Perturbative-pion interaction
- Simplification of chiral nuclear forces near unitarity limit

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In collaboration with Songlin Lyu, Lin Zuo, Rui Peng & Sebastian Koenig

Outline

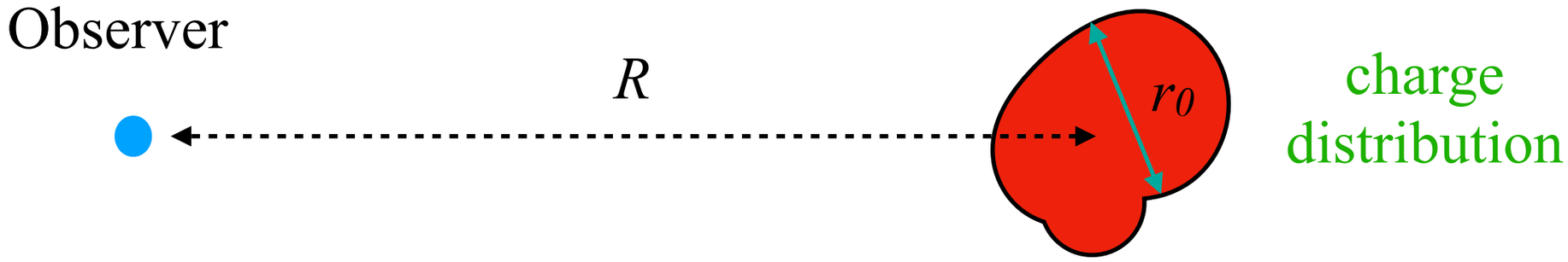
- Chiral nuclear forces

- Recent ab initio calculations based on short-range interactions and SU(4)-Wigner symmetry at odds with chiral forces

- If pions are perturbative, the tension can be resolved, but how to argue perturbative pions, given the difficulty encountered by KSW?

- Simplify LO nuclear interactions to manifest accidental symmetries like SU(4)-Wigner, unitarity limits, conservations of L & S, etc.

Multipole expansion
A classical example of EFT

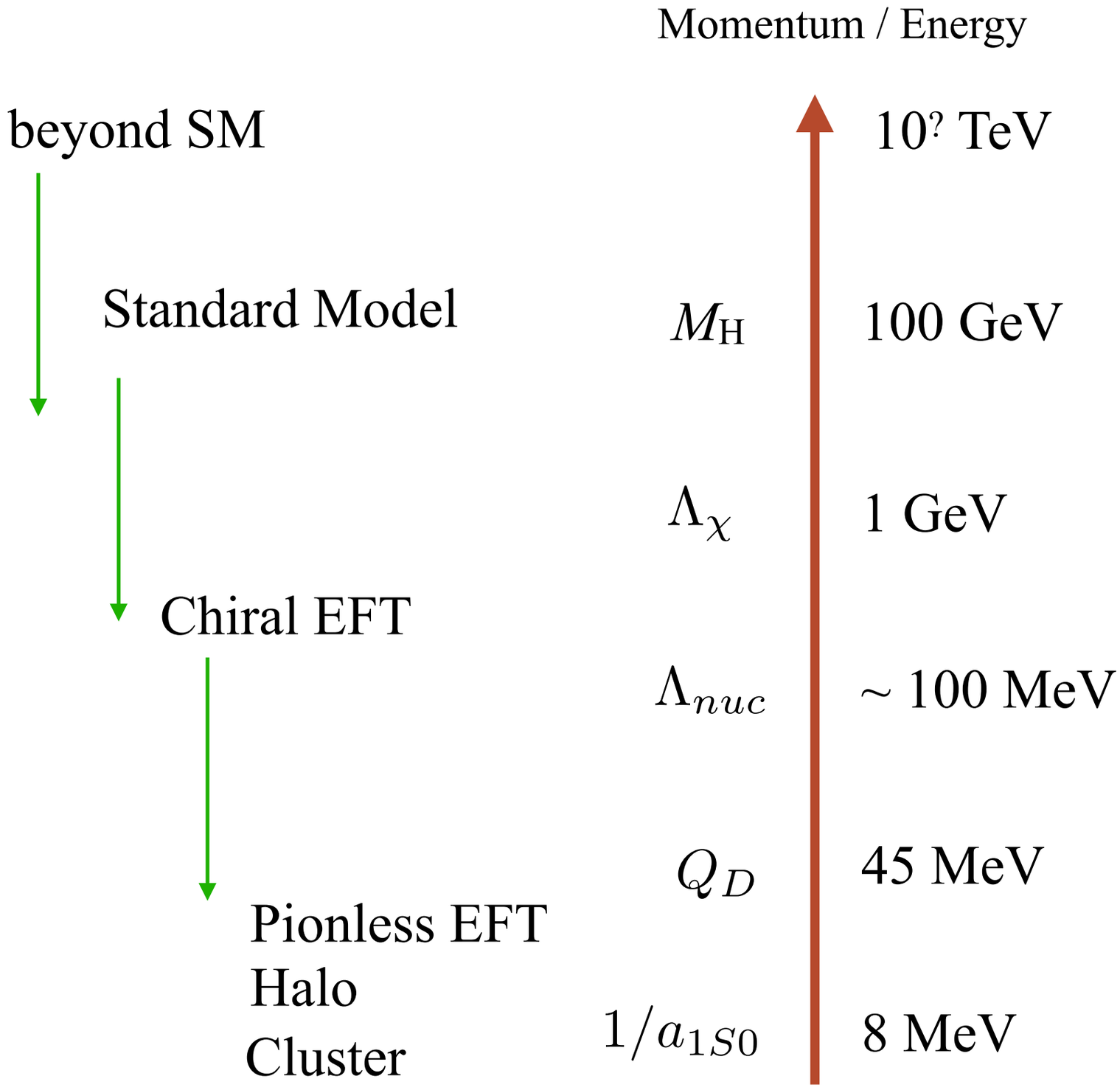


- Separation of scales: $R >> r_0$
- Controlled approximation, able to estimate uncertainty

$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \cdots$$

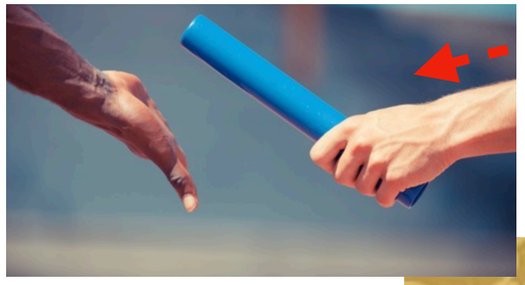
- Naturalness $|d_i| \sim q r_0 \qquad |Q_{ij}| \sim q r_0^2$ ⇒ power counting based on naive dim. analysis (NDA)
- What if it is a rod?
  Slow convergence of a regular PC ⇒ possible fine-tuning ⇒ change PC

Hierarchy of EFTs



Relay: from quarks & gluons to Uranium

Low-energy constants



Chiral EFTs

Lattice QCD

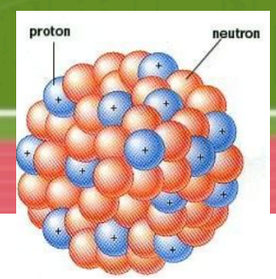
^{2}H & ^{3}H

A: number of nucleons ↑

Light nuclei :
A = 4 ~ 12

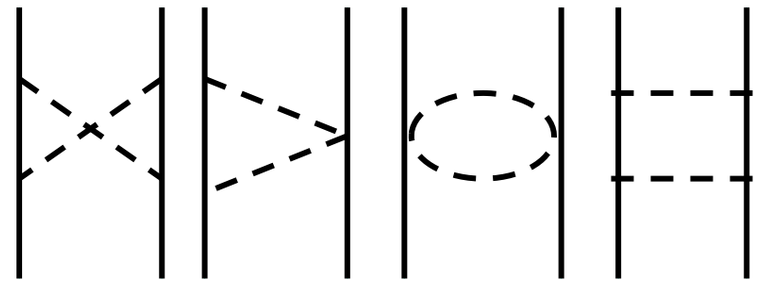
Cluster/Halo EFTs/Other EFTs

Many-body methods



Power counting for loops (HBChPT)

- Nucleon propagator — 1/Q or m_N/Q^2 $\quad \dfrac{i}{p_0 - \frac{p^2}{2m_N}}$
- Pion propagator — $1/Q^2$ $\quad \dfrac{i}{k_0^2 - \vec{k}^2 - m_\pi^2}$ $\quad Q \sim m_\pi$
- Loop integral — $Q^4/(16\pi^2)$ $\quad \displaystyle\int \frac{d^4 l}{(2\pi)^4}$



- Vertex with ν derivatives — Q^ν $\quad -\dfrac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$
- A pion loop brings a suppression factor of $\quad \left(\dfrac{Q}{4\pi f_\pi}\right)^2$

NN potentials from ChEFT

- Contact forces doing heavy lifting

24 contact terms
in NDA up to Q^4

NDA = naive dim. analysis

van Kolck et al. ’92

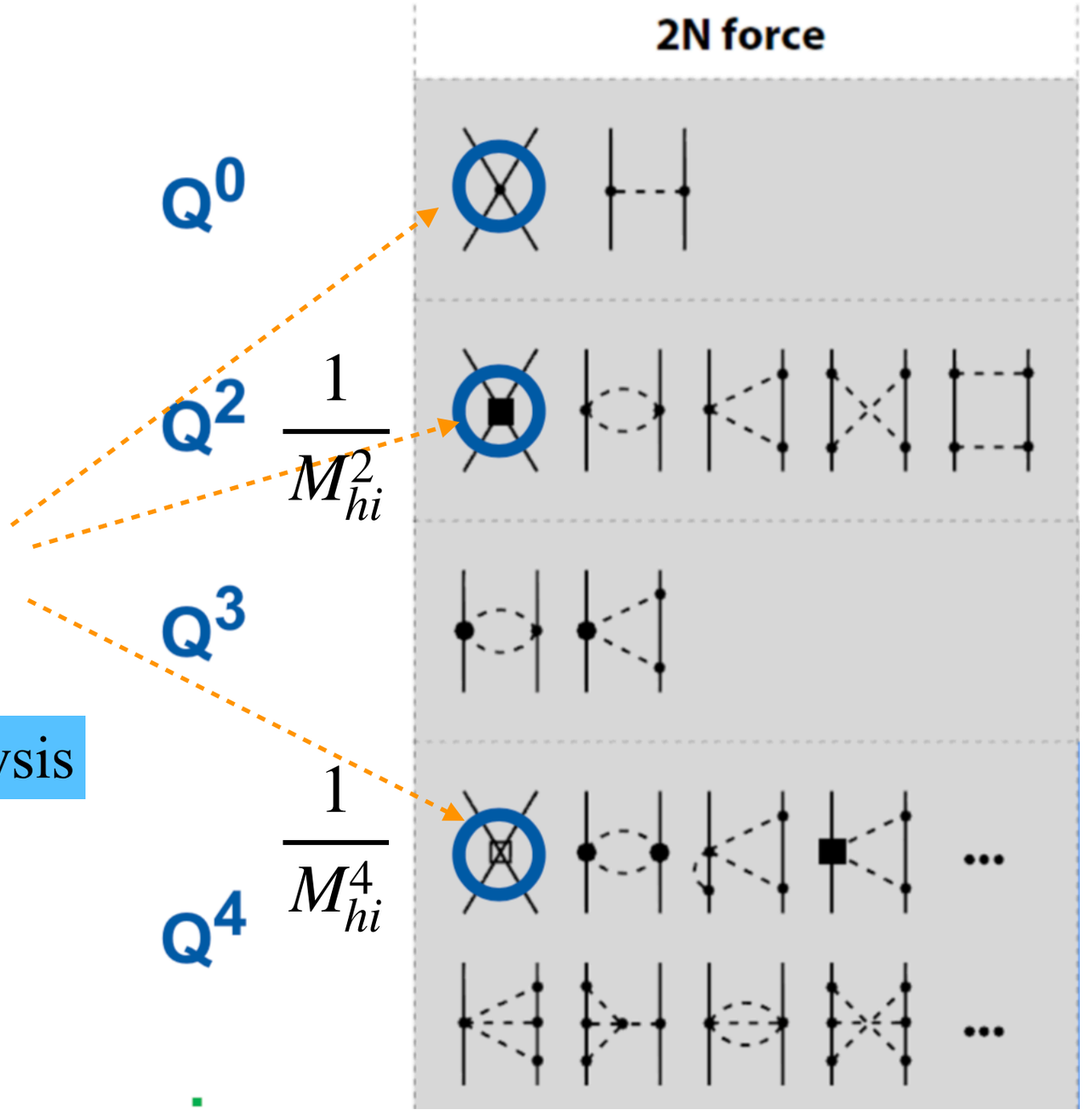
Entem & Machleidt ’03

Epelbaum et al. ’99

Pastore, Piarulli, et al. ’09

Ekstrom, et al. ’13

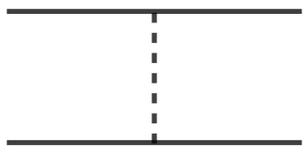
…



To promote, or not to promote ?

Potential diagrams

- One-pion exchange



Tensor F

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[T(r)S_{12} + Y(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]$$

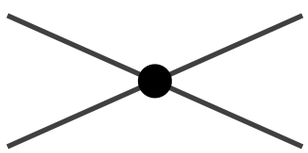
$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

- Tensor force dominant

$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right]$$

$$r = m_\pi^{-1}: \quad \frac{\langle {}^3S_1|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\hat{S}_{12}T(m_\pi^{-1})|{}^3D_1\rangle}{\langle {}^3S_1|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\sigma_1\cdot\sigma_2 Y(m_\pi^{-1})|{}^3S_1\rangle} = 14\sqrt{2}$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$



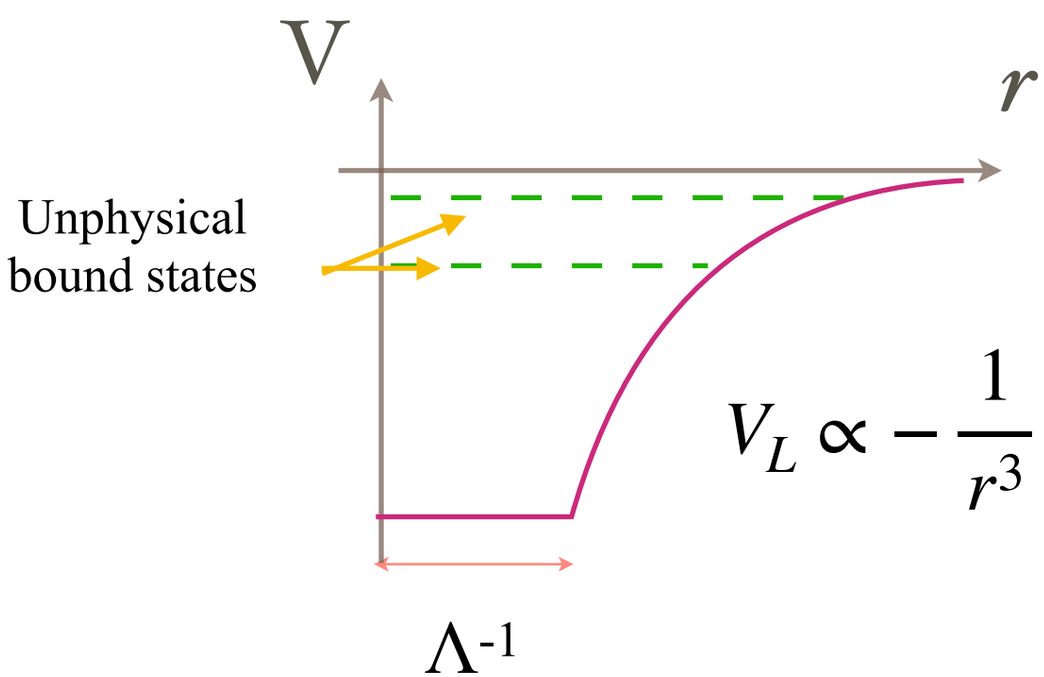
- Many contact terms parametrizing NN short-range forces, e.g.,

$$V_{1S0} = c_0^{1S0} + c_2^{1S0}(p^2 + p'^2) + \cdots$$

$$V_{3P0} = c_0^{3P0} pp' + \cdots$$

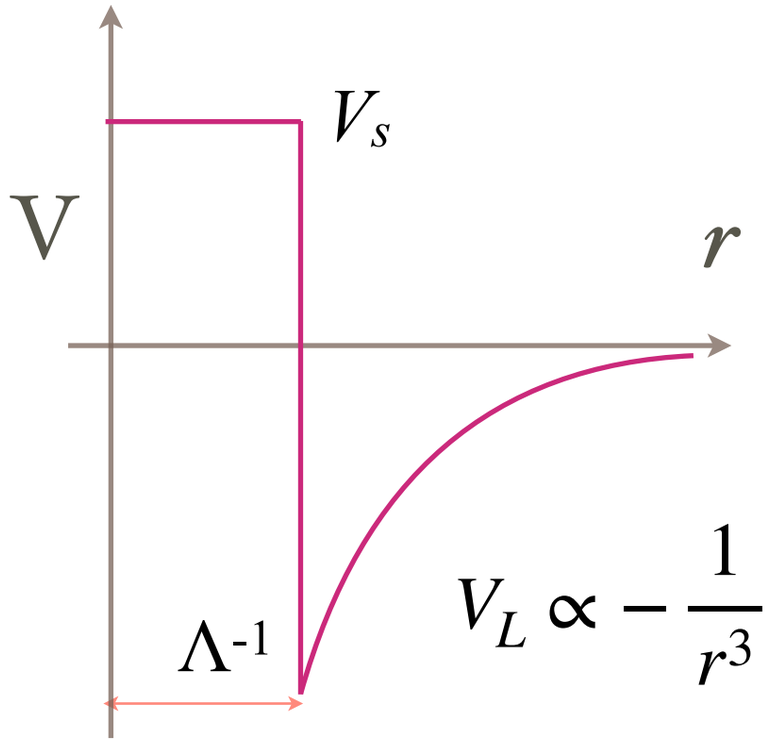
Renormalizing singular attraction

Beane et al ’01
Pavon Valderrama & Ruiz Arriola ’05, ’07
Nogga et al ’05
Birse ’05
BwL & van Kolck ’07
Gantenberg & Kaplan ’26



Modify PC

$\Longrightarrow$

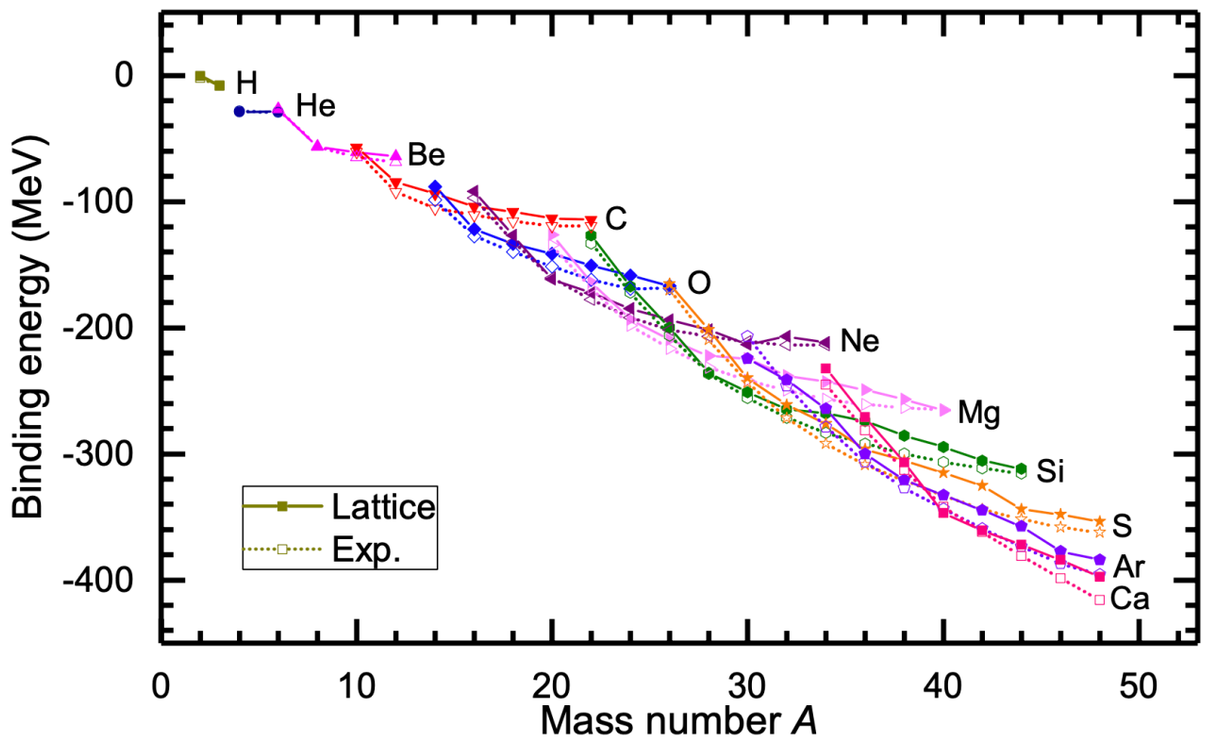


Success of pionless-like interactions

- Microscopic explanation for recent pionless-like (short-ranged) structure calculations:

Koenig et al. ’16
Lyu BN et al. ’18
Gattobigio et al. ‘19
…

Lyu et al. ’18, Lattice EFT calaboration



- Wigner SU(4) symmetric short-range potential (No pions)

Wigner SU(4) symmetry

- Approximate SU(4) invariance of nuclear forces

$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$ SU(4) transformation

JANUARY 15, 1937 PHYSICAL REVIEW VOLUME 51

On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei

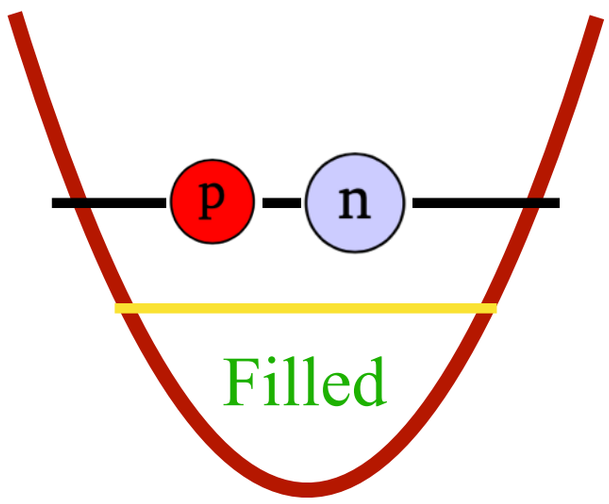
E. WIGNER*
Princeton University, Princeton, New Jersey
(Received October 23, 1936)

- Interactions between NN pairs : (S = 0, T = 1) ≈ (S = 1, T = 0)

2-body: $a_{1S0} \simeq -20$ fm ; $a_{3S1} \simeq 5$ fm

$$\frac{1}{-1/a + \frac{r}{2}k^2 + \cdots - ik}$$

Approximate degenerate states of nuclei:



^{4}He, ^{12}C, ^{16}O, etc. can be viewed as alpha clusters

Wigner symmetry via Chiral EFT?

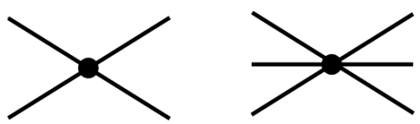
- OPE tensor force breaks SU(4), badly

Tensor F

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[T(r)S_{12} + Y(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

- SU(4) can implemented in pionless EFT by letting C_T to be perturbation



$$-\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2 \qquad -h_0(N^\dagger N)^3$$

- Approximiate SU(4) if OPE is perturbative

Fleming, Mehen & Stewart ’99

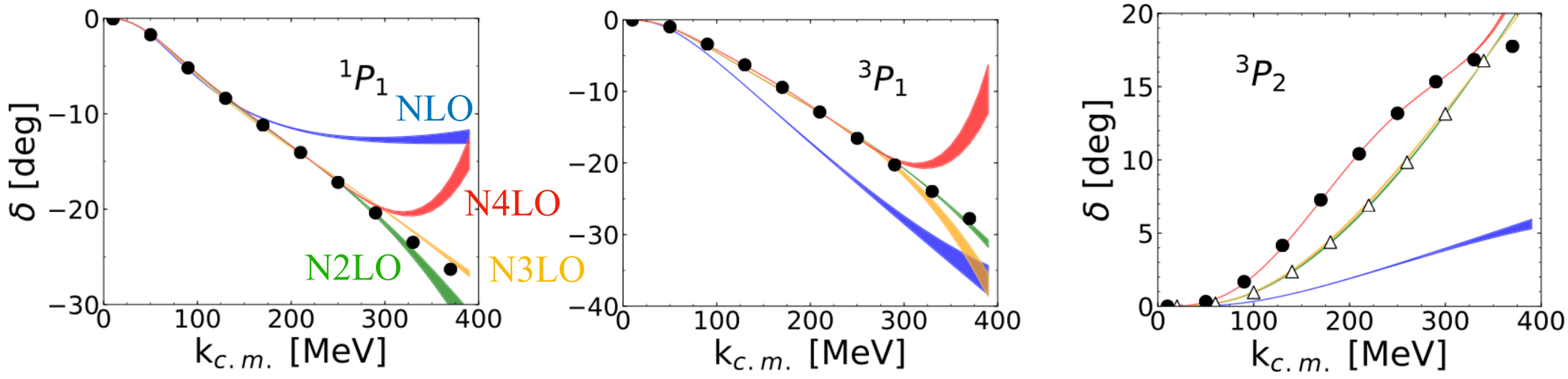
Kaplan, Savage & Wise ’98

Pion-exchange is perturbative for most waves

Birse ’05
Wu & Long ’18
Kaplan ‘19

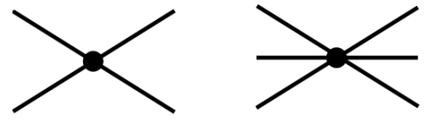
- OK for $k < \Delta \simeq 290$ MeV for $l > 0$ except 3P0

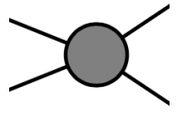
- TPEs included in N3LO & N4LO



Wu & Long ’18

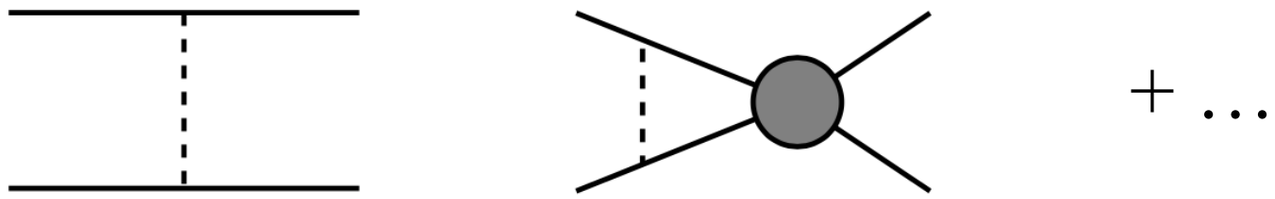
Perturbative pions?

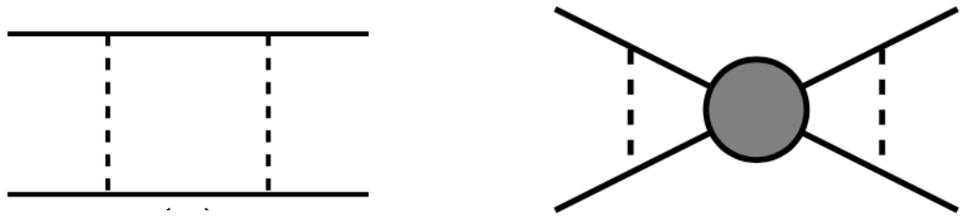
- Same LO as pionless: 

- LO of NN = bubble sums = 

- Simple LO has useful symmetries:
  Unitarity; Wigner SU(4);
  preserving L & S , first-order correction by tensor force to s-shell nuclei vanishes

Kaplan, Savage & Wise ’98
Fleming, Mehen & Stewart ’99
Beane, Kaplan & Vuorinen ’08
Teng & Griesshammer ’24

NLO:  + …

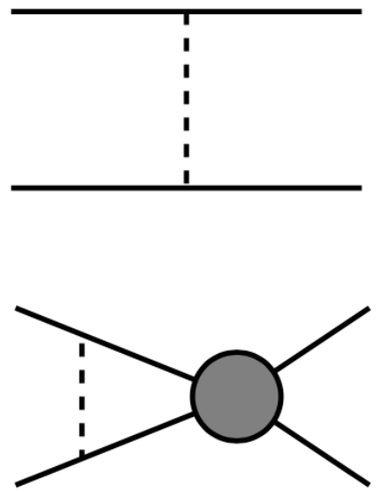
N2LO:  + …

Why mixing angle so small?

Lyu, Zuo, Peng, Koenig & BwL ‘25

- Mixing angle vanishing at unitarity & chiral limits despite strong OPE tensor force

$$\epsilon_{1,\pi} = \frac{m_N k}{4\pi} \frac{k}{(a^{-2}+k^2)^{\frac{1}{2}}} \times \left[\frac{1}{ak} \langle k, {}^3S_1 | V_\pi | k, {}^3D_1 \rangle - \frac{g_A^2}{\sqrt{2} f_\pi^2} \Pi\left(\frac{m_\pi}{k}\right) \right]$$

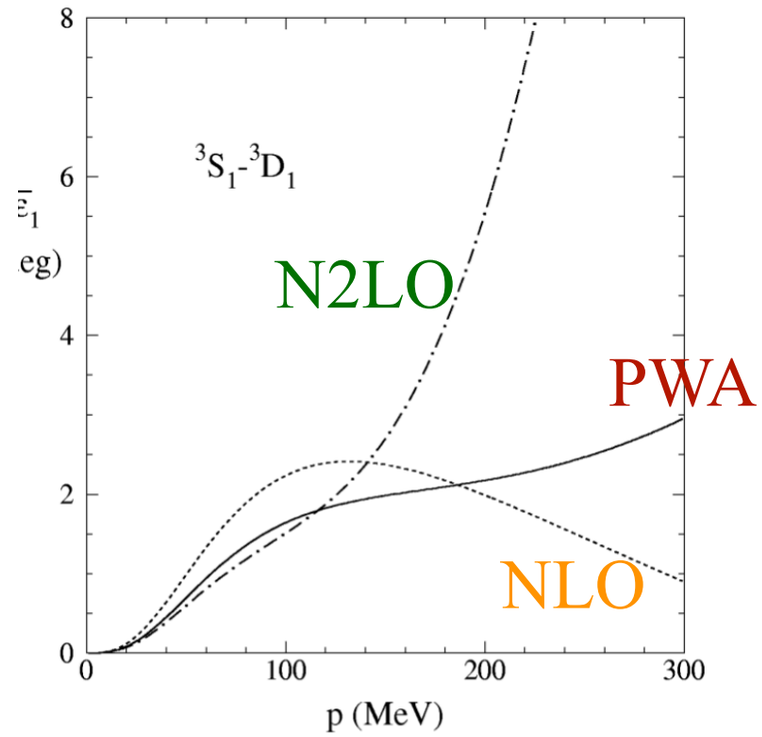


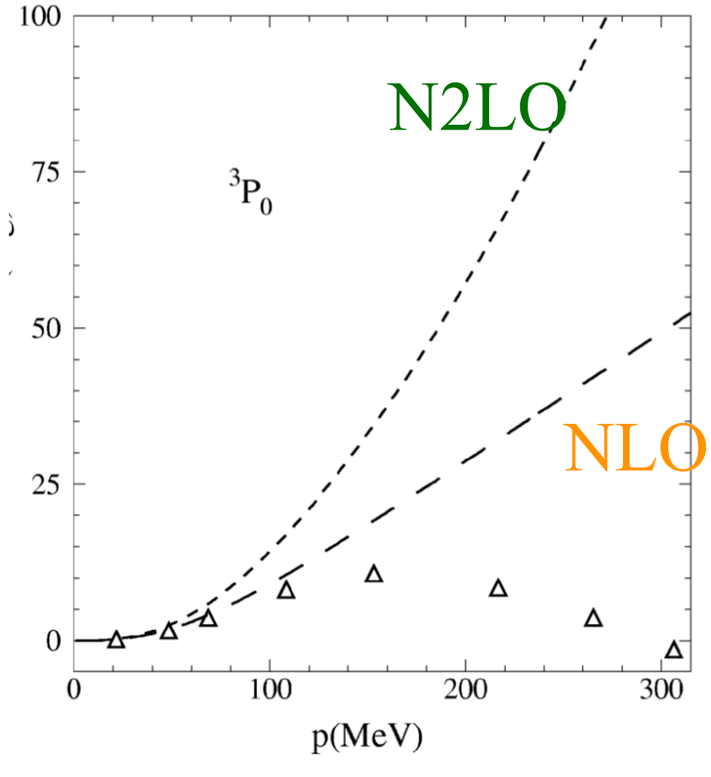
$$\Pi\left(\frac{m_\pi}{k}\right) \equiv \int_0^\infty \mathrm{d}r' r'^2 n_0(kr') T(r') j_2(kr')$$

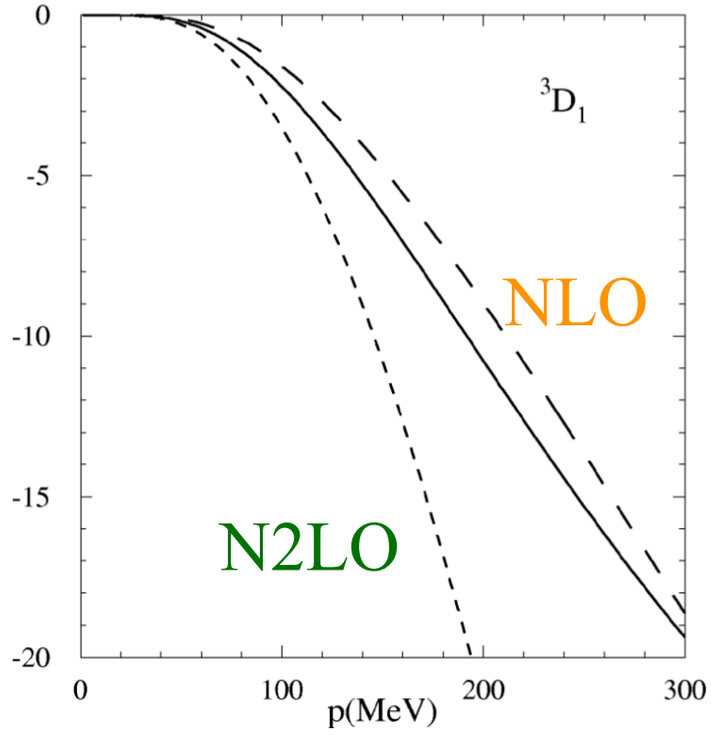
$$m_\pi \to 0: \quad \int_0^\infty dr r^2 n_0(kr) \frac{1}{r^3} j_2(kr) = 0$$

PPI: 3S1-3D1

- Not enough though
- Only works for on-shell kinematics
- N2LO 3D1, mixing angle are large
- No unitarity in 3P0

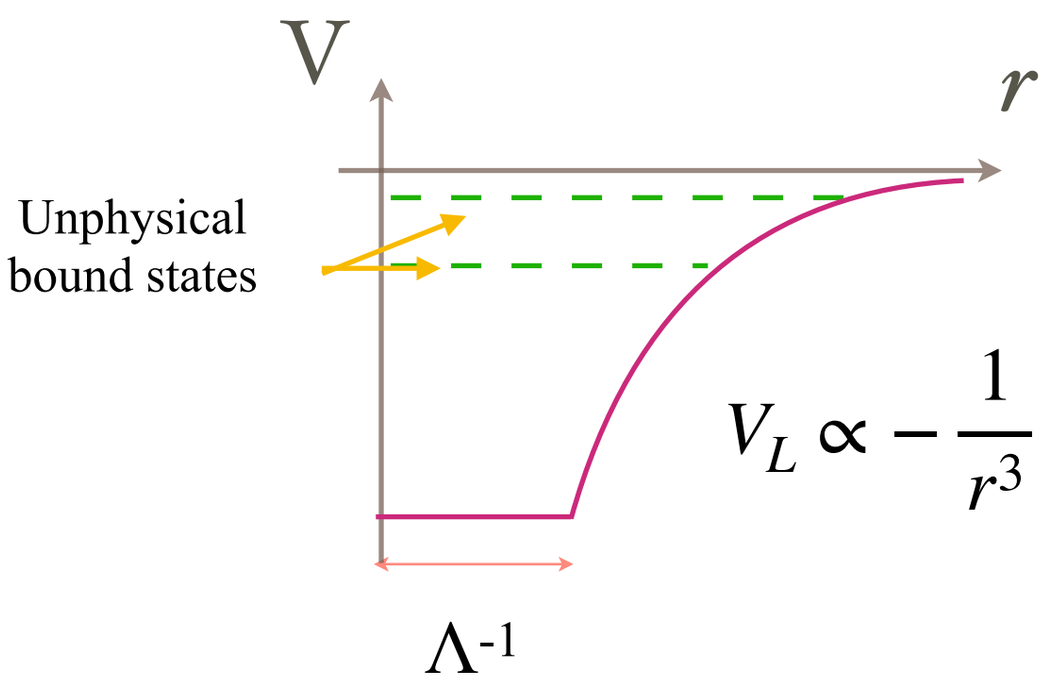






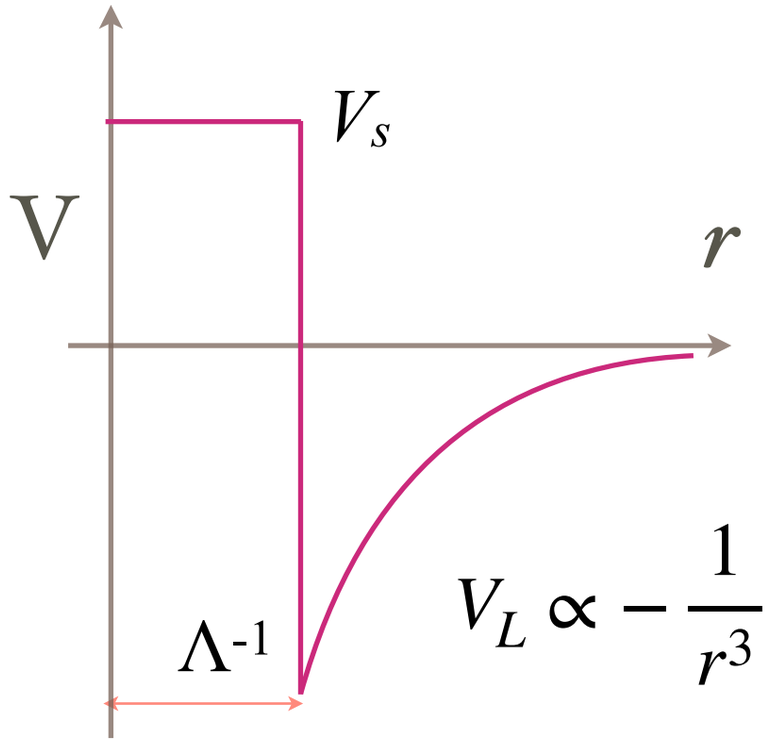
Renormalizing singular attraction

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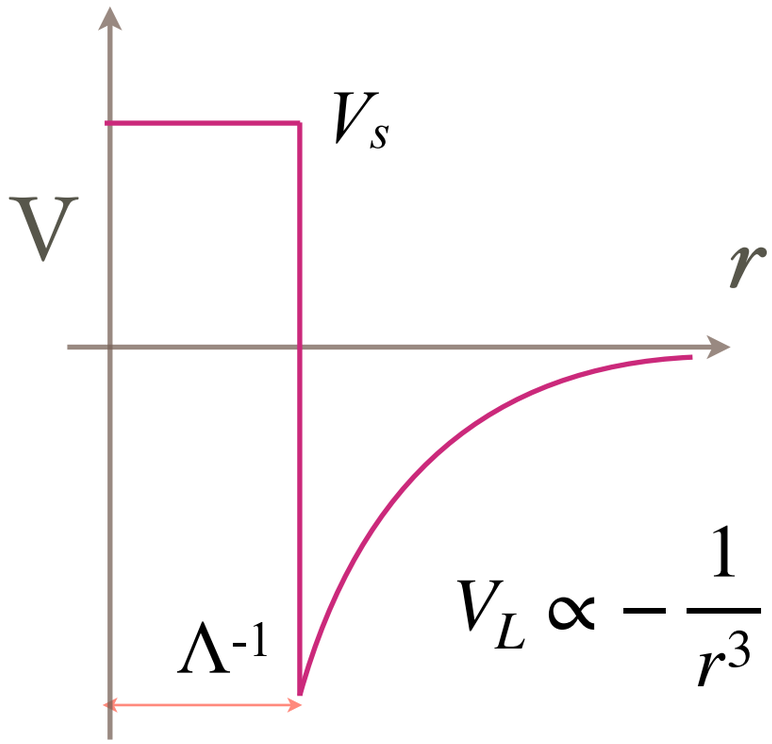
Modify PC

$\Longrightarrow$



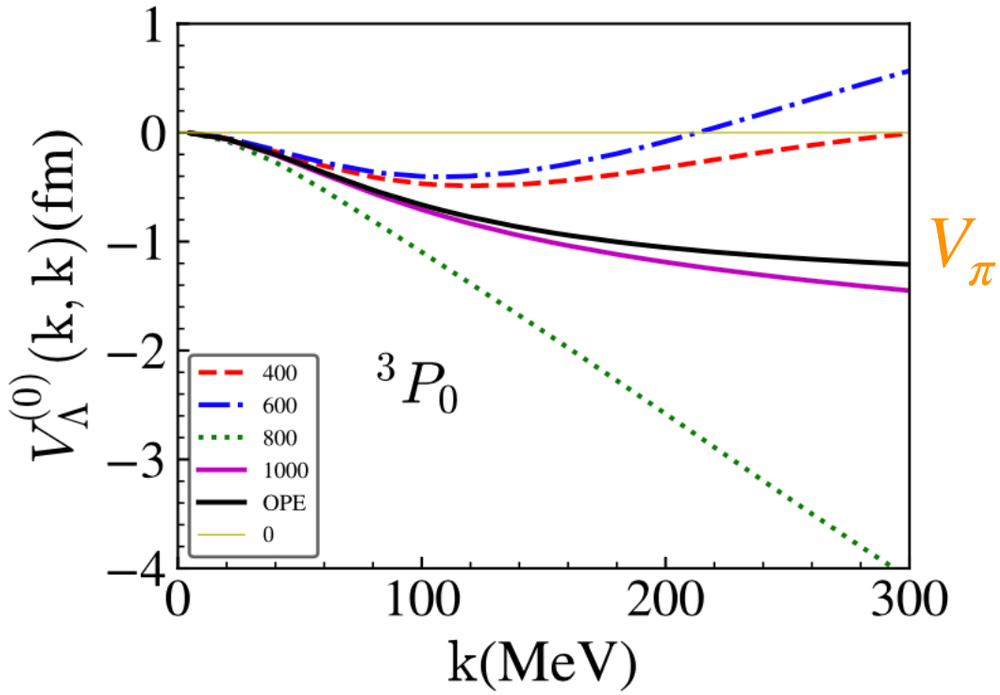
Pushing pert-pion interactions

- Re-organizing higher contact terms



Nonpert ren of OPE

$V_\pi + C_{3P0} p' p$



- From nonpert renormalization: V_s always repulsive (towards lower Λ)

- Using V_s to moderate the OPE tensor force

PPI in 3P0

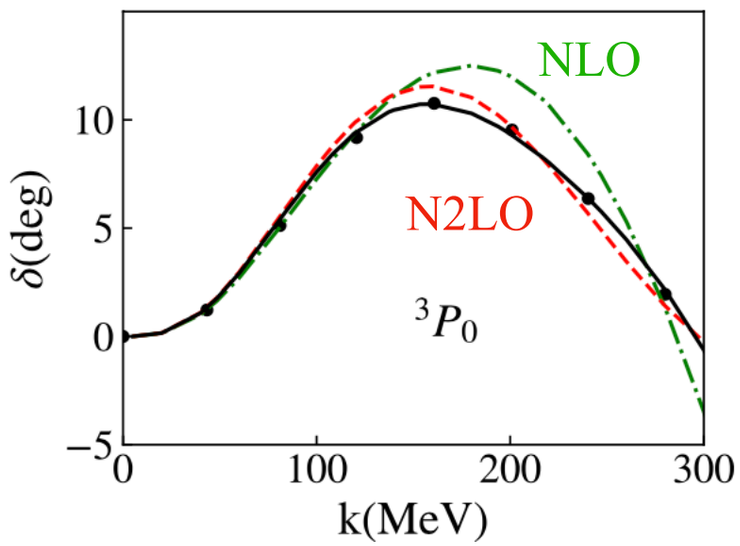
- Expansion in $V_\pi + C_{3P0} p'p$ (Born approximation) Peng, Lyu & BwL ’20

$$V^{(1)}({}^3P_0) = V_\pi + C_2^{{}^3P_0} p'p$$

- Higher-order contacts are identified when needed for renormalization at second, third-order Born approximation

	LO	NLO	N^2LO	N^3LO
π	OPE			TPE0
${}^3P_0$	C_2		C_4	C_6

Updated fit $\Lambda = 800$ MeV

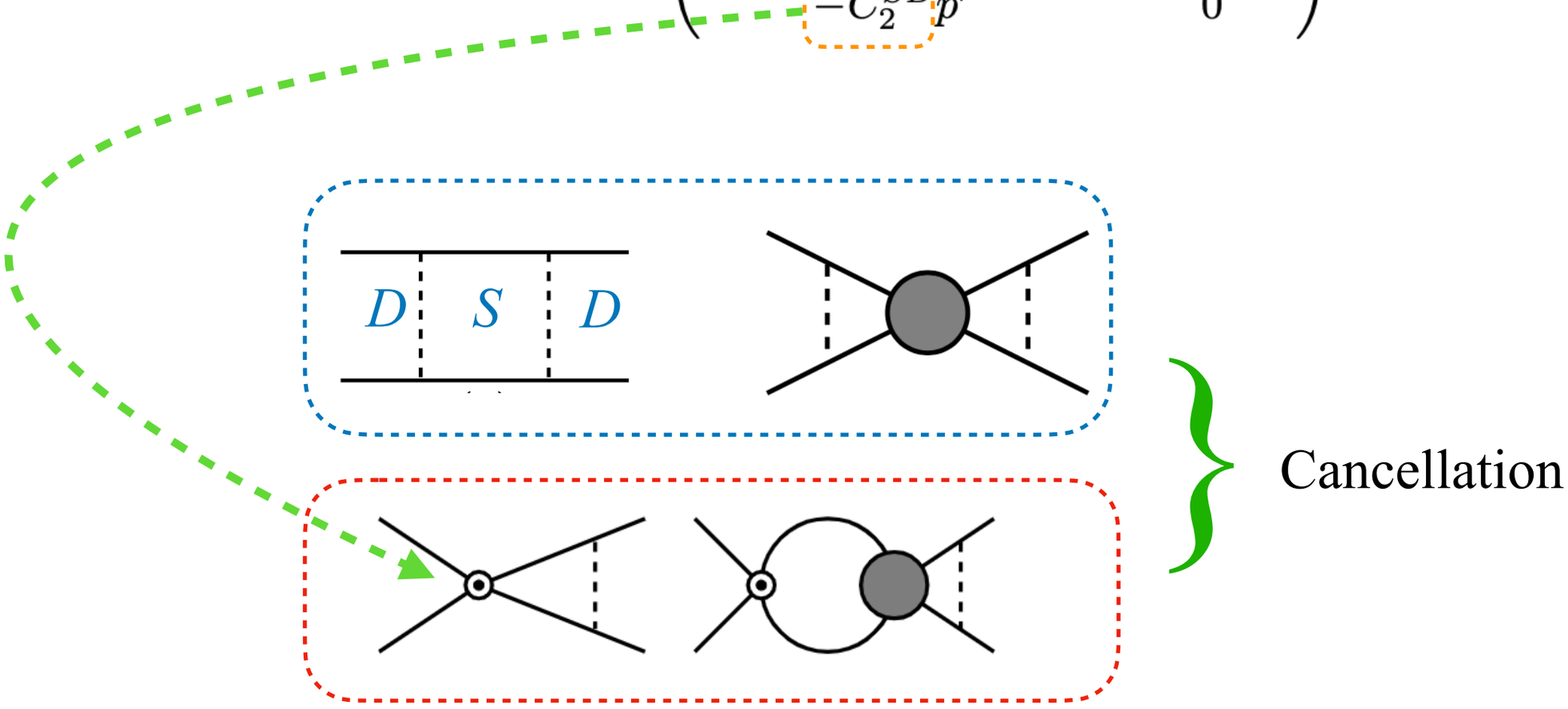


Peng, Lyu & BwL ’20

PPI: 3S1-3D1

- Expansion in (V_π + *SD* mixing contact)

$$V^{(1)}({}^3S_1 - {}^3D_1) = V_\pi + \begin{pmatrix} C_0^{(1)} + C_2^{^3S_1}(p'^2 + p^2) & -C_2^{SD} p^2 \\ -C_2^{SD} p'^2 & 0 \end{pmatrix}$$



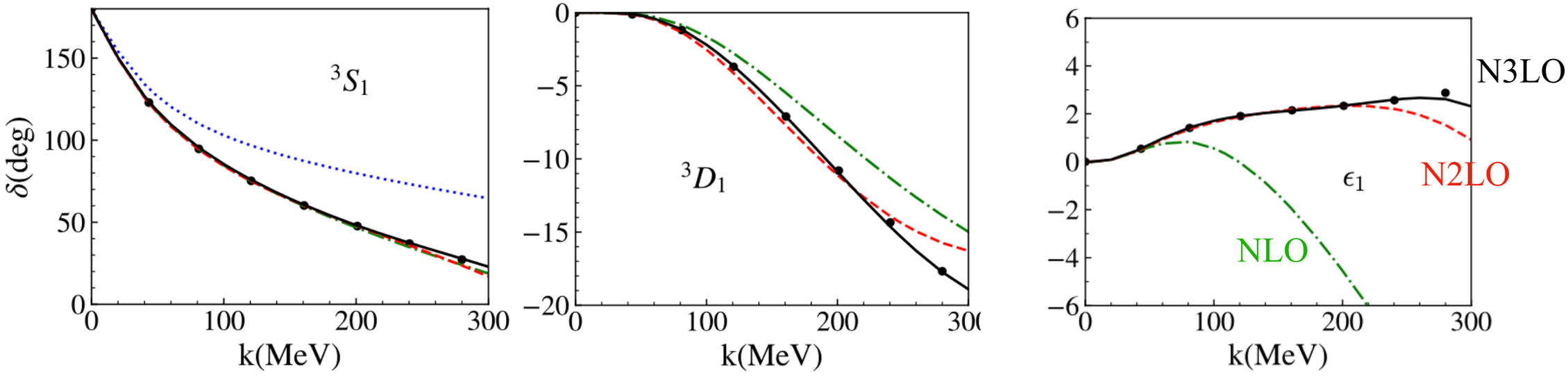
Cancellation

PPI: 3S1-3D1

- PC of high-order contacts by counting divergence

	LO	NLO	N^2LO	N^3LO
π		OPE		TPE0
${}^3S_1$ ${}^1S_0$	C_0	C_2	C_4	C_6
SD		C_2	C_4	C_6
${}^3D_1$				C_4

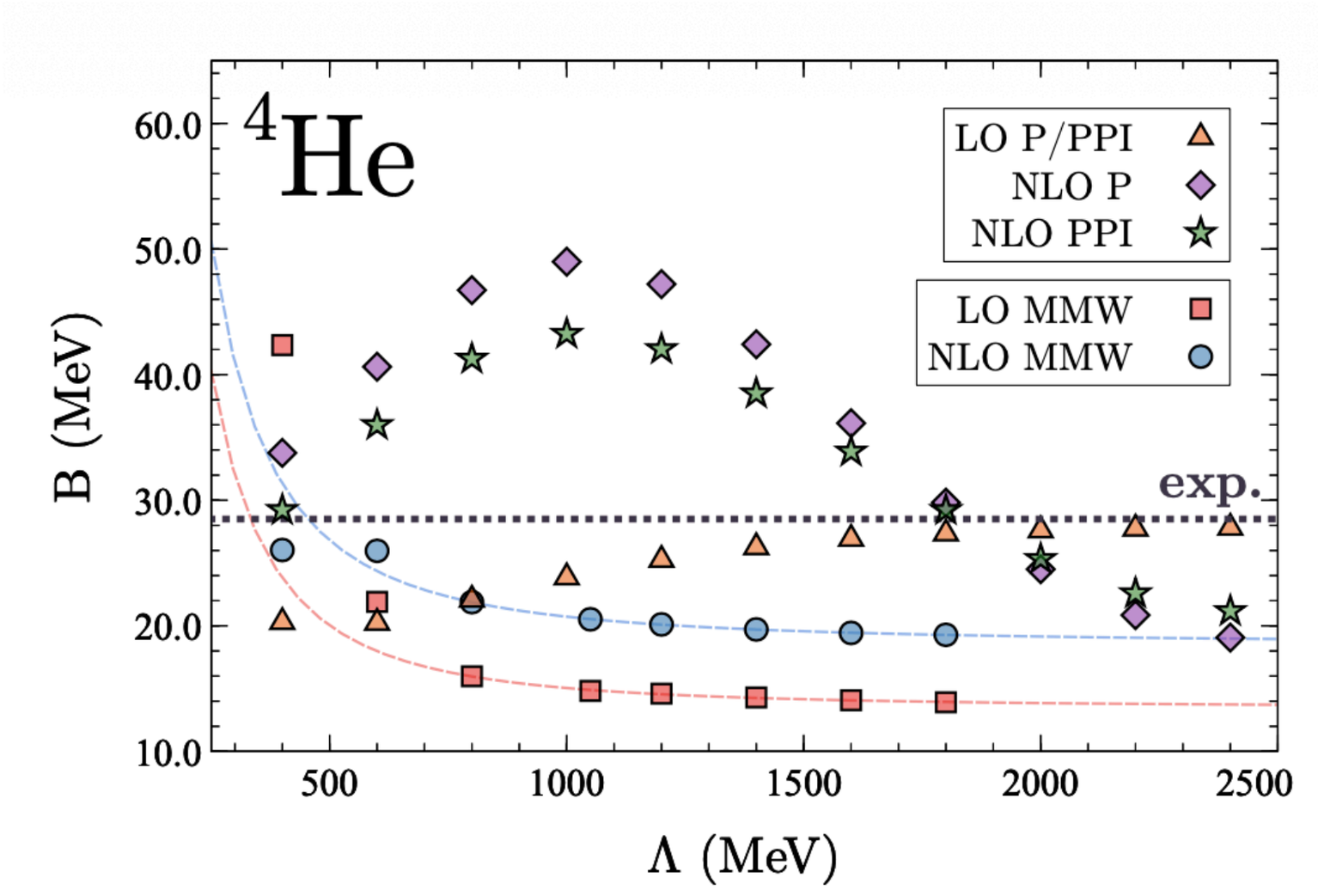
$\Lambda = 800$ MeV



Lyu, Zuo, Peng, Koenig & BwL

4He BE vs cutoff

Lyu, Zuo, Peng, Koenig & BwL, ‘25



P: Pionless

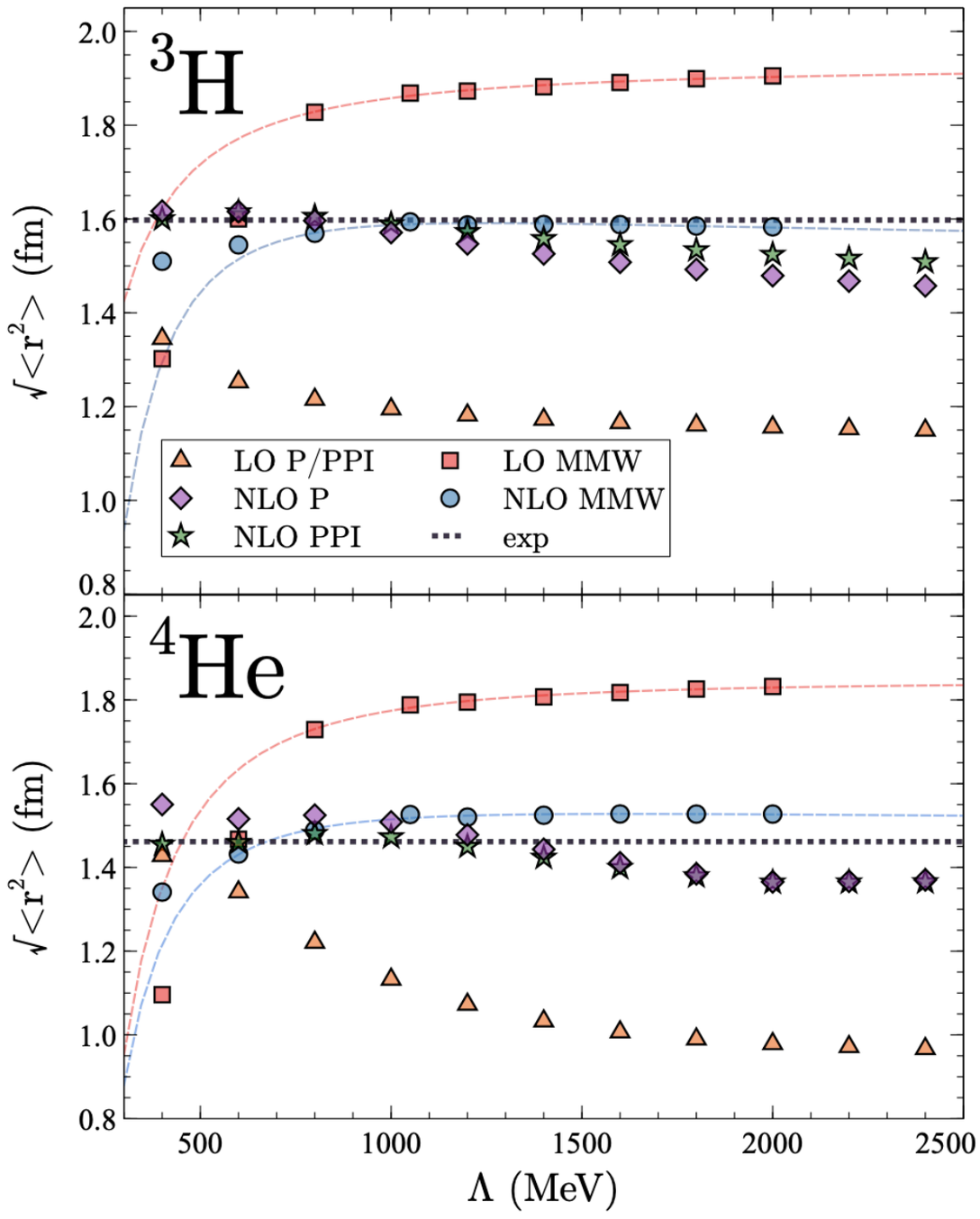
PPI: pert.-pion

MMW: nonpert-pion

- 4He energy shows large fluctuation w.r.t. Lambda at NLO
- Still needs 4NF at NLO, just as in pionless (Bazak et al. ’18)

Lyu, Zuo, Peng, Koenig & BwL

Charge radii



P: Pionless

PPI: pert.-pion

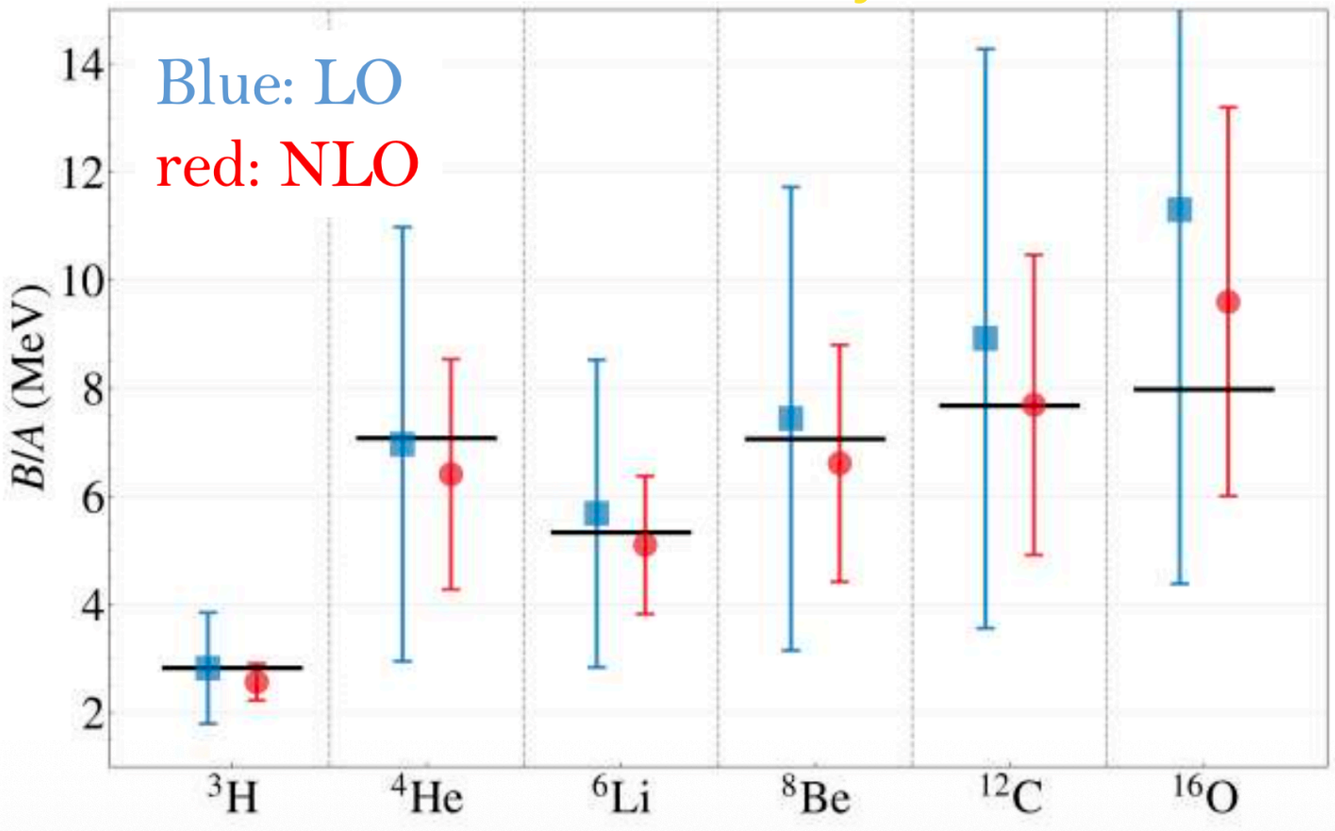
MMW: nonpert-pion

- NLO radius needs tensorF to correct w.f.
- Diff. of pionless vs perturbative-pion is small

Heavier nuclei

- Choose a cutoff where 3NF & 4NF are moderate

Preliminary



$$\Delta_{\mathrm{EFT}}^{B/A} = (B^{(0)}/A) \times (Q_A/M_{\mathrm{hi}})^i$$

$$\alpha_\pi^{-1} \rightarrow M_{\mathrm{hi}} = 200\ MeV$$

. LO: ^{3}H reproduced

. NLO corrections → all nuclei less bound

. NLO lacks: 4NF & 3NF corrections

Summary

- Chiral EFT does not even have approximate SU(4)

- Recent ab initio calculations with simply contact interactions are quite successful

- A new perturbative-pion scheme with re-organized contacts

Simplify LO nuclear interactions to manifest accidental symmetries like SU(4)-Wigner, unitarity limits, conservations of L & S, etc.

W/ larger validity range and 3NF at LO, implies 3NF important even for Q > m_π