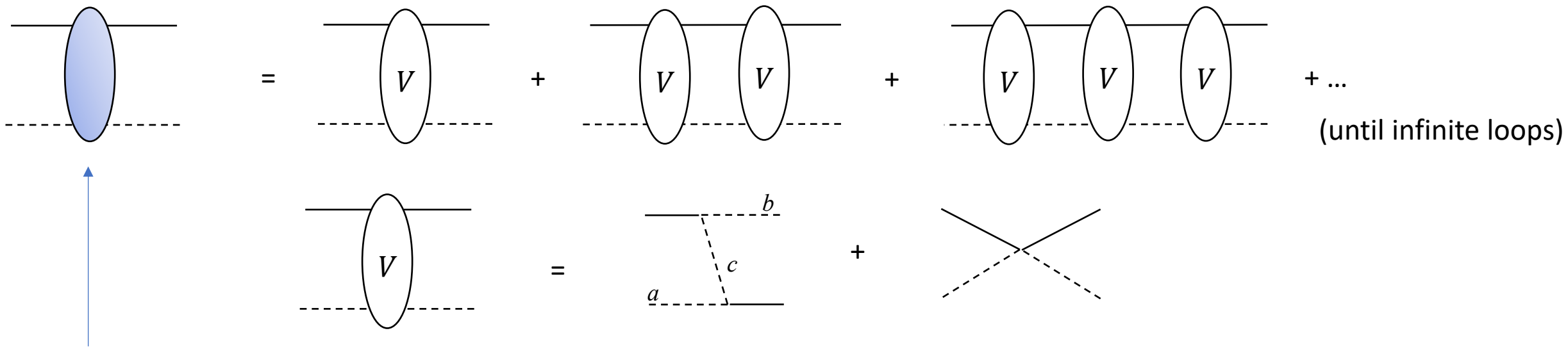
$$\sigma^{\mathrm{DR}}_{e^+e^-\to\mu^+\mu^-}(s) = \sigma^{\mathrm{tree}}_{e^+e^-\to\mu^+\mu^-}\,(1 + 2\mathrm{Re}[\Pi(s)])$$



$$\Pi(s) = \Pi_{\mathrm{lepton}} + \Pi_{uds} + \Pi_{J/\psi} + \Pi_{\psi'} + \Pi_c$$

Hadron-molecule poles





This amplitude may include hadron-molecule poles

$$\propto \frac{1}{E - E_a - E_b - E_c}$$

E-independent

Resonance parameters

This work		PDG(ψ) [4], BESIII [16, 38, 88]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3764.2 ± 2.0	47.3 ± 2.6	3751.9 ± 3.8	32.8 ± 5.8	$\mathcal{R}(3760)$
3780.2 ± 1.2	29.9 ± 2.3	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
3898.4 ± 0.9	127.5 ± 6.7	3872.5 ± 14.2	179.7 ± 14.1	$G(3900)$
3956.1 ± 1.0	96.8 ± 10.4	–	–	${}^{\mathrm{r}}D_s\bar{D}_s$
4029.2 ± 0.4	26.3 ± 1.0	4039 ± 1	80 ± 10	$\psi(4040)$
4052.4 ± 0.4	49.0 ± 0.3	–	–	${}^{\mathrm{v}}D_s^*\bar{D}_s$
4192.2 ± 2.2	129.3 ± 4.2	4191 ± 5	70 ± 10	$\psi(4160)$
4216.2 ± 0.5	40.3 ± 1.0	–	–	${}^{\mathrm{v}}D_s^*\bar{D}_s^*$
4229.9 ± 0.9	46.4 ± 2.6	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4308.1 ± 2.2	138.2 ± 4.4	4298 ± 12	127 ± 17	$Y(4320)$
4346.2 ± 3.8	122.8 ± 6.7	4374 ± 7	118 ± 12	$\psi(4360)$
4390.1 ± 2.0	106.5 ± 4.1	4421 ± 4	62 ± 20	$\psi(4415)$
4496.3 ± 3.1	16.4 ± 2.1	–	–	${}^{\mathrm{b}}D_{s1}\bar{D}_s$
4579.6 ± 1.7	-5.2 ± 7.6	–	–	${}^{\mathrm{b}}\Lambda_c\bar{\Lambda}_c$
4655.9 ± 3.0	134.9 ± 5.9	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

$M = \mathrm{Re}[E_\psi]$

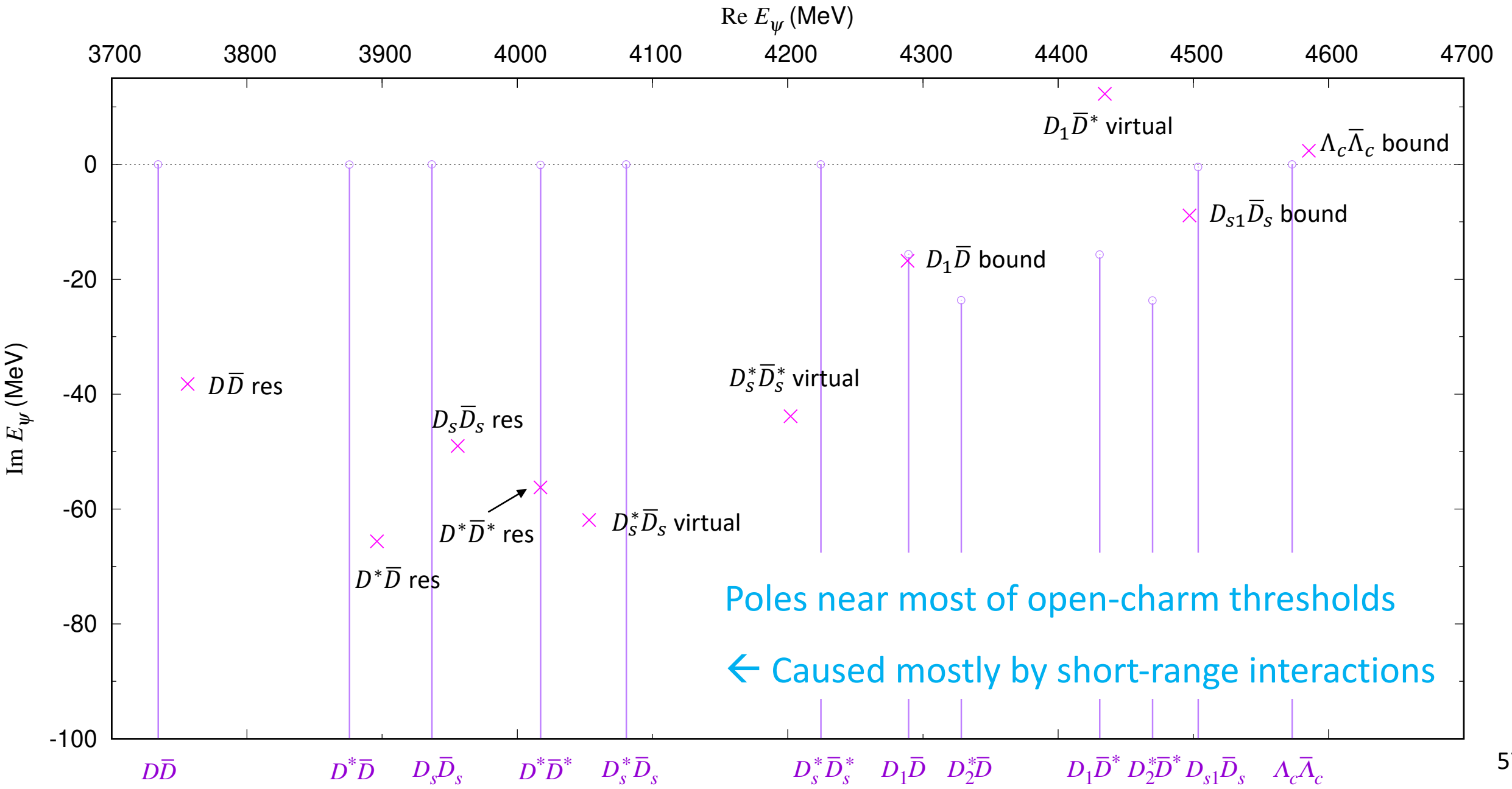
$\Gamma = -2 \times \mathrm{Im}[E_\psi]$

No Y-width puzzle by construction:

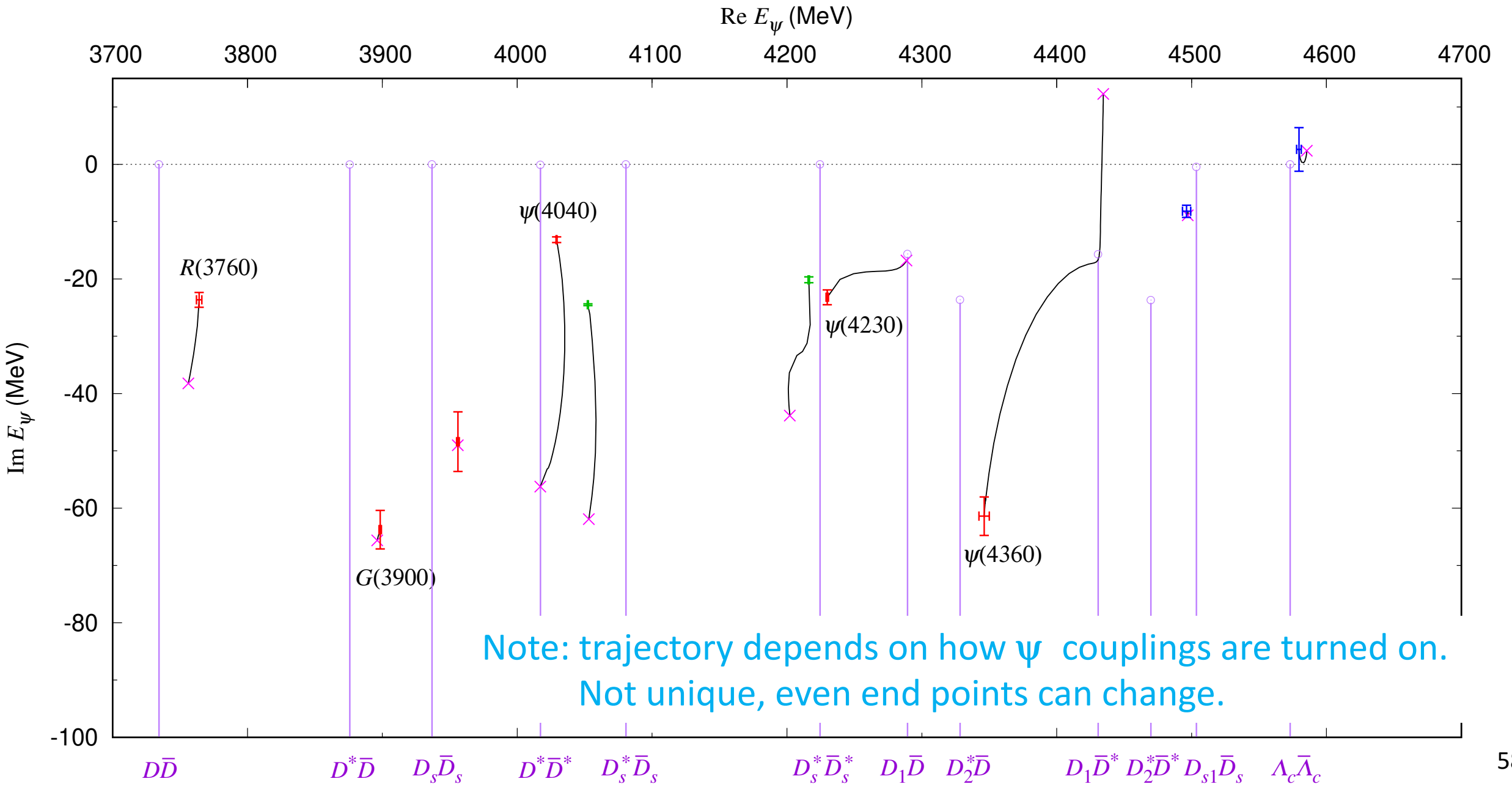
same Y-widths for all final states

← BW fit

Pole locations (no coupling to bare ψ)



Pole trajectories (0 → full coupling to bare ψ)



Note: trajectory depends on how ψ couplings are turned on.
Not unique, even end points can change.

Compositeness

T. Sekihara et al., PTEP 2015 (2015) 063D04

Normalization of resonance (Gamow) state $(\psi|\psi) = 1$ $\quad H\,|\psi) = \left(M - i\frac{\Gamma}{2}\right)|\psi)$

Insert complete set

$$1 = \sum_a |\psi_a\rangle\langle\psi_a| + \sum_j \int \frac{d^3q}{(2\pi)^3} |\boldsymbol{q}_j\rangle\langle\boldsymbol{q}_j|$$

bare state label

Two-body continuum channel label

$$1 = \sum_a (\psi|\psi_a\rangle\langle\psi_a|\psi) + \sum_j \int \frac{d^3q}{(2\pi)^3} (\psi|\boldsymbol{q}_j\rangle\langle\boldsymbol{q}_j|\psi) = \sum_a Z_a + \sum_j X_j$$

elementariness of bare state a

Compositeness of channel j

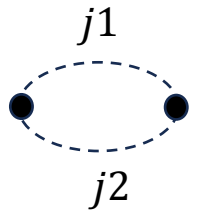
$$X_j = -g_j^2 \left[\frac{dG_j}{dE}\right]_{E=M_R - i\Gamma_R/2}$$

residue

with $\quad G_j = \int \frac{d^3q}{(2\pi)^3} \frac{[q^L \times f(q)]^2}{E - E_{j1} - E_{j2}}$

form factor

=



Caveats on compositeness

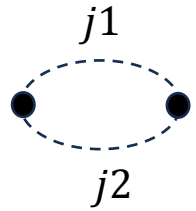
$$X_j = -g_j^2 \left[\frac{dG_j}{dE}\right]_{E=M_R-i\Gamma_R/2}$$

residue

with $$G_j = \int \frac{d^3q}{(2\pi)^3} \frac{[q^L \times f(q)]^2}{E - E_{j1} - E_{j2}}$$

form factor

=

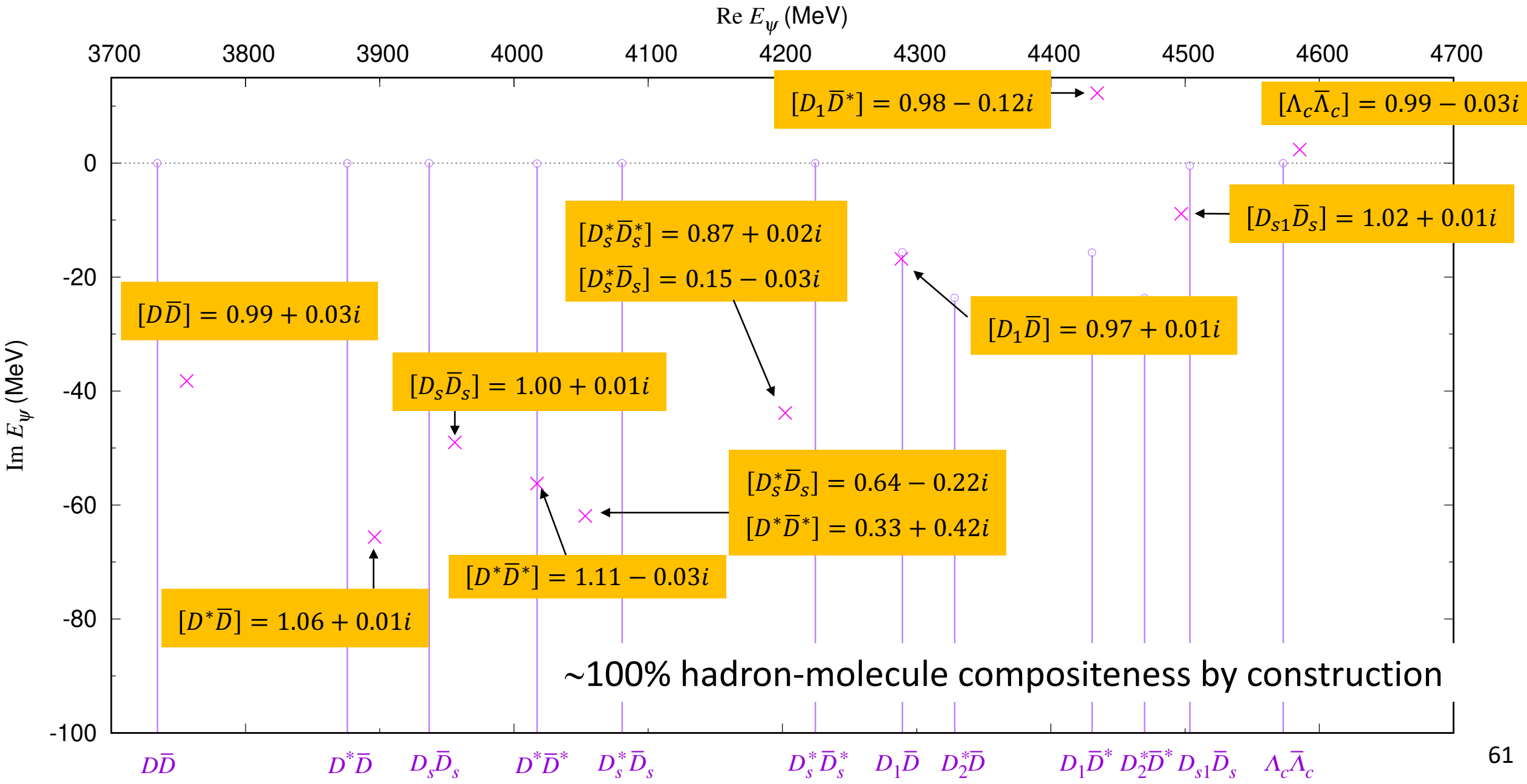


- Compositeness depends on form factors (model dependent)

  → model-independent at shallow bound state limit (X_j can be expressed with scattering length)
- For p-wave ($L = 1$) channels ($D\overline{D}, D^*\overline{D}, D^*\overline{D}^*$, etc.), X_j is more dependent on form factor
- Difficult to interpret $\mathrm{Im}[X]$, X < 0, X >1
- When $|\mathrm{Im}[X]| << 1$, 0 < X < 1, compositeness may suggest hadron-molecule contents in resonance
- Above formula is modified to account for three-body channels (small effect)

Pole locations (no coupling to bare ψ)

Compositeness



Compositeness

$c\bar{c}$ dominated states

$\psi(3770)$ $[D\bar{D}] = -0.66 - 0.41i$ $\rightarrow [c\bar{c}] \sim 1.7$

$\psi(4160)$ $[D^*\bar{D}^*] = 0.26 - 0.03i$

$[D_1\bar{D}] = 0.01 - 0.02i$ $\rightarrow [c\bar{c}] \sim 0.85$

$[D_s^*\bar{D}_s^*] = -0.10 - 0.04i$

$\Upsilon(4320)$ $[D^*\bar{D}^*] = 0.02 - 0.15i$

$[D_s^*\bar{D}_s] = 0.05 + 0.06i$ $\rightarrow [c\bar{c}] \sim 1.06$

$[D_s^*\bar{D}_s^*] = -0.11 - 0.03i$

$\psi(4415)$ $[D_1\bar{D}] = -0.03 + 0.04i$

$[D_1\bar{D}^*] = 0.08 + 0.13i$ $\rightarrow [c\bar{c}] \sim 0.90$

$[D_2^*\bar{D}^*] = 0.06 - 0.03i$

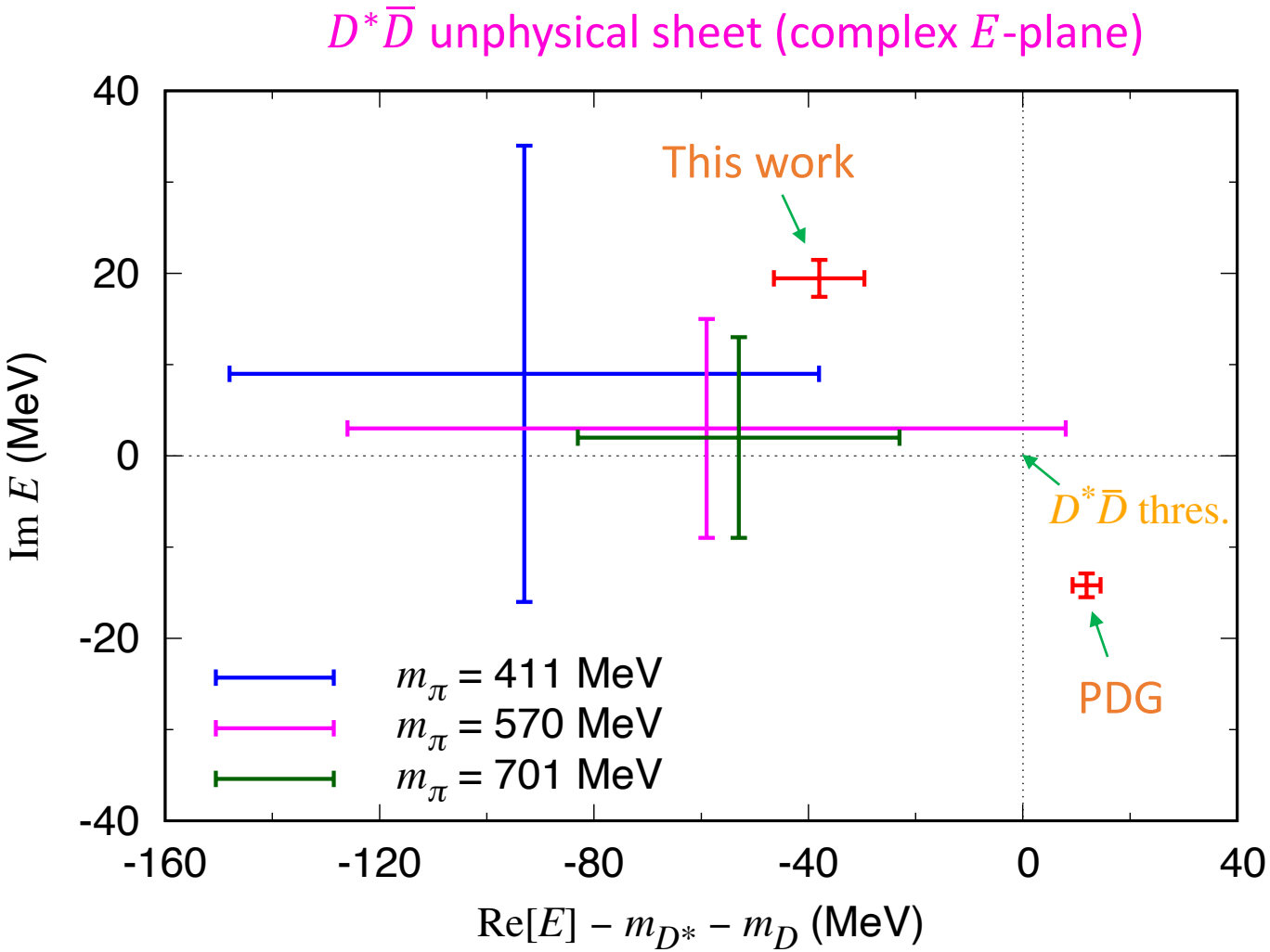
Zc poles

from $J^{PC} = 1^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ couple—channel amplitude

$E_{Z_c}^{\text{This work}}$	$M_{Z_c}^{\text{PDG}}$	$\Gamma_{Z_c}^{\text{PDG}}$	
$(3837.7 \pm 7.4) + (19.4 \pm 1.6)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9 \pm 5.6) + (26.1 \pm 4.3)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

$D^*\bar{D}$, $D^*\bar{D}^*$ virtual poles
(below thresholds)

Zc(3900) pole: comparison with LQCD result



LQCD ($m_\pi = 411$ MeV)
HAL QCD, J. Phys. G 45, 024002 (2018)

$m_{D^*} + m_D - (93 \pm 55 \pm 21) + (9 \pm 25 \pm 7)i$ MeV

$S(\{-k_i^*\}) = S^*(\{k_i\})$ applied; Yamada et al.
PRD 105, 014034 (2022)

This work

$m_{D^*} + m_D - (38 \pm 7.4) + (19 \pm 1.6)i$ MeV

PDG

$m_{D^*} + m_D + (11.9 \pm 2.6) - (14.2 \pm 1.3)i$ MeV

LQCD and this work are fairly consistent (virtual poles)

Zc(3900) pole: virtual or resonance

In literature, various models are used to describe Zc(3900)

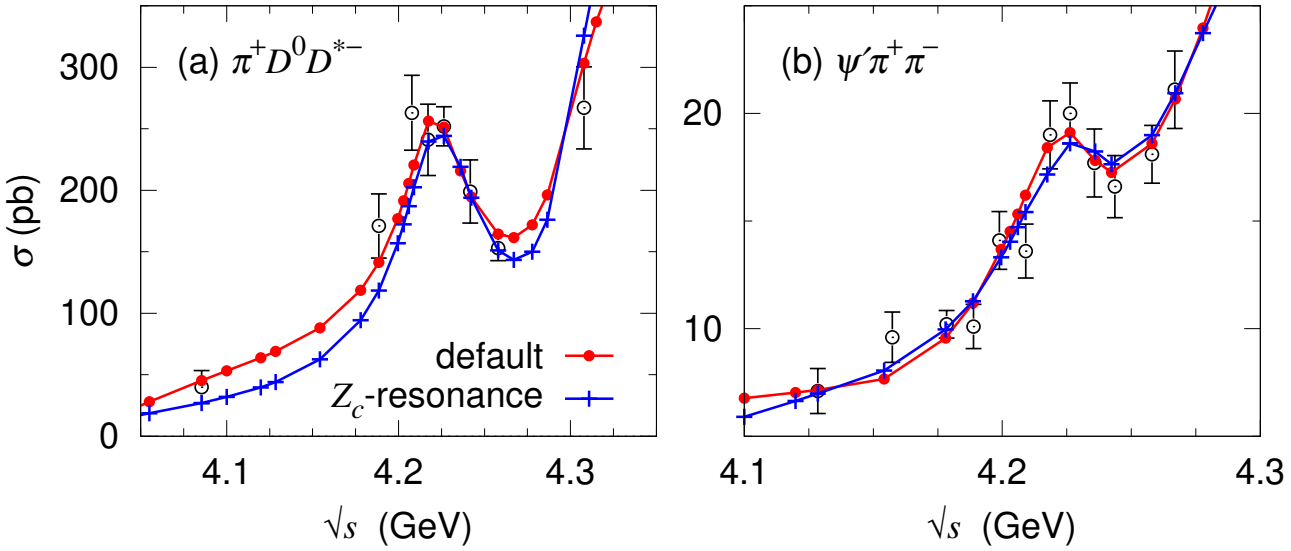
→ fit $J/\psi\pi$ and $D^*\overline{D}$ invariant mass distributions (clear Zc signals), NOT cross-section data

→ either virtual or resonance solutions How to discriminate ?

Searching Zc-resonance solution in global fit

energy-dependent $D^*\overline{D}^{(*)}$ interactions in default-fit model → refit all parameters for global fit → $\chi^2 \sim 2510$

→ not comparable to the default model with χ^2 = 2320 (Zc-virtual solution)



Suggestion:

Fitting cross section data may discriminate

Zc pole location

→ Testing Zc residue, Zc production mechanism