Three-body hadronic states of $KD_{s0}^{*}(2317)$ and $Kf_1(1285)$

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Based on: W.H. Jia, H.P. Li, WHL, J. Song, E. Oset, PRD 114 (2026) 1035042;

W.H. Jia, J. Song, WHL, E. Oset, EPJC 86 (2026) 761.

Outline

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Introduction

The $D_{s0}^{*}(2317)$ state

Experimental observation of $D_{s0}^{*}(2317)$

- First observed by BaBar Collaboration in 2003, in the $D_s^+\pi^0$ invariant mass distribution from B meson decays.
- Confirmed by CLEO, BaBar, Belle, …

Information of $D_{s0}^{*}(2317)$ from PDG

$$D_{s0}^{*}(2317): S=+1, \quad C=+1, \quad I(J^P)=0(0^+),$$

$$M_{D_{s0}^{*}(2317)} = 2317.8 \pm 0.5\,\text{MeV},$$

$$\Gamma_{D_{s0}^{*}(2317)} < 3.8\,\text{MeV}.$$

$$\frac{\mathcal{B}(D_{s0}^{*}(2317)^+ \to D_s^{*+}\gamma)}{\mathcal{B}(D_{s0}^{*}(2317)^+ \to D_s^+\pi^0)} = (7.14 \pm 0.70 \pm 0.23) \times 10^{-2}$$

Decay mode	BR
$D_s^+\pi^0$	$(100^{+0}_{-20})\%$
$D_s^+\gamma$	$< 5\%$
$D_s^*(2112)\gamma$	$< 6\%$
$D_s^+\gamma\gamma$	$< 18\%$
$D_s^*(2112)^+\pi^0$	$< 11\%$
$D_s^+\pi^+\pi^-$	$< 4 \times 10^{-3}$

- In quark model: $D_{s0}^*(2317)$ is a P-wave excited state of the $c\bar{s}$ system, corresponding to the 1^3P_0 state.

  The observed mass of $D_{s0}^*(2317)$ is at least 100 MeV lower than lower than the quark model predictions.

Regarding $D_{s0}^*(2317)$ as a $c\bar{s}$ state, it is difficult to explain the mass, narrow width and decay properties of this state.

↪ $D_{s0}^*(2317)$ is considered as a candidate of exotic states.

➢ $D_{s0}^*(2317)$ could be a DK bound state

$$M_{D^+K^0} = 2367.3\,\text{MeV}, \quad M_{D^0K^+} = 2358.5\,\text{MeV},$$

Kolomeitsev, Lutz, PLB582(2004)39;
Guo, Shen, Chiang, Ping, Zou, PLB641(2006)278;
Gamermann, Oset, Strottman, Vacas, PRD76(2007)074016;
Cleven, Grießhammer, Guo, Hanhart, Meißner, EPJA50(2014)149.

The $f_1(1285)$ state $I^G(J^{PC}) = 0^+(1^{++})$

- The internal structure of axial-vector mesons has long been a subject of debate.
- In chiral unitary approach (ChUA), $f_1(1285)$ is dynamically generated from pseudoscalar-vector interaction, being a $(K\bar{K}^*+K^*\bar{K})$ molecular state.

[11] L. Roca, E. Oset, and J. Singh, Low lying axial-vector mesons as dynamically generated resonances, Phys. Rev. D **72**, 014002 (2005), arXiv:hep-ph/0503273.

TABLE I. Classification of the axial vector resonances. The old $h_1(1380)$ has been updated in the PDG to the $h_1(1415)$.

J^{PC}	$I=1$	$I=0$
1^{+-}	$b_1(1235)$	$h_1(1170)$, $h_1(1380)$ $[h_1(1415)]$
1^{++}	$a_1(1260)$	$f_1(1285)$

$$f_1(1285) = \frac{1}{\sqrt{2}}\left[K\bar{K}^*(I=0) + \bar{K}K^*(I=0)\right].$$

TABLE II. Couplings g_i of the 1^+ resonances to the $K\bar{K}^*$ component, taken from Ref. [11].

Resonance	g_i [MeV]
$h_1(1415)$	$6147 + i\,183$
$f_1(1285)$	7230
$b_1(1235)$	$6172 - i\,75$
$a_1(1260)$	$1872 - i\,1486$

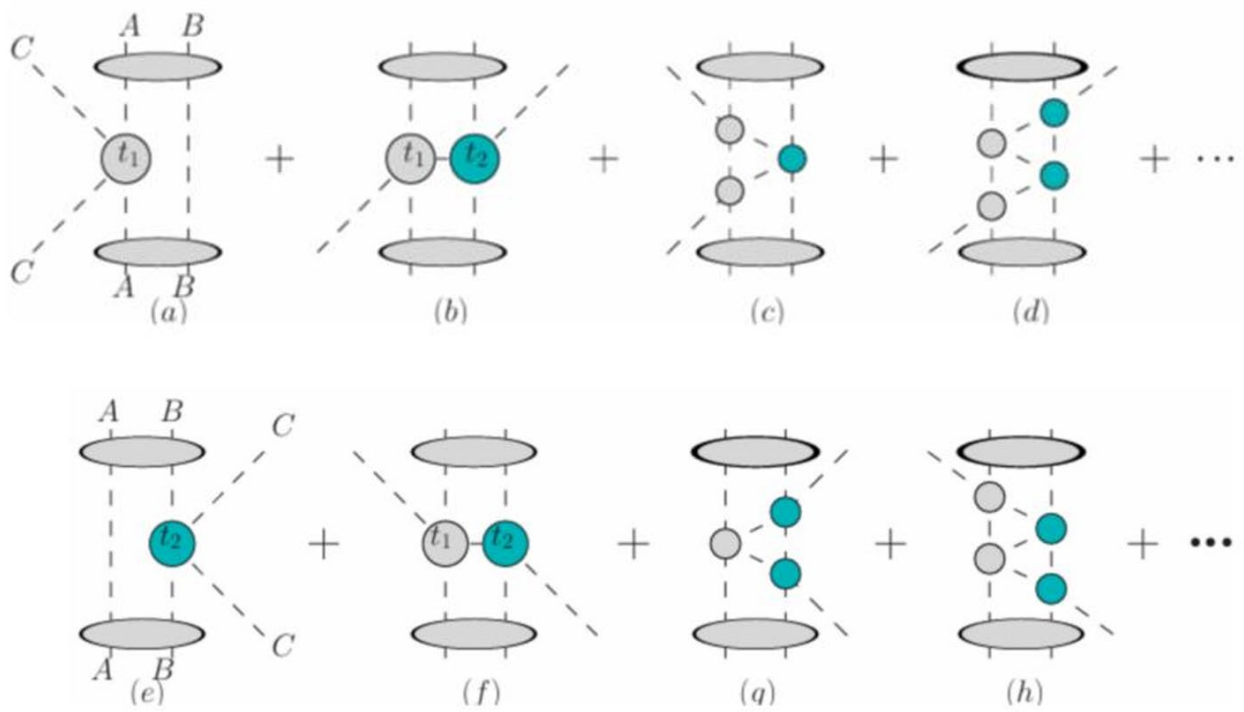
One can expect a $KD_{s0}^*(2317)$ (or KDK) bound state and a $Kf_1(1285)$ [or $K(K\bar{K}^* + K^*\bar{K})$] bound state.

Fixed Center Approximation (FCA) to Faddeev equation for three-body hadronic states

- A simplified theoretical method used in few-body scattering problems, particularly in hadron physics.
- Applied to 3-body systems where two of the particles form a tightly bound cluster, and the third particle scatters off this cluster.
- Key assumption: the two particles forming the cluster (the "fixed center") remain stable during the scattering process.
- Three-body problem

  ⬇ FCA

  two-body-like problem
- The interaction is described: summing the scattering amplitudes of particle C with A,B constituents of the cluster, taking into account the multiple scattering between them.



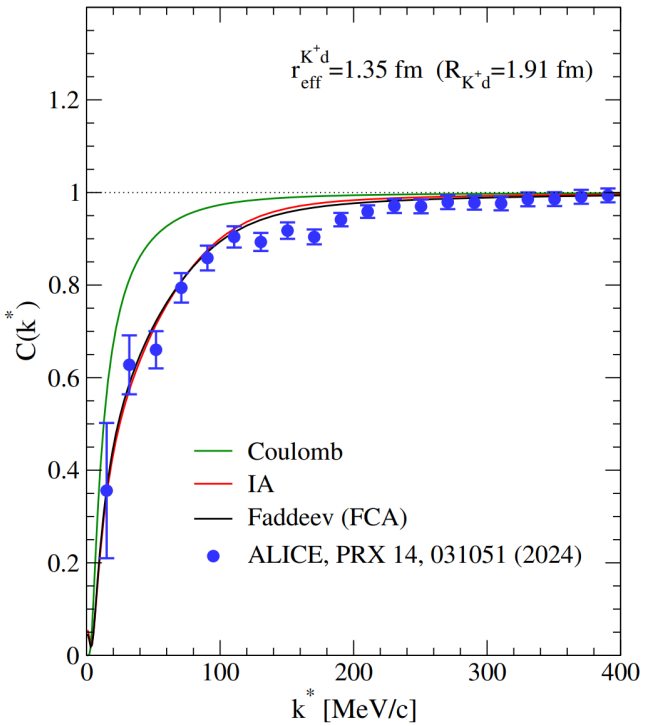
Representation of FCA

FCA has been applied to many three-body systems: *Kd*, *p*f1(1285), *K*f1(1285), *π*f1(1285), …….

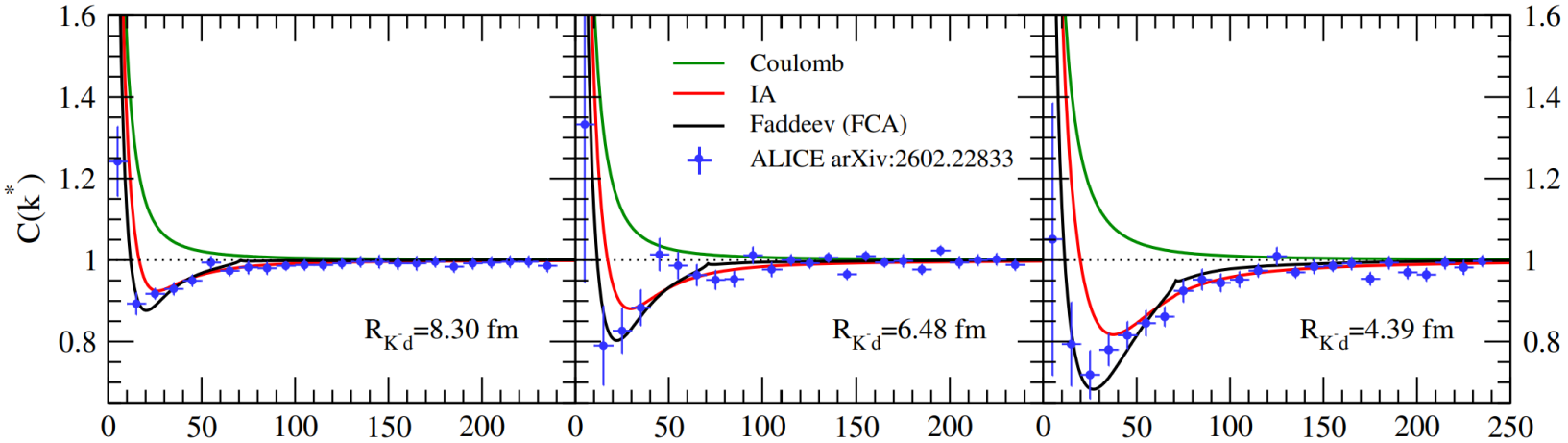
Alice collaboration got data for the correction functions of hadronic pair, providing good place to test FCA

- *Kd* correction functions

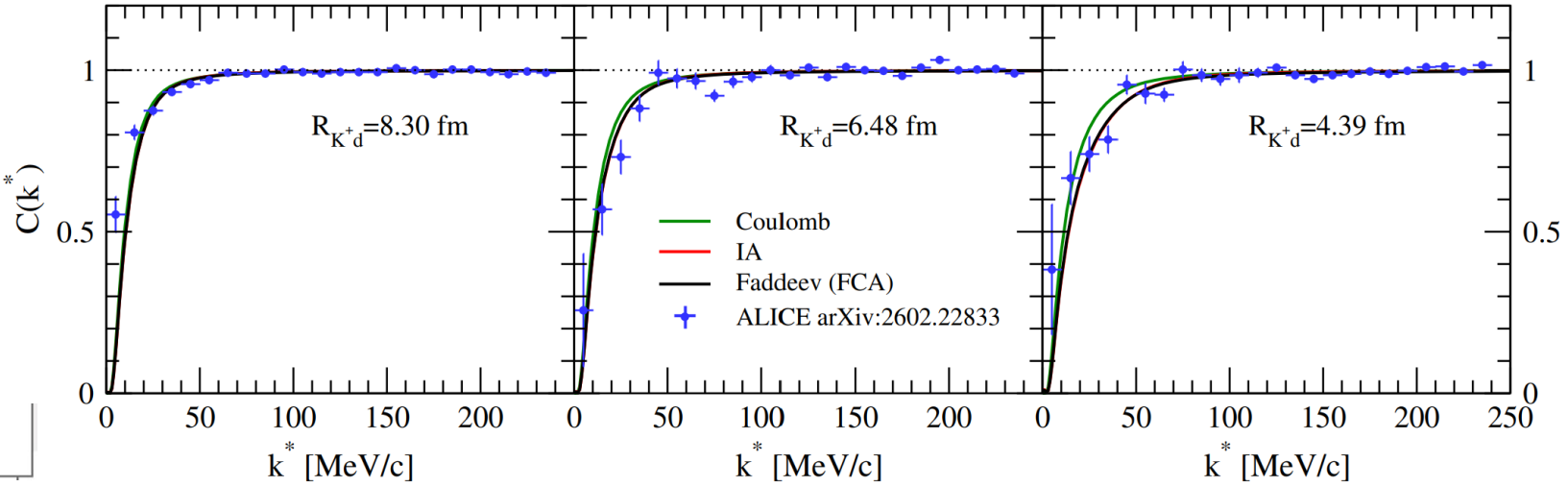
[A. Ramos, et al., PRD113(2026)3]



ALICE pp high mult. $\sqrt{s}$ = 13 TeV



ALICE Pb-Pb $\sqrt{s_{NN}}$ = 5.02 TeV



The FCA results show a better agreement with data

(IA = Impulse Approximation)

➢ $pf_1(1285)$ correction function

PHYSICAL REVIEW D **113,** L111502 (2026)

Letter

Scattering observables and correlation function for $p\,f_1(1285)$ revisited

P. Encarnación,[1] A. Feijoo,[2] and E. Oset[1,3]

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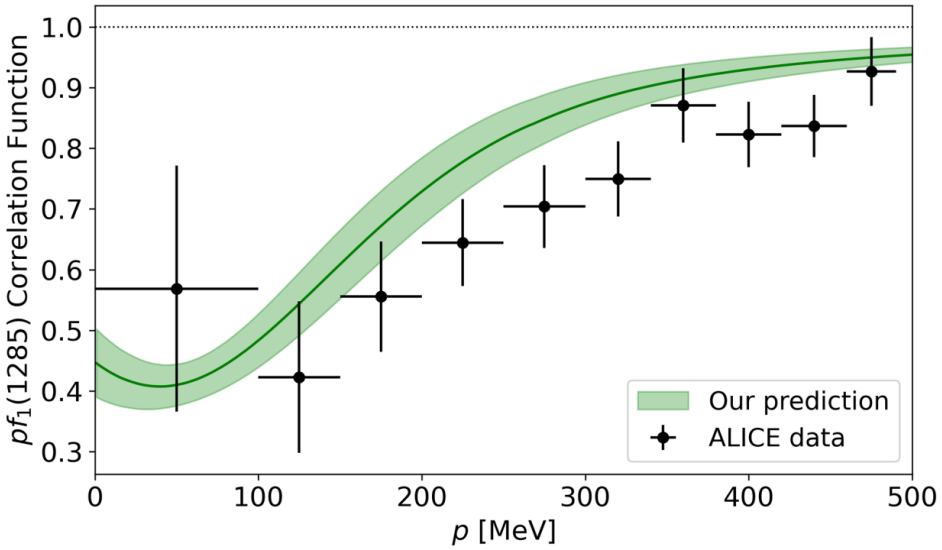


FIG. 5. $p\,f_1(1285)$ CF for $R = 0.75$ fm with the corresponding error band, compared with the ALICE experimental data taken from [54].

- FCA is very suitable for describing the interaction between a conventional hadron and a composite hadron (hadronic molecule).

◆ Three-body hadronic state of $KD_{s0}^{*}(2317)$

☐ $D_{s0}^{*}(2317)$ as a DK molecular state in ChUA

➢ Chiral Unitary Approach (ChUA)

- Use the interaction kernel derived from the chiral Lagrangian (e.g., from leading-order or next-to-leading order ChPT) as input for a non-perturbative scattering equation that enforces exact unitarity.

- Coupled-channel **Bethe-Salpeter (BS) equation** for the scattering amplitude T:

$$T = [1 - V \cdot G]^{-1} V$$

- ✓ V: the interaction kernel derived from the chiral Lagrangian;
- ✓ G: loop function representing the propagation of intermediate states

For meson-meson loop,

$$G(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2 + i\epsilon)\left[(P-q)^2 - m_2^2 + i\epsilon\right]},$$

in a three-momentum cutoff renormalization scheme,

$$G(s) = \frac{1}{4\pi^2} \int_0^{q_{\max}} d|\vec{q}\,|\,\vec{q}^{\,2} \frac{\omega_1 + \omega_2}{\omega_1\omega_2\left[s - (\omega_1+\omega_2)^2 + i\epsilon\right]}. \qquad \omega_1 = \sqrt{\vec{q}^{\,2} + m_1^2}, \qquad \omega_2 = \sqrt{\vec{q}^{\,2} + m_2^2},$$

The cutoff q_{max} is the only free parameter in our framework.

➢ Dynamical generation of resonances

- Instead of being introduced as fundamental fields in the Lagrangian, resonances appear as **poles** of the scattering amplitude in the complex energy plane.

- The resonances are interpreted as having a strong **molecular** or **composite** component, generating from the interactions of the hadrons in the coupled channels. (dynamically generated states)

- Poles of the scattering amplitude T:

$$\det(1 - VG) = 0 \quad \Rightarrow \quad \textbf{pole:} \quad \sqrt{s_R} = M_R - i\frac{\Gamma_R}{2},$$

M_R and Γ_R are mass and width of the resonace.

- Couplings of the dynamically generated states to coupled channels:

For meson-meson interaction,

$$g_i^2 = \lim_{s \to s_R} (s - s_R) T_{ii}, \qquad g_i g_j = \lim_{s \to s_R} (s - s_R) T_{ij}.$$

➢ $D_{s0}^*(2317)$ as a DK molecular state

$$D_{s0}^*(2317): S=+1, \quad C=+1, \quad I(J^P)=0(0^+),$$

- Coupled channels with same quantum numbers of $D_{s0}^*(2317)$:

$$D^0K^+(1), \quad D^+K^0(2), \quad D_s^+\eta(3), \quad D_s^+\pi^0(4),$$

- Chiral Lagrangian for the interation between two pseudoscalars:

$$\mathcal{L}_{VPP} = -ig\,\langle[\mathcal{P},\partial_\mu\mathcal{P}]\,\mathcal{V}^\mu\rangle,$$

where $g = \frac{M_V}{2f_\pi}$ with $M_V \approx 800$ MeV and $f_\pi = 93$ MeV.

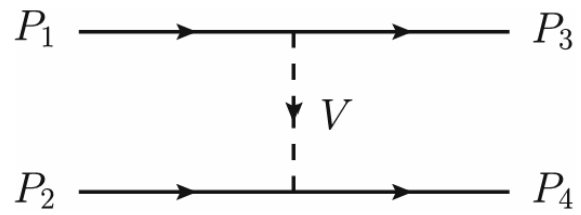


Fig. 1 Diagrammatic representation of vector exchange as a source of the interaction between two pseudoscalars

$$\mathcal{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{3}}\eta+\frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{3}}\eta+\frac{1}{\sqrt{6}}\eta' & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta+\sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix} \quad \mathcal{V} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0+\frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0+\frac{1}{\sqrt{2}}\omega & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}.$$

- Transition potential V_{ij} between the coupled channels:

$$
\begin{aligned}
&V_{11} = C_{11} f(p_i), && V_{12} = V_{21} = C_{12} f(p_i),\\
&V_{22} = C_{22} f(p_i), && V_{13} = V_{31} = C_{13} f(p_i) + C'_{13} g(p_i),\\
&V_{23} = V_{32} = V_{13}, && V_{14} = V_{41} = C_{14} f(p_i) + C'_{14} g(p_i),\\
&V_{24} = V_{42} = -V_{14}, && V_{34} = V_{43} = 0,\\
&V_{33} = 0, && V_{44} = 0,
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
C_{11} &= -\frac{1}{2}\left(\frac{1}{m_\rho^2} + \frac{1}{m_\omega^2}\right), & C_{12} &= -\frac{1}{m_\rho^2},\\
C_{13} &= \frac{2}{\sqrt{3}}\frac{1}{m_{K^*}^2}, & C_{22} &= -\frac{1}{2}\left(\frac{1}{m_\rho^2} + \frac{1}{m_\omega^2}\right)\\
C_{23} &= \frac{2}{\sqrt{3}}\frac{1}{m_{K^*}^2}, & C_{14} &= \frac{1}{\sqrt{2}}\frac{1}{m_{K^*}^2},\\
C'_{13} &= \frac{1}{\sqrt{3}}\frac{1}{m_{D^*}^2}, & C'_{14} &= \frac{1}{\sqrt{2}}\frac{1}{m_{D^*}^2}.
\end{aligned}
$$

$$
f(p_i) = g^2\ (p_1 + p_3)\cdot(p_2 + p_4),
$$

$$
g(p_i) = g^2\ (p_1 + p_4)\cdot(p_2 + p_3),
$$

We consider the *DK* interaction in S-wave, and have

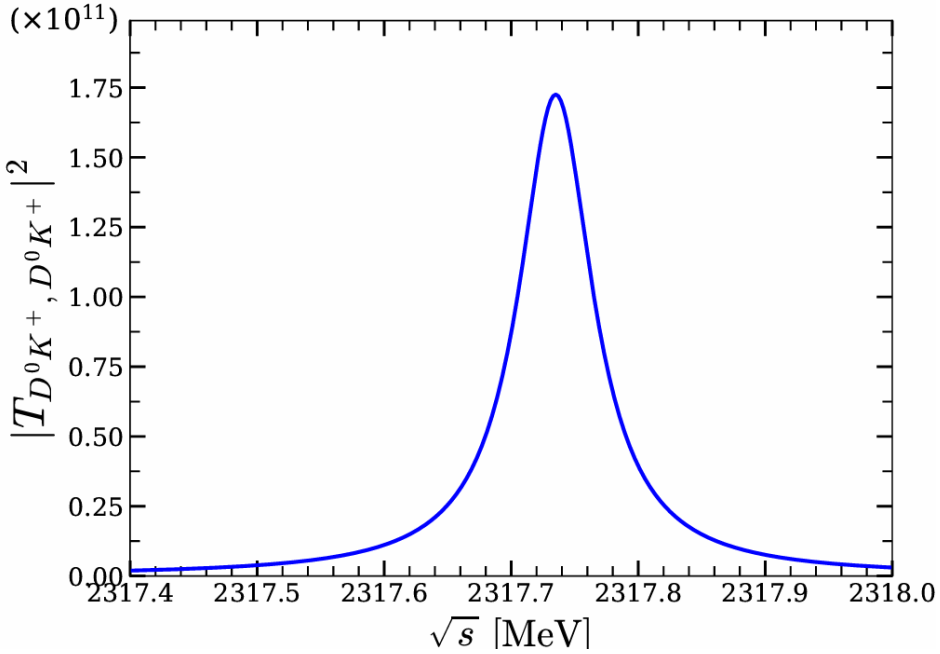
$$
\begin{aligned}
(p_1 + p_3)\cdot(p_2 + p_4) \to \frac{1}{2}\big[3s - (m_1^2 + m_2^2 + m_3^2 + m_4^2)\\
- \frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2)\big],\\
(p_1 + p_4)\cdot(p_2 + p_3) \to \frac{1}{2}\big[3s - (m_1^2 + m_2^2 + m_3^2 + m_4^2)\\
+ \frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2)\big],
\end{aligned}
$$

- Scattering amplitude, pole and coupling constants

$$T = [1 - V \cdot G]^{-1} V \qquad \det(1 - VG) = 0$$

$$\sqrt{s_R} = (2317.74 + 0.03537i)\ \text{MeV}.$$

$$\Gamma^{(\text{pole})}_{D^*_{s0}(2317)} = 70.75\ \text{keV}.$$



Pole position and couplings of $D^*_{s0}(2317)$ (with qmax=687 MeV)

$\sqrt{s_R}$	$g_{D^0K^+}$	$g_{D^+K^0}$	$g_{D_s^+\eta}$	$g_{D_s^+\pi^0}$
$(2317.74 + 0.035i)$	$8252.26 - 69.15i$	$8129.49 + 75.70i$	$-6312.66 - 2.50i$	$-174.59 + 39.95i$

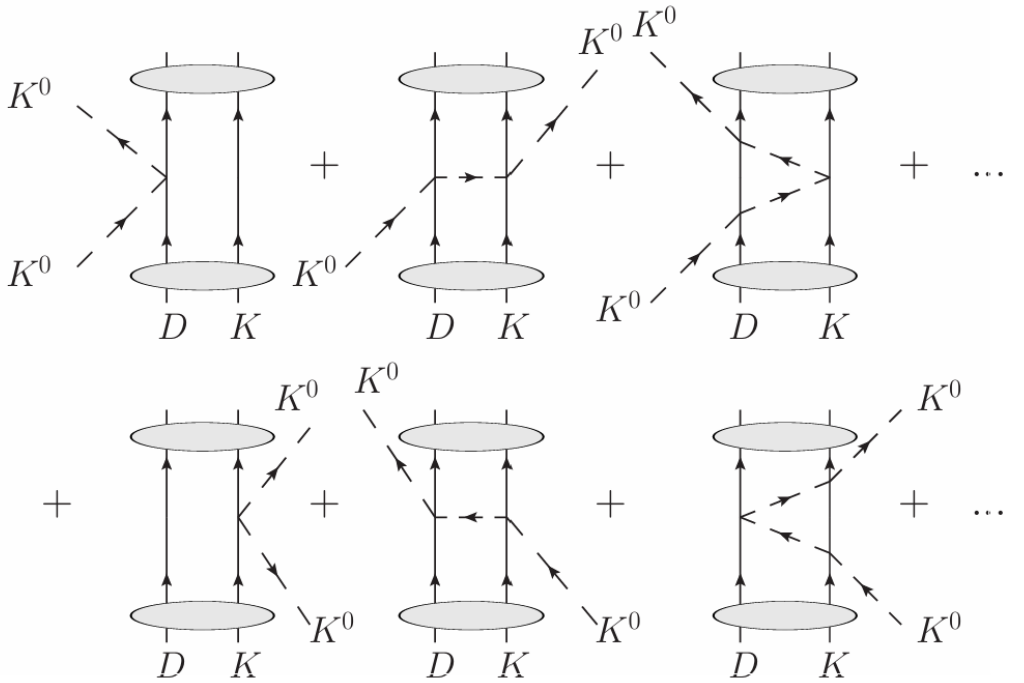
We tested the DK molecular picture of $D^*_{s0}(2317)$ in B decays and the radiative decay of $D^*_{s0}(2317)$, and obtained a good agreement with new data, supporting the DK molecular picture.

[47] W.-H. Liang, Z.-R. Hu, and E. Oset, The $B^{+(0)} \to \bar{D}^{0(-)} D^*_{s0}(2317)^+$ decays and the molecular structure of $D^*_{s0}(2317)$, Eur. Phys. J. A **62**, 104 (2026), arXiv:2512.22604 [hep-ph].

[73] P.-S. Su, W.-T. Lyu, W.-H. Liang, and E. Oset, The $D^*_{s0}(2317)^+$ decay to $D_s^+\pi^0$ and $D_s^{*+}\gamma$, Eur. Phys. J. C **86**, 242 (2026), arXiv:2512.07421 [hep-ph].

FCA to $KD_{s0}^{*}(2317)$

$$t_1 = \frac{3}{4}t_{DK}^{I=1} + \frac{1}{4}t_{DK}^{I=0}, \qquad t_2 = \frac{3}{4}t_{KK}^{I=1} + \frac{1}{4}t_{KK}^{I=0}.$$



Two-body amplitudes:

$$t_{DK}^{I=0} = \frac{1}{2}\left(t_{D^+K^0,D^+K^0} + 2\,t_{D^+K^0,D^0K^+} + t_{D^0K^+,D^0K^+}\right),$$

$$t_{DK}^{I=1} = 0, \qquad t_{KK}^{I=0} = 0,$$

$$t_{KK}^{I=1} = \frac{V_{KK}^{I=1}}{1 - \frac{1}{2}V_{KK}^{I=1}G}, \qquad V_{KK}^{I=1} = \frac{4g^2}{M_V^2}\left(\frac{3}{2}s - 2M_K^2\right),$$

The external K^0 can interact with the D and the K of the $D_{s0}^{*}(2317)$ cluster, leading to a three-body bound state of DKK nature.

For reasons of normalization, by referring the final amplitudes to the interaction of K^0 with the cluster as a whole of mass M_C, we define the amplitudes

$$\tilde{t}_1 = \frac{M_C}{m_D}t_1, \qquad \tilde{t}_2 = \frac{M_C}{m_K}t_2.$$

with arguments:

$$s_1(DK) = (p_{K^0} + p_D)^2 = m_K^2 + (\xi m_D)^2 + 2\xi m_D q_0,$$

$$s_2(KK) = (p_{K^0} + p_K)^2 = m_K^2 + (\xi m_K)^2 + 2\xi m_K q_0,$$

$$q_0 = \frac{s - m_{K^0}^2 - M_C^2}{2M_C}, \qquad \xi = \frac{M_C}{m_D + m_K},$$

It is useful to separate the terms $\tilde{T}_{ij}$ which include all diagrams where the K^0 interacts first with particle i of the cluster and finishes with particle j,

$$\tilde{T}_{11} = \frac{\tilde{t}_1}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, \qquad \tilde{T}_{22} = \frac{\tilde{t}_2}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, \qquad \tilde{T}_{12} = \tilde{T}_{21} = \frac{\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, \quad \Rightarrow \quad \tilde{T} = \begin{pmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \tilde{T}_{21} & \tilde{T}_{22} \end{pmatrix}$$

G_0 is the K^0 propagator from one particle to the other in the cluster,

$$G_0(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_K(\boldsymbol{q})} \frac{1}{2\omega_C(\boldsymbol{q})} \times \frac{F_C(\boldsymbol{q})}{\sqrt{s} - \omega_K(\boldsymbol{q}) - \omega_C(\boldsymbol{q}) + i\epsilon} \times \Theta(q_{\max}^{(1)} - q_1^*)\Theta(q_{\max}^{(2)} - q_2^*),$$

with $F_C(\boldsymbol{q})$ the form factor of the cluster,

$$F_C(\boldsymbol{q}) = \frac{F(\boldsymbol{q})}{\mathcal{N}}, \qquad F(\boldsymbol{q}) = \int_{\substack{|\boldsymbol{p}|<q_{\max}, \\ |\boldsymbol{p}-\boldsymbol{q}|<q_{\max}}} \frac{d^3p}{(2\pi)^3} \frac{1}{M_C - \omega_D(\boldsymbol{p}) - \omega_K(\boldsymbol{p})} \frac{1}{M_C - \omega_D(\boldsymbol{p}-\boldsymbol{q}) - \omega_K(\boldsymbol{p}-\boldsymbol{q})}, \qquad \mathcal{N} = F(0)$$

$q_{max}^{(1)}$ and $q_{max}^{(2)}$ refer to the cutoffs corresponding to DK and KK amplitudes;
q_1^* and q_2^* are the momenta in the rest frame of the propagating K^0 and the respective particle 1, 2 of the cluster.

$$\boldsymbol{q}_i^* = \boldsymbol{q}\left(1 - \frac{1}{2}\frac{m_{K^0}}{m_{K^0} + m_i}\right).$$

➢ Improved FCA: to consider the coherent $KD_{s0}^{*}(2317)$ propagation

Implement the coherent propagation of the $\tilde{T}_{ij}$ terms, omitting the terms $\tilde{T}_{ij}G_c\tilde{T}_{k\ell}$ with $j \neq k$ because this involves K^0 going from particle 1 to 2 (or 2 to 1) in the cluster, which is already accounted for by the $\tilde{t}_1\tilde{t}_2G_0$ terms of $\tilde{T}$.

The sum of the terms of Fig. 2 is given by

$$\tilde{T}' = [1 - \tilde{T}G_C]^{-1}\tilde{T}, \quad G_C = \begin{pmatrix} G_C^{(1)} & 0 \\ 0 & G_C^{(2)} \end{pmatrix},$$

$$G_C^{(i)}(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_K(\boldsymbol{q})} \frac{1}{2\omega_C(\boldsymbol{q})} \times \frac{[F_C^{(i)}(\boldsymbol{q})]^2}{\sqrt{s} - \omega_K(\boldsymbol{q}) - \omega_C(\boldsymbol{q}) + i\epsilon} \Theta(q_{\max}^{(i)} - q_i^*),$$

$$F_C^{(1)}(\boldsymbol{q}) = F_C\left(\frac{m_K}{m_D + m_K}\boldsymbol{q}\right), \quad F_C^{(2)}(\boldsymbol{q}) = F_C\left(\frac{m_D}{m_D + m_K}\boldsymbol{q}\right).$$

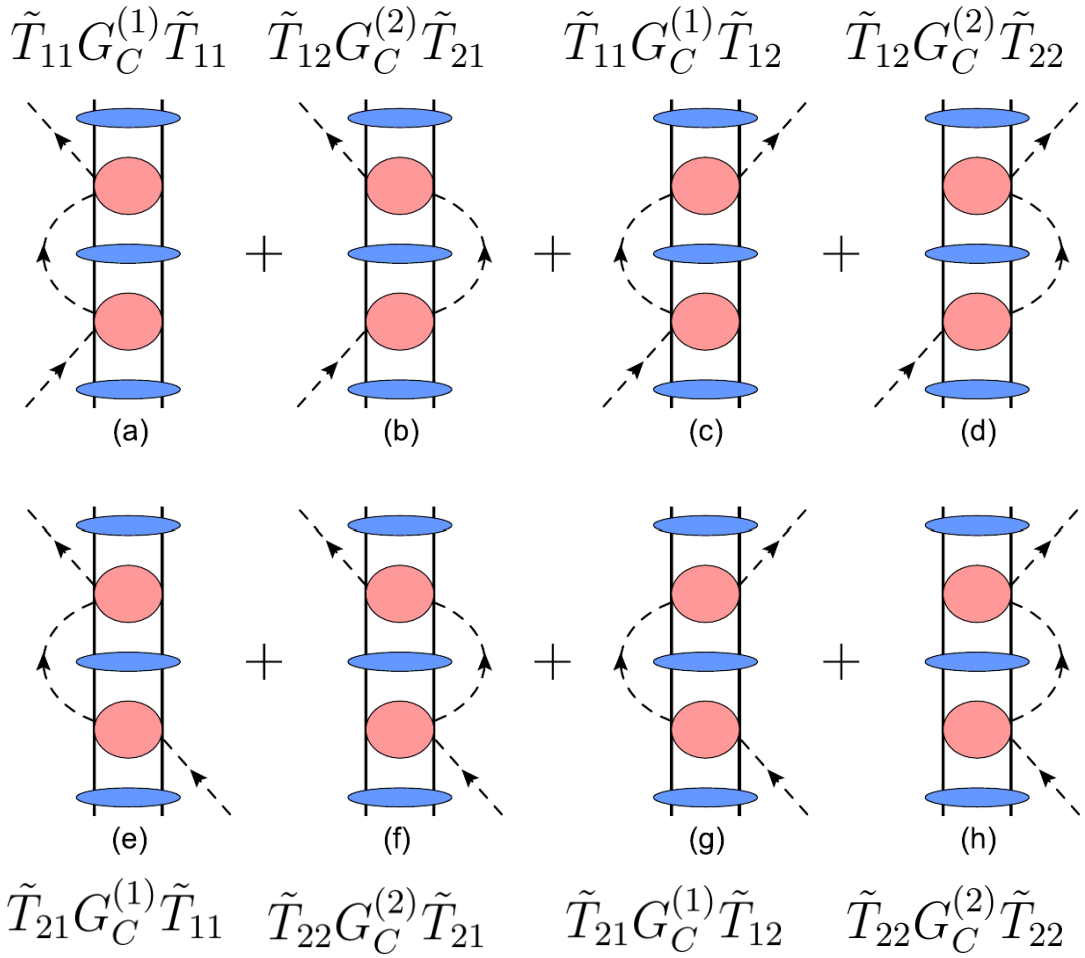


Fig. 2. Diagrams showing the elastic propagation of the external particle and the cluster $D_{s0}^{*}(2317)$ as a whole.

The final amplitude is then given by

$$T^{\mathrm{tot}}(\sqrt{s}) = \sum_{i,j} \tilde{T}'_{ij}$$

$$= \frac{\tilde{t}_1 + \tilde{t}_2 + (2G_0 - G_C^{(1)} - G_C^{(2)})\tilde{t}_1\tilde{t}_2}{1 - G_C^{(1)}\tilde{t}_1 - G_C^{(2)}\tilde{t}_2 - (G_0^2 - G_C^{(1)}G_C^{(2)})\tilde{t}_1\tilde{t}_2}\,.$$

➢ Results for observables

- The total scattering amplitude

✓ The absolute value of T^{tot} shows a neat and narrow peak, around 40 MeV below the $KD_{s0}^{*}(2317)$ threshold.

✓ From | Im T | at half height of its peak, we find the width of the state to be around 200 keV.

✓ The source of the $KD_{s0}^{*}(2317)$ bound state stems from the DK attraction.

✓ We have checked the effect of the KK repulsion, which moves the position of the three-body bound state 6 MeV closer to the $KD_{s0}^{*}(2317)$ threshold.

✓ Since the $D_{s0}^{*}(2317)$ decays into $D_s^{+}\pi^0$, the $KD_{s0}^{*}(2317)$ bound state that we predicted should be searched for in the invariant mass distributions of the $K^0 D_s^{+}\pi^0$ system.

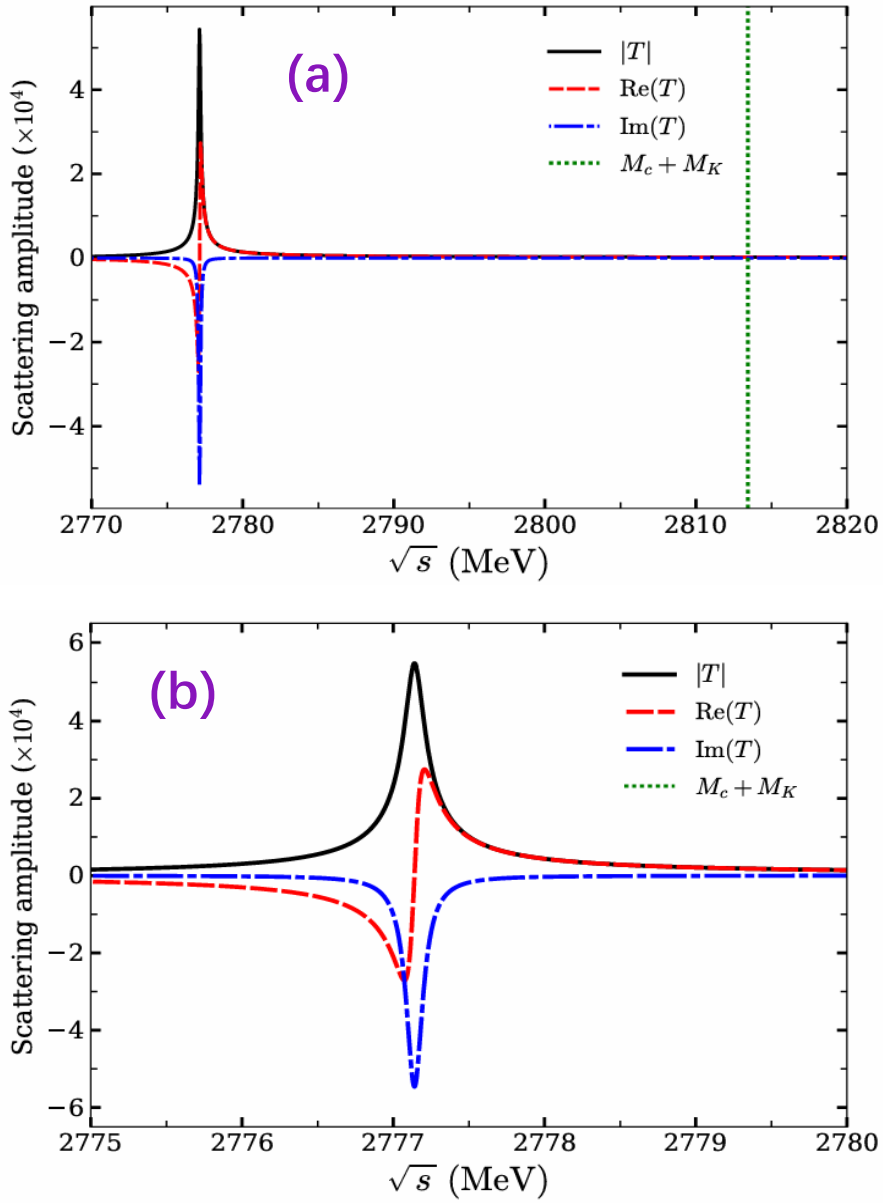


FIG. 3. Results for $T^{\text{tot}}(\sqrt{s})$ as a function of $\sqrt{s}$: (a) relative to the $KD_{s0}^{*}(2317)$ threshold, (b) blow up in a narrow energy window.

- Scattering length and effective range

$$a = \left. \frac{T^{\rm tot}}{8\pi\sqrt{s}} \right|_{\rm th},$$
$$r_0 = \frac{1}{\mu}\left[\frac{\partial}{\partial\sqrt{s}}\left(-8\pi\sqrt{s}(T^{\rm tot})^{-1}+iq_{\rm cm}\right)\right]_{\rm th},$$

with μ the $K^0 D_{s0}^*(2317)$ reduced mass.

$$a = (0.517 - 0.00016\,i)\,\text{fm},$$
$$r_0 = (-0.511 + 0.102\,i)\,\text{fm}.$$

✓ The small imaginary part of a reflects the small imaginary part of the DK amplitude arising from the small width of the $D_{s0}^*(2317)$.

- Femtoscopic correlation function (CF)

the zeroth order spherical Bessel function

$$C_{KD^*_{s0}}(p) = 1 + 4\pi \int_0^\infty dr\, r^2 S_{12}(r) \times \{|j_0(pr) + TG|^2 - j_0^2(pr)\},$$

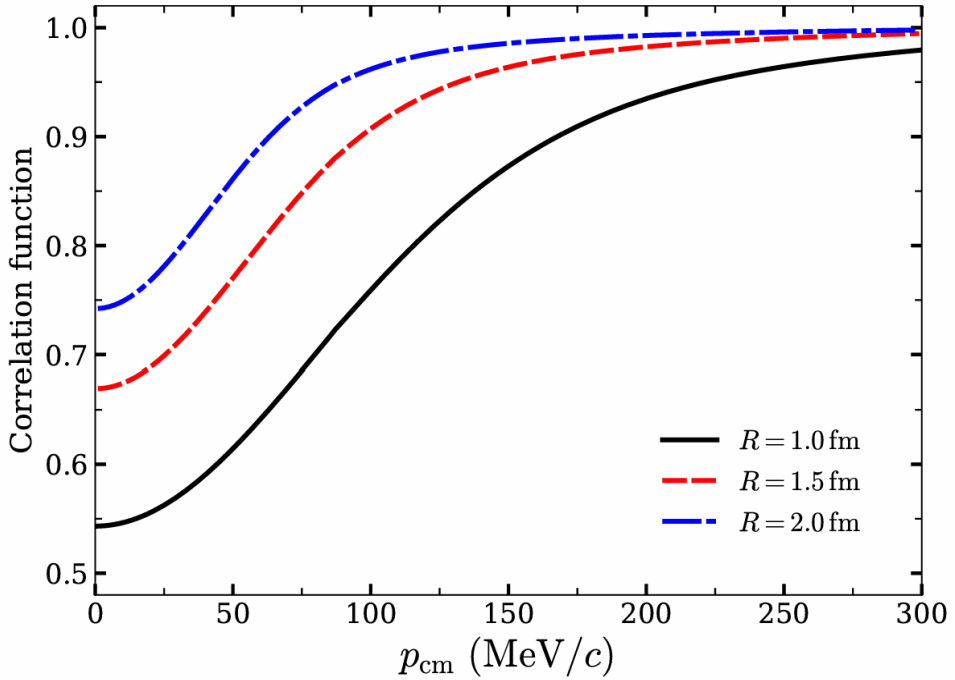
source function:

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp(-r^2/4R^2),$$ (in Gaussian with radius R)

R is determined in the experiment, or can also be obtained from a fit to the correlation function data.

$$TG = (\tilde{T}'_{11} + \tilde{T}'_{21}) G_1(\sqrt{s}, r) + (\tilde{T}'_{12} + \tilde{T}'_{22}) G_2(\sqrt{s}, r)$$

$$G_i(\sqrt{s}, r) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_K(\boldsymbol{q})} \frac{1}{2\omega_C(\boldsymbol{q})} \Theta(q^{(i)}_{\max} - q^*_i) \times \frac{j_0(qr) F^{(i)}_C(\boldsymbol{q})}{\sqrt{s} - \omega_K(\boldsymbol{q}) - \omega_C(\boldsymbol{q}) + i\epsilon}.$$



The CFs have a typical feature of deeply bound states.

◆ Three-body hadronic state of $Kf_1(2317)$

☐ $f_1(1285)$ as a $(K\bar{K}^*+K^*\bar{K})$ molecular state in ChUA

TABLE I. Classification of the axial vector resonances. The old $h_1(1380)$ has been updated in the PDG to the $h_1(1415)$.

J^{PC}	$I=1$	$I=0$
1^{+-}	$b_1(1235)$	$h_1(1170)$, $h_1(1380)$ $[h_1(1415)]$
1^{++}	$a_1(1260)$	$f_1(1285)$

$$f_1(1285) = \frac{1}{\sqrt{2}}\left[K\bar{K}^*(I=0) + \bar{K}K^*(I=0)\right].$$

[11] L. Roca, E. Oset, and J. Singh, Low lying axial-vector mesons as dynamically generated resonances, Phys. Rev. D **72**, 014002 (2005), arXiv:hep-ph/0503273.

TABLE II. Couplings g_i of the 1^+ resonances to the $K\bar{K}^*$ component, taken from Ref. [11].

Resonance	g_i [MeV]
$h_1(1415)$	$6147 + i\,183$
$f_1(1285)$	7230
$b_1(1235)$	$6172 - i\,75$
$a_1(1260)$	$1872 - i\,1486$

FCA to $Kf_1(2317)$

$$f_1(1285) \equiv \frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\left(K^+K^{*-}+K^0\bar{K}^{*0}\right)\right.$$
$$\left.+\frac{1}{\sqrt{2}}\left(K^-K^{*+}+\bar{K}^0K^{*0}\right)\right]$$
$$=\frac{1}{\sqrt{2}}\left[K\bar{K}^*(I=0)+\bar{K}K^*(I=0)\right].$$

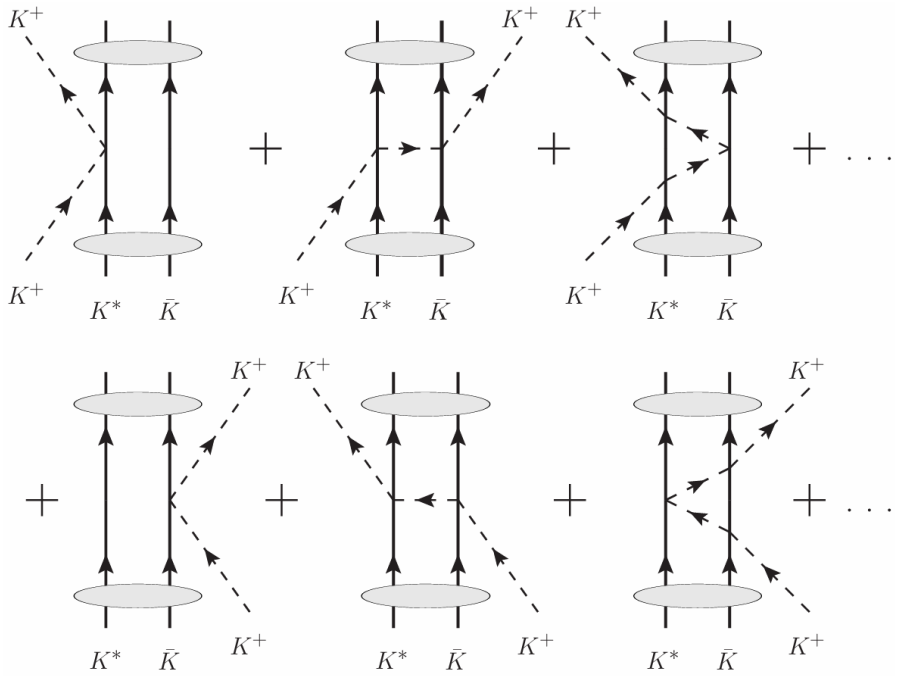
$$T^{\rm tot}_{K^*\bar{K}}=\frac{\tilde{t}_1+\tilde{t}_2+\left(2\,G_0-G_c^{(1)}-G_c^{(2)}\right)\tilde{t}_1\,\tilde{t}_2}{1-G_c^{(1)}\,\tilde{t}_1-G_c^{(2)}\,\tilde{t}_2-\left(G_0^2-G_c^{(1)}\,G_c^{(2)}\right)\tilde{t}_1\,\tilde{t}_2}.$$

$$T^{\rm tot}=\frac{1}{2}\left(T^{\rm tot}_{K^*\bar{K}}+T^{\rm tot}_{\bar{K}^*K}\right)$$

$$\tilde{t}_1=\frac{M_c}{M_{K^*}}\,t_1\ ,\qquad \tilde{t}_2=\frac{M_c}{M_K}\,t_2\ ,$$

$$t_1=\frac{3}{4}\,t^{I=1}_{KK^*}+\frac{1}{4}\,t^{I=0}_{KK^*},$$

$$t_2=\frac{3}{4}\,t^{I=1}_{K\bar{K}}+\frac{1}{4}\,t^{I=0}_{K\bar{K}}.$$



K^0 interacting with the cluster of $K^*\overline{K}$.

- Two-body amplitudes

1. The KK interaction

$$T = \frac{V}{1 - \frac{1}{2}VG}. \qquad V_{KK}^{I=1} = \frac{4g^2}{M_V^2}\left(\frac{3}{2}s - 2M_K^2\right). \qquad V_{KK}^{I=0} = 0.$$

2. The KK^* interaction

$$\mathcal{L}_{VVV} = ig\,\langle\,(V^\mu \partial_\nu V_\mu - \partial_\nu V^\mu V_\mu)\,V^\nu\,\rangle.$$

$$V_{KK^*}^{I=0} = 0, \qquad V_{KK^*}^{I=1} = \frac{2g^2}{M_V^2}\left[\frac{3}{2}s - (M_K^2 + M_{K^*}^2) - \frac{1}{2s}(M_{K^*}^2 - M_K^2)\right],$$

3. The $K\bar{K}$ amplitudes

$$\langle K\bar{K},\, I=0\,|\,t\,|\,K\bar{K},\, I=0\rangle = \frac{1}{2}\left(t_{K^+K^-,\,K^+K^-} + 2\,t_{K^+K^-,\,K^0\bar{K}^0} + t_{K^0\bar{K}^0,\,K^0\bar{K}^0}\right),$$

$$\langle K\bar{K},\, I=1|\,t\,|K\bar{K},\, I=1\rangle = \frac{1}{2}\left(t_{K^+K^-,\,K^+K^-} - 2\,t_{K^+K^-,\,K^0\bar{K}^0} + t_{K^0\bar{K}^0,\,K^0\bar{K}^0}\right).$$

4. The $K\bar{K}^*$ amplitudes

1) $I=0$, $C=+$, $f_1(1285)$

$$\begin{aligned} f_1(1285) \equiv & \frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\left(K^+K^{*-}+K^0\bar{K}^{*0}\right)\right. \\ & \left.+\frac{1}{\sqrt{2}}\left(K^-K^{*+}+\bar{K}^0K^{*0}\right)\right] \\ = & \frac{1}{\sqrt{2}}\left[K\bar{K}^*(I=0)+\bar{K}K^*(I=0)\right]. \end{aligned}$$

2) $I=1$, $C=+$, $a_1(1260)$

$$\begin{aligned} a_1(1260) \equiv & \frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}}\left(K^+K^{*-}-K^0\bar{K}^{*0}\right)\right. \\ & \left.+\frac{1}{\sqrt{2}}\left(K^-K^{*+}-\bar{K}^0K^{*0}\right)\right] \\ = & \frac{1}{\sqrt{2}}\left[K\bar{K}^*(I=1)-\bar{K}K^*(I=1)\right]. \end{aligned}$$

3) $I=0$, $C=-$, $h_1(1170)$, $h_1(1415)$

$$h_1 \equiv \frac{1}{\sqrt{2}}\Big[K\bar{K}^*(I=0)-\bar{K}K^*(I=0)\Big],$$

4) $I=1$, $C=-$, $b_1(1235)$

$$b_1(1235) = \frac{1}{\sqrt{2}}\Big[K\bar{K}^*(I=1)+\bar{K}K^*(I=1)\Big],$$

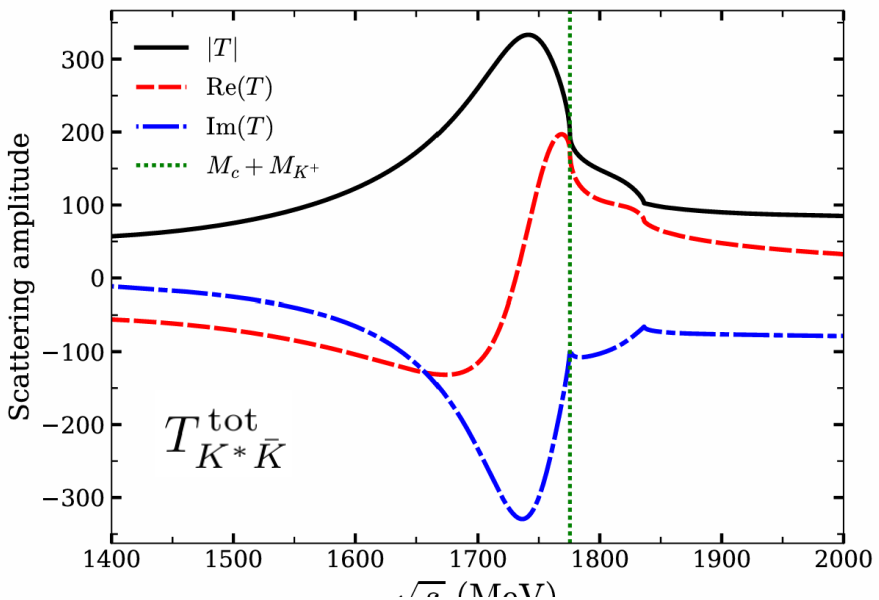
$$\begin{aligned} & \langle f_1|\,t\,|f_1\rangle + \langle h_1|\,t\,|h_1\rangle \\ =& \langle K\bar{K}^*,\, I=0|\,t\,|K\bar{K}^*,\, I=0\rangle \\ & + \langle \bar{K}K^*,\, I=0|\,t\,|\bar{K}K^*,\, I=0\rangle \\ =& 2\langle K\bar{K}^*,\, I=0|\,t\,|K\bar{K}^*,\, I=0\rangle, \end{aligned}$$

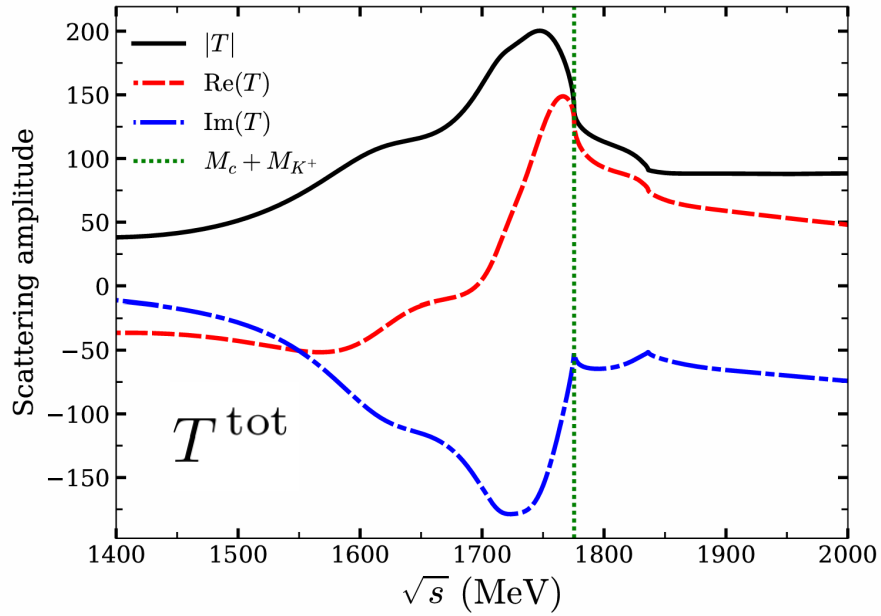
$$\langle a_1|\,t\,|a_1\rangle + \langle b_1|\,t\,|b_1\rangle = 2\left\langle K\bar{K}^*,\, I=1\right|t\left|K\bar{K}^*,\, I=1\right\rangle.$$

$$\langle R_i|\,t\,|R_i\rangle = \frac{g_i^2}{s-M_{R_i}^2+iM_{R_i}\Gamma_{R_i}},$$

➢ Results for observables

- The total scattering amplitude





✓ The amplitude has the approximate shape of a resonance, where the absolute value of the imaginary part has a peak and the real part goes through zero at the peak of | Im $T^{\rm tot}_{K^*\bar{K}}$ |.

✓ This resonant-like structure below threshold, corresponds to a binding with respect to $M_{K^0} + M_c$ of about 10 MeV and a width of around 15 MeV. The structure is clear and stable and deserves some experimental search.

✓ The structure is clear and stable and deserves some experimental search.

- Scattering length and effective range

$$a = (0.57 - i\,0.22)\ \text{fm},$$

$$r_0 = (-0.84 + i\,0.24)\ \text{fm}.$$

- Femtoscopic correlation function (CF)

$$C_{Kf_1}(p) = 1 + 4\pi \int_0^\infty \mathrm{d}r\, r^2\, S_{12}(r) \times \left\{ |j_0(pr) + TG|^2 - j_0^2(pr) \right\},$$

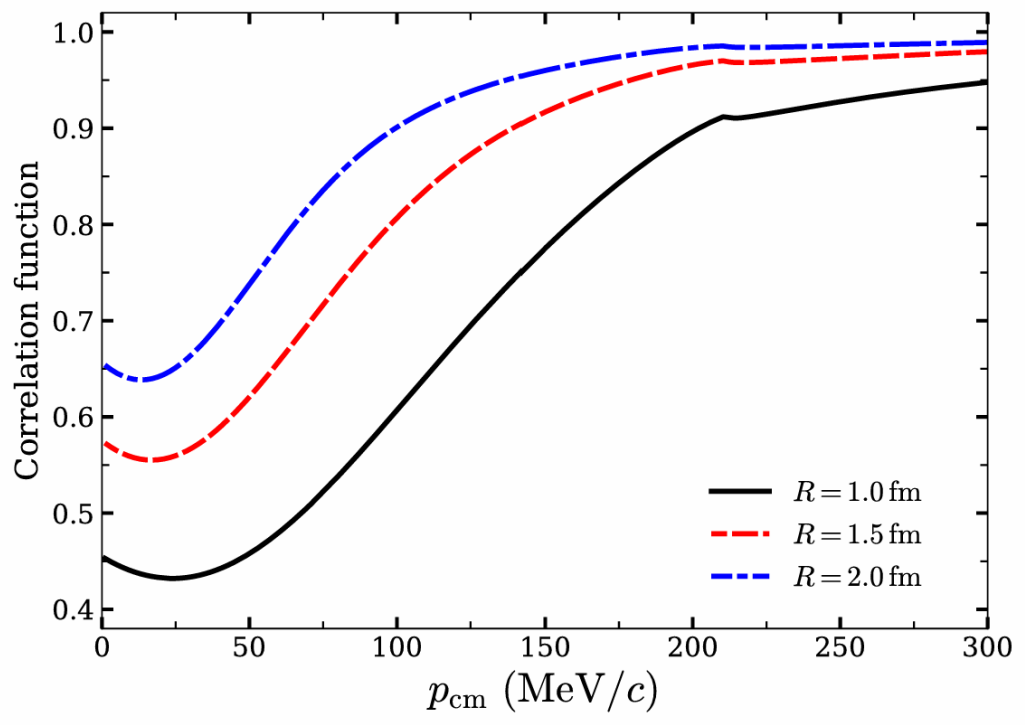


FIG. 4. Correlation function of the component $K^*\bar{K}$ of the $f_1(1285)$ wave function.

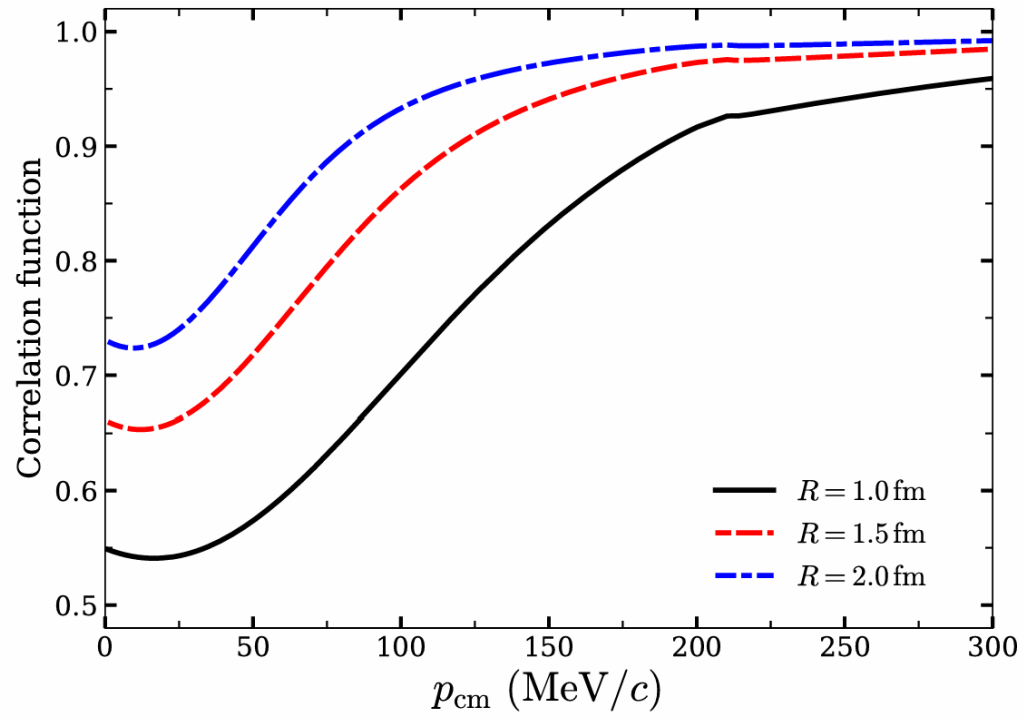


FIG. 6. Correlation function considering the two components $K^*\bar{K}$ and $\bar{K}^*K$ of the f_1 wave function.

✓ The correlation function has a structure corresponding to having a bound state below the Kf_1 threshold, or resonant state, since it has a width.

Summary

- ✓ Anticipating results in the new wave of ALICE experiments on particle-resonance correlation functions, we have carried out the evaluation of observables related to the $KD_{s0}^{*}(2317)$ and $Kf_1(2317)$ system, using ChUA and FCA.

- ✓ In the ChUA, the $D_{s0}^{*}(2317)$ is consider as a DK molecular stat, and $f_1(2317)$ is a $(K\bar{K}^{*}+K^{*}\bar{K})$ molecular state.

- ✓ We predict a three-body hadronic $KD_{s0}^{*}(2317)$ state, around 40 MeV below the $KD_{s0}^{*}(2317)$ threshold and with a width of about 200 keV; and a three-body hadronic $Kf_1(2317)$ state, around 10 MeV below the $Kf_1(2317)$ threshold and with a width of about 15 MeV.

- ✓ We calculate the related observables, including scattering length, effective range and correlation functions, which have a shape corresponding qualitatively to a strongly attractive potential.

Thank you for your attention!