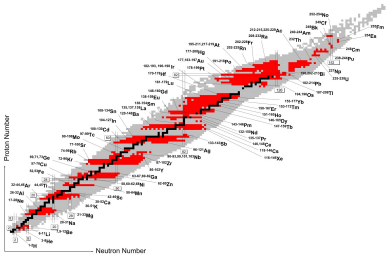
Nuclear structure effects to high-precision spectroscopy in hydrogen-like atoms

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Central China Normal University
Institute of Particle Physics

Workshop on Few-Body Problems in Nuclear and Hadron Physics
CCAST, Beijing · September 22, 2026

Hydrogen-like atoms as a probe of nuclear structure

- Atomic spectroscopy delivers nuclear structure observables
  - charge radius R_E, Zemach radius R_Z, magnetic radius R_M
  - magnetic dipole and electric quadrupole moments
- Precision constrains the nuclear interaction itself
  - tensor forces, meson-exchange and three-nucleon forces
- Light systems ($A \lesssim 9$) are where ab initio few-body theory can meet that precision

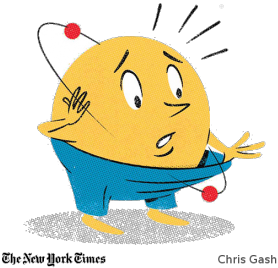


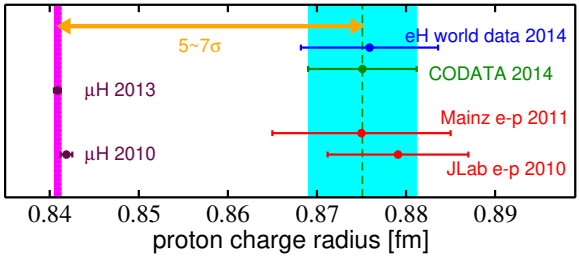
Laser spectroscopy reach across the nuclear chart

Nörtershäuser (TU Darmstadt)

Proton radius puzzle

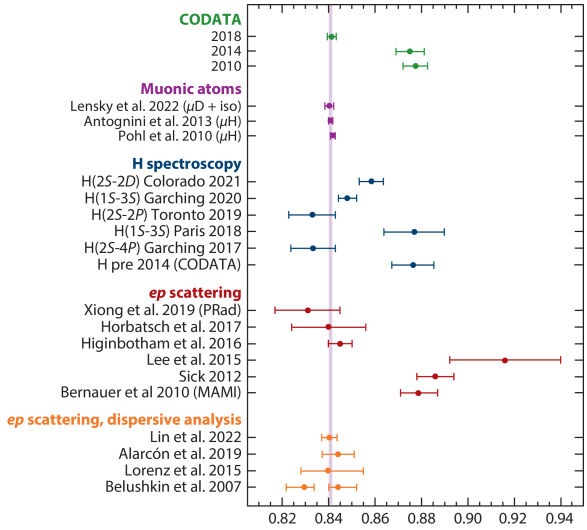
- Electron–proton experiments: $r_p = 0.8770(45)$ fm
  - eH atomic spectroscopy
  - $e-p$ elastic scattering
- Muon–proton experiments: $r_p = 0.8409(4)$ fm
  - μH atom Lamb shift (ΔE_{2S-2P}) [PSI-CREMA]
    Pohl *et al.*, Nature (2010); Antognini *et al.*, Science (2013)





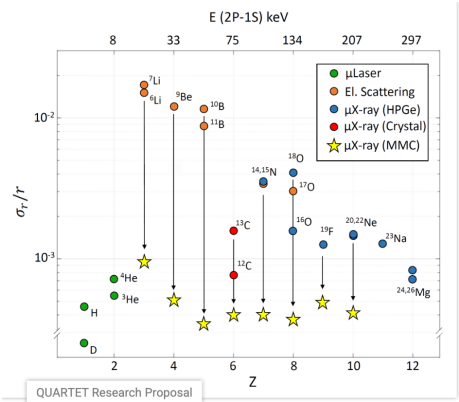
Resolving the proton radius puzzle

- New experiments to measure the proton radius
  - $e - p$ scattering (JLab, Mainz, Tohoku U.)
  - $\mu - p$ scattering (PSI-MUSE)
  - eH spectroscopy (MPQ, LKB, York U.)



Nuclear charge radii across light muonic atoms

- Muonic atoms/ions: Lamb shift (PSI–CREMA)
  - $\mu^2\mathrm{H}$ [Pohl *et al.*, Science 2016]
  - $\mu^4\mathrm{He}^+$ [Krauth *et al.*, Nature 2021]
  - $\mu^3\mathrm{He}^+$ [Schuhmann *et al.*, Science 2025]
- Few-electron atoms: isotope shift spectroscopy
  - $e^{1-2}\mathrm{H}$, $e^{3-4}\mathrm{He}$, $e^{4-6,8}\mathrm{He}$, $e^{7-6,8,9,11}\mathrm{Li}$
- Coming: QUARTET — microcalorimeter X-ray spectroscopy, Li to Ne



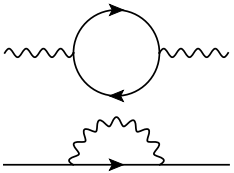
QUARTET precision goal

Nuclear structure in atomic Lamb shift spectroscopy

- Extracting nuclear charge radii from muonic atom Lamb shift

$$\delta E_{\rm LS} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\, R_E^2 + \delta_{\rm TPE}$$

- **QED corrections:**
  - Vacuum polarization
  - Lepton self-energy corrections
  - Relativistic recoil corrections

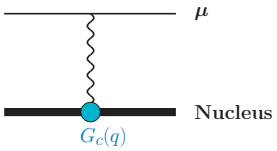


Nuclear structure in atomic Lamb shift spectroscopy

- Extracting nuclear charge radii from muonic atom Lamb shift

$$\delta E_{\rm LS} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\, R_E^2 + \delta_{\rm TPE}$$

- **Nuclear structure corrections:**
  - $\propto R_E^2 \Longrightarrow$ **nuclear structure in one-photon exchange**
    $\mathcal{A}_{\rm OPE} \approx m_\mu^3 (Z\alpha)^4/12$

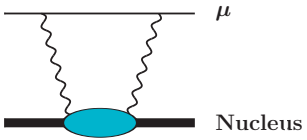


Nuclear structure in atomic Lamb shift spectroscopy

- Extracting nuclear charge radii from muonic atom Lamb shift

$$\delta E_{\rm LS} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\, R_E^2 + \delta_{\rm TPE}$$

- **Nuclear structure corrections:**
  - $\delta_{\rm TPE} \Longrightarrow$ **nuclear structure in two-photon exchange**
    - Elastic contribution: Zemach moment $\delta_{\rm Zem}$
    - Inelastic contribution: nuclear polarizability $\delta_{\rm pol}$



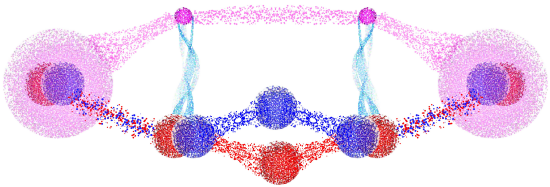
- **Accuracy of extracted nuclear radius R_E depends on theoretical input of $\delta_{\rm TPE}$**

  μ^2H experiment: $\delta_{\rm pol}$ requires 1% theoretical precision

  $\mu^{3,4}$He$^+$ experiments: $\delta_{\rm pol}$ requires 5% theoretical precision

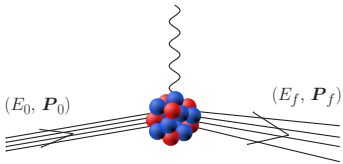
Nuclear polarizability from photonuclear sum rules

$$\delta_{\rm pol} = \sum_{g,\, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega \; \underbrace{g\,(\omega)}_{\text{weight}} \; \underbrace{S_{\hat{O}}\,(\omega)}_{\text{nuclear response function}}$$



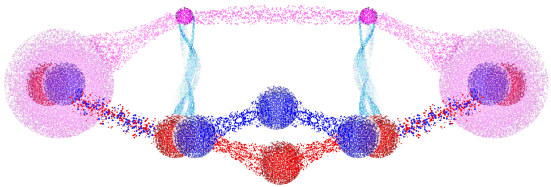
- Energy sum-rule weight $g\,(\omega)$
- Nuclear response function $S_{\hat{O}}\,(\omega)$

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$$



Nuclear polarizability from photonuclear sum rules

$$\delta_{\rm pol} = \sum_{g, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega \underbrace{g(\omega)}_{\text{weight}} \underbrace{S_{\hat{O}}(\omega)}_{\text{nuclear response function}}$$



Contributions to nuclear polarizability $\delta_{\rm pol}$ in muonic atoms:

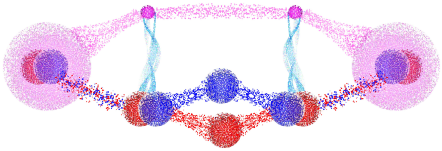
- Electromagnetic multipole expansion
  - E0, E1, E2 response sum rules
- Relativistic and Coulomb distortion corrections
- Intrinsic nucleon structure corrections

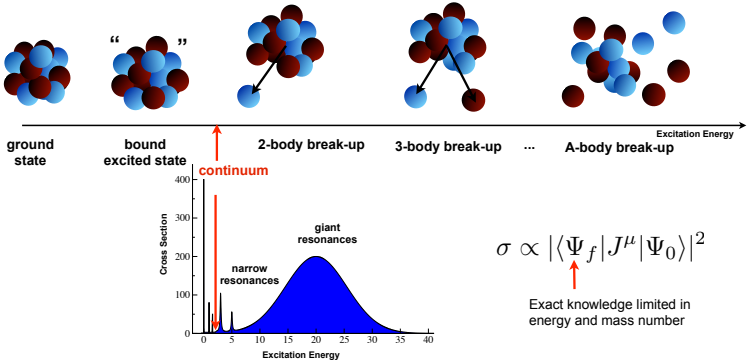
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45, 093002 (2018)

Nuclear response function: continuum spectrum

- The nucleus is virtually excited in two-photon exchange

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$





$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Exact knowledge limited in energy and mass number

Ab initio calculations of nuclear polarizability $\delta_{\rm pol}$

- $\mu^{2,3}$H, $\mu^{3,4}$He$^+$:
  - Ab initio numerical methods

    Effective Interaction Hyperspherical Harmonics (EIHH)
    Lorentz Integral Transform (nuclear response functions)
    Lanczos Algorithm (sum rules)

    bound states $\rightarrow$ resonance/scattering states
  - Nucleon–nucleon interactions

    AV18+UIX
    χEFT NN(N^{3}LO)+NNN(N^{2}LO)
    $\not\pi$EFT (N^{2}LO)
    Compare $\delta_{\rm pol}$ from different nuclear force models to estimate nuclear theory uncertainties

CJ, Nevo-Dinur, Bacca, Barnea, PRL 111, 143402 (2013)
Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, PLB 736, 344 (2014)
Nevo Dinur, CJ, Bacca, Barnea, PLB 755, 380 (2016)
Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, PLB 788, 377 (2018)
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45, 093002 (2018)
Hernandez, Ekström, Nevo, CJ, Bacca, Barnea, PLB 788, 377 (2018)
Nevo, Hernandez, Bacca, Barnea, CJ, Pastore, Piarulli, Wiringa, PRC 99, 034004 (2019)
Emmons, CJ, Platter, JPG 48, 035101 (2021)
CJ, Zhang, Platter, PRL 133, 042502 (2024)
Bonilla, Richardson, Bacca, CJ, Platter, PLB 874, 140257 (2026)

Effective Interaction Hyperspherical Harmonics Method

For heavier nuclei one must go beyond the Lippmann–Schwinger equation and adopt more powerful few-body methods, able to treat bound and continuum states on the same footing.

- Many-body Schrödinger equation
  $[T+V]\psi(\vec{\eta}_1,\vec{\eta}_2) = E\psi(\vec{\eta}_1,\vec{\eta}_2)$
- Hyperspherical coordinates
  $\rho = \sqrt{\eta_1^2+\eta_2^2}$, $\Omega = [\theta_1,\phi_1,\theta_2,\phi_2,\arctan(\frac{\eta_2}{\eta_1})]$
  $T = T_\rho + \hat{K}^2/\rho^2$
- Hyperspherical Harmonics Basis Expansion

$$\psi(\vec{\eta}_1,\vec{\eta}_2) \sim \sum_{[K]}^{K_{max}} R_{[K]}(\rho)\ \mathcal{Y}_{[K]}(\Omega)$$

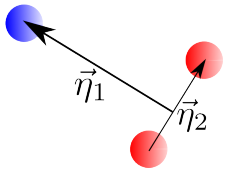
$$\hat{K}^2\mathcal{Y}_{[K]}(\Omega) = K(K+4)\mathcal{Y}_{[K]}(\Omega)$$

- Hamiltonian in finite model space P: $K = \{0,1,\cdots,K_{max}\}$

$$H = PH_0P + PV^{eff}P$$

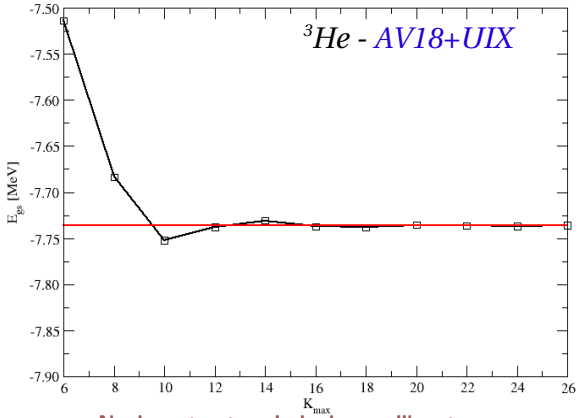
V^{eff} Lee-Suzuki transformation (soften interactions)

three-body system



Convergence in Hyperspherical Harmonics Expansion

$$|\psi\rangle = \sum_{K}^{K_{max}} c_K \, \mathrm{HH}(K)$$



Lorentz Integral Transform

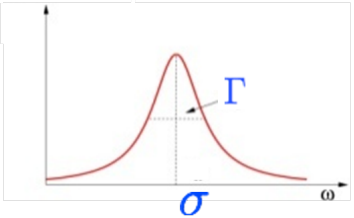
- Response function involves continuum states

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$$

- LIT maps it onto a bound-state-like equation

$$\mathcal{L}(\sigma,\Gamma) = \int d\omega \frac{S_O(\omega)}{(\sigma-\omega)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

$$(H - E_0 - \sigma + i\Gamma)|\tilde{\psi}\rangle = \hat{O}|\psi_0\rangle$$

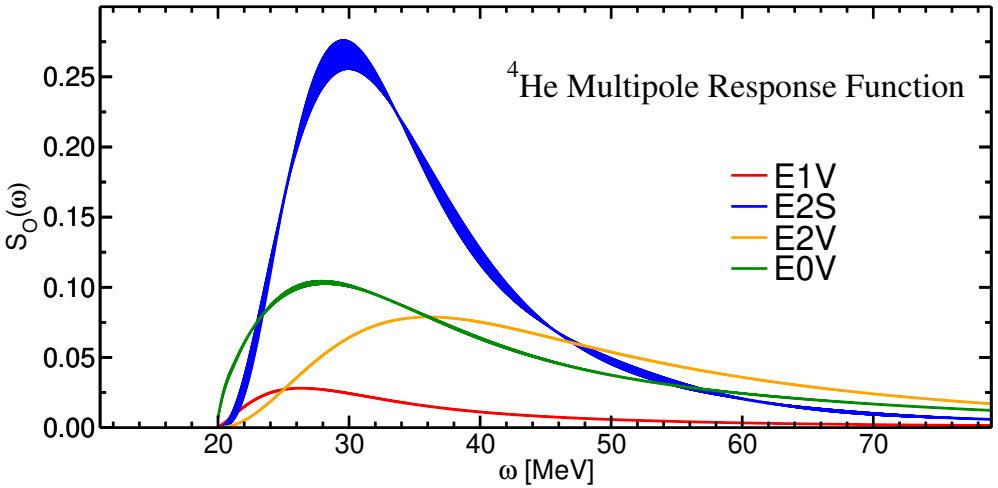


Efros *et al.*, J. Phys. G 34, R459 (2007)

- Invert the transform to recover $S_O(\omega)$

> $|\tilde{\psi}\rangle$ has bound-state asymptotics — so the LIT is compatible with most few-body bound-state basis expansions.

Response functions from the LIT



Generalized Lanczos sum rules

- $\delta_{\rm pol} \Longrightarrow$ photonuclear response sum rules converge slowly via direct diagonalization

$$I_{\hat{O}} = \int_0^\infty d\omega\, S_{\hat{O}}(\omega)\, g(\omega) \approx \sum_{n\neq 0}^{N_{max}} |\langle 0|\hat{O}|n\rangle|^2 g(\omega_n)$$

- Use the Lanczos method to project I_O onto an effective subspace, avoiding direct calculation of $S_{\hat{O}}(\omega)$
  - Map the Hamiltonian onto a Krylov subspace

$$V = \{|\phi_0\rangle, |\phi_1\rangle, \cdots, |\phi_M\rangle\}$$

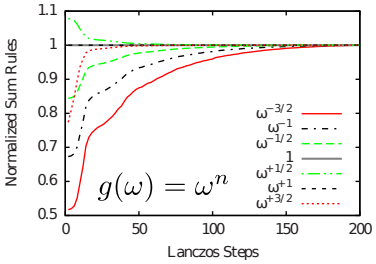
$$b_{i+1}|\phi_{i+1}\rangle = \hat{H}|\phi_i\rangle - a_i|\phi_i\rangle - b_i|\phi_{i-1}\rangle$$

$$|\phi_{-1}\rangle = 0; \quad |\phi_0\rangle = \frac{\hat{O}|0\rangle}{\sqrt{\langle 0|\hat{O}^\dagger\hat{O}|0\rangle}}; \quad \langle\phi_i|\phi_j\rangle = \delta_{ij}$$

$$\tilde{H}_M = QDQ^\dagger; \quad D = \text{diag}\{E_0, E_1, \cdots, E_\nu, \cdots, E_M\}$$

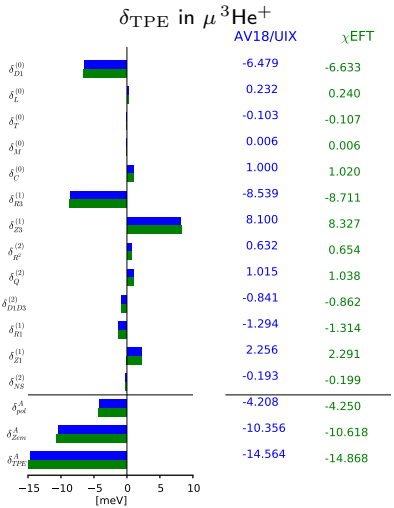
  - $I_{\hat{O}}$ converges as the number of Lanczos iterations increases

$$I_{\hat{O}} \approx \langle 0|\hat{O}^\dagger\hat{O}|0\rangle \sum_{\nu\neq 0}^{M} |Q_{\nu 0}|^2 g(\omega_\nu); \quad M \ll N_{max}$$

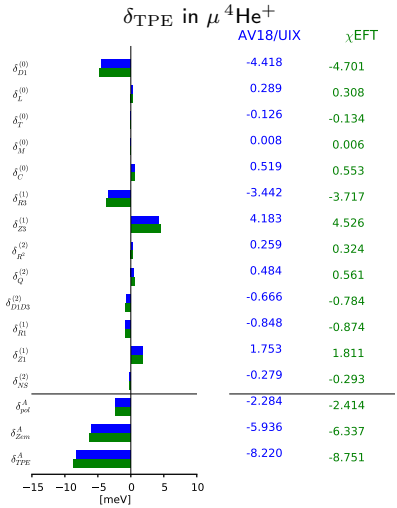


Nevo-Dinur, Barnea, CJ, Bacca, PRC 89, 064317 (2014)

TPE & nuclear polarizability: nuclear-model uncertainty



$\delta_{\rm TPE} = -14.72$ meV $\pm 1.5\%(1\sigma)$

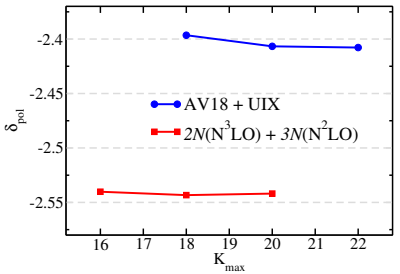


$\delta^{A}_{\rm TPE} = -8.49$ meV $\pm 4.4\%(1\sigma)$

TPE & nuclear polarizability: other uncertainty

Numerical uncertainty

- convergence of EIHH model space (μ^4He$^+$)



Atomic-physics uncertainty

- $(Z\alpha)^6$ correction three-photon exchange
- relativistic and Coulomb distortion beyond E1
- higher-order nucleonic-structure corrections
- Overall atomic-physics uncertainty
  - 1.5% in μ^3He$^+$
  - 1.3% in μ^4He$^+$

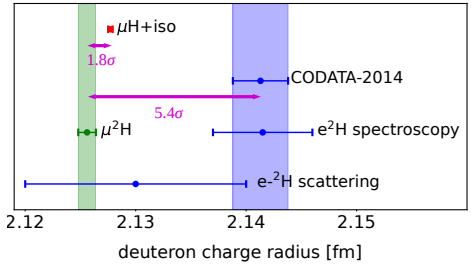
- Combine all uncertainties:

  $\delta_{\rm TPE}(\mu^3{\rm He}^+) = -14.72$ meV $\pm 2.1\%$

  $\delta_{\rm TPE}(\mu^4{\rm He}^+) = -8.49$ meV $\pm 4.6\%$
- The TPE prediction fulfills the 5% accuracy requirements from $\mu^{3,4}$He$^+$ experiments

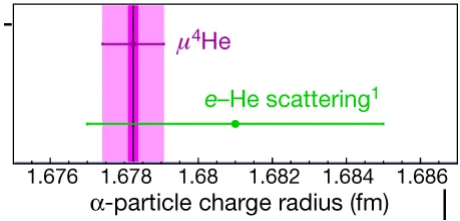
Charge radii from μ^2H and μ^4He$^+$ Lamb shift

- Nuclear polarizability predictions used by PSI experiments to extract nuclear radii
- Uncertainty of extracted charge radii is dominated by nuclear polarizability theory uncertainty



$r_d = 2.12562(13)_{\rm exp}(77)_{\rm theo}$ fm

Pohl, *et al.*, Science (2016)



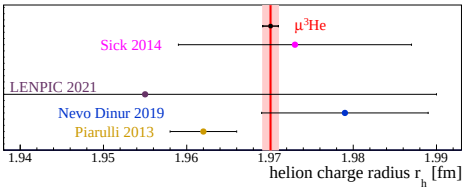
$r_\alpha = 1.67824(13)_{\rm exp}(82)_{\rm theo}$ fm

Krauth et al., Nature (2021)

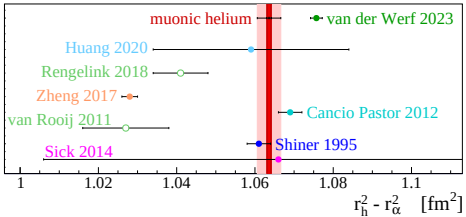
Our nuclear polarizability studies:
Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, PLB 736, 344 (2014); PRC 100, 064315 (2019) (μ^2H)
Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, PLB 788, 377 (2018) (μ^2H)
CJ, Nevo-Dinur, Bacca, Barnea, PRL 111, 143402 (2013) (μ^4He$^+$)
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45, 093002 (2018) ($\mu^{2,3}$H, $\mu^{3,4}$He$^+$)

Charge radius from $\mu^3\mathrm{He}^+$ Lamb shift

- Nuclear polarizability predictions used by PSI experiments to extract nuclear radii
- Uncertainty of extracted charge radii is dominated by nuclear polarizability theory uncertainty



$r_h = 1.97007(12)_{\mathrm{exp}}(93)_{\mathrm{theo}}$ fm

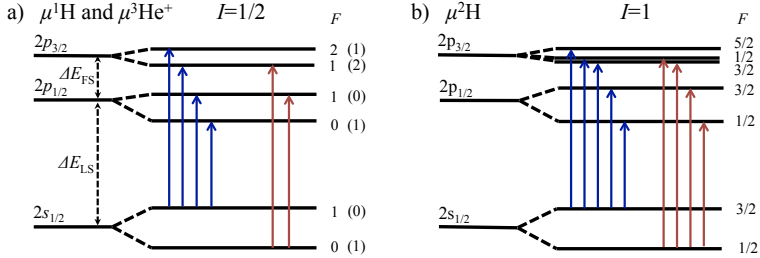


$r_h^2 - r_\alpha^2 = 1.0636(6)_{\mathrm{exp}}(30)_{\mathrm{theo}}$ fm^2

Schuhmann et al. (CREMA) Science (2025)

Our nuclear polarizability studies:
Nevo Dinur, CJ, Bacca, Barnea, PLB 755, 380 (2016) (μ^3H, $\mu^3\mathrm{He}^+$)
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45, 093002 (2018) ($\mu^{2,3}$H, $\mu^{3,4}\mathrm{He}^+$)

Effective Zemach radius from atomic HFS

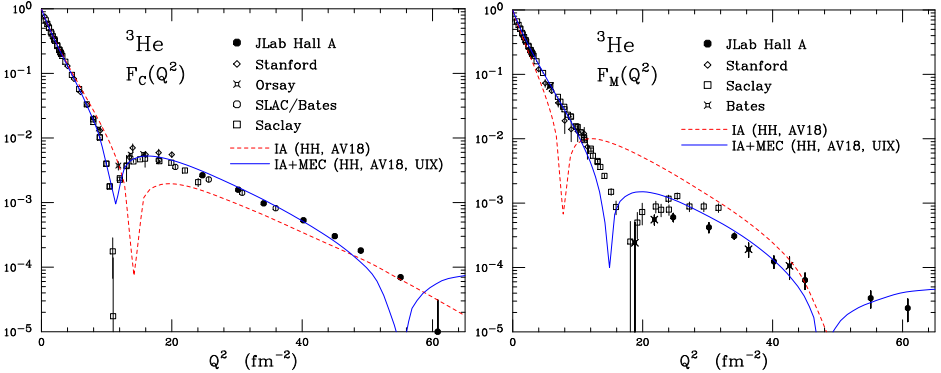


- Zemach radius R_Z is determined by both nuclear charge and magnetization distributions

$$R_Z = \iint d\boldsymbol{r} d\boldsymbol{r}' \rho_E(\boldsymbol{r}) \rho_M(\boldsymbol{r}') |\boldsymbol{r} - \boldsymbol{r}'|$$
$$= -\frac{4}{\pi} \int_0^\infty \frac{dq}{q^2} [G_E(q) G_M(q) - 1] \qquad \neq \text{ effective Zemach radius from HFS}$$

Discrepancy in studies of nuclear magnetization form factor

- ^{3}**He** charge form factor F_C: experiment agrees with theory
- ^{3}**He** magnetic form factor F_M: discrepancy between experiment and theory at high momentum
- Muonic atom spectroscopy measures the magnetization distribution more precisely



Nuclear effects in atomic HFS: challenges

- Lack of systematic studies of nuclear effects in hyperfine spectra
  - How does nuclear polarizability scale with Z and A?

- In the absence of direct calculations, extract nuclear polarizability via:
  - HFS spectroscopy − QED theory → nuclear effects (effective Zemach radius $\tilde{R}_Z$)
  - Form factors from electron–nucleus scattering: compute Zemach radius R_Z
  - $\tilde{R}_Z - R_Z$: nuclear polarizability

Effective Zemach radius: p and d

- HFS spectroscopy $\rightarrow$ effective Zemach radius $\tilde{R}_Z$:

$$\nu_{\rm TPE} = \nu_{\rm exp} - \nu_{\rm QED} = -2Z\alpha m_R E_F \tilde{R}_Z; \qquad m_R = \frac{m_l m_{nucl}}{m_l + m_{nucl}}$$

	$\nu_{\rm TPE} = \nu_{\rm exp} - \nu_{\rm QED}$	$\tilde{R}_Z$ [fm]	R_Z [fm]	$\Delta\%$
$e\mathrm{H}_{1S}$	-47.4(1) kHz	0.883(2)	1.054(3)	-16%
$\mu\mathrm{H}_{1S}$	-1.161(2) meV	0.906(2)		-14%
$e^2\mathrm{H}_{1S}$	45.2 kHz	-3.66	2.593(16)	-240%
$\mu^2\mathrm{H}_{2S}$	0.0966(73) meV	-2.13(16)		-180%

- $e^2\mathrm{H}_{1S}$: Wineland, Ramsey, PRA (1972)
- $\mu^2\mathrm{H}_{2S}$: Krauth *et al.*, Ann. Phys. 366, 168 (2016); Pohl *et al.*, Science 353, 669 (2016)

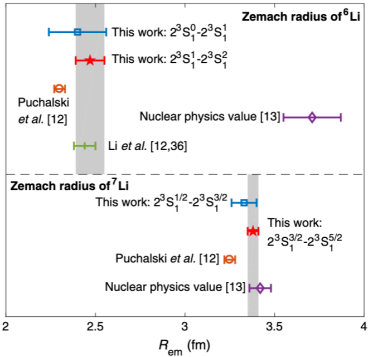
Effective Zemach radius: light nuclei

- HFS spectroscopy $\rightarrow$ effective Zemach radius $\tilde{R}_Z$

	$\nu_{\mathrm{TPE}} = \nu_{\mathrm{exp}} - \nu_{\mathrm{QED}}$	$\tilde{R}_Z$ [fm]	R_Z [fm]	$\Delta\%$
$e^3\mathrm{He}^+_{1S}$	1701.0(5.5) kHz	2.600(8)	2.528(16)	3%
$\mu^3\mathrm{He}^+_{2S}$	6.25(10) meV	2.42(39)		-4%
$e^9\mathrm{Be}^+_{1S}$		4.03(5)	3.38(6)	19%
$e^9\mathrm{Be}^{3+}_{1S}$		4.048(2)		20%

- ^{7}Li: $\tilde{R}_Z$ agrees with R_Z from nuclear models
- ^{6}Li: large discrepancy between $\tilde{R}_Z$ and R_Z
- $\delta_{pol}(^7\mathrm{Li}) \ll \delta_{pol}(^6\mathrm{Li})$?

Schneider '22; Schuhmann '25; Puchalski & Pachucki '14; Dickopf '24; Puchalski & Pachucki '13; Sun '23



Nuclear structure effects in hyperfine splitting spectra

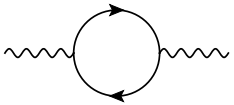
- Extracting Zemach radius from e/μ atom HFS

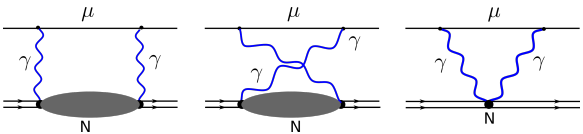
$$E_{\rm HFS}(nS) = E_F(1 + \delta_{\rm QED} + \delta_{\rm TPE})$$

- Fermi contact term
  - Nuclear–lepton spin coupling

$$E_F = \frac{2\pi\alpha g_m}{3 m_\ell m_N} \phi_n^2(0) \langle \vec{\sigma}^{(\ell)} \cdot \vec{I} \rangle$$

- QED corrections
  - Vacuum polarization
  - Lepton self-energy
  - Relativistic recoil
- Two-photon exchange (nuclear structure effects)
  - Nuclear polarizability
  - Elastic contribution (Zemach radius)



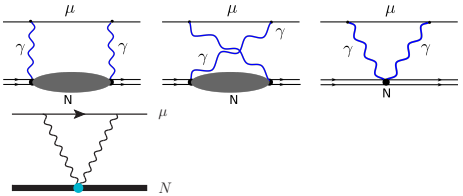


Two-photon corrections in ^{2}H and μ^2H hyperfine spectra

- Two-photon exchange corrections

$$E_{\rm TPE} = E_{\rm el} + E_{\rm pol} + E_{\rm 1N}$$

- Elastic part: $F_c(q)$, $F_m(q)$, $F_Q(q)$: $\sim r_Z$
- Inelastic vector polarizability
- $E_{\rm 1N}$: single-nucleon TPE (nucleon structure)



$$\delta^{(0,1)}_{\rm pol} \propto \int d\omega \int dq h^{(0,1)}(\omega,q) S^{(0,1)}(\omega,q)$$

Charge–magnetization current coupling:

$$S^{(0)}(\omega,q) = -\frac{1}{q^2}{\rm Im} \sum_{N\neq N_0} \int \frac{d\hat{q}}{4\pi} \langle N_0 II| \left[\vec{q}\times \vec{J}^\dagger_m(\vec{q})\right]_3 |N\rangle\langle N|\rho(\vec{q})|N_0 II\rangle\, \delta\Big(\omega - \tfrac{q^2}{2m_A} - \omega_N\Big)$$

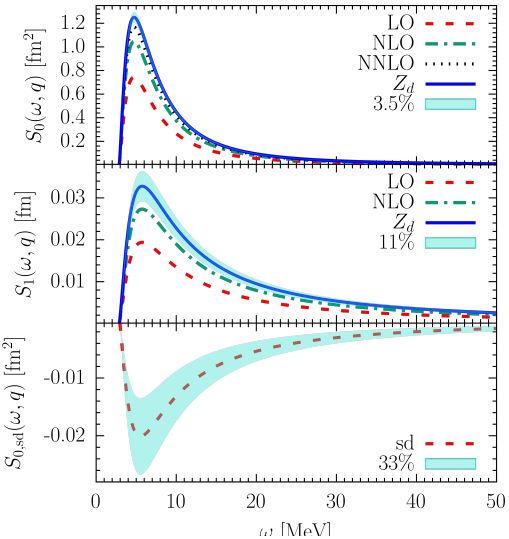
Convection–magnetization current coupling:

$$S^{(1)}(\omega,q) = -{\rm Im} \sum_{N\neq N_0} \int \frac{d\hat{q}}{4\pi} \epsilon^{3jk} \langle N_0 II|\vec{J}^\dagger_{m,j}(\vec{q})|N\rangle\langle N|\vec{J}_{c,k}(\vec{q})|N_0 II\rangle\, \delta\Big(\omega - \tfrac{q^2}{2m_A} - \omega_N\Big)$$

- Pionless EFT calculation CJ*, Zhang, Platter, PRL 133, 042502 (2024)

Polarizability response functions for HFS TPE

- $S^{(0)}(\omega, q)$: charge-density to magnetization-current transition (LO)
- $S^{(1)}(\omega, q)$: convection-current to magnetization-current transition (NLO)
- $S^{(0)}_{\rm sd}(\omega, q)$: SD-coupling correction (NNLO)
- Nuclear response functions converge order by order



CJ*, Zhang, Platter, PRL 133, 042502 (2024)

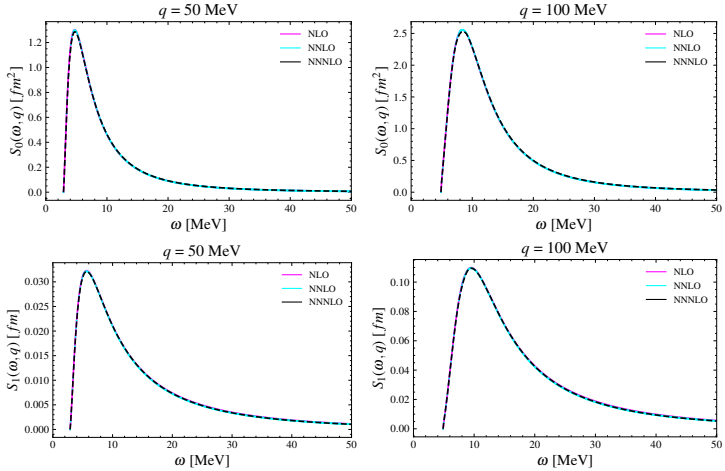
^{2}H and μ^2H HFS two-photon corrections (pionless EFT)

	^{2}H (1S)	μ^2H (1S)	μ^2H (2S)
$E_{1\mathrm{p}}$ (Antognini 2022)	$-35.54(8)$	$-1.018(2)$	$-0.1272(2)$
$E_{1\mathrm{n}}$ (Tomalak 2019)	$9.6(1.0)$	$0.08(3)$	$0.010(4)$
E_{el}	$-42.1(2.1)$	$-0.984(46)$	$-0.123(6)$
E_{pol}	$109.8(4.5)$	$2.86(12)$	$0.358(14)$
E_{TPE}	kHz	meV	meV
This work	$41.7(4.4)$	$0.94(11)$	$0.117(13)$
Khriplovich, Milstein 2004	43		
Friar, Payne 2005 $_{\mathrm{mod}}$	64.5		
Kalinowski, Pachucki 2018		$0.304(68)$	$0.0383(86)$
$\nu_{\mathrm{exp}} - \nu_{\mathrm{qed}}$	45.2		$0.0966(73)$

- Calculations explain the experiment–QED discrepancy within $0.8 - 1.3\sigma$
- Theoretical uncertainty of single-nucleon corrections may be underestimated (ChPT and dispersion-relation calculations differ by a factor of 10)

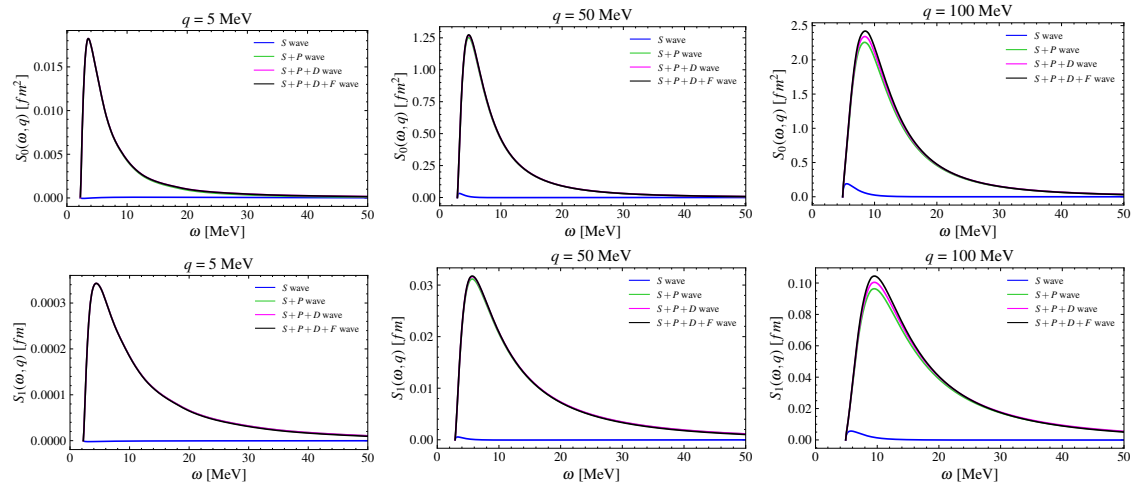
CJ*, Zhang, Platter, PRL 133, 042502 (2024)

Nuclear response functions from χEFT (convergence analysis)



Bonilla, Richardson, Bacca, CJ, Platter, PLB 874, 140257 (2026)

Nuclear response functions from χEFT (partial-wave analysis)



Bonilla, Richardson, Bacca, CJ, Platter, PLB 874, 140257 (2026)

^{2}H, μ^2H HFS polarizability: χEFT vs $\not\pi$EFT

	e^2H (1S) [kHz]	μ^2H (1S) [meV]	μ^2H (2S) [meV]
$\nu_{\rm EXP} - \nu_{\rm QED}$	45	-	0.0966(73)
$\Delta_{2\gamma}$ ($\not\pi$EFT)	41.7(2.6)	0.940(73)	0.118(9)
$\Delta_{2\gamma}$ (χEFT)	44.5(1.1)	0.995(57)	0.124(7)
Khriplovich 2004	43		
Friar 2005	64.5		
Kalinowski, Pachucki 2018		0.304(68)	0.0383(86)

- χEFT prediction of TPE effects agrees with $\not\pi$EFT prediction
- Uncertainty mainly from higher-order χEFT terms and 3γ-exchange contributions

Bonilla, Richardson, Bacca, CJ, Platter, PLB 874, 140257 (2026)

Nuclear effects in other atomic hyperfine spectra

- Exact approach: Lanczos sum rules
  - Nuclear effects in HFS: EM operators and excitation modes more complex than Lamb shift
  - Excited-state calculations converge slowly with large numerical cost

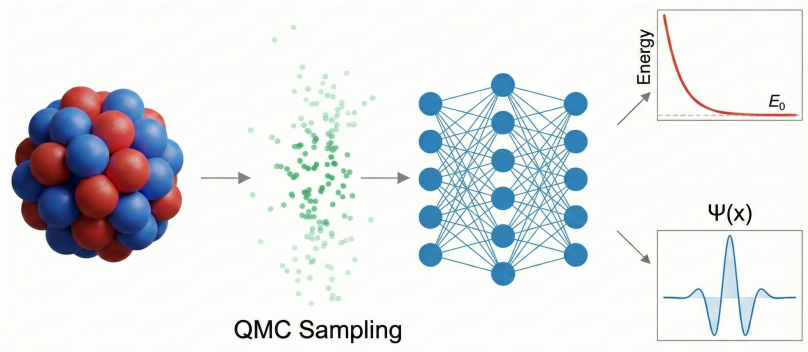
- Approximate approach: closure approximation
  - Zero-excitation-energy approximation $\omega \to 0$ (Low term)

  - Finite-excitation-energy closure:
    - Set $\omega = \bar{\omega}$ as a finite but fixed parameter
    - Apply closure $\sum |N\rangle\langle N| = 1 - |0\rangle\langle 0|$ at fixed $\bar{\omega}$
    - Reduce nuclear polarizability to ground-state observables

Yang, Epelbaum, CJ, Zhao, arXiv:2509.01303

Neural-network variational Monte Carlo

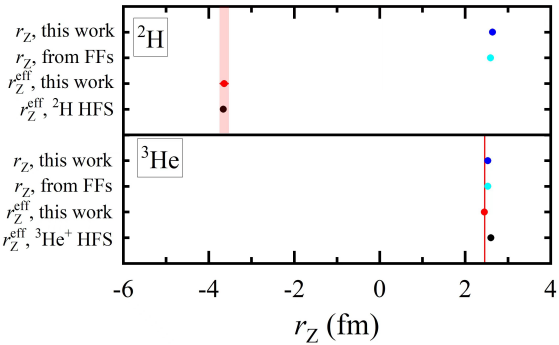
- Use neural networks to optimize parameters of quantum many-body wave functions
- Train the network variationally to minimize the ground-state binding energy
- Use the high-precision ground-state wave function to compute TPE observables

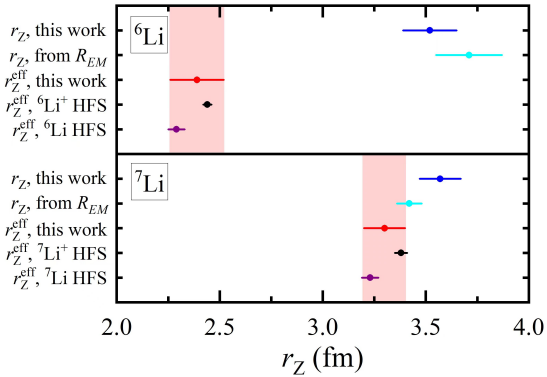


Yang, Epelbaum, CJ, Zhao, arXiv:2509.01303

Effective Zemach radius via closure approximation

- Elastic R_Z: Monte Carlo results agree with experimental extraction from form factors
- Effective Zemach radius ($\tilde{R}_Z$): weak dependence on closure energy (stable for ω: 10–30 MeV)
- Tune $\bar{\omega}$ to best-fit $\tilde{R}_Z$



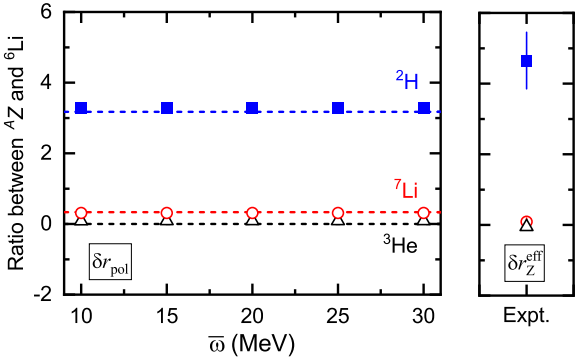


Yang, Epelbaum, CJ, Zhao, arXiv:2509.01303

SU(4) symmetry of $E_{\rm pol}$ in HFS

- The inelastic (polarizability) effect to HFS unveils SU(4) symmetry:
  - spins of even protons and neutrons saturate
  - nuclear magnetic moment is dominated by unpaired proton and neutron
  - $E_{\rm pol}$ is driven by unpaired proton δr_p only ($\delta r_n \approx 0$)

$$\delta r_{\rm pol}(\bar{\omega}) \simeq \begin{cases} -\frac{1}{Z}\frac{\mu_p}{\mu_p+\mu_n}\delta r_p(\bar{\omega}), & \text{odd } Z, \text{odd } N, \\ -\frac{1}{Z}\delta r_p(\bar{\omega}), & \text{odd } Z, \text{even } N, \\ 0, & \text{even } Z, \text{odd } N, \end{cases}$$



Yang, Epelbaum, CJ, Zhao, arXiv:2509.01303

Summary

- **Few-body methods deliver the nuclear input**
  - HH expansion + LIT + Lanczos sum rules → *ab initio* response functions, bound and continuum on the same footing
  - Uncertainty quantified, not estimated: model spread, model-space convergence, atomic input
- **Lamb shift: this is the bottleneck**
  - TPE / nuclear polarizability limits charge-radius extraction in μ^2H and $\mu^{3,4}\mathrm{He}^+$
  - Theory input at percent-level precision is *more* accurate than δ_{pol} from photonuclear data
- **The same machinery extends to hyperfine splitting**
  - Effective Zemach radii for p, d, ^{3}He, ^{6,7}Li, ^{9}Be — discrepancies against nuclear-model R_Z
  - χEFT vs $\not\pi$EFT for ^{2}H and μ^2H HFS polarizability
  - SU(4) symmetry: E_{pol} driven by the unpaired proton
- **Outlook**
  - Nuclear-model uncertainty remains the dominant systematic
  - Open: the ^{6}Li effective-Zemach discrepancy; QUARTET data from Li to Ne

Collaborators

Ab initio muonic-atom Lamb shift

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S. Bacca *(Mainz)*
N. Nevo-Dinur
N. Barnea *(Hebrew U.)*
A. Ekström *(Chalmers)*

Two-nucleon currents

S. Pastore, M. Piarulli *(Wash. U.)*
R.B. Wiringa *(Argonne)*

Pionless EFT

S.B. Emmons *(Carson-Newman)*
L. Platter *(U. Tennessee)*
X. Zhang (CCNU)

χEFT response functions

S. Bacca, J. Richardson *(Mainz)*
A. Bonilla, L. Platter *(U. Tennessee)*

Neural-network VMC

E. Epelbaum *(Bochum)*
Y.L. Yang, P. Zhao *(Peking U)*

Thank you for your attention!