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Form factors of the ρ-meson

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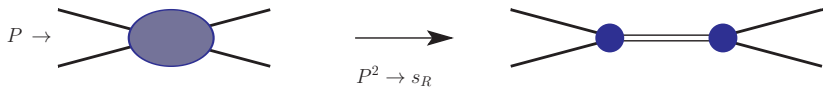
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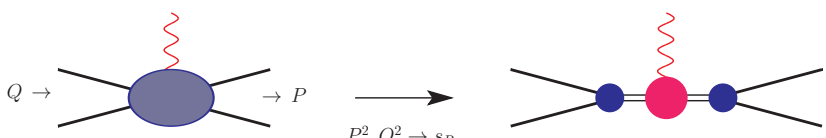
Plan

- Introduction: What are the resonance form factors and how to calculate them on the lattice? What to they tell us about the nature of a state?
- Calculation of the form factors in NREFT: peculiar behavior at $k^2 \simeq 0$: emergence of the small scale, stable states vs. resonaces
- ρ-meson form factors in ChPT: infrared regularization and the counting rules
- Choice of the formalism: unnatural coupling constants
- External fields on the lattice and the form factors
- Conclusions: the role of the external probes in the study of the structure of QCD resonances and bound states

Resonance matrix elements



$$\sqrt{s_R} = m_R - \frac{i}{2}\Gamma_R$$



- Neither the pole position, nor the matrix element depend on the external legs: *process-independence*
- Invariance under *field redefinitions*

The wave function renormalization constant

- $\psi_\mu(x)$ is the ρ-meson interpolating field (composite or elementary)
- Two-point function of the ρ-meson:

$$\begin{aligned} D_{\mu\nu}(P) &= i\int d^4x e^{iPx}\langle 0|T\psi_\mu(x)\psi_\nu^\dagger(0)|0\rangle \to \frac{Z}{s_R - P^2}\sum_s \varepsilon_\mu(P,s)\tilde{\varepsilon}_\nu(P,s) \\ &= \frac{Z}{s_R - P^2}\left(-g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2}\right) \end{aligned}$$

- Z is the wave function renormalization constant

Electromagnetic form factors of the ρ-meson

- The $\gamma\rho\rho$ vertex:

$$\begin{aligned} G^{\alpha}_{\mu\nu}(P,Q) &= \int d^4x d^4y e^{iPx-iQy}\langle 0|\,T\psi_\mu(x)J^\alpha(0)\psi^\dagger_\nu(0)|0\rangle \\ &\to -\sum_{s,s'}\frac{Z^{1/2}\varepsilon_\mu(P,s)}{s_R-P^2}\,\langle P,s|J^\alpha(0)|Q,s'\rangle\,\frac{Z^{1/2}\tilde{\varepsilon}_\nu(Q,s')}{s_R-Q^2} \end{aligned}$$

- Form factors (here, $k^\mu=(P-Q)^\mu$):

$$\begin{aligned} \langle P,s|J^\alpha(0)|Q,s'\rangle &= \tilde{\varepsilon}_\mu(P,s)\Big\{-G_1(k^2)g^{\mu\nu}(P+Q)^\alpha - G_2(k^2)(k^\mu g^{\nu\alpha}-k^\nu g^{\mu\alpha}) \\ &+ \frac{1}{2s_R}\,G_3(k^2)k^\mu k^\nu(P+Q)^\alpha\Big\}\varepsilon_\nu(Q,s') \end{aligned}$$

Physical meaning

- The FFs describe moments of charge distribution inside a given state:
  - charge
  - magnetic moment
  - quadrupole moment
- For a *molecular state*, but not for the *tight quark compound*, the distributions are characterized by anomalously large radii
- An intrinsic small mass scale for the resonances is given by the *width*, and not by the *binding energy*.

$\hookrightarrow$ FFs provide a convenient tool to study the nature of the QCD bound states!

NREFT formalism

- Pion-pion interactions in all partial waves:

$$\mathscr{L} = \sum_{a=+,0} \phi_a^\dagger 2w(iv\cdot\partial - w)\phi_a + \sum_\ell \sigma_\ell \psi^{(\ell)\dagger}_{\mu_1\cdots\mu_\ell}\psi^{(\ell),\mu_1\cdots\mu_\ell}\,, \qquad \sigma_\ell = \pm 1$$

$$w = \sqrt{m^2 + \partial^2 - (v\cdot\partial)^2}\,, \qquad w^\mu = v^\mu w + i(\partial^\mu - v^\mu(v\cdot\partial))$$

$$\psi^{(\ell)}_{\mu_1\cdots\mu_\ell}(x) = \left\{\phi_+(x)\mathscr{Y}^{(\ell)}_{\mu_1\cdots\mu_\ell}(\overleftrightarrow{w})f_\ell(-\overleftrightarrow{w}^2)\phi_0(x)\right\}$$

- Matching to the scattering phase shift:

$$q\cot\delta_\ell(q) = \left(\frac{\sigma_\ell c^{(\ell)} q^{2\ell} f_\ell^2(q^2)}{8\pi\sqrt{s}}\right)^{-1}$$

Gauging the Lagrangian

- How does one gauge the square root?

$$w^2 \to m^2 + (\partial - ieA)^2 - (v\cdot\partial - iev\cdot A)^2 = w^2 + B + O(e^2)$$

$$w \to w + D\,, \qquad D = \int_0^\infty dt e^{-tw} B e^{-tw}\,, \qquad \langle p|D|q\rangle = \frac{\langle p|B|q\rangle}{w(p)+w(q)}$$

- The vertex:

$$\begin{aligned}\Gamma^\mu(p,q) &= e(p+q)^\mu\left(2 - \frac{v\cdot p + v\cdot q}{w(p)+w(q)}\right)\\ &+ ev^\mu\left((w(p)+w(q)) + \frac{(v\cdot p + v\cdot q)^2}{w(p)+w(q)} - 2(v\cdot p + v\cdot q)\right) \to e(p+q)^\mu\end{aligned}$$

- Ward identity:

$$(p-q)^\mu\Gamma^\mu(p,q) = eS^{-1}(q) - eS^{-1}(p)\,, \qquad S^{-1}(p) = 2w(p)(w(p) - v\cdot p)$$

Non-minimal couplings

- Three linearly independent, gauge-invariant operators at the lowest order:

$$\begin{aligned}
\mathscr{O}_1 &= g_1\, \psi^{(1)\dagger}_\mu \{\partial^\beta F_{\beta\alpha}, w^\alpha\} g^{\mu\nu} \psi^{(1)}_\nu \\
\mathscr{O}_2 &= g_2\, \psi^{(1)\dagger}_\mu F^{\mu\nu} \psi^{(1)}_\nu \\
\mathscr{O}_3 &= \frac{g_3}{2}\, \psi^{(1)\dagger}_\mu \{(\partial^\mu F^{\nu\alpha} + \partial^\nu F^{\mu\alpha}), w_\alpha\} \psi^{(1)}_\nu
\end{aligned}$$

  $+$ higher-order terms with more derivatives

- No need to consider gauging of the 4-pion Lagrangian at this order

Form Factors

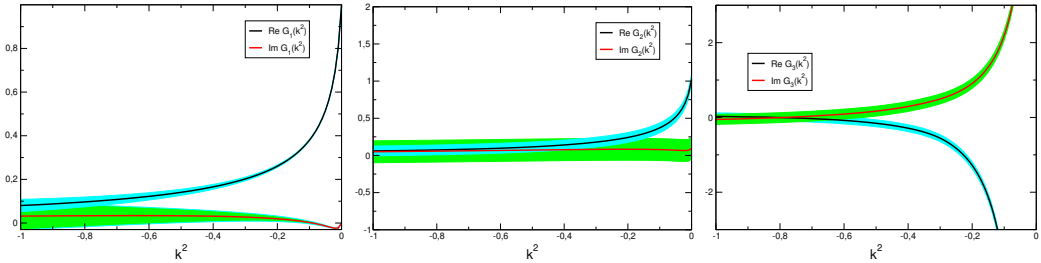


$$
\begin{aligned}
G_1(k^2) &= -eZg_1k^2 + eZ\langle F_1\rangle \\
G_2(k^2) &= -eZg_2 + eZ\langle F_2\rangle \\
G_3(k^2) &= 2es_RZg_3 + eZ\langle F_3\rangle
\end{aligned}
$$

- Calculations done in the Breit frame

$$\langle F_i\rangle = \int \frac{d^d\boldsymbol{l}}{(2\pi)^d}\,\frac{f_{ini}(\boldsymbol{l})f_{fin}(\boldsymbol{l})N_i(\boldsymbol{l})}{D(\boldsymbol{l})}$$

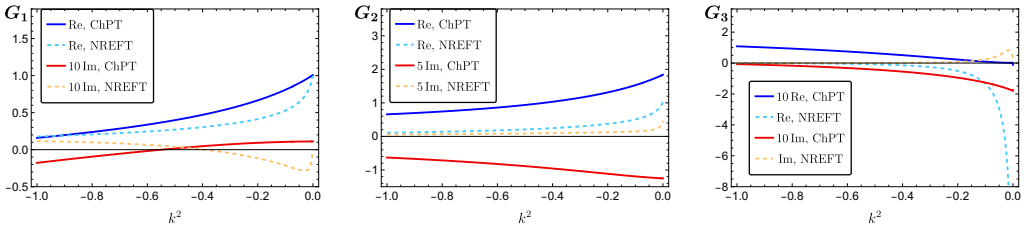
Invariant form factors



- Two different parameterizations of the phase shift ✓
- Bands: assuming natural size for g_i *(perturbative matching to ChPT)*
- Small scale emerging, proportional to width, present in $f_{ini}(\boldsymbol{I}), f_{fin}(\boldsymbol{I})$
  - Anomalously large charge radius
  - Large correction to the magnetic moment
  - Large quadrupole moment

The same form factors in ChPT

- ρ and ω fields explicitly included in the chiral Lagrangian
- Consistent power counting (infrared regularization)
- All contributions at $O(p^3)$, 22 different topologies



The result shows no peculiar behavior near $k^2 = 0$!!!

Comparison of the results in two frameworks

- NREFT: No explicit ρ-meson field
  - 4-pion couplings are summed to all orders, yielding $\tan\delta_1(p)/p$. The K-matrix is singular near the resonance pole, the power-counting is broken
  - A small scale emerges in the triangle diagram, leading to the anomalously large radii in the moments of charge distribution
- ChPT: Explicit ρ-meson field
  - $\rho\pi\pi$ couplings can be treated perturbatively, no anomalously small scale in the triangle diagram
- Note: Both relativistic and non-relativistic approaches can be formulated with/without explicit resonance fields. Mathematically, these two descriptions are equivalent

Exploring the nature of the QCD states

- The choice of the appropriate degrees of freedom leads to the couplings of the natural size, no breaking of the counting rules
- It is expected that a molecular state has a large extension (an inherent small scale emerges), whereas the size of a tight quark compound is determined by the QCD scale
- If appropriate degrees of freedom are not chosen, the non-minimal $\gamma\pi\pi\pi\pi$ and/or $\gamma\rho\rho$ couplings should be *unnatural*
- Thus, the answer about the nature of a given QCD state can be, in particular, inferred from the values of the pertinent EFT couplings $\rightarrow$ lattice QCD

The calculation of the form factors on the lattice: preliminaries

- Calculation of the FFs on the lattice enables one to judge about the nature of a state
- The pioneering calculation, stable ρ: Gürtler *et al.*, 2008
- The problem with a resonance: the finite-volume matrix elements do not possess a regular $L \to \infty$ limit (triangle diagram)
  - Subtract triangle diagram by hand and perform the infinite-volume limit in the remainder: Hoja *et al.*, 2010; Briceno *et al.*, 2020;
    Implementation on the lattice: Ortega-Gama *et al.*, arXiv2609.04142
  - Feynman-Hellman method – placing the system in the periodic external field: Lozano *et al.*, 2022

Configuration of the external field

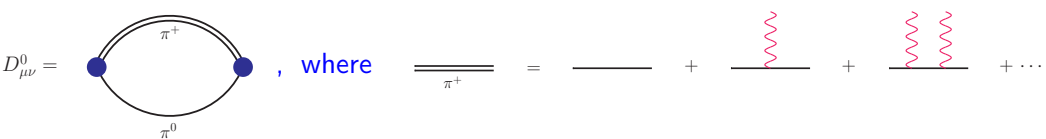
$$A^\mu(\boldsymbol{x}, r) = \underline{A}^\mu \cos(\boldsymbol{\omega x}), \qquad \boldsymbol{\omega} = (0, 0, \omega), \qquad \omega = \frac{2\pi}{L}, \qquad n \in \mathbb{Z}$$

- Momentum transfer: $k^2 = -\boldsymbol{\omega}^2$
- Temporal field: $A^0 \neq 0, \quad \boldsymbol{A} = 0$
- Transverse field: $A^0 = A^3 = 0$ and $A^1 \neq 0,\ A^2 = 0$ or $A^1 = 0,\ A^2 \neq 0$
  Both configurations should be included, in order to extract all form factors $G_i(k^2)$
- Calculations should be carried out in the Breit frame $\boldsymbol{P} = -\boldsymbol{Q} = \dfrac{\boldsymbol{\omega}}{2}$
- Inclusion of the external field leads to the shift/splitting of energy levels at $O(e)$

The Lüscher equation in the presence of the external field

$$\det(\mathscr{A}) = 0\,, \qquad \mathscr{A}_{\mu\nu}(\boldsymbol{P},\boldsymbol{Q}) = \Pi^{-1}_{\mu\nu}(\boldsymbol{P},\boldsymbol{Q}) - D^0_{\mu\nu}(\boldsymbol{P},\boldsymbol{Q})$$

- $\Pi^{-1}_{\mu\nu}(\boldsymbol{P},\boldsymbol{Q})$ contains non-minimal couplings g_1, g_2, g_3 only, is a *polynomial*
- $D^0_{\mu\nu}(\boldsymbol{P},\boldsymbol{Q})$ can be interpreted as the *Lüscher function in the external field*



The essentials/workflow

- The triangle diagram does not appear explicitly and the limit $L \to \infty$ can be readily performed
- The function $D^{(0)}_{\mu\nu}(\boldsymbol{P}, \boldsymbol{Q})$ contains $\pi\pi$ phase shift in the vertices. This should be known prior to the calculation of the FFs
- The calculation implies the determination of the non-minimal couplings $g_i = g_i(k^2)$ at different k^2 separately. The infinite-volume FF is reconstructed by the use of the known expression
- Projection on the different irreps of the cubic group for various external field configurations: done

Lüscher function in the external field

- Example: temporal field $A^0 \neq 0$
- One-particle propagator:

$$S(\boldsymbol{p}, \boldsymbol{q}; p_0) = \sum_{\alpha} \frac{\psi_\alpha(\boldsymbol{p})\psi^*_\alpha(\boldsymbol{q})}{E_\alpha - p_0 - i\varepsilon}$$

- Eigenfunctions:

$$2w(\boldsymbol{p})(w(\boldsymbol{p}) - E_\alpha)\psi_\alpha(\boldsymbol{p}) - \frac{e}{L^3}\sum_{\boldsymbol{q}} \left(w(\boldsymbol{p})A^0(\boldsymbol{p}-\boldsymbol{q}) + A^0(\boldsymbol{p}-\boldsymbol{q})w(\boldsymbol{q})\right)\psi_\alpha(\boldsymbol{q}) = 0\,.$$

↪ Can be reduced to Mathieu equation at $O(e^2)$

↪ Lowest energy level in the Breit frame splits. At order e, splitting can be related to the form factor at $k^2 = -\omega^2$!!!

Conclusions, outlook

- We present the calculation of the ρ-meson form factor in the NREFT and ChPT
  - NREFT framework: no explicit ρ-field
  - ChPT framework: an explicit ρ-field in the Lagrangian
- The calculated results are strikingly different. The form factors rapidly vary in the vicinity of $k^2 = 0$, if no explicit ρ is present
- This variation can be canceled only by unnaturally large effective couplings

$\hookrightarrow$ The calculation of the form factors on the lattice can shed light on the structure of the QCD resonances

$\hookrightarrow$ Implementing an external field on the lattice, it is possible to directly extract the short-range part of the form factors, which feature only exponentially suppressed finite-volume corrections