Workshop on Few-Body Problems in Nuclear and Hadron Physics
21-24 September 2026, ITP, CAS

北京航空航天大學
BEIHANG UNIVERSITY

Femtoscopy: basics and recent development

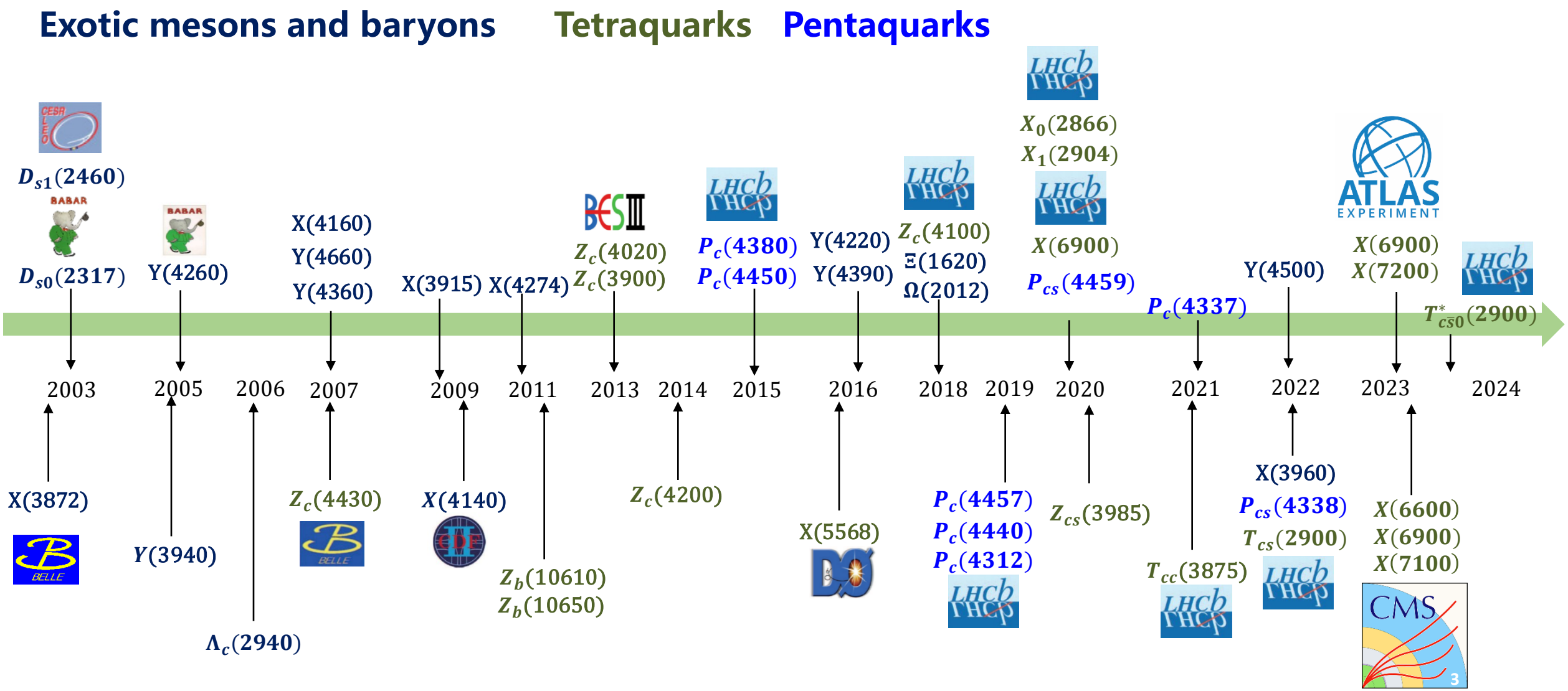
Lisheng Geng, Beihang University

Zhi-Wei Liu, Jun-Xu Lu, LSG*, PRD 107 (2023) 074019
Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, LSG*, Sci.Bull. 70 (2025) 3515-3521
Duo-Lun Ge, Zhi-Wei Liu, LSG*, PRD 114 (2026) 014054
Hao-Nan Liu, Duo-Lun Ge, Zhi-Wei Liu, Jun-Xu Lu, and LSG*, 2609.19161
Zhi-Wei Liu, Jun-Xu Lu, Duo-Lun Ge, and LSG*, 2607.04351

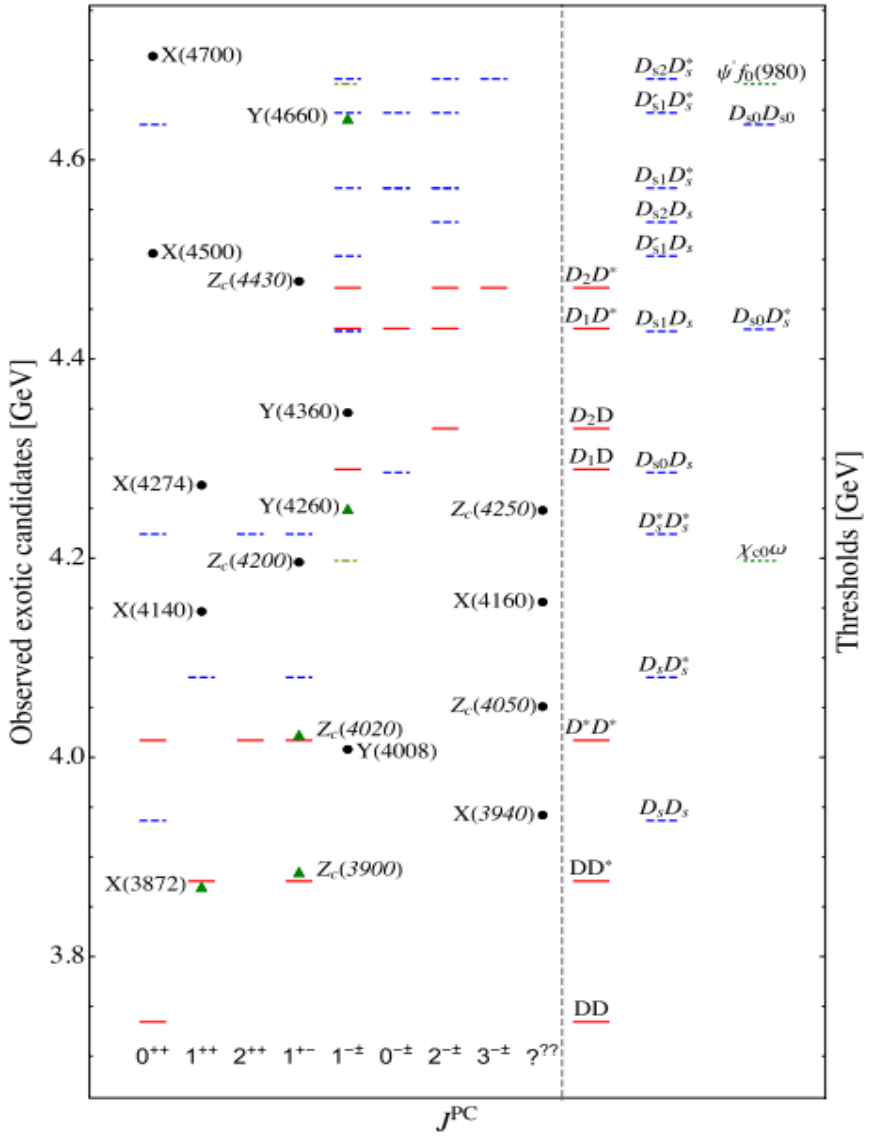
Contents

- Brief introduction: exotic hadrons and femtoscopy
- Femtoscopic correlation functions (CFs)—general features
- Recent applications
  - $D_{s0}^{*}(2317)$, $Z_c(3900)$ and $Z_{cs}(3985)$, and $Tcc(3875)$
  - Two interference mechanisms at play in $\pi^{+}p$ CF
- Summary and outlook

Many so-called exotic hadrons discovered since 2003



Many (if not all) of them close to thresholds—molecules



Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, Bing-Song Zou. Rev.Mod.Phys. 90 (2018) 015004

Richard F. Lebed, Ryan E. Mitchell, Eric S. Swanson, Prog.Part.Nucl.Phys. 93 (2017) 143

Atsushi Hosaka, Toru Iijima, Kenkichi Miyabayashi, Yoshihide Sakai , Shigehiro Yasui, PTEP 2016 (2016) 062C01

Hua-Xing Chen, Wei Chen, Xiang Liu Shi-Lin Zhu, Phys. Rept.639 (2016) 1

……

How to check the molecular picture?



How to check the molecular picture?

arXiv:2406.17006v1 [hep-ex] 24 Jun 2024

Probing the nature of the $\chi_{c1}(3872)$ state using radiative decays

LHCb collaboration[†]

Abstract

The radiative decays $\chi_{c1}(3872) \to \psi(2S)\gamma$ and $\chi_{c1}(3872) \to J/\psi\gamma$ are used to probe the nature of the $\chi_{c1}(3872)$ state using proton-proton collision data collected with the LHCb detector, corresponding to an integrated luminosity of $9\,\text{fb}^{-1}$. Using the $B^+ \to \chi_{c1}(3872)K^+$ decay, the $\chi_{c1}(3872) \to \psi(2S)\gamma$ process is observed for the first time and the ratio of its partial width to that of the $\chi_{c1}(3872) \to J/\psi\gamma$ decay is measured to be

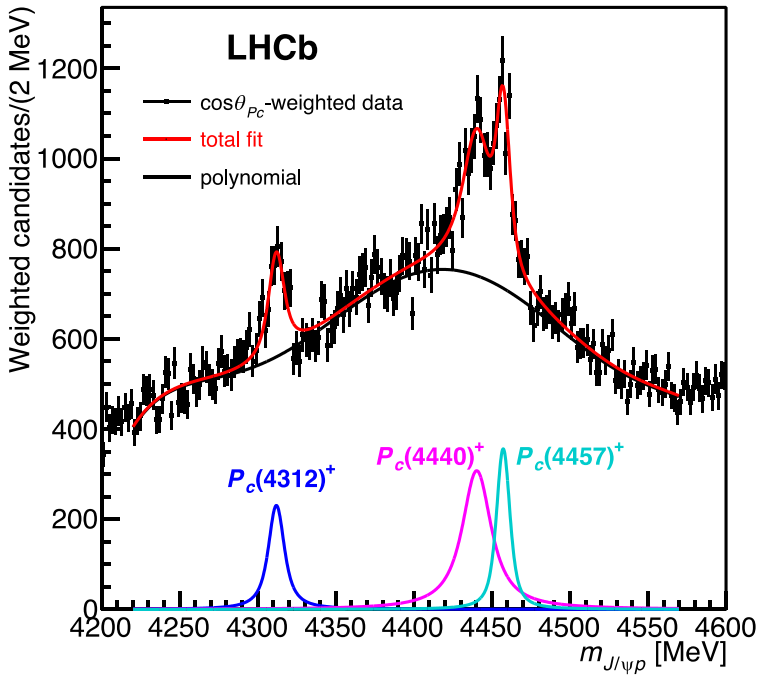
$$\frac{\Gamma_{\chi_{c1}(3872)\to\psi(2S)\gamma}}{\Gamma_{\chi_{c1}(3872)\to J/\psi\gamma}} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04\,,$$

where the first uncertainty is statistical, the second systematic and the third is due to the uncertainties on the branching fractions of the $\psi(2S)$ and J/ψ mesons. The measured ratio makes the interpretation of the $\chi_{c1}(3872)$ state as a pure $D^0\overline{D}^{*0} + \overline{D}^0 D^{*0}$ molecule questionable and strongly indicates a sizeable compact charmonium or tetraquark component within the $\chi_{c1}(3872)$ state.

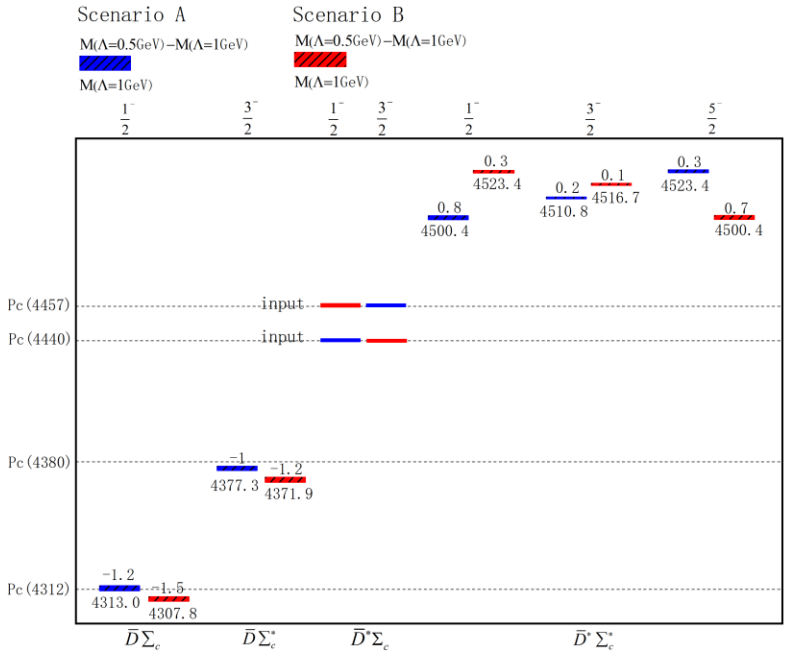
How to verify the molecular picture

Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, LSG, Phys.Rept. 1108 (2025) 1-108*

- **Symmetries in the two-hadron interactions imply the existence of multiplets of hadronic molecules**
  - **Heavy-quark spin/flavor** symmetry: there are **seven Pc** states



LHCb, PRL 122 (2019) 222001

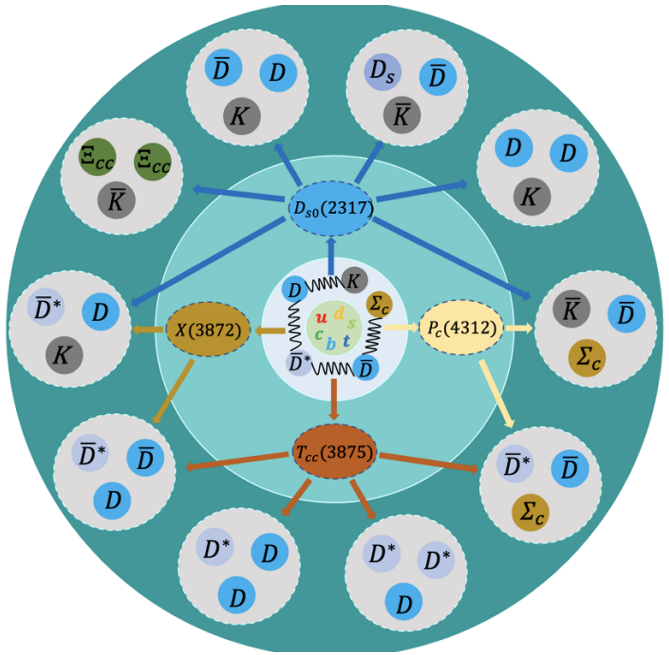


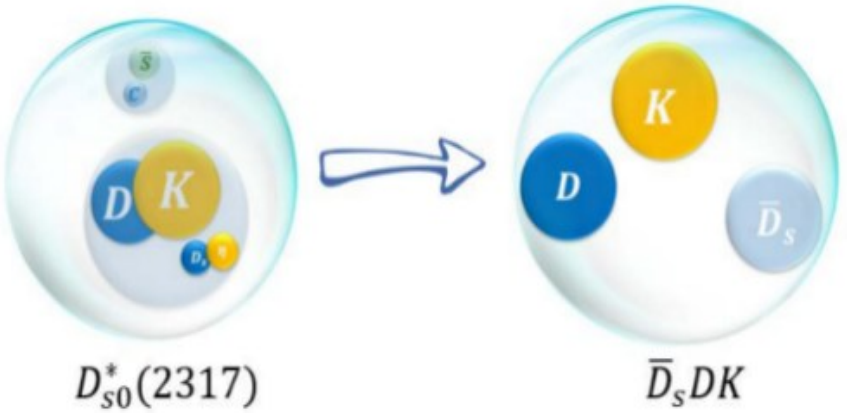
Liu, Pan, Peng, Sánchez, LSG, Hosaka, Valderrama, PRL122, 242001 (2019)

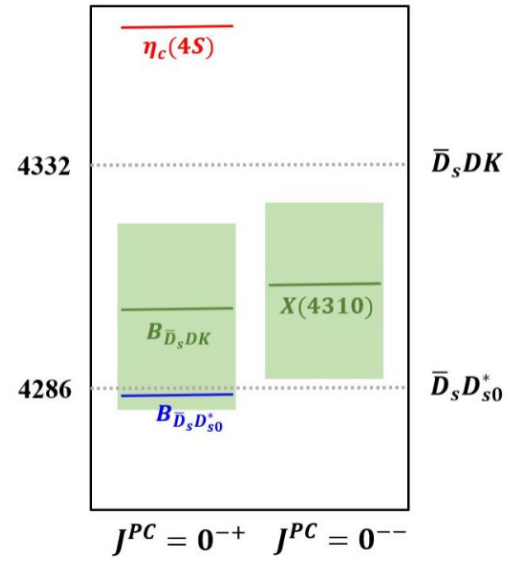
How to verify the molecular picture

Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, LSG, Phys.Rept. 1108 (2025) 1-108*

- **Three hadrons experience pairwise two-body attractions can form three-body molecules**
  - Treating $\boldsymbol{D_{s0}^*(2317)}$ as a DK state, one can expect a $\bar{D}_s DK$ state







T.W. Wu and LSG, Sci.Bull. 67 (2022) 1735-1738

T.W Wu, M.Z Liu, and LSG, PRL(2025) 031902

How to verify the molecular picture

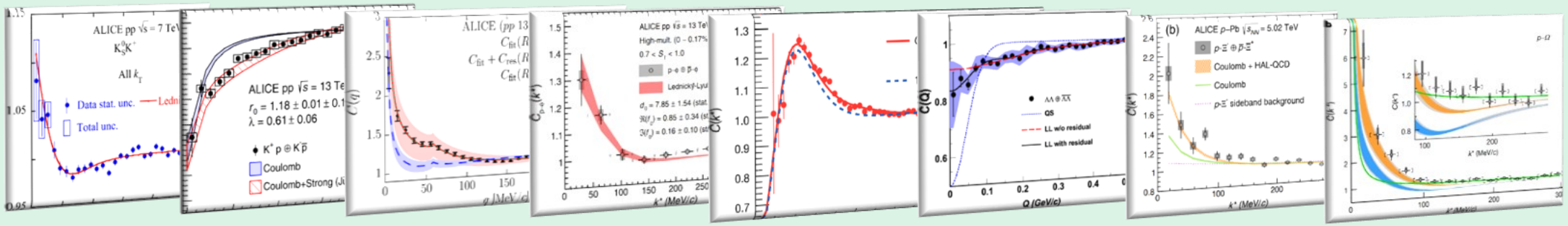
Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, LSG, Phys.Rept. 1108 (2025) 1-108*

☐ Direct measurement of the two-hadron interactions

CERN The Large Hadron Collider

Brookhaven National Laboratory RHIC Relativistic Heavy Ion Collider

$K^0_S K^\pm$ $K^\pm p$ $K^- p$ ϕp $\overline{p}\overline{p}$ $\Lambda\Lambda$ $\Xi^- p$ $\Omega^- p$



PLB 2019 PRL 2020 PRL 2020 PRL 2021 Nature 2015 PRL 2015 PRL 2019 Nature 2020

$K^- d$

$\Sigma^0 p$

$\Lambda\Xi$

$\Xi\Xi$

$\Omega\Omega$

pD

ppp

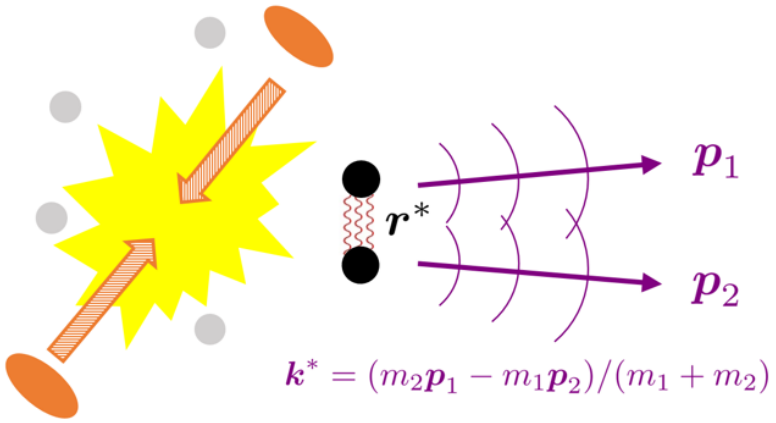
$pp\Lambda$

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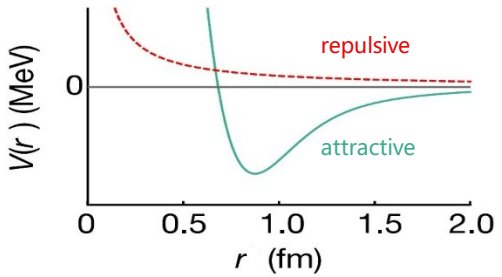
Femtoscopic correlation functions (CFs)

$$C(p_1, p_2) = \frac{\mathrm{P}(p_1, p_2)}{\mathrm{P}(p_1) \cdot \mathrm{P}(p_2)}$$



Emission source $S_{12}(r^*)$ Two-particle wavefunction $\psi(k^*, r^*)$

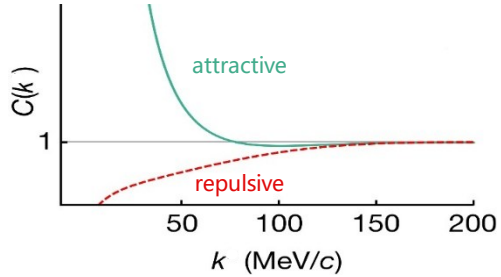
Interacting potential



Schrödinger equation

Two-particle wavefunction $\psi(k, r)$

Correlation function



Exp. measurement

mixed-event technique

$$C(k) = \xi(k) \frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

N_{same}: the same event distributions

N_{mixed}: the mixed event distributions

ξ: the corrections for experimental effects

Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(r) \, |\psi(k, r)|^2 \, \mathrm{d}r$$

spacial structure

final-state interactions

quantum statistics effects

coupled-channel effects

Basic Properties

$C(k)$

- >1 if the interaction is attractive
- =1 if there is no interaction
- <1 if the interaction is repulsive

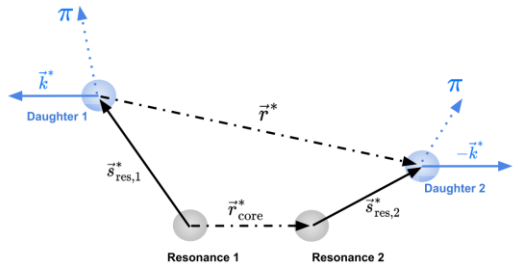
Constraining the source function

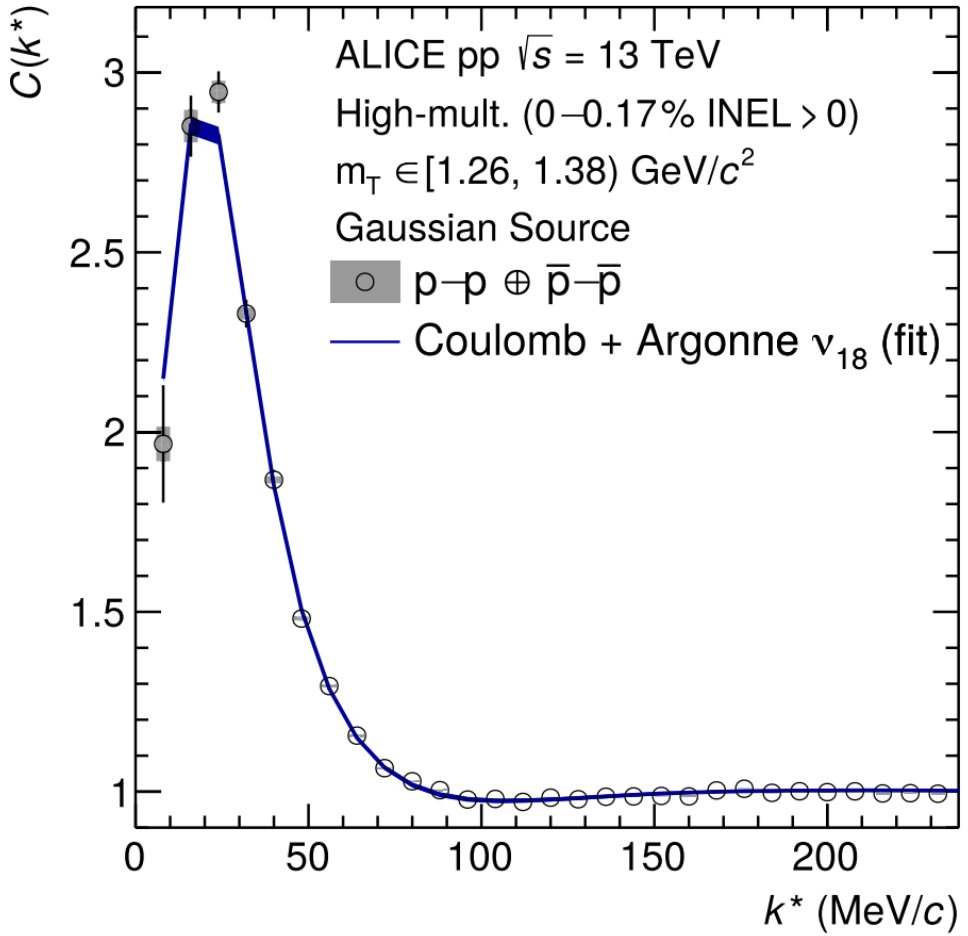
- **Using the well-known proton-proton interaction to calibrate the source**

$$C(k^*) = \int S(\vec{r}^*) \left|\psi\left(\vec{k}^*, \vec{r}^*\right)\right|^2 d^3\vec{r}^*$$

- **For CFs involving short-lived resonances ($c\tau \sim 1\text{fm}$)—Resonance Source Model**

$$S(r^*) = \frac{1}{\left(4\pi r_{\text{core}}^2\right)^{3/2}} \exp\left(-\frac{r^{*2}}{4r_{\text{core}}^2}\right) \times \text{SLR}$$

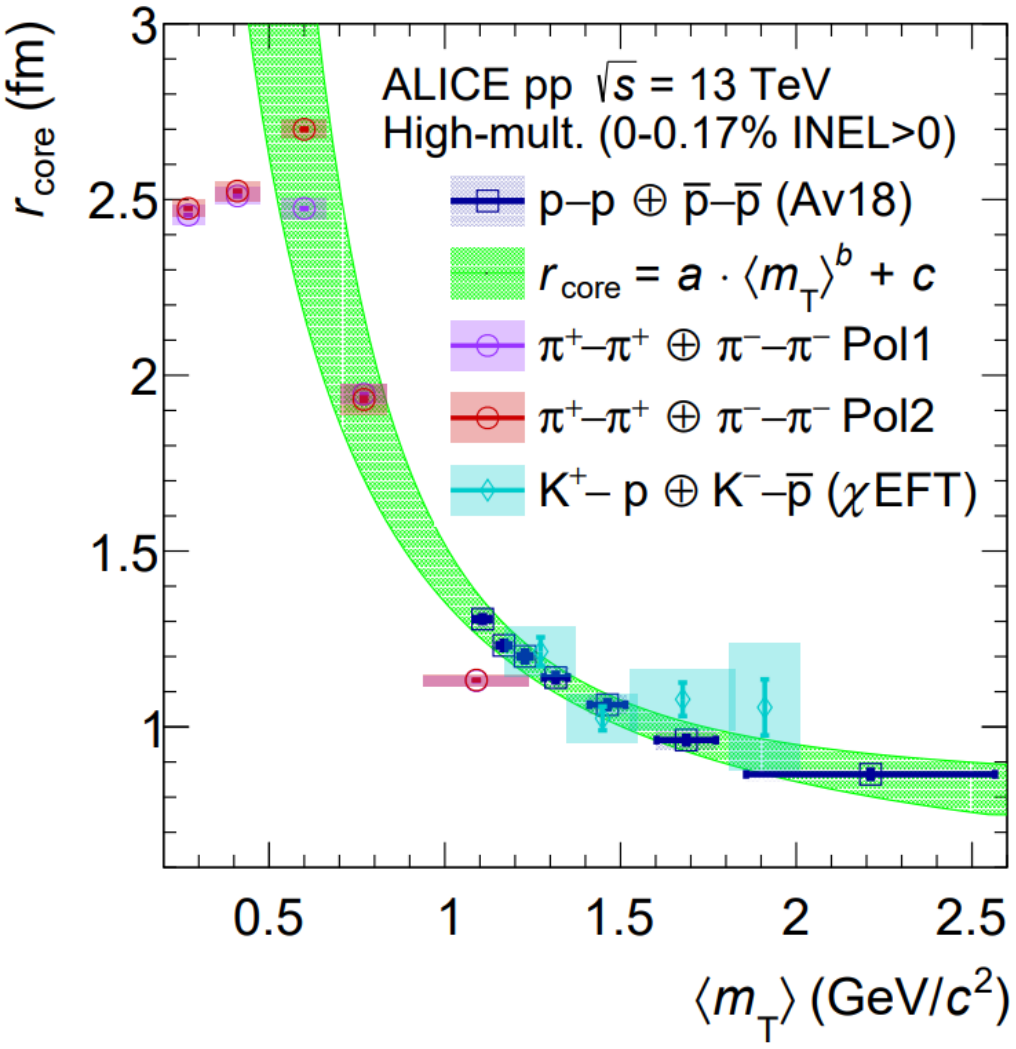




ALICE, PLB, 811 (2020) 135849

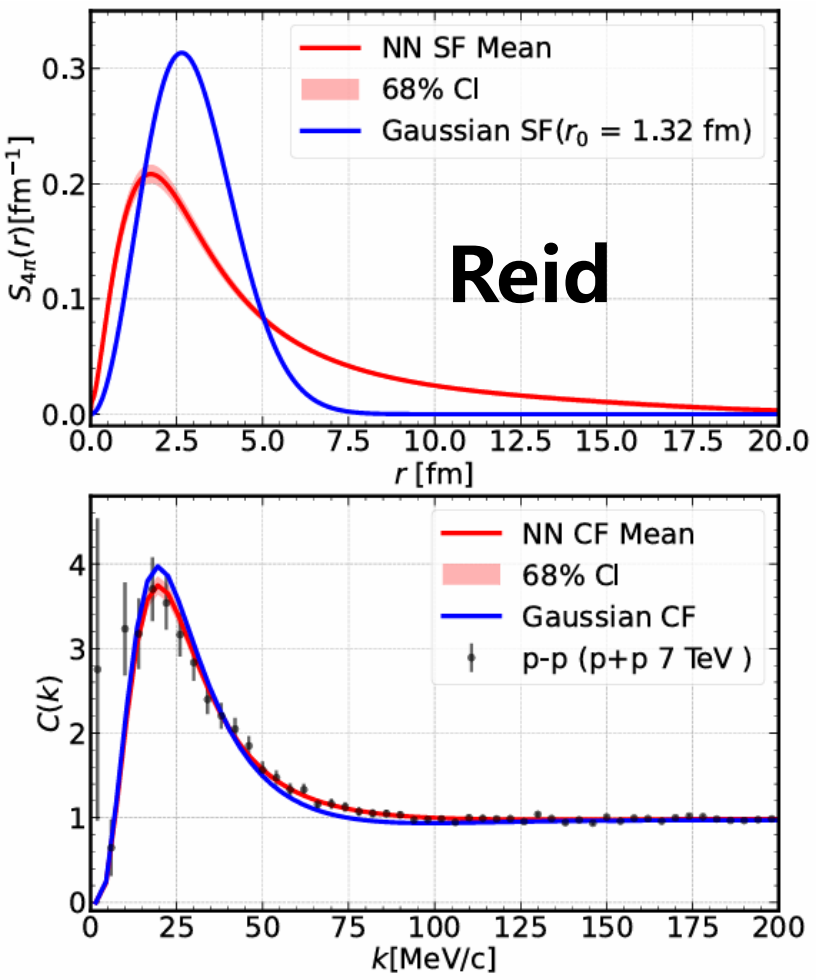
Constraining the source function

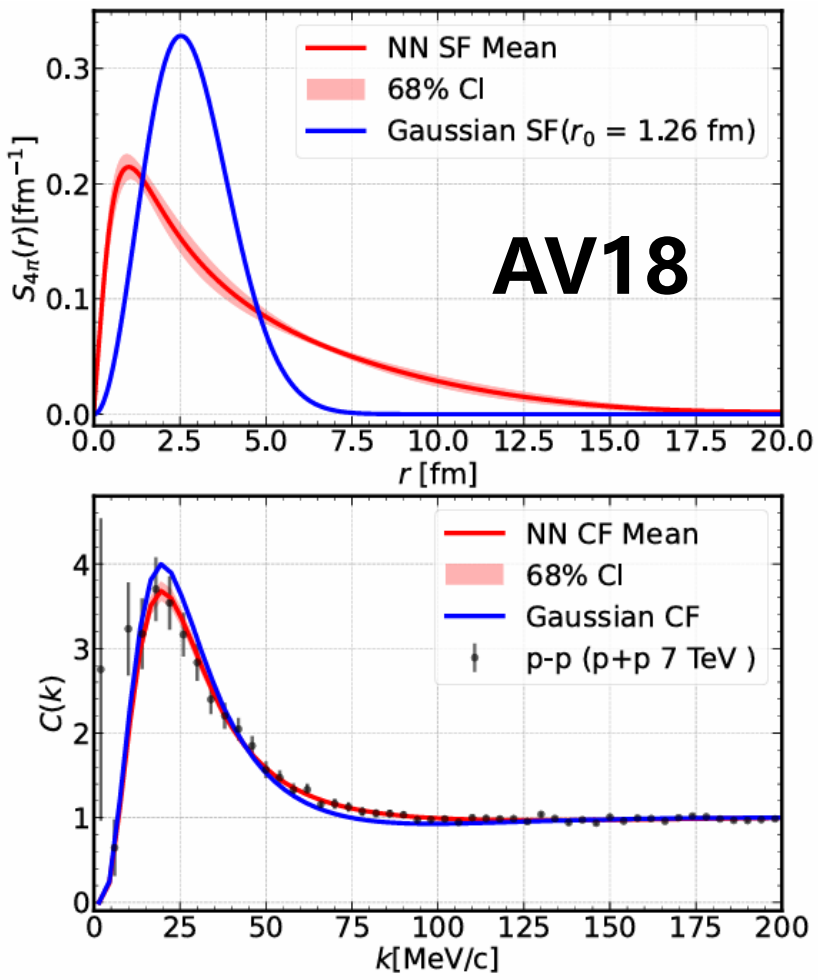
2311.14527v1.



… "providing compelling evidence for the presence of a **common** emission source for all hadrons in small collision systems at the LHC …"

Constraining the source function—Machine Learning

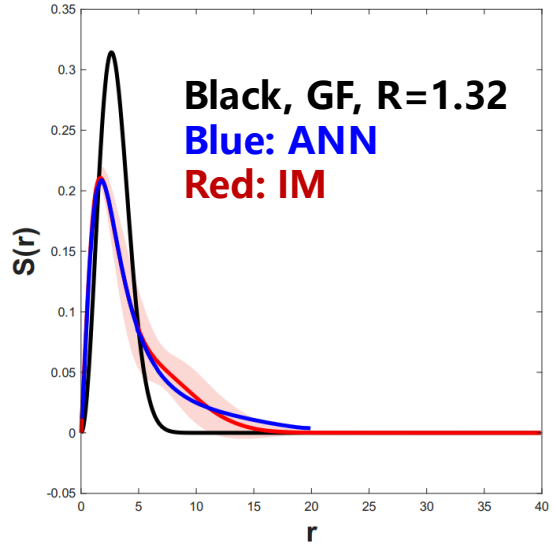


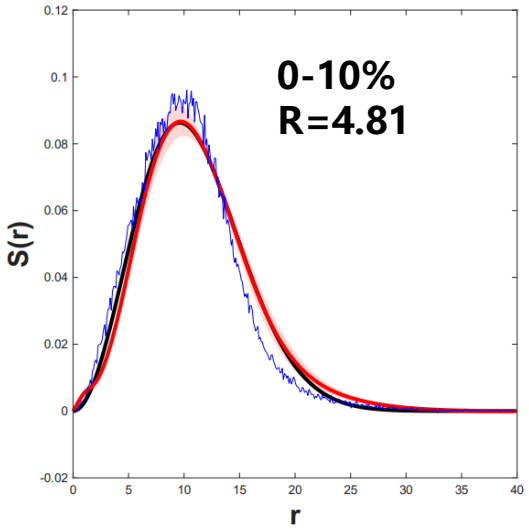


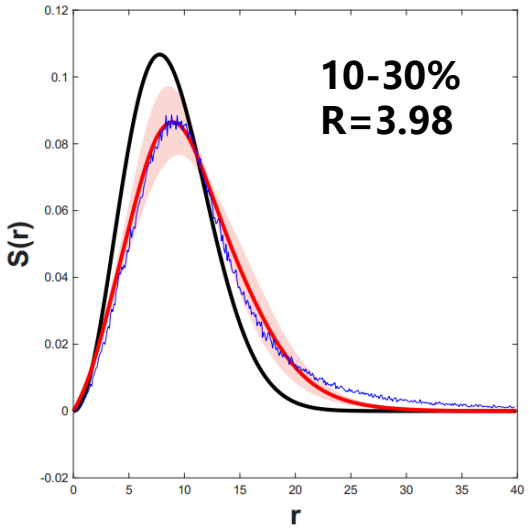
Lingxiao Wang and Jiaxing Zhao, Commun.Phys. 9 (2026) 1, 90

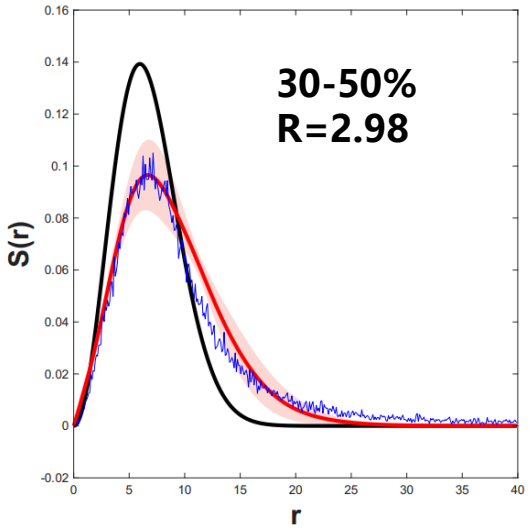
Constraining the source function—Inverse Method

Blue: Parton–Hadron–String Dynamics (PHSD) transport model









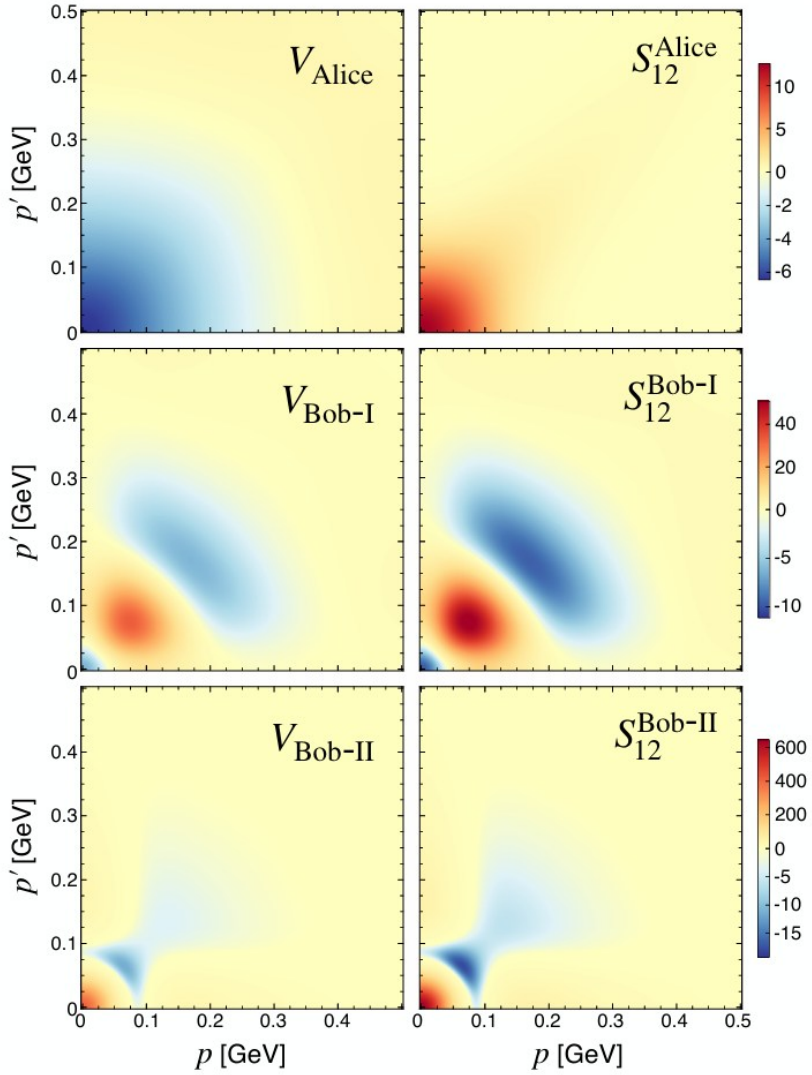
pp, Sqrt[s]=7 TeV

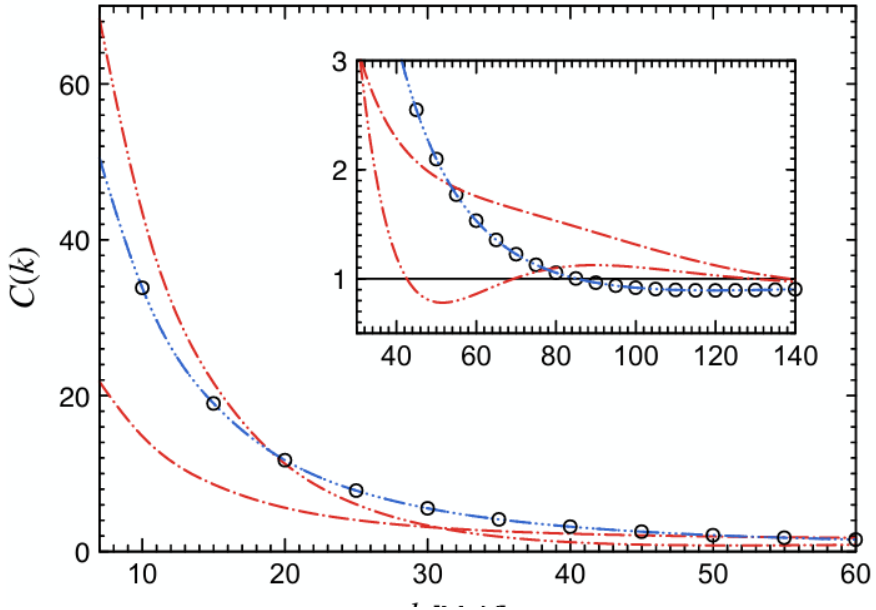
Pb-pb, Sqrt[s]=5.02TeV

Ao-Sheng Xiong, Qi-Wei Yuan, Ming-Zhu Liu, Fu-Sheng Yu, Zhi-Wei Liu, LSG, CPCC50 (2026)081001
Ao-Sheng Xiong, Duo-Lun Ge, Qi-Wei Yuan, Ming-Zhu Liu, Fu-Sheng Yu, Zhi-Wei Liu, and LSG, in preparation

Universal/common source function questioned

Evgeny Epelbaum , Sven Heihoff ,Ulf-G. Meißner , and Alexander Tscherwon1, PRL136 (2026) 21





$$\hat{V}_{\rm Bob} = \hat{U}\left(\frac{\hat{p}^2}{2\mu} + \hat{V}_{\rm Alice}\right)\hat{U}^\dagger - \frac{\hat{p}^2}{2\mu}\,,$$

$$\hat{U} = 1 - 2|g\rangle\langle g|, \quad \langle g|g\rangle = 1\,,$$

$$g(\mathbf{r}) \equiv \langle \mathbf{r}|g\rangle = Cr(1-\beta r)e^{-\alpha r}\,,$$

Off-shell ambiguities

PLB 869 (2025) 139835

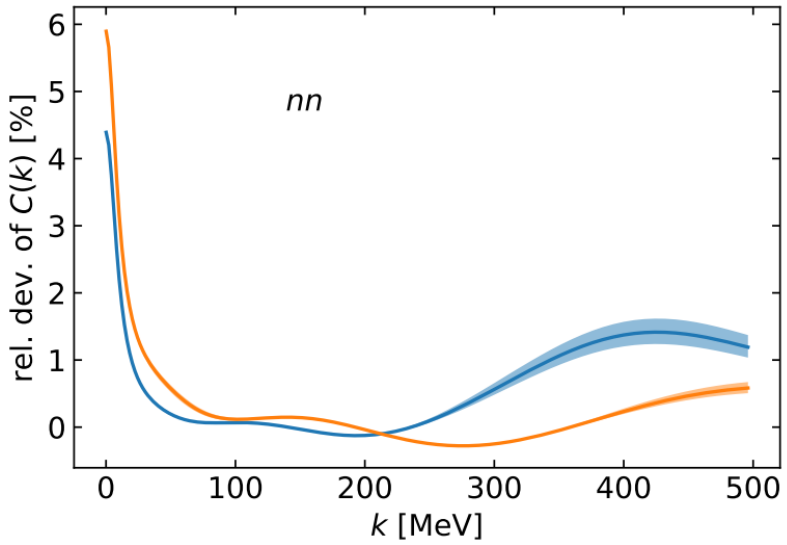
Nucleon-nucleon correlation functions from different interactions in comparison

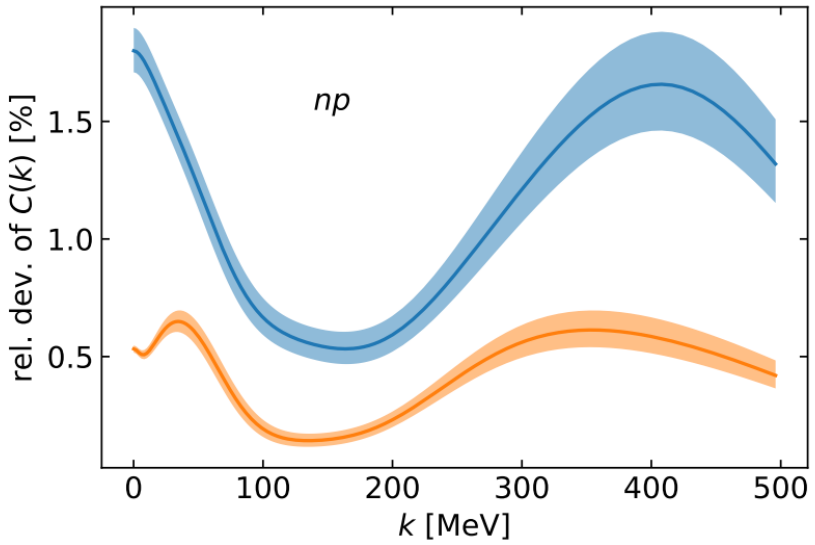
Matthias Göbel [a], Alejandro Kievsky [a]

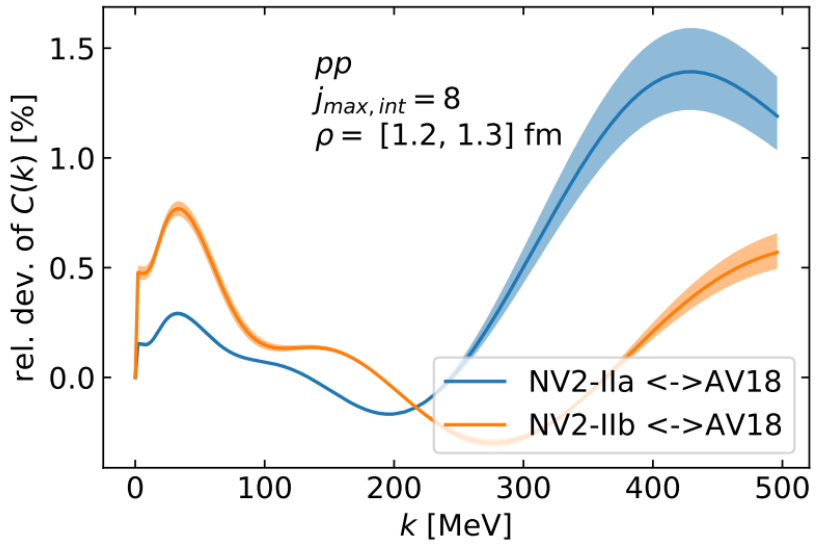
[a] Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy

19.05.2025

"The analysis of the differences between the correlations of the various interactions shows that for momenta between 0 and 500 MeV there are variations of up to 5.9 % for the nn system, of up to 1.8 % for the np system, and of 1.4 % for the pp system."







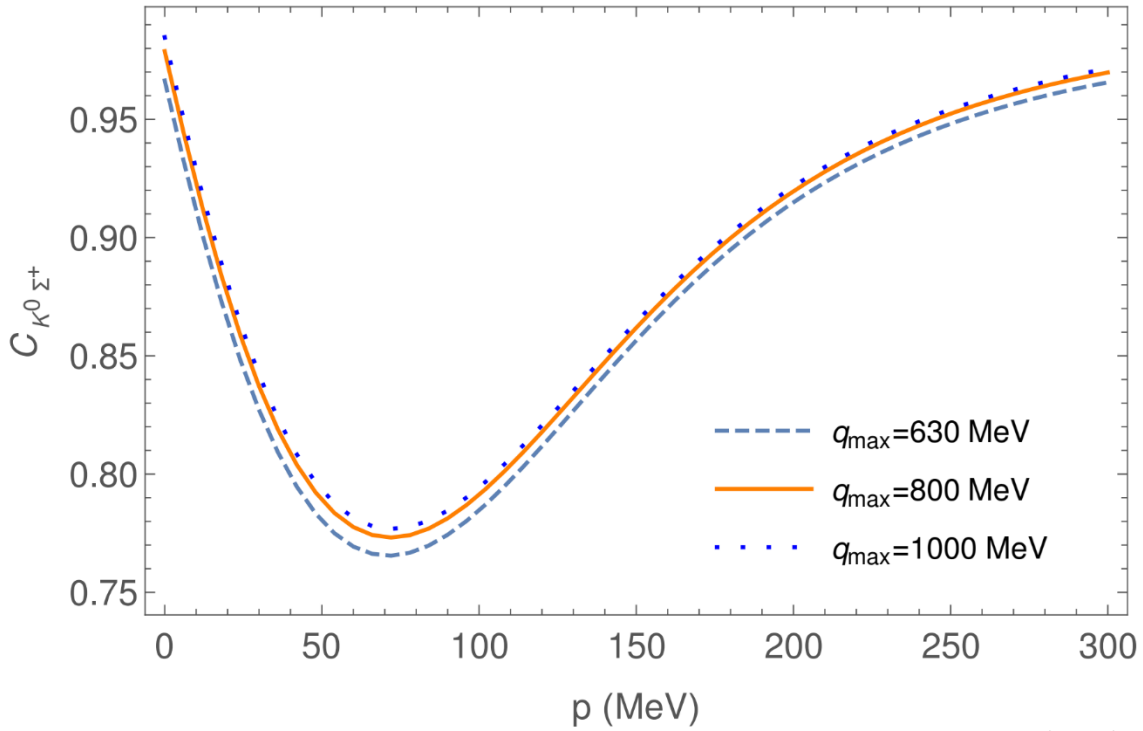
Off-shell ambiguities

Off-shell ambiguities in correlation functions: strategies to minimize them

PRD 112 (2025)096006

R. Molina[1,*] and E. Oset[1,†]

[1] *Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Parc Científic UV, C/ Catedrático José Beltrán, 2, 46980 Paterna, Spain*



"We find that using realistic interactions based on chiral dynamics the uncertainties are small, of the order of 2 - 3 %, but could be much bigger if other methods are used."

$$\mathcal{V}(\sqrt{s};k,k') = V(\sqrt{s})\cdot\theta(q_{\max}-k)\cdot\theta(q_{\max}-k'), \tag{1}$$

$$\mathcal{T}(\sqrt{s};k,k') = T(\sqrt{s})\cdot\theta(q_{\max}-k)\cdot\theta(q_{\max}-k'), \tag{2}$$

$$\psi(r,k) = j_0(kr) + T(\sqrt{s})\cdot\theta(q_{\max}-k)\cdot\int_0^\infty \frac{\mathrm{d}^3k'}{(2\pi)^3}\frac{E_1'+E_2'}{2E_1'E_2'}\frac{\theta(q_{\max}-k')\cdot j_0(k'r)}{s-(E_1'+E_2')^2+i\varepsilon}, \tag{3}$$

Femtoscopic CFs in Density Operator Representation

Hao-Nan Liu, Duo-Lun Ge, Zhi-Wei Liu, Jun-Xu Lu, and LSG, 2609.19161

We formulate CFs at the operator level, with the observed subsystem described by a reduced density operator and subsequent dynamics absorbed into an effective measurement operator

Sudden, equal-time, on-shell approximations:

$$C(\mathbf{q},\mathbf{P}) = \langle \psi_{\mathbf{q}}^{(-)} | \, \hat{\rho}_{\mathbf{P}} \, | \psi_{\mathbf{q}}^{(-)} \rangle \,, \quad \hat{\rho}_{\mathbf{P}} = \int \mathrm{d}^3 r \mathrm{d}^3 r' \, \rho_{\mathbf{P}}(\mathbf{r},\mathbf{r}') \, |\mathbf{r}\rangle \, \langle \mathbf{r}'|$$

Diagonal or decoherence approximation: $\hat{\rho}_{\mathbf{P}} \simeq \int \mathrm{d}^3 r \, S_{\mathbf{P}}(\mathbf{r}) \, |\mathbf{r}\rangle \, \langle \mathbf{r}|$

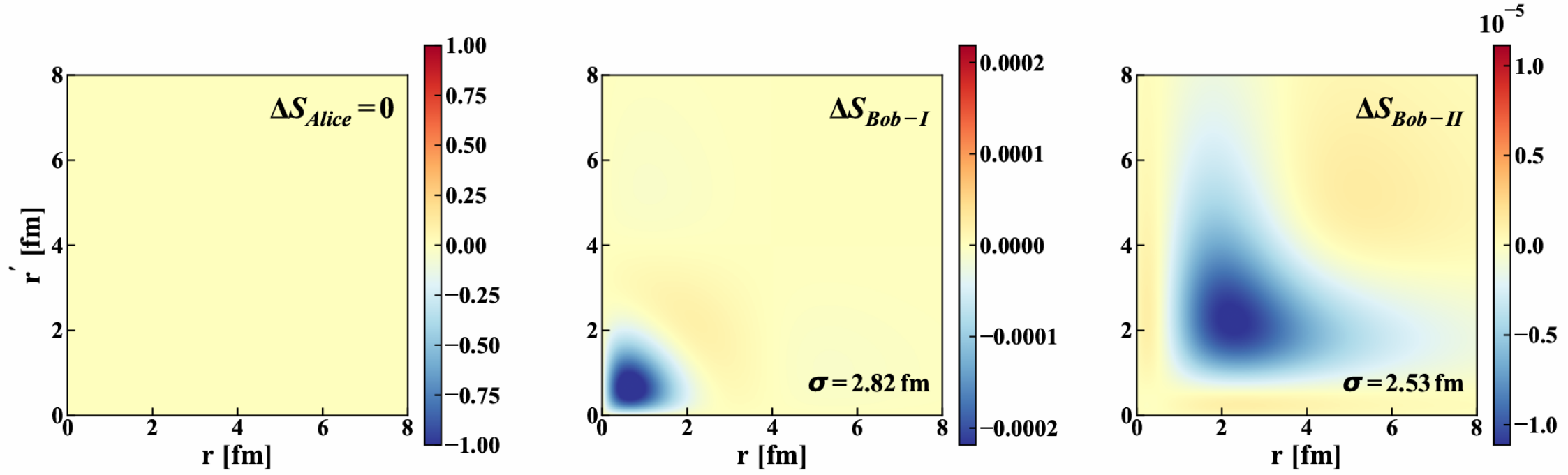
Konin-Pratt formula

$$C(\mathbf{q}) = \int \mathrm{d}^3 r \, S(\mathbf{r}) \, |\psi_{\mathbf{q}}(\mathbf{r})|^2$$

Femtoscopic CFs in Density Operator Representation

Hao-Nan Liu, Duo-Lun Ge, Zhi-Wei Liu, Jun-Xu Lu, and LSG, 2609.19161

- A unitary transformation of the interaction is equivalent to a unitary transformation of the source.
- As a result, one needs to consider the source and interaction simultaneously.
- Calibrating the source using a known interaction chooses a particular reference

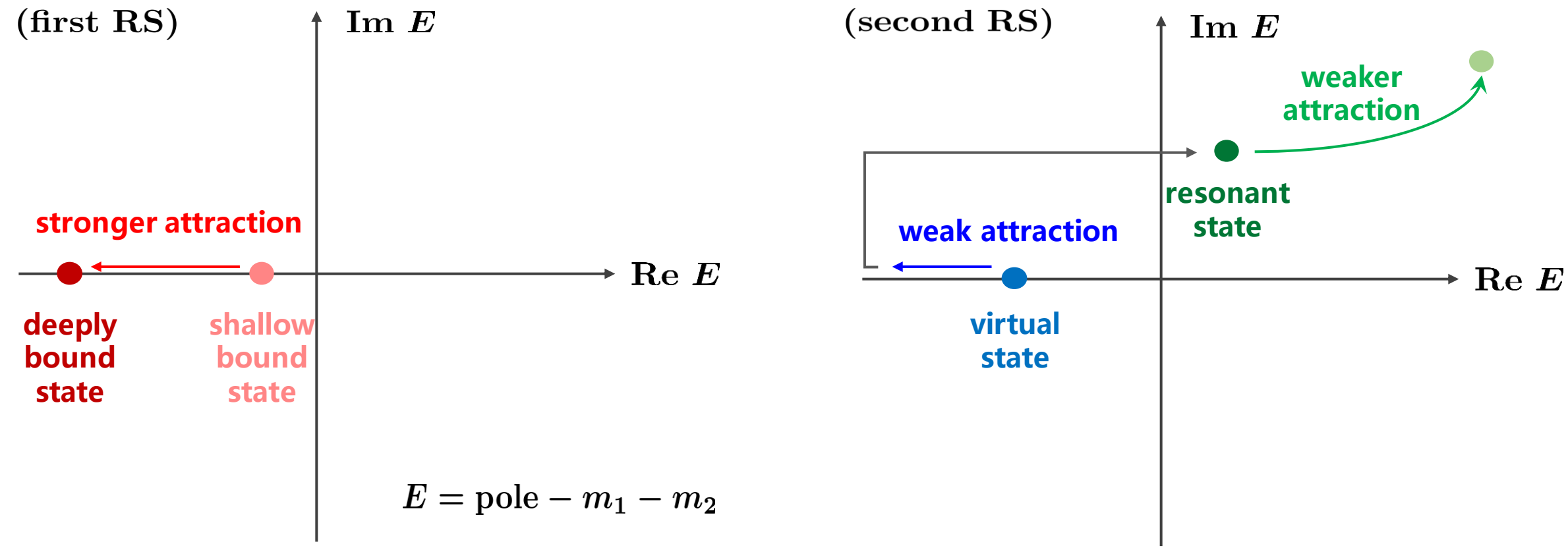


Classification of hadron-hadron interactions

repulsive

attractive

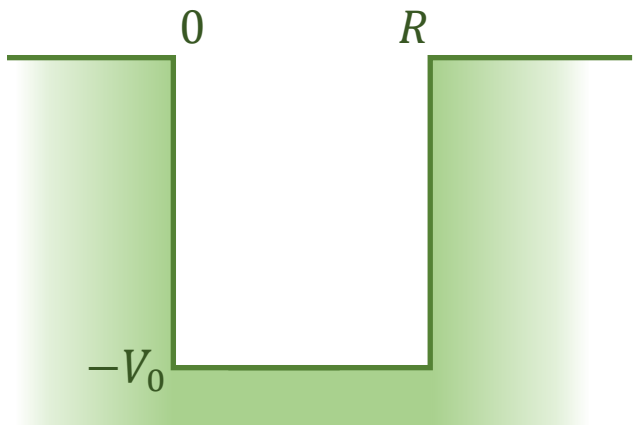
nonexistent



CFs for interactions capable of generating bound states

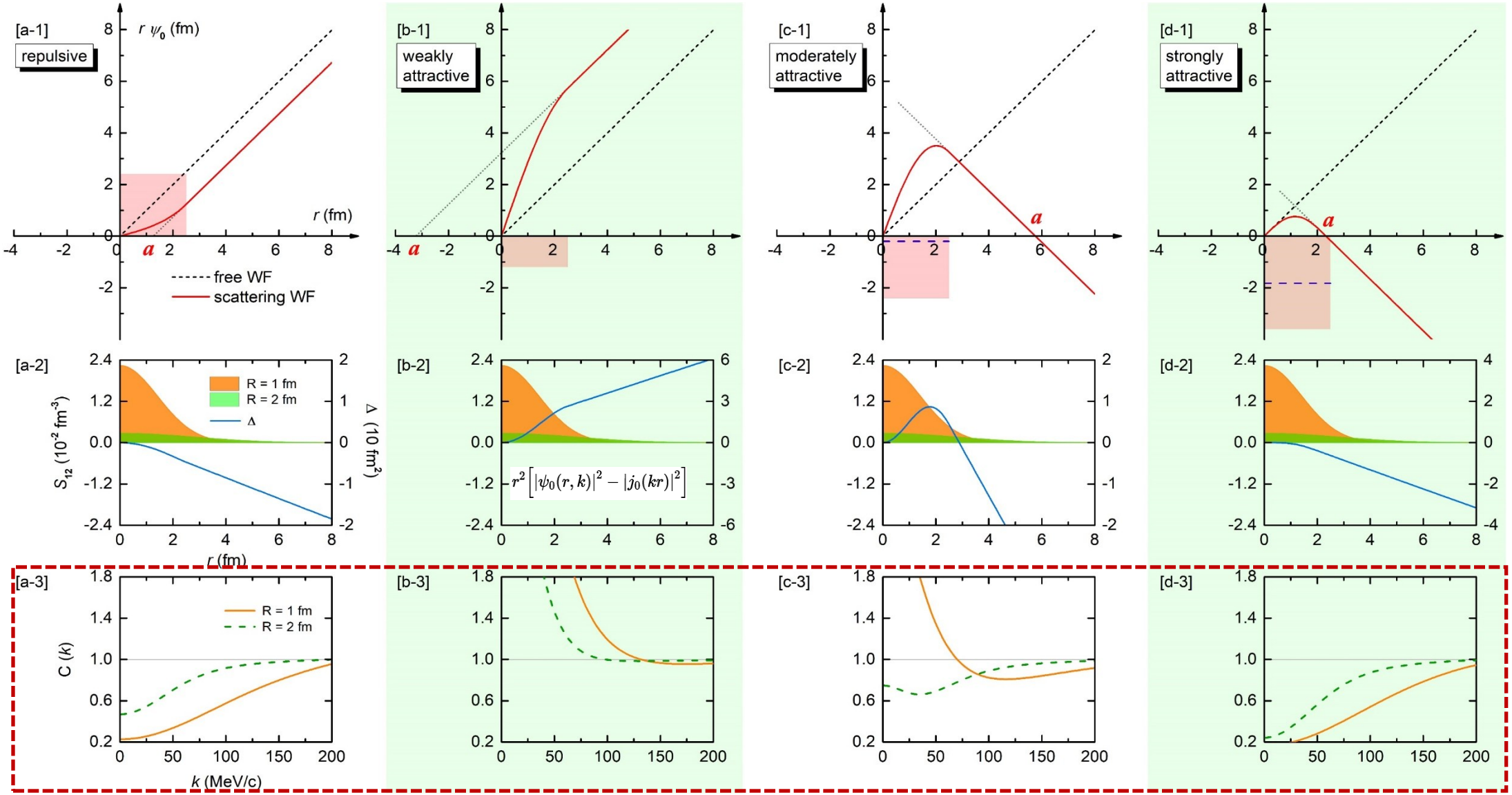
*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD 107, 074019 (2023)*

Spherical potential well—physically intuitive



CFs for interactions capable of generating bound states

*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD 107, 074019 (2023)*



CFs for interactions capable of generating resonant/virtual states

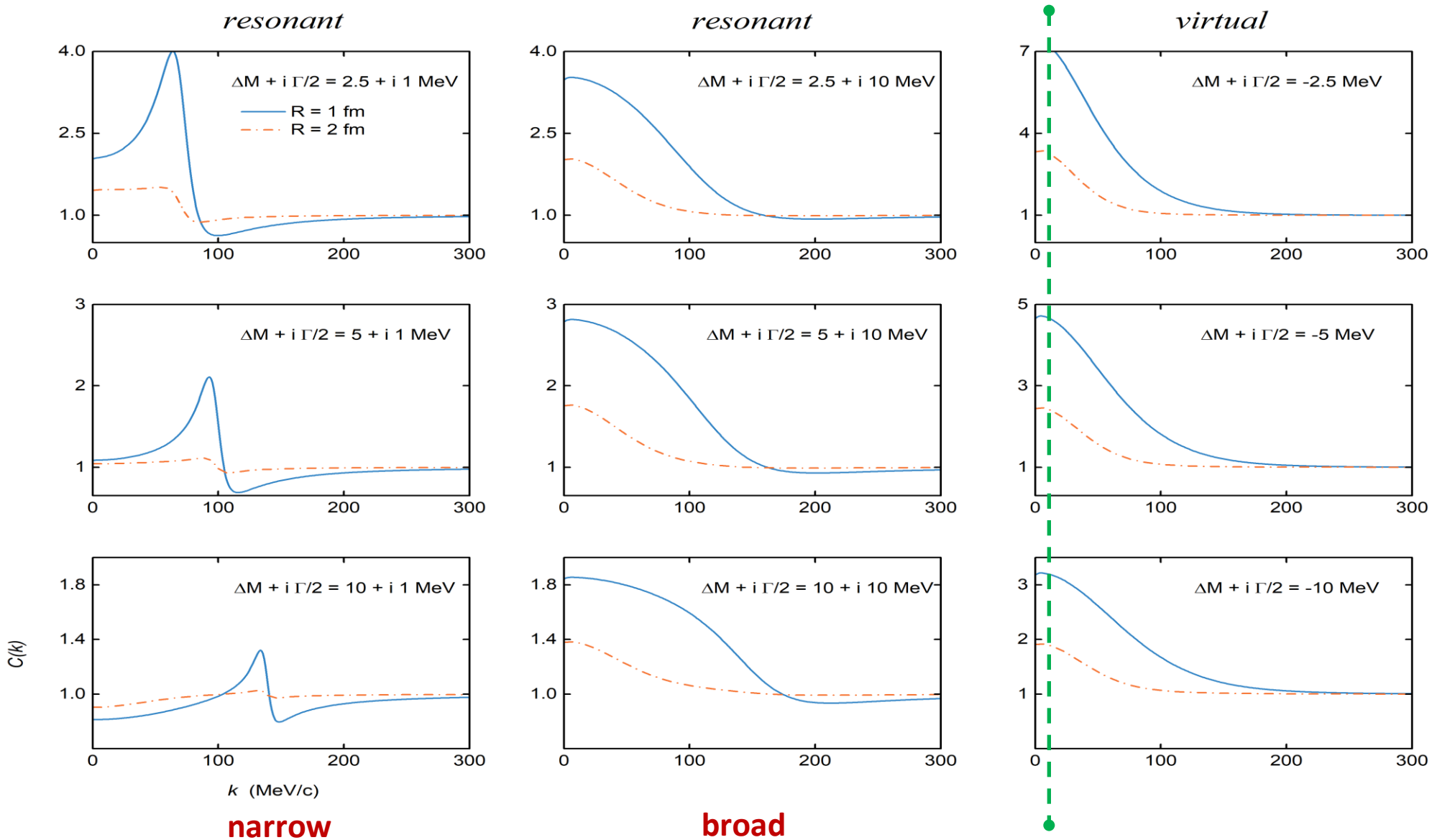
Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and LSG, Sci.Bull. 70 (2025) 3515-3521*

EFT potential—model independent:
capable of generating bound, virtual, and resonant states

$$\mathbf{V} = \mathbf{a} + \mathbf{b} \cdot \mathbf{k}^2$$

CFs for interactions capable of generating resonant/virtual states

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and LSG, Sci.Bull. 70 (2025) 3515-3521*

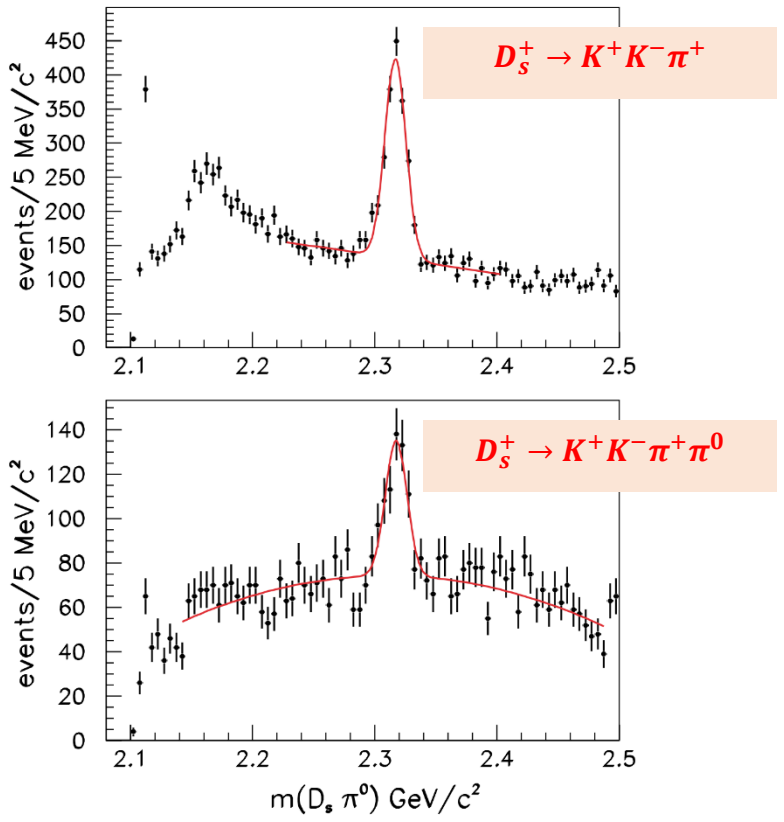


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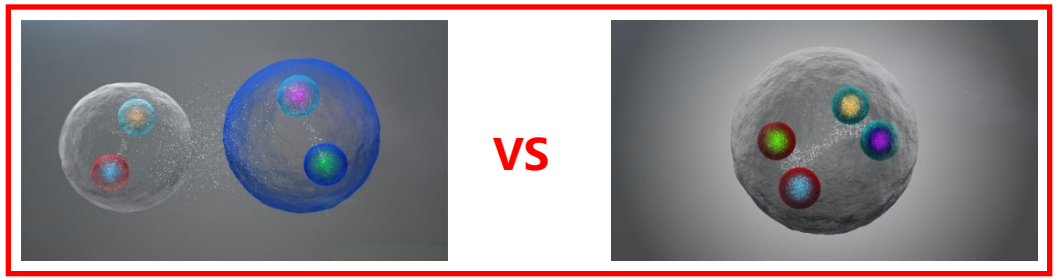
Mysterious exotic hadron $D_{s0}^{*}(2317)$

$\mathrm{M} = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV



BABAR, PRL90 (2003) 242001

- **160 MeV lower than the quark model predictions – difficult to understand as a conventional charm-strange meson**
- **It could be a DK bound state**
  - ✓ E. E. Kolomeitsev 2004
  - ✓ F. K. Guo 2006
  - ✓ D. Gamermann 2007



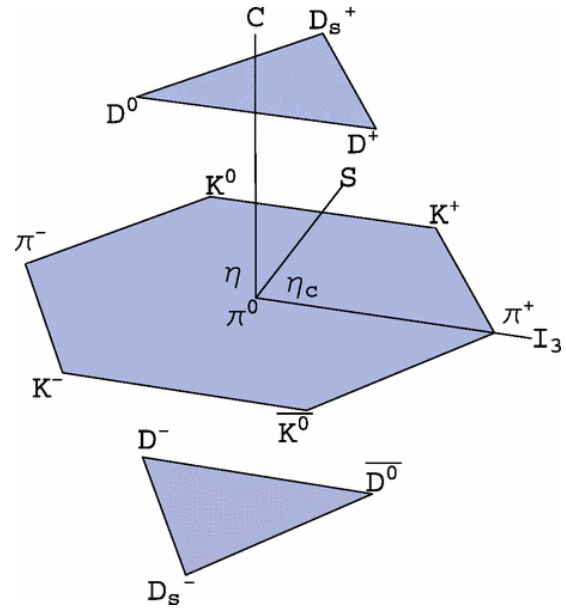
Weinberg-Tomozawa Interaction (leading order)

- **LO interaction between a NGB and a heavy pseudoscalar boson**

$$\mathcal{L} = \frac{1}{4f_\pi^2}\left(\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger\right)$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$P = \left(D^0, D^+, D_s^+\right)$$

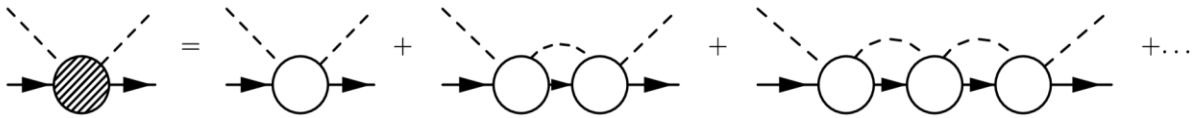


- **Weinberg-Tomozawa (WT) potential – parameter free**

$$V_{\nu'\nu} = \frac{C_{\nu'\nu}}{4f_0^2}\left[(p_1 + p_2)^2 - (p_1 - p_4)\right] \qquad p_{1(3)} = (E_{1(3)}, \boldsymbol{p}^{(\prime)}), \quad p_{2(4)} = (\sqrt{s} - E_{1(3)}, -\boldsymbol{p}^{(\prime)})$$

Scattering wave function

- **Coupled-channel scat. eq.**



$$T_{\nu'\nu}(k',k) = V_{\nu'\nu}\cdot f_{\Lambda_F}(k',k) + \sum_{\nu''}\int_0^\infty \frac{\mathrm{d}k''k''^2}{8\pi^2}\frac{V_{\nu'\nu''}\cdot f_{\Lambda_F}(k',k'')\cdot T_{\nu''\nu}(k'',k)}{E_{P,\nu''}E_{\Phi,\nu''}(\sqrt{s}-E_{P,\nu''}-E_{\Phi,\nu''}+i\epsilon)}$$

$$f_{\Lambda_F}(k',k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2-\left(\frac{k}{\Lambda_F}\right)^2\right]$$

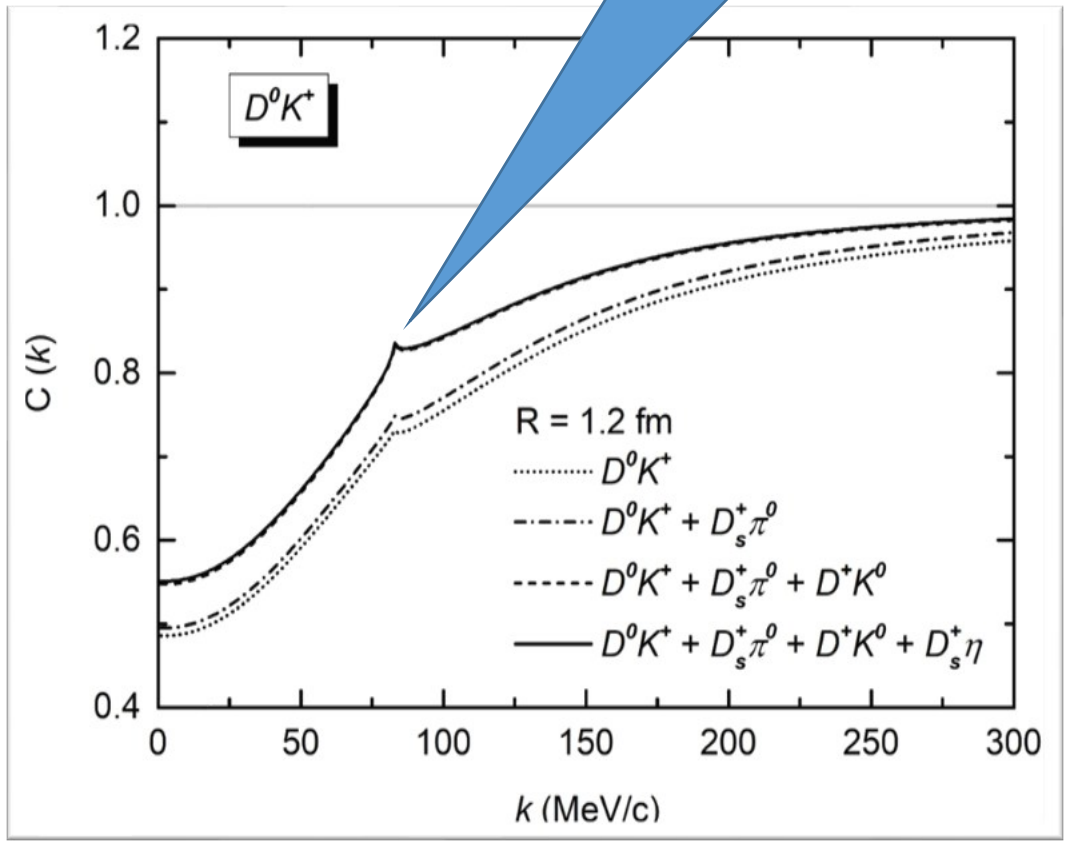
$M_{D_{s0}^*} = 2317.8$ MeV → $\Lambda_F = 1107$ MeV

- **S-wave scattering wave function (including off-shell effect)**

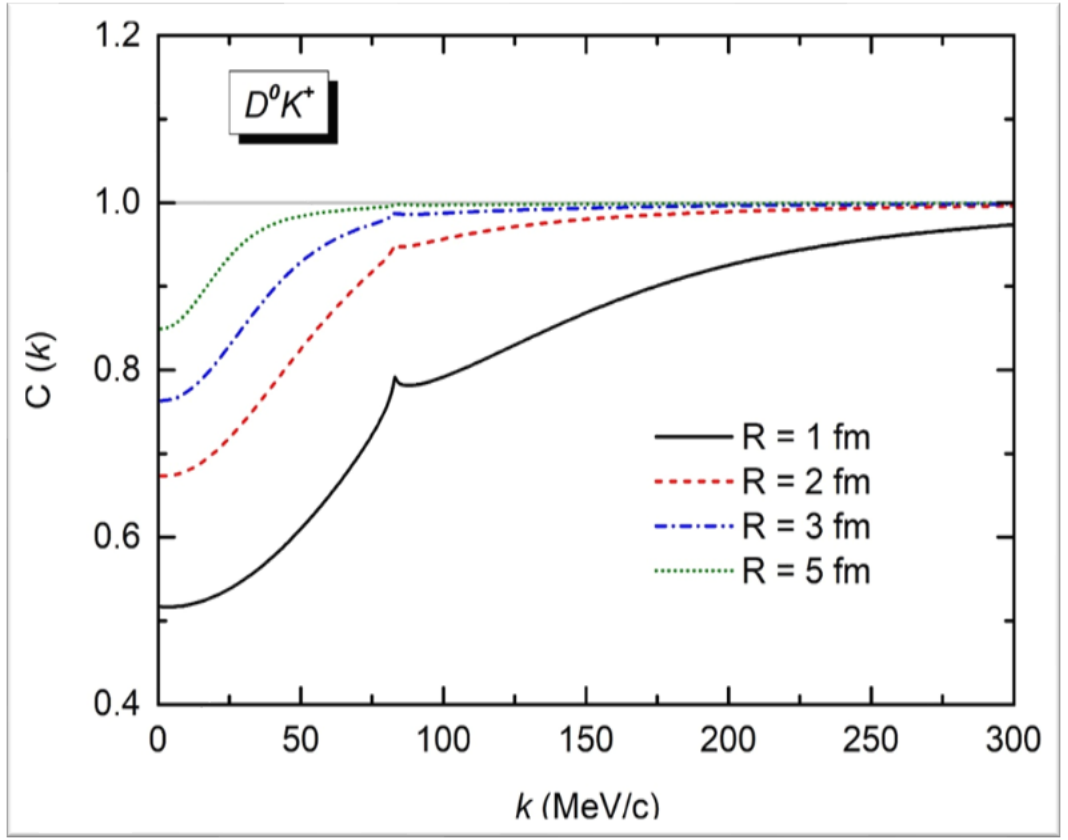
$$\psi_{\nu'\nu}(k,r) = \delta_{\nu'\nu}j_0(kr) + \int_0^\infty \frac{\mathrm{d}k'k'^2}{8\pi^2}\frac{T_{\nu'\nu}(k',k)\cdot j_0(k'r)}{E_{P,\nu'}E_{\Phi,\nu'}(\sqrt{s}-E_{P,\nu'}-E_{\Phi,\nu'}+i\epsilon)}$$

DK CFs and its source size dependence

$D^+K^0 \rightarrow D^0K^+$



Typical feature of deeply bound states

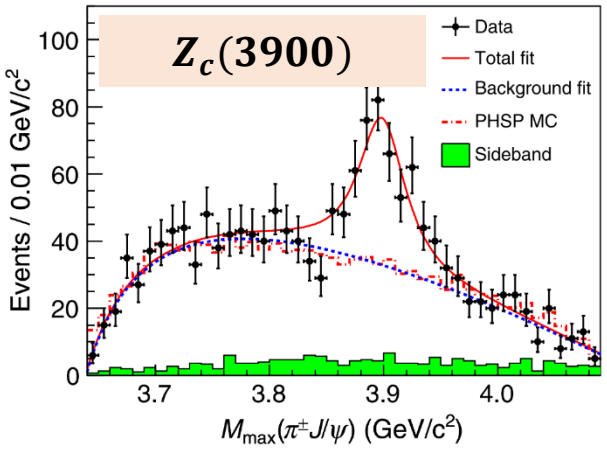


*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD107(2023)074019*

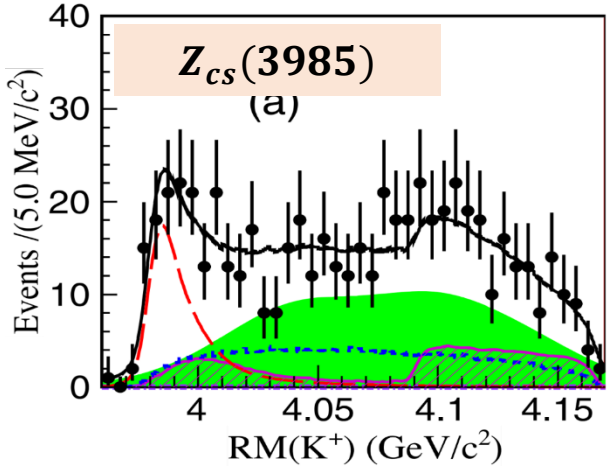
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- ☞ Brief introduction: exotic hadrons and femtoscopy
- ☞ Femtoscopic correlation functions (CFs)—general features
- ☞ Recent applications
  - ➢ $D_{s0}^{*}(2317)$, $Z_c(3900)$ and $Z_{cs}(3985)$, and $Tcc(3875)$
  - ➢ Two interference mechanisms at play in $\pi^{+}p$ CF
- ☞ Summary and outlook

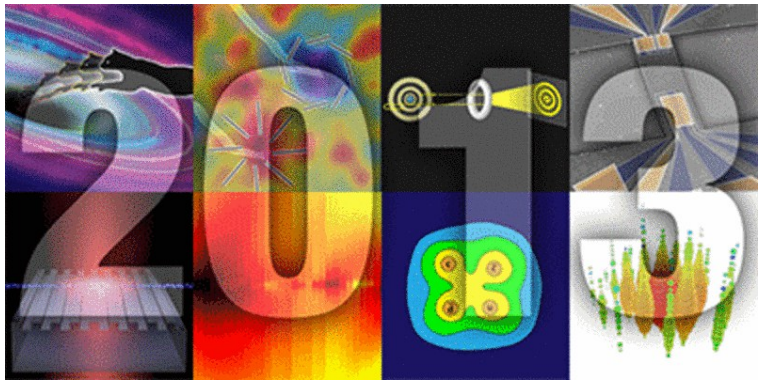
Tetraquark states $Z_c(3900)$ & $Z_{cs}(3985)$



BESIII, PRL110 (2013) 252001



BESIII, PRL126 (2021) 102001



Tetraquark states • 2013 APS Highlights

$Z_c(3900)$ & $Z_{cs}(3985)$: Resonant VS Virtual states

Particle Data Group, PTEP 2022 (2022) 083C01

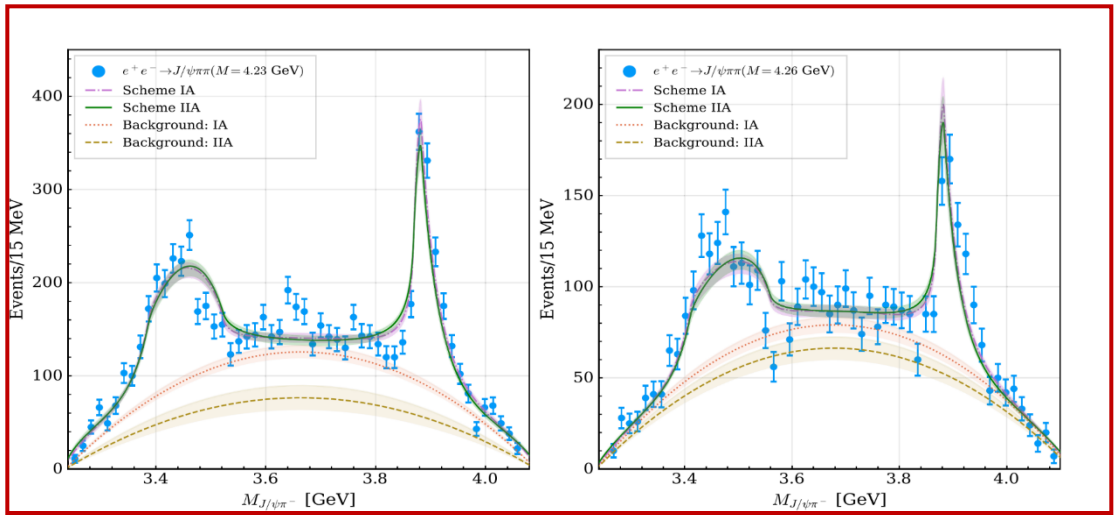
M.-L. Du, M. Albaladejo, F.-K. Guo and J. Nieves, PRD 105 (2022) 074018

T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo and J. Nieves, PRD106 (2022) 094002

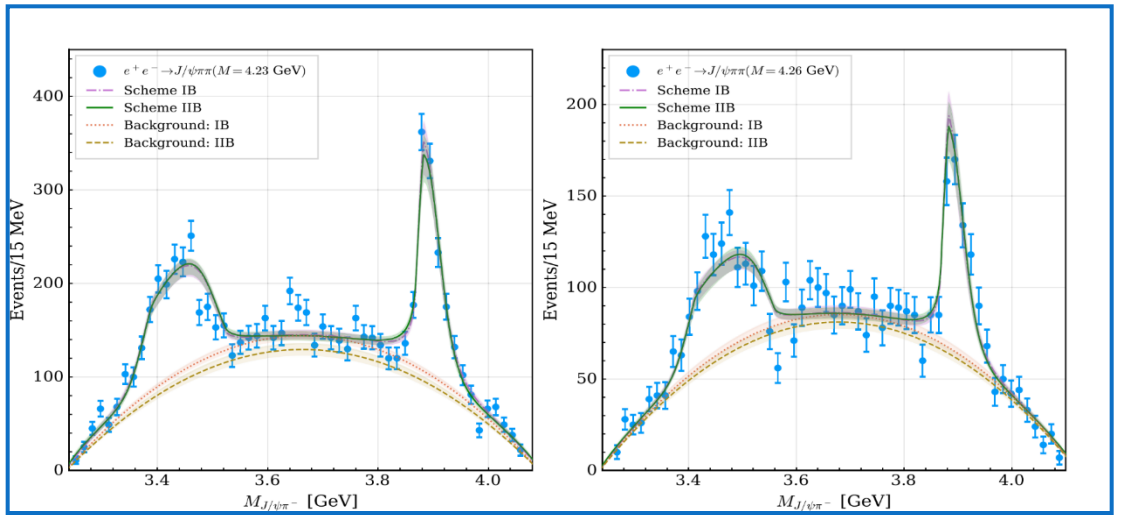
L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao and Z.-Y. Zhou, PRD109 (2024) 014026

Invariant mass distributions fail to distinguish vir. or res.

Virtual state scenario



Resonant state scenario



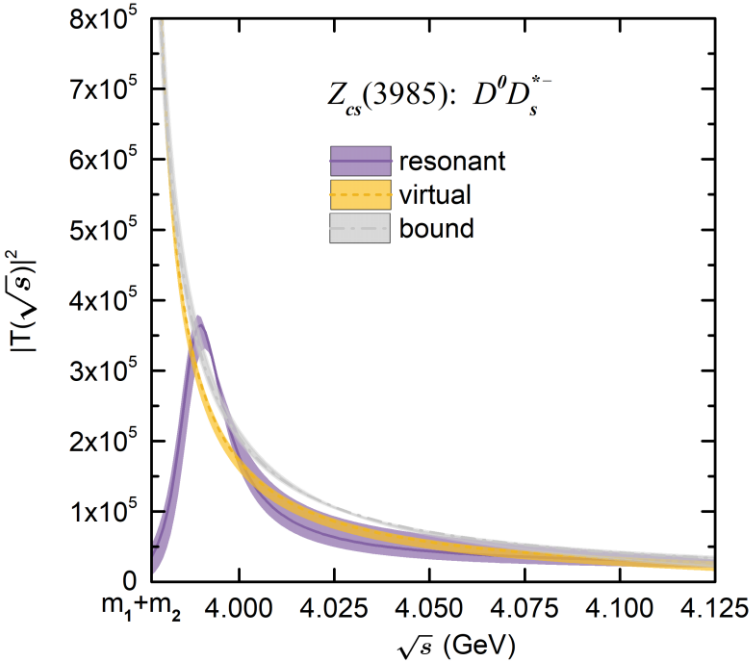
M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

Data are compatible with either a resonant or virtual state.

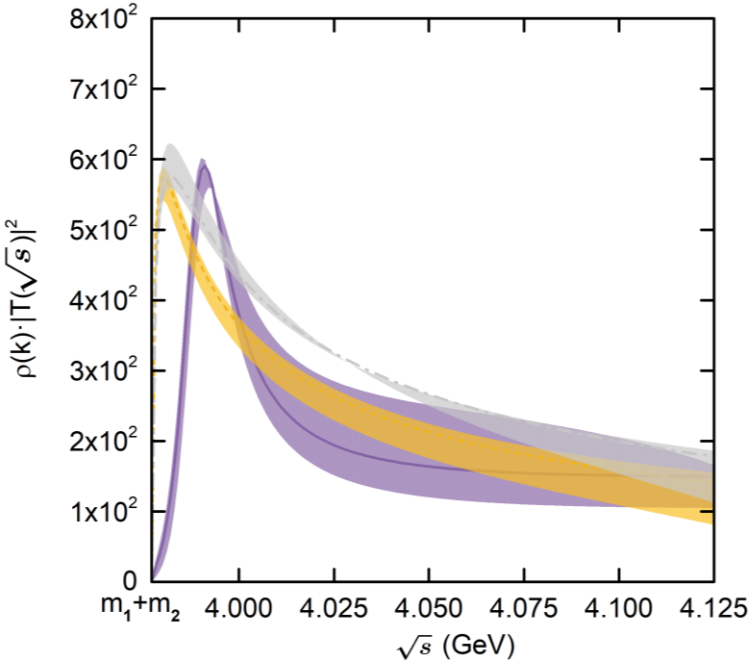
How to tell which is reality?

Phase-space suppression effect

Modulus squared of T-matrix



Line shape of invariant mass spectra



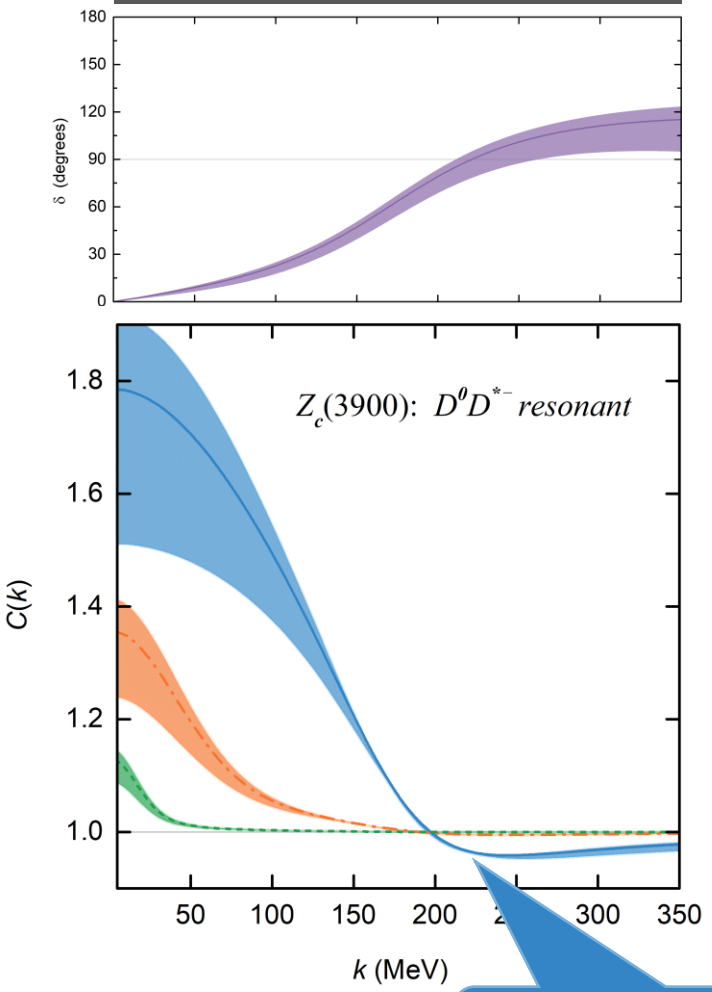
Phase-space suppression makes near-threshold states hard to distinguish using invariant mass spectra alone.

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and LSG, Sci.Bull. 70 (2025) 3515-3521*

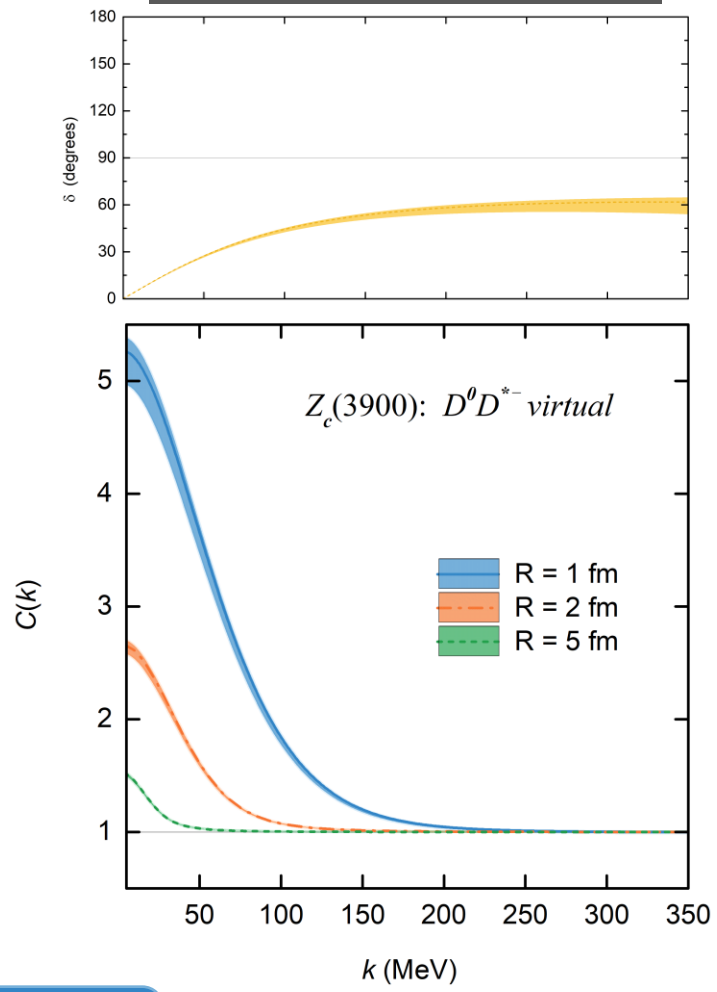
D^0D^{*-} CFs for $Z_c(3900)$

*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, Sci.Bull. 70 (2025) 3515-3521*

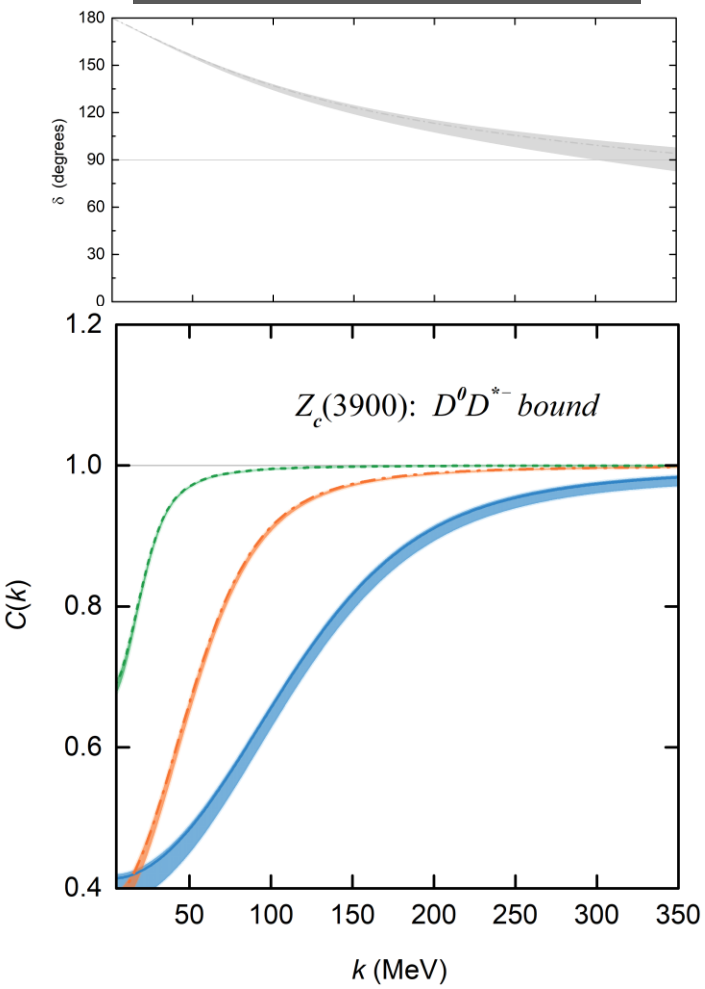
Resonant state scenario



Virtual state scenario



Bound state scenario



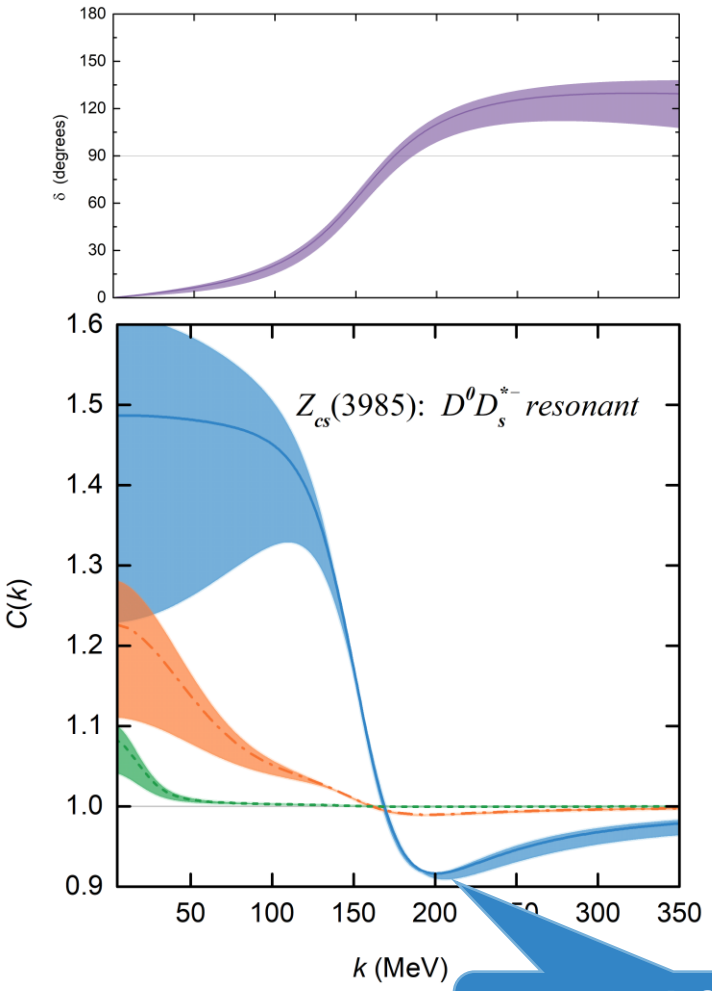
Typical feature of broad resonant state

$$C(k) = 1 + \mathcal{F}_1 \sin^2\delta + \mathcal{F}_2 \sin\delta\cos\delta, \qquad \mathcal{F}_2 > 0$$

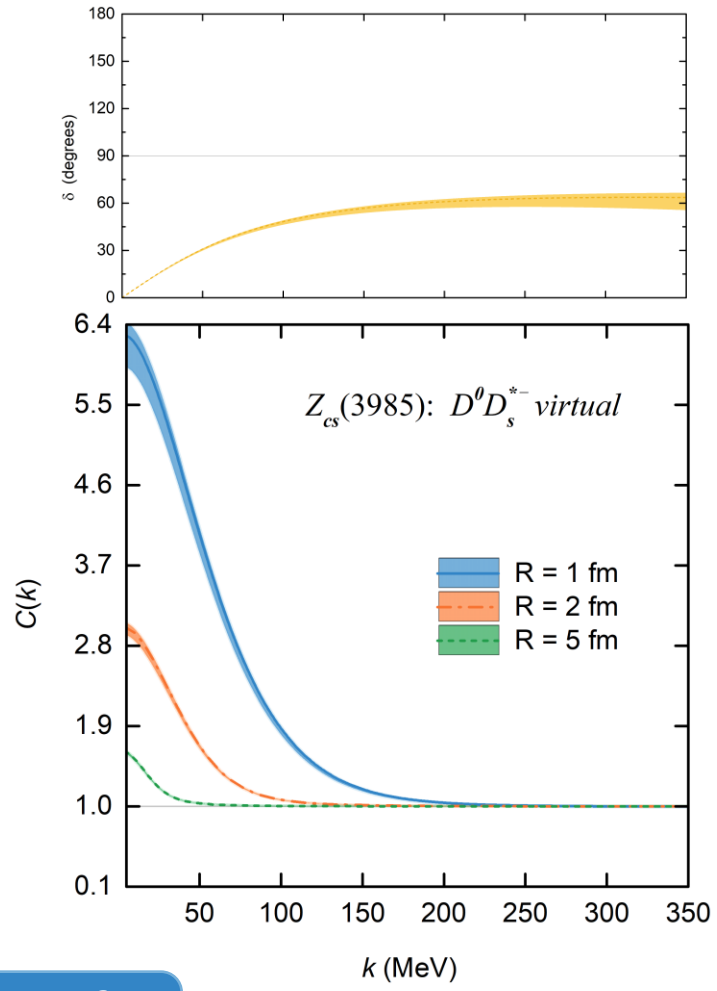
$D^0D_s^{*-}$ CFs $Z_{cs}(3985)$

*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, Sci.Bull. 70 (2025) 3515-3521*

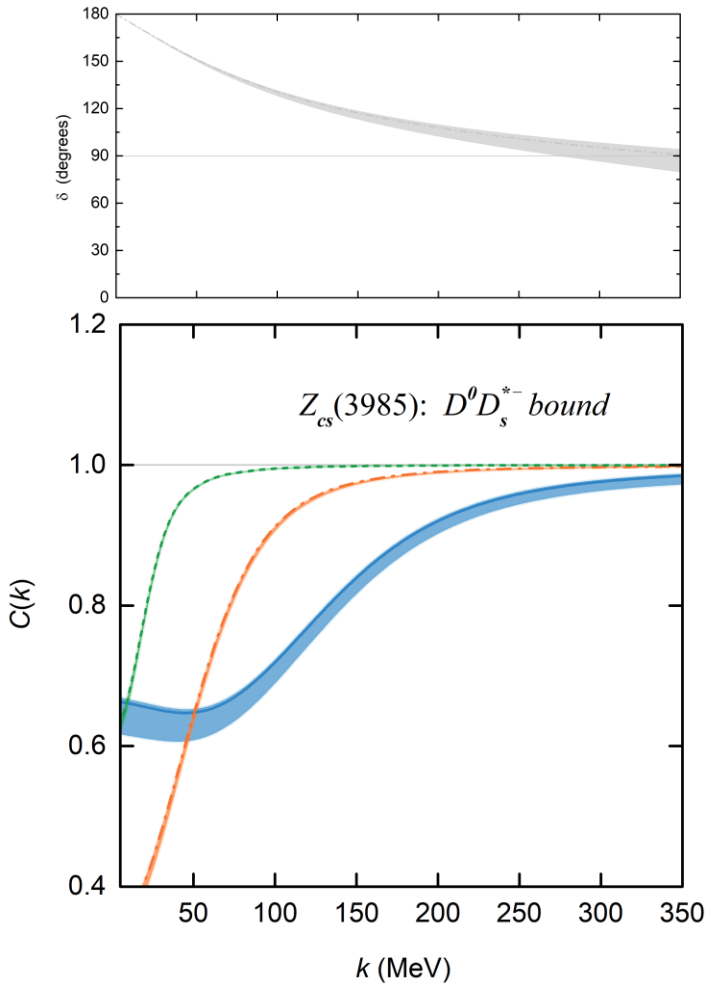
Resonant state scenario



Virtual state scenario



Bound state scenario



Typical feature of narrow resonant state

$$C(k) = 1 + \mathcal{F}_1 \sin^2\delta + \mathcal{F}_2 \sin\delta\cos\delta, \qquad \mathcal{F}_2 > 0$$

A femtoscopic tale of two C-parities: the $Z_c(3900)$ and the isovector partner of the $X(3872)$

Pan-Pan Shi,[1,*] Miguel Albaladejo,[1,†] Feng-Kun Guo,[2,3,4,‡] and Juan Nieves[1,§]

[1]*Instituto de Física Corpuscular (centro mixto CSIC-UV), Institutos de Investigación de Paterna, Apartado 22085, 46071, Valencia*
[2]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100*
[3]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing*
[4]*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Chinese Academy of Sciences, Huizhou 516000, China*

Femtoscopy probes of Zc(3900) and Zcs(3985) and the isovector X(3872) partner

4:50 PM - 5:10 PM

Presenter Pan-Pan Shi

Understanding the nature of the exotic $Z_c(3900)$ and $Z_{cs}(3985)$ states, and search isovector partner W_{c1} of the $X(3872)$, remain central challenges in exotic-hadron spectro the femtoscopic correlation functions (CFs) of the $D^{(*)0}D^{(*)-}_{(s)}$ systems. Since these charm-meson–antimeson pairs are not G-parity eigenstates, their CFs contain contributions from both the C-odd and C-even sectors, providing direct access to the dynamics underlying the Z_c, Z_{cs}, and the predicted isovector exotic W_{c1}. Within a heavy-quark-spin-symmetric coupled-channel framework, we show that the C-even admixture enhances the low-momentum CFs by more than 2.5σ in the vicinity of their thresholds. Free from Coulomb distortions and accessible in high-multiplicity pp collisions at the LHC, these channels offer the first direct femtoscopic probe of the isovector C-even sector and of the elusive W_{c1} state.

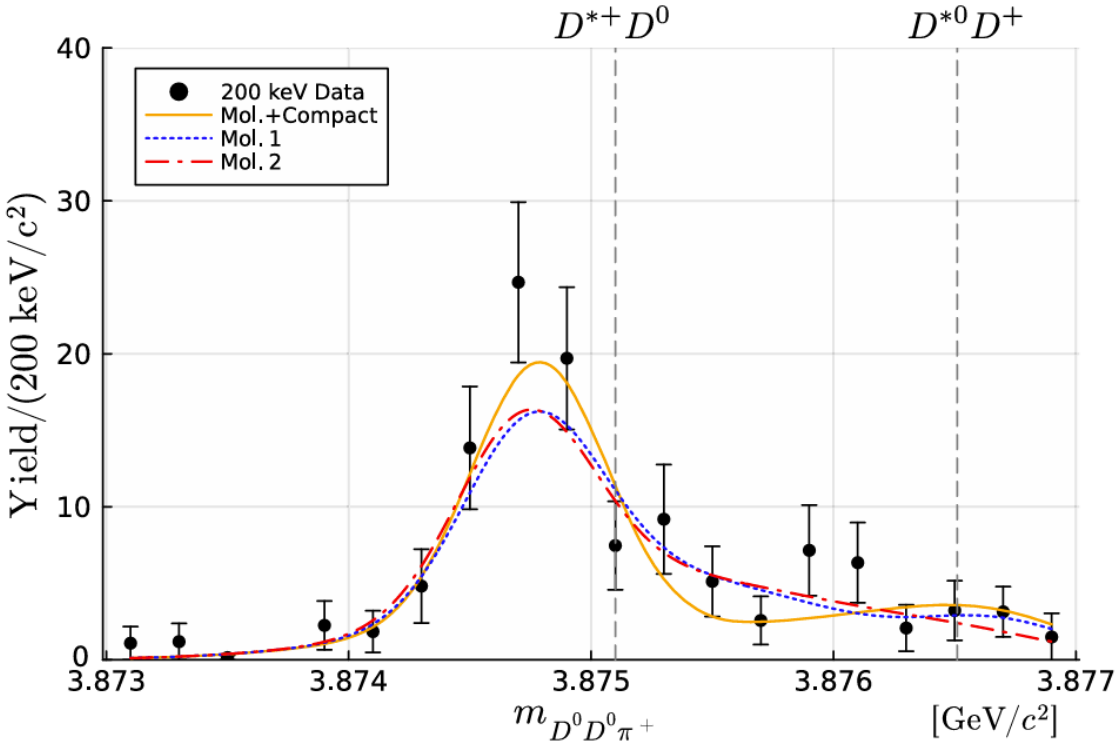
Contents

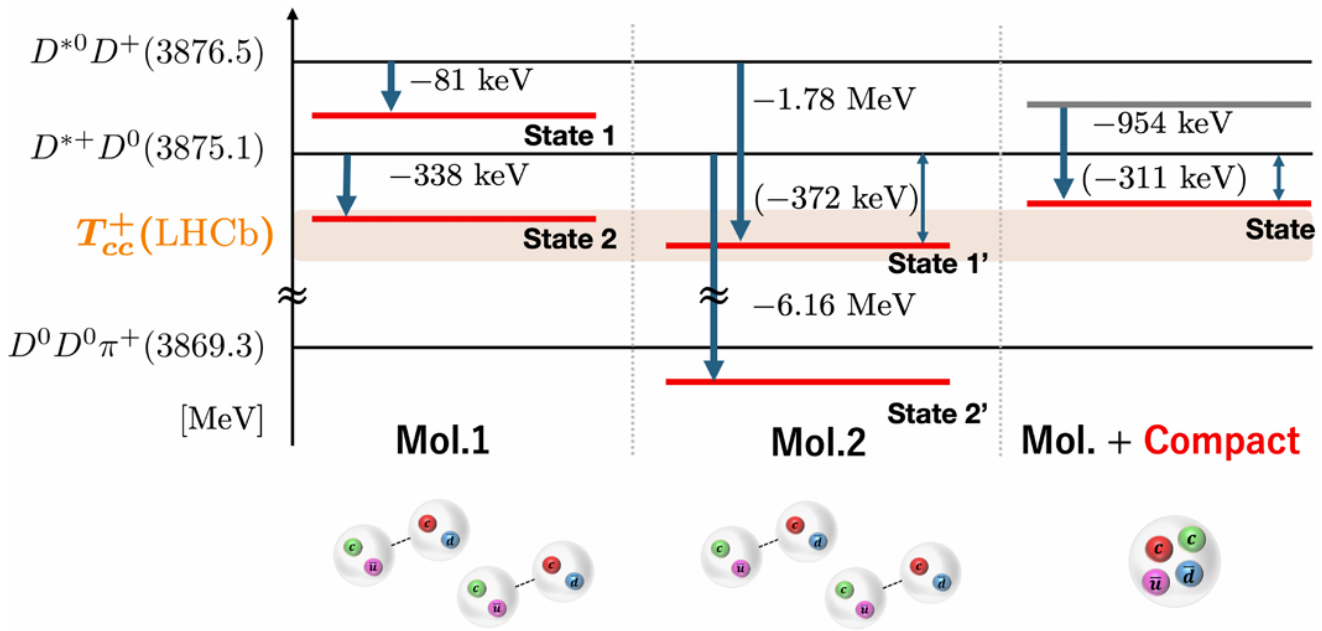
- Brief introduction: exotic hadrons and femtoscopy
- Femtoscopic correlation functions (CFs)—general features
- Recent applications
  - $D_{s0}^{*}(2317)$, $Z_c(3900)$ and $Z_{cs}(3985)$, and $Tcc(3875)$
  - Two interference mechanisms at play in $\pi^{+}p$ CF
- Summary and outlook

More recent applications—Tcc(3875)

Distinguishing the compact versus molecular nature of $T_{cc}(3875)^+$

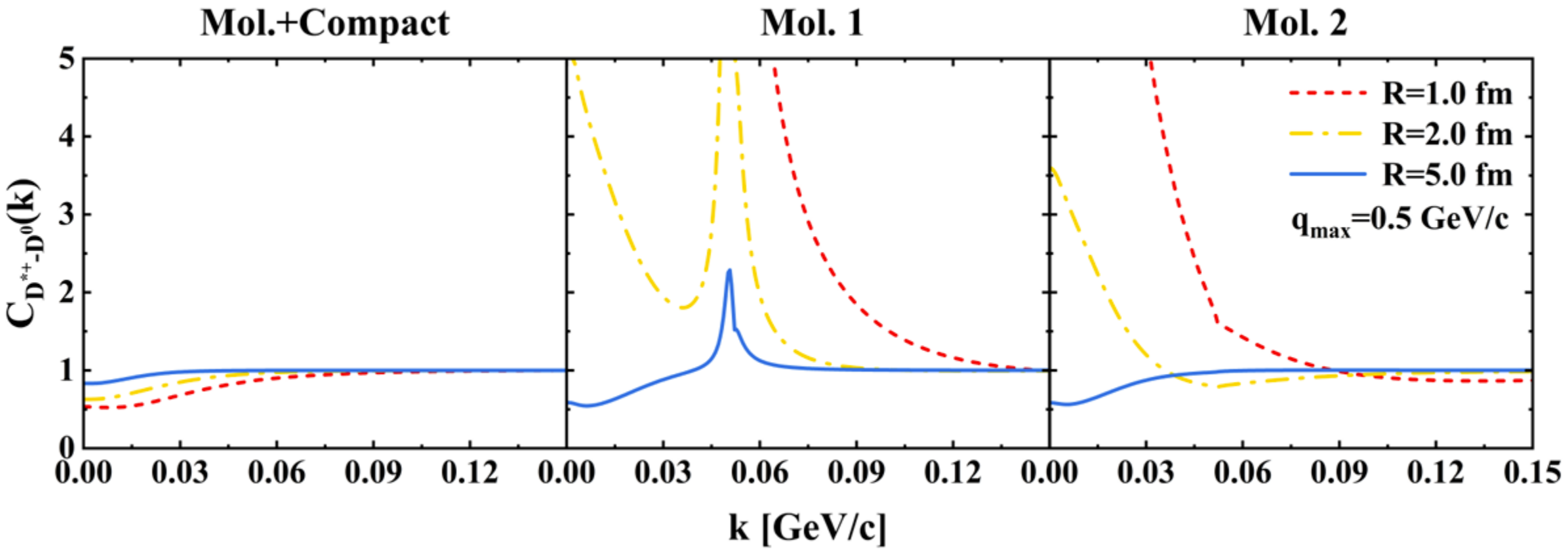
Shota Ampuku, Yasuhiro Yamaguchi, Masayasu Harada, 2511.13003





More recent applications—Tcc(3875)

DD^* correlation functions in deciphering the nature of $T_{cc}(3875)^+$



Duo-Lun Ge, Zhi-Wei Liu, LSG, PRD 114 (2026) 014054*

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Why study the π^+p CF?

☐ π^+p CF as critical input for π^+d femtoscopy studies

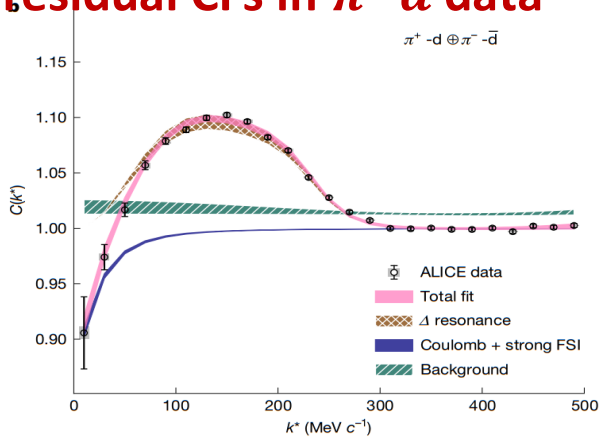
- ➢ Deuteron formation in high-energy hadronic collisions: a long-standing key question
- ➢ Deuteron formation follows resonance decays, evidenced by π^+p residual CFs in π^+d data

Observation of deuteron and antideuteron formation from resonance-decay nucleons

The ALICE Collaboration

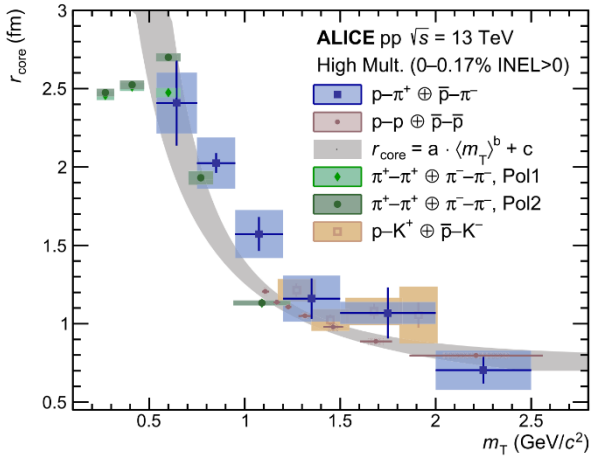
Nature **648**, 306–311 (2025) | Cite this article

25k Accesses | **2** Citations | **282** Altmetric | Metrics



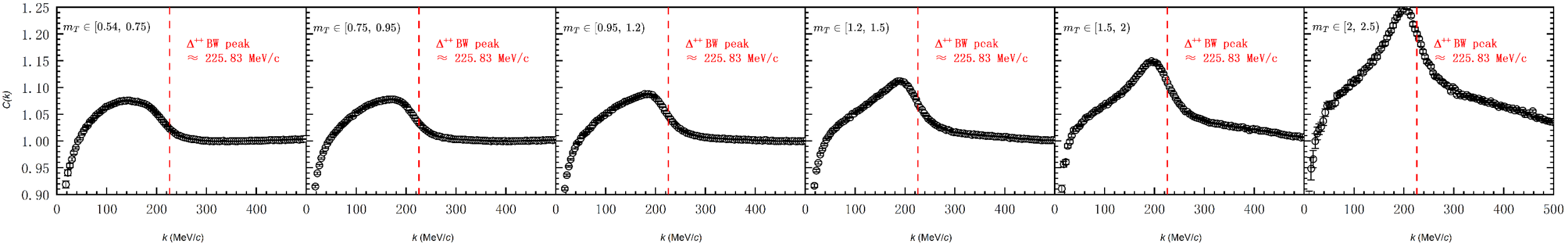
☐ π^+p CF anchors emission-source calibration

- ➢ Abundant πN scattering data allow model-independent source calibration, which lays the groundwork for reliable extractions of other hadron pair interactions.



ALICE data difficult to understand

ALICE Collaboration, EPJA61 (2025) 194



- **Widely used resonance description in Femtoscopy seems to contradict the anomalous $\pi^+ p$ peak shift**

$$C_{\mathrm{res}}(k) = \frac{a\Gamma_R^2}{(\sqrt{s} - M_R)^2 + \Gamma_R^2/4}$$

M_R: **Breit-Wigner mass;** Γ_R: **Breit-Wigner width**

- *Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, and W. Weise, PRL 124 (2020) 132501* **[$\Lambda(1520)$]**
- *ALICE Collaboration, PRC 107 (2023) 054904* **[$\phi(1020)$]**
- *ALICE Collaboration, PLB 845 (2023) 138145* **[excited Ξ]**
- *ALICE Collaboration, PLB 856 (2024) 138915* **[$K^*(892)$]**

Exp.: temperature-dependent spectral function

➢ Basic formalism

ALICE Collaboration, EPJA61 (2025) 194

$$C_{\mathrm{res}}(k) = N_{\Delta^{++}} \times PS(p_{T,\Delta^{++}}, T) \times \mathrm{Sill}(M_{\Delta^{++}}, \Gamma_{\Delta^{++}})$$

$$PS(p_{T,\Delta^{++}}, T) \propto \frac{M_{p\pi^+}}{\sqrt{M_{p\pi^+}^2 + p_{T,\Delta^{++}}^2}} \exp\left[-\frac{\sqrt{M_{p\pi^+}^2 + p_{T,\Delta^{++}}^2}}{T}\right]$$

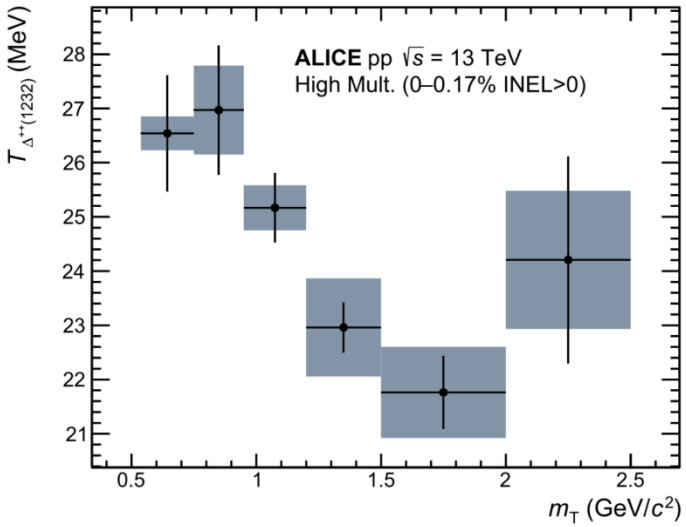
$$\mathrm{Sill}(M_{\Delta^{++}}, \Gamma_{\Delta^{++}}) = \frac{2\sqrt{s}}{\pi} \frac{\sqrt{s - m_{\mathrm{th}}^2}\tilde{\Gamma}_R}{(s - M_R^2)^2 + (\sqrt{s - m_{\mathrm{th}}^2}\tilde{\Gamma}_R)^2} \theta(\sqrt{s} - m_{\mathrm{th}}), \quad \tilde{\Gamma}_R = \frac{\Gamma_R M_R}{\sqrt{s - m_{th}^2}}, \quad m_{th} = M_p + m_{\pi^+}$$

$N_{\Delta^{++}}$: **overall normalization**

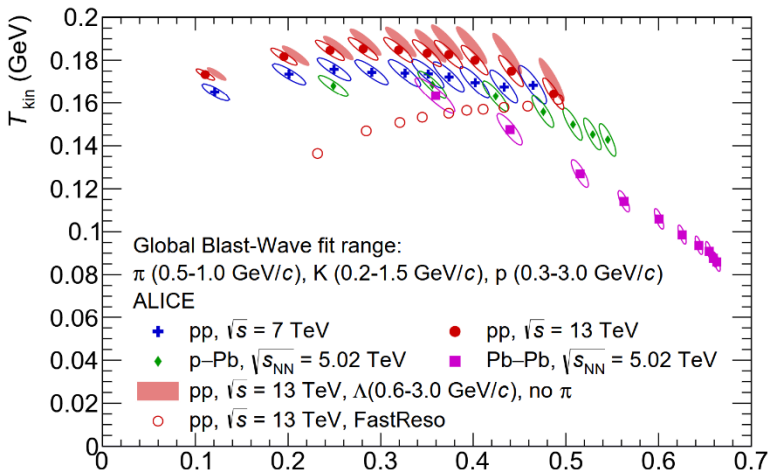
$M_{p\pi^+}$: **invariant mass**

$p_{T,\Delta^{++}}$: **transverse momentum**

➢ Extracted temperature contradicts blast-wave freeze-out



≠



ALICE Collaboration, Eur. Phys. J. C 80 (2020) 693

Theo. 1: relativistic kinetic equations

➢ Basic formalism

L.-Y. Zhang, C. M. Ko, Y.-G. Ma, Q.-Y. Shou, K.-J. Sun, R. Wang, and S. Zhang, 2511.10298

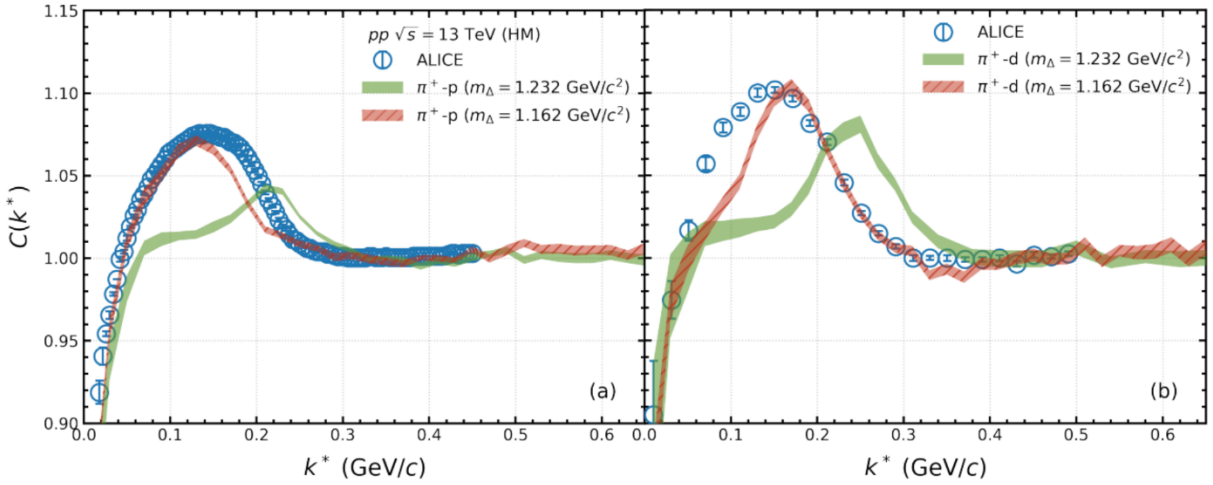
$$\frac{\partial f_d}{\partial t} + \frac{\boldsymbol{P}}{E_d} \cdot \frac{\partial f_d}{\partial \boldsymbol{R}} = -\mathcal{K}^{>} f_d + \mathcal{K}^{<}(1 + f_d)$$

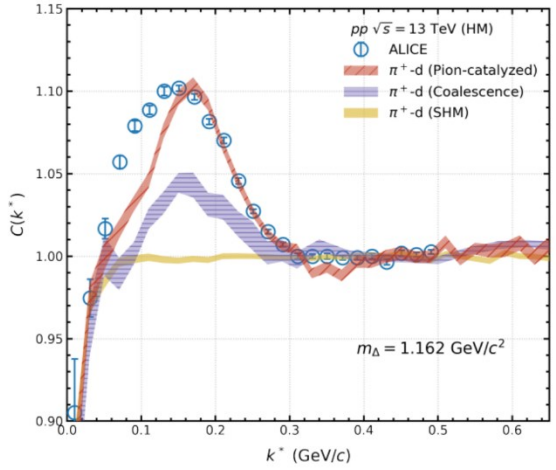
f_d: **phase space distribution**
$K^{>}$ $(K^{<})$: **dissociation and regeneration rates**

$$C(k) = \mathcal{N} \frac{\sum_{\text{pairs}}^{\text{same}} \delta(\boldsymbol{k}_{\text{pair}} - \boldsymbol{k}) |\Psi_k(\boldsymbol{r})|^2}{\sum_{\text{pairs}}^{\text{mixed}} \delta(\boldsymbol{k}_{\text{pair}} - \boldsymbol{k})}$$

$\Psi_{\mathbf{k}}(\mathbf{r})$: **Bethe-Salpeter amplitude**

➢ Freeze-out require a ≈70 MeV/c² mass reduction of the $\Delta(1232)$ resonance





Theo. 2: Friedrichs-Lee model T-matrix

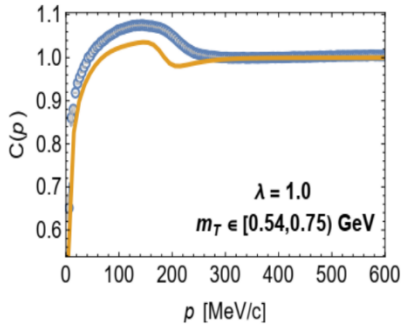
- **Basic formalism** *L. Zhang, T. Shao, S. Zhang, K.-J. Sun, and Y.-G. Ma, (2026), 2606.08230*

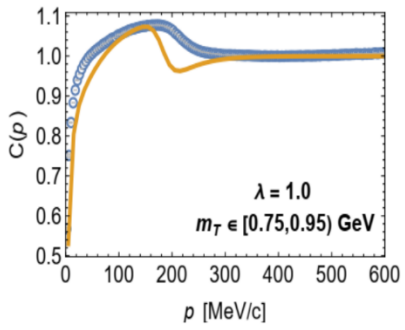
$$C(p) = \int d^3k d^3k' \langle \psi_p^{(+)}|k\rangle\langle k|\hat{S}|k'\rangle\langle k'|\psi_p^{(+)}\rangle + \langle \psi_p^{(+)}|R\rangle\langle R|\hat{S}|R\rangle\langle R|\psi_p^{(+)}\rangle$$

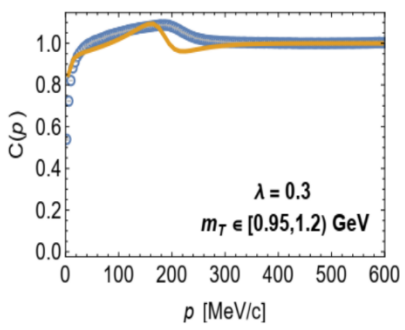
$$\langle k|\psi_p^{(+)}\rangle = \delta^{(3)}(k-p) + G^{(+)}(k;E)T(k,p;E), \qquad T(k,p;E) = \frac{g(k)g^*(p)}{E - E_0 - \Sigma_0(E) + i\epsilon}$$

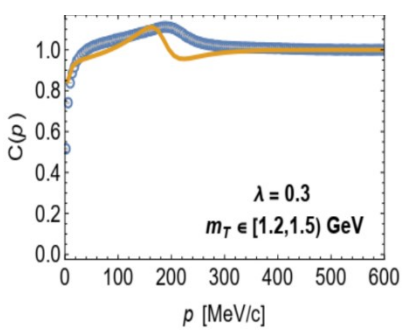
$$\langle \psi_p^{(+)}|R\rangle = g(p)\frac{E - E_0}{E - E_0 - \Sigma_0(E) + i\epsilon}$$

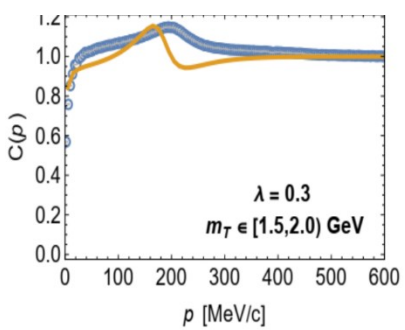
- **Underestimates strength, predicts dip absent**

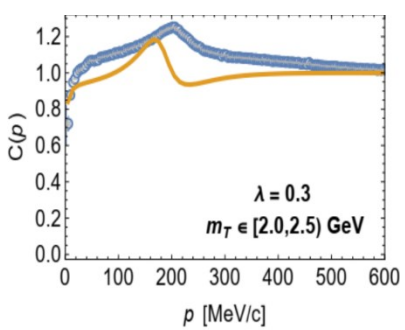












A new attempt

Zhi-Wei Liu, Jun-Xu Lu, Duo-Lun Ge, LSG, 2607.04351*

Quantum interference effects enhanced in $\pi^+ p$ femtoscopic correlation functions

Zhi-Wei Liu (Michigan U., Math. Dept. and Shenzhen U.), Jun-Xu Lu (Beihang U.), Duo-Lun Ge (Beihang U.), Li-Sheng Geng (Beihang U. and Beijing, CSRC and Hui Zhou, Inst. Modern Phys.) (Jul 5, 2026)

e-Print: 2607.04351 [hep-ph]

1. **Model-independent treatment of the $\pi^+ \mathrm{p}$ interaction**
2. **Consistent treatment of two mechanism: scattering and decay**
3. **Economical fitting—only four parameters**

Strong interaction

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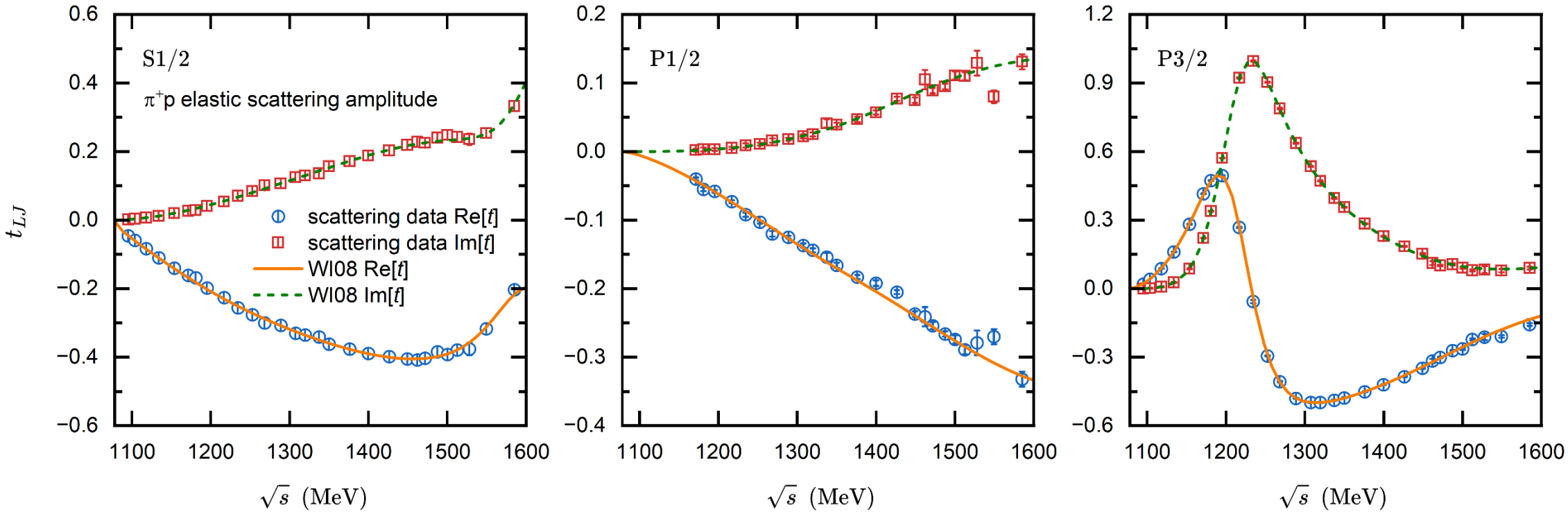
control off-shell behavior

$$T^{\rm str}_{LJ}(p,p') = -1/\rho \cdot t_{LJ} \cdot \theta(\Lambda_F - p)\theta(\Lambda_F - p'),\ \ \Lambda_F \in [0.6, 1.4]{\rm GeV}/c$$

$$\rho = 2Mk/(8\pi\sqrt{s})$$

$$k = \lambda^{1/2}(s, m^2, M^2)/(2\sqrt{s})$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$



Coulomb interaction

Fourier-transforming the r-space Coulomb potential into p-space

$$V^{\rm cou}(|\boldsymbol{p'}-\boldsymbol{p}|)=\int_0^{|\boldsymbol{r}|<R_c} {\rm d}\boldsymbol{r}\ e^{{\rm i}(\boldsymbol{p'}-\boldsymbol{p})\cdot\boldsymbol{r}}\frac{\epsilon\alpha}{r}=\frac{4\pi\epsilon\alpha}{|\boldsymbol{p'}-\boldsymbol{p}|^2}\left[1-\cos(|\boldsymbol{p'}-\boldsymbol{p}|R_c)\right], \qquad R_c=60\ {\rm fm}$$

Partial Wave Projection

$$V_L^{\rm cou}(p,p')=\frac{1}{2}\int_{-1}^{1}{\rm d}\cos\theta\ V^{\rm cou}(q)P_L(\cos\theta), \qquad q^2=p^2+p'^2-2pp'\cos\theta$$

Relativistic correction (relativistic propagator & relativistic normalization convention)

$$T_L^{\rm cou}(p,p')=\sqrt{\frac{2E_M(p)2E_m(p)\xi(p)}{2M}}\sqrt{\frac{2E_M(p')2E_m(p')\xi(p')}{2M}}V_L^{\rm cou}$$

where $\xi(p)=[\sqrt{s}-E_M(p)-E_m(p)]/[k^2/(2\mu)-p^2/(2\mu)]$

CF from the $\pi^+ p$ scattering

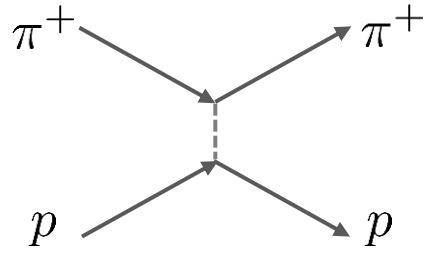
Outgoing wave function

$$\mathcal{R}_{LJ}(k,r) = \int_0^{|\boldsymbol{k'}|<\Lambda_F} \mathrm{d}\boldsymbol{k'}\ \mathcal{G}(k')\ T_{LJ}(k,k')\ j_L(k'r)$$

where $T_{LJ}(k,k') = T_{LJ}^{\mathrm{str}}(k,k') + T_L^{\mathrm{cou}}(k,k'),$ $\mathcal{G}(k') = \dfrac{2M}{(2\pi)^3}\dfrac{E_M+E_m}{2E_M E_m}\dfrac{1}{s-(E_M+E_m)^2+\mathrm{i}\varepsilon}$

Correlation function from the $\pi^+ p$ scattering process

$$C_{\mathrm{sca}}(k) = \underbrace{C_{\mathrm{S1/2}} + C_{\mathrm{P1/2}} + C_{\mathrm{P3/2}}}_{\text{strong + coulomb}} + \underbrace{C_{\mathrm{hpw}}}_{\text{coulomb only}}$$

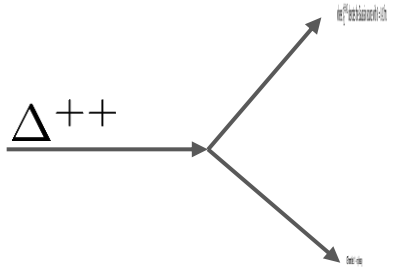


where $C_{LJ}(k) = \omega_{LJ}\int_0^\infty \mathrm{d}\boldsymbol{r}\ S_{12}\ (2L+1)|j_L+\mathcal{R}_{LJ}|^2,$ $\omega_{LJ} = (2J+1)/[2(2L+1)]$

$$C_{\mathrm{hpw}}(k) = \int_0^\infty \mathrm{d}\boldsymbol{r}\ S_{12}\ \left(|\Phi^{\mathrm{cou}}|^2 - \sum_{L=0,1}|\Phi_L^{\mathrm{cou}}|^2\right)$$

CF from the $\Delta^{++} \to \pi^+ p$ decay

Correlation function from the $\Delta^{++} \to \pi^+ p$ decay process



$$C_{\rm dec}(k) = \frac{dN_{\rm BW}}{dm_{12}} \frac{dm_{12}}{dk}, \qquad \frac{dN_{\rm BW}}{dm_{12}} = \frac{\Gamma^2_{\Delta^{++}}/4}{(\sqrt{s} - M_{\Delta^{++}})^2 + \Gamma^2_{\Delta^{++}}/4}.$$

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, and W. Weise,Phys. Rev. Lett. 124, 132501 (2020)

Complete CF

$$C(k) = C_{\mathrm{bg}}[1 + \lambda Q(C_{\mathrm{sca}} - 1)] + \lambda P C_{\mathrm{dec}},$$

- ✓ Q, P : weighting factors for the scattering and decay processes
- ✓ λ: misidentification and feed-down effects
- ✓ C_{bg}: experimental background (fitted)

$$C_{\mathrm{bg}} = aC_{\mathrm{C}} + (1-a)C_{\mathrm{NC}} + \sum_{i=0}^{3} a_i k^i,$$

S. Acharya et al. (ALICE), Eur. Phys. J. A 61, 194 (2025),arXiv:2502.20200 [nucl-ex].

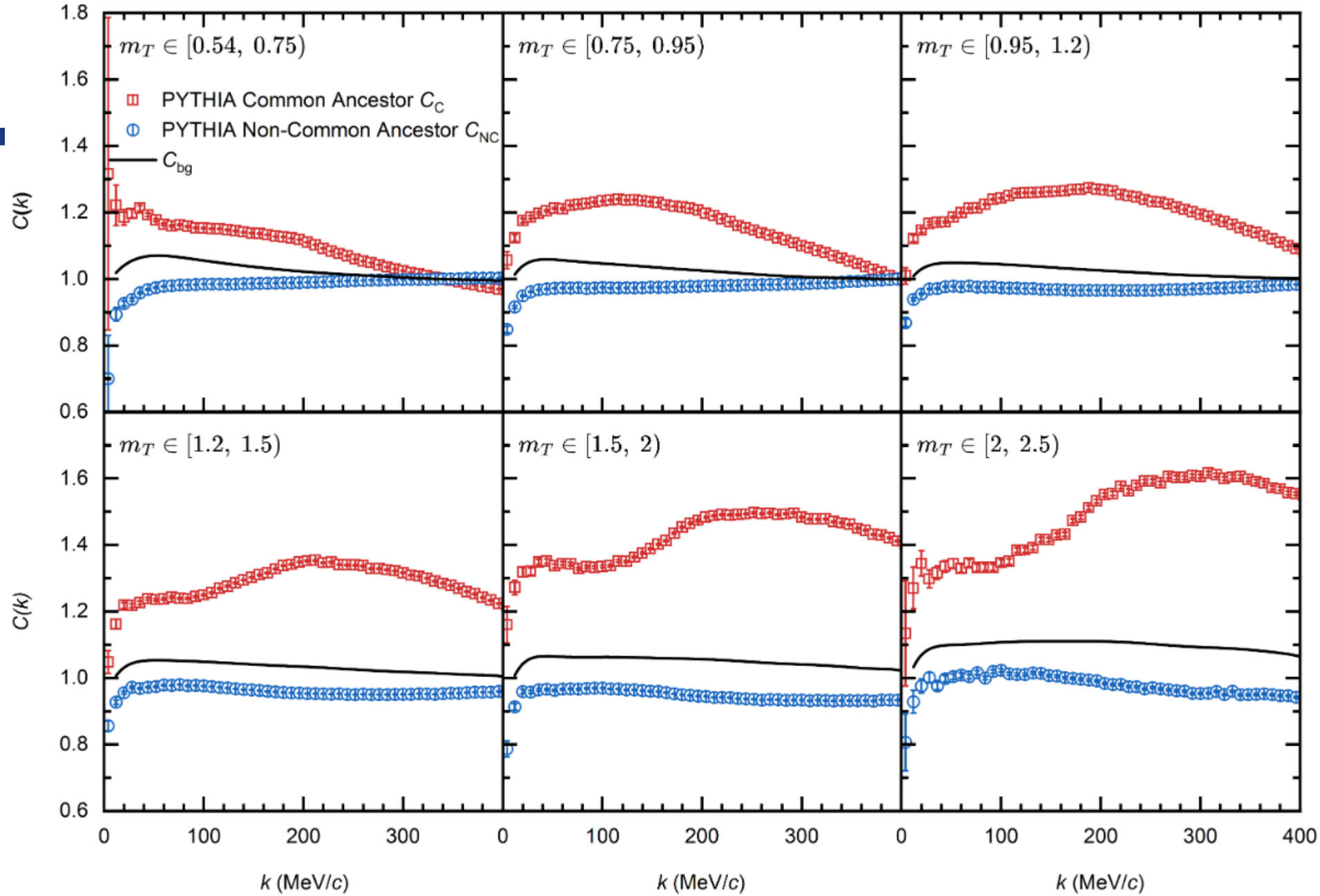
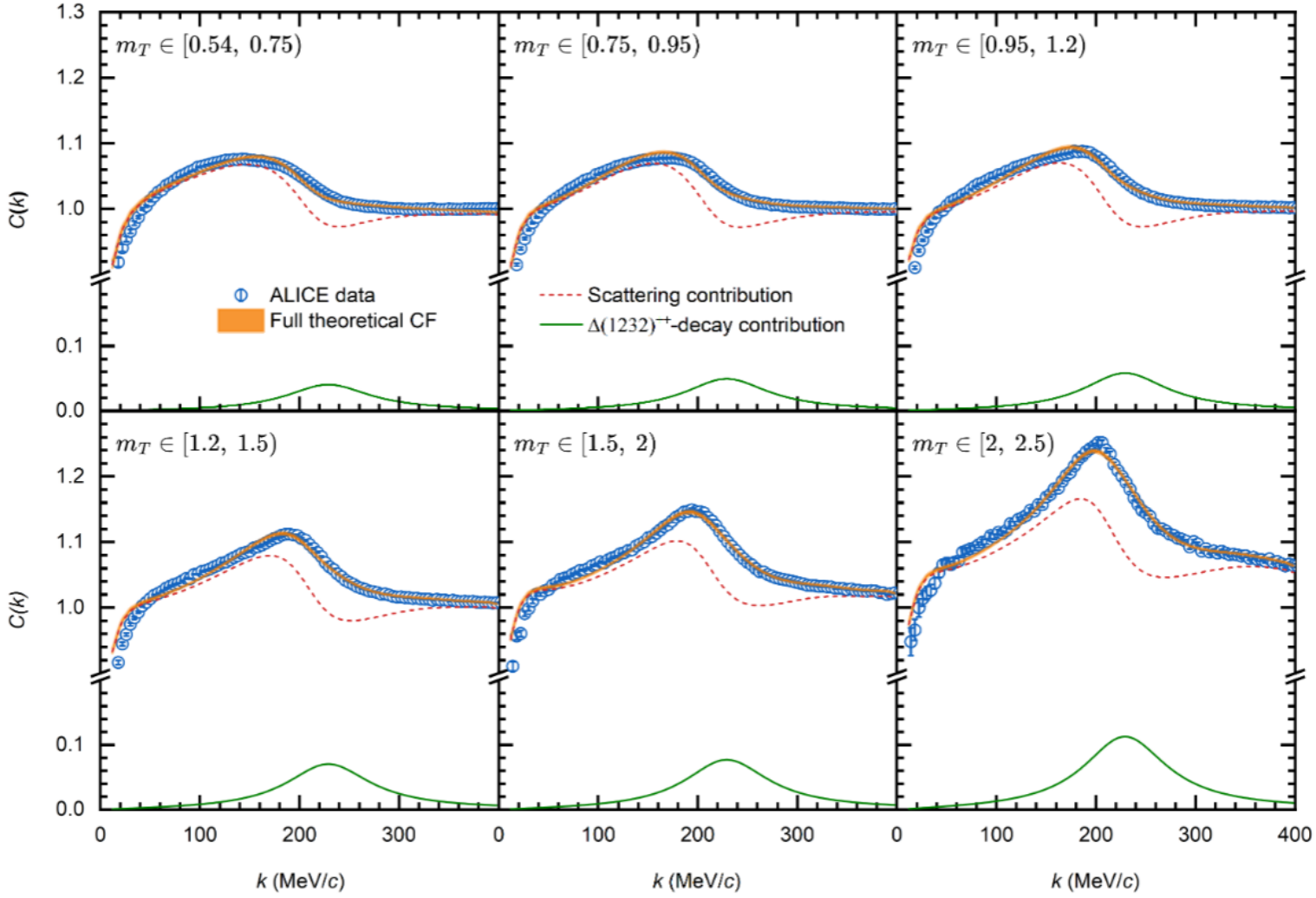


FIG. 3. The correlation function of $\pi^+ p$ pairs for common and non-common ancestor simulated by the ALICE Collaboration [35] as a function of the pair relative momentum k for six intervals of m_T. The background function $C_{\rm bg}$ used in this work is also shown.

Comparison with the experimental data



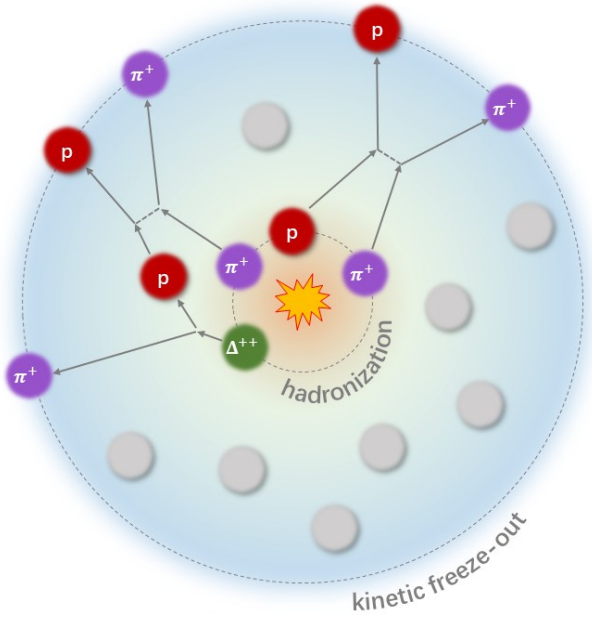
- ✓ The theory matches data and reproduces the peak shift from broad low-k to narrow high-k with increasing transverse-mass m_T.
- ✓ The observed peak is a weighted sum of the scattering and Δ-decay parts; their relative weights P drive the m_T-dependent shift.

Physical picture of the peak shift

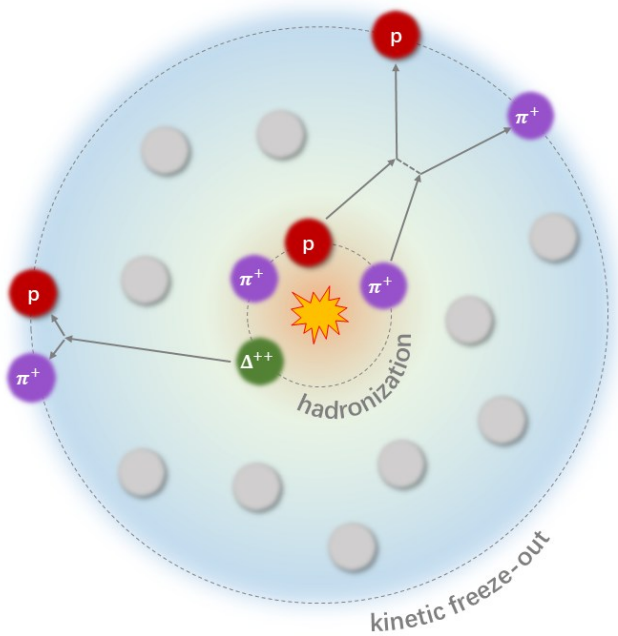
TABLE 1. Transverse mass m_T intervals and parameter λ from the ALICE measurements [35], together with the fit parameters P and Q (weight factors), R (effective source size), and a (background parameter) obtained in the present analysis.

No.	m_T (GeV/c^2) [35]	λ [35]	$P \times 10^7$	Q	R (fm)	a
I	[0.54, 0.75)	0.677	0.59±0.02	0.74±0.04	1.20±0.02	0.03±0.01
II	[0.75, 0.95)	0.687	0.71±0.02	0.70±0.03	1.14±0.02	0.12±0.00
III	[0.95, 1.20)	0.689	0.83±0.03	0.57±0.03	1.04±0.02	0.14±0.00
IV	[1.20, 1.50)	0.688	1.01±0.03	0.48±0.02	0.94±0.02	0.14±0.00
V	[1.50, 2.00)	0.661	1.15±0.04	0.36±0.02	0.81±0.02	0.14±0.00
VI	[2.00, 2.50)	0.616	1.80±0.07	0.38±0.02	0.73±0.02	0.19±0.00

Lower transverse mass

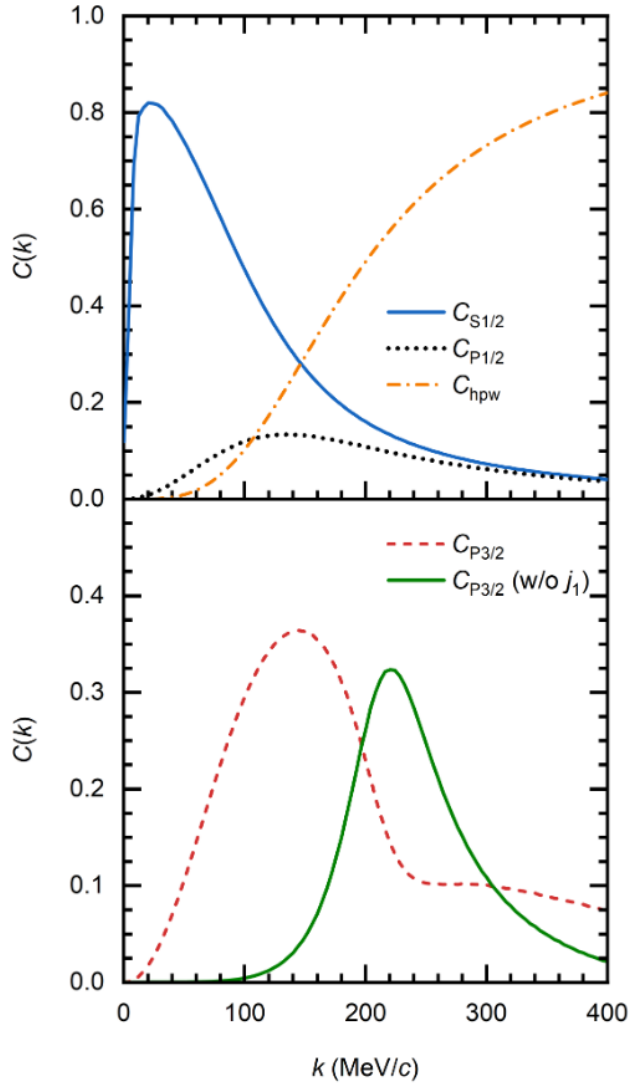


Higher transverse mass



At low m_T, Δ decays inside source, mixing with direct pairs; at high m_T, escapes before Δ decay. Increasing m_T raises P (escape fraction) supporting the picture.

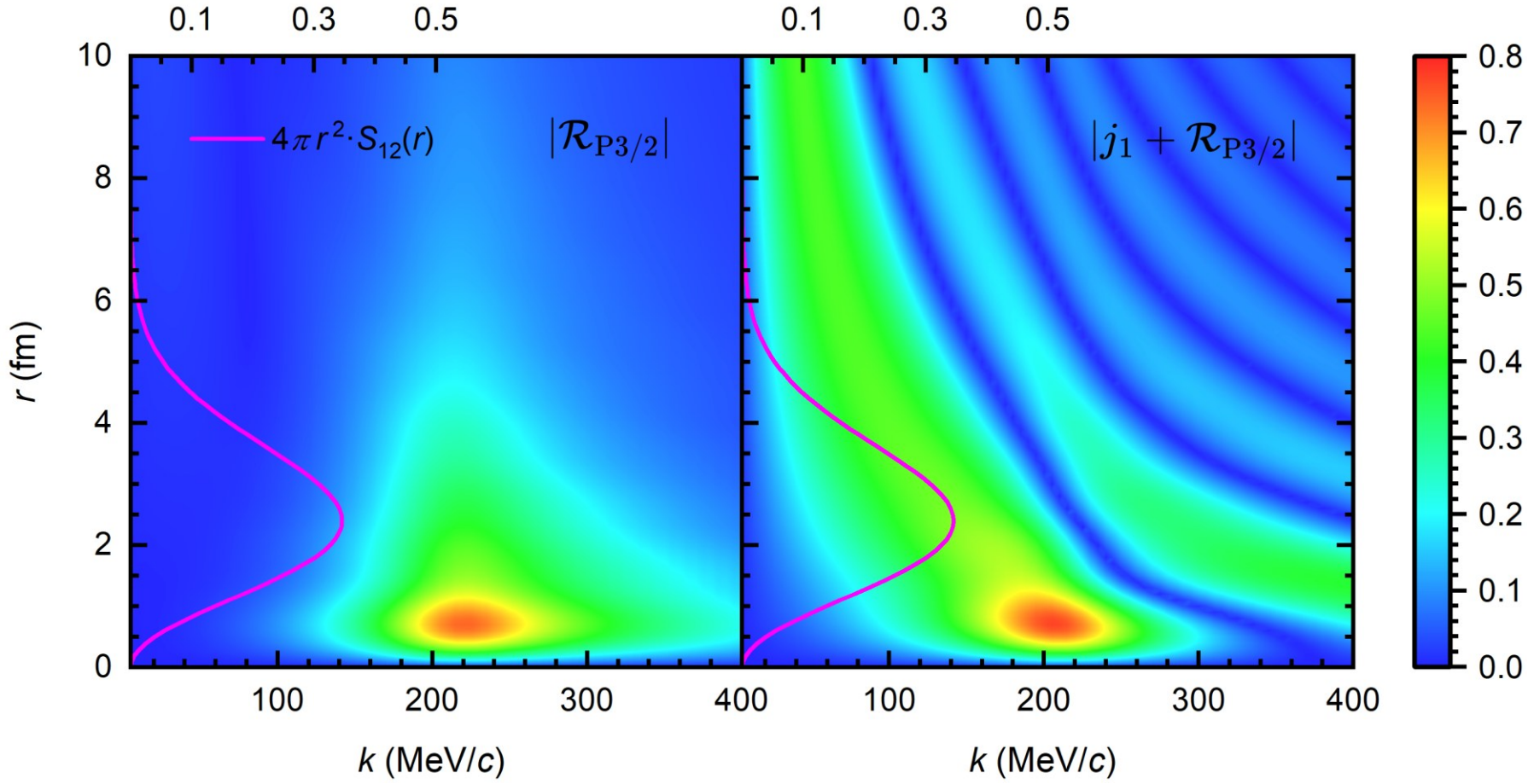
Why scattering and decay peaks differ



- ✓ **The scattering peak near 140 MeV/c originates from the P3/2 partial wave, while the decay peak near 220 MeV/c resembles the conventional Breit-Wigner form.**

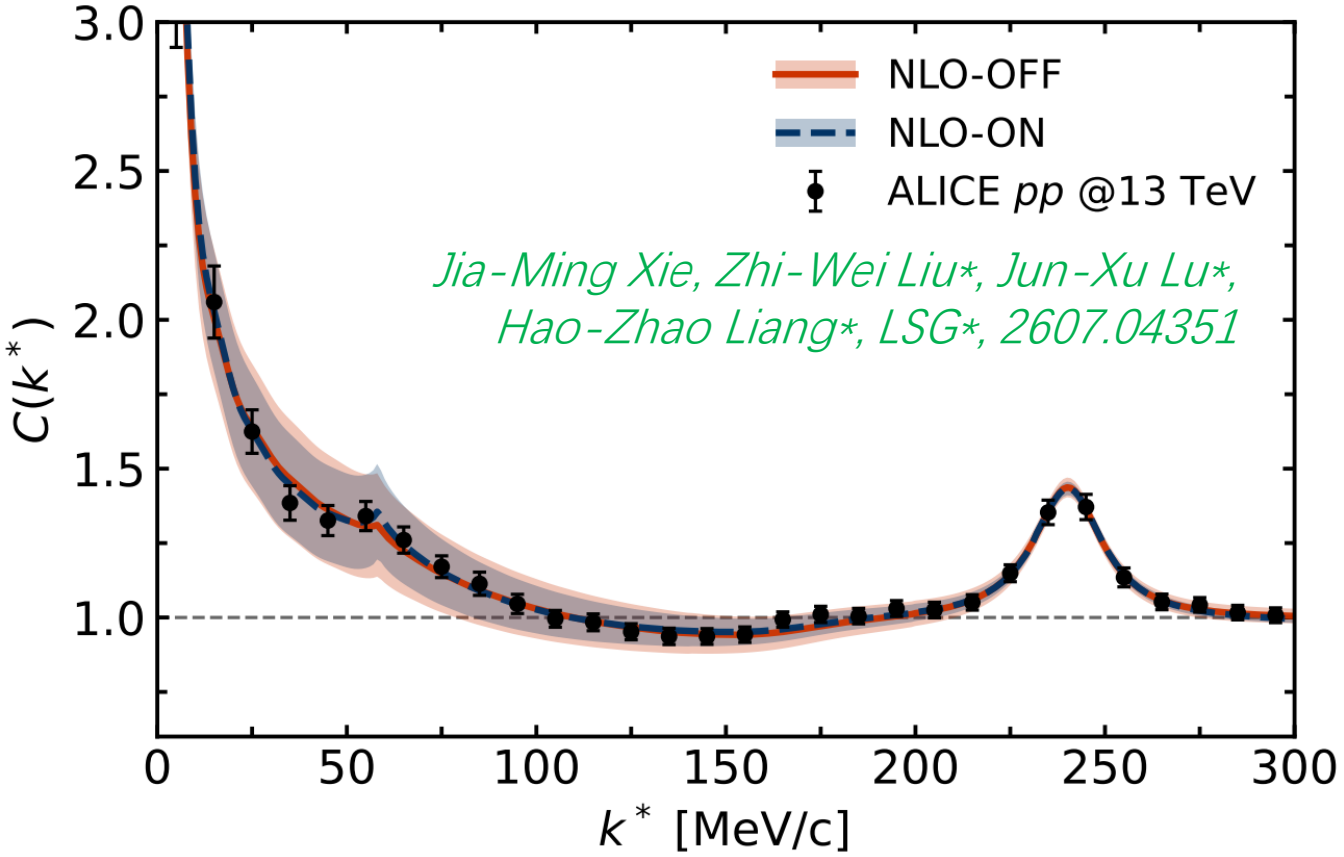
- ✓ **The shift arises because scattering includes the incident wave j_1 in the outgoing wave function, while the decay does not.**

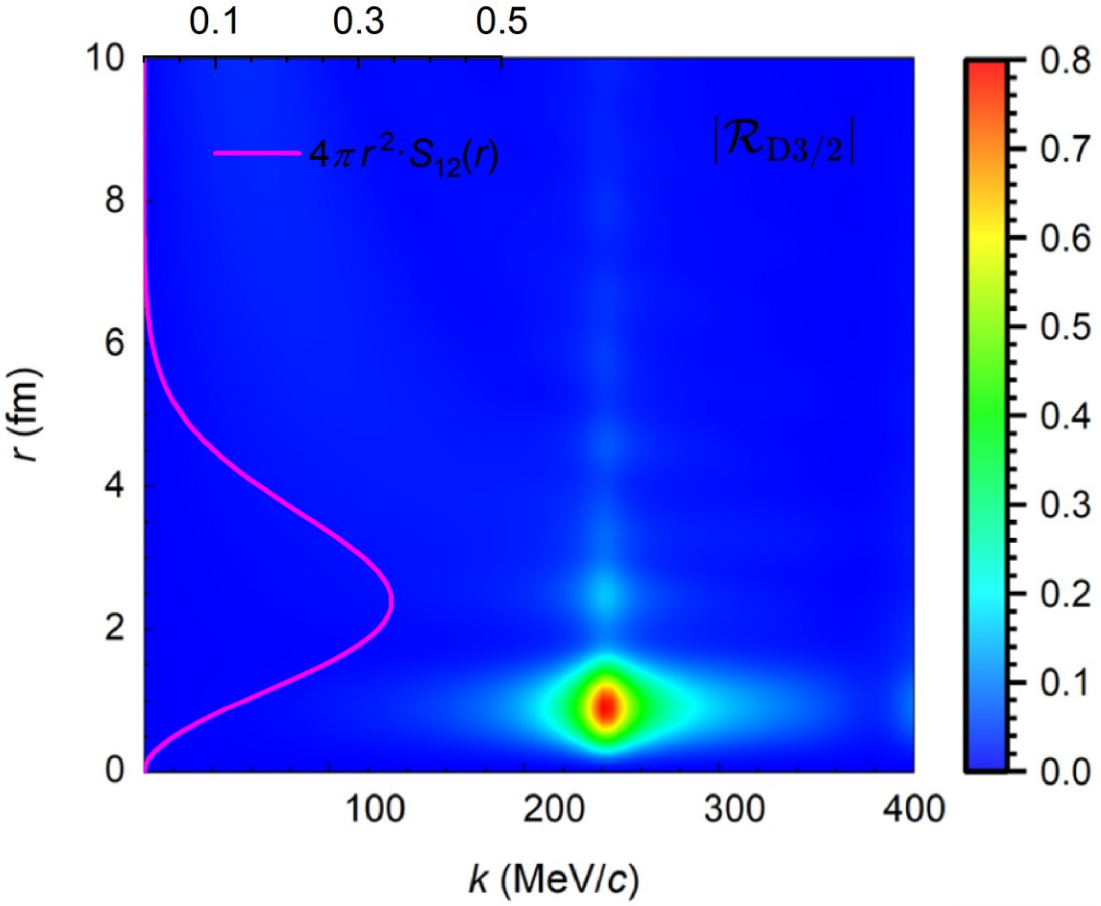
Interference-induced peak shift



- ✓ **Comparison of $|j_1 + R_{P3/2}|$ and $|R_{P3/2}|$ in phase space shows that interference shifts amplitude to low k.**

Interference from broad vs narrow resonances





- ✓ **Large $\Delta(1232)$ width enhances $\pi^+ p$ interference, while narrow $\Lambda(1520)$ width causes negligible $K^- p$ peak shift (only 2 MeV/c shift).**

Contents

- ☞ Brief introduction: exotic hadrons and femtoscopy
- ☞ Femtoscopic correlation functions (CFs)—general features
- ☞ Recent applications
  - ➢ $D_{s0}^{*}(2317)$, $Z_c(3900)$ and $Z_{cs}(3985)$, and $Tcc(3875)$
  - ➢ Two interference mechanisms at play in $\pi^{+}p$ CF
- ☞ Summary and outlook

Summary and outlook

- **Femtoscopy offers high-precision tests of the strong interaction between pairs of (un)stable particles and can be valuable to decipher the nature of the many exotic hadrons discovered so far.**
  - ✓ $D_{s0}^{*}(2317)$, $Z_c(3900)$ and $Z_{cs}(3985)$, $Tcc(3875)$
  - ✓ **Two interference mechanisms at play in $\pi^{+}p$ CF**

- **With more data from LHC Run3, the future is very promising:**

- Pp data taking: event count increase: ~3000×
- Pb-pb data taking: event count increase: ~70×



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