LIGHT HYPERNUCLEI

Andreas Nogga, IAS-4, Forschungszentrum Jülich

Workshop on Few-Body Problems in Nuclear and Hadron Physics, ITP, CAS, Beijing, China

- YN & YY interactions
- Faddeev-Yakubovsky, J-NCSM & SRG
- Uncertainty of Λ separation energies & chiral YNN interactions
- $S = -2$ hypernuclei: predictions for $A \leq 7$
- Conclusions & Outlook

in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

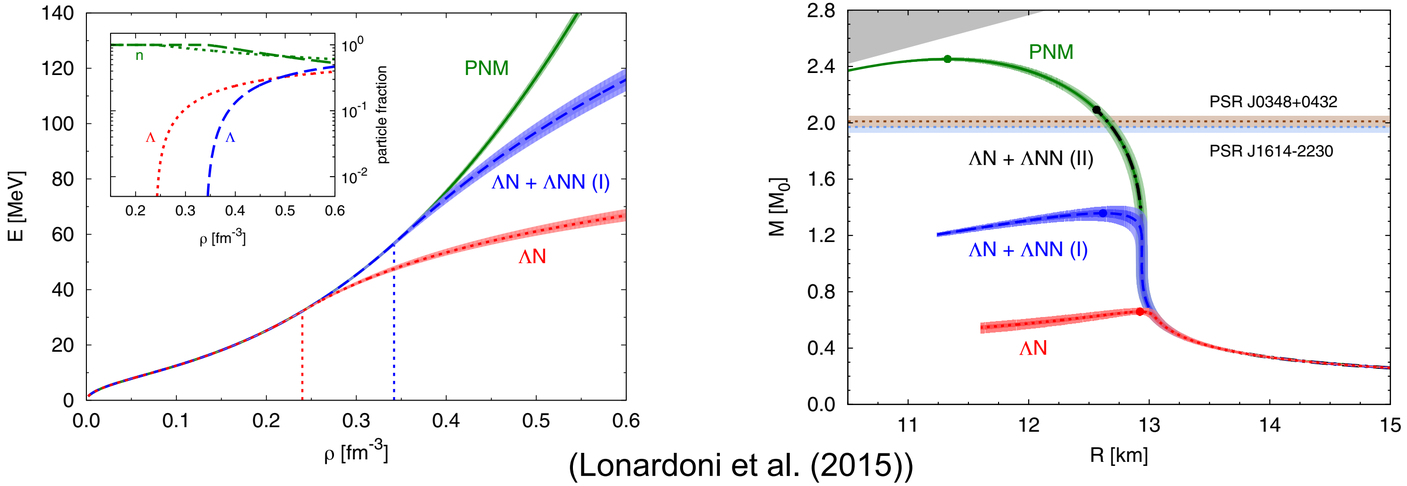
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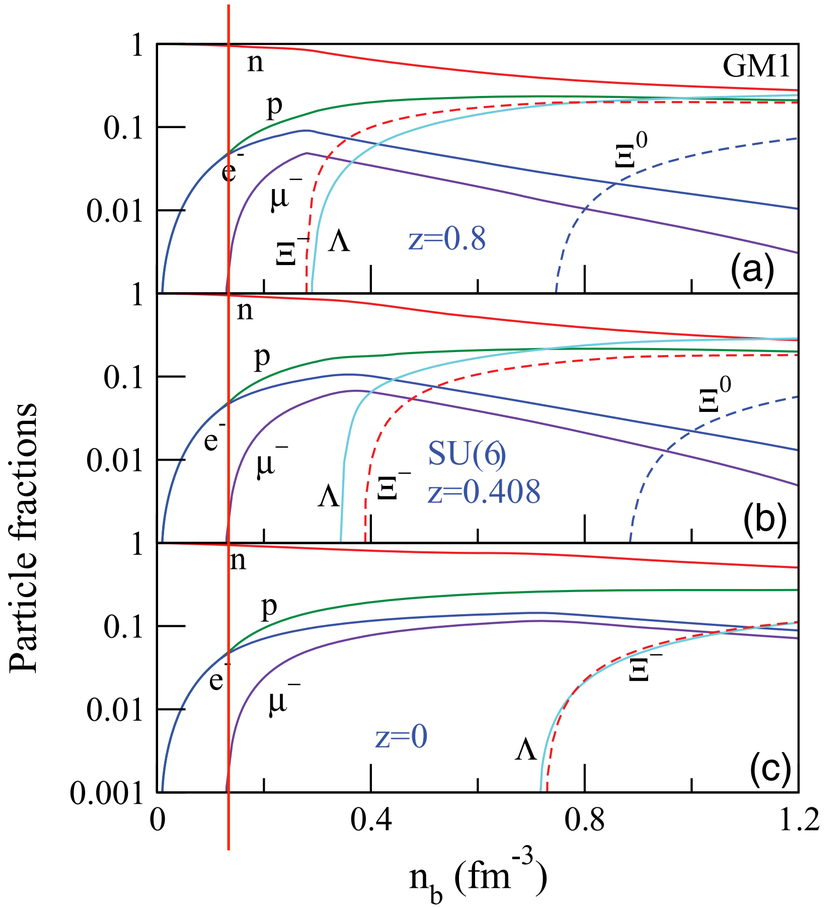
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HYPERNUCLEAR INTERACTIONS

- **Why is understanding hypernuclear interactions interesting?**
  - hyperon contribution to the EOS, neutron stars, supernovae
  - "hyperon puzzle"
  - Λ as probe to nuclear structure
  - **flavor dependence of baryon-baryon interactions**



(Lonardoni et al. (2015))



(Weissenborn et al. 2012)

HYPERNUCLEI

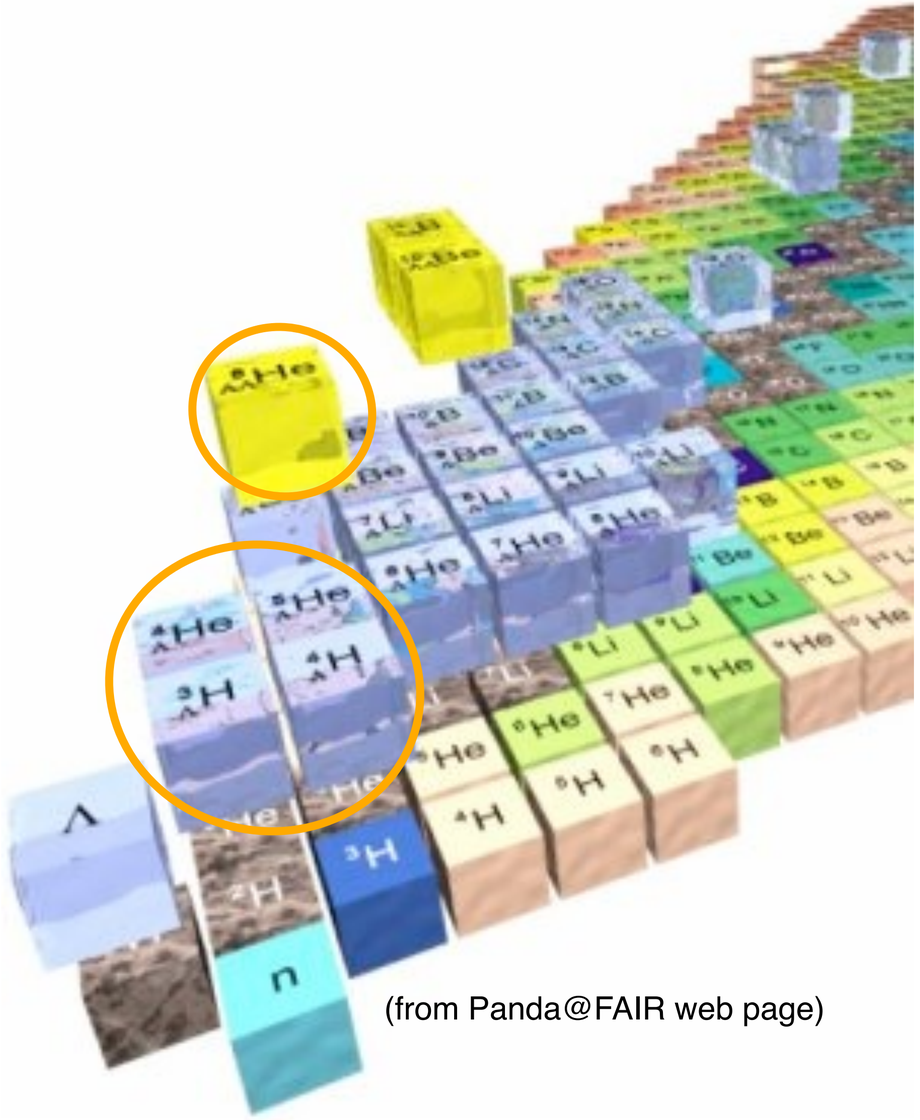
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Only few YN data. Hypernuclear data provides additional constraints.

- ΛN interactions are generally weaker than the NN interaction
  - naively: **core nucleus + hyperons**
  - separation energies **quite** independent from NN(+3N) interaction

- no Pauli blocking of Λ in nuclei
  - good to study nuclear structure
  - even light hypernuclei exist in **several spin states**

- **non-trivial constraints** on the YN interaction
- size of **YNN** interactions? → need to include **Λ-Σ conversion**!

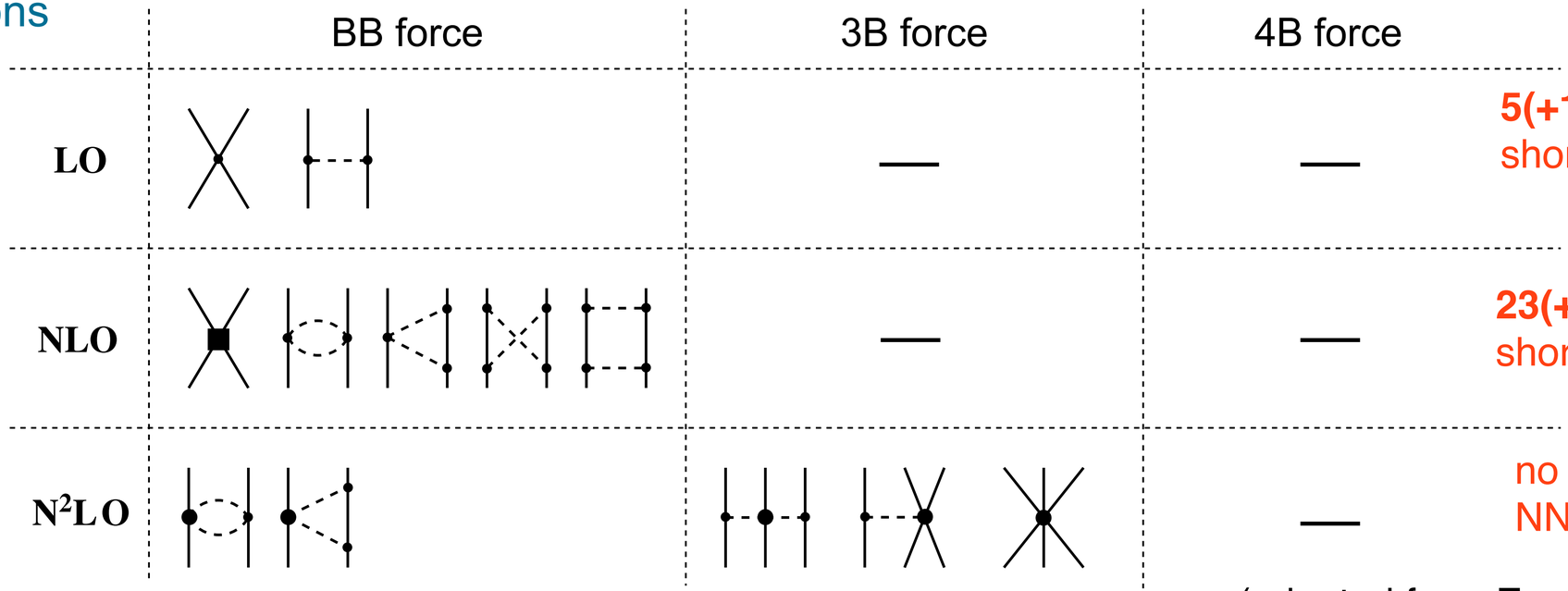


(from Panda@FAIR web page)

CHIRAL NN & YN & YY INTERACTIONS

- Chiral EFT implements **chiral symmetry of QCD**
- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions

- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N^{2}LO (for YN)



5(+1) NN/YN (YY)
short range parameters

23(+5) NN/YN (YY)
short range parameters

no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

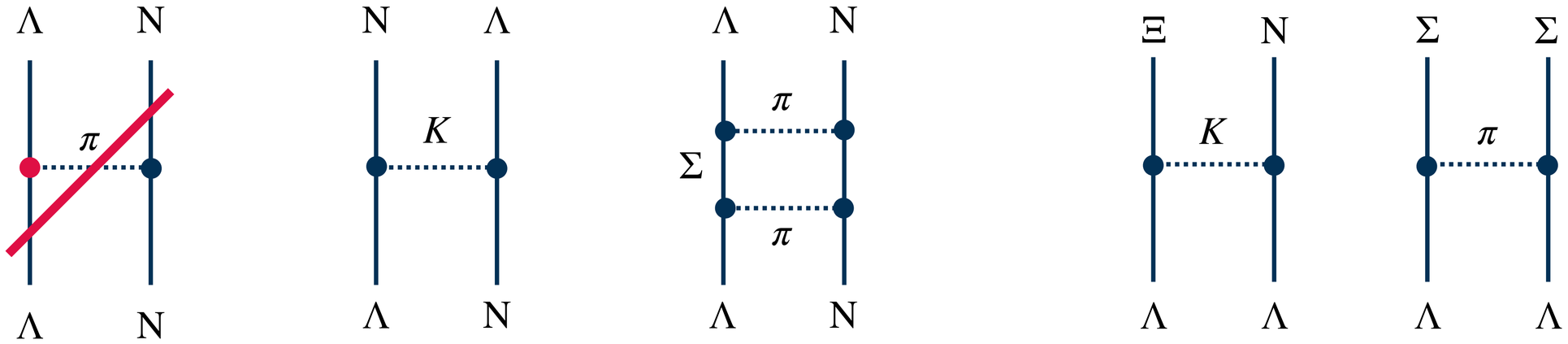
Retain flexibility to adjust to data due to counter terms — LECs are not uniquely determined yet

Regulator required — cutoff/different orders often used to estimate uncertainty

$\Lambda - \Sigma$ ($\Lambda\Lambda - \Sigma\Sigma - \Xi$N) **conversion** is explicitly included (3BFs only in N^{2}LO)

Hypertriton used to determine spin dependence

PARTICLE CONVERSION



ΛN interaction: pion exchange
suppressed by isospin symmetry

2π exchange with Σ
$2M_\pi + \Delta M_\Sigma \approx 350$ MeV

K exchange short ranged
$M_K \approx 500$ MeV

Particle conversion is driving **longest range part of the interaction**!

Particle conversion leads to interesting spin-isospin dependencies

Mass difference $\Delta M_{\Xi\mathrm{N}} \approx 25$MeV

Channels of **identical** and **non-identical** particles

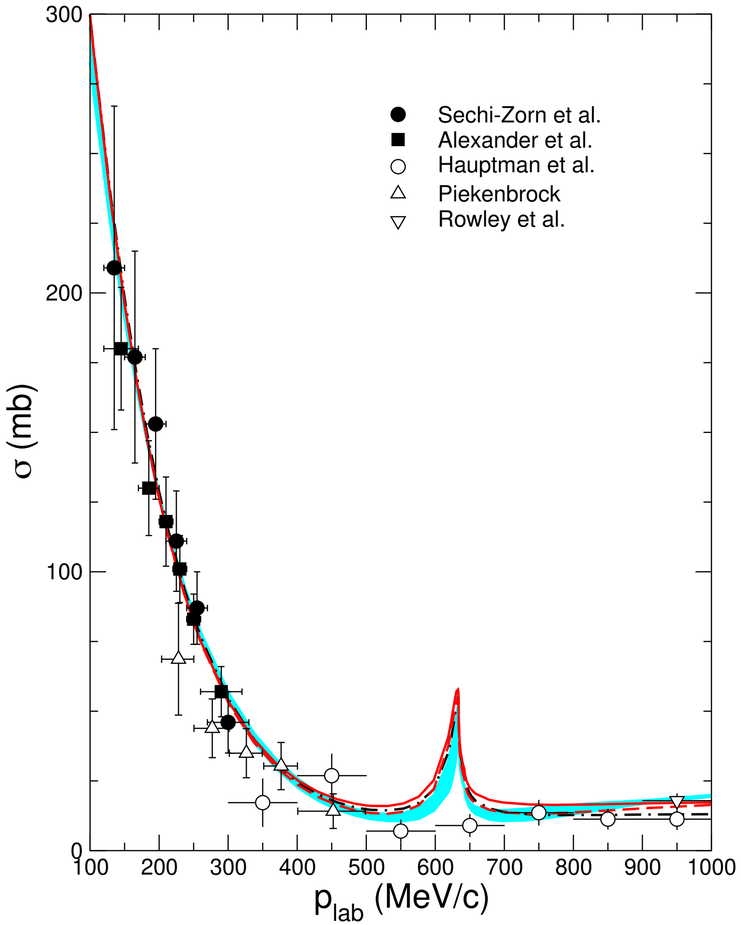
Indication of suppression of $\Xi\mathrm{N} \rightarrow \Lambda\Lambda$
(Ξ-hypernuclei)

SMS NLO/N²LO INTERACTION

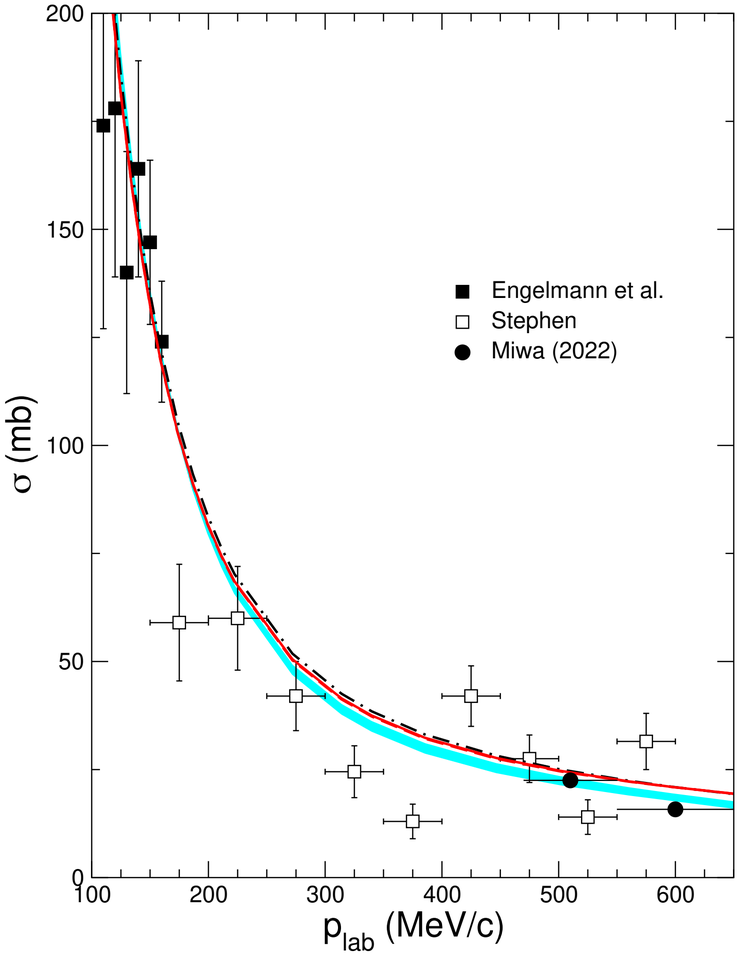
selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)

- total cross sections very similar in NLO and N²LO
- similar to NLO19 (and NLO13)
- alternative fit
- new data provides **new constraints**!

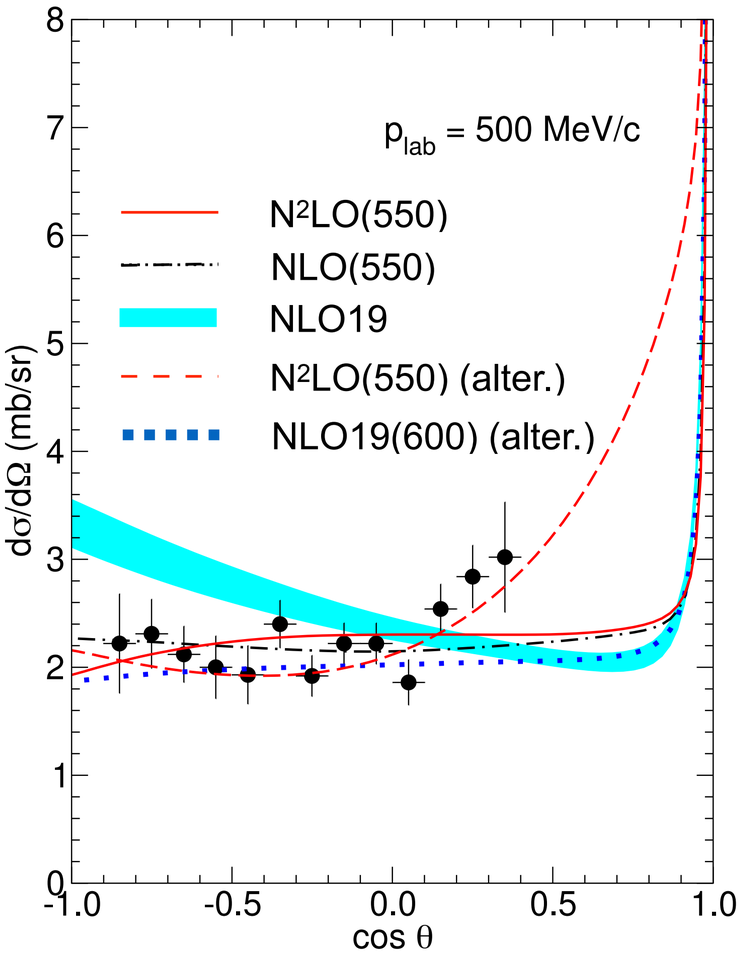
$\Lambda p \to \Lambda p$



$\Sigma^- p \to \Lambda n$



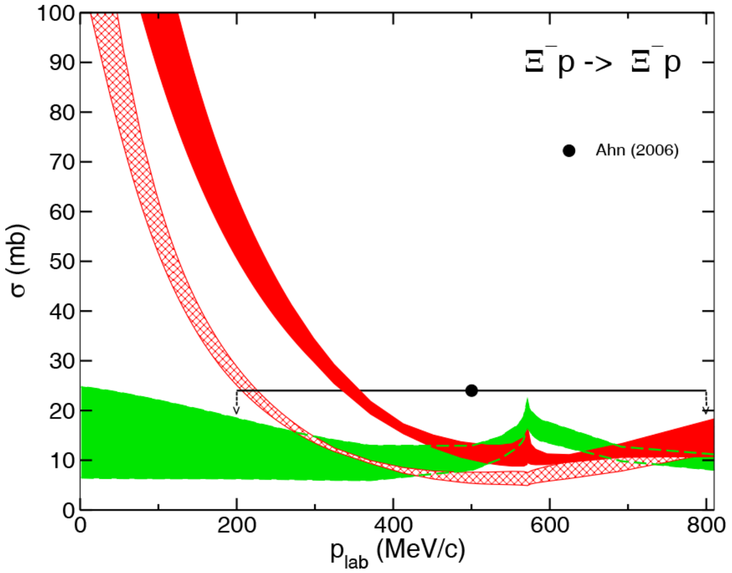
$\Sigma^+ p \to \Sigma^+ p$

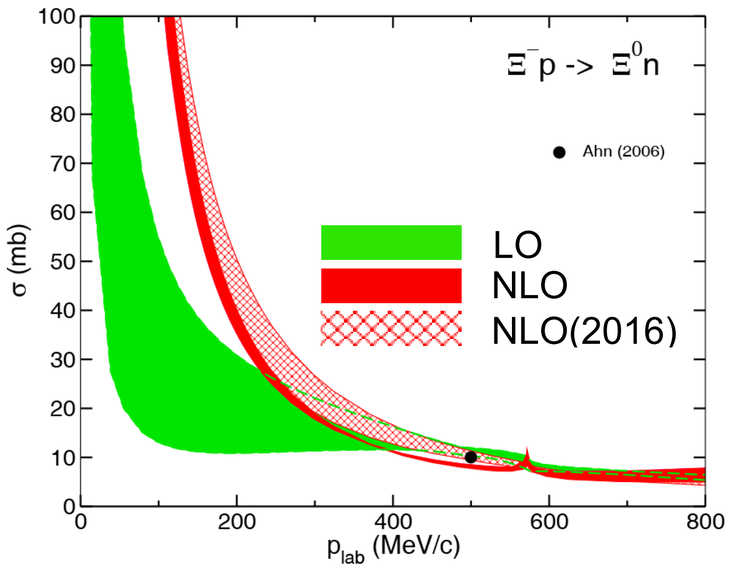


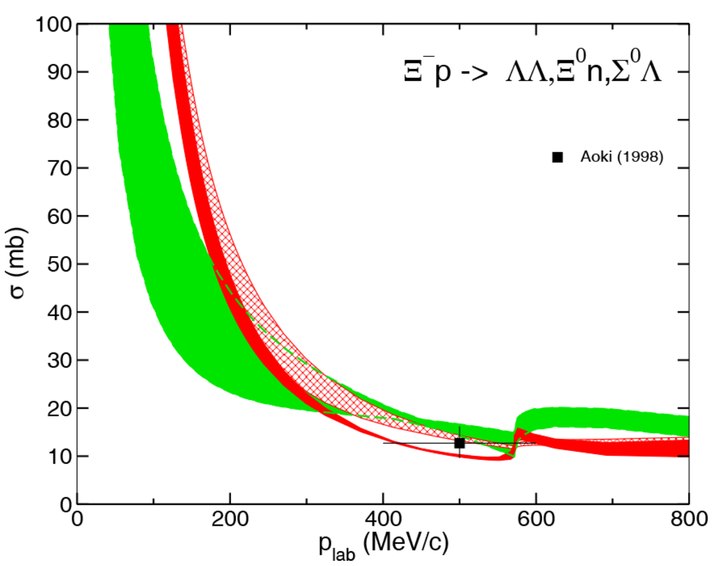
J. Haidenbauer et al. EPJ A 59, 63 (2023)

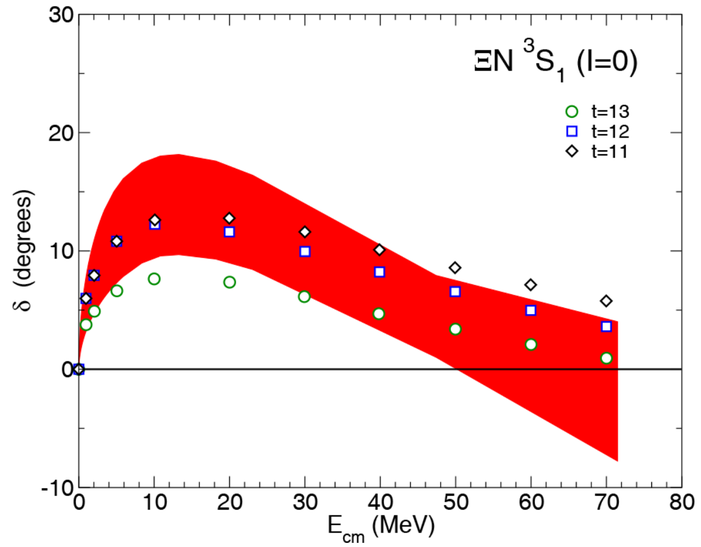
YY INTERACTION

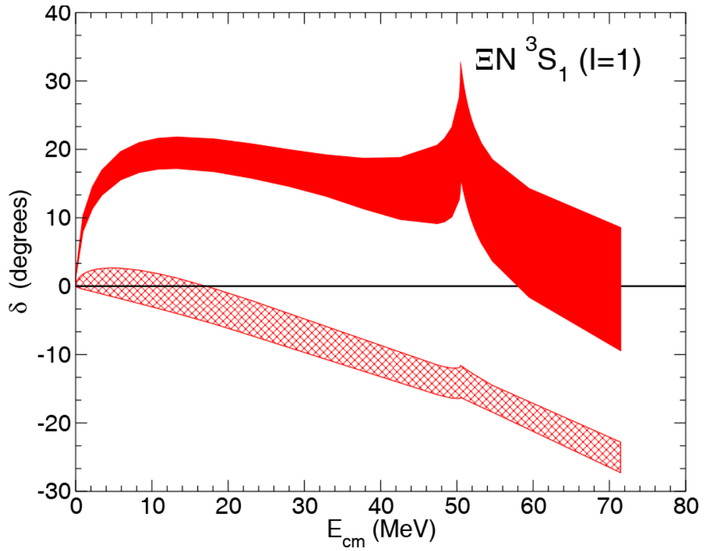
- adjusted to scarce data & LQCD (HAL QCD)
- updated version consistent with ΞN $^{33}S_1$)

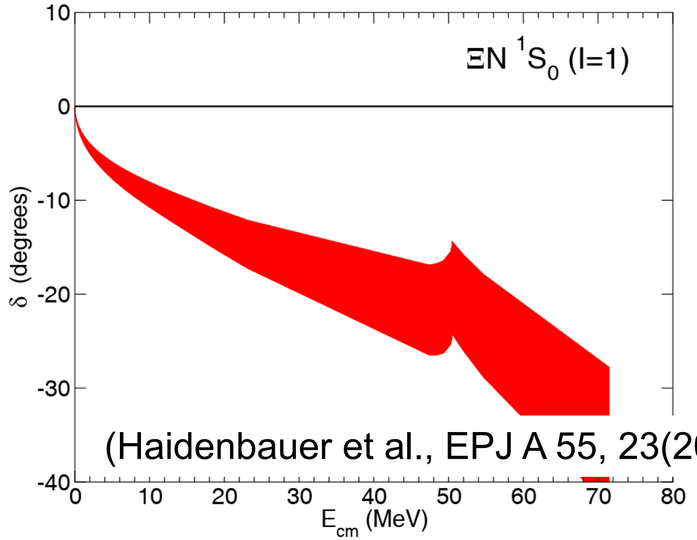












(Haidenbauer et al., EPJ A 55, 23(2019))

SRG INTERACTIONS

The evolution naturally separated in 2- and 3-body,… parts.

$$\frac{dV_{ij}(s)}{ds} = \Big[\Big[T_{ij}, V_{ij}(s)\Big], T_{ij} + V_{ij}(s)\Big]$$

easily done — we use momentum space

$$\frac{dV_{ijk}(s)}{ds} = \Big[\Big[T_{ij}, V_{ij}(s)\Big], V_{ki}(s) + V_{jk}(s) + V_{ijk}(s)\Big] + \Big[\Big[T_{jk}, V_{jk}(s)\Big], V_{ki}(s) + V_{ij}(s) + V_{ijk}(s)\Big]$$

$$+ \Big[\Big[T_{ki}, V_{ki}(s)\Big], V_{ij}(s) + V_{jk}(s) + V_{ijk}(s)\Big]$$

more involved but implemented

$$+ \Big[\Big[T_{ij} + T_k, V_{ijk}(s)\Big], T_{ij} + T_k + V_{ij}(s) + V_{ij}(s) + V_{ki}(s) + V_{ijk}(s)\Big]$$

SRG parameter s or $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ variation allows to assess size of missing more-body induced interactions

4-body SRG-induced interactions small (see later)

For 3N forces: induced interactions are of similar size a chiral 3N forces

For ΛNN: SRG induced-3BFs are large, much larger than chiral ones! (see also Wirth et al. (2016))

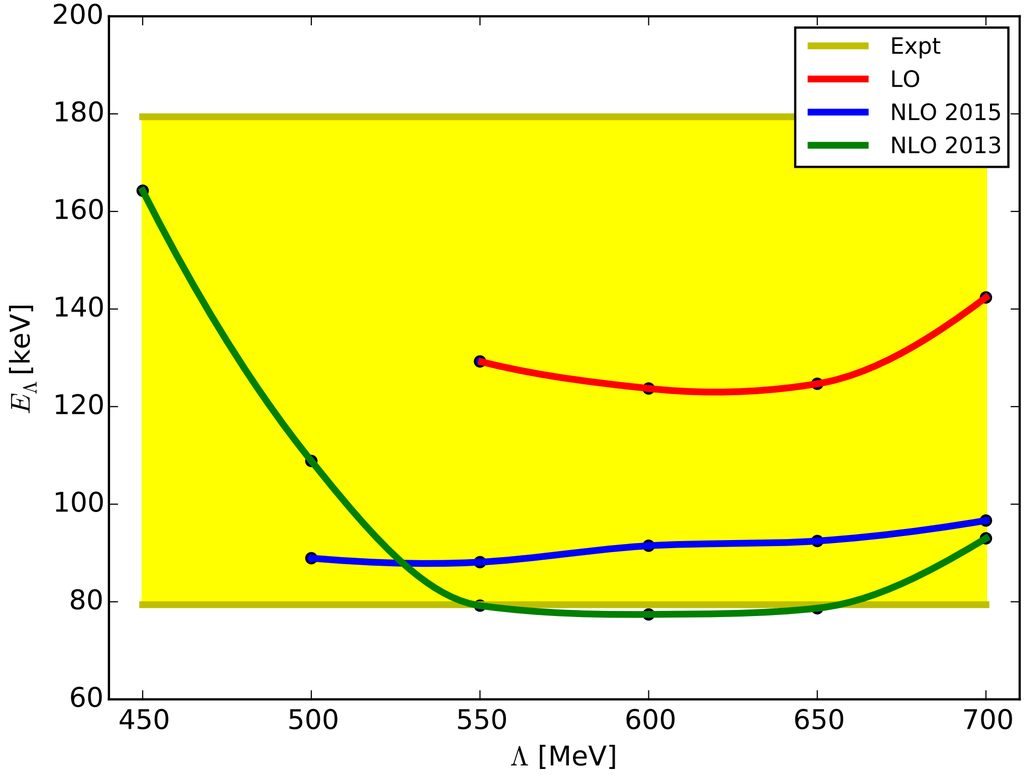
SRG DEPENDENCE OF RESULTS (2B ONLY)

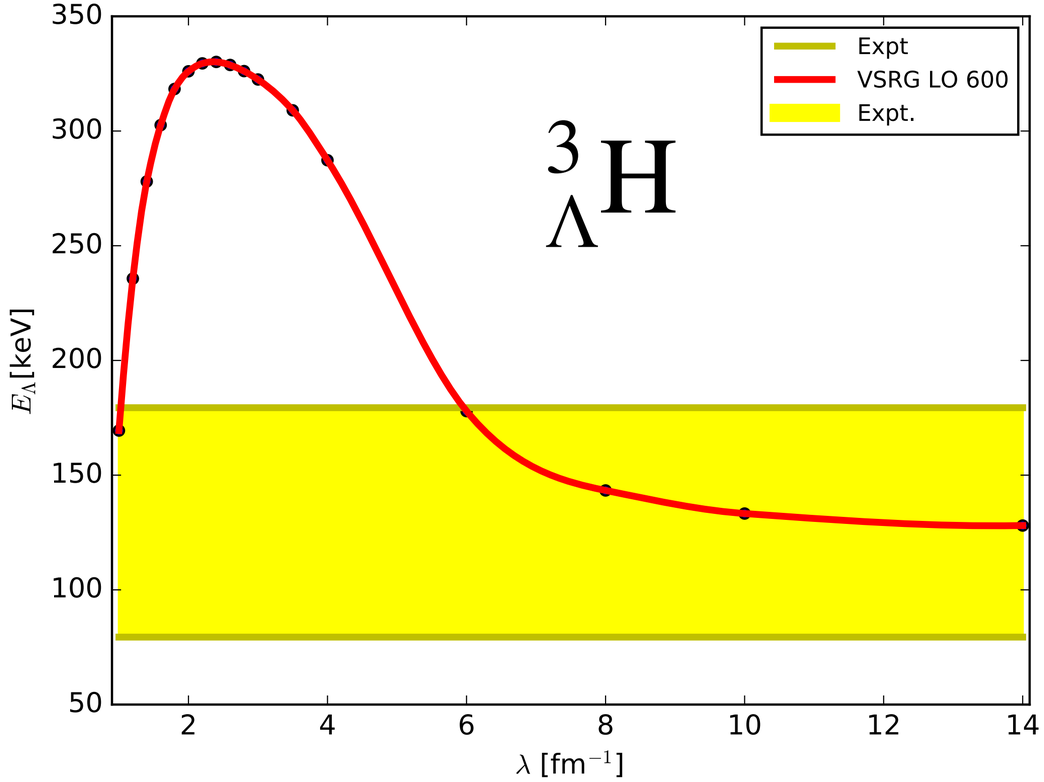
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SRG parameter dependence is significant when NN and YN interactions are evolved

missing 3N and YNN interactions

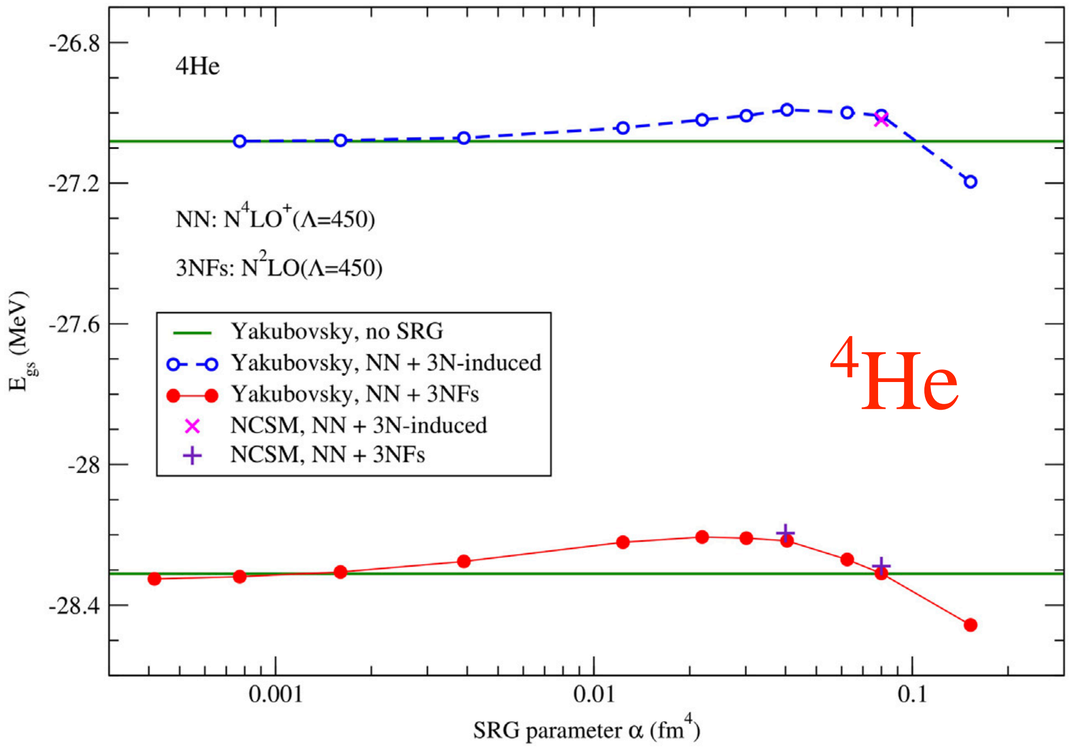
→
- 3NF is comparable to chiral 3NF
- YNN is much larger than chiral YNN





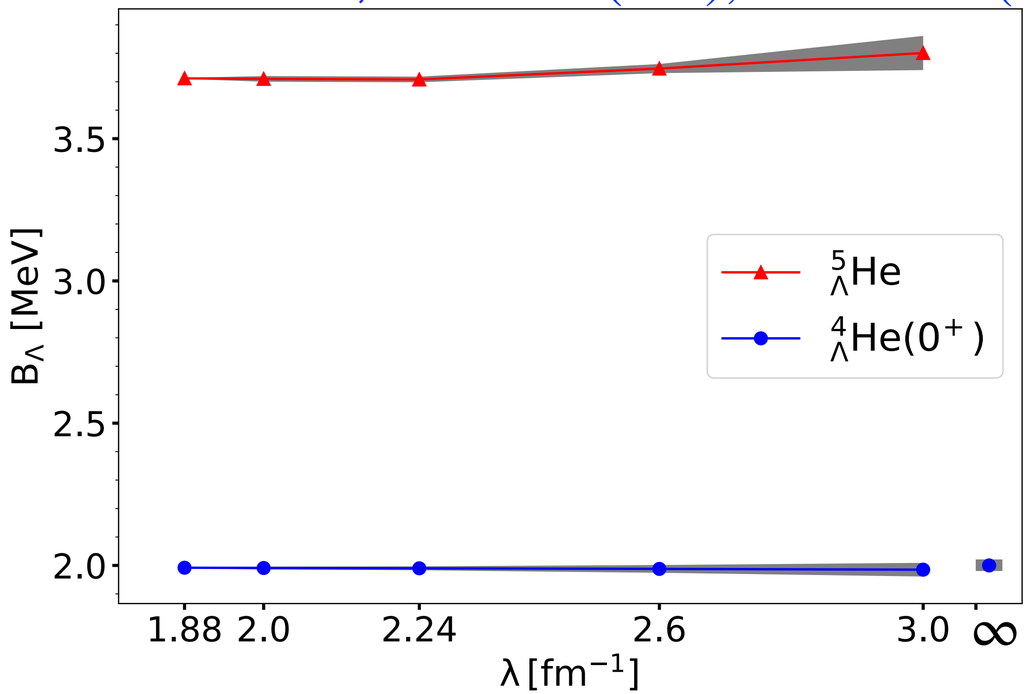
SRG DEPENDENCE OF RESULTS

- SRG-induced 3N and YNN interactions (see Wirth et al. 2016)
- ^{4}He binding energies varies by $\approx 100 - 200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris et al. Front. Phys. 11,1098262 (2023))

NN: N^4LO^+, **3N**: $N^2LO(450)$; **YN**: $N^2LO(550)$



(see Le et al. PRC 107,024002 (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

UNCERTAINTY ANALYSIS FOR $A = 3$ TO 5

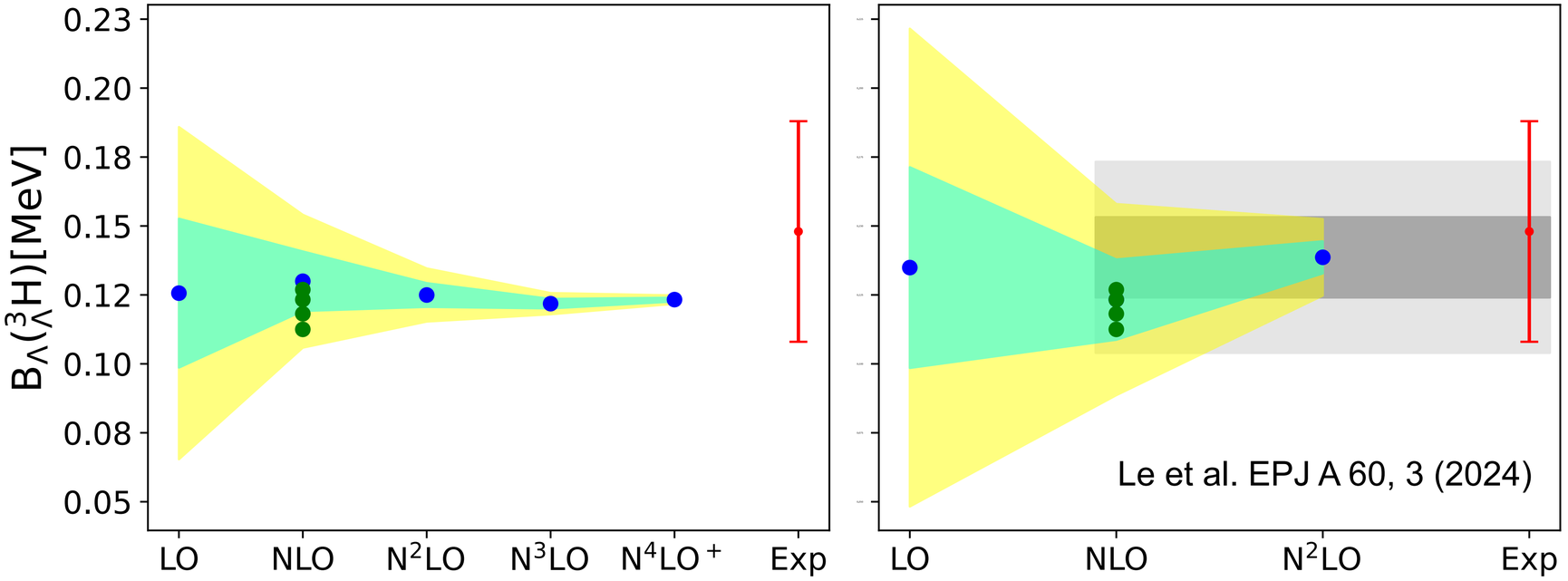
Results for **different orders** enable uncertainty estimate:

SMS YN interactions up order N²LO, but neglected YNN interaction here

- Ansatz for the order by order convergence (Melendez et al. PRC 2017,2019):

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, …})$$

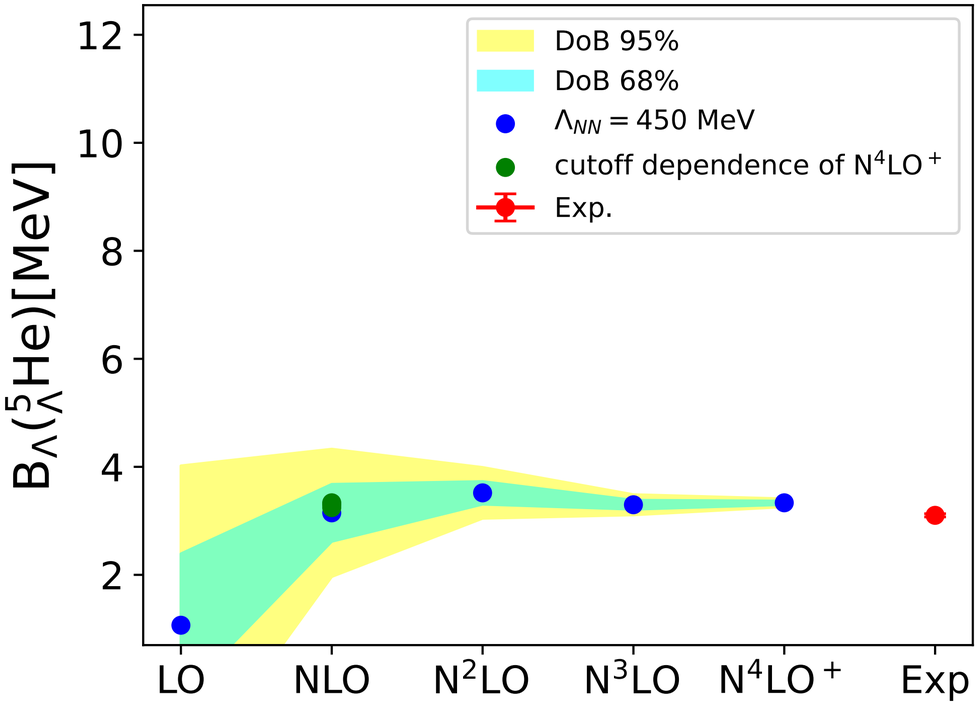
- **Extracting c_k for $k \leq K$ from calculation → probability distributions for c_k →** $\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$

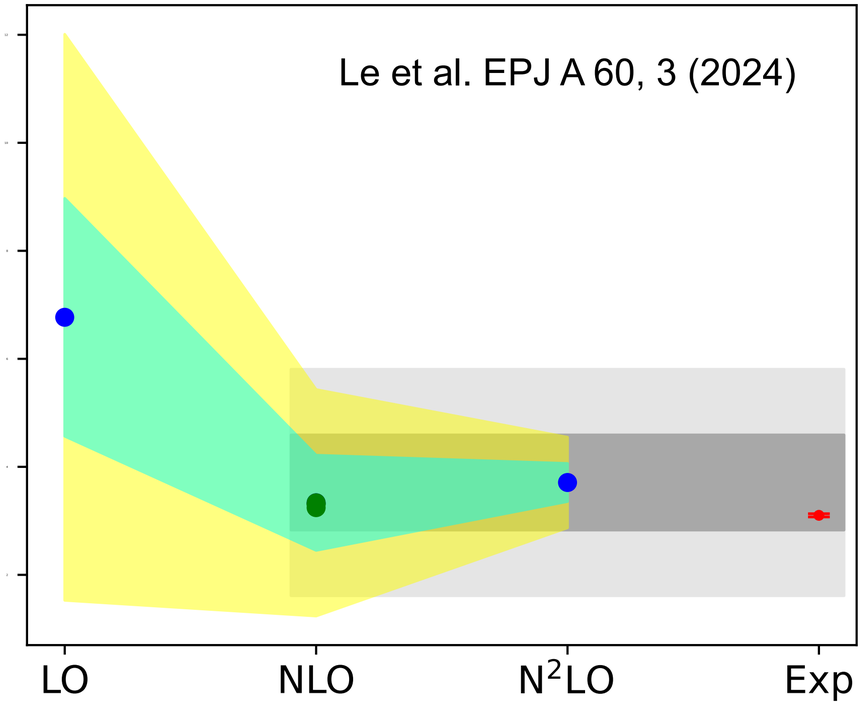


APPLICATION TO ${}^{5}_{\Lambda}\mathrm{He}$ AND SUMMARY

- **without YNN**: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$
- at the same time: obtain estimate of YNN contribution!

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^+)$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^+)$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881

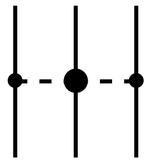




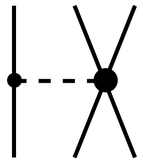
YNN (ΛNN) INTERACTIONS

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

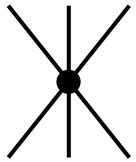
ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN
(up to 10)



2 LECs in ΛNN
(up to 14)

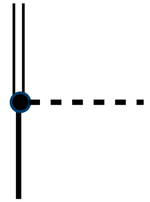


3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

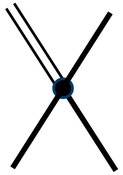
only few data → need to keep the **# of LECs** small

Decuplet baryons ($\Sigma^*\ldots$) might enhance YNN partly to NLO (Petschauer et al., NPA 957, 347 (2017))

By decuplet saturation all LECs can be related to the following leading octet-decuplet transitions
(Petschauer et al. Front. Phys. 8,12 (2020))



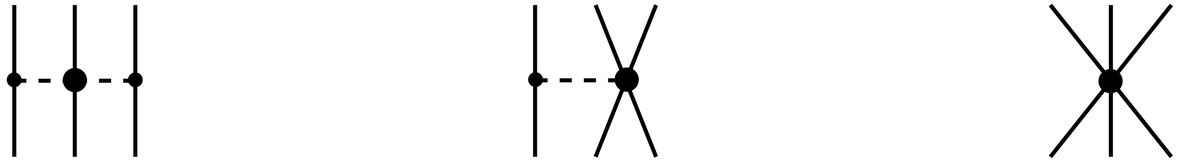
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) INTERACTIONS

Decuplet saturation relates all LECs to G_1 and G_2



$\propto C^2$ $\propto CG_1, CG_2$ $\propto (G_1)^2, (G_2)^2, G_1G_2$

$\propto C^2$ $\propto C(G_1 + 3G_2)$ $\propto (G_1 + 3G_2)^2$ for ΛNN: 1 LEC

→ density dependent BB interactions (Petschauer et al., NPA 957, 347 (2017))

→ application to nuclear matter (Haidenbauer et al., EPJ A 53, 121 (2017))

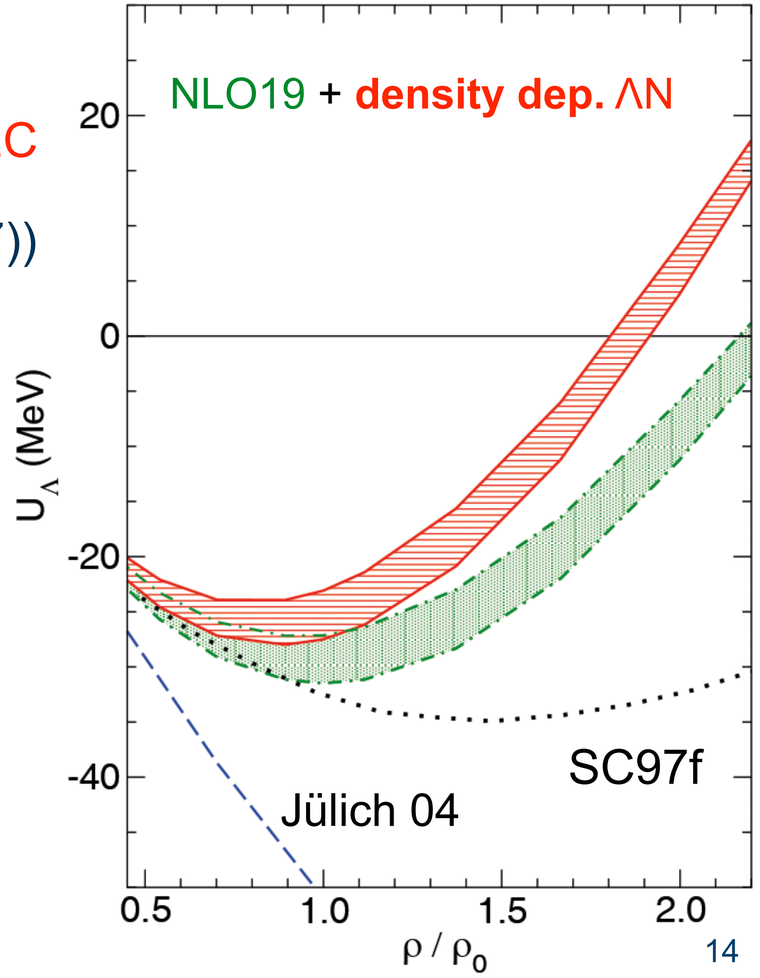
neutron stars (Logoteta et al., EJA 55, 207 (2019))

- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of 2π and 1π exchange

(fixes sign of $G_1 + 3G_2$!)

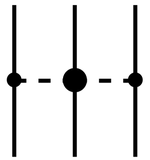
Consistent with light hypernuclei?

Overcome approximation of density dependent YN interactions

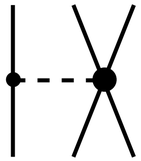


YNN (ΛNN) INTERACTIONS IN PRACTICE

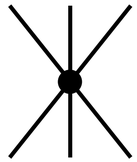
Decuplet approximation in YNN



$\propto C^2$



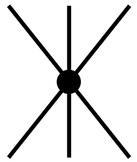
$\propto CG_1, CG_2$



$\propto (G_1)^2, (G_2)^2, G_1 G_2$

→ fit shows: not sufficient to fix spin dependence

Decuplet approximation for ΛNN contact terms



ΛNN $\propto C_1', C_2', C_3'$

$$C_1' = C_3' = \frac{(G_1 + 3G_2)^2}{72\Delta} \qquad C_2' = 0$$

***ad hoc* choice:** alter C_2:

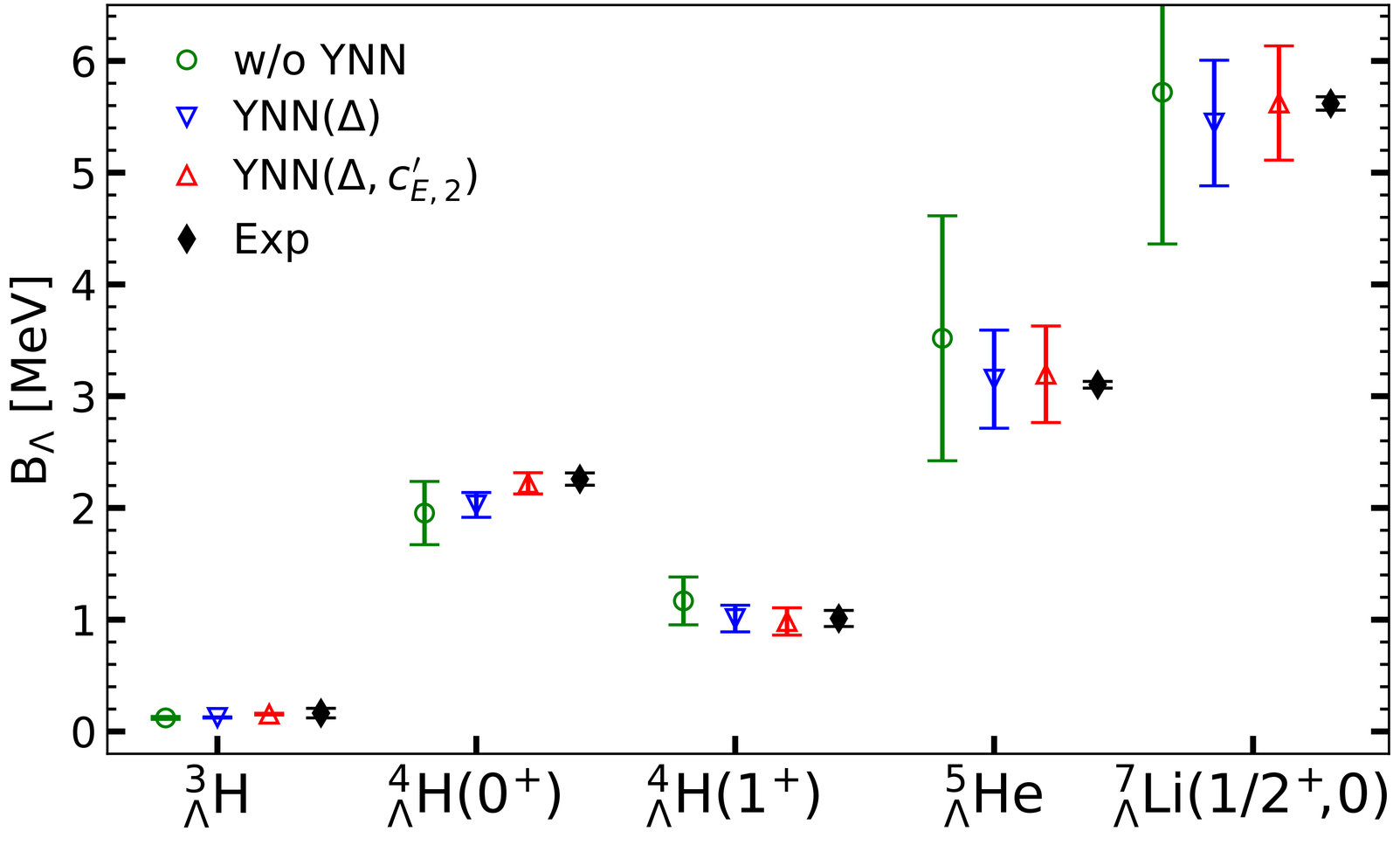
$$V_{\Lambda NN} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

C_2' introduces a spin dependent interaction in the most relevant particle channel

YNN (ΛNN) INTERACTIONS IN PRACTICE

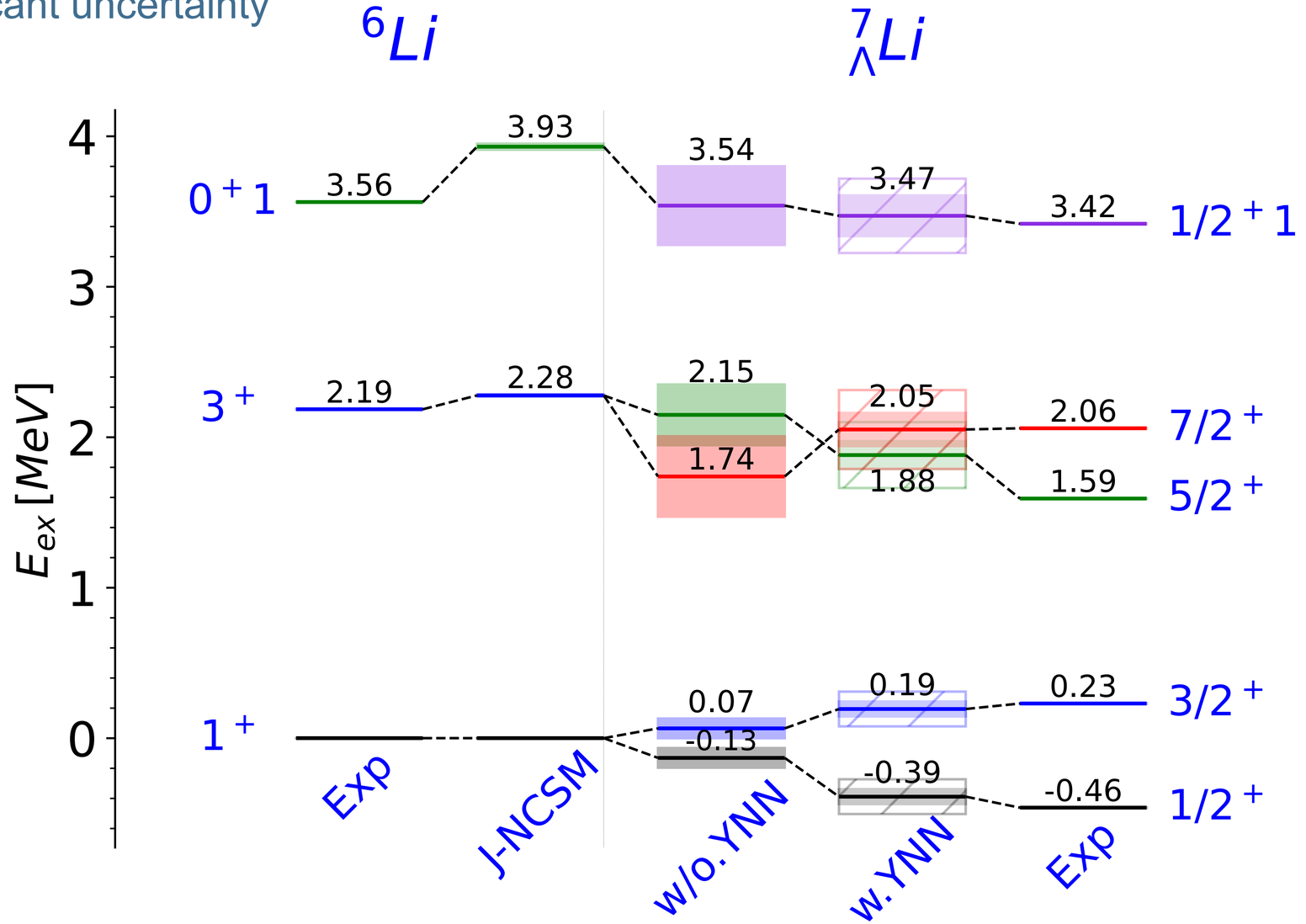
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- Fit to 0^+ and 1^+ state of ${}^{4}_{\Lambda}\mathrm{He}$ and/or ${}^{5}_{\Lambda}\mathrm{He}$
- spin-dependence in $A = 4$ not well explained by decuplet saturation
- C_2' term improves 0^+ of ${}^{4}_{\Lambda}\mathrm{He}$ and $1/2^+$ of ${}^{7}_{\Lambda}\mathrm{Li}$
- agreement generally much better than $\mathrm{N^2LO}$ uncertainty



YNN PREDICTION FOR SPECTRUM OF ${}^{7}_{\Lambda}\mathrm{Li}$

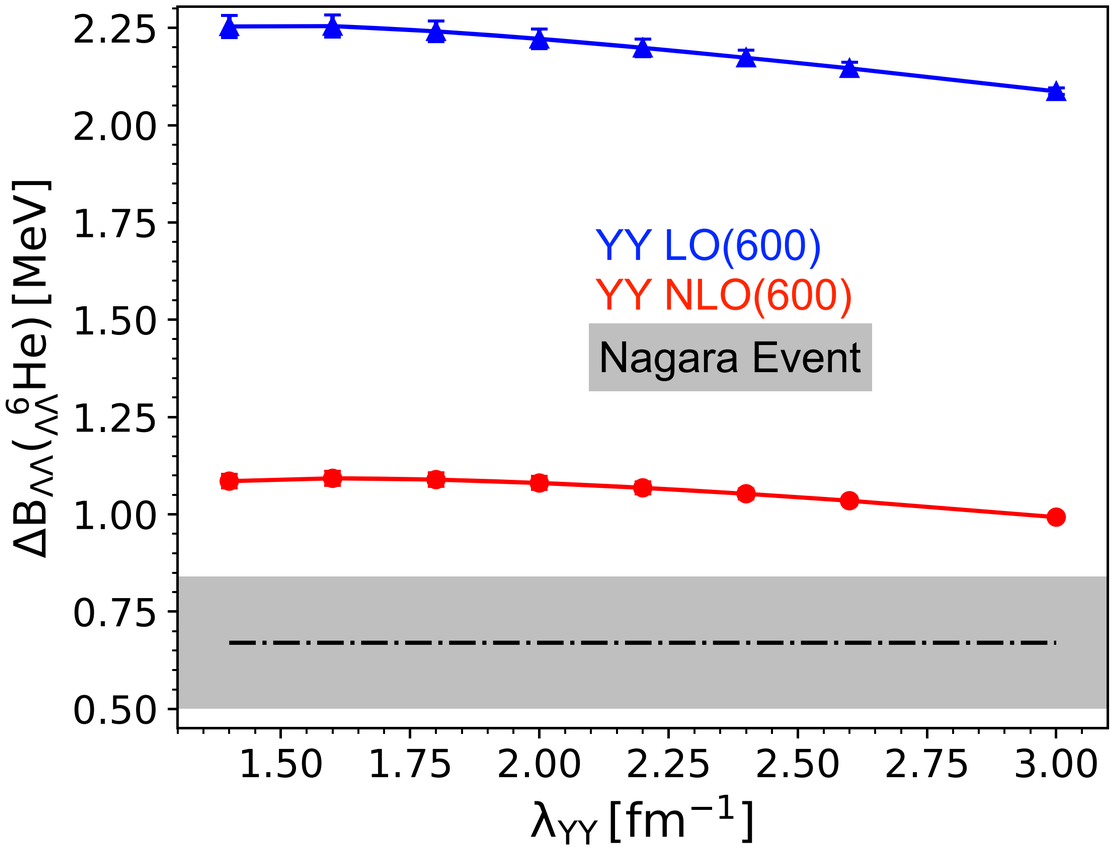
- good agreement
- C_2' term included, but not very important (not shown)
- higher states have significant uncertainty



$S=-2$ HYPERNUCLEI — ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

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- $\Lambda\Lambda$ excess binding energy $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda} = 2E\left({}^{A-1}_{\Lambda}X\right) - E\left({}^{A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$
- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernucle
- small λ_{YY} dependence
- uncertainties are numerical only!
- LO overbinds YY
- NLO predicts binding fairly well

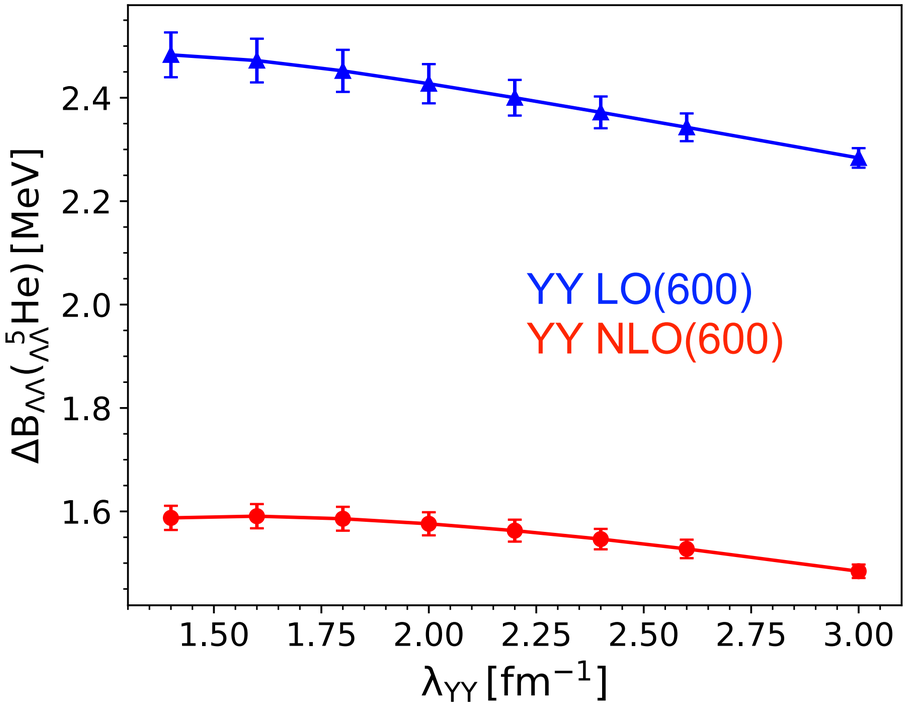


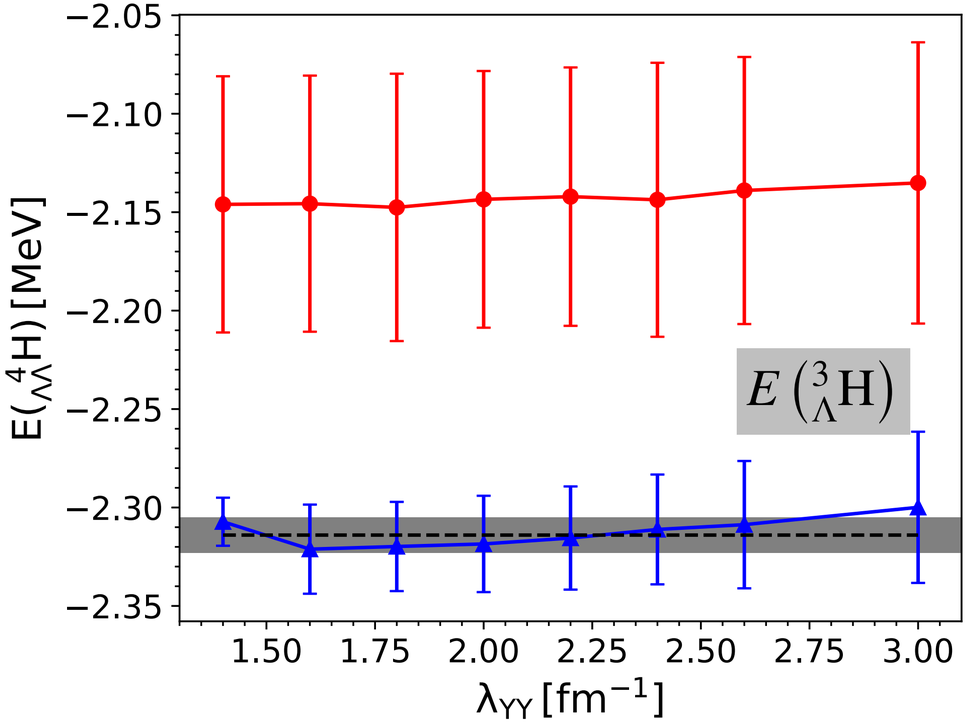
(Le et al., EPJA 57,217 (2021))

Can an $S=-2$ bound state for $A=4,5$ be expected?

$S = -2$ HYPERNUCLEI — ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ & ${}^{4}_{\Lambda\Lambda}\mathrm{H}$

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- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., PLB 797, 134893 (2019))
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., PRC 65, 041001 (2002)!)

$S = -2$ bound state for $A = 5$ can be expected, for $A = 4$ less likely but not ruled out!

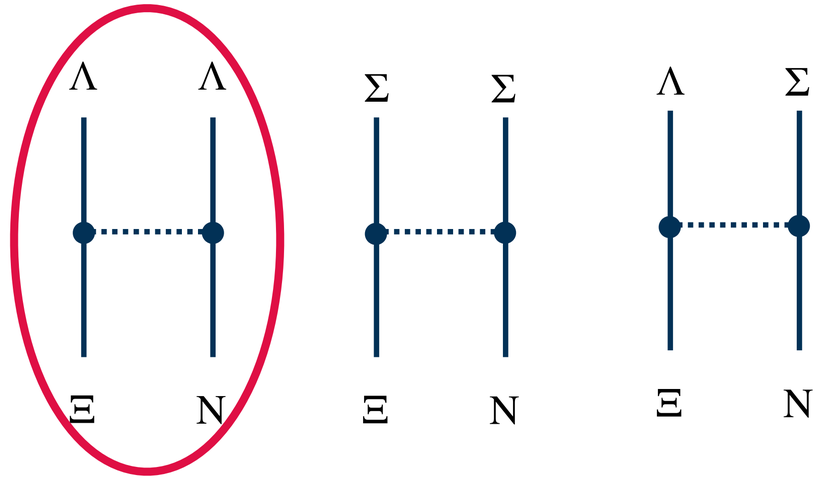
Ξ HYPERNUCLEI

- experimentally accessible: Ξ^- capture process (experimental data for ${}^{15}_{\Xi}\mathrm{C}$ and ${}^{12}_{\Xi}\mathrm{Be}$)
- $\Xi\mathrm{N} - \Lambda\Lambda$ conversion channel open: possibly short life times/difficult calculations
- HAL QCD & chiral YY interactions: suppression $\Xi\mathrm{N} - \Lambda\Lambda$ transition
- $\Xi\mathrm{N}$ interaction relevant: Ξ is often the second hyperon to appear in neutron matter

Identify possibly interesting states:

calculations based on chiral interactions neglecting $\Xi\mathrm{N} - \Lambda\Lambda$ transitions

(keeping $\Xi\mathrm{N} - \Lambda\Sigma, \Sigma\Sigma$) → states are bound states



finetuning of ${}^{11}\mathrm{S}_0$ interaction to correct for missing $\Lambda\Lambda$ channel

neglect YN interaction to avoid transitions to $\Lambda\Lambda$

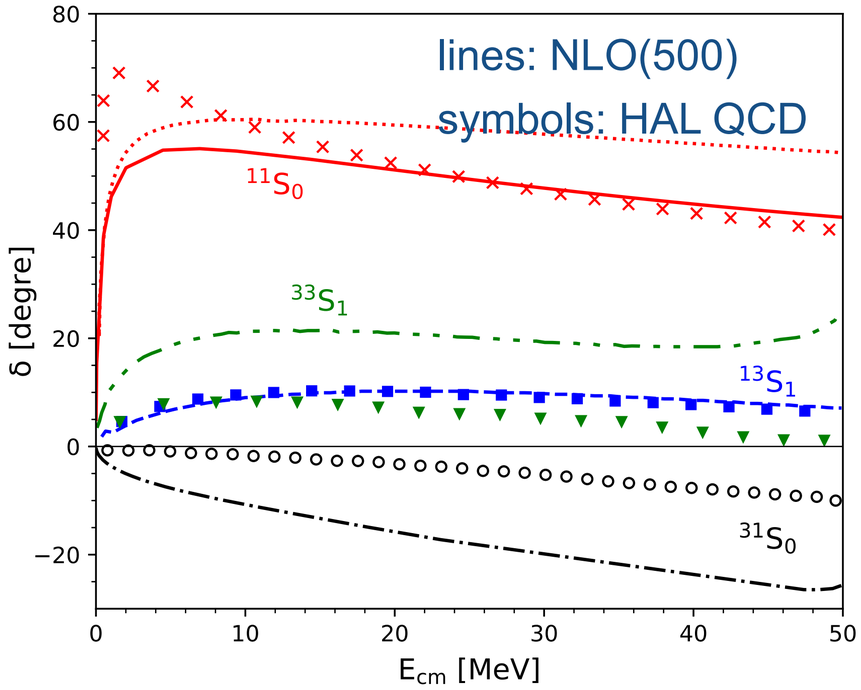
perturbative width estimates indicate small widths (see Hiyama et al., PRC 78, 054316 (2008))

Here: look at ${}^{7}_{\Xi}\mathrm{H}$ (exp. expected), ${}^{5}_{\Xi}\mathrm{H}$, ${}^{4}_{\Xi}\mathrm{H}$ and ${}^{4}_{\Xi}\mathrm{n}$ explore possible bound states and their widths

Ξ HYPERNUCLEI

Ξ separation energies (NLO(500) and SMS N⁴LO⁺(450))

	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	–	–
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2



	$V^{S=-2}$			
	$^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	− 1.95	0.02	− 0.7	− 2.31
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	− 0.6	0.25	− 0.004	− 0.74
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	− 0.02	0.16	− 0.13	− 1.14
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	− 0.002	0.08	− 0.01	− 0.006
${}^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$	− 0.96	0.94	− 0.58	− 3.63
${}^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$	− 1.23	1.79	− 0.79	− 6.74

(Le et al. EPJ A 57, 339 (2021))

CONCLUSIONS & OUTLOOK

JÜLICH Forschungszentrum
Mitglied der Helmholtz-Gemeinschaft

- **YN/YY interactions not well understood**
  - scarce YN/YY data
  - more information necessary to solve "hyperon puzzle"

- **Hypernuclei provide important constraints**
  - $^1S_0$ $\Lambda\Lambda$ scattering length & $^{\;\;6}_{\Lambda\Lambda}\mathrm{He}$ & predictions for A=4,5
  - Light $A = 4 - 7$ Ξ-hypernuclei could help to determine spin/isospin dependence in $\Xi\mathrm{N}$

- **New SMS YN interactions**
  - order LO, NLO and N^{2}LO allow **uncertainty quantification**
  - have a **no unique** determination of contact interactions (**data necessary**)

- **Chiral 3BF**
  - decuplet saturation sufficient for overall strength / spin-dependent $\Lambda\mathrm{NN}$ leads to further improvement
  - study cutoff dependence / application to more p-shell hypernuclei necessary