Isospin symmetry breaking effects for near-threshold states





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University of Connecticut[A], RCNP, The University of Osaka[B]

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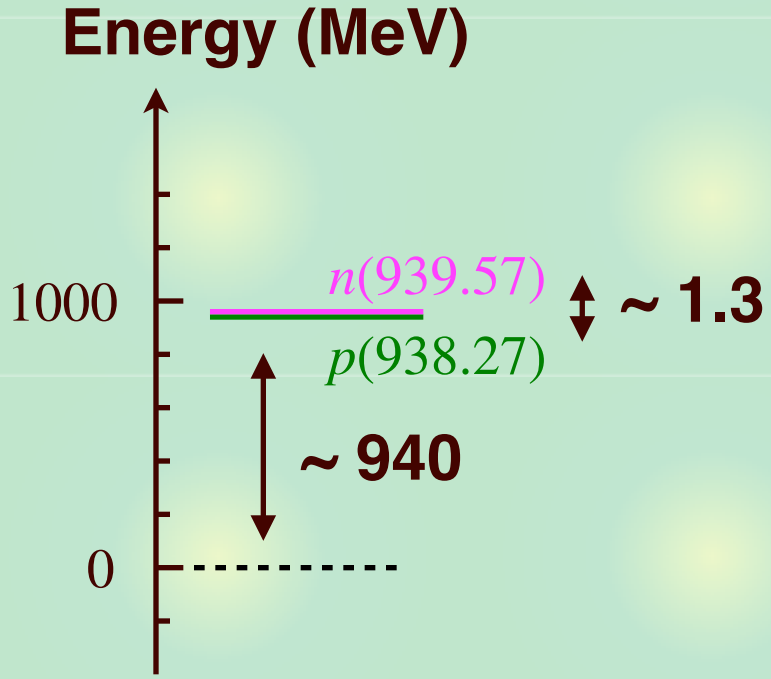
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Isospin symmetry breaking

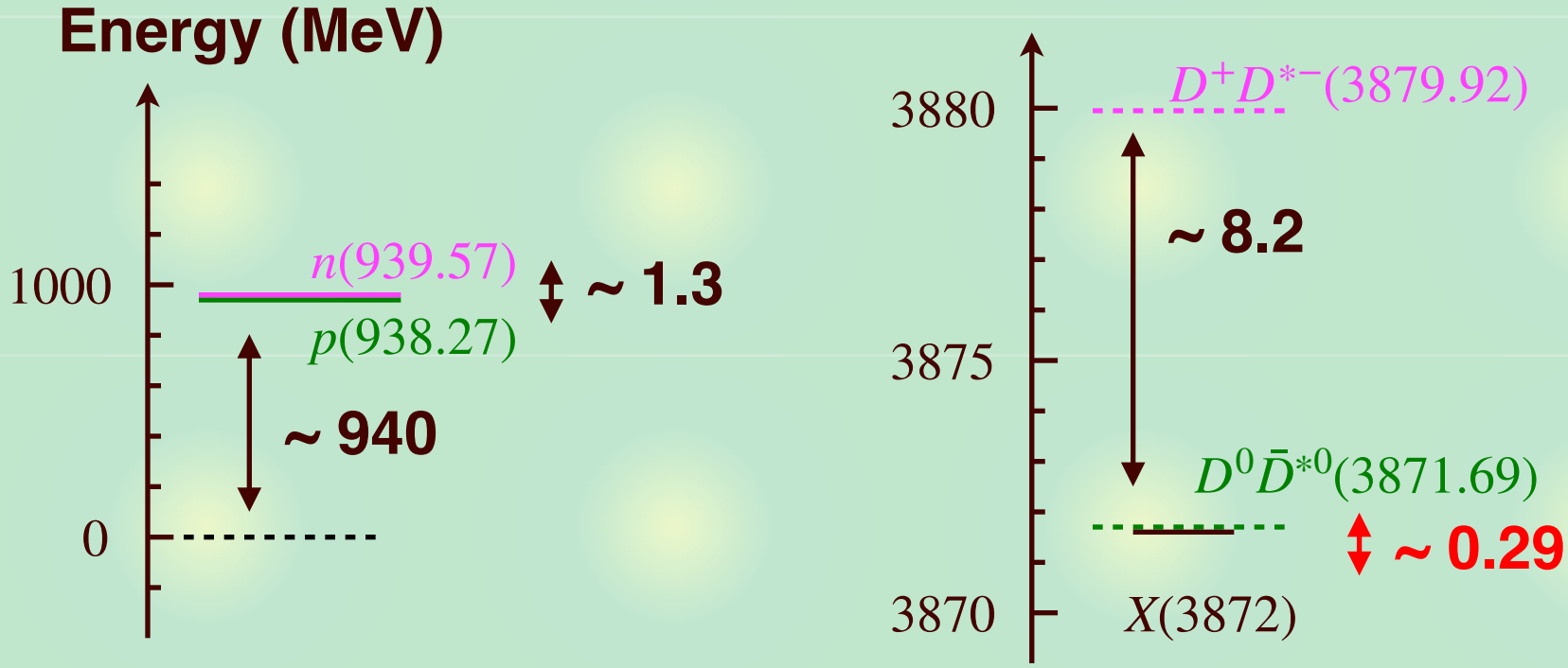
Isospin symmtry holds approximately well (up to a few MeV)



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- **Nucleon: 1.3 MeV compared with mass (940 MeV)**
- $X(3872)$**: 8.2 MeV compared with binding energy (0.29 MeV)**

Important symmetry breaking for near-threshold eigenstates

$X(3872)$ and η_{c2}

Isospin symmetry breaking for $X(3872)$

$$\mathrm{Br}(\, X(3872) \to \rho J/\psi\,) = 3.4 \pm 1.1\,\%\,, \quad \mathrm{Br}(\, X(3872) \to \omega J/\psi\,) = 5.0 \pm 1.9\,\%$$

<— Dominance of neutral channel: $X_0 \gg X_c$

D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, PRD 81, 014029 (2010);
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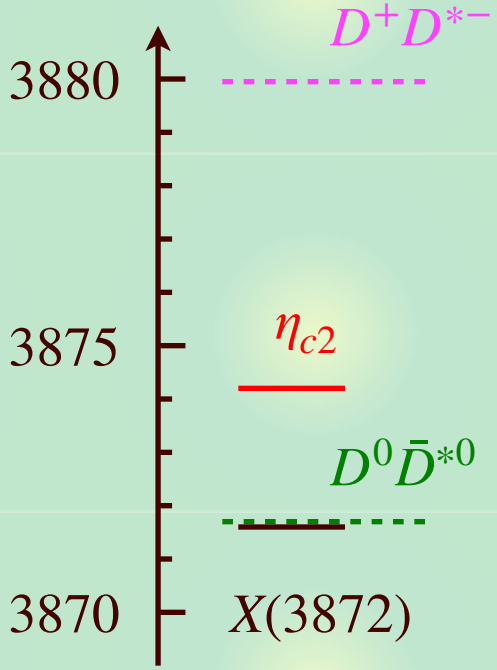
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η_{c2} : $J^{PC} = 2^{-+}$ **(not yet identified)**

- η_{c2} **near** $D\bar{D}^*$ **threshold in addition to** $X(3872)$

S.X. Nakamura, PRD 113, 114031 (2026)



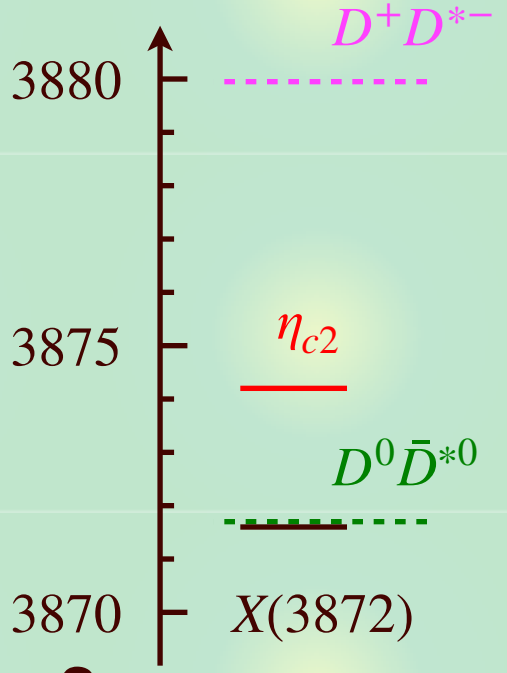
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Questions

- **How is the isospin symmetry breaking for** η_{c2} **?**
- $X(3872)$ **couples to s-wave** $D\bar{D}^*$**, while** η_{c2} **to p-wave** $D\bar{D}^*$

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Hamiltonian and self-energy

$c\bar{c}$ state $|\psi_0\rangle$ coupled with $D^0\bar{D}^{*0}$ state $|\boldsymbol{p},0\rangle$ and D^+D^{*-} state $|\boldsymbol{p},c\rangle$

V. Baru, *et al.*, PLB 586, 53 (2004); T. Hyodo, PRC 90, 055208 (2014)

$$\begin{pmatrix} M_0 & V & V \\ V & \boldsymbol{p}^2/(2\mu_0) & 0 \\ V & 0 & \boldsymbol{p}^2/(2\mu_c)+\Delta \end{pmatrix} |\Psi\rangle = E|\Psi\rangle, \quad |\Psi\rangle = \begin{pmatrix} c|\psi_0\rangle \\ \int d\boldsymbol{q}\,\chi_0(\boldsymbol{q})|\boldsymbol{q},0\rangle \\ \int d\boldsymbol{q}\,\chi_c(\boldsymbol{q})|\boldsymbol{q},c\rangle \end{pmatrix}$$

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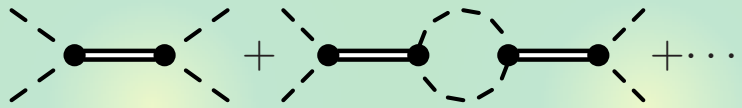
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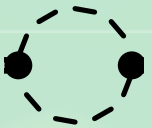
T-matrix

$$T_{ij}(\boldsymbol{p},\boldsymbol{p}',E) = \frac{\langle \boldsymbol{p},i\,|\,V\,|\,\psi_0\rangle\langle\psi_0\,|\,V\,|\,\boldsymbol{p}',j\rangle}{E-M_0-\Sigma(E)} \sim$$



Self-energy

$$\Sigma(E) = \Sigma_0(E)+\Sigma_c(E), \quad \Sigma_i(E) = \int d\boldsymbol{q}\,\frac{\langle\psi_0\,|\,V\,|\,\boldsymbol{q},i\rangle\langle\boldsymbol{q},i\,|\,V\,|\,\psi_0\rangle}{E_i-\Delta\delta_{ic}-\boldsymbol{q}^2/(2\mu_i)+i0^+} \sim$$



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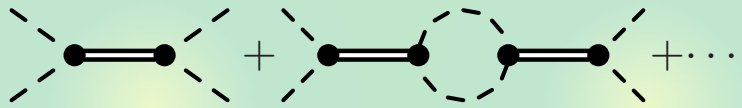
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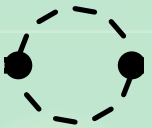
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Pole condition: zero of the denominator

$$E_h - M_0 - \Sigma(E_h) = 0$$

Compositeness

Pole corresponds to discrete eigenstate (outgoing b.c.)

$$H|\Psi_h\rangle = E_h|\Psi_h\rangle$$

- Left eigenstate for resonances and normalization

$$\langle\widetilde{\Psi_h}|H = E_h\langle\widetilde{\Psi_h}|, \quad \langle\widetilde{\Psi_h}|\Psi_h\rangle = 1.$$

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Compositeness X_i (weight of scattering channel i)

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, IJMPA 28, 1330045 (2013); F.K. Guo *et al.*, RMP 90, 015004 (2018);
T. Kinugawa, T. Hyodo, EPJA 61, 154 (2025)

$$X_i = \int d\boldsymbol{q}\,\langle\widetilde{\Psi_h}|\boldsymbol{q},i\rangle\langle\boldsymbol{q},i|\Psi_h\rangle, \quad Z = \langle\widetilde{\Psi_h}|\psi_0\rangle\langle\psi_0|\Psi_h\rangle \in \mathbb{C}, \quad \sum_i X_i + Z = 1$$

- Expression in terms of the self-energy

$$X_i = \left.\frac{-\Sigma_i'(E)}{1-\Sigma_0'(E)-\Sigma_c'(E)}\right|_{E=E_h}, \quad Z = \left.\frac{1}{1-\Sigma_0'(E)-\Sigma_c'(E)}\right|_{E=E_h}$$

Transition matrix element

Angular momentum structure of transition

- Bare state $|\psi_0\rangle$: spin J, polarization M
- $|p\rangle$: orbital angular momentum ℓ, D^* polarization λ

$$\langle \boldsymbol{p}, \ell, \lambda, i \,|\, V \,|\, \psi_0; JM \rangle = g_i^{(\ell)} \sqrt{4\pi}\, p^{\ell} F^{(\ell)}(p) \sum_m \langle \ell m, 1\lambda \,|\, JM \rangle Y_{\ell m}(\hat{\boldsymbol{p}})$$

- Coupling constant: $g_i^{(\ell)}$
- Form factor normalized as $F^{(\ell)}(0) = 1$

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For s-wave $X(3872)$ and p-wave η_{c2}

- $X(3872)$: $\langle \boldsymbol{p}, 0, \lambda, i | V | \chi_{c1}; 1M \rangle = g_i^{(0)} F^{(0)}(p) \delta_{\lambda M}$
- η_{c2} : $\langle \boldsymbol{p}, 1, \lambda, i | V | \eta_{c2}; 2M \rangle = g_i^{(1)} \sqrt{4\pi}\, p F^{(1)}(p) \sum_m \langle 1m, 1\lambda | 2M \rangle Y_{1m}(\hat{\boldsymbol{p}})$

Explicit model

Isospin symmetric coupling and sharp cutoff form factor

$$g_0^{(\ell)} = g_c^{(\ell)} \equiv g^{(\ell)}, \quad F^{(\ell)}(p) = \Theta(\Lambda - p)$$

- For a given M, reduced matrix element reads

$$\langle \boldsymbol{p}, i \,|\, V \,|\, \psi_0 \rangle = \begin{cases} g^{(0)}\,\Theta(\Lambda - p) & X(3872) \\ g^{(1)}\, p\,\Theta(\Lambda - p) & \eta_{c2} \end{cases}$$

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Explicit expression of self-energy

$$\Sigma_i(E) = \begin{cases} -8\pi\mu_i (g^{(0)})^2 \left[\Lambda + ik_i \arctan\left(-\frac{\Lambda}{ik_i}\right)\right] & X(3872) \\ -8\pi\mu_i (g^{(1)})^2 \left[\frac{\Lambda^3}{3} + k_i^2\Lambda + ik_i^3 \arctan\left(-\frac{\Lambda}{ik_i}\right)\right] & \eta_{c2} \end{cases}$$

$$k_i = \sqrt{2\mu_i(E - \Delta\delta_{ic})}$$

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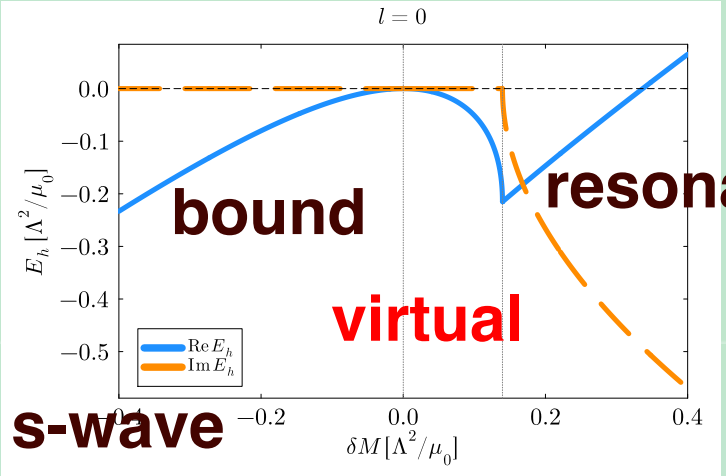
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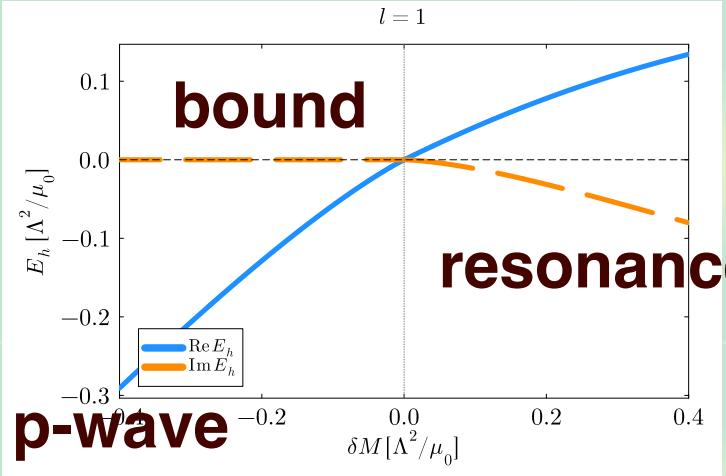
Near-threshold eigenenergy (single-channel)

Eigenenergy as function of bare mass for fixed $g^{(\ell)}$

T. Hyodo, PRC 90, 055208 (2014)

Energy



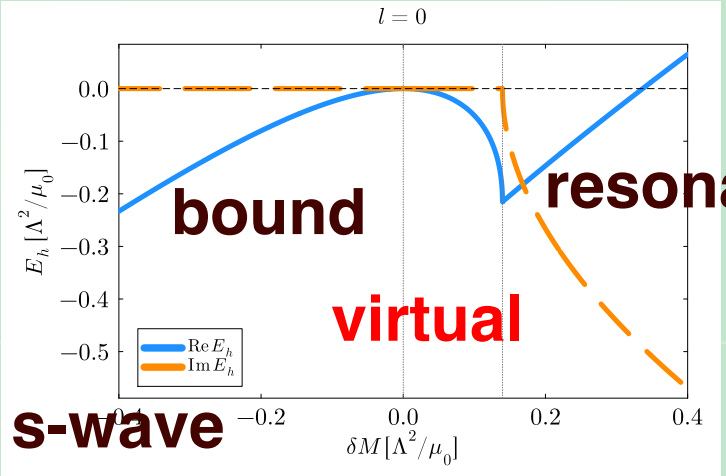


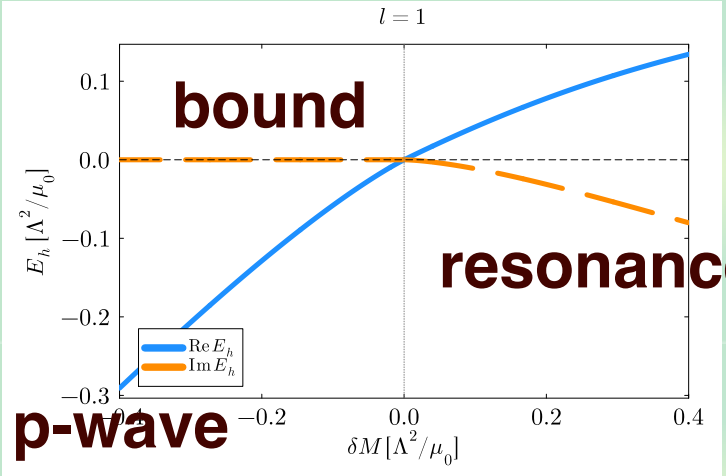
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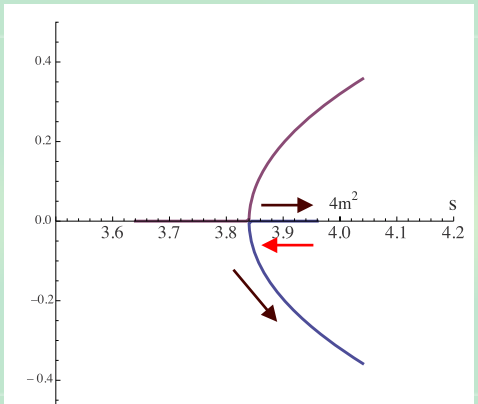
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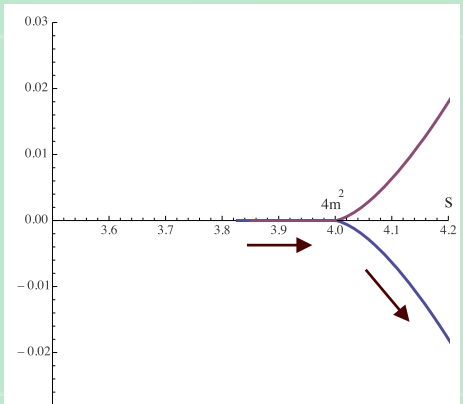




c.f.) pole trajectory

C. Hanhart, J.R. Pelaez, G. Rios, PLB 739, 375 (2014)



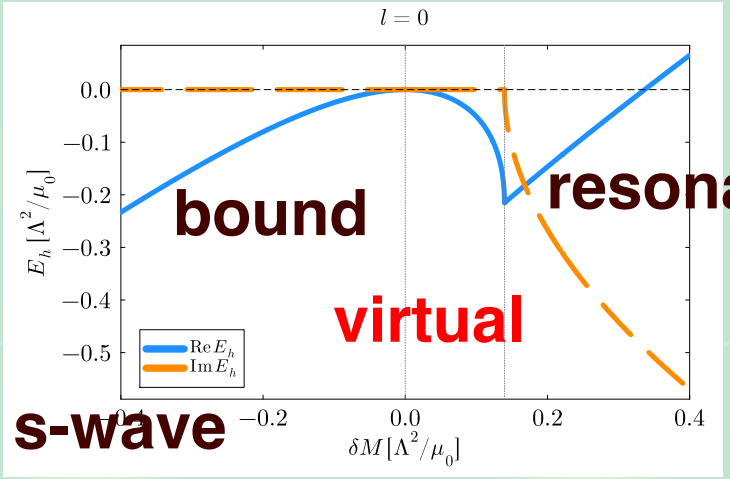


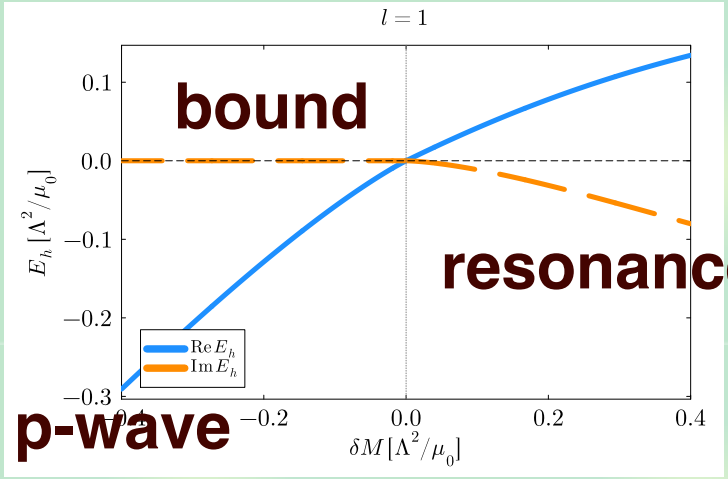
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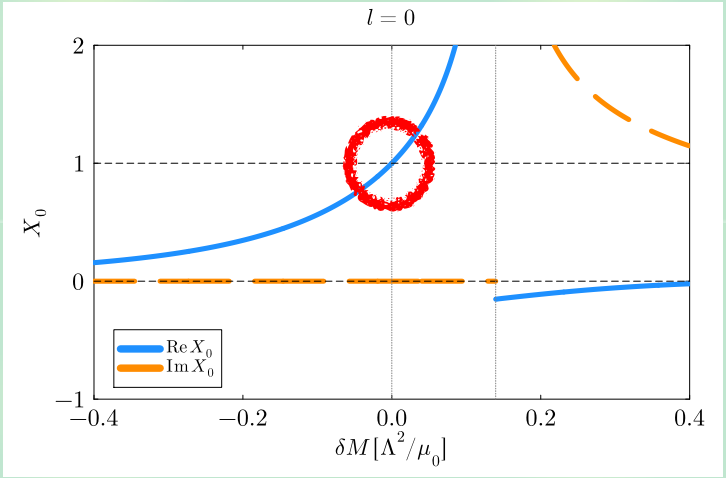
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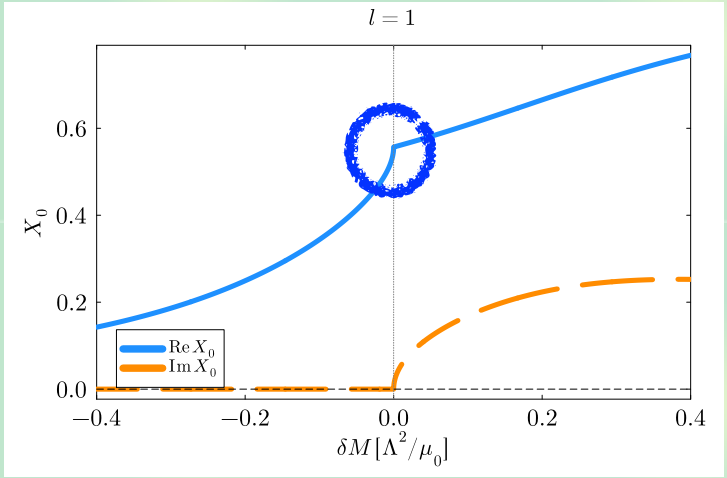
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Compositeness



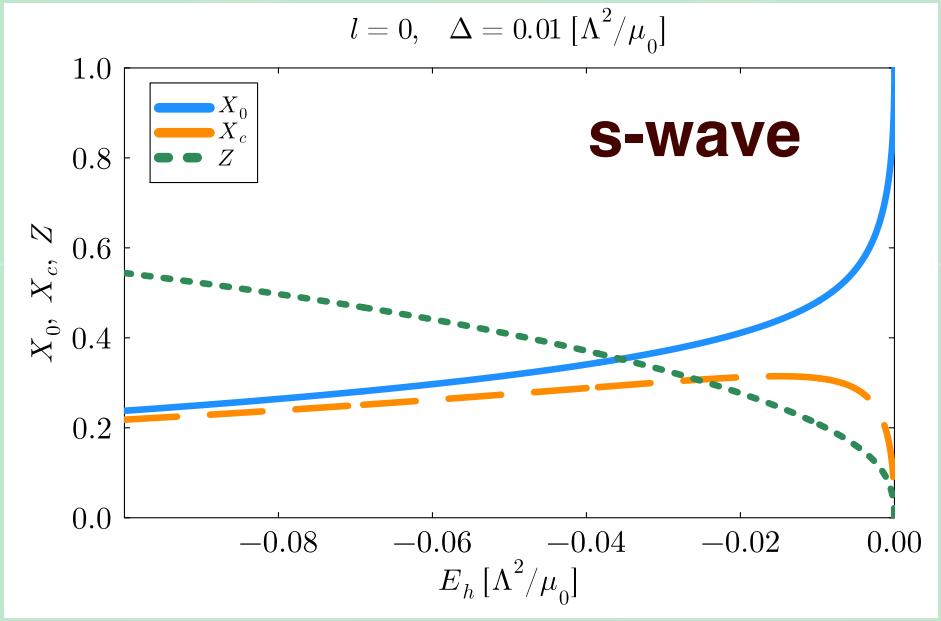


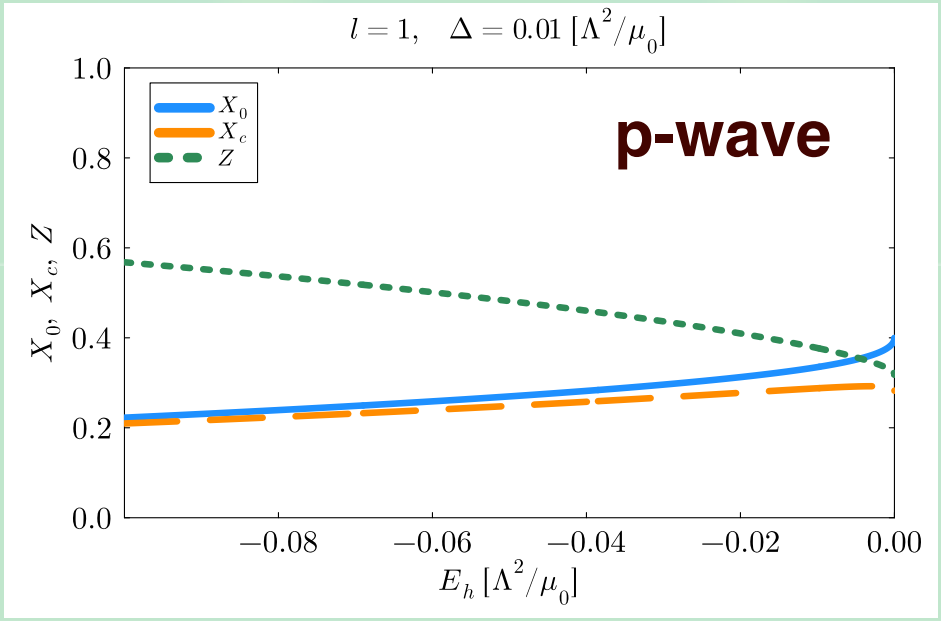
s-wave: $X \to 1$ at threshold, p-wave: arbitrary X at threshold

Ispspin symmetry breaking (weak)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.01\ [\Lambda^2/\mu_0^2]$

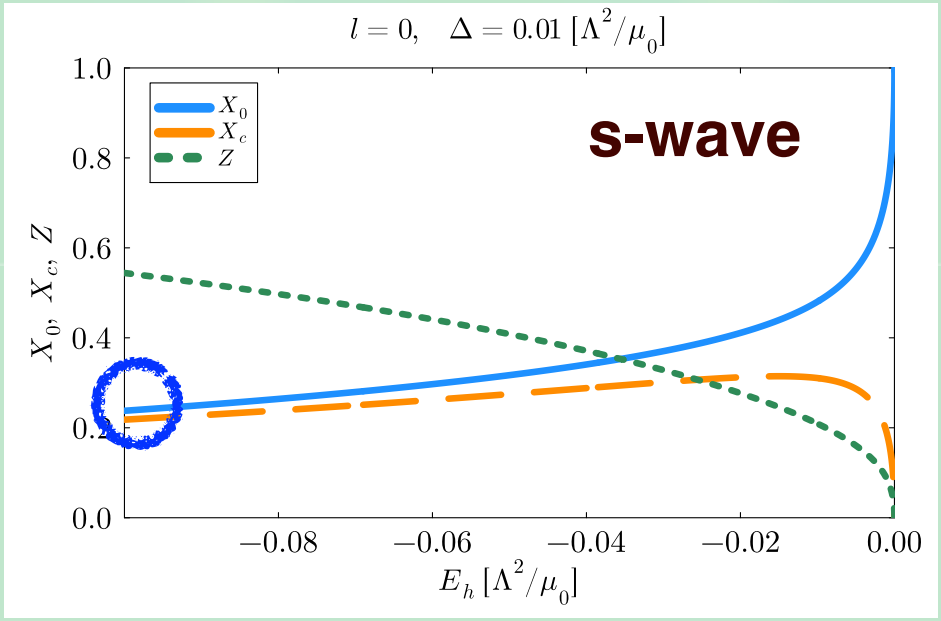


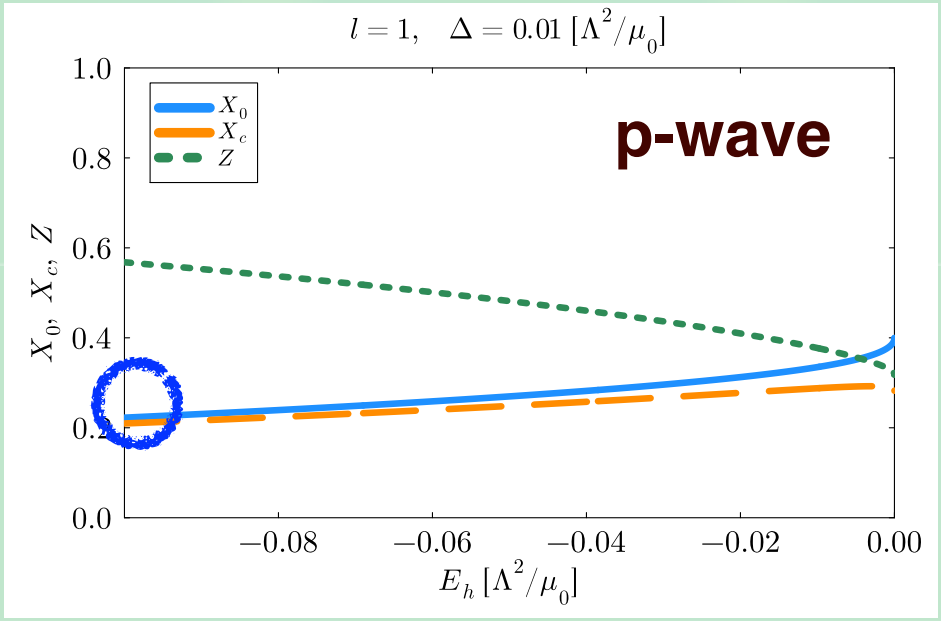


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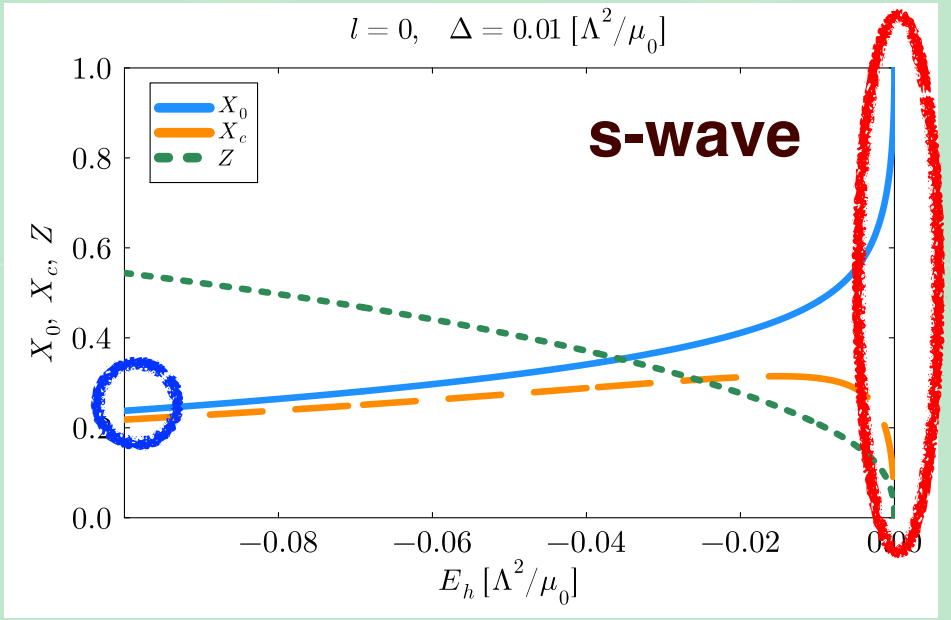


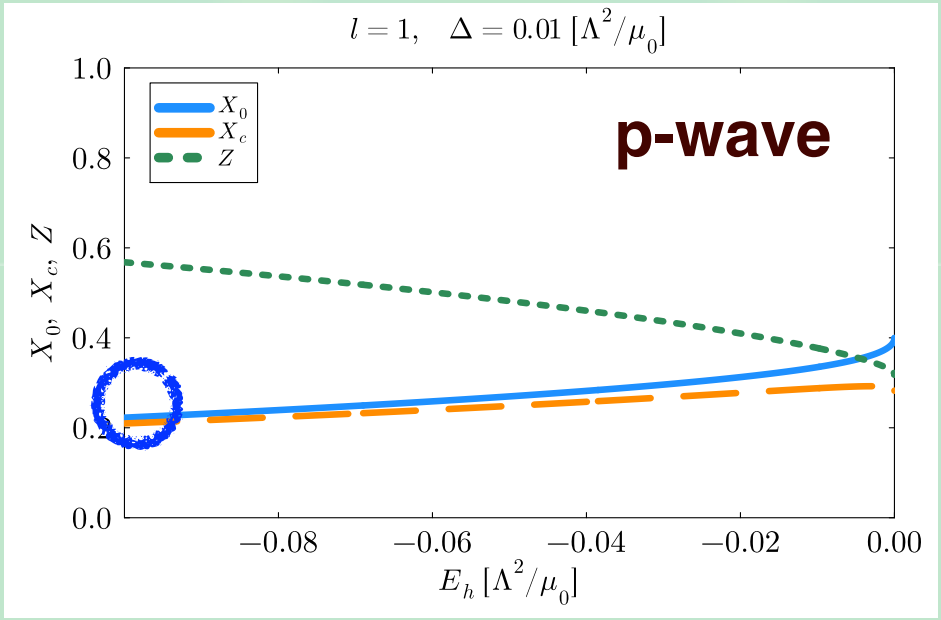
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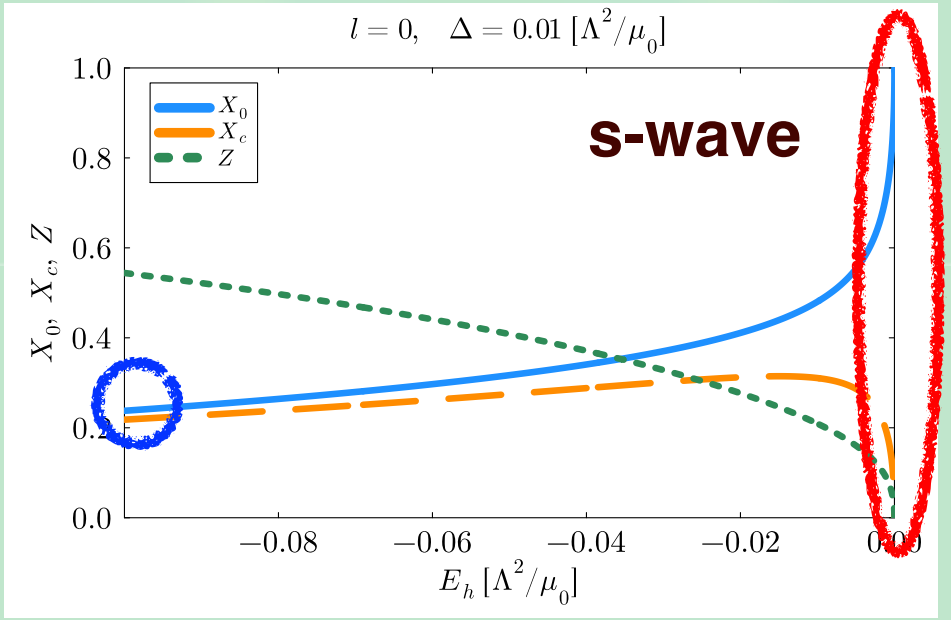


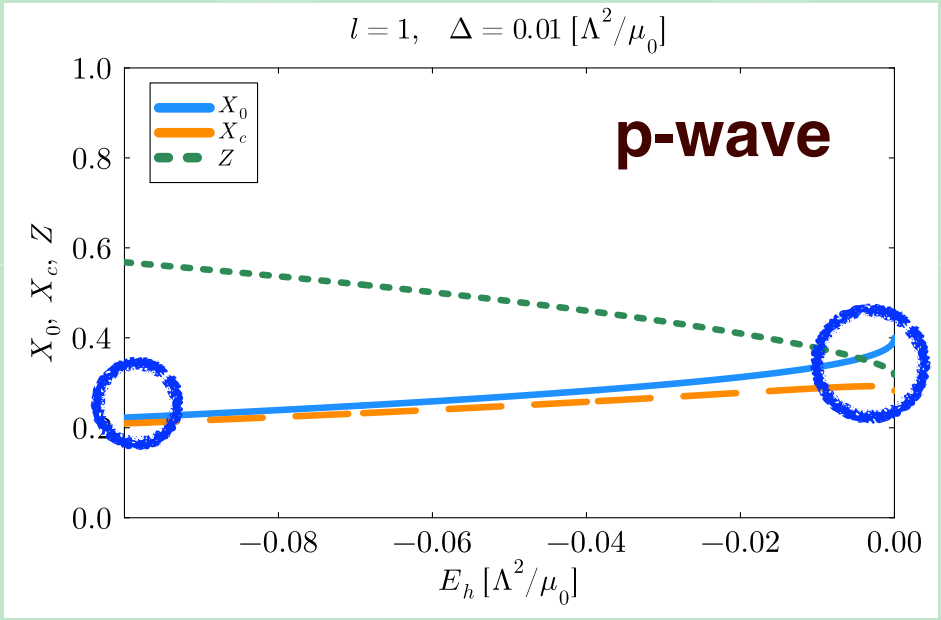
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- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)

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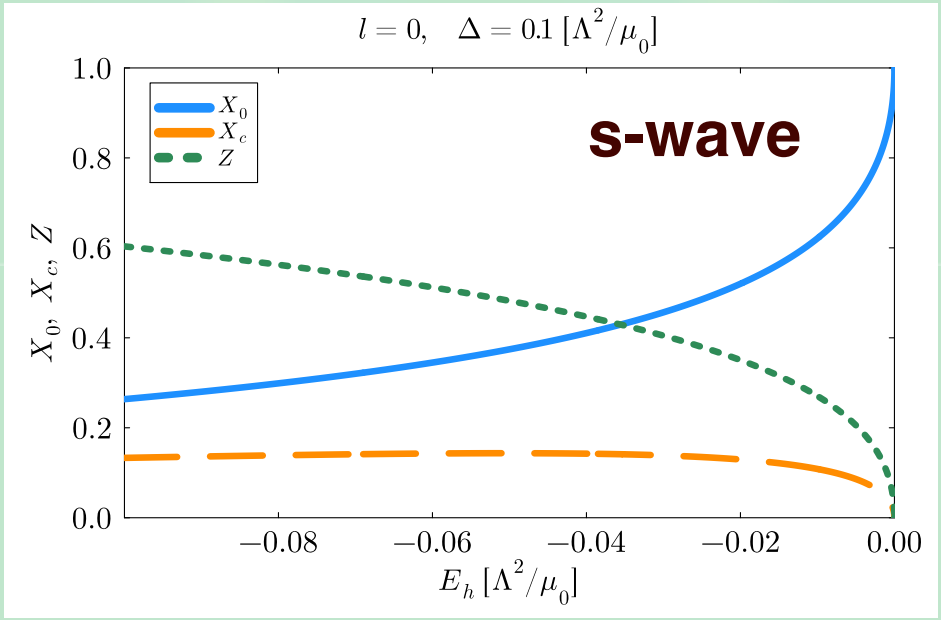


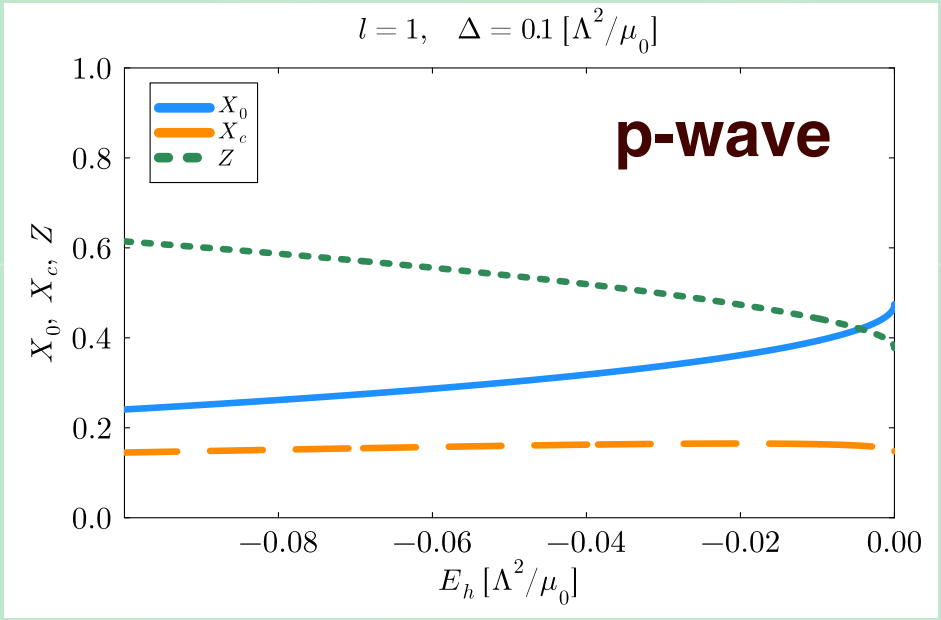
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- p-wave shallow bound state: almost symmetric ($X_0 \approx X_c$)

Ispspin symmetry breaking (large)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.1\ [\Lambda^2/\mu_0^2]$

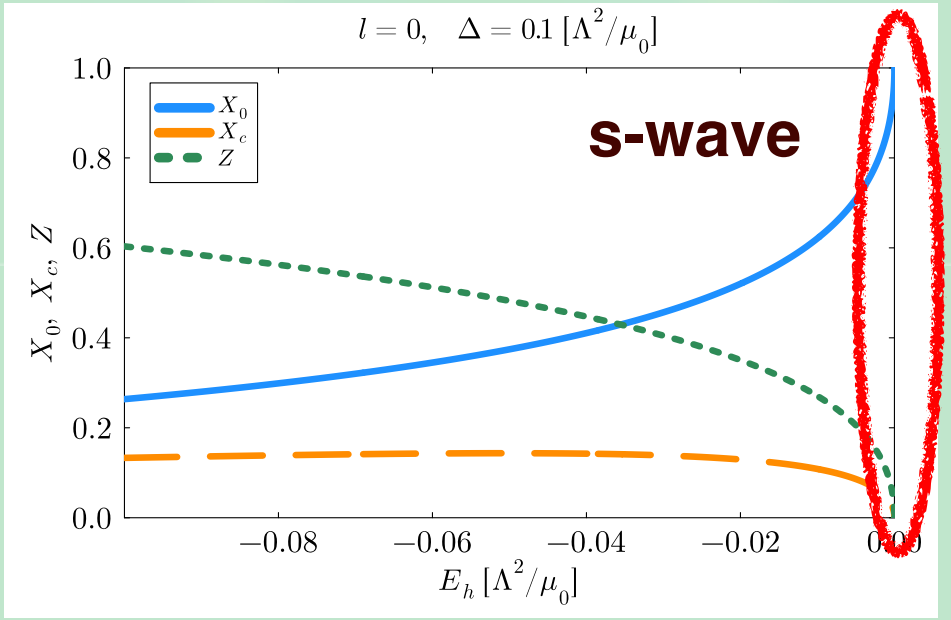


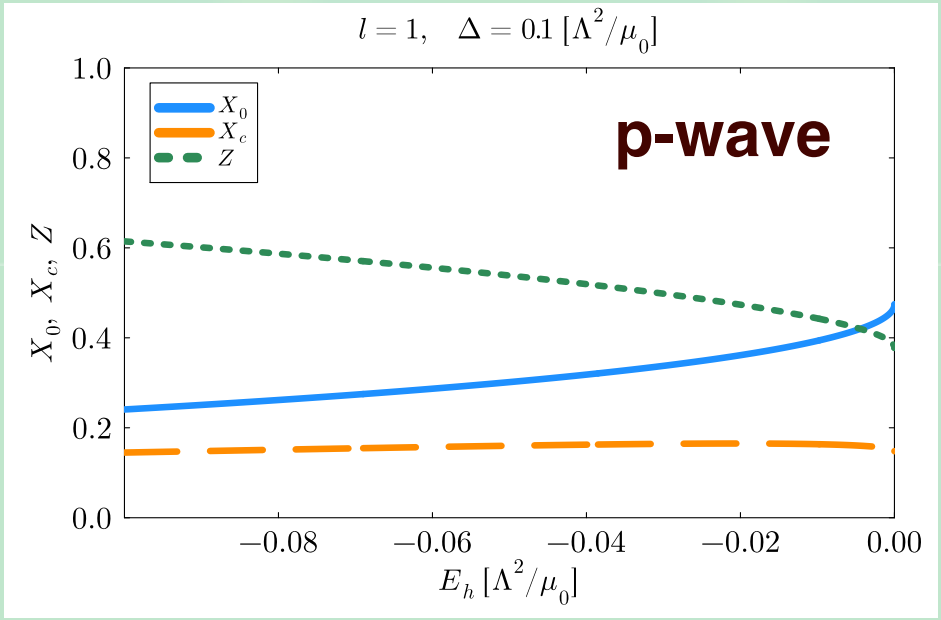


Ispspin symmetry breaking (large)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.1\ [\Lambda^2/\mu_0^2]$



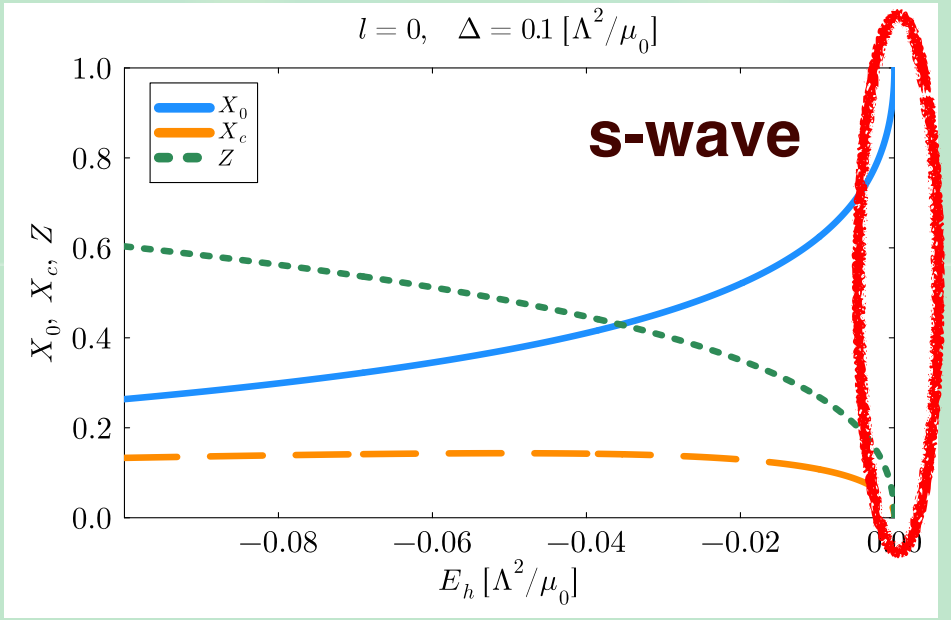


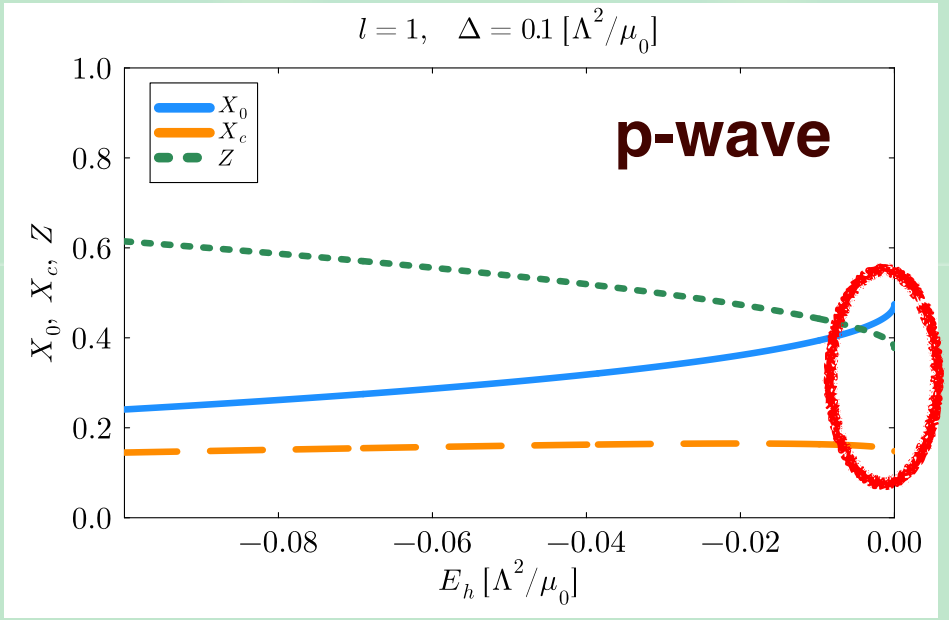
- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)

Ispspin symmetry breaking (large)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.1\ [\Lambda^2/\mu_0^2]$



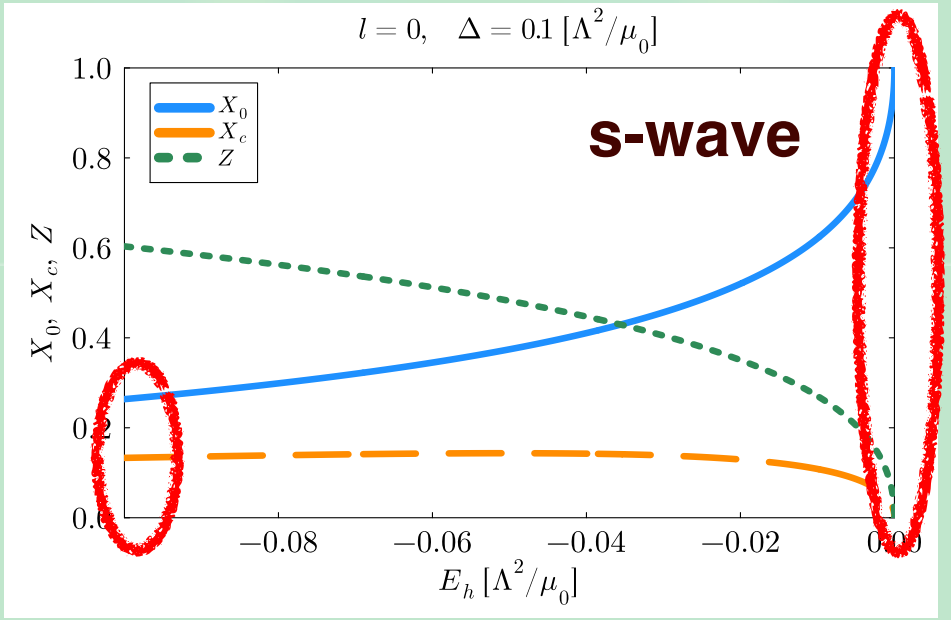


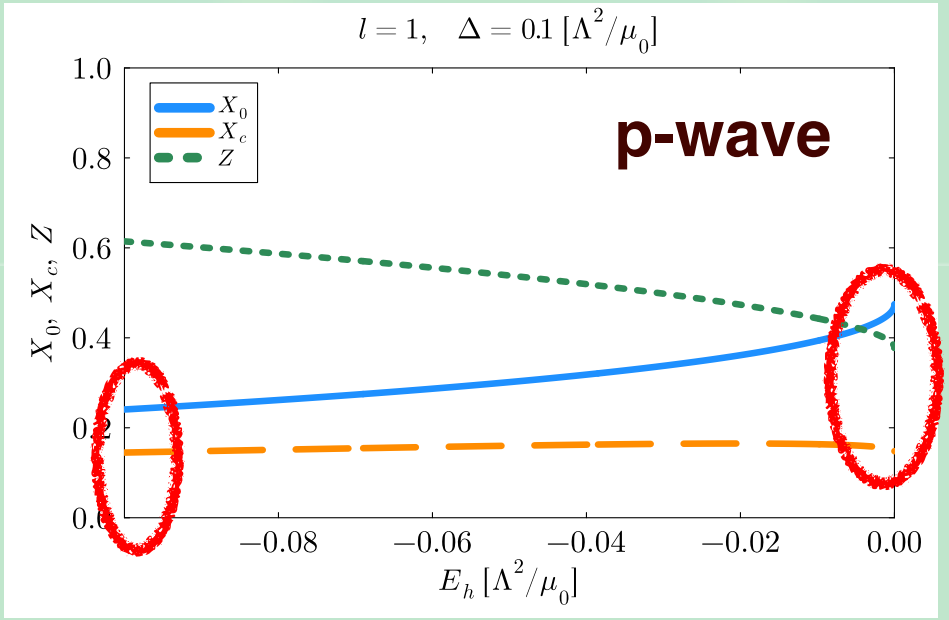
- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)
- p-wave shallow bound state: substantial breaking ($X_0 > X_c$)

Ispspin symmetry breaking (large)

Compositeness of bound state (two-channel)

- Threshold energy difference $\Delta = 0.1\ [\Lambda^2/\mu_0^2]$





- s-wave shallow bound state: large breaking ($X_0 \gg X_c$)
- p-wave shallow bound state: substantial breaking ($X_0 > X_c$)
- Deep bound states: substantial breaking ($X_0 > X_c$)

Realistic setup

Input

- **Eigenenergy E_h : $X(3872)$ from PDG, η_{c2} from Nakamura**
- **Bare mass M_0 : constituent quark model**

  S. Godfrey and N. Isgur, PRD 32, 189 (1985)

- **Cutoff** $600\ \text{MeV} \leq \Lambda \leq 1000\ \text{MeV}$

Input Resonance	Bare Mass (GeV)	Z	X_0	X_c
$X(3872)$	3.95	0.19 ± 0.04	0.69 ± 0.04	0.117 ± 0.004
η_{c2}	3.84	(1.49 ± 0.29) $-(0.14 \pm 0.12)i$	(-0.29 ± 0.17) $+(0.16 \pm 0.09)i$	(-0.20 ± 0.11) $+(0.03 \pm 0.02)i$
η_{c2}	3.95	(0.61 ± 0.11) $(0.03 \pm 0.01)i$	$+(0.23 \pm 0.07)$ $-(0.04 \pm 0.02)i$	(0.16 ± 0.04) $+(0.01 \pm 0.01)i$

$X(3872)$: large ispsin symmetry breaking $X_0 \gg X_c$

η_{c2} : almost isospin symmetric $X_0 \approx X_c$

Summary

- We study isospin symmetry breaking for near-threshold states.
- Near-threshold s-wave bound state is complete composite at threshold, while p-wave state has arbitrary structure even at threshold

  T. Hyodo, PRC 90, 055208 (2014)
- Isospin symmetry is largely broken for near-threshold s-wave states, while it is not always broken for p-wave states
- Near-threshold η_{c2} follows isospin symmetry

  M. Bradshaw, T. Hyodo, in preparation