

中国科学院高能物理研究所

Institute of High Energy Physics, CAS





中国科学院

CHINESE ACADEMY OF SCIENCES

Kinematic triangle singularity associated with dynamical couplings

Qiang Zhao

Theoretical Physics Division

Institute of High Energy Physics, Chinese Academy of Sciences

zhaoq@ihep.ac.cn

Workshop on Few-Body Problems in Nuclear and Hadron Physics
CCAST, Sept. 21-24, 2026

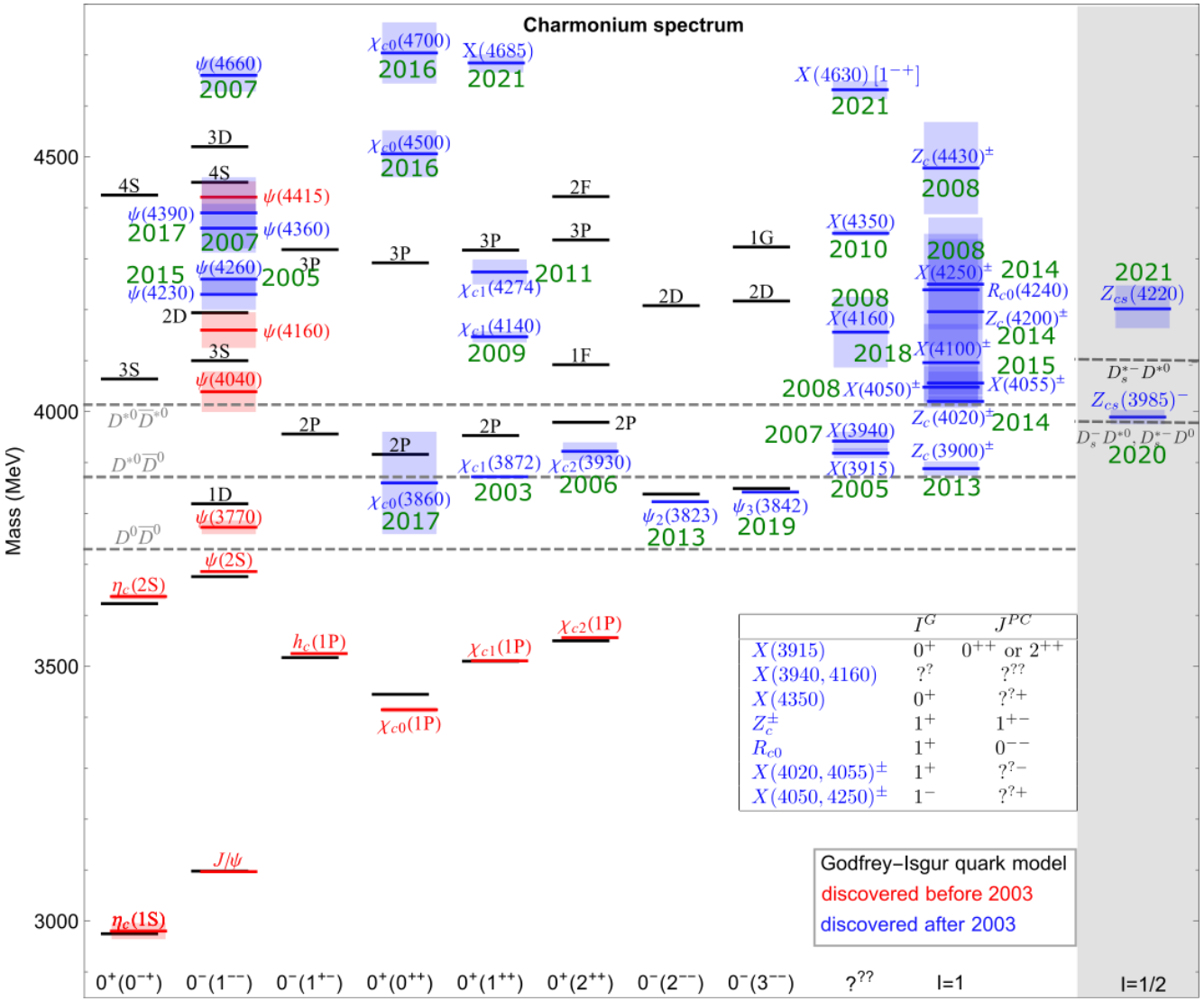
Outline

1. About the triangle singularity (TS)
2. Manifestations of the TS in various processes
3. Dynamical aspects of the TS mechanism
4. Summary

1. About the triangle singularity (TS)

Conventional vs. Exotic

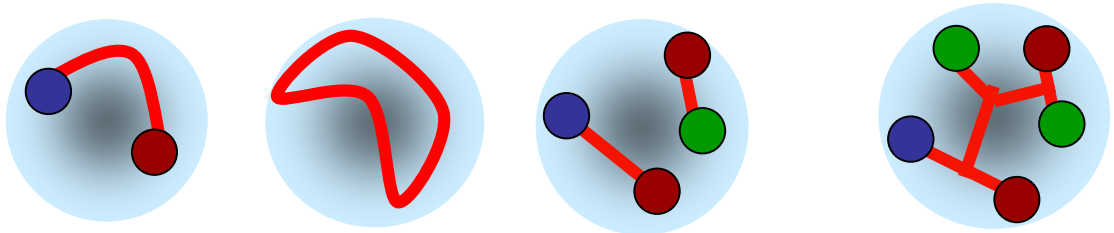
Charmonium and Charmonium-like meson spectrum



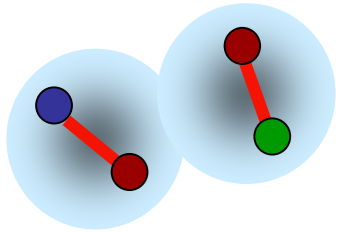
Plot from F.K. Guo

Exotic hadrons

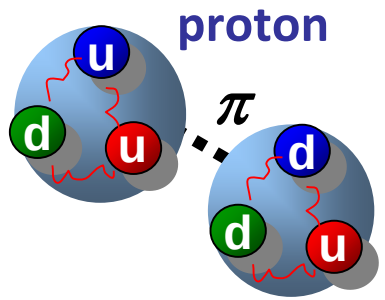
Hybrid glueball Tetraquark Pentaquark



Hadronic molecule



Deuteron

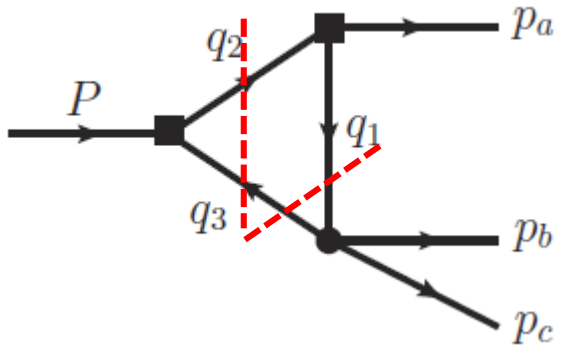


Evidence for QCD exotic states is a missing piece of knowledge about the Nature of strong QCD.

Triangle singularity: Peak structures caused by kinematic effects

$$\Gamma_3(s_1,s_2,s_3) = \frac{1}{i(2\pi)^4}\int \frac{d^4q_1}{(q_1^2-m_1^2+i\epsilon)(q_2^2-m_2^2+i\epsilon)(q_3^2-m_3^2+i\epsilon)}$$
$$= \frac{-1}{16\pi^2}\int_0^1\int_0^1\int_0^1 da_1\,da_2\,da_3\,\frac{\delta(1-a_1-a_2-a_3)}{D-i\epsilon},$$

$$D \equiv \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2}\left[m_i^2+m_j^2-(q_i-q_j)^2\right]$$

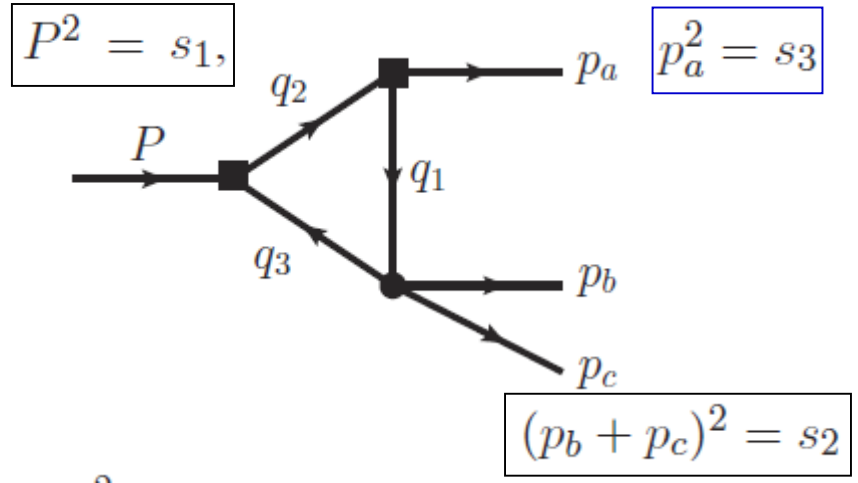


The **TS** occurs when all the three internal particles can approach their on-shell condition simultaneously:

$\partial D/\partial a_j = 0$ for all j=1,2,3. ⇒ $\det[Y_{ij}] = 0$

L. D. Landau, Nucl. Phys. 13, 181 (1959);
J.J. Wu, X.-H. Liu, Q. Zhao, B.-S. Zou, Phys. Rev. Lett. 108, 081003 (2012);
Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013); Phys. Lett. B 725, 106 (2013)
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016);
F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv:1705.00141[hep-ph], Rev. Mod. Phys. 90, 015004 (2018) ; F.-K. Guo, X.-H. Liu and S. Sakai, Prog. Part. Nucl. Phys. 112, 103757 (2020)
M.-C. Du and Q. Zhao, PRD 104, 036008 (2021); PRD 100, 036005 (2019); Y. Cheng, L. Qiu, and Q. Zhao, PRD111, 114015 (2025), arXiv:2407.10234

Kinematics :



The TS condition for fixed s_1, m_j, and s_3 is:

$$s_2^{\pm} = (m_1+m_3)^2 + \frac{1}{2m_2^2}[(m_1^2+m_2^2-s_3)(s_1-m_2^2-m_3^2) - 4m_2^2 m_1 m_3 \pm \lambda^{1/2}(s_1,m_2^2,m_3^2)\lambda^{1/2}(s_3,m_1^2,m_2^2)],$$

Or for fixed s_2, m_j, and s_3 :

$$s_1^{\pm} = (m_2+m_3)^2 + \frac{1}{2m_1^2}[(m_1^2+m_2^2-s_3)(s_2-m_1^2-m_3^2) - 4m_1^2 m_2 m_3 \pm \lambda^{1/2}(s_2,m_1^2,m_3^2)\lambda^{1/2}(s_3,m_1^2,m_2^2)].$$

with $\lambda(x,y,z) \equiv (x-y-z)^2 - 4yz.$

X.-H. Liu, M. Oka and Q. Zhao, PLB (2016); arXiv:1507.01674 [hep-ph]

Single dispersion relation in s_2:

$$\Gamma_3(s_1,s_2,s_3)=\frac{1}{\pi}\int_{(m_1+m_3)^2}^{\infty}\frac{ds_2'}{s_2'-s_2-i\epsilon}\,\sigma(s_1,s_2',s_3)$$

For a typical scalar loop integral the spectral function $\sigma(s_1,s_2,s_3)$ can be obtained by either the Cutkosky's rules (absorptive part of the loop amplitude) or the following formula

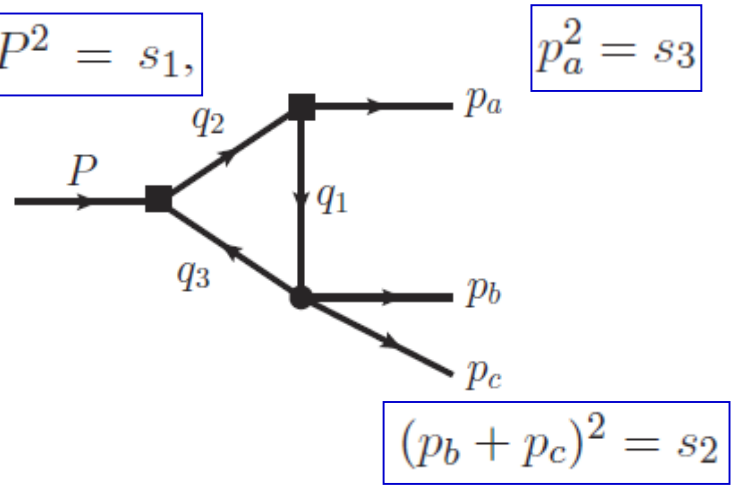
$$\sigma(s_1,s_2,s_3) = \frac{-1}{16\pi}\int_0^1\int_0^1\int_0^1 da_1\,da_2\,da_3\,\delta(1-a_1-a_2-a_3)\delta(D).$$

which reads

$$\begin{aligned}\sigma(s_1,s_2,s_3) &= \sigma_+-\sigma_-,\\ \sigma_\pm(s_1,s_2,s_3) &= \frac{-1}{16\pi\lambda^{1/2}(s_1,s_2,s_3)}\log[-s_2(s_1+s_3-s_2+m_1^2+m_3^2-2m_2^2)\\ &- (s_1-s_3)(m_1^2-m_3^2)\pm\lambda^{1/2}(s_1,s_2,s_3)\lambda^{1/2}(s_2,m_1^2,m_3^2)].\end{aligned}$$

For fixed s_1, s_3 and m_i, the spectral function $\sigma(s_1,s_2,s_3)$ has logarithmic branch points $s_2^{\pm}$, which correspond to the anomalous thresholds by solving the Landau equation.

How the logarithmic branch points $s_2^{\pm}$ move as s_1 increases from the threshold of $(m_2+m_3)^2$, with s_3 and m_i fixed?



Substituting $s_1 \to s_1 + i\varepsilon$, $s_2^{\pm}$ in the s'-plane are then located at

$$s_2^{\pm}(s_1+i\epsilon) = s_2^{\pm}(s_1) + i\epsilon\frac{\partial s_2^{\pm}}{\partial s_1},$$

The normal thresholds and critical values for s_1 and s_2 as follows,

$$s_{1N} = (m_2+m_3)^2,\ s_{1C} = (m_2+m_3)^2 + \frac{m_3}{m_1}[(m_2-m_1)^2 - s_3],$$
$$s_{2N} = (m_1+m_3)^2,\ s_{2C} = (m_1+m_3)^2 + \frac{m_3}{m_2}[(m_2-m_1)^2 - s_3],$$

With $\partial s_2^{\pm}/\partial s_1 = 0$ $(\partial s_1^{\pm}/\partial s_2 = 0)$.

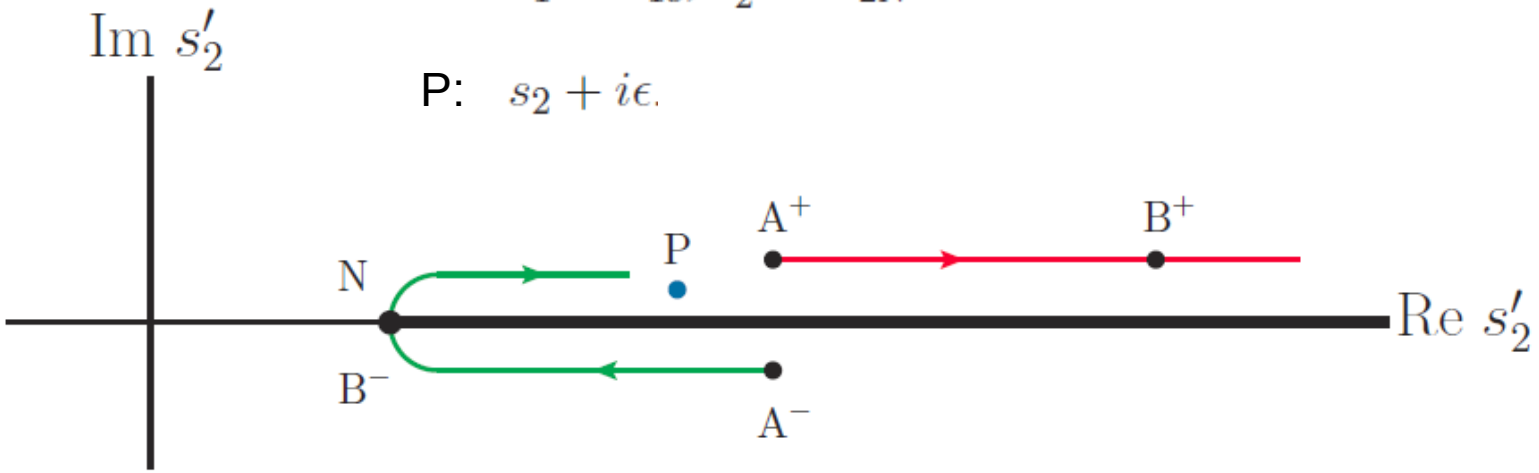
Trajectory of $s_2^{\pm}$ in the complex s_2'-plane with s_1 increasing from s_{1N} to ∞.

$$\mathrm{A}^{\pm}\colon\ s_1 = s_{1N},\ s_2^{\pm} = s_{2C} \pm i\epsilon,$$

$$\mathrm{B}^{+}\colon\ s_1 = s_{1c},\ s_2^{+} = s_{2N} + \frac{m_3}{m_1 m_2^2}\lambda(s_3, m_1^2, m_2^2) + i\epsilon$$

$$\mathrm{B}^{-}\colon\ s_1 = s_{1c},\ s_2^{-} = s_{2N}$$

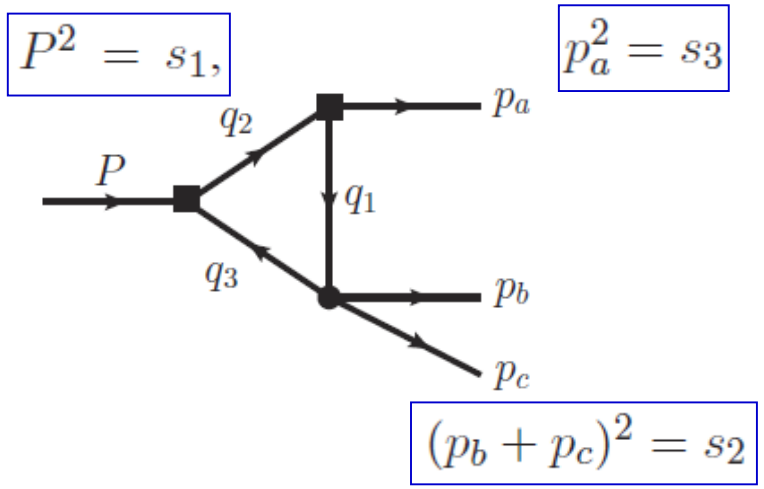
P: $s_2 + i\epsilon$.



$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi}\int_{(m_1+m_3)^2}^{\infty} \frac{ds_2'}{s_2' - s_2 - i\epsilon}\,\sigma(s_1, s_2', s_3)$$

The difference (distance) between the normal and anomalous thresholds:

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$
$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$



When $s_2 = s_{2N}$ ($s_1 = s_{1N}$), we will obtain the maximum value of Δs_1 (Δs_2),

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

- The larger Δs_1 (Δs_2), the more importance of the TS mechanism!
- For a specific transition process which fulfills the TS kinematic the detailed dynamics, such as the widths and coupling vertices, will affect the behavior of the TS mechanism.

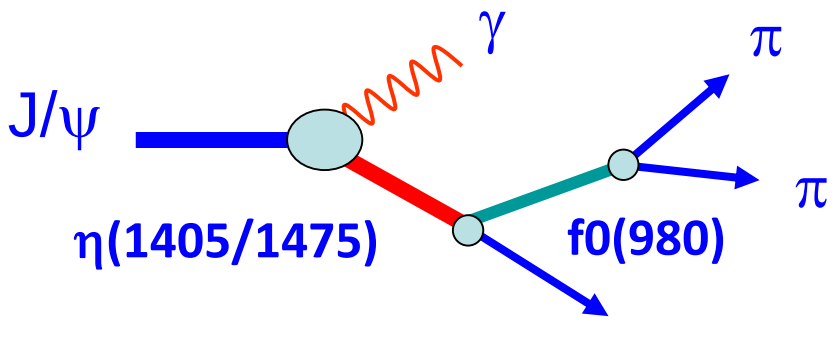
2. Manifestations of the TS in various processes

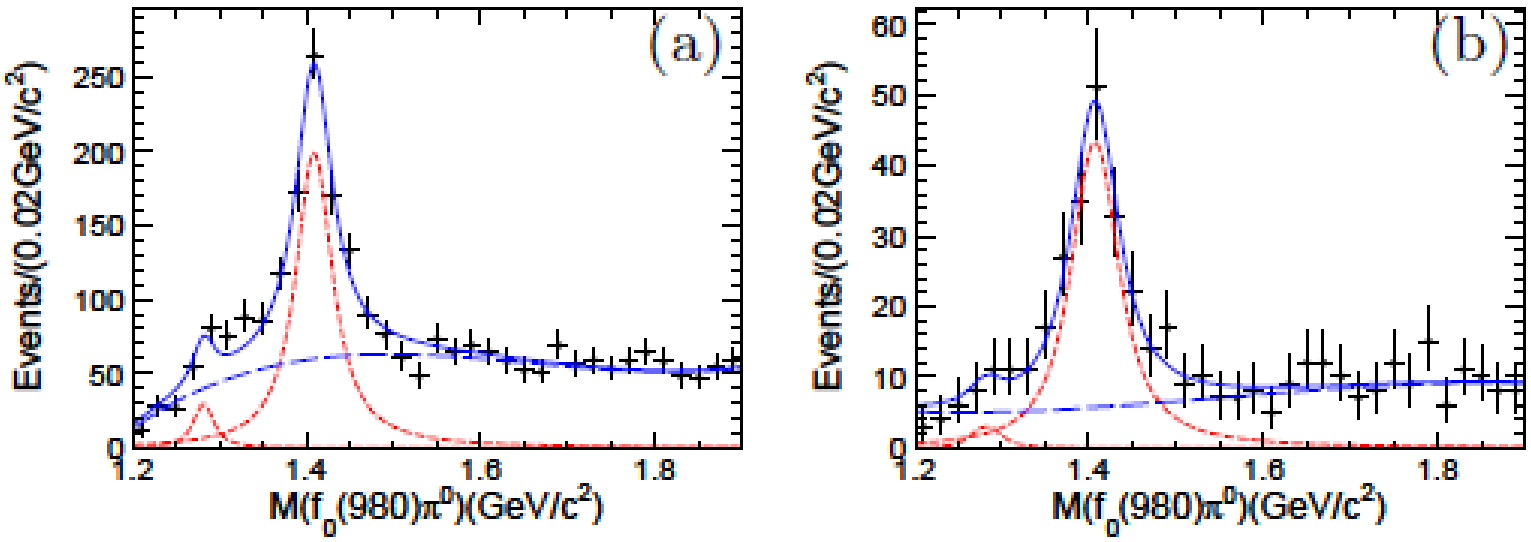
The presence of the "triangle singularity"

PRL 108, 182001 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

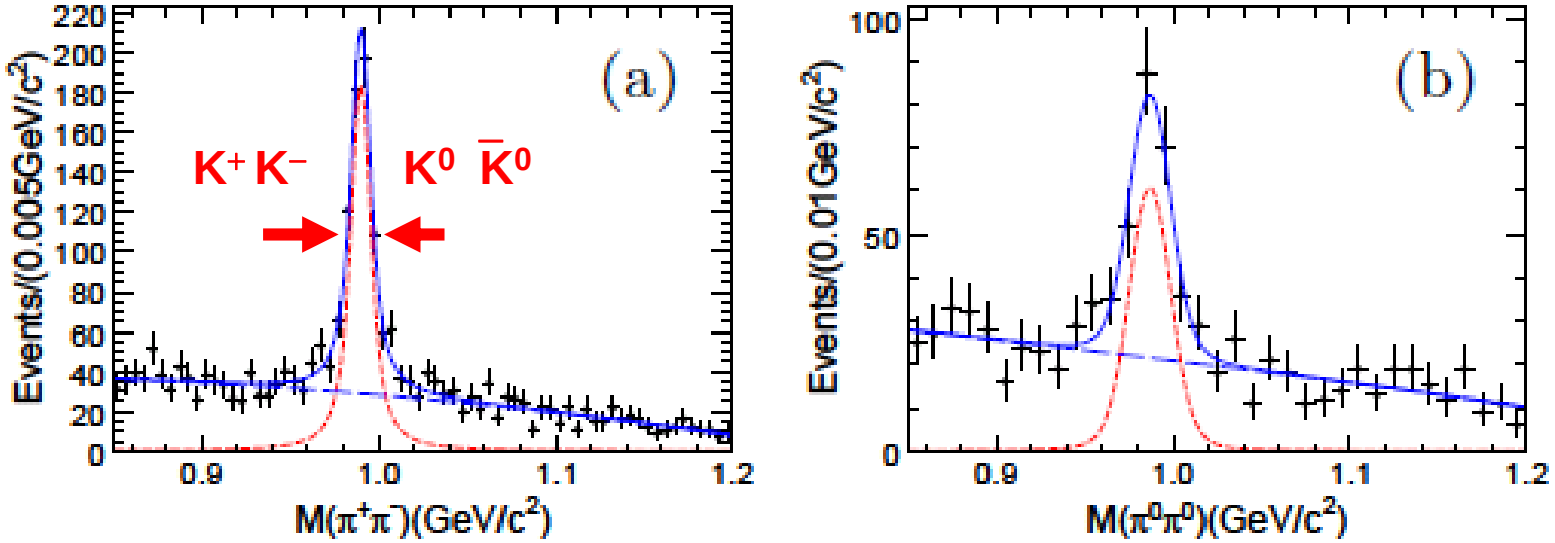
First Observation of $\eta(1405)$ Decays into $f_0(980)\pi^0$

Isospin-violating decay of J/ψ → γ η(1405) → γππ





BES-III Collaboration, Phys. Rev. Lett. 108, 182001 (2012)



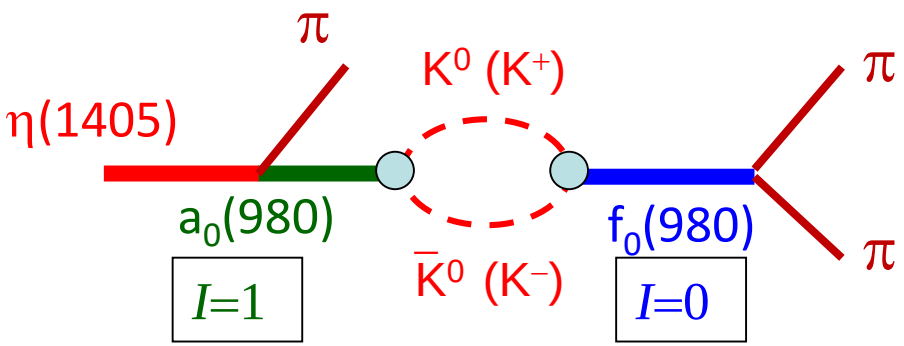
- $f_0(980)$ is extremely narrow: $\Gamma \cong 10$ MeV !

PDG: $\Gamma \cong 40$~100 MeV.

- **Anomalously large isospin violation!**

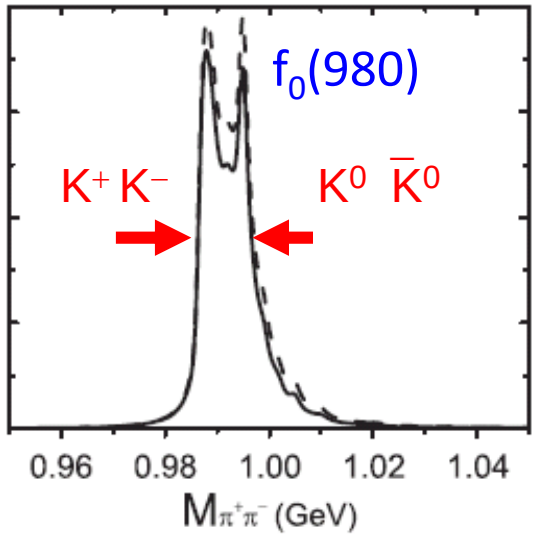
$$\frac{Br(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\eta(1405) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

"$a_0(980)$-$f_0(980)$ mixing" gives only ~1% isospin violation effects !



$$g(a_0K^+K^-)\, g(f_0K^+K^-) = -g(a_0K^0\ \bar{K}^0)\, g(f_0K^0\ \bar{K}^0)$$

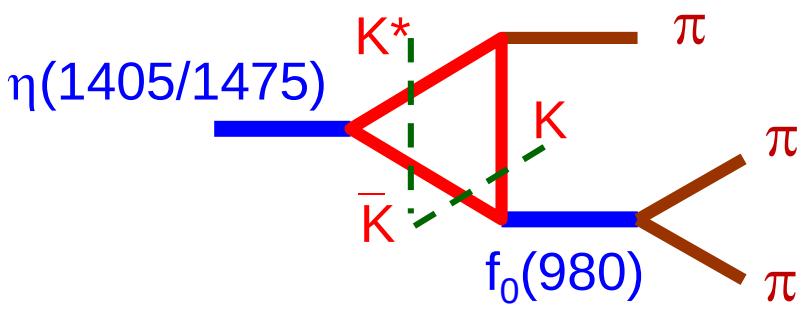
$$M(K^0)-M(K^\pm) = m_d-m_u$$



V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B586, 53-61 (2004)

V. Baru, J. Haidenbauer, C. Hanhart, A.E. Kudryavtsev and U.-G. Meissner, Eur. Phys. J. A23, 523-533 (2005)

"Triangle singularity"

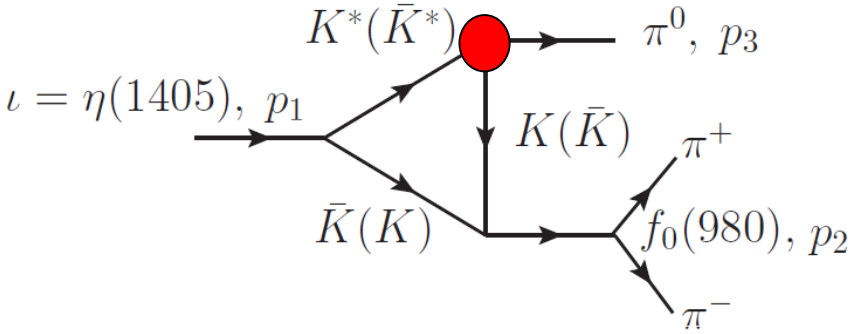


Internal $\bar{K}K^*(K)$ approach the on-shell condition simultaneously!

Manifestation of the TS singularity!

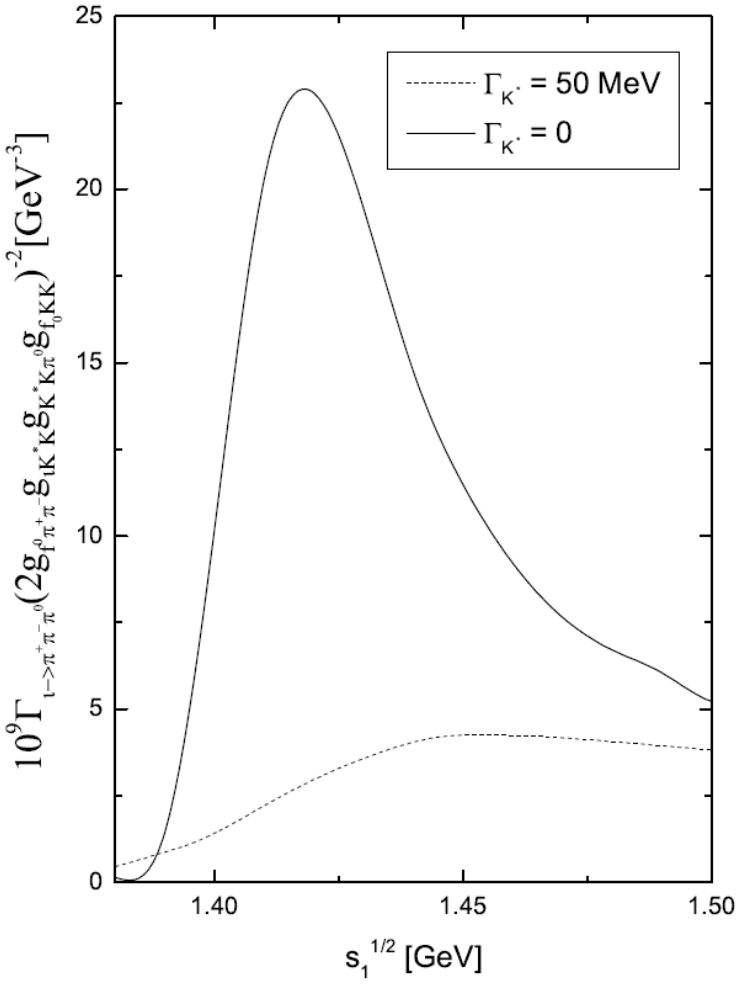
J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012);
X.G. Wu, J.-J. Wu, Q. Z., and B.-S. Zou, PRD87, 014023 (2013)

Understanding the width effects from the intermediate K* in η(1405/1475)→ 3π, K $\bar{K}$π, ηππ



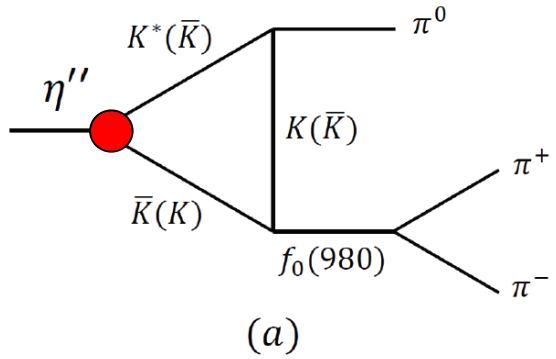
$a_0(980) - f_0(980)$ mixing is required to be enhanced.

However, experimental data do not support large b.r. for $\eta(1405)/\eta(1475) \rightarrow a_0(980)\pi$!

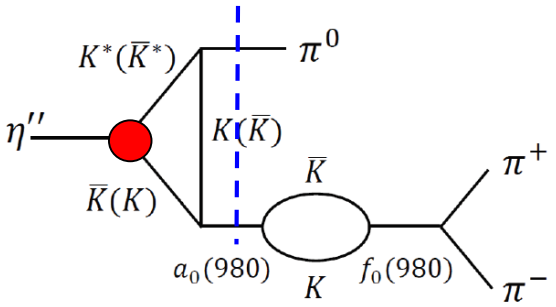


N.N. Achasov et al., PRD92, 036003 (2015)

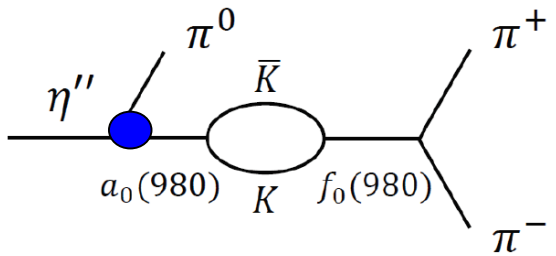
Updated study of η(1405/1475)→ 3π, K $\bar{K}$π, ηππ with width effects



(a)



(b)



(c)

- **Direct isospin breaking via the TS mechanism**

- **a0-f0 mixing enhanced by the TS mechanism**

-- Unitarized treatment for a0 and f0;

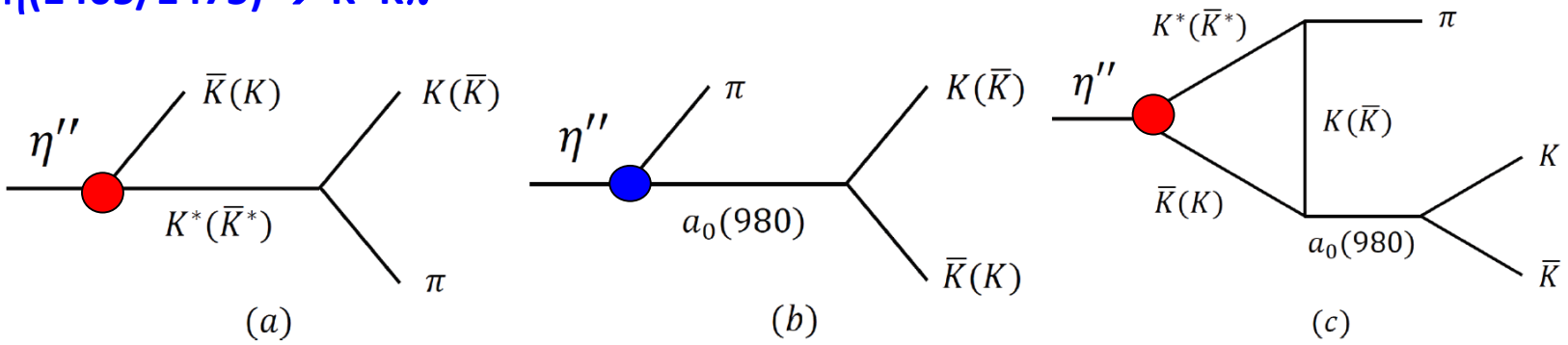
-- To separate (b) and (c) allows a self-contained evaluation of the TS and a0-f0 mixing contributions.

- **a0-f0 mixing at tree level**

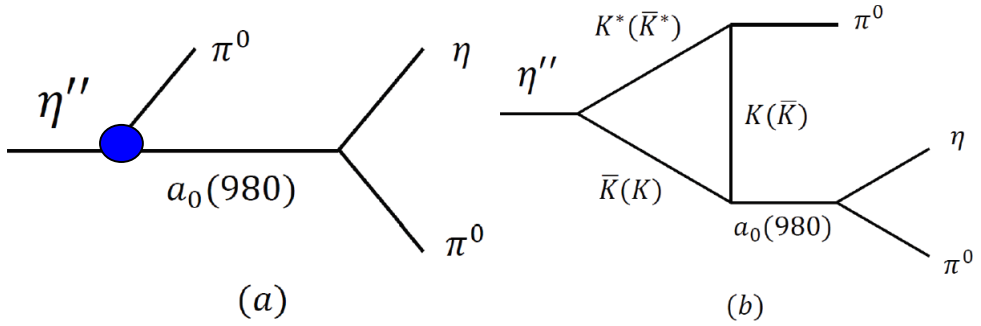
M.C. Du and Q.Z., PRD100, 036005 (2019).
See also N.N. Achasov et al., PRD92, 036003 (2015)

Updated study of η(1405/1475)→ 3π, K $\bar{K}$π, ηππ with width effects

η(1405/1475) → K $\bar{K}$π



η(1405/1475) → ηππ

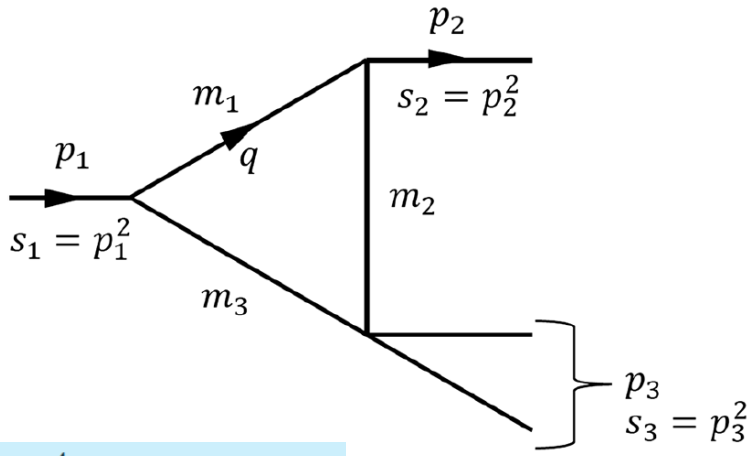


Schmidt theorem is broken

M.C. Du and Q.Z., PRD100, 036005 (2019).
M.-C. Du, Y. Cheng, and Q. Z., Phys. Rev. D 106, no.5, 054019 (2022)
Y. Cheng and Q.Z., Phys. Rev. D 105, 076023 (2022)
Y. Cheng, L. Qiu, and Q. Z., Phys.Rev.D 111, 114015 (2025)

Dynamical features arising from the coupling vertices in association with the TS mechanism

$$I = -i\int \frac{d^4q}{(2\pi)^4}\frac{(2p_1-q)_\mu(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2})(q-2p_2)_\nu}{(q^2-m_1^2+im_1\Gamma)[(p_2-q)^2-m_2^2][(q-p_1)^2-m_3^2]}$$



$$= -i\int \frac{d^4q}{(2\pi)^4}\frac{(2p_1-q)_\mu(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2})(q-2p_2)_\nu}{D_1D_2D_3}$$

$$= -i\left(s_1-m_1^2+im_1\Gamma_1+s_2-2s_3+2m_K^2-\frac{(s_1-m_K^2)(s_2-m_K^2)}{m_1^2-im_1\Gamma_1}\right)\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_2D_3}$$

$$-i\left(\frac{(s_1-m_K^2)(s_2-m_K^2)}{m_1^2-im_1\Gamma_1}\right)\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_2D_3}-i\left(1+\frac{s_1-m_K^2}{m_1^2-im_1\Gamma_1}\right)\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_3}$$

$$-i\left(1+\frac{s_2-m_K^2}{m_1^2-im_1\Gamma_1}\right)\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_1D_2}+i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_1}-i\frac{m_K^2-s_1}{m_1^2-im_1\Gamma_1}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_3}$$

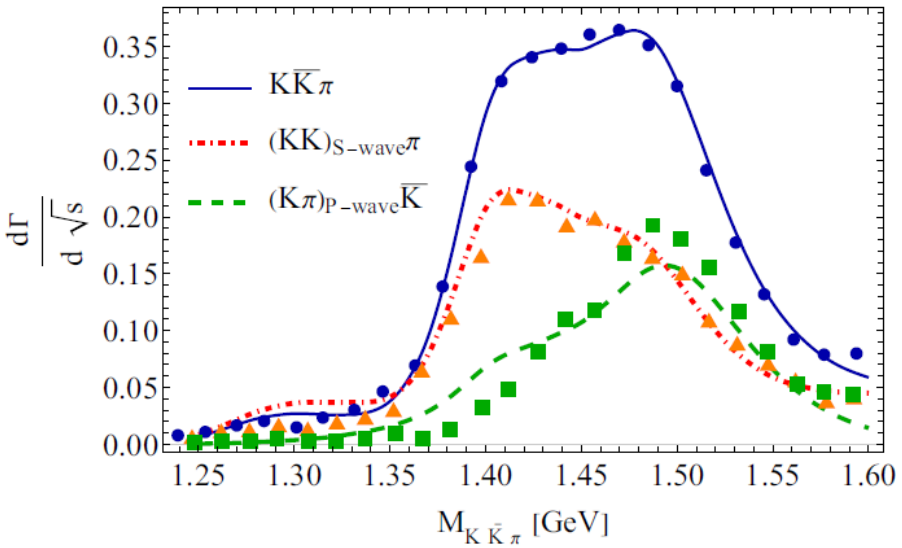
$$-i\frac{m_K^2-s_2}{m_1^2-im_1\Gamma_1}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2D_2}+i\int\frac{d^4q}{(2\pi)^4}\frac{1}{D_2D_3}$$

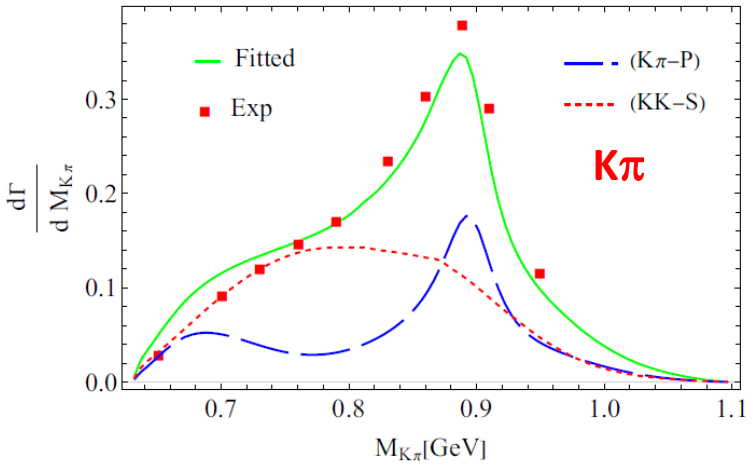
M.C. Du and Q.Z., PRD100, 036005 (2019).

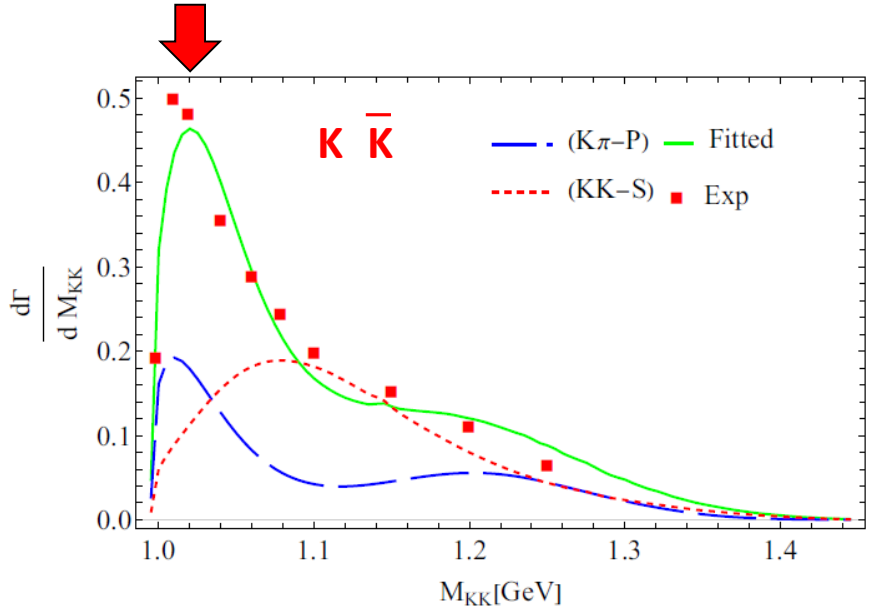
Understanding the recent PWA results from BESIII

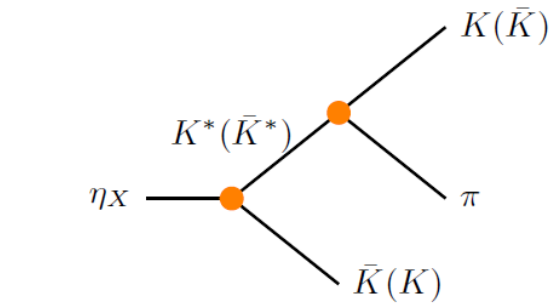
BESIII, JHEP03(2023)121

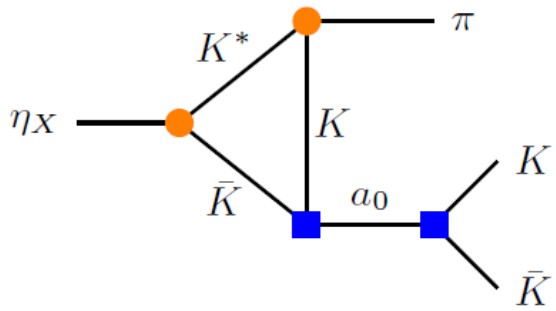
J/psi → γ η(1405/1475) → γ K $\bar{K}\pi$











- Manifestation of the TS mechanism, which violates the Schmidt theorem
- Partial waves can be accounted for in an isobar model with two-body unitarity [Y. Cheng, L. Qiu and Q. Z., Phys.Rev.D 111, 114015 (2025)]
- Coupled-channel approach with three-body unitarity [L. Qiu, and Q. Zhao, in progress]
- The $\eta\pi\pi$ channel can also be accounted for

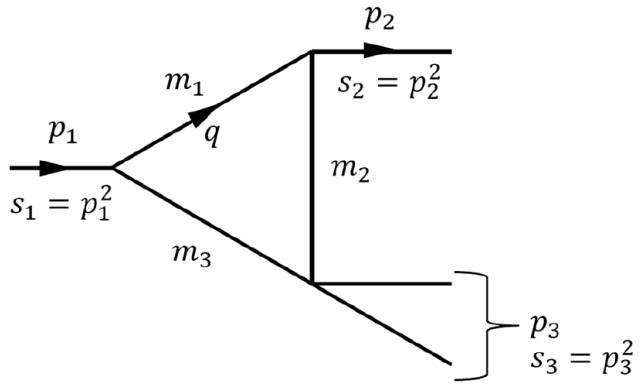
See also S.X. Nakamura et al., PRD109, 014021; PRD107, L091505

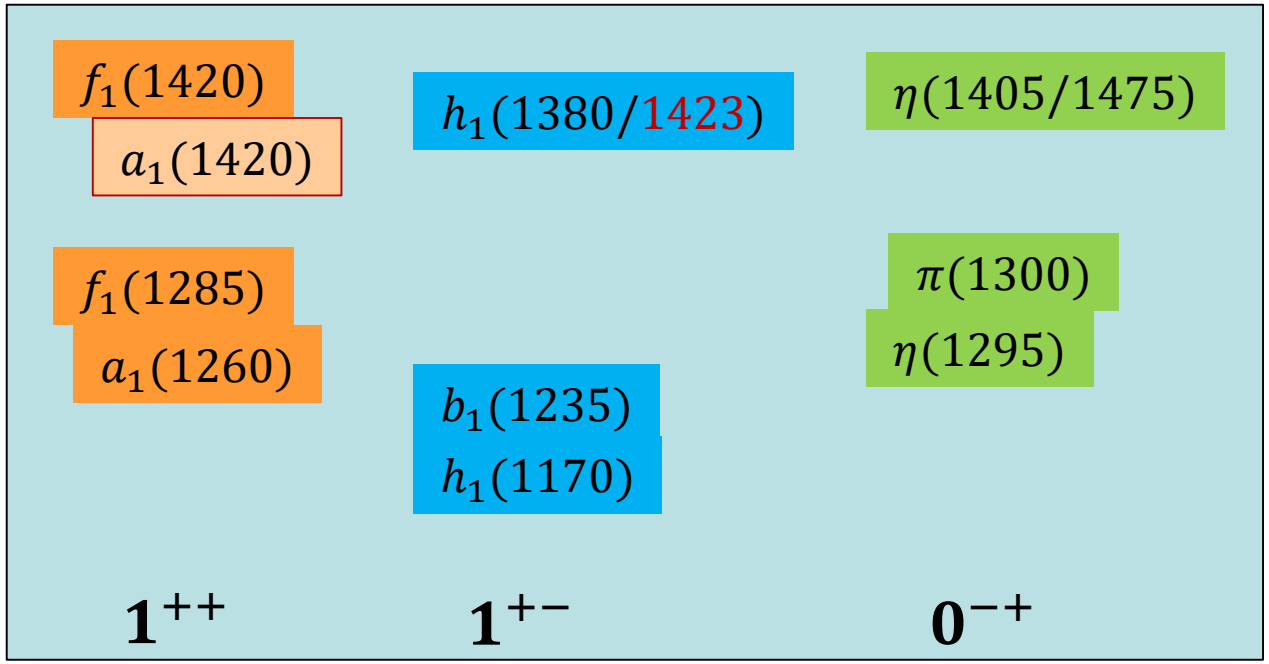
3. Dynamical aspects of the TS mechanism

Similar kinematics in the $K^*\bar{K}+\mathrm{c.c.}$ and $D^*\bar{D}+\mathrm{c.c.}$ system, but with diff. dynamic properties

The narrow two-body open thresholds

	$S-wave(L=0)$	$P-wave(L=1)$
PP	$0^{+(\pm)}$	$1^{-(\pm)}$
PV	$1^{+(\pm)}$	$0^{-(\pm)},\ 1^{-(\mp)},\ 2^{-(\pm)}$
VV	$0^{+(+)},\ 1^{+(-)},\ 2^{+(+)}$	$1^{-(+)};$ $0^{-(-)},\ 1^{-(-)},\ 2^{-(-)};$ $1^{-(+)},\ 2^{-(+)},\ 3^{-(+)}$
PA	$1^{-(-)}$	.....
VA	$0^{-(\pm)},\ 1^{-(\mp)},\ 2^{-(\pm)}$	.....

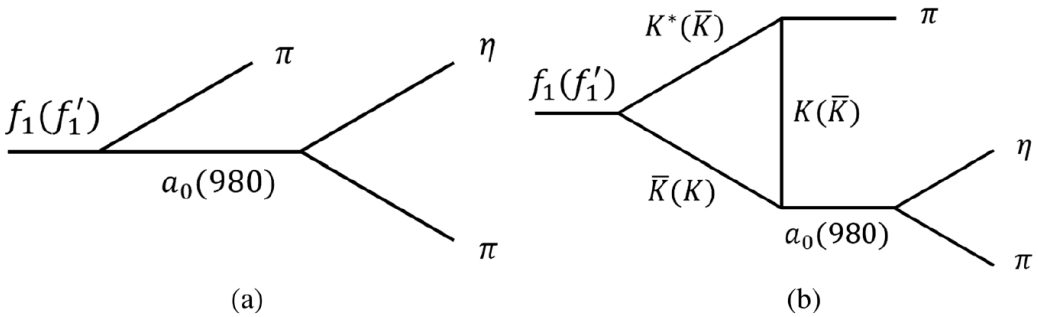




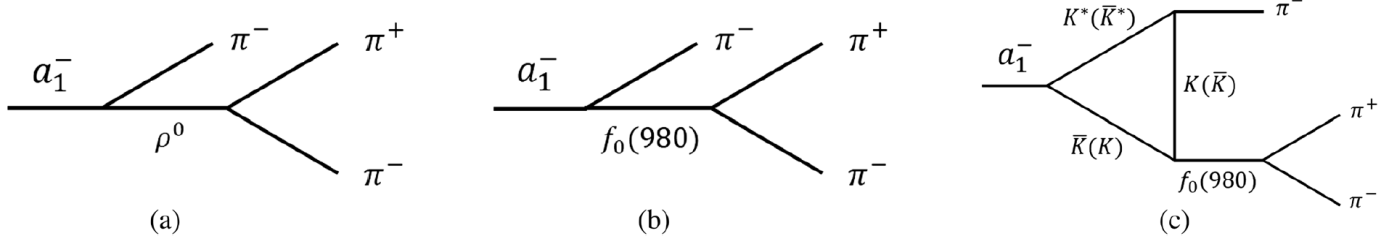
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016)
M.-C. Du and Q. Zhao, Phys.Rev.D 104, 036008 (2021)
J. Wang and Q. Zhao, to be submitted.

BESIII, PRD **98**, 072005 (2018)

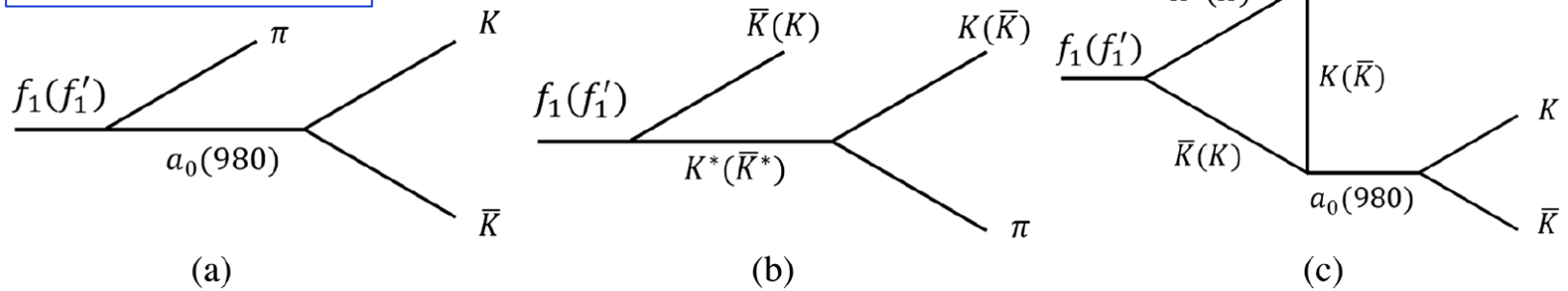
$f_1(f_1') \to \eta\pi\pi$



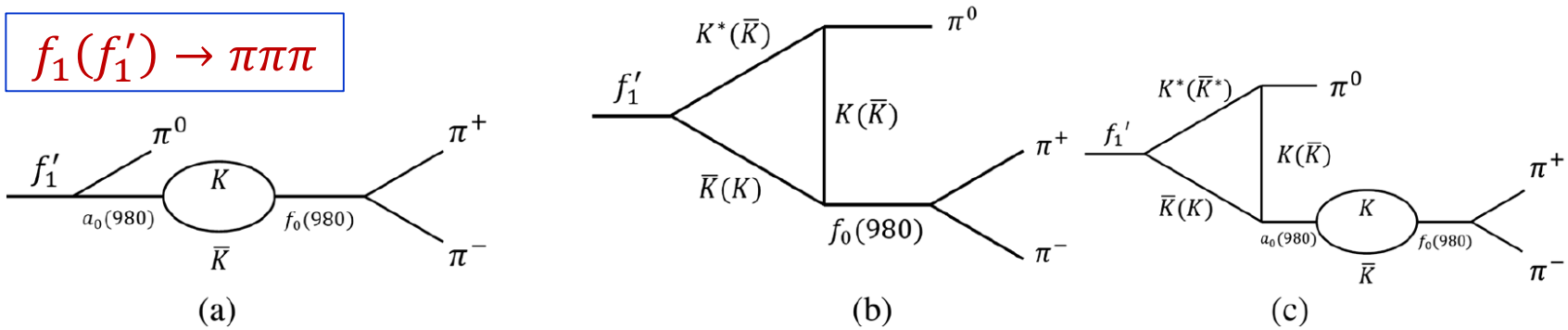
$a_1(1420) \to \pi\pi\pi$



$f_1(f_1') \to K\bar{K}\pi$

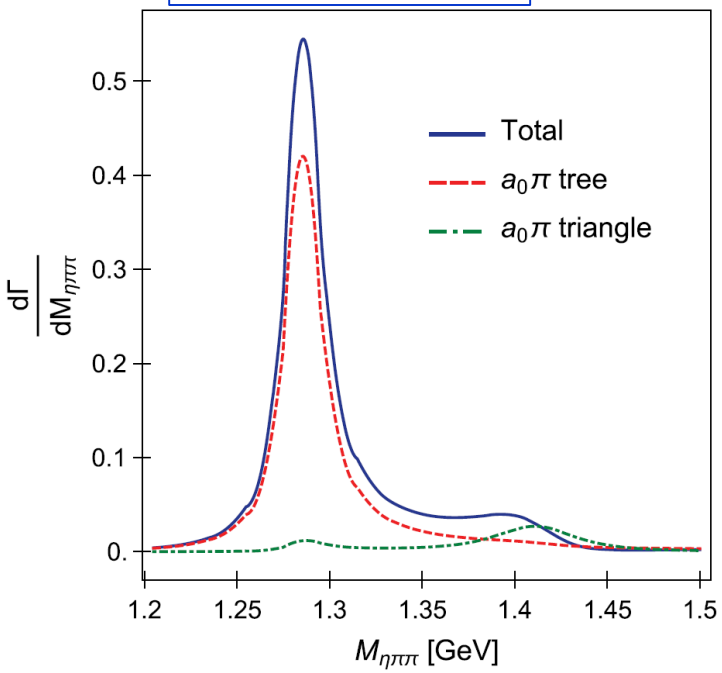


$f_1(f_1') \to \pi\pi\pi$

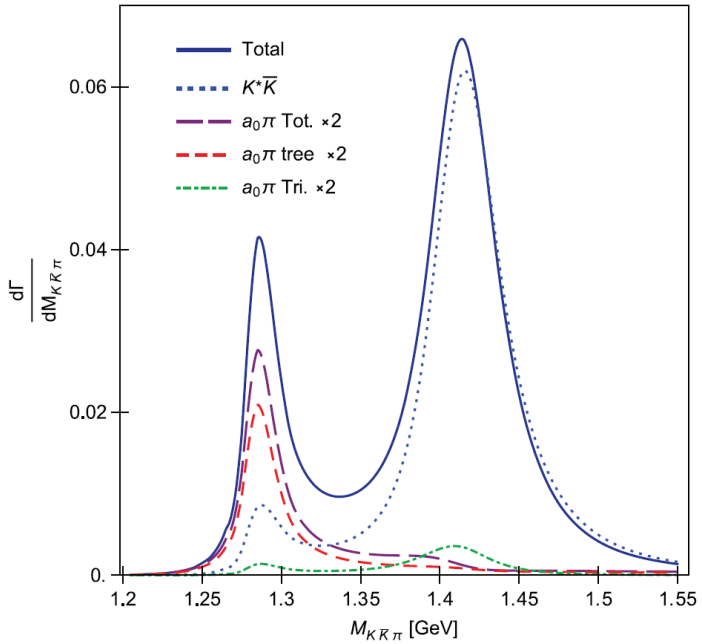


M.C. Du and Q. Zhao, PRD 104, 036008 (2021)

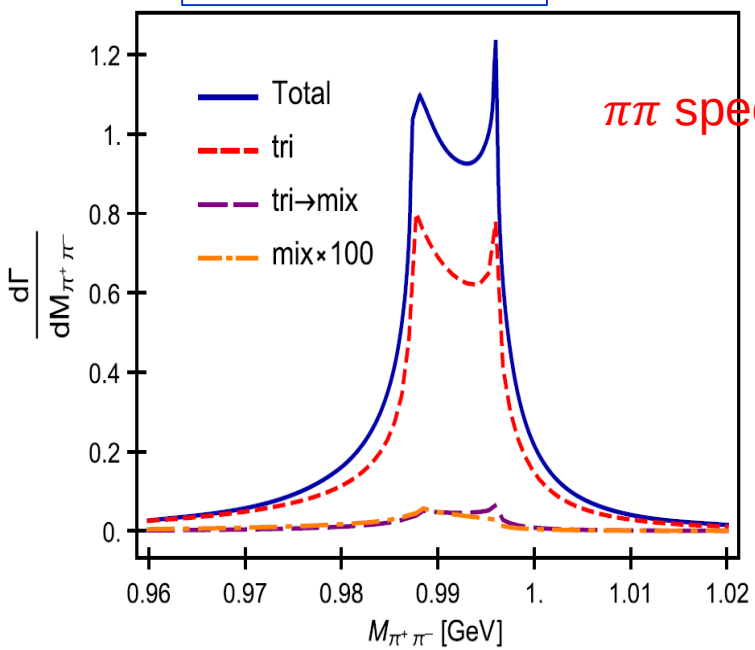
$f_1(f_1') \to \eta\pi\pi$



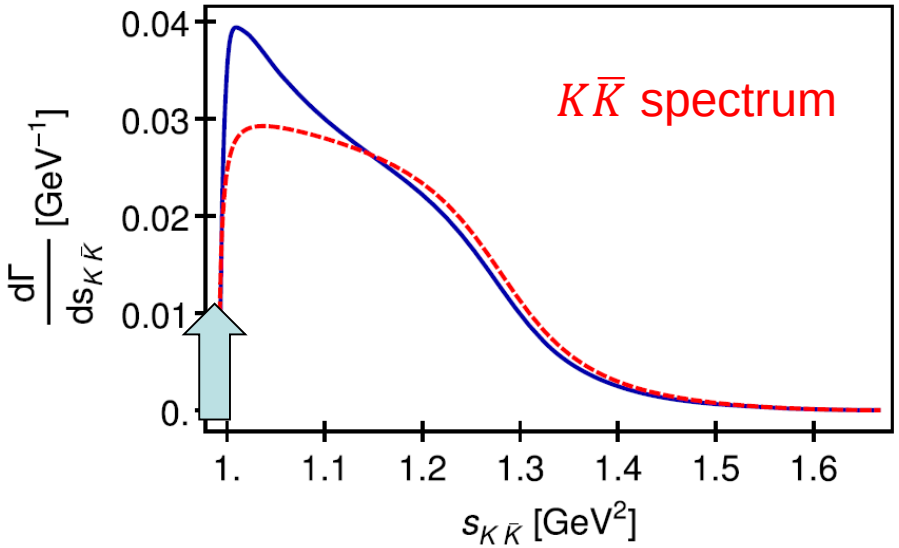
$f_1(f_1') \to K\bar{K}\pi$



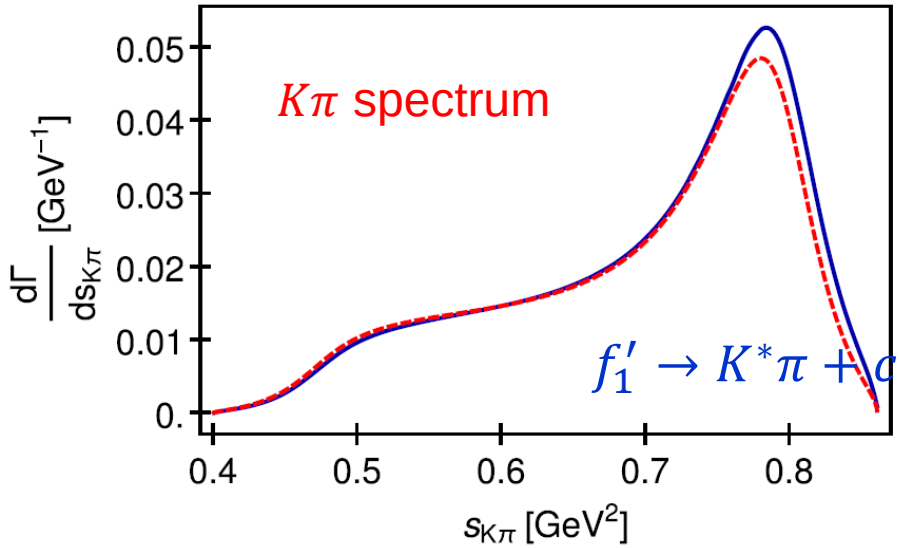
$f_1(f_1') \to \pi\pi\pi$



$\pi\pi$ spectrum



$K\bar{K}$ spectrum



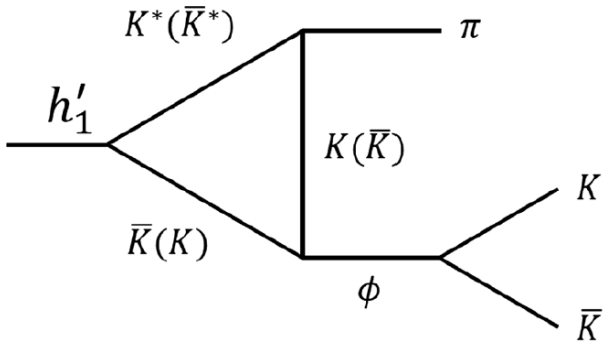
$K\pi$ spectrum

$f_1' \to K^*\pi + c.c. \to K\bar{K}\pi$ dominant

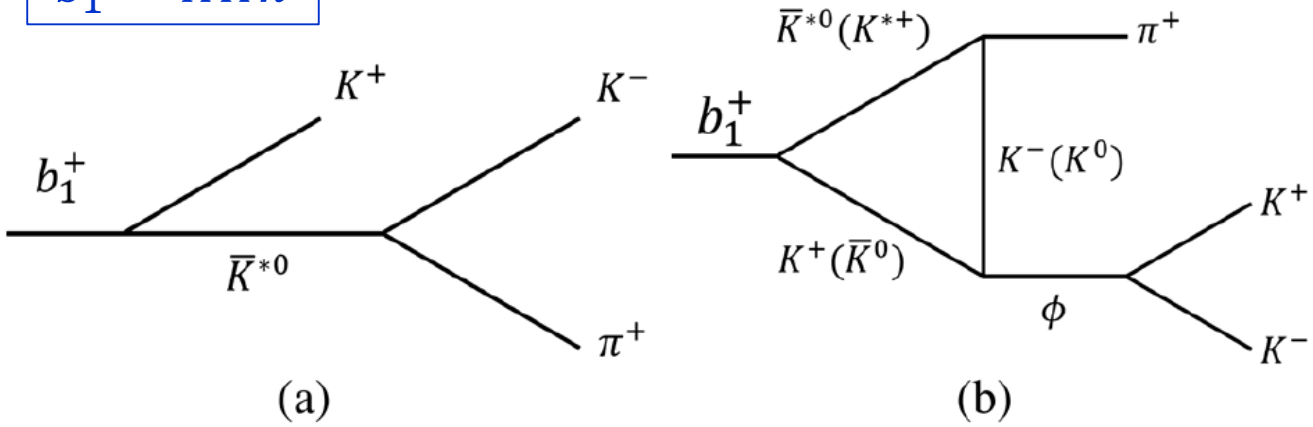
M.C. Du and Q. Zhao, PRD 104, 036008 (2021)

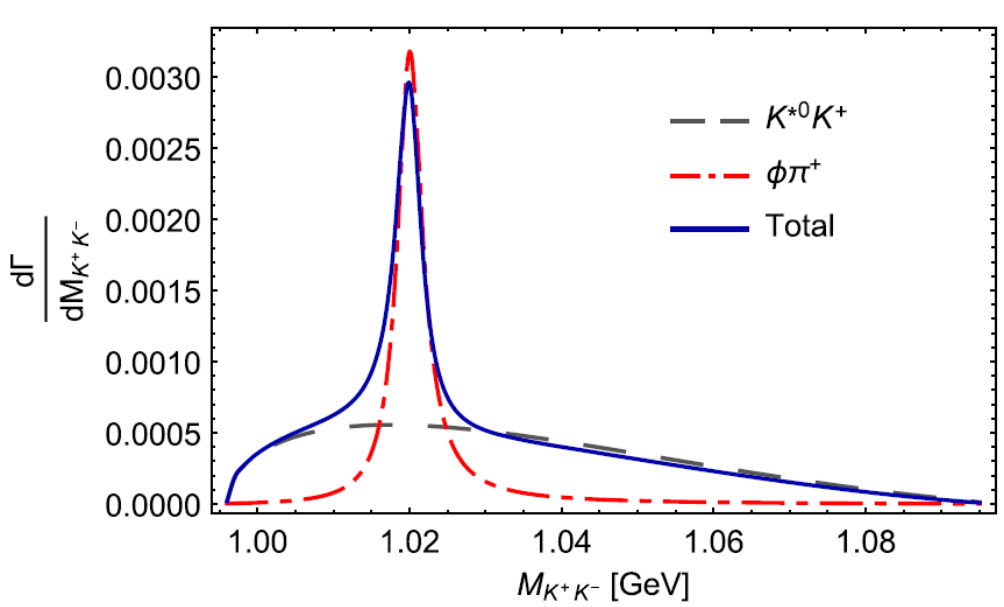
Unobvious effects from TS in $1^{+-} \to 0^{-} + 1^{-}$ if no pole present

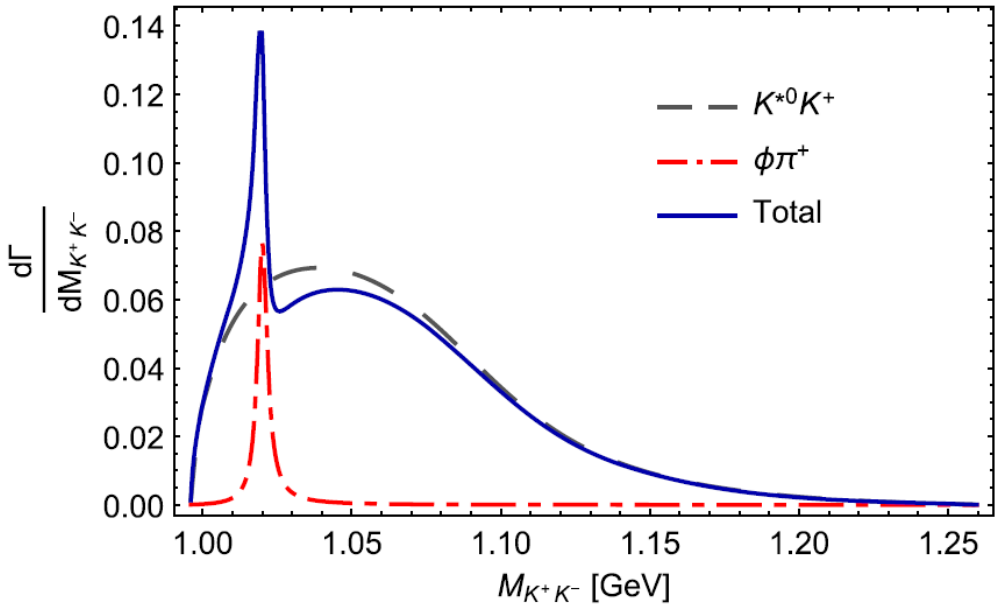
$h_1' \to K\bar{K}\pi$



$b_1 \to K\bar{K}\pi$

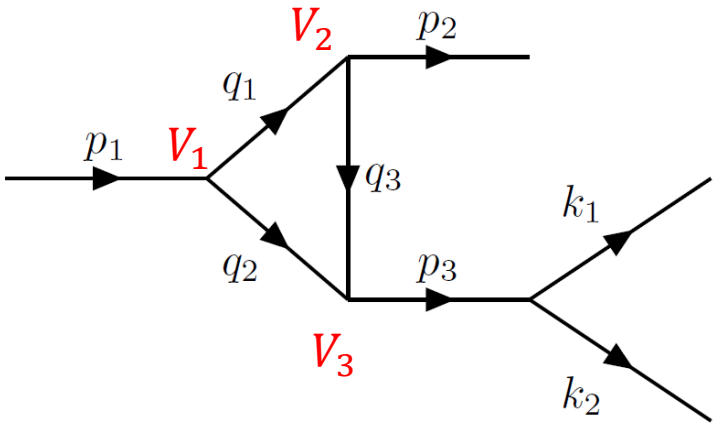






M.C. Du and Q. Zhao, PRD 104, 036008 (2021)

Possible quantum numbers involving pseudoscalar and vector mesons in the triangle loops



$V_{1,2,3}$ represent the partial wave types of the vertex couplings $p_1q_1q_2$, $p_2q_1q_3$, and $p_3q_2q_3$, respectively.

Serial Number	p_1	p_2	p_3	q_1	q_2	q_3	k_1	k_2	V_1	V_2	V_3
1	1^+	0^-	0^+	1^-	0^-	0^-	0^-	0^-	S	P	S
2	1^+	0^-	1^-	1^-	0^-	0^-	0^-	0^-	S	P	P
3	1^+	0^-	1^-	1^-	1^-	0^-	0^-	0^-	S	P	P
4	1^+	0^-	1^+	1^-	1^-	0^-	1^-	0^-	S	P	S
5	1^-	0^-	0^-	1^-	1^-	0^-	1^-	0^-	P	P	P
6	1^-	0^-	1^-	1^-	0^-	0^-	0^-	0^-	P	P	P
7	1^-	0^-	1^-	1^-	1^-	0^-	0^-	0^-	P	P	P
8	1^-	0^-	1^+	1^-	1^-	0^-	1^-	0^-	P	P	S
9	0^+	0^-	1^+	1^-	1^-	0^-	1^-	0^-	S	P	S
10	0^-	0^-	0^+	1^-	0^-	0^-	0^-	0^-	P	P	S
11	0^-	0^-	1^-	1^-	0^-	0^-	0^-	0^-	P	P	P
12	0^-	0^-	1^-	1^-	1^-	0^-	0^-	0^-	P	P	P

The one-loop amplitudes can be decomposed into scalar loop integrals with Lorentz-scalar coefficients

$$i\mathcal{M}_i/g_i = \left(\sum_j A_i^j \mathbf{A}_{0j} + \sum_j B_i^j \mathbf{B}_{0j} + C_i \mathbf{C}_0\right) \frac{1}{s_3 - M_3^2 + iM_3\Gamma_3},$$

The scalar loop integrals $\boldsymbol{A_{0j}}$, $\boldsymbol{B_{0j}}$, and $\boldsymbol{C_0}$ are defined as

$$\mathbf{A}_{01} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_1}, \quad \mathbf{A}_{02} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_2}, \quad \mathbf{A}_{03} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_3},$$

$$\mathbf{B}_{01} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_2D_3}, \quad \mathbf{B}_{02} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_1D_3}, \quad \mathbf{B}_{03} = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_1D_2},$$

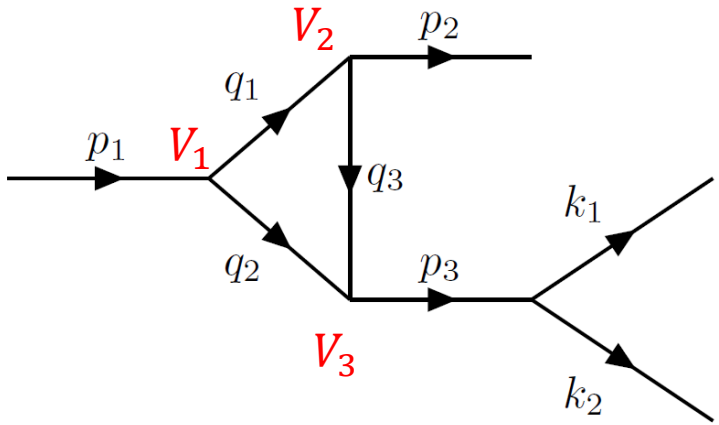
$$\mathbf{C}_0 = \int \frac{d^4q_3}{(2\pi)^4}\frac{1}{D_1D_2D_3},$$

Coefficients A_i^j, B_i^j, and C_i are linear combinations of Lorentz scalars in terms of the external momenta and polarization vectors

$$A_i^j = \sum_k L_i^k f_{Ai}^{jk}, \quad B_i^j = \sum_k L_i^k f_{Bi}^{jk}, \quad C_i = \sum_k L_i^k f_{Ci}^k,$$

where L_i^k, is the k-th independent Lorentz scalar in the i-th diagram, and f_{Ai}^{jk}, f_{Bi}^{jk}, and f_{Ci}^k are its coefficients multiplying the corresponding scalar integrals.

The effective Lagrangians for the vertices:



$$\mathcal{L}_{PPS} = g_{PPS} P_1 P_2 S\,, \quad \mathcal{L}_{VPP} = i g_{VPP}(P_1 \partial_\mu P_2 - \partial_\mu P_1 P_2) V^\mu, \quad \mathcal{L}_{VPA} = g_{VPA} V_\mu A^\mu P$$

$$\mathcal{L}_{VVP} = g_{VVP} \varepsilon_{\alpha\beta\mu\nu} \partial^\alpha V_1^\mu \partial^\beta V_2^\nu P\,, \quad \mathcal{L}_{VVA} = i g_{VVA} \varepsilon_{\alpha\beta\mu\nu} \partial^\alpha A^\beta V_1^\mu V_2^\nu\,,$$

$$\mathcal{L}_{VVV} = i g_{VVV}(V_{1,\nu} \overleftrightarrow{\partial^\mu} V_2^\nu V_{3,\mu} + V_{1,\mu} V_2^\nu \overleftrightarrow{\partial^\mu} V_{3,\nu} + V_{2,\mu} V_3^\nu \overleftrightarrow{\partial^\mu} V_{1,\nu})\,, \quad \mathcal{L}_{VVS} = g_{VVS} V_1^\mu V_{2\mu} S\,.$$

Form factor is introduced to regularize the UV divergence:

$$\mathcal{F}(q^2) = \prod_{i=1}^{3} \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2}\,,$$

with $\Lambda_i = m_i + \alpha \Lambda_{QCD}$, with $\Lambda_{QCD} = 0.22$ GeV.

The form factor can be implemented by mass subtraction using: $\dfrac{1}{q_r^2 - m_r^2} \dfrac{\Lambda_r^2 - m_r^2}{\Lambda_r^2 - q_r^2} = \dfrac{1}{q_r^2 - m_r^2} - \dfrac{1}{q_r^2 - \Lambda_r^2}$

Define the eight-term subtraction operator:

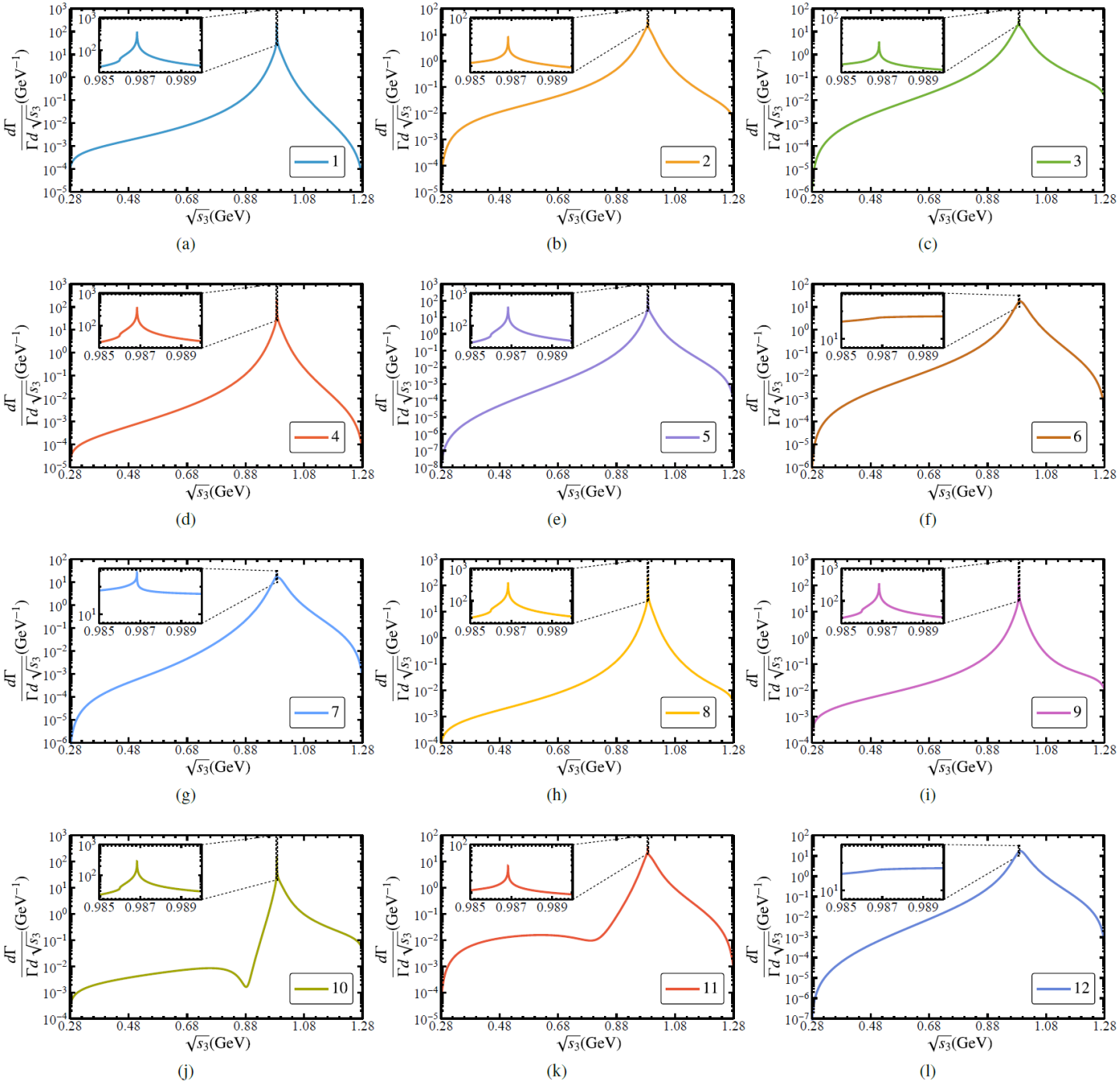
$$\mathcal{R}[X] = (1 - \mathsf{T}_1)(1 - \mathsf{T}_2)(1 - \mathsf{T}_3) X, \qquad \mathsf{T}_r X = X|_{m_r^2 \to \Lambda_r^2}$$

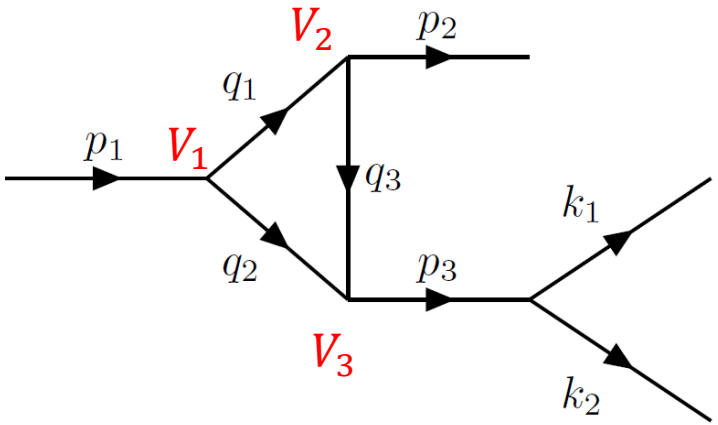
The dynamical effects on the TS with the regularized contributions from different scalar functions can be illustrated by the relative size of

$$a_i^{jk} = \frac{\left|\mathcal{R}[f_{Ai}^{jk}\mathbf{A}_{0j}]\right|}{\left|\mathcal{R}[f_{Ci}^{k}\mathbf{C}_{0}]\right|} \qquad b_i^{jk} = \frac{\left|\mathcal{R}[f_{Bi}^{jk}\mathbf{B}_{0j}]\right|}{\left|\mathcal{R}[f_{Ci}^{k}\mathbf{C}_{0}]\right|}$$

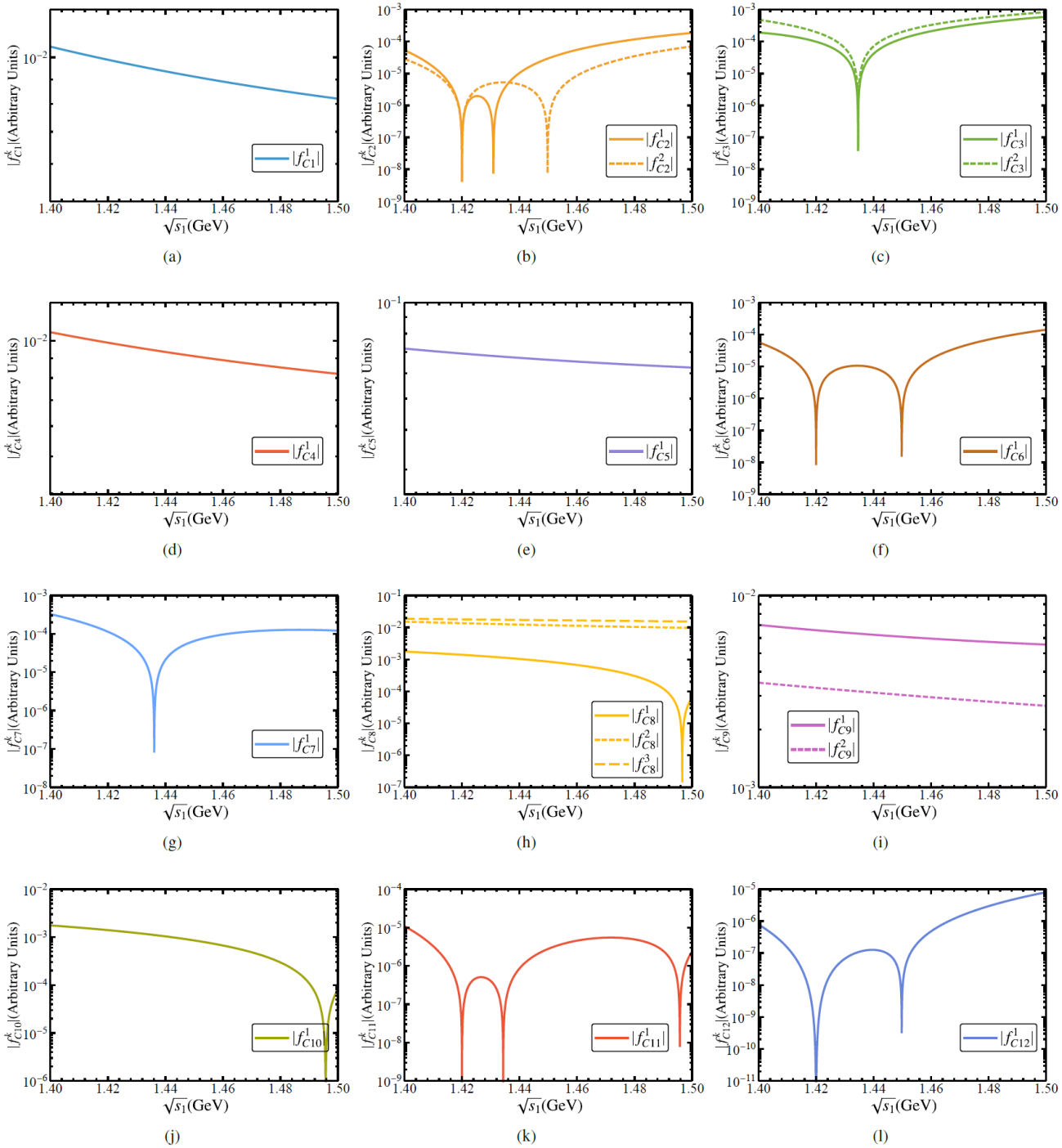
The possible Lorentz scalars obtained from the decomposition of the diagrams

Serial Number	L_i^k
1	$\varepsilon_{1i}\cdot p_2$
2	$\varepsilon_{1i}\cdot p_2,\quad \varepsilon_{1i}\cdot k_1$
3	$\varepsilon_{1i}\cdot p_2,\quad \varepsilon_{1i}\cdot k_1$
4	$\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{1i}^{\mu}\varepsilon_{1f}^{*\nu}p_1^{\alpha}p_2^{\beta}$
5	$\varepsilon_{1i}\cdot p_2\varepsilon_{1f}^{*}\cdot k_2$
6	$\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{1i}^{\mu}p_2^{\nu}k_1^{\alpha}k_2^{\beta}$
7	$\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{1i}^{\mu}p_2^{\nu}k_1^{\alpha}k_2^{\beta}$
8	$\varepsilon_{1i}\cdot\varepsilon_{1f}^{*},\quad \varepsilon_{1i}\cdot p_2\varepsilon_{1f}^{*}\cdot p_2,\quad \varepsilon_{1i}\cdot p_2\varepsilon_{1f}^{*}\cdot k_2$
9	$\varepsilon_{1f}^{*}\cdot p_2,\quad \varepsilon_{1f}^{*}\cdot k_2$
10	1
11	1
12	1

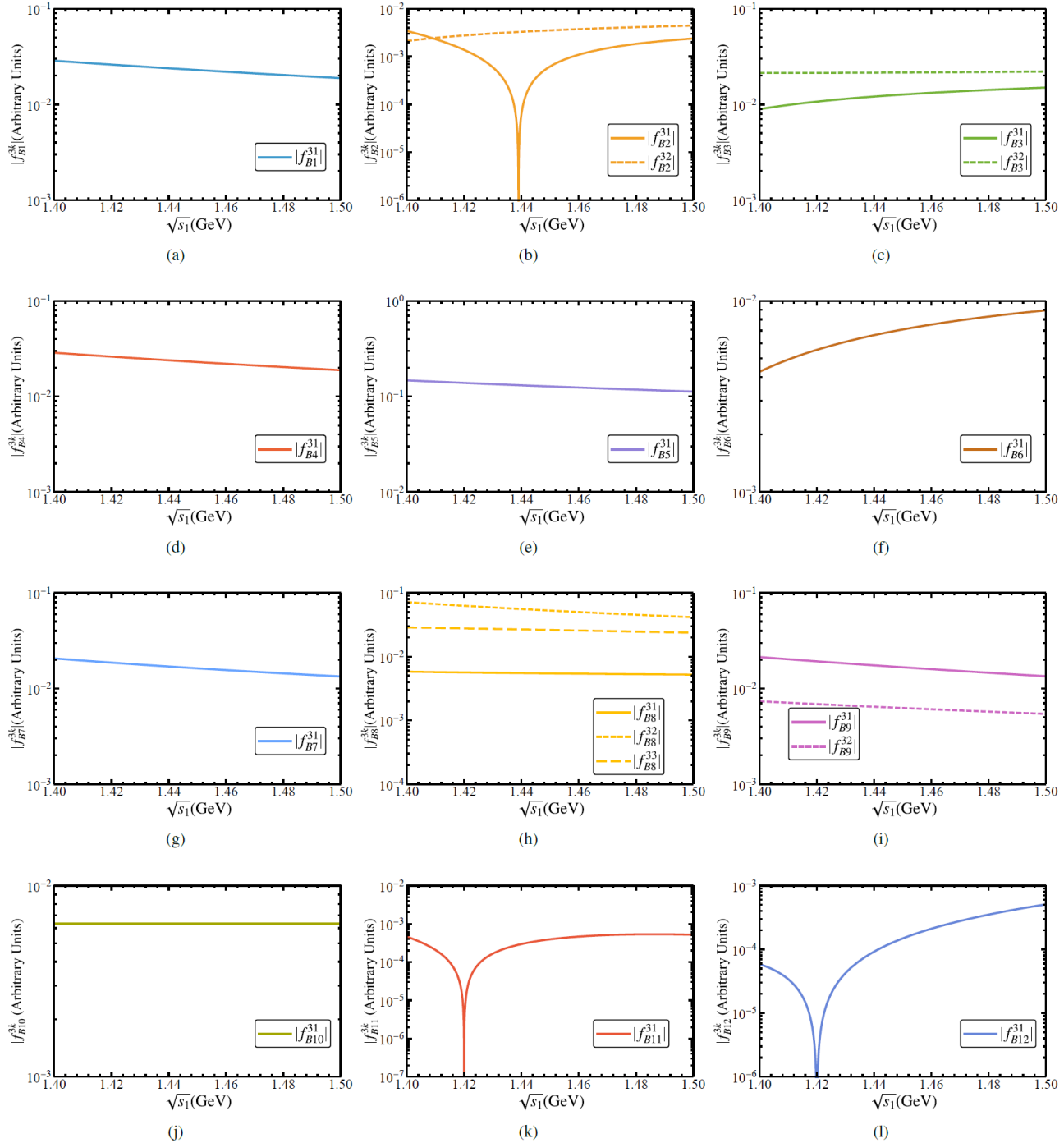




Invariant-mass spectra $\sqrt{s_3} = M(k_1k_2)$ for the twelve spin-parity assignments, using the common mass inputs and including the final-state resonance factor with the $f_0(980)$ parameters.

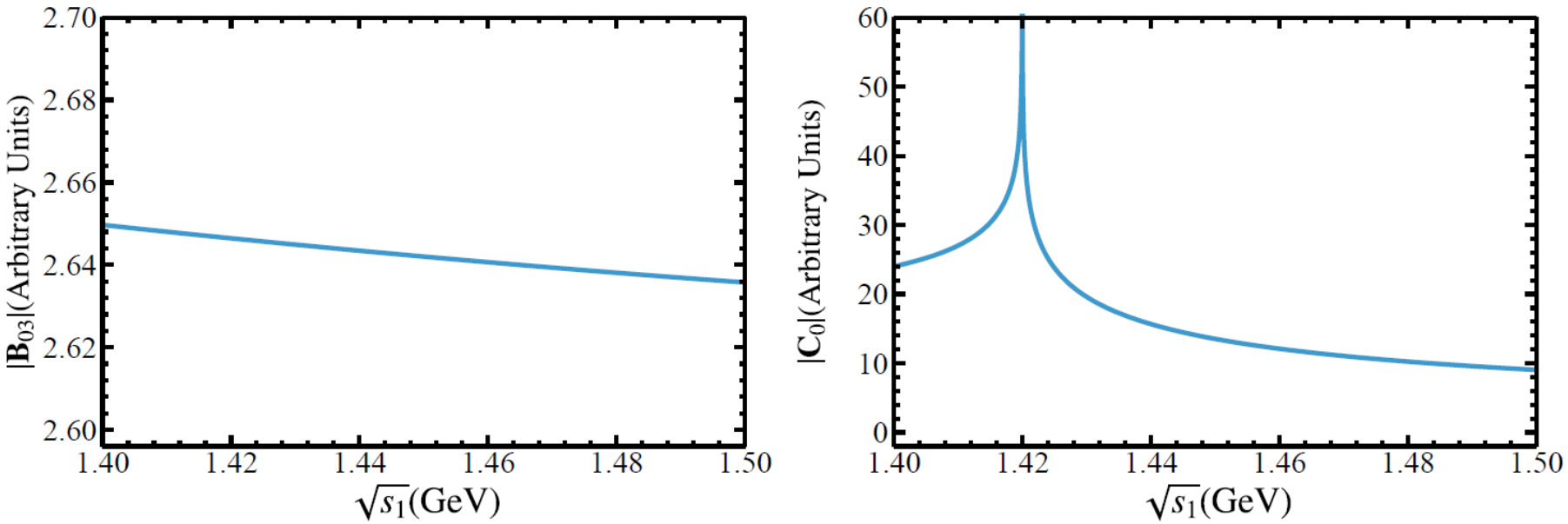


Magnitudes of the physical-mass C_0 coefficients f_{Ci}^k before the form-factor subtraction, as functions of the initial-state center-of-mass energy.



Magnitudes of the physical-mass coefficients f_{Bi}^{3k} of B_{03} before the form-factor subtraction, as functions of the initial-state center-of-mass energy.

Evolution of $|B_{03}|$ and $|C_0|$ with the initial-state center-of-mass energy, evaluated at the physical masses without the form-factor subtraction.



4. Summary

Three classes of behaviors from the TS mechanism due to different dynamic properties:

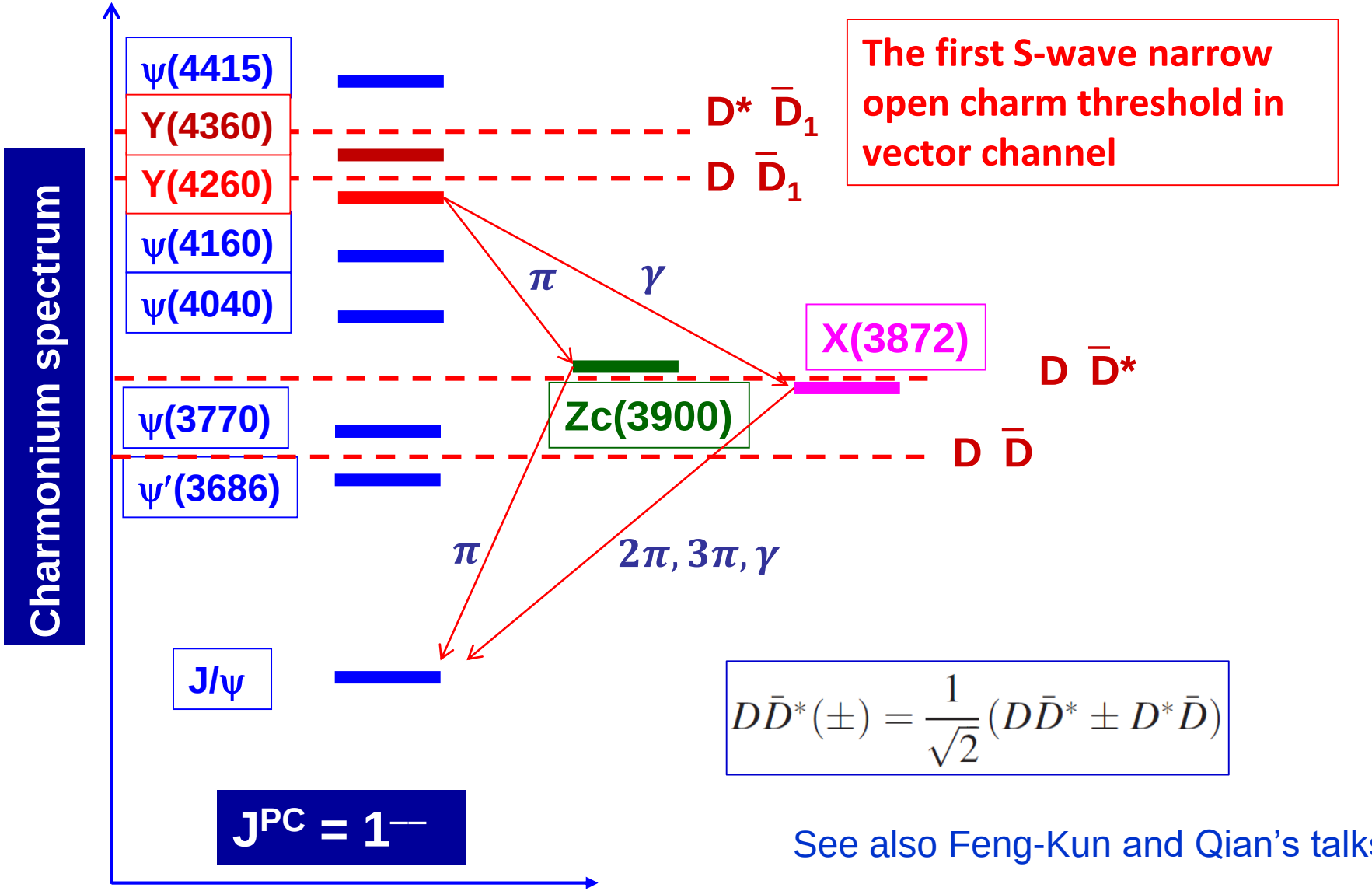
1) The C_0 contribution dominates and produces a pronounced narrow TS peak.
2) The bubble contributions are more competitive, and their interference with the triangle term can reduce/enhance the prominence of the peak.
3) The physical-mass coefficient of the leading Landau logarithm vanishes as a consequence of the vertex dynamics.

Rich and informative structures can arise from different dynamic systems and more detailed studies are needed.

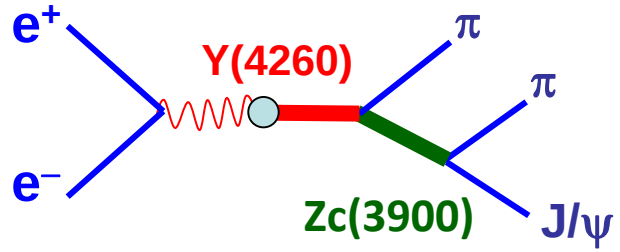
Thanks for your attention!

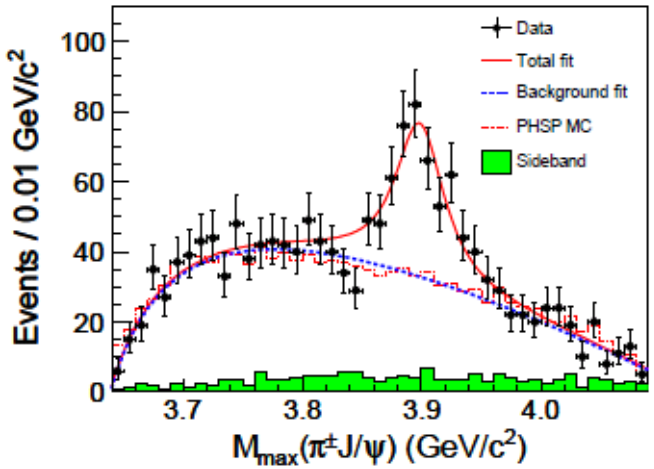
Case-II: The nature and property of Zc(3900)

Correlations between Y(4260) and Zc(3900)/X(3872) via the TS mechanism



The first S-wave narrow open charm threshold in vector channel



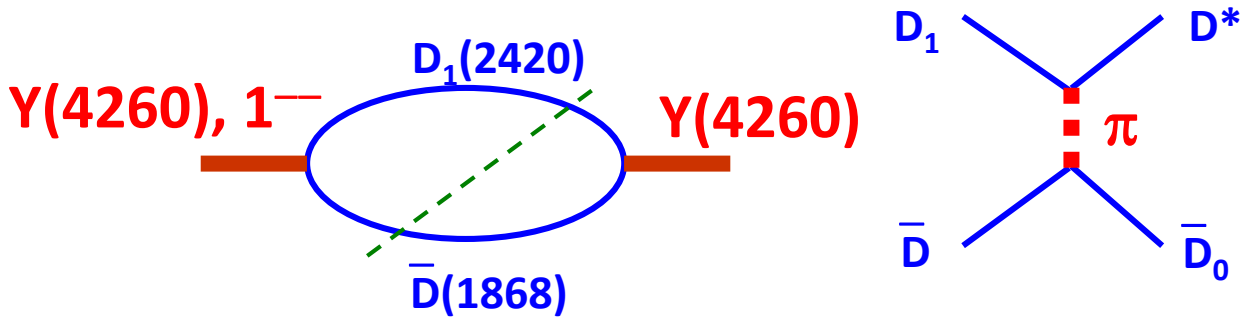


BESIII, PRL(2013)

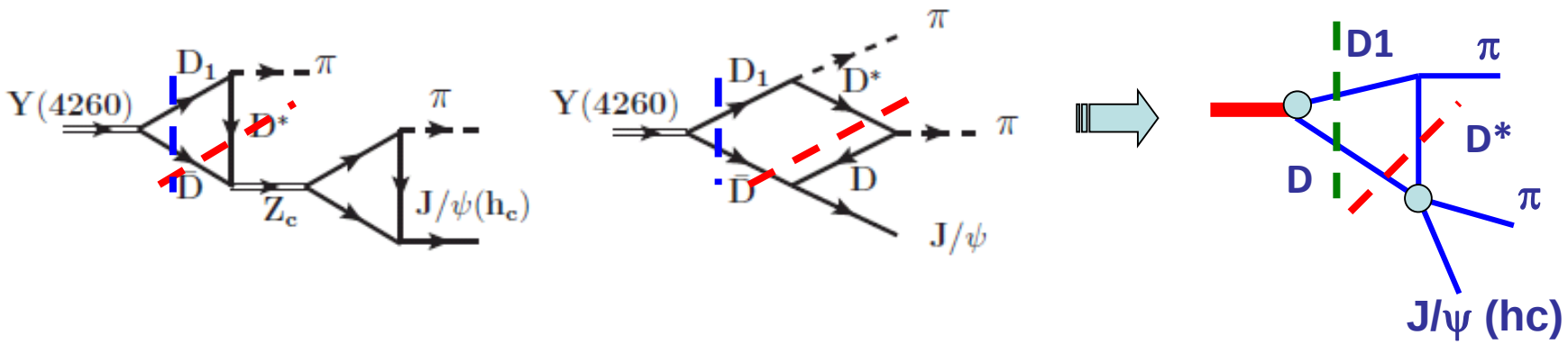
$$D\bar{D}^*(\pm) = \frac{1}{\sqrt{2}}(D\bar{D}^* \pm D^*\bar{D})$$

See also Feng-Kun and Qian's talks

- Y(4260) could be a hadronic molecule made of $DD_1(2420)$ with coupled channel effects.



- The production of Zc(3900) is strongly correlated with Y(4260) and enhanced by the triangle singularity kinematics.



Q. Wang, C. Hanhart, Q. Zhao, PRL111, 132003 (2013)
Q. Wang, C. Hanhart, Q. Zhao, PLB725, 106 (2013)