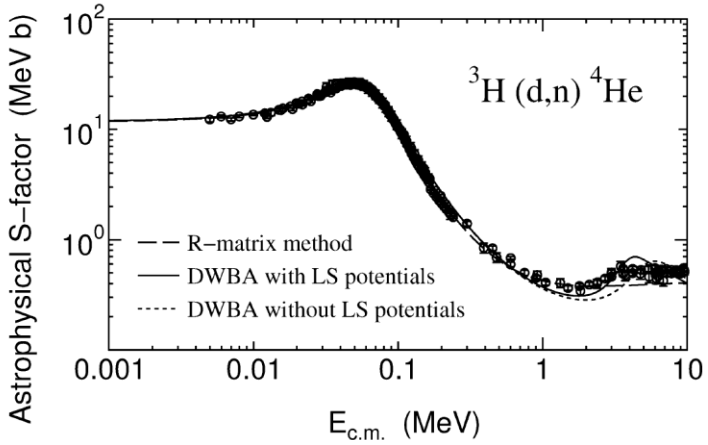
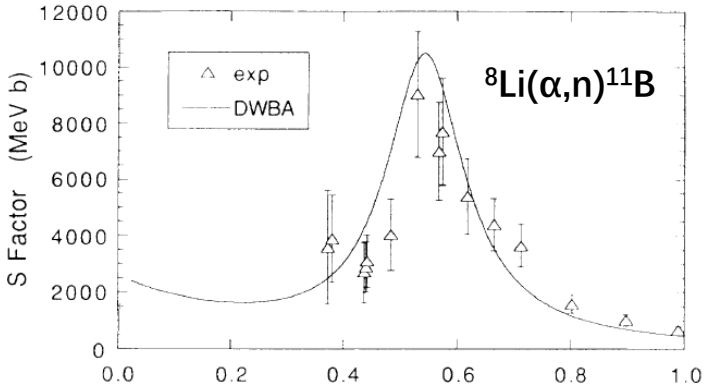




M. Katsuma, AIP Conf. Proc. 891, 355 (2007)



T. Rauscher, PRC 45, 1996 (1992)

Reaction mechanisms of low-energy nuclear reactions on light nuclei

Pang Danyang（庞丹阳）

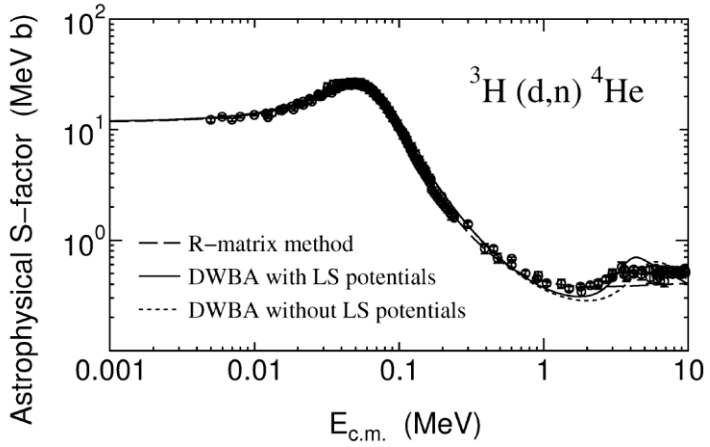
School of Physics, Beihang University

22th Sept. 2026

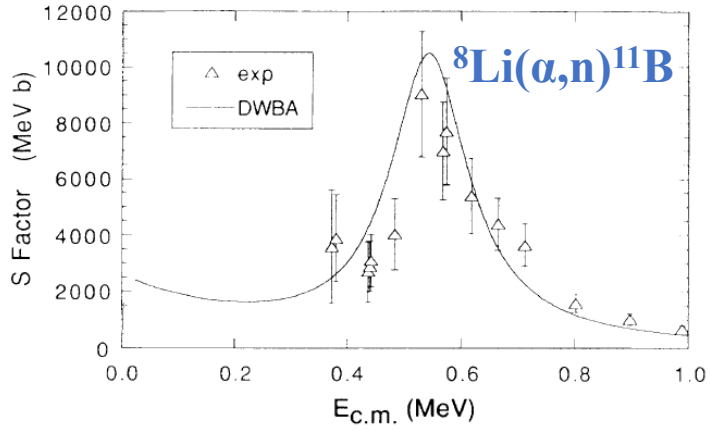
Low energy direct nuclear reactions on light nuclei

Low energy reactions described with the DWBA

targets	reactions	Q-value
^{3}H	(d,n)	17.59
^{3}He	(d,p)	18.35
^{3}H	(^{3}H,2n)	11.33
^{3}He	(^{3}He,2p)	12.86
^{6}Li	(p,^{3}He)	4.02
^{6}Li	(d,α)	22.37
^{7}Li	(p,α)	17.35
^{8}Li	(α,n)	6.63
^{9}Be	(p,d)	0.56
^{9}Be	(p,α)	2.15
^{10}B	(p,α)	1.15
^{11}B	(p,α)	8.59
^{19}F	(p,α)	8.11



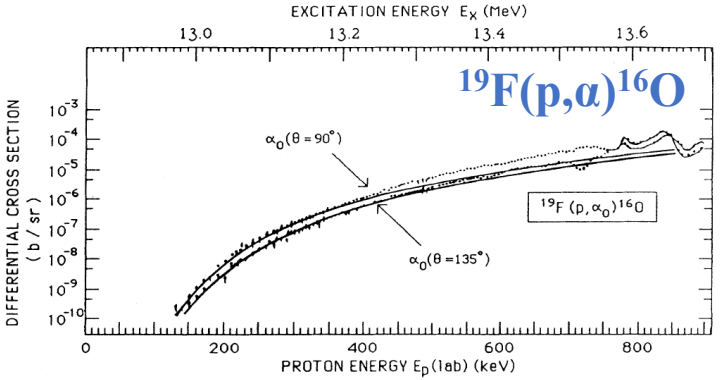
M. Katsuma, AIP Conf. Proc. 891, 355 (2007)



T. Rauscher, PRC 45, 1996 (1992)

Topics

- Direct *vs* compound reactions?
- Resonance states
- Cluster structure input
- Optical model potentials?



H. Herndl, PRC 44, R952 (1991)

Why direct nuclear reaction mechanism at such low energy?

Compound nuclear reactions

- Competition among many open reaction channels
- High level densities (statistical theory applicable)

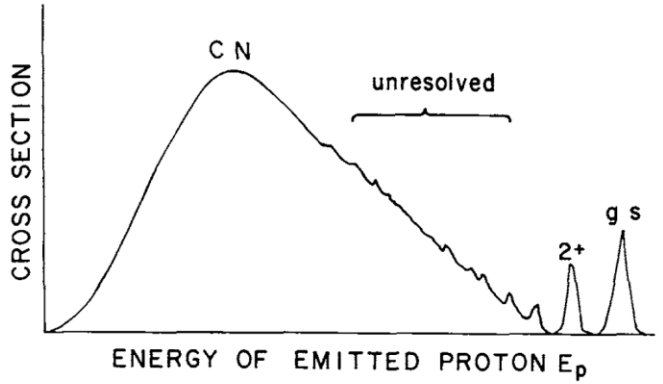


Fig. 1.1. Schematic proton spectrum from proton–nucleus scattering showing a few resolved levels near the beam energy, unresolved, but discrete, levels, and the compound-nucleus continuum. N.K. Glendenning, direct nuclear reactions, 1983

Reaction channels of the p+^{11}B system below 2 MeV

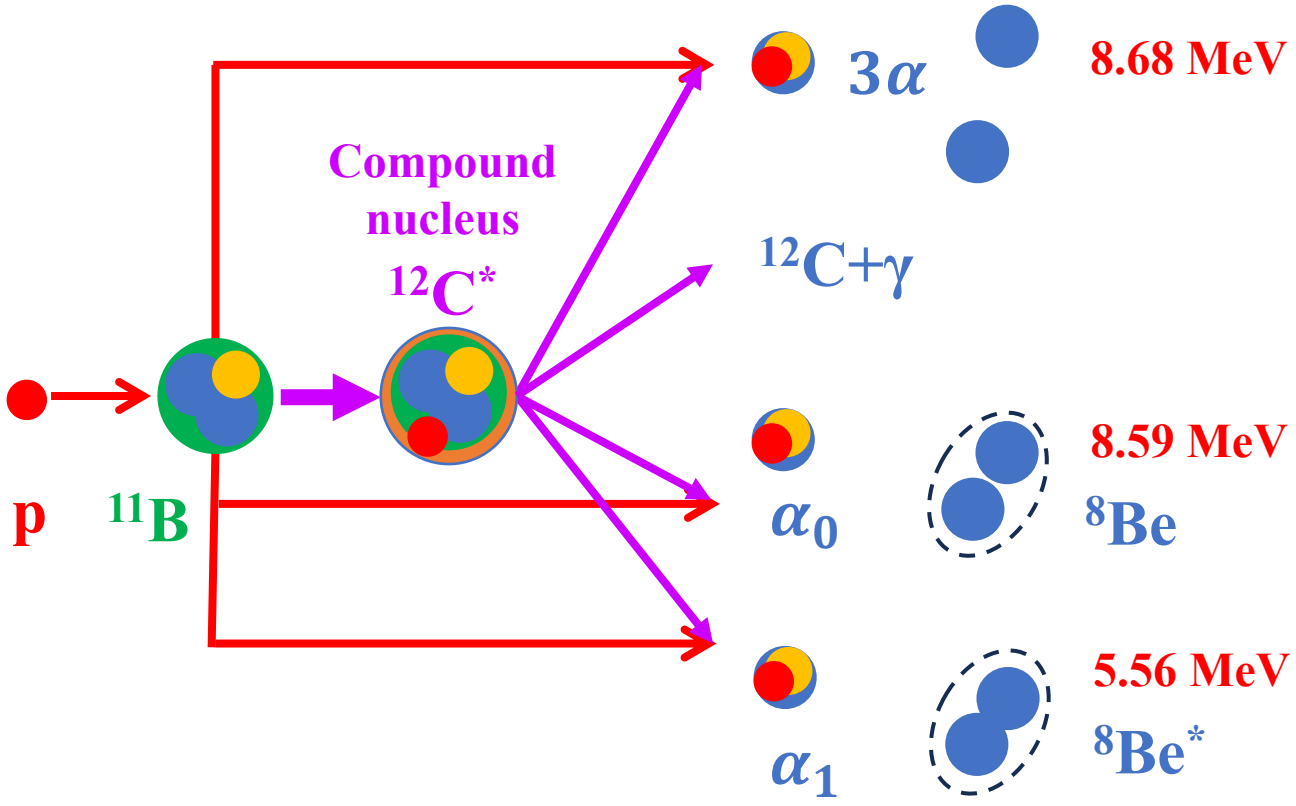
elastic scattering	$Q = 0$,	open
inelastic scattering	$Q = -2.12$ MeV,	closed
(p,n) reaction	$Q = -2.76$ MeV,	closed
(p,d) reaction	$Q = -9.23$ MeV,	closed
(p,t) reaction	$Q = -11.4$ MeV,	closed
(p,^{3}He) reaction	$Q = -10.3$ MeV,	closed
(p,α_1) reaction,	$Q = 5.56$ MeV,	open

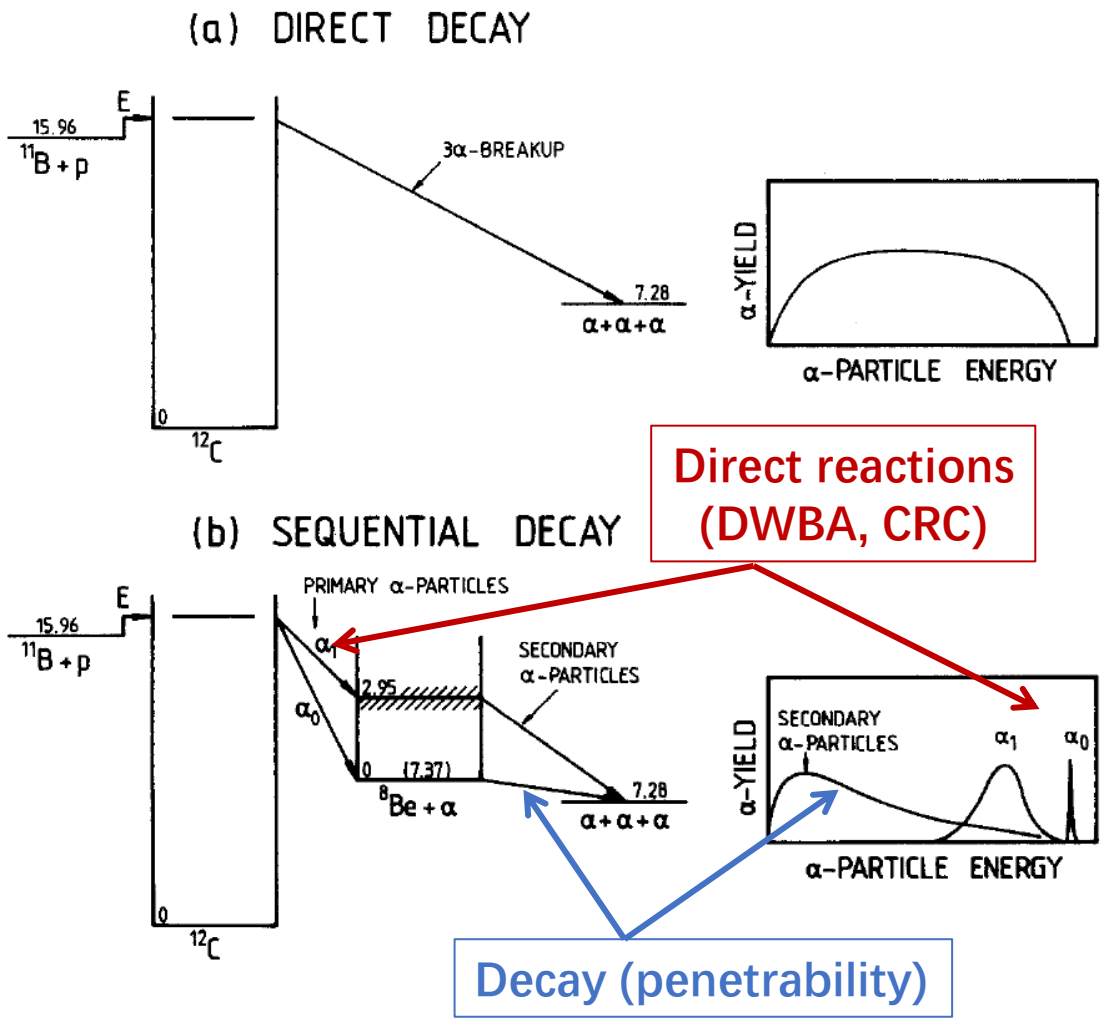
⇒ when $E_p < 2$ MeV (fusion energy region)

- only elastic scattering and (p,α) reactions are open
- number of relevant energy levels are small

⇒ proton-Boron fusion reaction could be described with direct reaction mechanism even at very low incident energies

Proton-Boron fusion: different processes

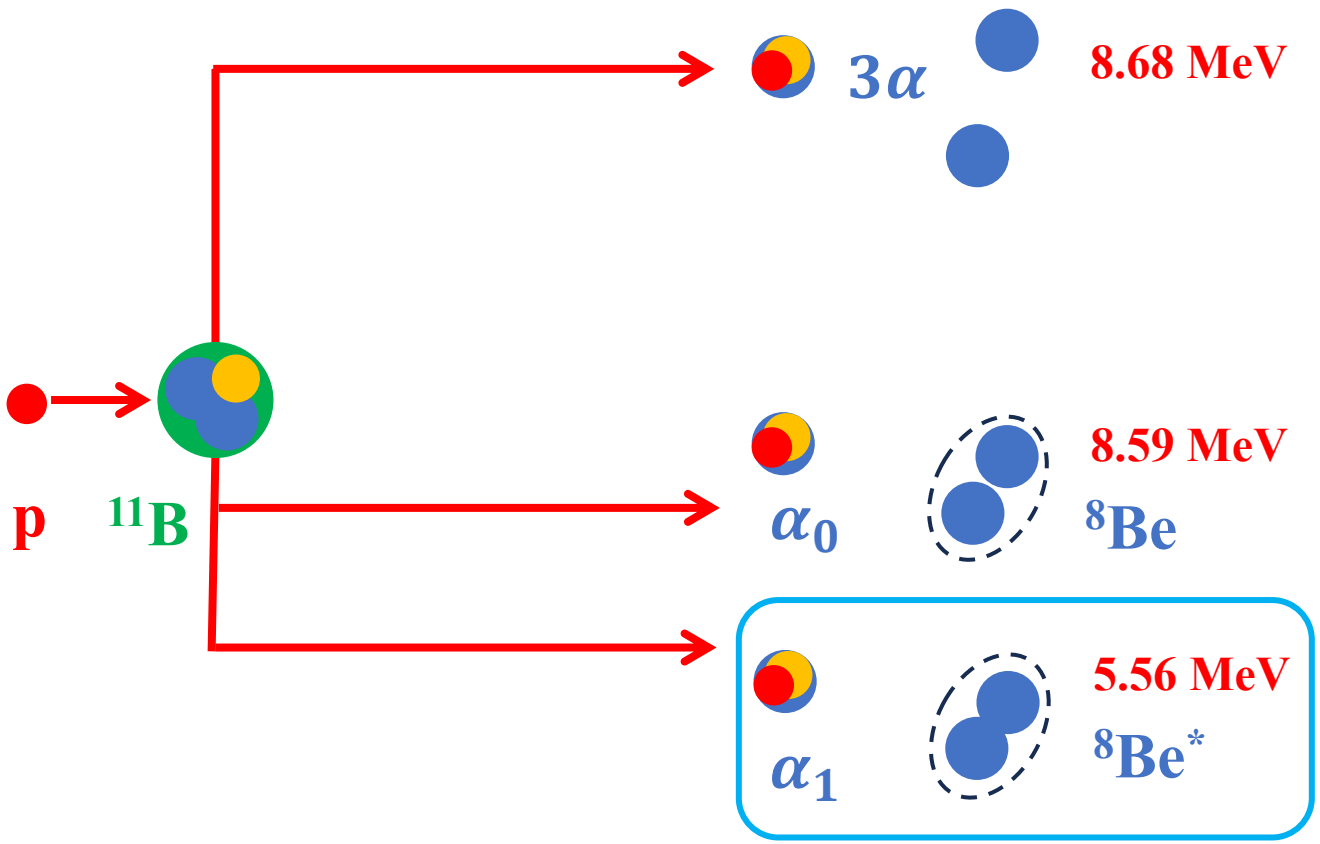




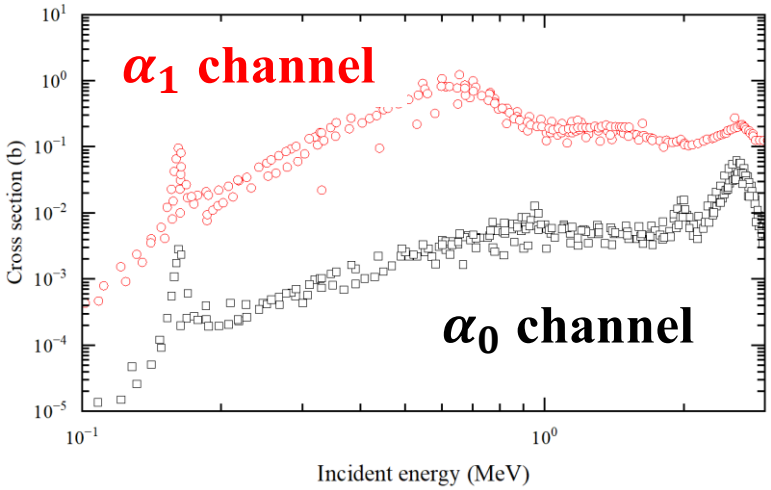
Conclusion: "the $^{11}B(p,3\alpha)$ reaction **proceeds predominantly by a sequential decay via ^{8}Be** and that the direct 3α-particle mechanism contributes less than 5% to the cross sections."

H.W. Becker, C. Rolfs, and H.P. Trautvetter, Z. Phys. A 327, 341 (1987)

Proton-Boron fusion: different processes



The most important channel



Main reaction channels

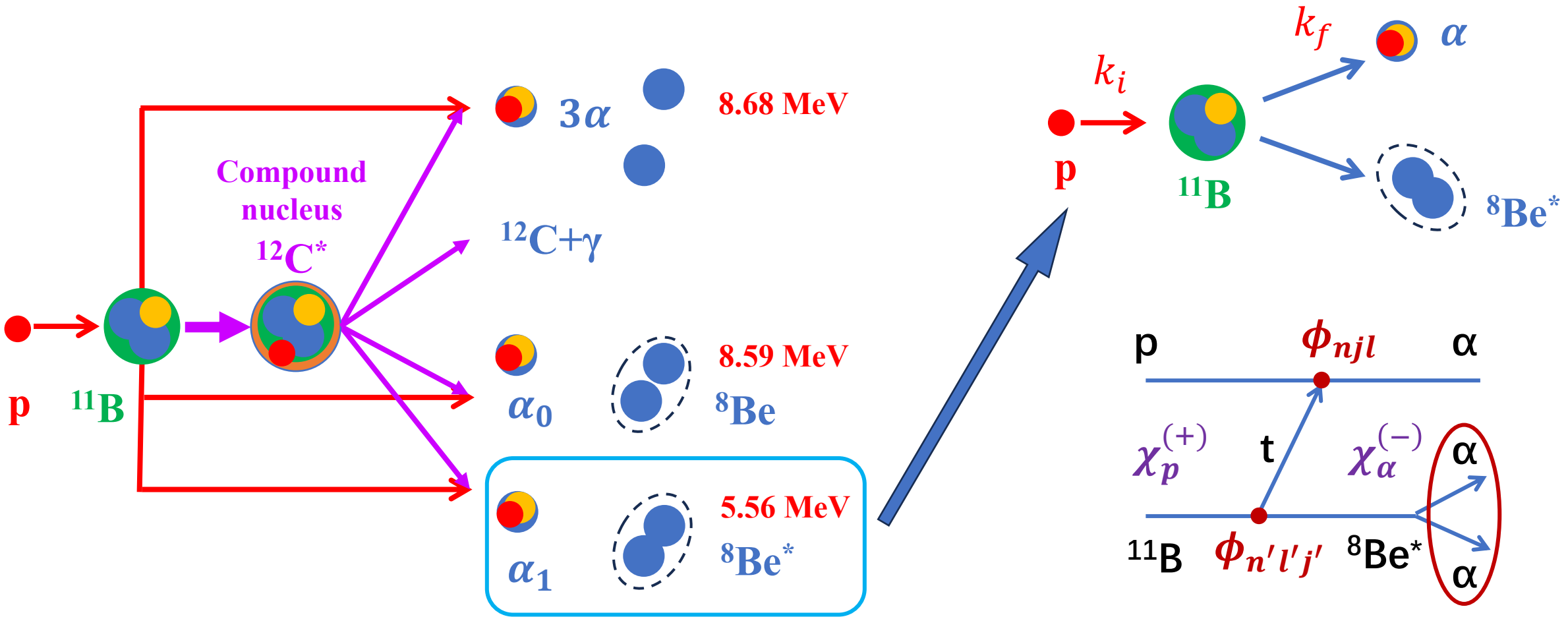
p + ^{11}B → α_0 + ^{8}Be + 8.59 MeV → α_0 + α_{01} + α_{02}

p + ^{11}B → α_1 + ^{8}Be* + 5.65 MeV → α_1 + α_{11} + α_{12}

p + ^{11}B → 3α + 8.68 MeV (5% contribution)

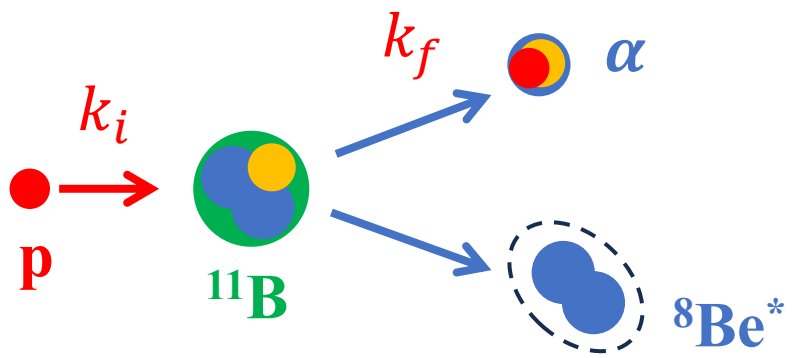
from Dr. Liu Bing, ENN

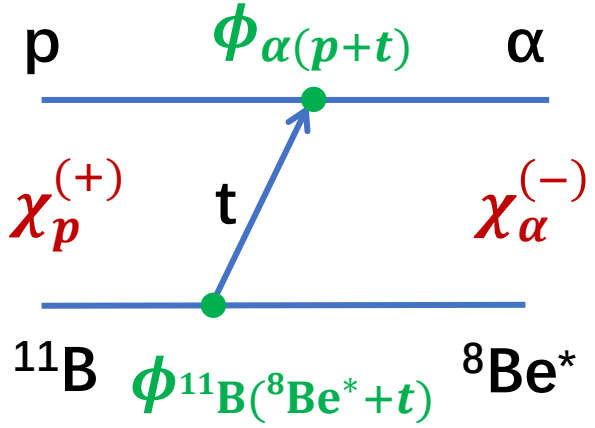
Proton-Boron fusion: different processes

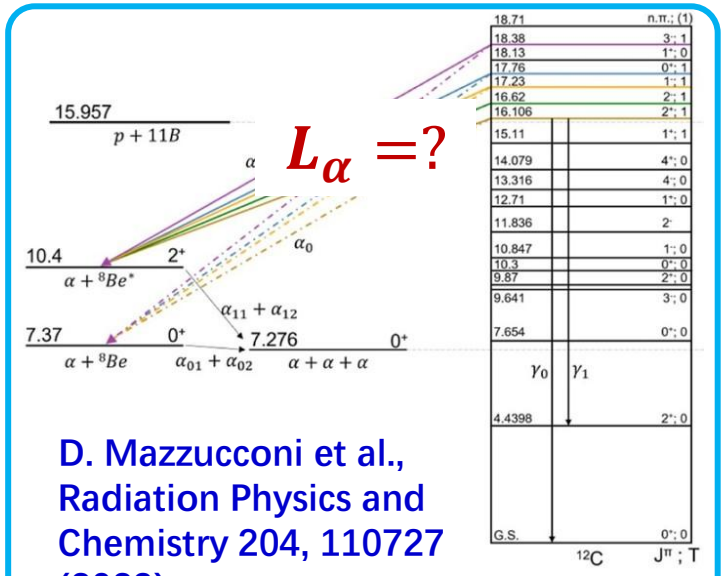


The most important channel

Theoretical description of the ^{11}B(p,α)^{8}Be* reaction







D. Mazzucconi et al., Radiation Physics and Chemistry 204, 110727 (2023)

Structure info of ^{12}C in resonance states are crucial for describing this reaction

$\beta_{^{11}\mathrm{B}(^{8}\mathrm{Be}^{*}+t)}\phi_{^{11}\mathrm{B}(^{8}\mathrm{Be}^{*}+t)}$ $\chi^{(+)}_{p-^{11}\mathrm{B}}$

incident channel

nuclear structure:
bound state wavefunctions ϕ
& spectroscopic amplitudes β

Reaction dynamics:
distorted waves χ

$\beta_{\alpha(p+t)}\phi_{\alpha(p+t)}$ $\chi^{(-)}_{\alpha-^{8}\mathrm{Be}^{*}}$

outgoing channel

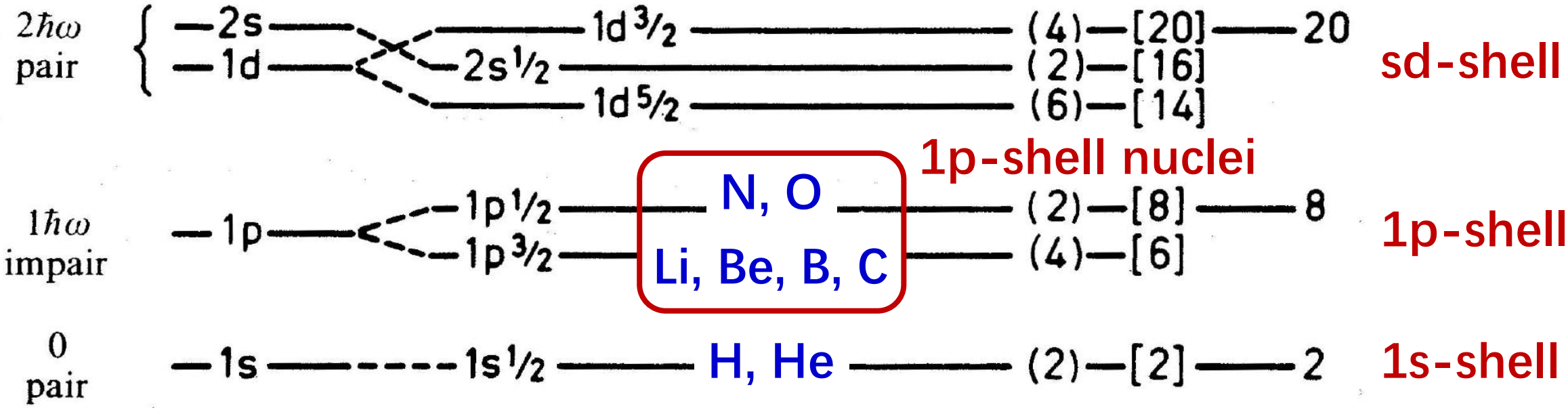
Transition amplitude

$$T_{fi}^{\mathrm{DWBA}} = \left\langle \beta_{^{11}\mathrm{B}(^{8}\mathrm{Be}+t)}\phi_{^{11}\mathrm{B}(^{8}\mathrm{Be}+t)}\chi^{(+)}_{p-^{11}\mathrm{B}} \middle| V_\alpha - U_\alpha \middle| \chi^{(-)}_{\alpha-^{8}\mathrm{Be}}\beta_{\alpha(p+t)}\phi_{\alpha(p+t)} \right\rangle^{\dagger} \Rightarrow \frac{d\sigma}{d\Omega} \propto \sum \left|T_{fi}^{\mathrm{DWBA}}\right|^2$$

③ ① ② ④

①: interaction in the entrance-channel ($p+^{11}B$)

- **Optical model potentials (OMPs) ⇐ fitting elastic scattering data**
- **OMPs from individual data sets have great uncertainty**
- **It is always better to use Systematic OMPs from large sets of experimental data**
- **The only existing systematic nucleon-nucleus OMP (WSS, B.A. Watson, P.P. Singh, and R.E. Segel, Phys. Rev. 182, 977 (1960)) does not describe $p+^{11}B$ data sufficiently well ⇒ make our own systematic OMP for 1p-shell nuclei**



①: A new systematic nucleon-nucleus OMP: PN1P

PN1P includes energy (E) and target mass (A) dependent terms

$$U(r,E) = -V - i[W_V + W_S] - [V_{SO} + iW_{SO}]\ \ell\cdot\sigma + V_C(r)$$

real part: $V = v_1[1 - v_2(E - E_f)] + V_c v_1 v_2$

volume imaginary part: $W_V = w_1 \dfrac{(E-E_f)^2}{(E-E_f)^2 + w_2^2}$

surface imaginary part: $W_S = d_1 \dfrac{(E-E_f)^2}{(E-E_f)^2 + d_3^2} \exp(-d_2(E-E_f))$

real spin − orbit part: $V_{SO} = vso_0$

imaginary spin − orbit part: $W_{SO} = wso_0 \dfrac{(E-E_f)^2}{(E-E_f)^2 + wso_1^2}$

Coulomb correction term: $V_C(E) = \begin{cases} \dfrac{1.73 \times Z \times A^{-1/3}}{R_C}, & \text{for protons} \\ 0, & \text{for neutrons} \end{cases}$

Fermi energies: $E_f = \begin{cases} -\dfrac{1}{2}[S_p(Z,N) + S_p(Z+1,N)], & \text{for protons} \\ -\dfrac{1}{2}[S_n(Z,N) + S_n(Z,N+1)], & \text{for neutrons} \end{cases}$

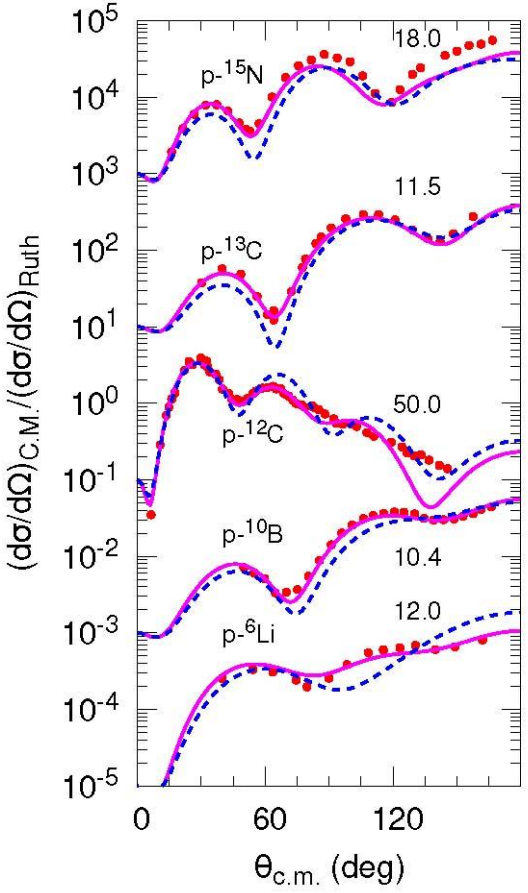
mass dependence is allocated to the radius parameters

TABLE I: Proton elastic scattering differential cross section data used in this work.

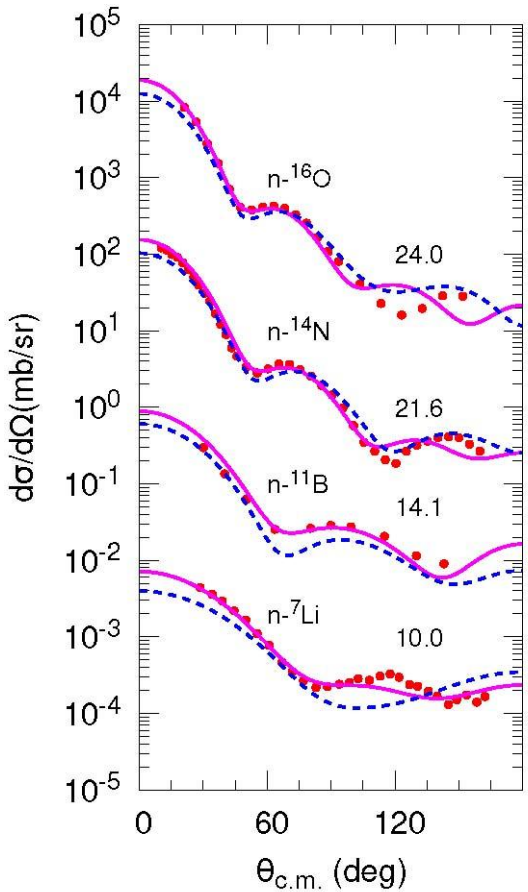
Target	E (MeV)	References
^{6}Li	5, 6, 7, 8, 9, 10, 11, 12, 30.5, 39.7, 50	[17, 19]
^{7}Li	5, 6, 8, 9, 10, 39.7, 50	[17, 20, 21]
^{9}Be	10, 11.9, 13, 14, 25, 29.1, 30.3, 35.2	[22–26]
^{10}B	6.5, 8.5, 9.4, 10.4, 11.4, 12.8, 13.2	[17]
^{11}B	6.46, 7.36, 13, 30.5	[27–29]
^{12}C	4.1, 5.0, 6.8, 7, 9.95	[30, 31]
^{12}C	14.8, 16.8, 17.7, 18.6	[32]
^{12}C	15.6, 19.4, 20, 21.1, 22.3, 23.5, 29.7 32.7, 34.5, 44.7, 54.4, 59.5, 69.5	[33–35]
^{12}C	24.1, 26, 27, 27.8, 28.85, 30.4	[36–40]
^{12}C	35.2, 40, 48.5, 49.4, 50, 51.93, 61, 65	[26, 41–47]
^{13}C	7, 10, 11, 11.5, 12.5, 13.5, 14.5, 16.0 16.75, 17.5, 22, 30.4, 30.95, 35, 35.2	[26, 40, 48–52]
^{14}N	7.73, 8.52, 10.5, 11.9, 12.5, 13.56, 14.5 21, 21.02, 23, 26, 31, 35.2, 49.4, 50 4.526, 5.722, 6.585	[26, 43, 44, 53–57]
^{15}N	18.0, 20.0, 22.4, 24.5, 26.0, 28.1, 30.0, 32.1, 34.9, 37.1, 39.2, 41.7, 44.2	[26, 58]
^{16}O	6.87, 7.53, 8.66, 9.42, 10.5, 11.4, 12.9, 13.9	[59]
^{16}O	14.8, 15.8, 16.8, 17.8, 19.2	[60]
^{16}O	19.8, 21.4, 22.1, 26.2, 30.5	[61]
^{16}O	24.5, 27.3, 30.1, 34.1, 36.8, 39.7, 43.1, 46.1	[62]
^{16}O	31, 35, 35.2	[26, 57, 63]

①: A new systematic nucleon-nucleus OMP: PN1P

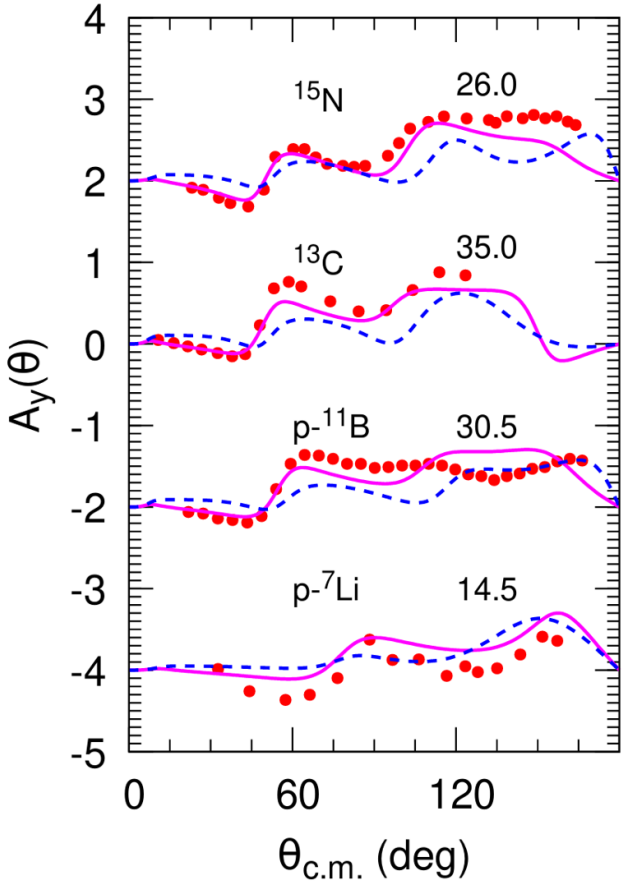
Comparisons with experimental data. Solid curves: PN1P, dashed curves: WSS



Proton elastic scattering



neutron elastic scattering



Analyzing power

Important for polarization cross section calculations

Remarks:

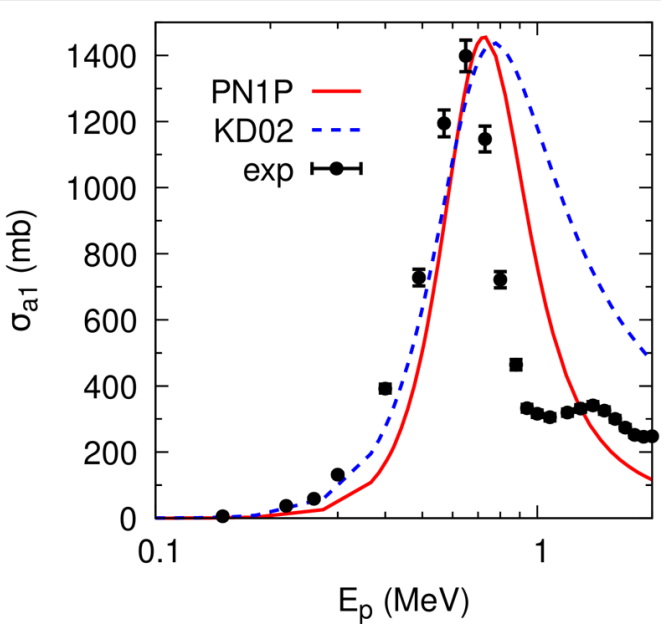
- Why mostly high-energy data? (low energy data are rare and not sensitive to OMPs)
- Nuclear parts of OMPs are important for light-ion involved reactions even at low energies (different from heavy-ion reactions).
- We make extrapolations

Cluster EFT might be very helpful here!

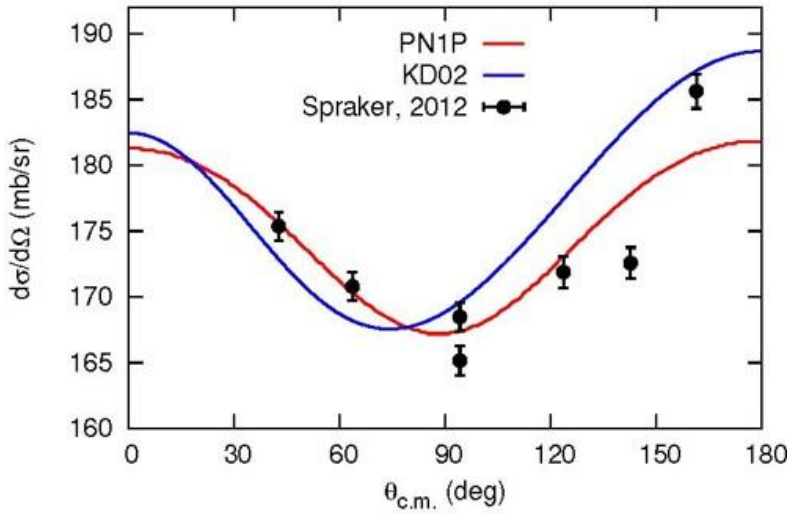
(cf. Ando-san's talk)

$^{11}B(p,\alpha_1)^{8}Be^*$ reaction with direct reaction mechanism

Compare PN1P with other systematic OMPs (KD02) for p-^{11}B reactions!



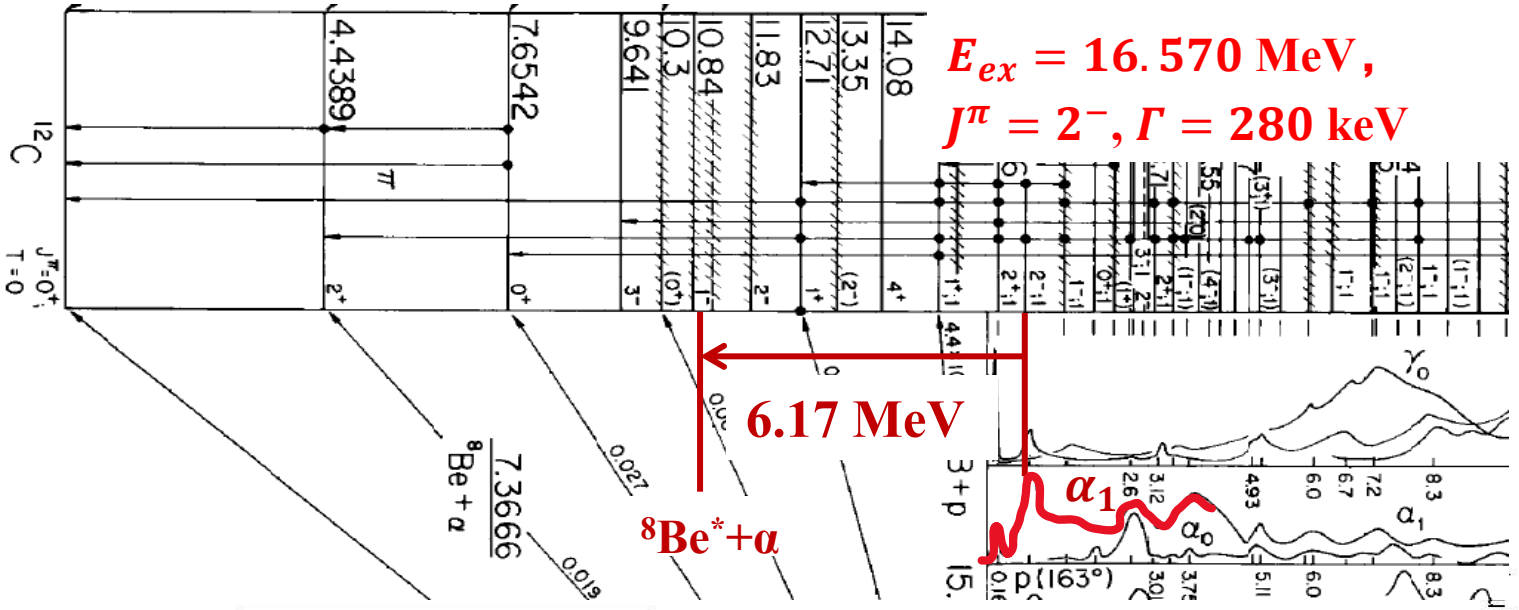
Integrated cross sections



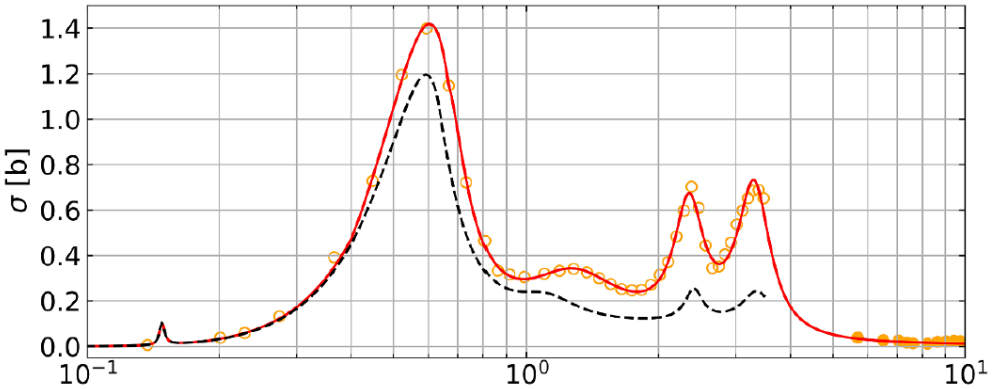
Differential cross sections at $E_p = 0.8$ MeV

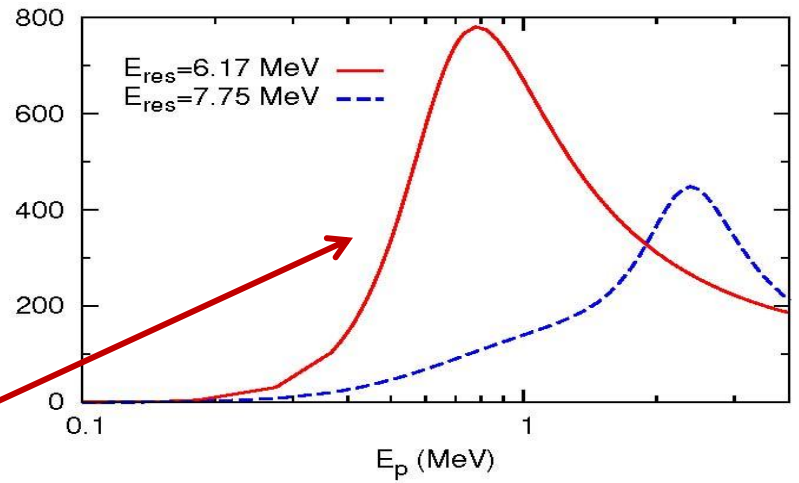
Low energy p-^{11}B reactions are sensitive to nuclear part of optical model potentials!

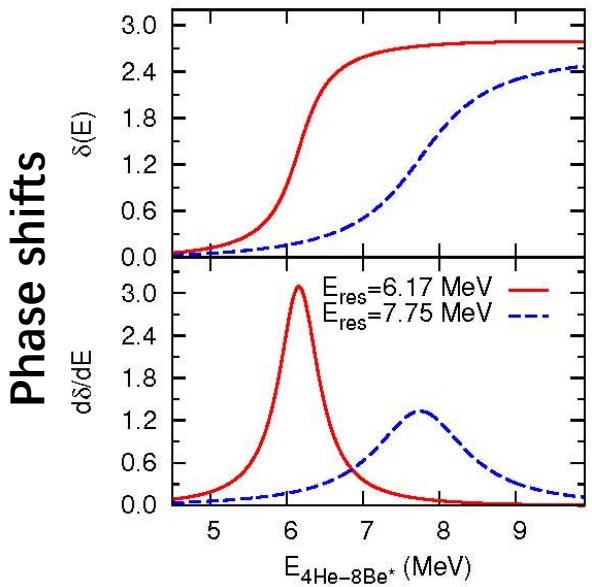
②: interaction in the exit-channel (between ^{4}He and ^{8}Be*)

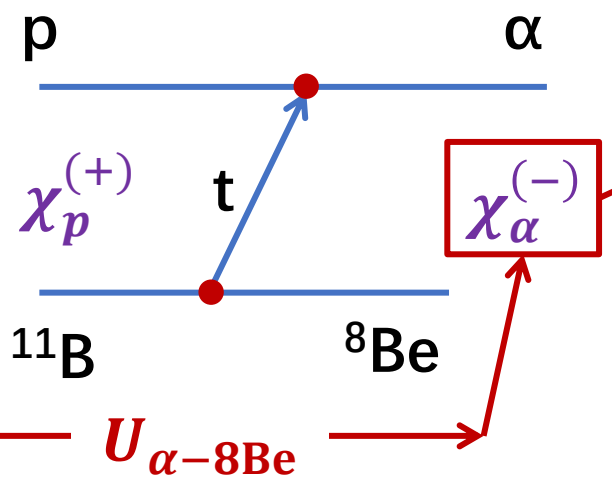


$E_{ex} = 16.570$ MeV,
$J^\pi = 2^-, \Gamma = 280$ keV









⇒ Potentials must support ^{12}C resonance states to describe the ^{11}B(p,α)^{8}Be* reaction

③ and ④: bound-state wave functions

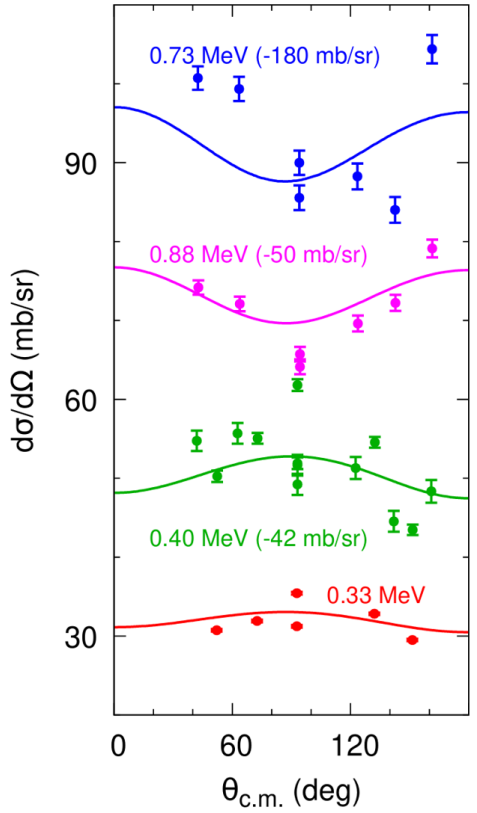
✓ For wave function of triton cluster in ^{4}He, we use result of *ab initio* calculation of I. Brida, et al., Physics Review C 84, 024319 (2011).

☐ We don't have reliable structure theory (yet) for triton cluster in ^{11}B. So we use parameters (spectroscopic amplitudes):

$$\Phi_{^{11}\mathrm{B}} = \alpha \Phi_{^{8}\mathrm{Be}^*} \Phi_{^{3}\mathrm{H}} \phi_p(r) + \beta \Phi_{^{8}\mathrm{Be}^*} \Phi_{^{3}\mathrm{H}} \phi_f(r)$$

Currently, we take spectroscopic amplitudes $\alpha = \beta = 1$ (a rather common practice when we don't know their values), and match the calculated cross sections to the experimental data.

Shapes of angular distributions may change when α and β are changed ⇒ need reliable experimental data to fix them or reliable structure theory (cf. Prof. Zhou Bo's talk)

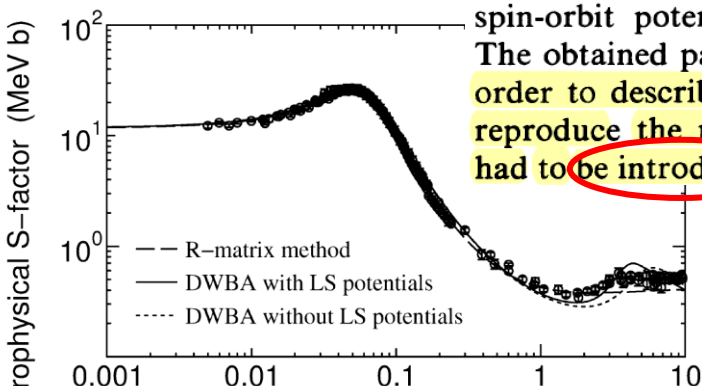


Low energy direct nuclear reactions on light nuclei

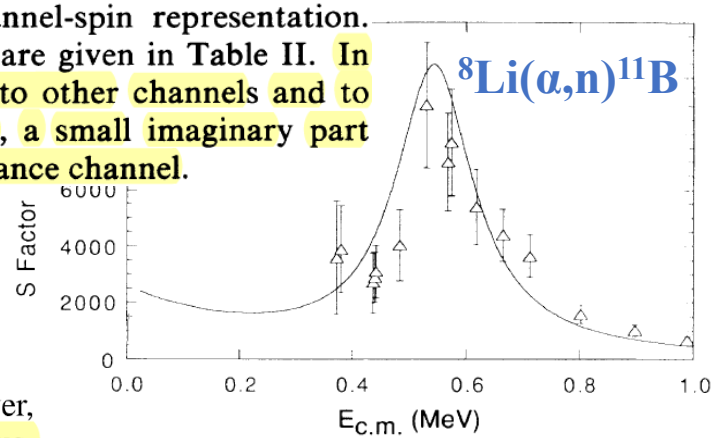
Low energy reactions described with the DWBA

targets	reactions	Q-value
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^{3}He	(d,p)	18.35
^{3}H	(^{3}H,2n)	11.3
^{3}He	(^{3}He,2p)	12.8
^{6}Li	(p,^{3}He)	4.02
^{6}Li	(d,α)	22.37
^{7}Li	(p,α)	17.35
^{8}Li	(α,n)	6.63
^{9}Be	(p,d)	0.56
^{9}Be	(p,α)	2.15
^{10}B	(p,α)	1.15
^{11}B	(p,α)	8.59
^{19}F	(p,α)	8.11

nel spin. We are not able to reproduce the fine structure of the resonance in our model since we did not include a spin-orbit potential in the channel-spin representation. The obtained parameter values are given in Table II. In order to describe absorption into other channels and to reproduce the resonance width, a small imaginary part had to be introduced in the entrance channel.

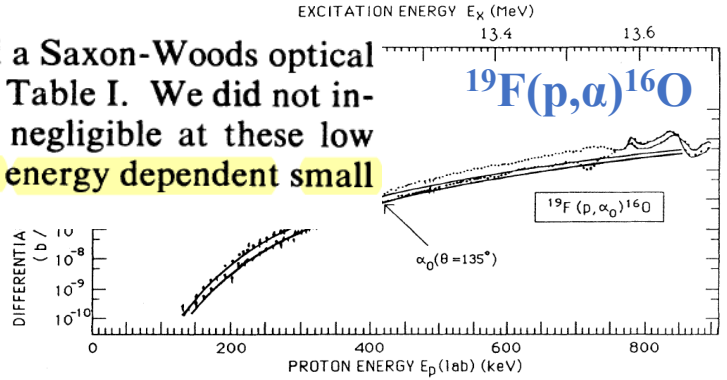


the empirical expression of the global potential were adopted as the first trial. However, we could not obtain good reproductions of the experimental data by this choice. Thus, we adjust the strengths of the potentials so as to reproduce the experimental data. In the parameter optimization, we find that the reproductions of the resonance energies in the ^{3}H(d,n)^{4}He and ^{3}He(d,p)^{4}He reactions are sensitive to the variation of the potential



T. Rauscher, PRC 45, 1996 (1992)

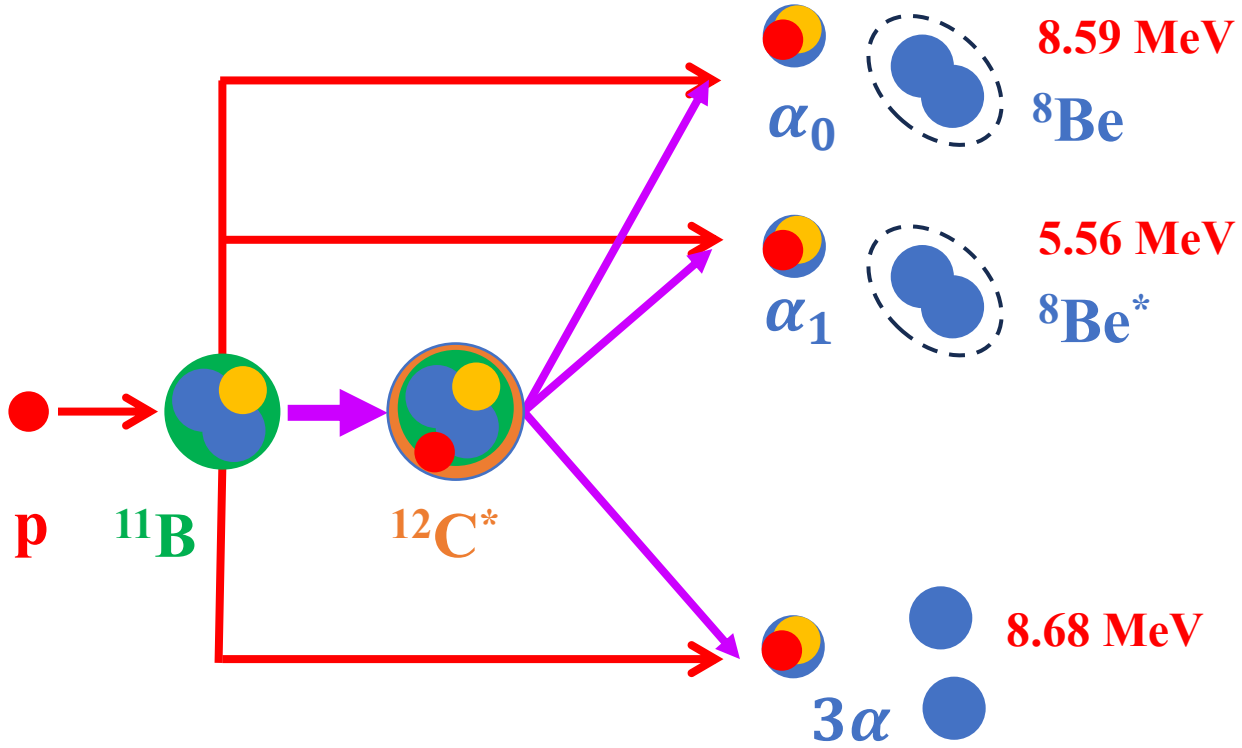
In the entrance channel we used a Saxon-Woods optical potential with parameters given in Table I. We did not include a spin-orbit term, which is negligible at these low energies. We needed, however, an energy dependent small volume absorption term.



H. Herndl, PRC 44, R952 (1991)

A better theory is needed

Microscopic coupled-reaction channel theory



when $E_p < 2$ MeV
(fusion energy region)
all other reactions
are closed
Except elastic scattering
⇒ NO NEED of OMPS

elastic scattering	$Q = 0$,	open
inelastic scattering	$Q = -2.12$ MeV,	closed
(p,n) reaction	$Q = -2.76$ MeV,	closed
(p,d) reaction	$Q = -9.23$ MeV,	closed
(p,t) reaction	$Q = -11.4$ MeV,	closed
(p,^{3}He) reaction	$Q = -10.3$ MeV,	closed

$$H = H_0 + V(\boldsymbol{r},\boldsymbol{\xi}) = H_0(\boldsymbol{\xi}) + T(\boldsymbol{r}) + V(\boldsymbol{r},\boldsymbol{\xi})$$

$$(H - E)\Psi = 0$$ Feshbach, 1962

$P\Psi = \Phi_0 = \phi_0\psi_0(r_0)$: $P-$space, treated explicitly

$Q = 1 - P$: Q − space, outside of the model space

$$\left\{E - T - \underbrace{(\phi_0|V|\phi_0)}_{U_{00}} - \underbrace{(\phi_0|VQ\frac{1}{E-QHQ}QV|\phi_0)}_{U_{DPP}}\right\}\psi_0(r) = 0$$

U_{00}: bare pot. U_{DPP}: dynamical pol. pot.

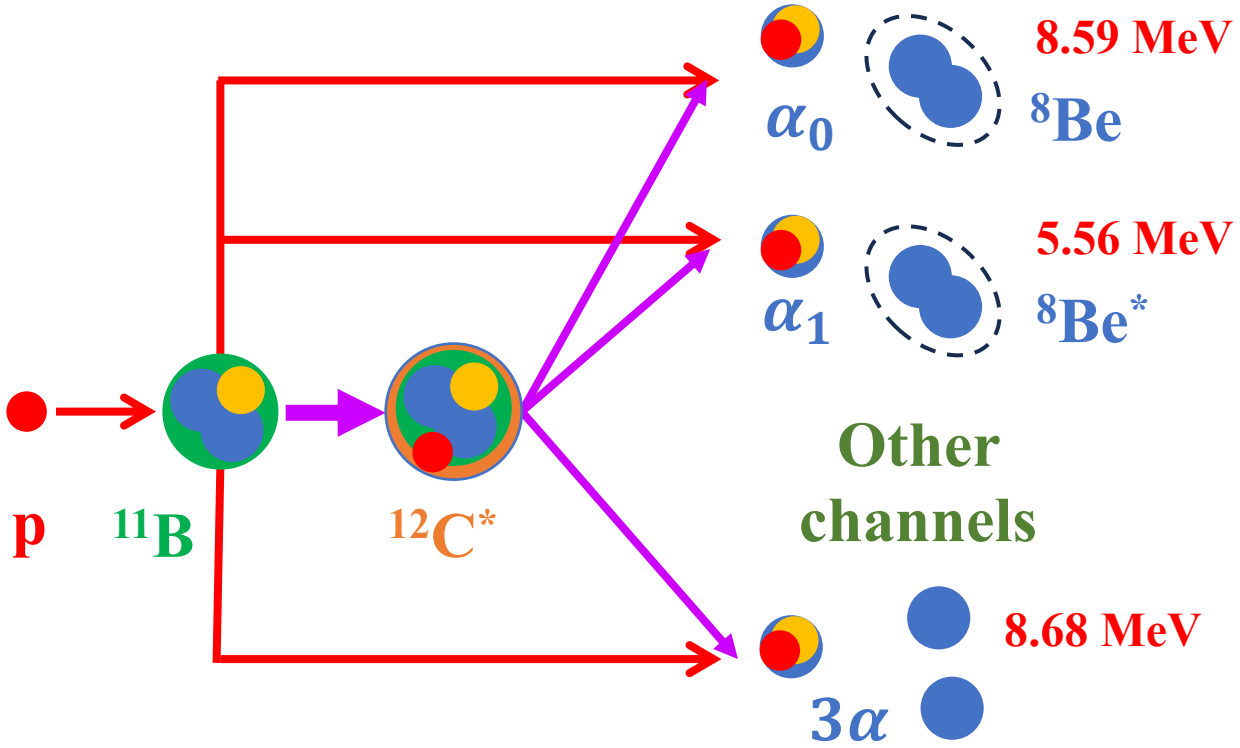
$U = U_{00} + U_{DPP}$: OMP
energy dependent, nonlocal

Phenomenological pot. (uncertainties)

$$U(r) = \frac{V_0}{1+\exp[(r-R_V)/a_V]} + i\frac{W_0}{1+\exp[(r-R_W)/a_W]}$$

Low-energy light-ion reactions, very few open channels, coupled-reaction channels: No OMPs needed!

Microscopic coupled reaction channel theory



when $E_p < 2$ MeV

(fusion energy region)

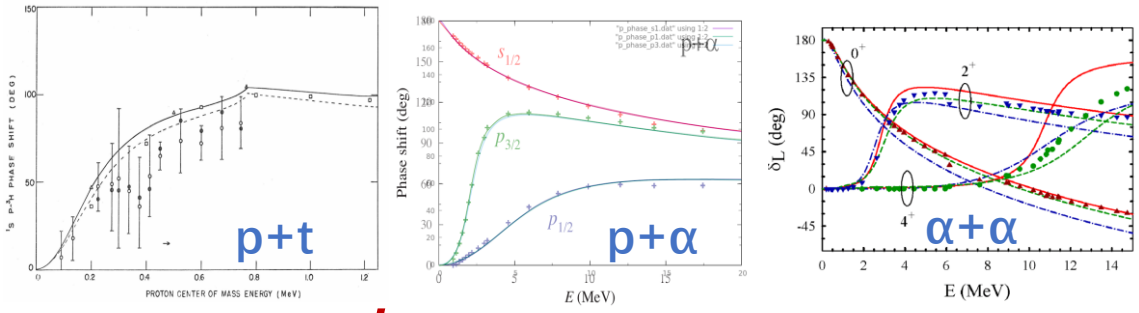
all other reactions are closed

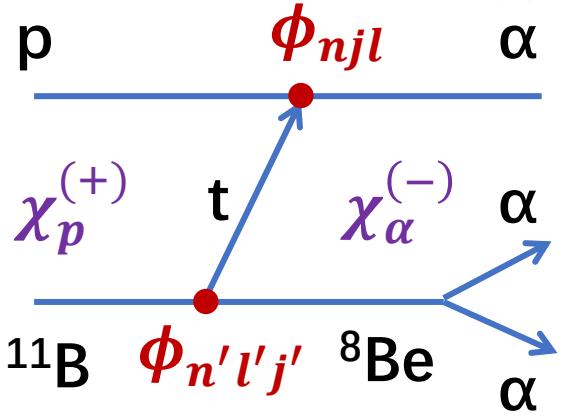
Except elastic scattering

⇒ NO NEED OMPS

elastic scattering	Q = 0,	**open**
inelastic scattering	Q = −2.12 MeV,	**closed**
(p,n) reaction	Q = −2.76 MeV,	**closed**
(p,d) reaction	Q = −9.23 MeV,	**closed**
(p,t) reaction	Q = −11.4 MeV,	**closed**
(p,^{3}He) reaction	Q = −10.3 MeV,	**closed**

"*building block*" interactions from phase shifts





Coupled reaction channels + Microscopic nuclear structure

$$U_{p+^{11}B} = \langle \Phi_{^{11}B} | V_{p+t} + V_{p+\alpha} | \Phi_{^{11}B} \rangle$$

$$U_{\alpha+^{8}Be} = \langle \Phi_{^{8}Be} | V_{\alpha+\alpha} | \Phi_{^{8}Be} \rangle$$

$\Phi_{^{11}B}$, $\Phi_{^{8}Be}$ from cluster structure theory

NO optical model potentials needed!

Microscopic coupled reaction channel theory

The coupled reaction channel (CRC) method

$$\Psi_{\text{model}} = u_\alpha(r_\alpha)\psi_\alpha(x_\alpha) + u_\beta(r_\beta)\psi_\beta(x_\beta).$$

$$(\psi_\alpha|(E-H)|\Psi_{\text{model}}) = 0, \qquad (\psi_\beta|(E-H)|\Psi_{\text{model}}) = 0,$$

$$H = H_\alpha + K_\alpha + V_\alpha = H_\beta + K_\beta + V_\beta.$$

Coupled equations

$$[(E-c_\alpha) - K_\alpha - (\alpha|V_\alpha|\alpha)]u_\alpha(r_\alpha) = \int dr_\beta K_{\alpha\beta}(r_\alpha, r_\beta)u_\beta(r_\beta),$$

$$[(E-\varepsilon_\beta) - K_\beta - (\beta|V_\beta|\beta)]u_\beta(r_\beta) = \int dr_\alpha K_{\beta\alpha}(r_\beta, r_\alpha)u_\alpha(r_\alpha).$$

$$K_{\alpha\beta}(r_\alpha, r_\beta) = J_{\beta\alpha}\int \mathrm{d}\zeta_\alpha \psi^*_\alpha(x_\alpha)(H-E)\psi_\beta(x_\beta),$$

$$K_{\beta\alpha}(r_\beta, r_\alpha) = J_{\beta\alpha}\int \mathrm{d}\zeta_\beta \psi^*_\beta(x_\beta)(H-E)\psi_\alpha(x_\alpha).$$

G.R. Satchler, *Direct Nuclear Reactions*, Clarendon Press, 1983.

The resonating group method (RGM)

$$\psi = \sum_\nu \mathcal{A}_\nu[\phi_{1\nu}(\xi_{1\nu})\phi_{2\nu}(\xi_{2\nu})g_\nu(r_\nu)],$$

$$\sum_{\nu'}\int \mathrm{d}^3\boldsymbol{r}'_{\nu'}[\mathcal{H}_{\nu\nu'}(\boldsymbol{r}_\nu, \boldsymbol{r}'_{\nu'}) - E\mathcal{N}_{\nu\nu'}(\boldsymbol{r}_\nu, \boldsymbol{r}'_{\nu'})]g_{\nu'}(\boldsymbol{r}'_{\nu'}) = 0.$$

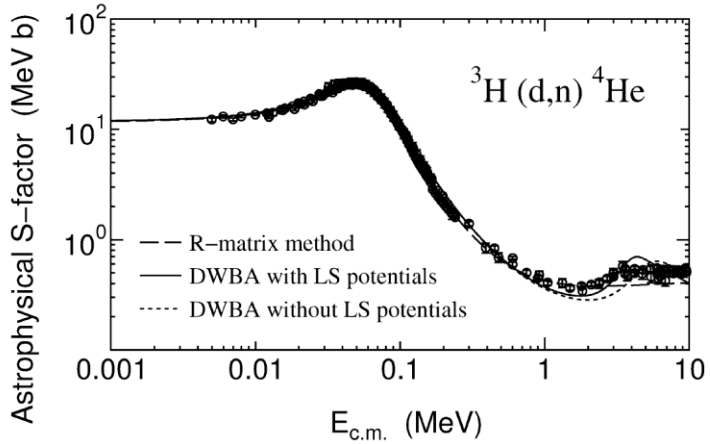
$$\begin{aligned}
\mathcal{N}_{\nu\nu'}(\boldsymbol{r}, \boldsymbol{r}') &= \langle\phi_{1\nu}\phi_{2\nu}\delta(\boldsymbol{r}_\nu - \boldsymbol{r})|\mathcal{A}_\nu\mathcal{A}_{\nu'}|\phi_{1\nu'}\phi_{2\nu'}\delta(\boldsymbol{r}_{\nu'} - \boldsymbol{r}')\rangle \\
&= \sum_{\mathrm{P}}^{\nu}(-)^{\mathrm{P}}\langle\phi_{1\nu}\phi_{2\nu}\delta(\boldsymbol{r}_\nu - \boldsymbol{r})|P|\phi_{1\nu'}\phi_{2\nu'}\delta(\boldsymbol{r}_{\nu'} - \boldsymbol{r}')\rangle \\
\mathcal{H}_{\nu\nu'}(\boldsymbol{r}, \boldsymbol{r}') &= \langle\phi_{1\nu}\phi_{2\nu}\delta(\boldsymbol{r}_\nu - \boldsymbol{r})|\mathcal{A}_\nu H\mathcal{A}_{\nu'}|\phi_{1\nu'}\phi_{2\nu'}\delta(\boldsymbol{r}_{\nu'} - \boldsymbol{r}')\rangle \\
&= \sum_{\mathrm{P}}^{\nu}(-)^{\mathrm{P}}\langle\phi_{1\nu}\phi_{2\nu}\delta(\boldsymbol{r}_\nu - \boldsymbol{r})|HP|\phi_{1\nu'}\phi_{2\nu'}\delta(\boldsymbol{r}_{\nu'} - \boldsymbol{r}')\rangle.
\end{aligned}$$

G. Bluge, K. Langanke, and H.-G. Reusch, *Microscopic Description of Nuclear Collisions*, Chapter 4 in K. Langanke, l.A. Maruhn, and S.E. Koonin, Computational Nuclear Physics 2, Nuclear Reactions, Springer-Verlag, (1993).

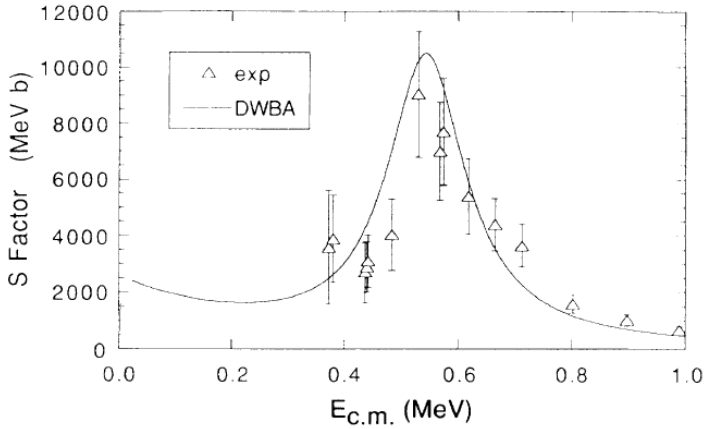
Summary

Hope: more microscopic, better predictivity

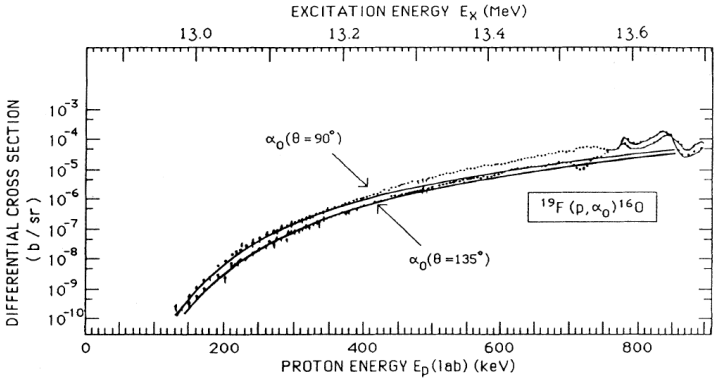
targets	reactions	Q-value
^{3}H	(d,n)	17.59
^{3}He	(d,p)	18.35
^{3}H	(^{3}H,2n)	11.33
^{3}He	(^{3}He,2p)	12.86
^{6}Li	(p,^{3}He)	4.02
^{6}Li	(d,α)	22.37
^{7}Li	(p,α)	17.35
^{8}Li	(α,n)	6.63
^{9}Be	(p,d)	0.56
^{9}Be	(p,α)	2.15
^{10}B	(p,α)	1.15
^{11}B	(p,α)	8.59
^{19}F	(p,α)	8.11



M. Katsuma, AIP Conf. Proc. 891, 355 (2007)



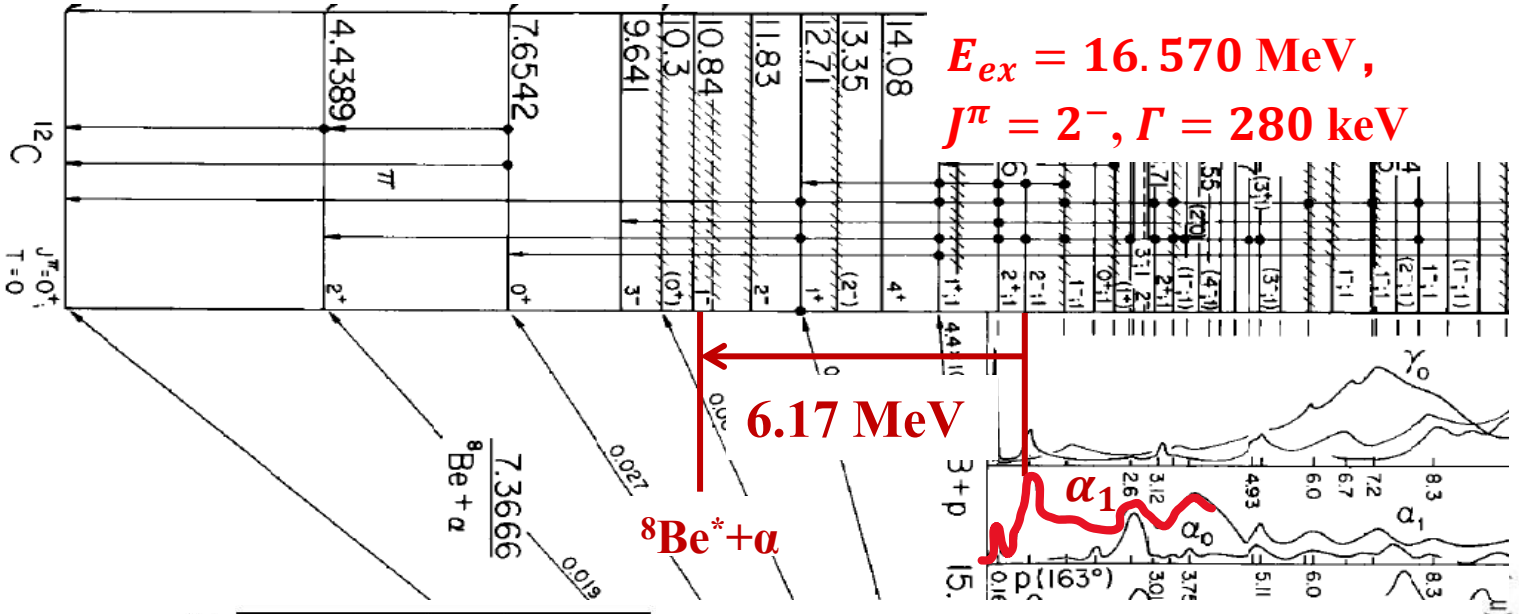
T. Rauscher, PRC 45, 1996 (1992)



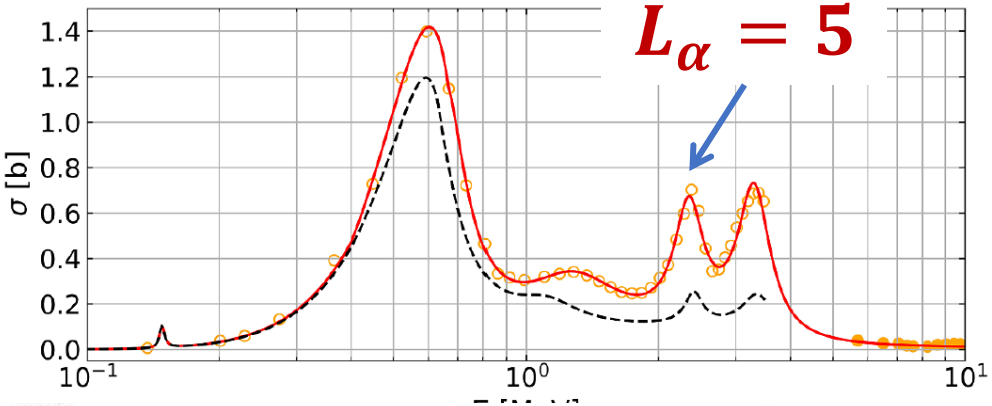
H. Herndl, PRC 44, R952 (1991)

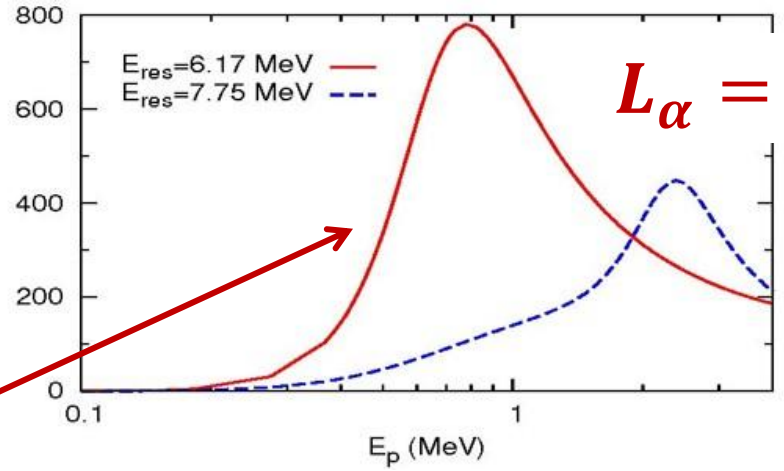
- Direct reaction mechanism can be important at low energies with light ions
- Potential resonance states vs CN resonances
- Can be described with the DWBA (OMPs needed)
- Microscopic coupled reaction channel theory may liberate us from OMPs

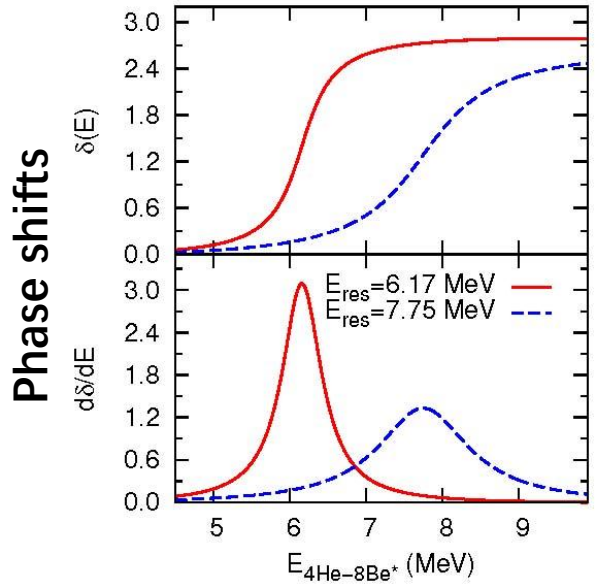
Backup material: need for ℓ-dependent potential

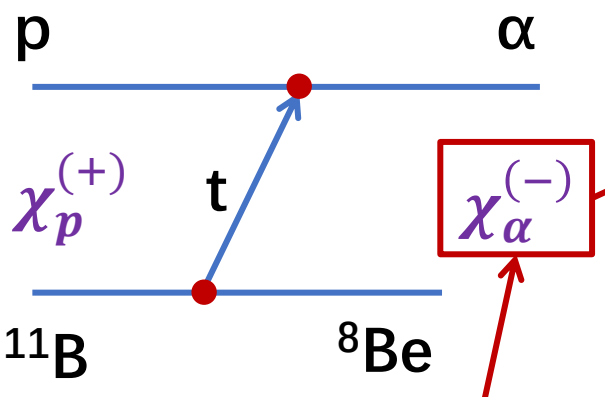


$E_{ex} = 16.570$ MeV,
$J^{\pi} = 2^-, \Gamma = 280$ keV





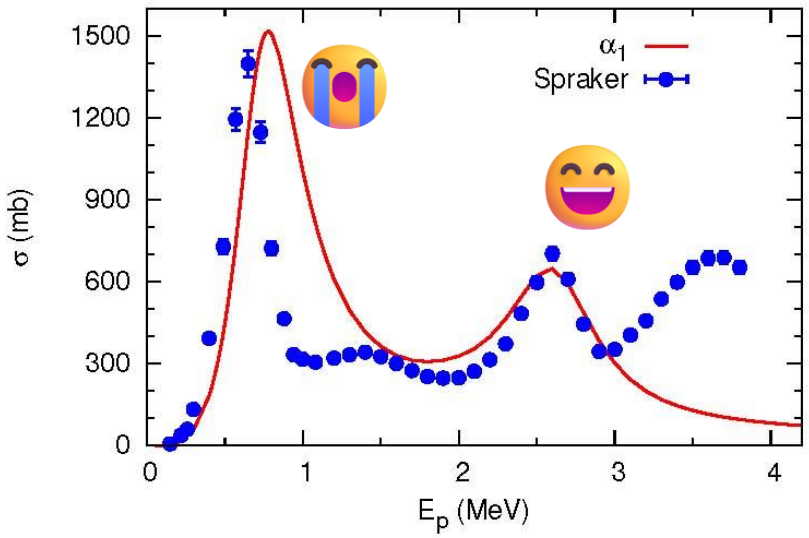




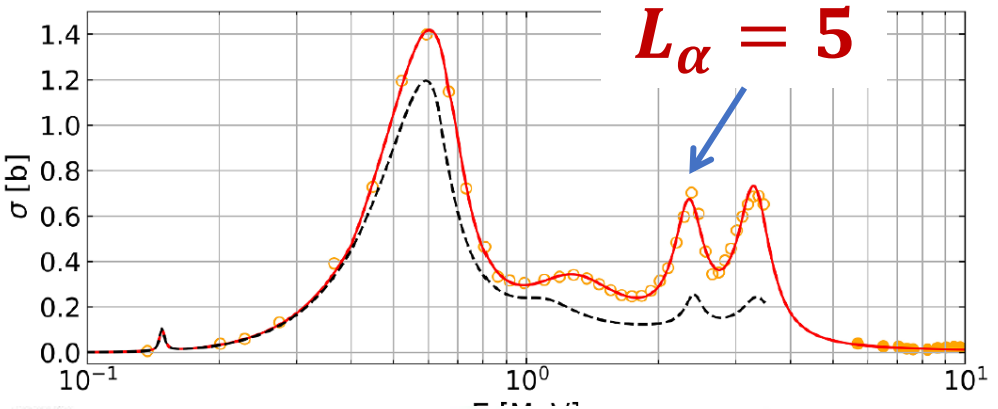
$U_{\alpha-8Be}$

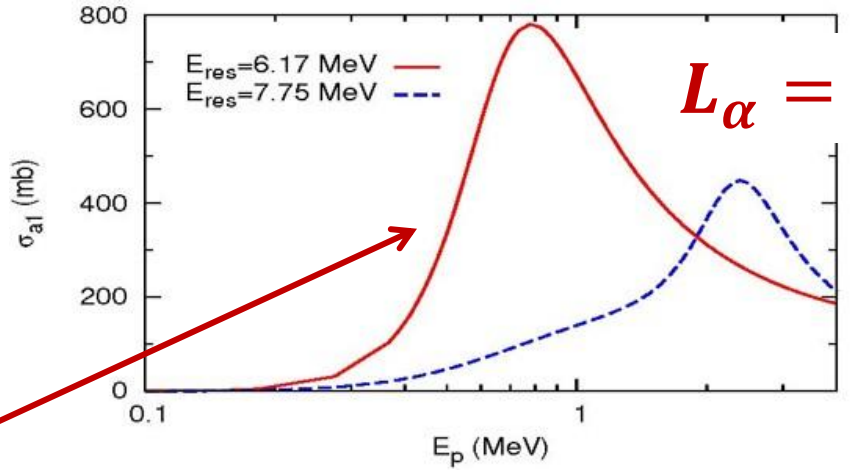
⇒ Potentials must support ^{12}C resonant states to describe the ^{11}B(p,α)^{8}Be* reaction

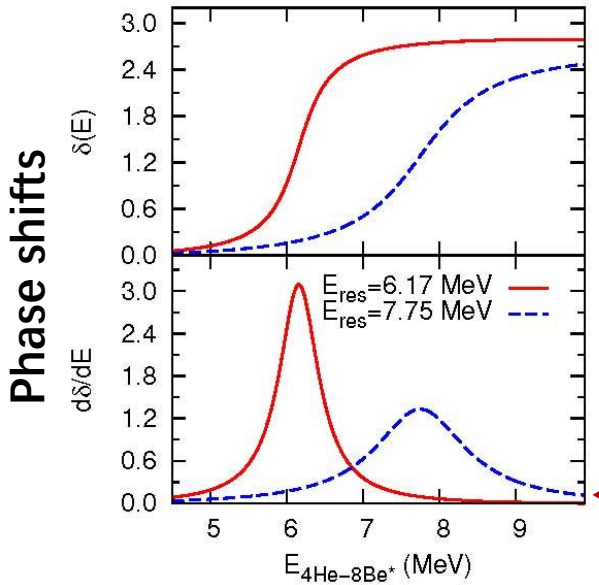
Backup material: need for ℓ-dependent potential

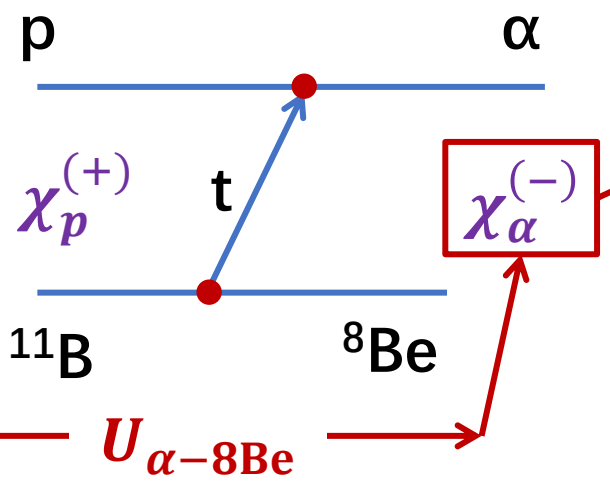


^{4}He+^{8}Be potential supports the $L_\alpha = 5$ resonant state also support a $L_\alpha = 3$ resonant state, but at wrong energy. Not we want! ⇒ ℓ-dependent potential









⇒ Potentials must support ^{12}C resonant states to describe the ^{11}B(p,α)^{8}Be* reaction