Structure and decay of α-condensate candidates from cluster EFT interactions

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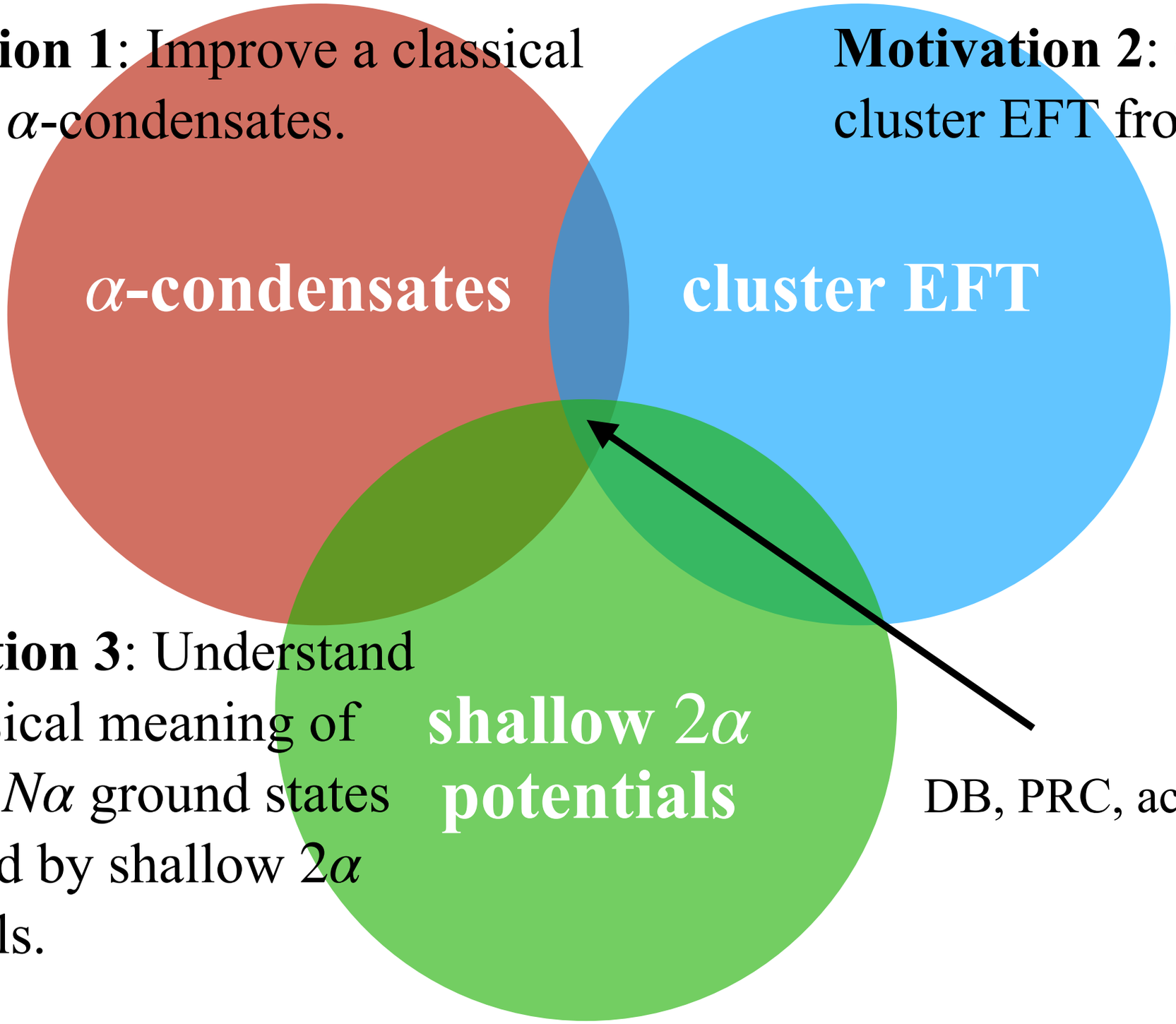
Workshop on Few-Body Problems in Nuclear and Hadron Physics

September 22, 2026

Three tales, one theme

Motivation 1: Improve a classical work on α-condensates.

Motivation 2: Generalize cluster EFT from 2α to $N\alpha$.



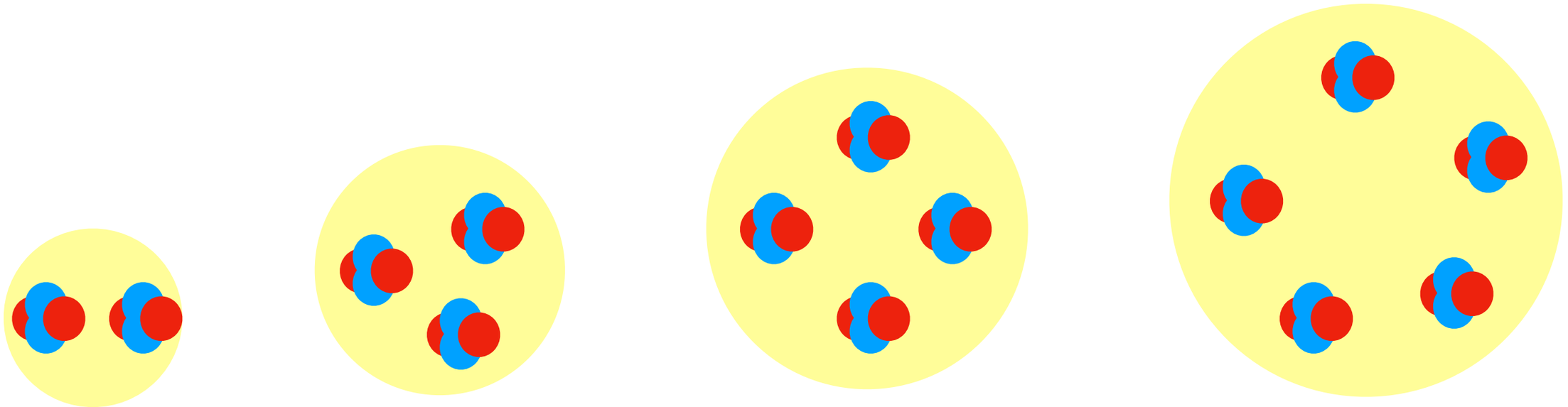
Motivation 3: Understand the physical meaning of shallow $N\alpha$ ground states predicted by shallow 2α potentials.

DB, PRC, accepted (2026)

α-condensates

Tohsaki, Horiuchi, Schuck, and Röpke (THSR, 2001+):

- There exists a family of **dilute** $N\alpha$ **states** in the spectrum of α-conjugate nuclei;
- 0^+ as "band head", large spatial extent, slightly above the $N\alpha$ threshold, dominated by $N\alpha$ configurations;
- Named as **α-condensates** under the inspiration of BEC.



$N\alpha$ threshold

α-condensate candidates:

- The 0_1^+ state of ^{8}Be, 0.9184 MeV above the 2α threshold;
- The 0_2^+ state of ^{12}C (the Hoyle state), 0.3795 MeV above the 3α threshold;
- The 0_6^+ state of ^{16}O, 0.66 MeV above the 4α threshold;
- …

Yamada & Schuck (YS, 2004) predicted the existence of α -condensates up to around ^{40}Ca, based on the $N\alpha$ model:

- α clusters as bosonic structureless particles;
- **Phenomenological** 2α (and 3α) interactions;
- The $N\alpha$ system is solved by **the Gross-Pitaevskii equation**, a **mean-field method**, with 2α interactions soften deliberately.
- **The dilute $N\alpha$ states identified as the lowest quasi-bound states within the Coulomb barriers.**

How to identify α-condensate candidates experimentally?

- Exploring its multi-α decay properties.
- …

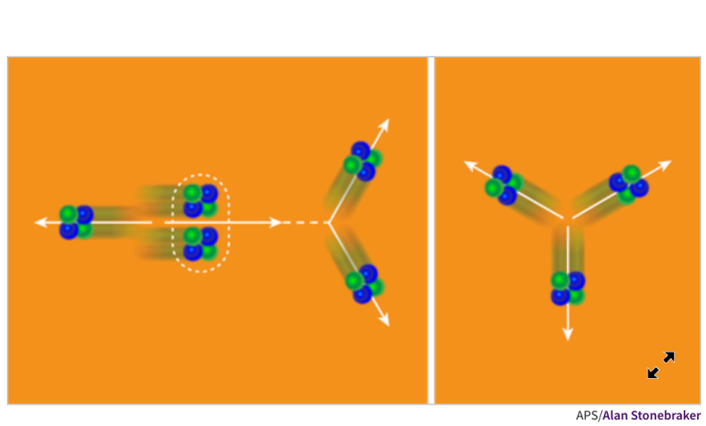
The Hoyle state: direct 3α decay

Watching the Hoyle State Fall Apart

Oliver Kirsebom
Department of Physics and Astronomy, Aarhus University, Ny Munkegade 120, Aarhus C Denmark, 8000

September 25, 2017 • *Physics* 10, 103

Two experiments provide the most precise picture to date of how an excited state of carbon decays into three helium nuclei.



APS/Alan Stonebraker

New Measurement of the Direct 3α Decay from the ^{12}C Hoyle State

R. Smith, Tz. Kokalova, C. Wheldon, J. E. Bishop, M. Freer, N. Curtis, and D. J. Parker

Phys. Rev. Lett. 119, 132502 (2017)
Published September 25, 2017

Read PDF

High-Precision Probe of the Fully Sequential Decay Width of the Hoyle State in ^{12}C

D. Dell'Aquila *et al.*

Phys. Rev. Lett. 119, 132501 (2017)
Published September 25, 2017

Read PDF

The 0_6^+ state of ^{16}O:

- decay to $\alpha + {}^{12}\mathrm{C}(0_2^+)$ *vs* ${}^{8}\mathrm{Be}(0_1^+) + {}^{8}\mathrm{Be}(0_1^+)$; M. Barbui *et al.*, Phys. Rev. C 98, 044601 (2018).

Cluster Effective Field Theory

For dilute $N\alpha$ systems,

- α clusters as relevant degrees of freedom, described by the scalar field Φ.
- UV scale: set by the pion mass & the first excited state of ^{4}He,

$$\Lambda_b \sim \sqrt{m_N E_\alpha^*} \sim m_\pi \simeq 140 \text{ MeV/c}.$$

- IR scale: the intercluster momemtum

$$Q \sim k_R = \sqrt{2\mu E_R} \ (\simeq 20 \text{ MeV/c for } ^8\text{Be GS}) \Rightarrow Q/\Lambda_b \simeq 1/7.$$

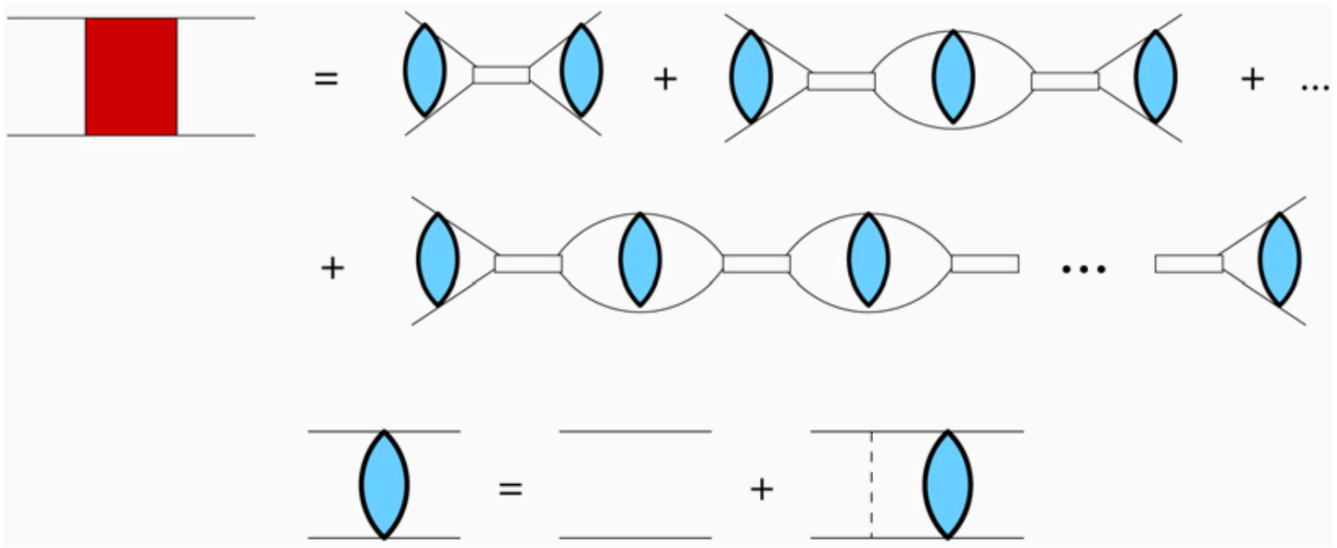
- The EFT Lagrangian is organized in powers of Q/Λ_b, systematic expansion, uncertainty quantification. Following Higa, Hammer, and van Kolck (2008), promoting $(Q/\Lambda_b)^2$ to LO,

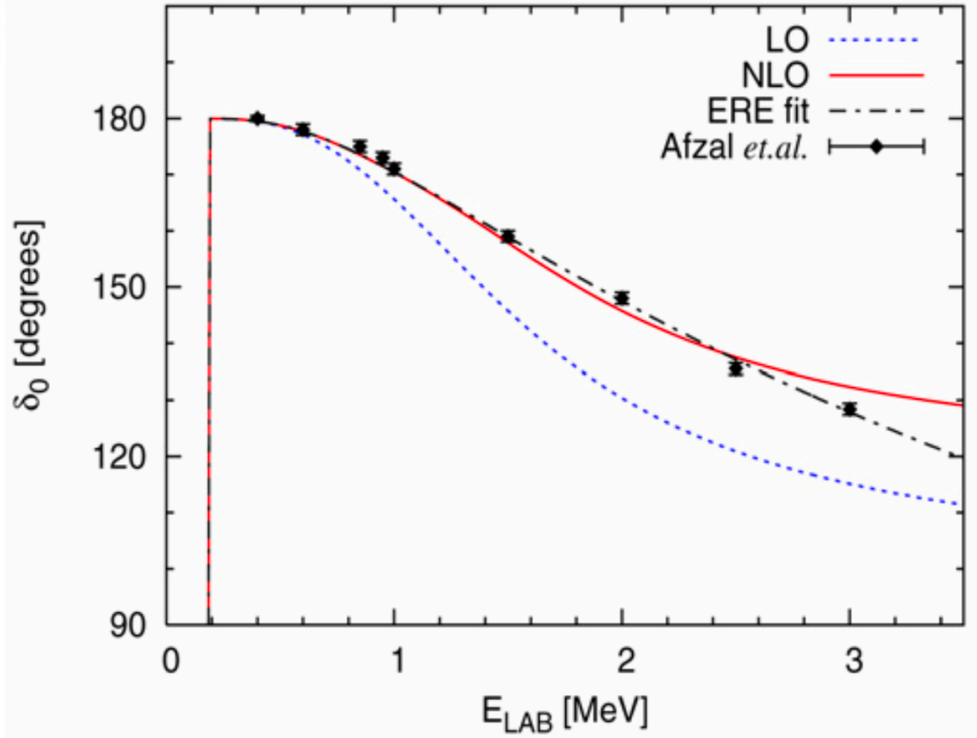
$$\mathscr{L}_{\text{LO}} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \Phi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_\alpha} \right) \Phi$$
$$- \tilde{C}_0 (\Phi^\dagger \Phi)^2 - \tilde{C}_2 \left[(\Phi^\dagger \Phi)(\Phi^\dagger \mathbf{D}^2 \Phi) + \text{h.c.} \right].$$

In **Higa, Hammer, and van Kolck (2008)**, cluster EFT is applied to the 2α system, the ground state (GS) of ^{8}Be and the α-α scattering, in terms of the **dimer** language.

$$\mathscr{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \Phi^\dagger\left(iD_t + \frac{\mathbf{D}^2}{2m_\alpha}\right)\Phi + \sigma d^\dagger\left(iD_t + \frac{\mathbf{D}^2}{4m_\alpha} - \Delta\right)d$$

$$+ g[d^\dagger\Phi\Phi + (\Phi\Phi)^\dagger d]$$





Q: How to generalize cluster EFT to the $N\alpha$ system?

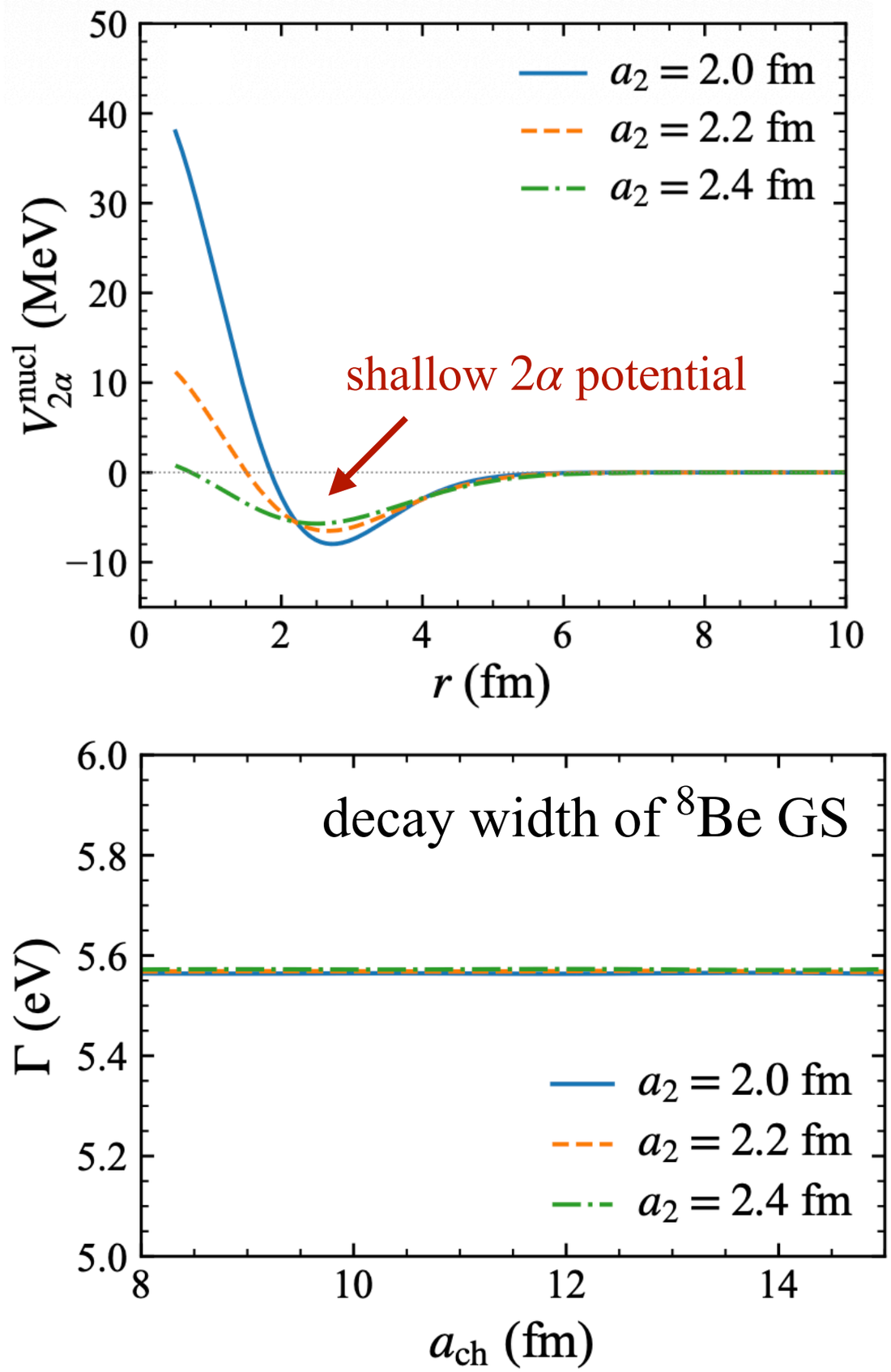
2α system from cluster EFT

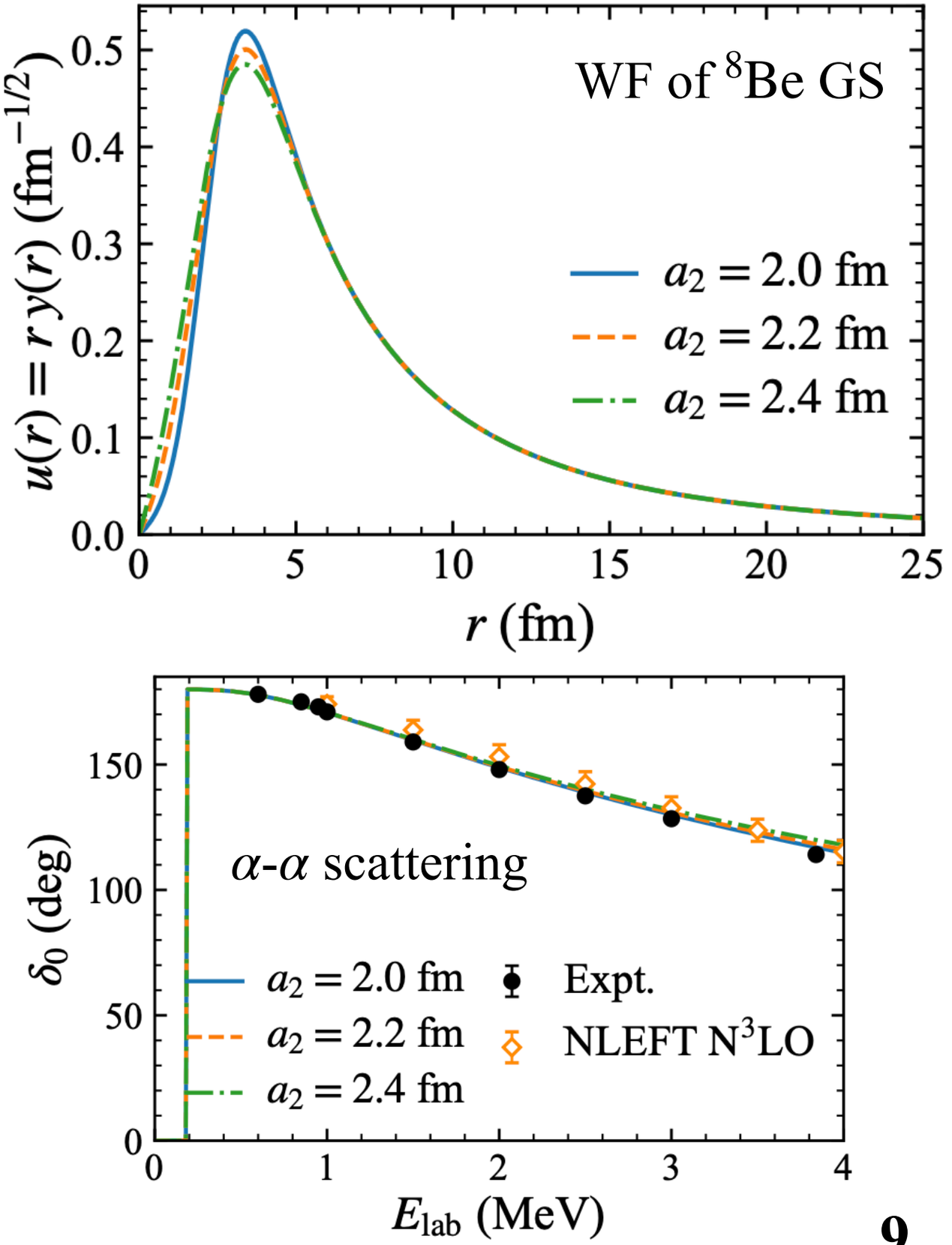
Difficult to generalize the QFT treatment of Higa *et al.* (2008) to the $N\alpha$ system; instead, the QM language is adopted by deriving effective potentials of α clusters from the cluster EFT Lagrangian,

$$V_{2\alpha}^{(N)} = \sum_{1\le i<j\le N} C_0 \exp(-r_{ij}^2/a_2^2) + C_2 r_{ij}^2 \exp(-r_{ij}^2/a_2^2) + \frac{4e^2}{r_{ij}} .$$

- a_2: the Gaussian parameter for regularization; $a_2 = 2, 2.2, 2.4$ fm; smaller a_2 will hit the Wigner bound.
- C_0, C_2: low-energy coefficients determined by fitting the ground-state **energy** and **decay width** of ^{8}Be and **phase shifts** of low-energy α-α scattering.

a_2 (fm)	C_0 (MeV)	C_2 (MeV·fm^{-2})
2.0	43.6214	−12.75
2.2	13.2742	−5.85
2.4	1.53604	−2.94531


$a_2 = 2.0$ fm
$a_2 = 2.2$ fm
$a_2 = 2.4$ fm
shallow 2α potential
$V_{2\alpha}^{\rm nucl}$ (MeV)
r (fm)


WF of ^{8}Be GS
$a_2 = 2.0$ fm
$a_2 = 2.2$ fm
$a_2 = 2.4$ fm
$u(r) = r\,y(r)$ (fm$^{-1/2}$)
r (fm)

decay width of ^{8}Be GS
$a_2 = 2.0$ fm
$a_2 = 2.2$ fm
$a_2 = 2.4$ fm
Γ (eV)
$a_{\rm ch}$ (fm)

α-α scattering
$a_2 = 2.0$ fm
$a_2 = 2.2$ fm
$a_2 = 2.4$ fm
Expt.
NLEFT N^{3}LO
δ_0 (deg)
$E_{\rm lab}$ (MeV)

Notably, the Italian group reported their efforts to generalize cluster EFT to 3α and 4α systems in E. Filandri *et al.*, PRC (2025).

PHYSICAL REVIEW C **112**, 044006 (2025)

3-α and 4-α systems within a short-range effective field theory–inspired approach

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[2]*INFN-Pisa, I-56127 Pisa, Italy*
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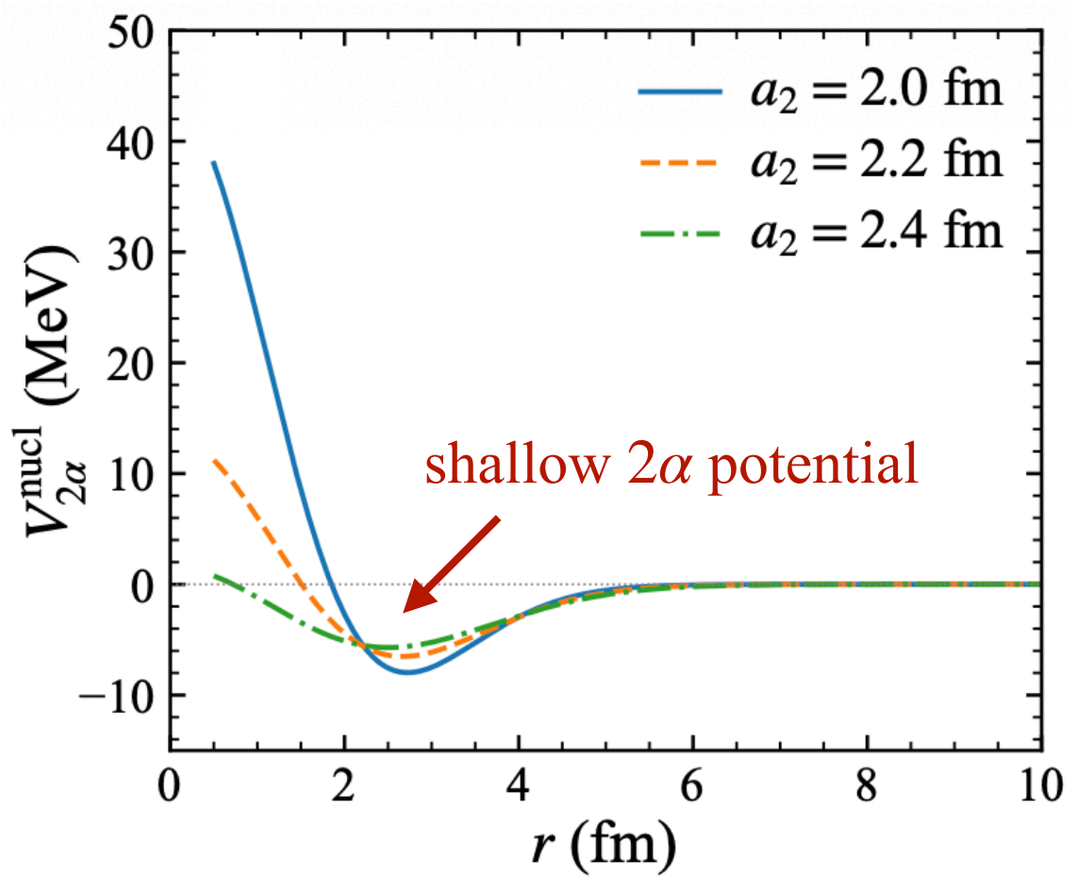
^{12}C and ^{16}O nuclei represent essential elements for life on Earth. The study of their formation plays a key role in understanding heavy element nucleosynthesis and stellar evolution. In this paper we present the study of ^{12}C and ^{16}O nuclei as systems composed of α-particle clusters using a short-range effective field theory–inspired approach. The fundamental and excited states of the studied nuclei are calculated within an *ab initio* approach using the hyperspherical harmonics method. Thanks to the two-body potential and fine-tuning of the three-body force, we have found the ^{12}C system nicely reproduced by theory. However, for the ^{16}O case, it is necessary to include a four-body force in order to achieve agreement with the experimental data.

Our work differs significantly in how it treats the consequences the consequences of shallow 2α potentials.

Shallow 2α potentials

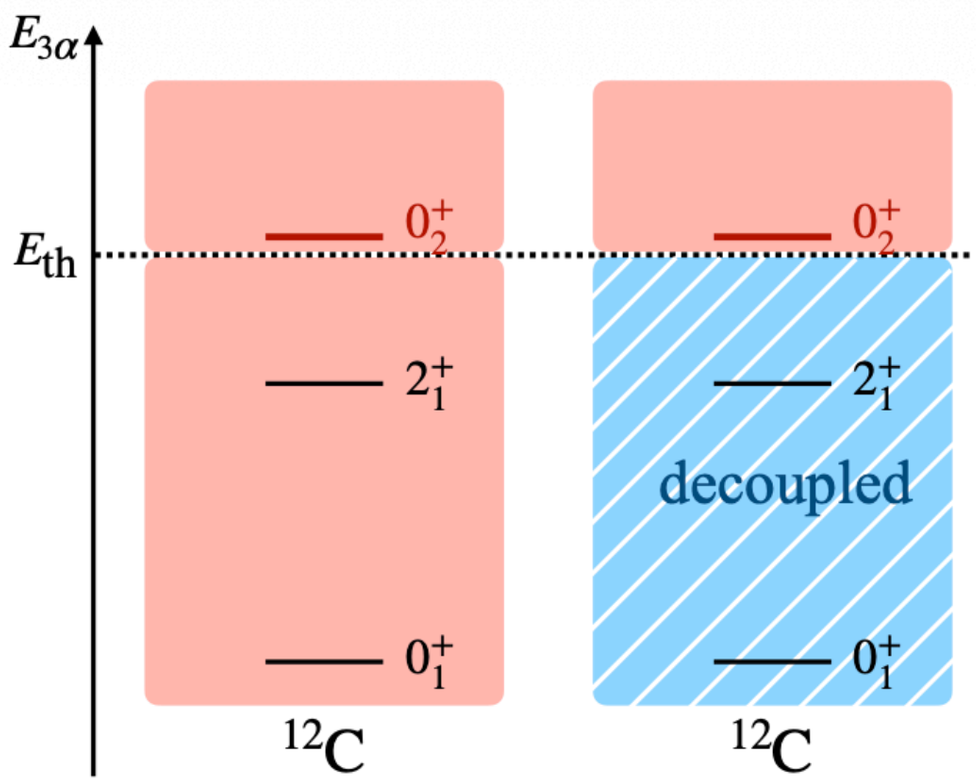
Shallow: the physical GS of ^{8}Be as the lowest state within the Coulomb barrier, similar to the **Ali-Bodmer potential**; in contrast to the **Buck-Friedrich-Wheatley potential**

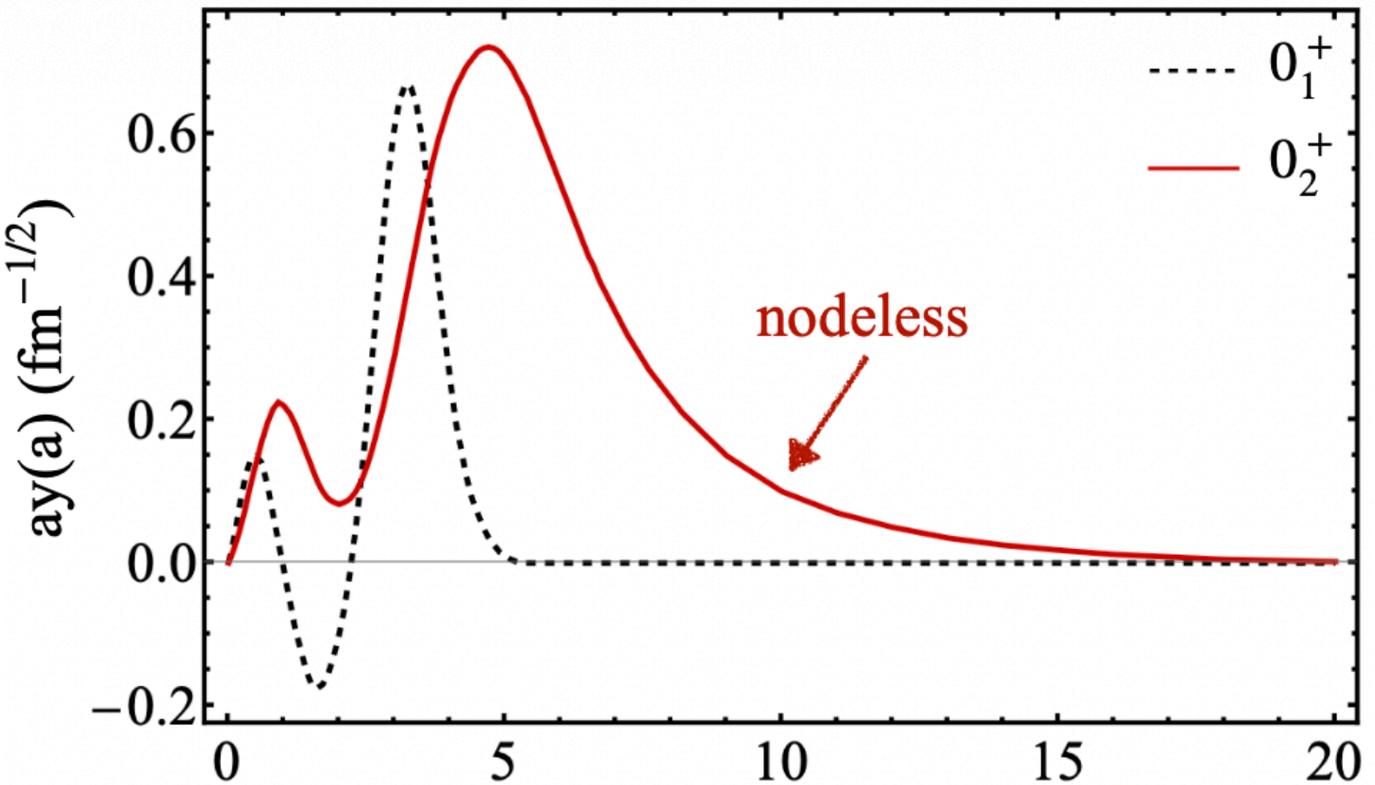
$$V_{\mathrm{BFW}} = v_0 \exp(-\rho_0 r^2) + \frac{4e^2}{r}\mathrm{erf}(\beta r)$$



which supports also unphysical Pauli forbidden states.

- When applied the 3α system, shallow 2α potentials lead to **a shallow ground state of the 3α system**, typically ~ **1-3 MeV beneath** the 3α disintegration threshold.
- In comparison, the physical GS of ^{12}C is 7.275 MeV beneath the 3α threshold.





conventional

suprathreshold

Add a **negative** 3α potential to pull **down** the lowest 0^+ state to the physical GS of ^{12}C; **different** 3α potentials for **different** spin-parities; adopted by the Italy group.

Add a **positive** 3α potential to push **up** the lowest 0^+ state to the physical Hoyle state of ^{12}C, decoupling the GS and 2_1^+ states from the model space; proposed by YS and inherited by our work.

3α potential

By pushing up the lowest 0^+ state to the Hoyle state, α-cluster EFT is proposed to be **an EFT specialized for α-condensates**, excluding compact states of $N\alpha$-nuclei.

Meanwhile, the physical GS of ^{12}C has its characteristic momentum $Q \sim 130$ MeV $\sim \Lambda_b \Rightarrow$ Whether cluster EFT is applicable to GS?

Assume the 3α interaction

$$V_{3\alpha}^{(N)} = \sum_{1\le i<j<k\le N} D_0 \exp\left[-(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/b_3^2\right],$$

with $b_3 = 5.0$ fm and D_0 determined by fitting **the resonant energy of the Hoyle state**.

a_2 (fm)	C_0 (MeV)	C_2 (MeV·fm^{-2})	D_0 (MeV)
2.0	43.6214	−12.75	7.679
2.2	13.2742	−5.85	7.5135
2.4	1.53604	−2.94531	7.2721

Stochastic variational method + real scaling method

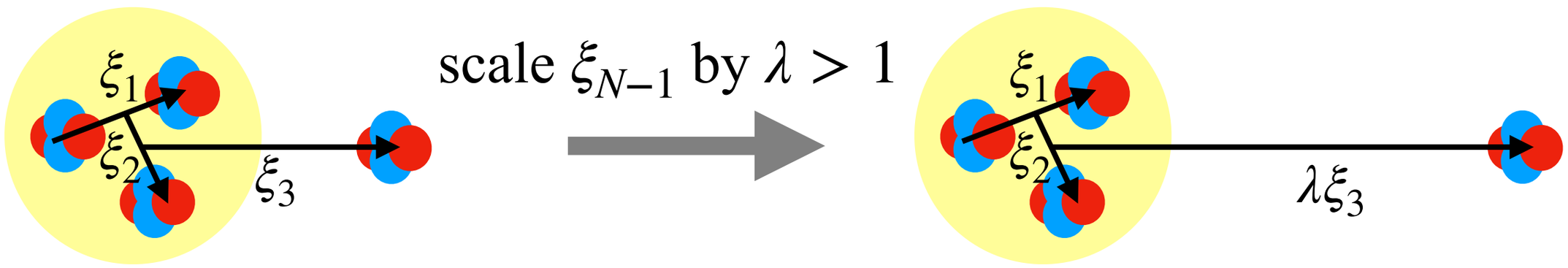
The Schrödinger equation of the $N\alpha$ system is solved by **the bound-state approach** to few-body resonant states; the $N\alpha$-condensates are **narrow resonances**.

- **Stochastic variational method (SVM)**: the $N\alpha$-condensates as quasi-bound states located within the Coulomb barriers, (Varga & Suzuki, 1995)

$$\Phi_N = \sum_{k=1}^{K} c_k \phi_k(\xi), \quad \phi_k(\xi) = \mathcal{S} \exp\left(-\frac{1}{2} \sum_{i,j=1}^{N-1} A_{ij}^{(k)} \xi_i \cdot \xi_j \right),$$

where $\xi = (\xi_1, \cdots, \xi_{N-1})$ are the Jacobi coordinates.

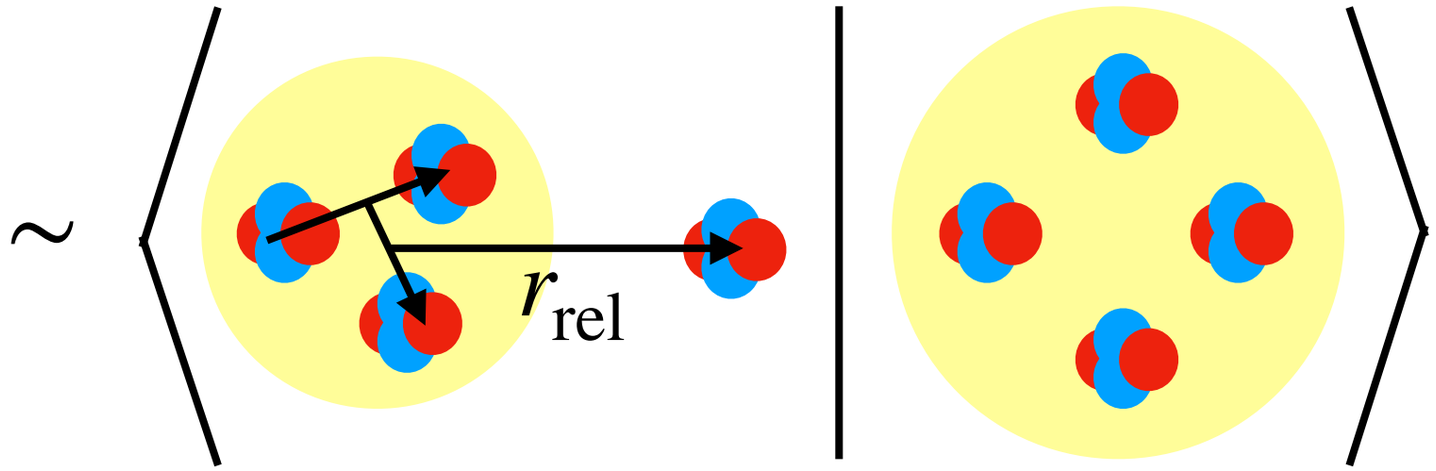
- **Real scaling method (RSM)**: Hiyama (2006), the stabilization method, identify the true resonant states from discretized continuum states generated by SVM



Reduced width amplitude + R-matrix theory

Reduced width amplitude (RWA)

$$y(r) = \sqrt{\frac{N!}{(1+\delta_{m,N-m})(N-m)!\,m!}} \left\langle \frac{\delta(r-r_{\rm rel})}{r_{\rm rel}^2} \left[\Phi_{N-m} \otimes \Phi_m \otimes Y_{00}(\hat{\mathbf{r}}_{\rm rel})\right]_{0^+} \middle| \Phi_N \right\rangle$$

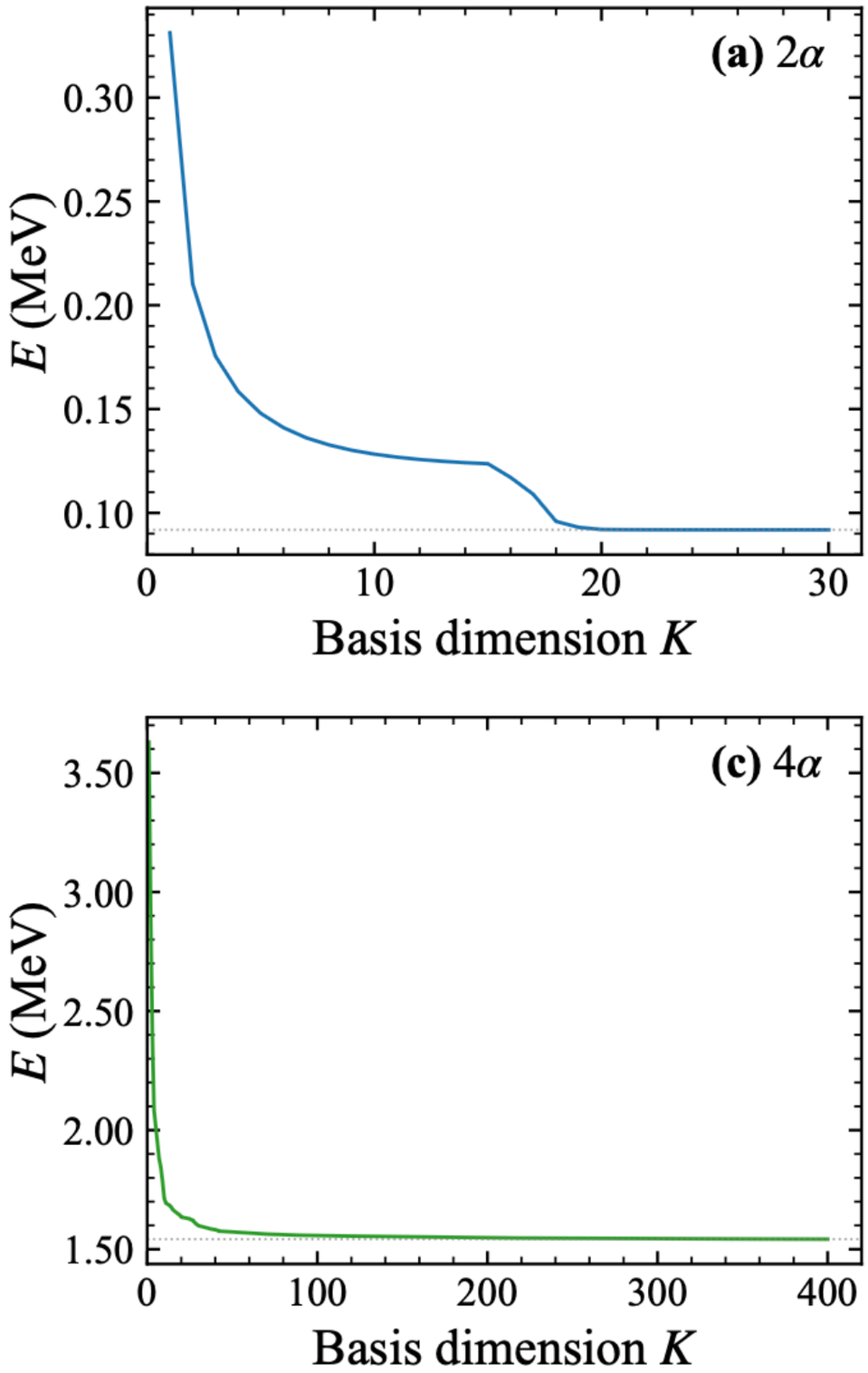


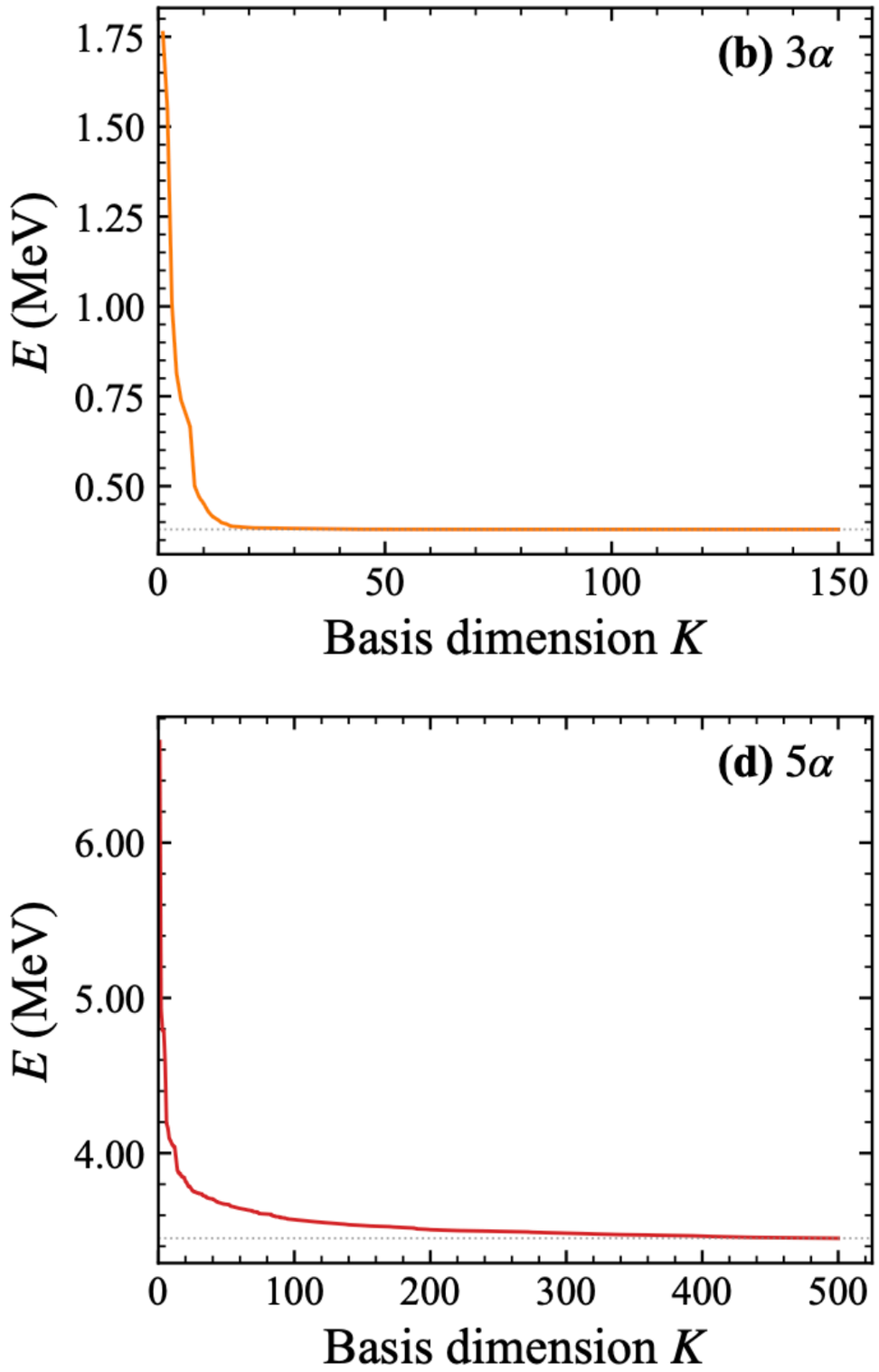
R-matrix theory

$$\Gamma = 2P_0(ka_{\rm ch})\,\gamma^2(a_{\rm ch}), \qquad \gamma^2(a_{\rm ch}) = \frac{\hbar^2}{2\mu a_{\rm ch}}[u(a_{\rm ch})]^2 = \frac{\hbar^2 a_{\rm ch}}{2\mu}[y(a_{\rm ch})]^2,$$

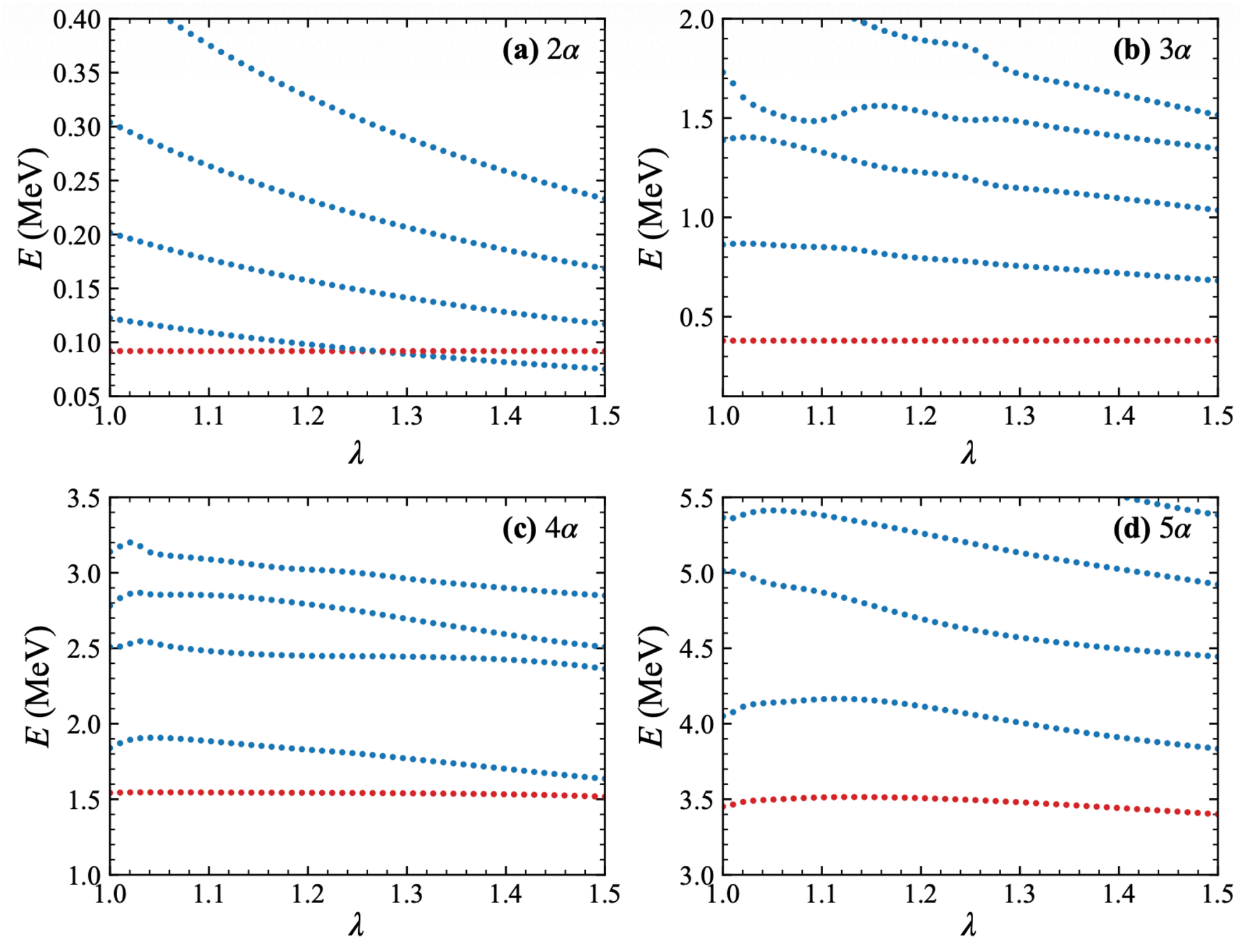
$$P_0(ka_{\rm ch}) = \frac{ka_{\rm ch}}{F_0^2(\eta, ka_{\rm ch}) + G_0^2(\eta, ka_{\rm ch})}, \qquad k = \frac{\sqrt{2\mu E_{\rm rel}}}{\hbar}, \qquad \eta = \frac{Z_1 Z_2 e^2 \mu}{\hbar^2 k}.$$

SVM calculations





RSM calculations



Main table + self-consistency check

Main table

	$a_2 = 2.0$ fm			$a_2 = 2.2$ fm			$a_2 = 2.4$ fm			YS [17]	
System	E	$R_{\rm pp}$	$R_{\rm ch}$	E	$R_{\rm pp}$	$R_{\rm ch}$	E	$R_{\rm pp}$	$R_{\rm ch}$	E	$R_{\rm rms}$
2α	0.09184^*	2.89	3.34	0.09184^*	2.87	3.32	0.09184^*	2.85	3.31	–	–
3α	0.3795^*	2.98	3.42	0.3795^*	2.96	3.40	0.3795^*	2.95	3.39	0.98	4.87
4α	1.5425	4.22	4.54	1.5403	4.25	4.57	1.5307	4.29	4.61	1.84	5.23
5α	3.4517	5.54	5.79	3.4287	5.50	5.75	3.3855	5.55	5.80	3.04	5.55

Energies and charge radii have weak dependence on the regularization parameter $a_2 \Rightarrow$ some kind of **universality** (limited by the Wigner bound).

Self-consistency check of diluteness

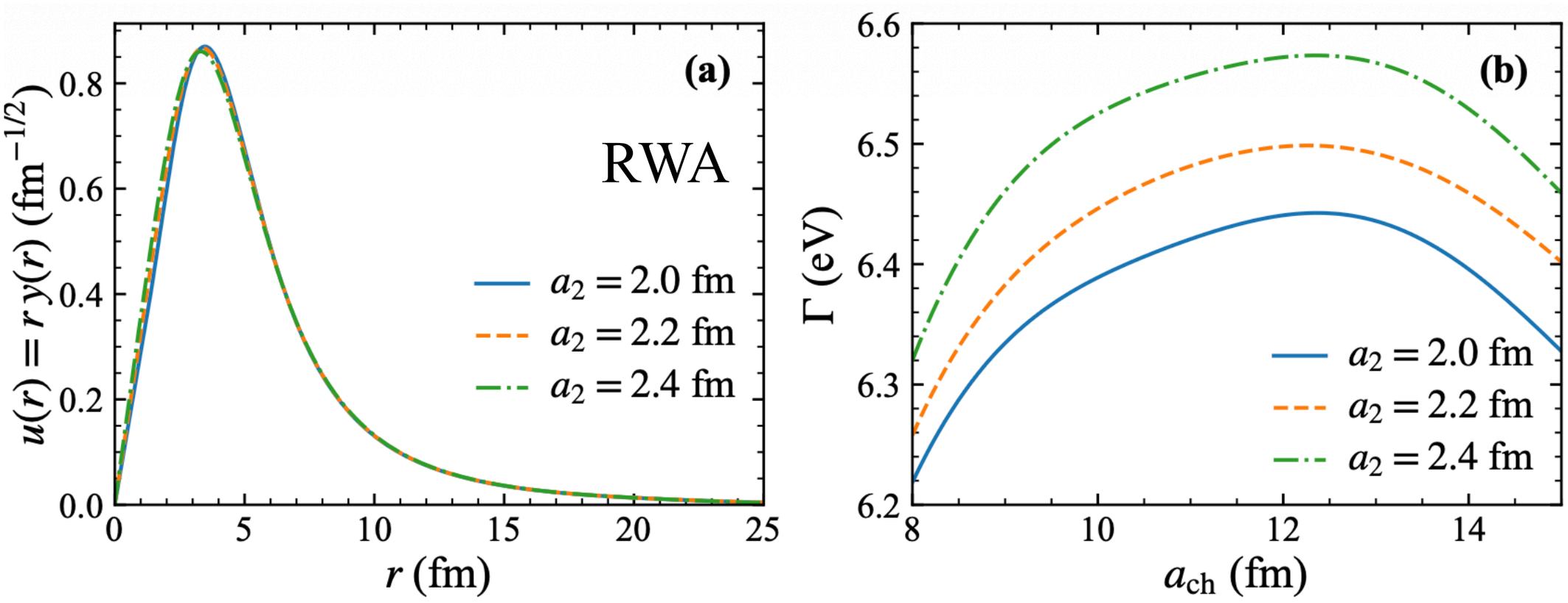
$$\bar{\rho}_{\rm eff} = \frac{4N}{(4\pi/3)R_{\rm eq}^3}, \qquad R_{\rm eq} = \sqrt{\frac{5}{3}} R_{\rm ch}$$

$$\frac{\bar{\rho}_{\rm eff}}{\rho_0} \simeq 0.21,\ 0.12,\ 0.07 \qquad \text{for} \qquad 3\alpha,\ 4\alpha,\ 5\alpha,$$

where $\rho_0 = 0.16\ \text{fm}^{-3}$ is the nuclear saturation density.

3α system

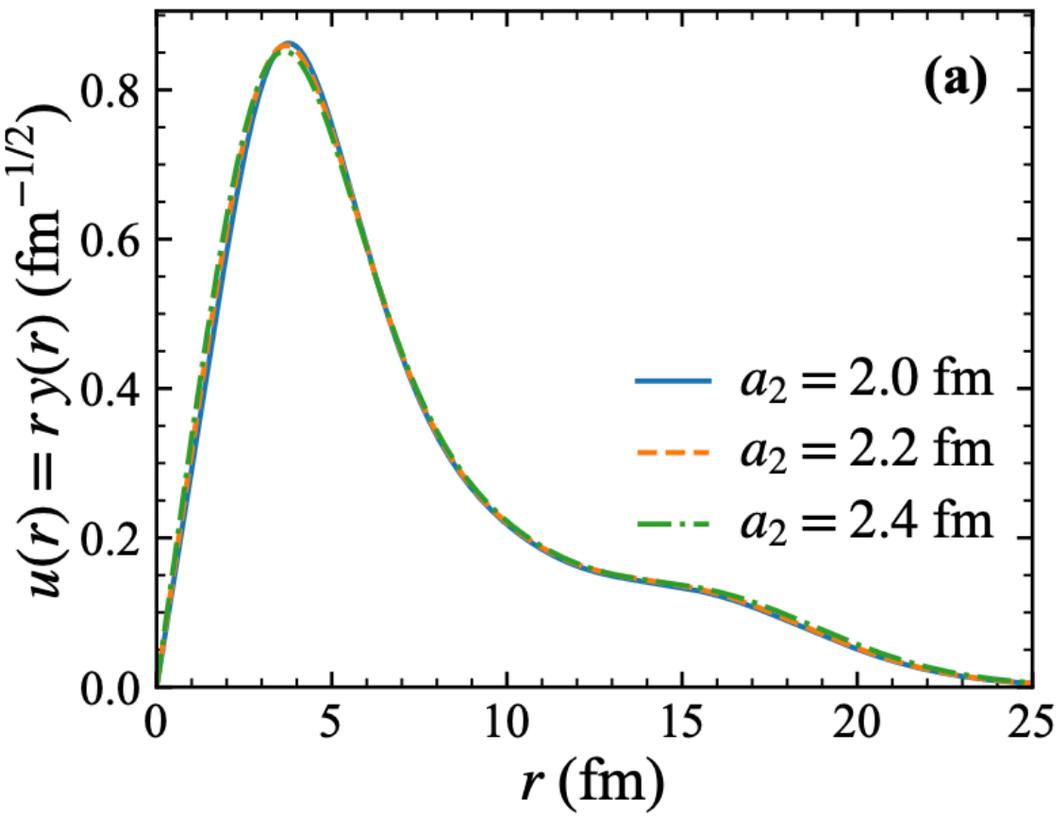
the Hoyle state $\rightarrow \alpha + {}^{8}\mathrm{Be}(0_1^+)$



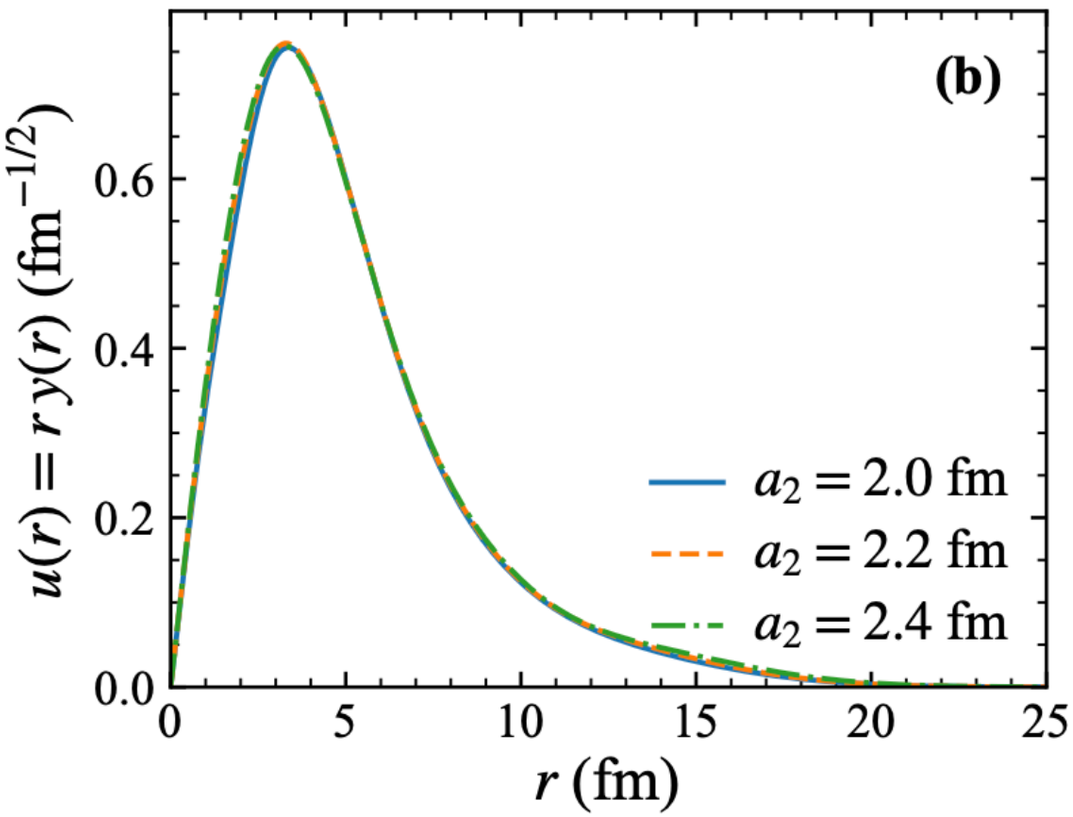
The decay width is $\Gamma \sim 6.4$ eV; the experimental decay with is 9.3(9) eV.

4α system

(a) $4\alpha \rightarrow {}^{12}\mathrm{C}(0_2^+) + \alpha$

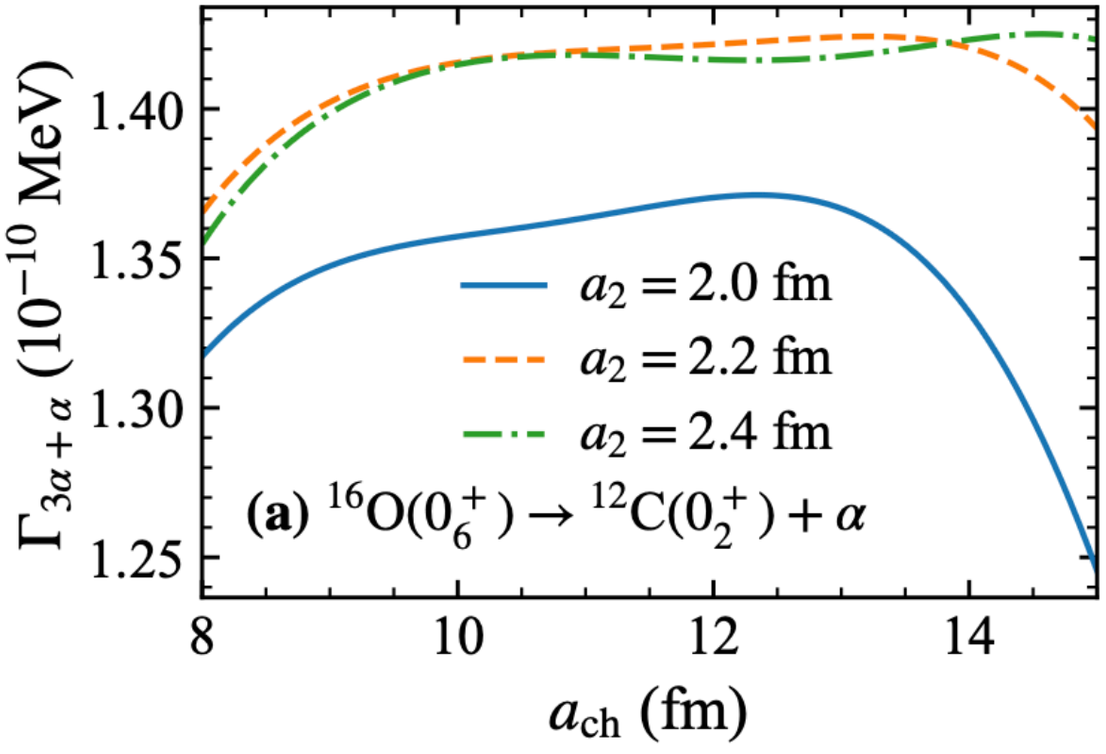


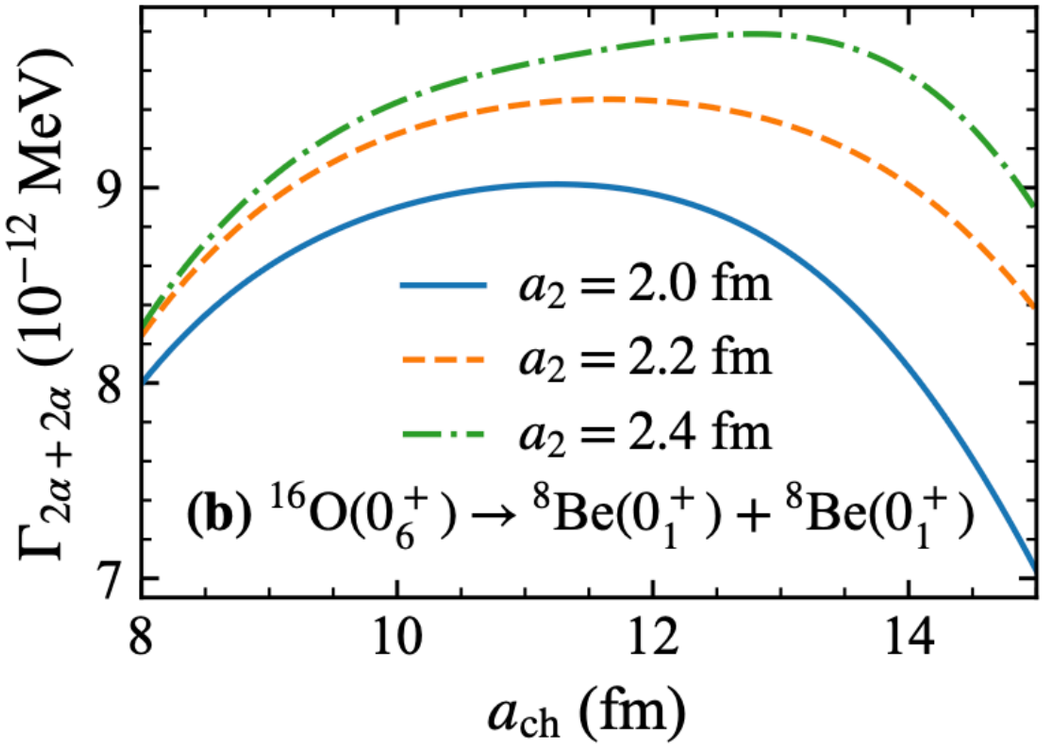
(b) $4\alpha \rightarrow {}^{8}\mathrm{Be}(0_1^+) + {}^{8}\mathrm{Be}(0_1^+)$

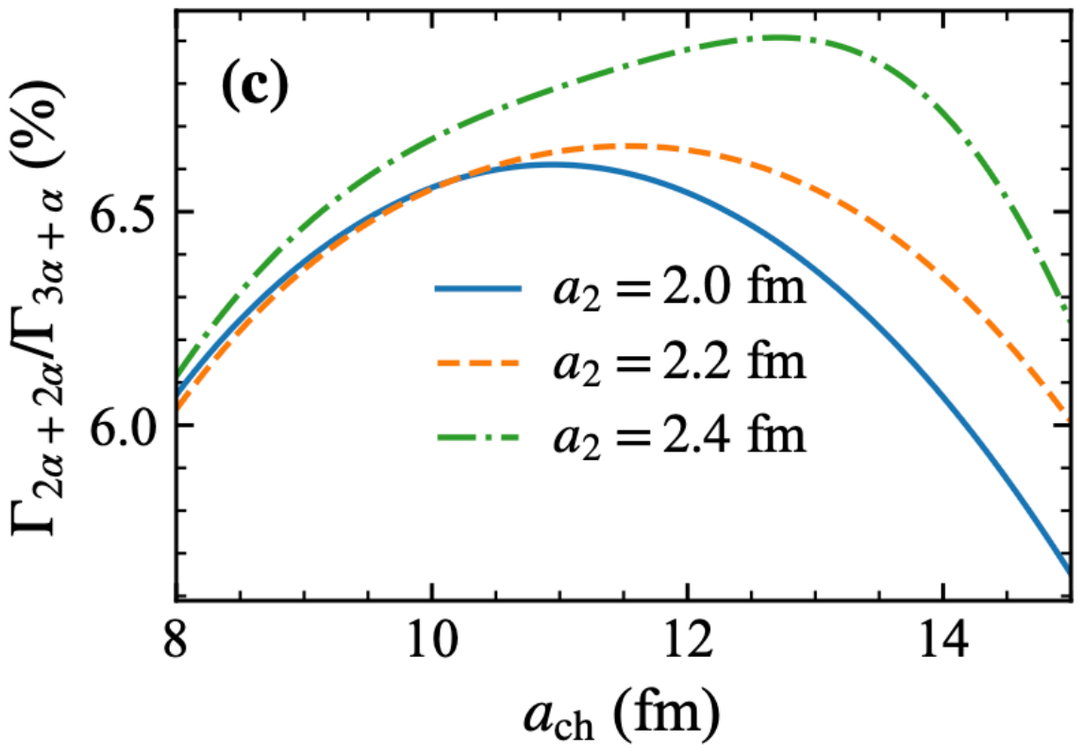


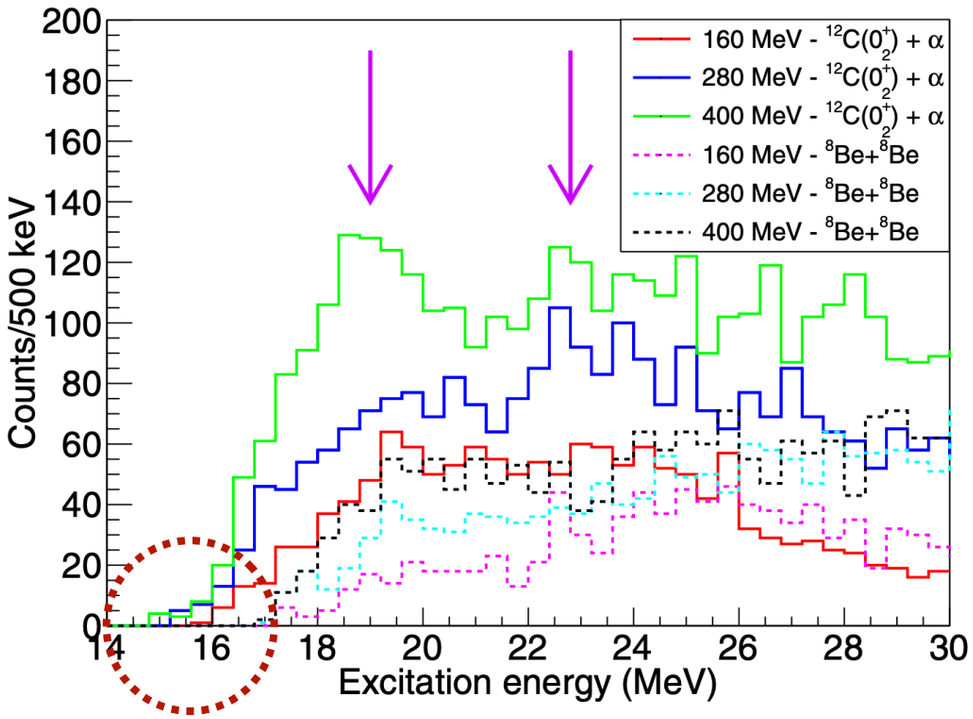
Experimental conflict on the decay branching ratio of ${}^{16}\mathrm{O}(0_6^+)$:

- M. Barbui *et al.* (2018): $\Gamma(\mathrm{Be})/\Gamma(\mathrm{Hoyle}) = 1 \pm 0.7$
- J. Bishop *et al.* (2019): may suggest $\Gamma(\mathrm{Be}) \ll \Gamma(\mathrm{Hoyle})$
- J. Chen et al. (2023): "Unfortunately, the cluster-decay measurements for the 15.1 MeV (0_6^+) state in ${}^{16}\mathrm{O}$ have been unsuccessful despite numerous experimental efforts made over the past two decades."





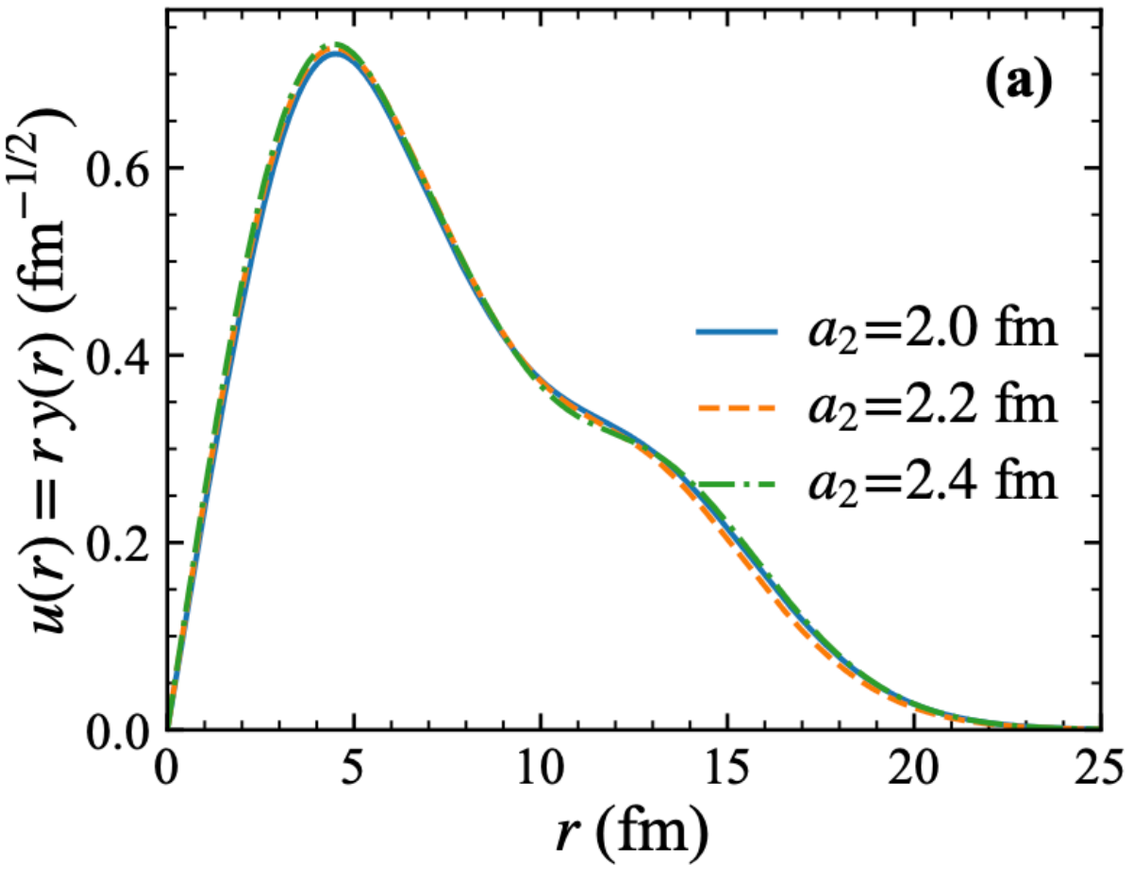




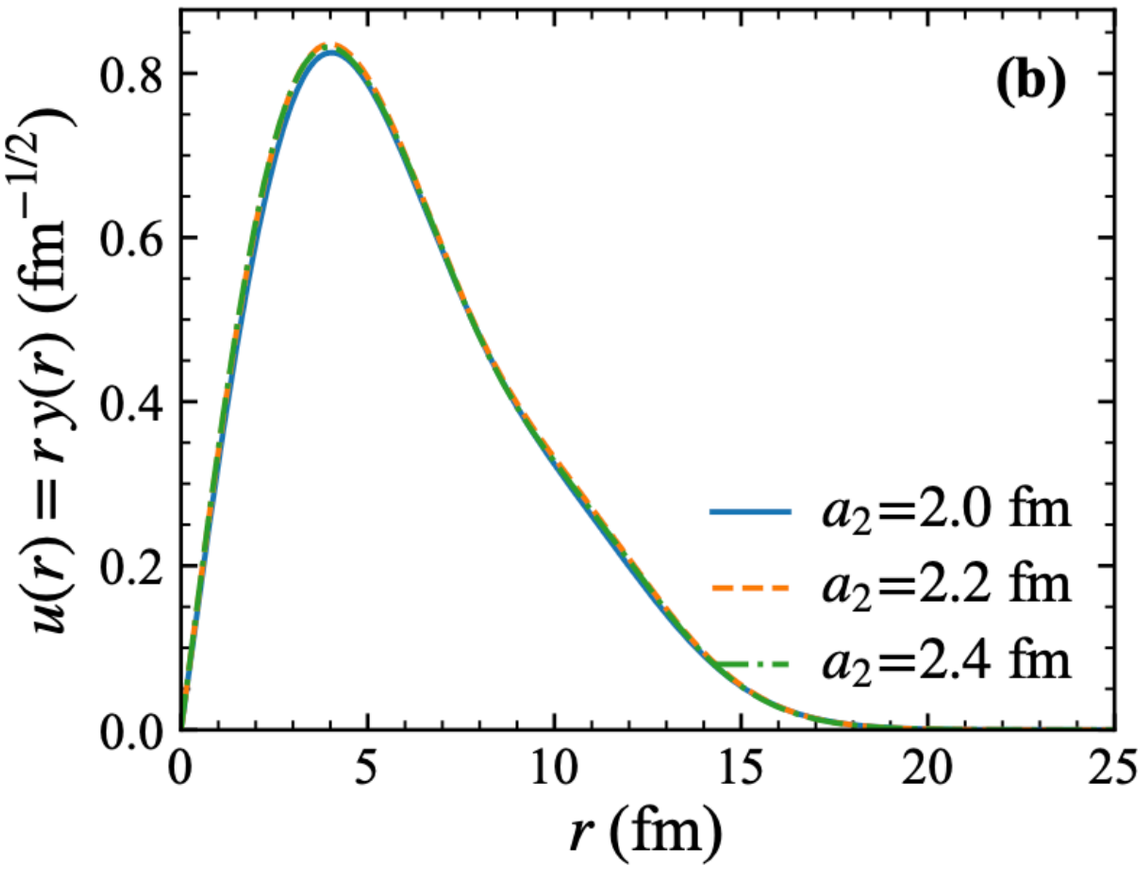
Exp data from Bishop *et al.* (2019)

5α system

(a) $5\alpha \rightarrow 4\alpha + \alpha$



(b) $5\alpha \rightarrow {}^{12}\mathrm{C}(0_2^+) + {}^{8}\mathrm{Be}(0_1^+)$



More on the 3α system (preliminary)

Excitations of the Hoyle state:

- $^{12}\mathrm{C}(2_2^+)$ $[(E_x, \Gamma) = (9.87\ \mathrm{MeV}, 0.85\ \mathrm{MeV})]$, 2.76 MeV above 3α threshold.

With cluster EFT interactions,

a_2 (fm)	$E(2^+)$
2.0	$2.73 - 1.49i$
2.2	$2.63 - 1.45i$
2.4	$2.54 - 1.40i$

Resonant energies comparable to the exp value, while decay widths 3.5-3.7 times larger; $Q/\Lambda_b \simeq 0.6$. No other resonant states well below Λ_b.

Direct 3α decay vs sequential decay (SD) from the Hoyle state

- SD: branching ratio $\approx 99.923\,\%$
- Direct: DDL $\lesssim 0.02\,\%$, DDE $\lesssim 0.002\,\%$
- Exp: Direct $< 0.047\,\%$ $(0.00057\,\%$, model dependent)

Summary

Cluster EFT as an EFT specialized for α-condensates.

- α-condensates as the lowest quasi-bound states located within the Coulomb barriers.
- Reasonable descriptions of α-condensate candidates.
- Weak dependence on regulators.
- A natural starting point for further exploration of the triple-α reaction and dilute α-matter based on cluster EFT.