Chiral Symmetry Preserving Nuclear Forces

Hermann Krebs
Ruhr-Universität Bochum

Workshop on Few-Body Problems in Nuclear and Hadron Physics
China Center of Advanced Science and Technology
Beijing, China
September 21, 2026





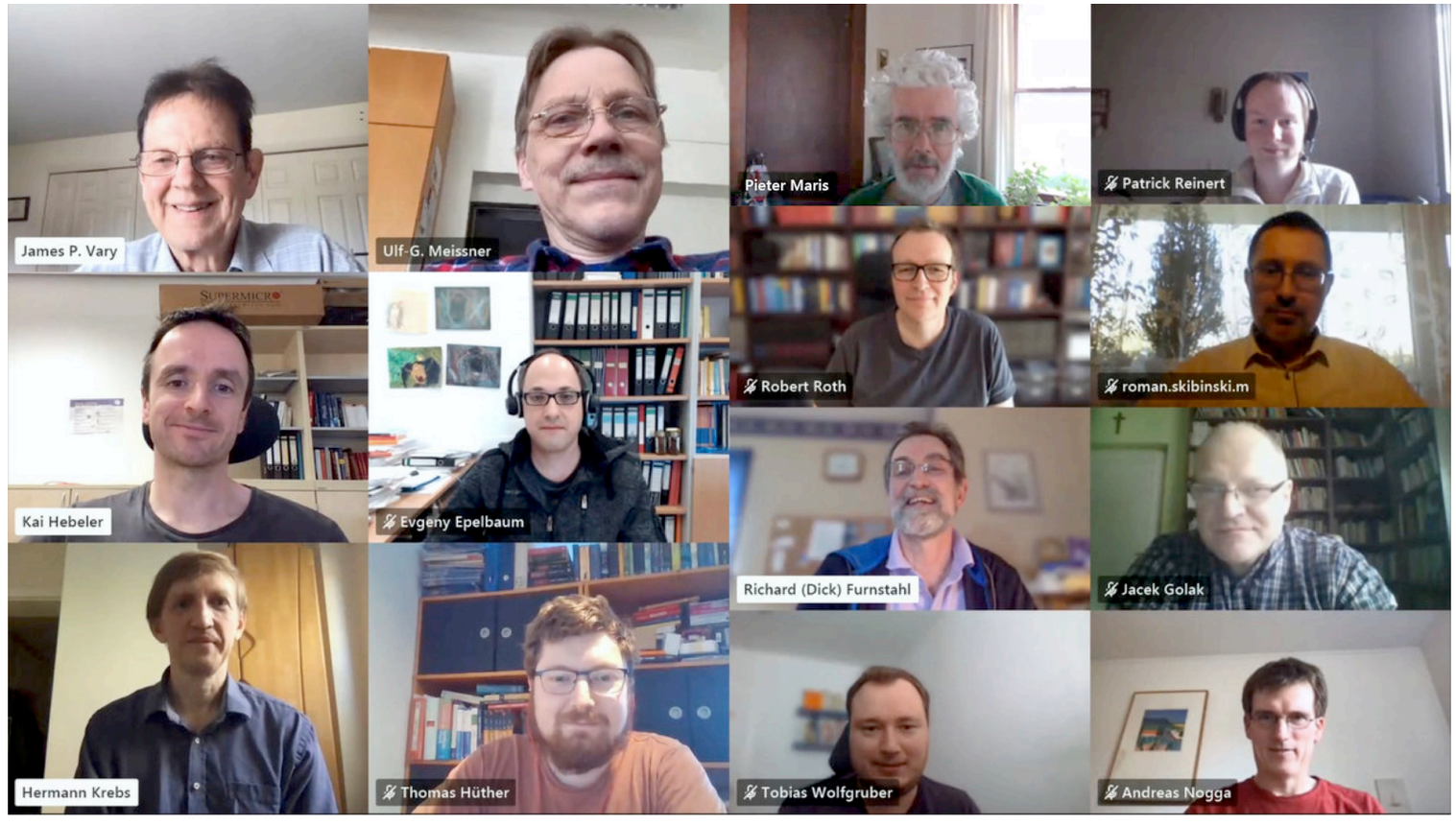


LENPIC

Low Energy Nuclear Physics International Collaboration

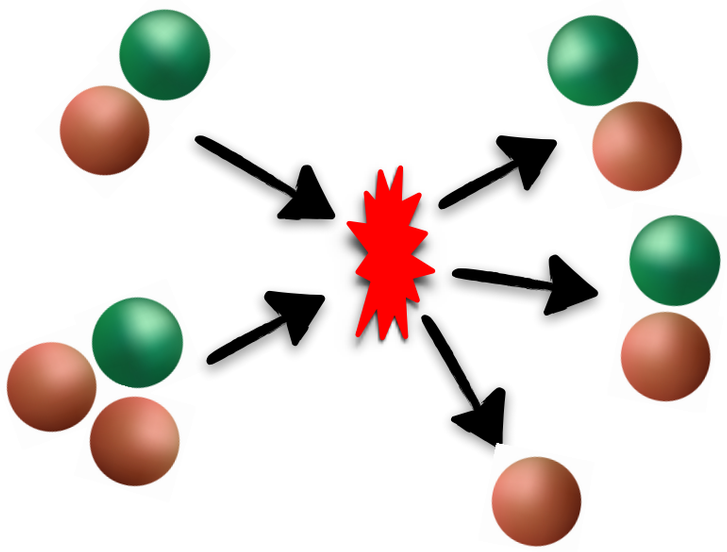
V. Bernard, E. Epelbaum, R. J. Furnstahl, J. Golak, K. Hebeler, T. Hüther, H. Kamada, H. Krebs, Ulf-G. Meißner,
P. Maris, J. A. Melendez, A. Nogga, P. Reinert, R. Roth, R. Skibinski, V. Soloviov, K. Topolnicki,
J. P. Vary, Yu. Volkotrub, H. Witala and T. Wolfgruber
(LENPIC Collaboration)

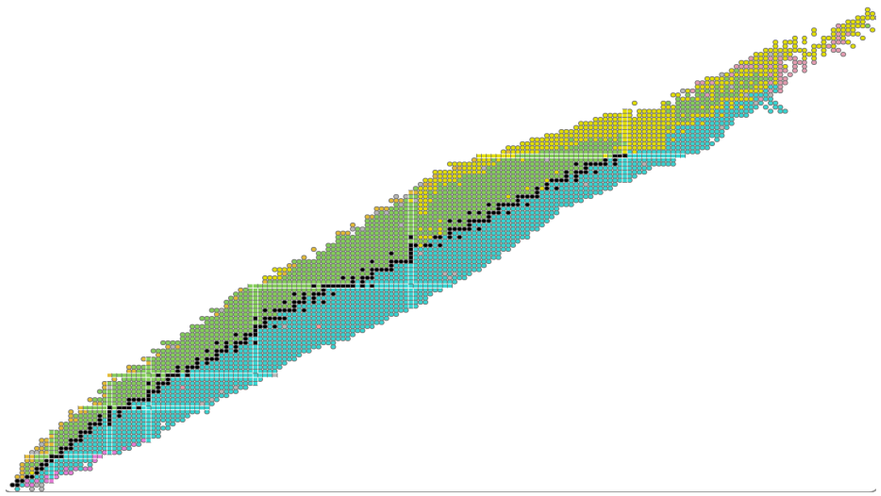
LENPIC aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents



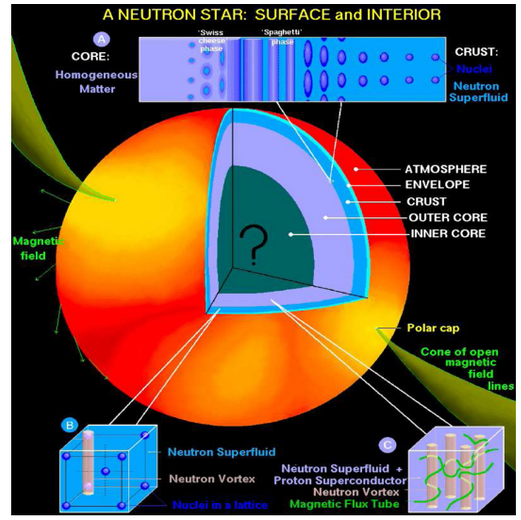
Outline

- SMS interaction
  - LENPIC activities for NN up to N^4LO^+ and 3NF at N^2LO
  - Tritium beta decay up to N^2LO
- Status report on 3NF at N^3LO and beyond
  - Method for derivation of nuclear forces in chiral EFT
  - PWD of 3NF at N^3LO
  - Short-range 3NF-LECs up to N^4LO
  - Woodbury emulator for 3N system





Livechart, IAEA: https://www-nds.iaea.org

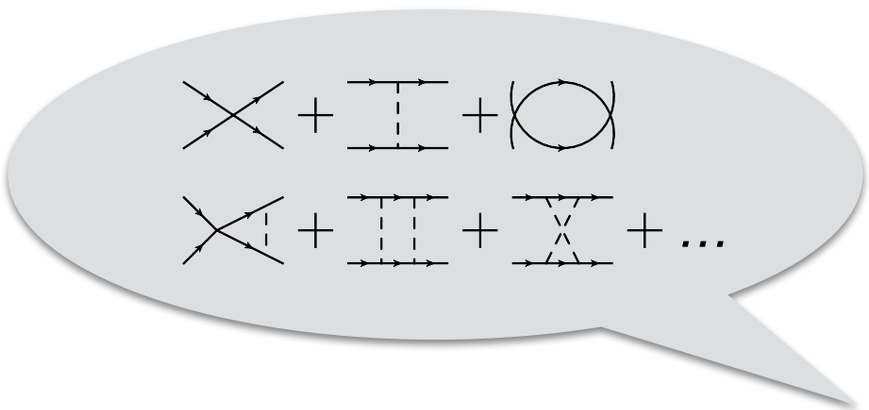


Lattimer: NAR54 (2010) 101

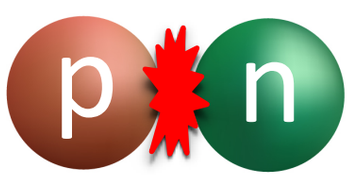
QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle=E|\Psi\rangle$$

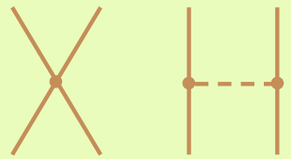
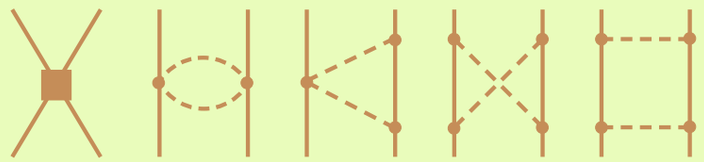
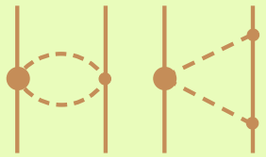
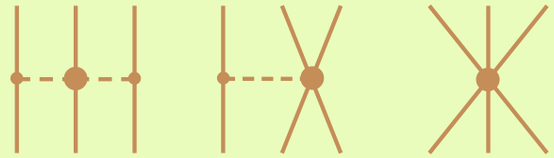
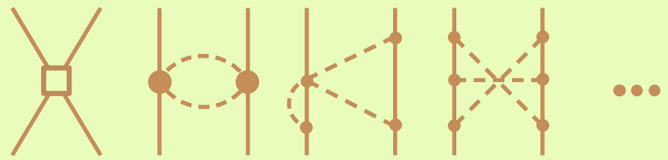
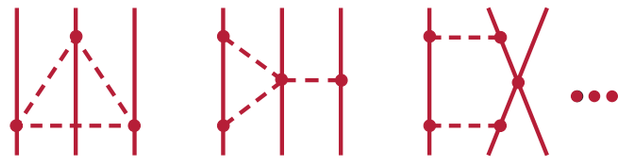
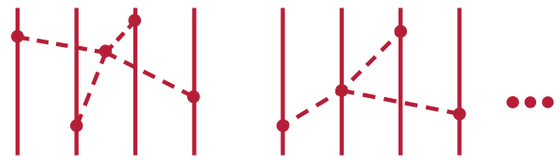
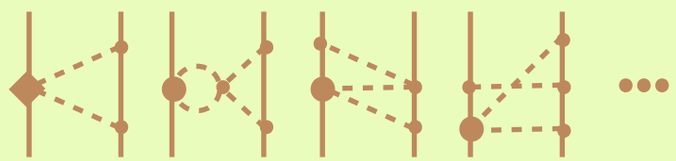
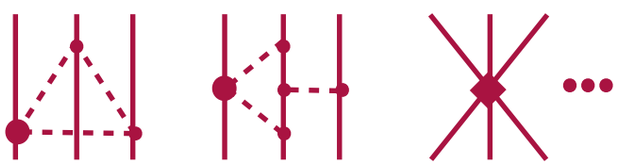
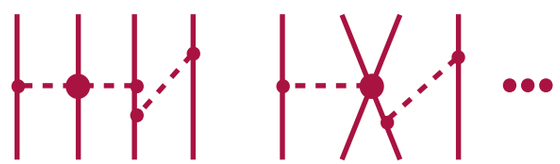
Weinberg ’91



Chiral EFT is a systematic tool for derivation of nuclear forces below pion-production threshold



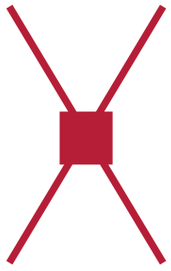
Chiral Expansion of the Nuclear Forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg ’90	—	—
NLO (Q^2)	 Ordonez, van Kolck ’92	—	—
N^2LO (Q^3)	 Ordonez, van Kolck ’92	 van Kolck ’94; Epelbaum et al. ’02	⇦ Available matrix elements LENPIC ´19 —
N^3LO (Q^4)	 Kaiser ’00 - ‘02	[parameter-free]  Bernard, Epelbaum, HK, Meißner,’08, ’11	[parameter-free]  Epelbaum ’06
N^4LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk ’15 Epelbaum, HK, Meißner ’15	 Girlanda, Kievsky, Viviani ’11 HK, Gasparyan, Epelbaum ’12,’13 Huessmann, HK, Epelsbaum, ´26 (short-range loop contrib. still missing)	 still have to be worked out

Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)

NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)

N^{2}LO [Q^3]: no new terms

N^{3}LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)

N^{4}LO [Q^5]: + 5 IB operators

N^{4}LO$^+$ [Q^6]: + 4 operators (F-waves)

# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

Summary on NN

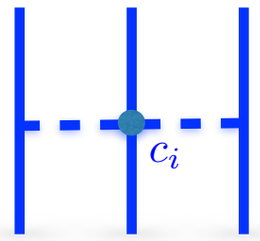
- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data

  χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

Beyond Two-Body System

Maris et al. (LENPIC) PRC103 (2021) 054001; PRC106 (2022) 064002

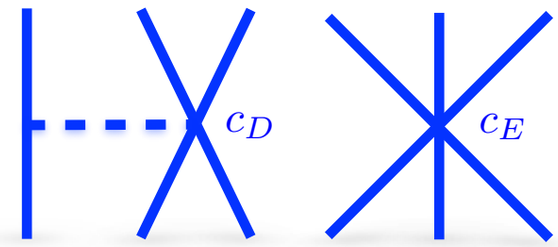
c_i´s are extracted from solutions of Roy-Steiner equation in pion-nucleon scattering: **Hoferichter et al. PRL115 (2015) 192301**

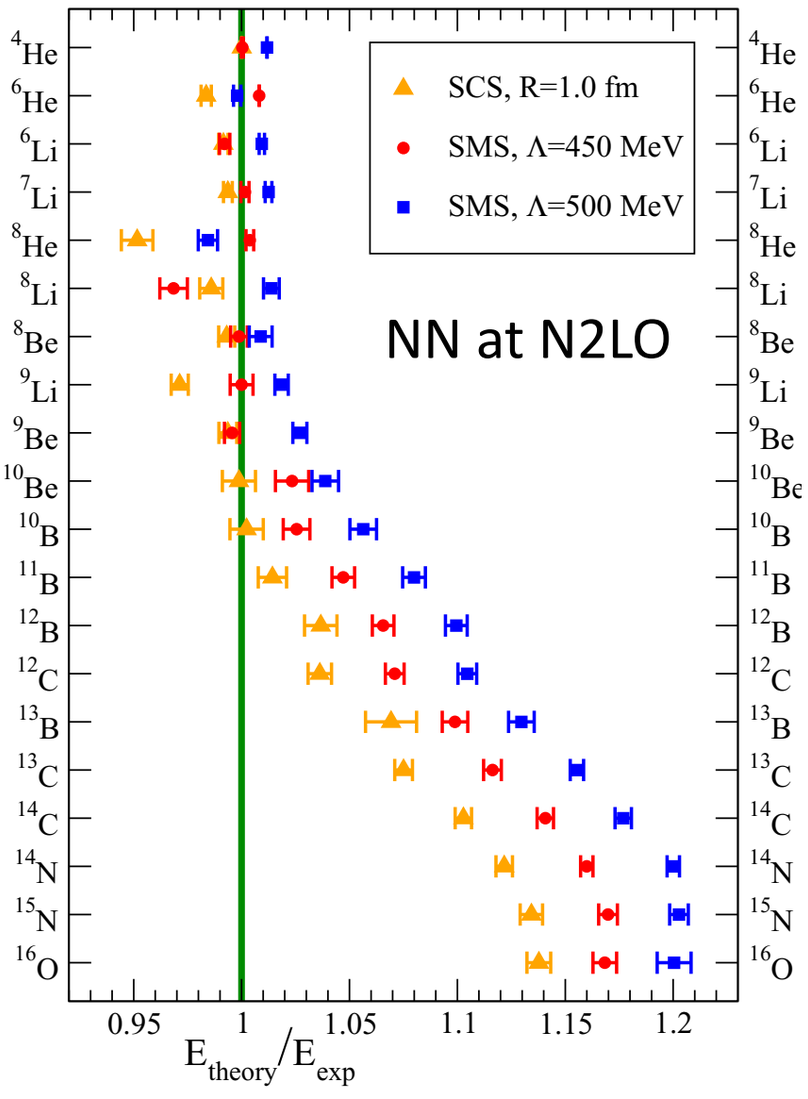


Requirement to reproduce ^{3}H correlates c_D & c_E

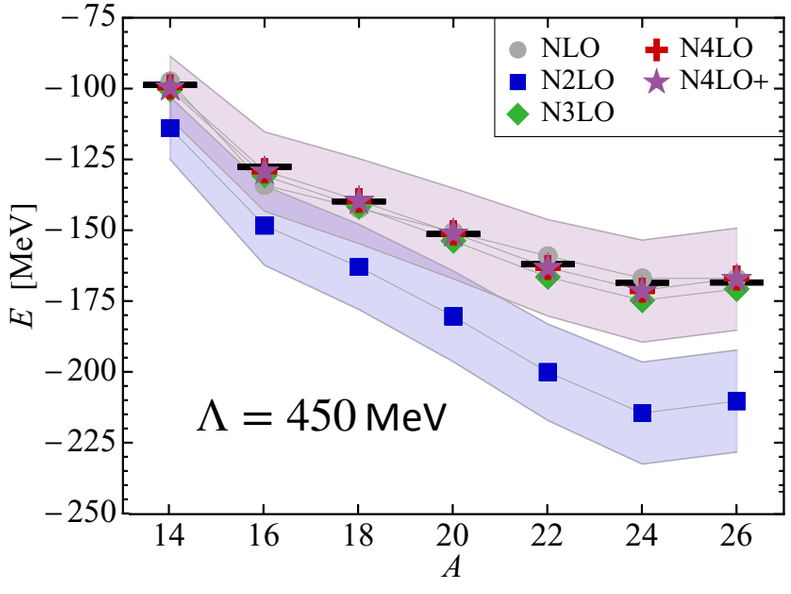
c_D is fitted to the minimum of Nd-scattering cross section at $E^N_{\rm lab} = 70\,{\rm MeV}$

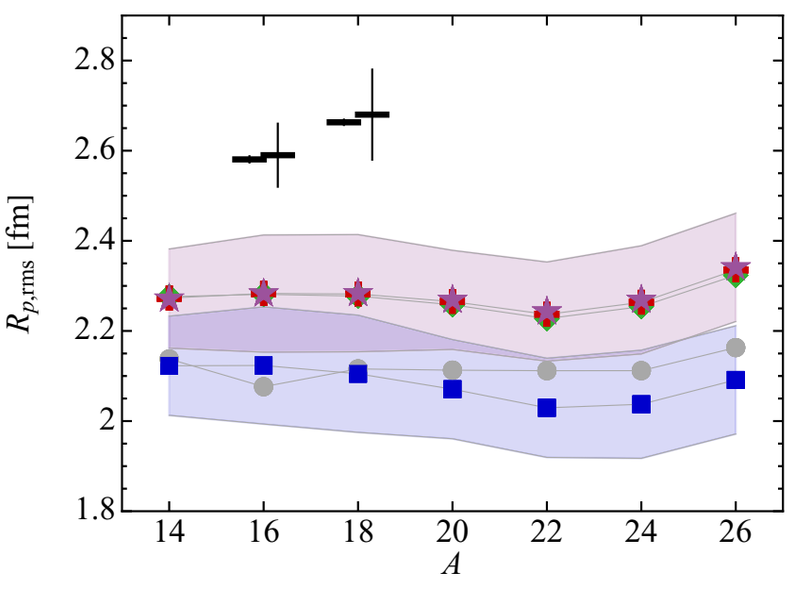
Sekiguchi et al. PRC65 (2002) 034003





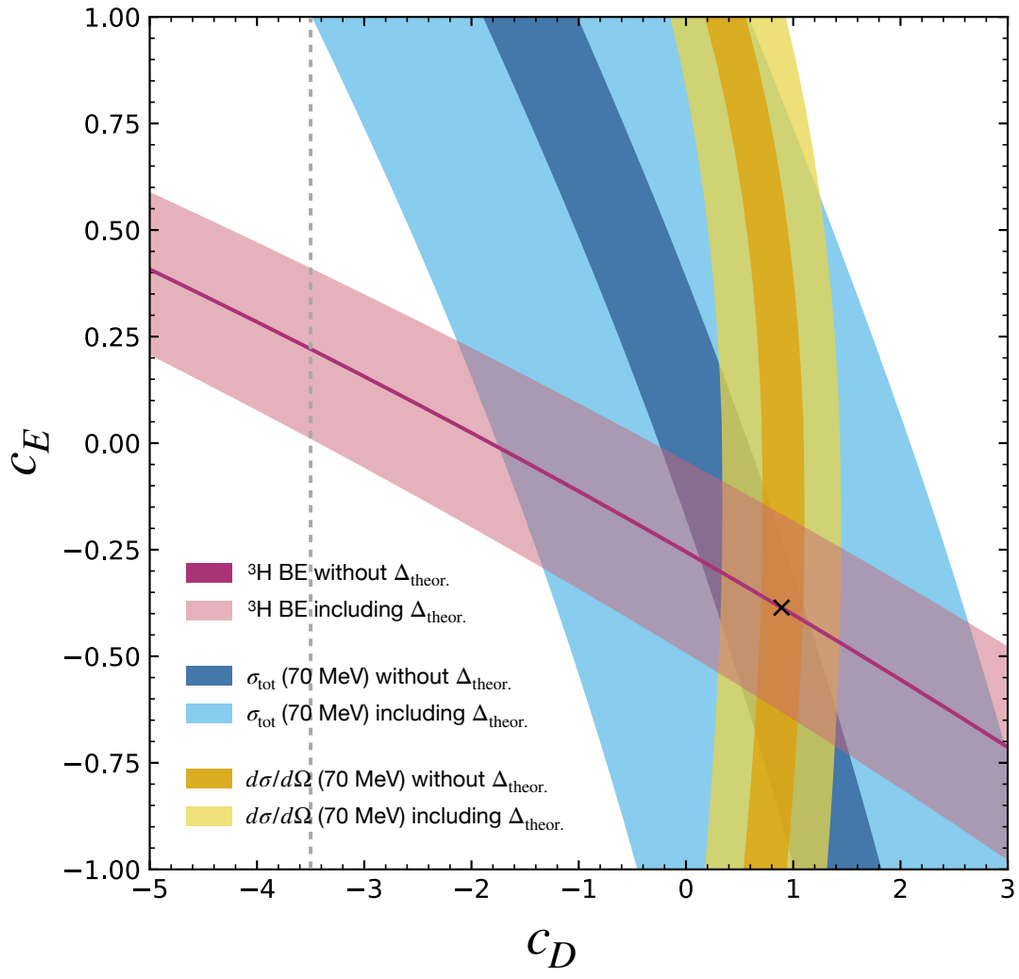
- Systematic overbinding removed for NN at N^{4}LO$^+$
- Well reproduced ground state energies of even oxygen isotopes (IM-NCSM)
- Systematic underestimation of point-radii





Tritium Beta Decay at N²LO

Jimenez, Heihoff, Golak, Epelbaum, HK, Reinert, Skibinski, Topolniki, Witała, arXiv:2606.26926v1



- Similar to

  Baroni et al. PRC94 (2015) 024003; PRC98 (2018) 044003,

  King et al. PRC102 (2020) 022501

  we find for single-nucleon current contribution

  $\text{GT} = (92.31 \ldots 93.78) \times 10^{-2}$ compared with

  $\text{GT}_{\text{emp}} = 94.84(19) \times 10^{-2}$

- Inclusion of 2N current operator at N²LO leads to

  $\delta\text{GT}_{\text{MEC}} = (5.81 \ldots 9.49) \times 10^{-2}$

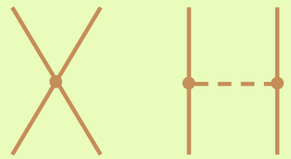
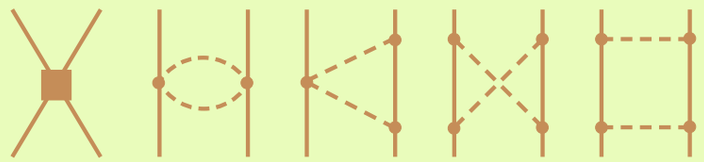
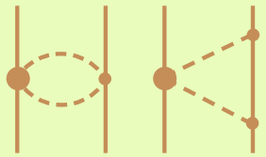
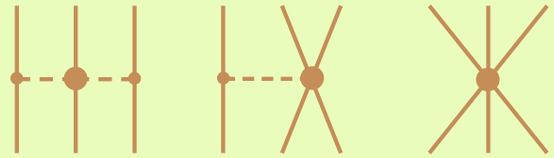
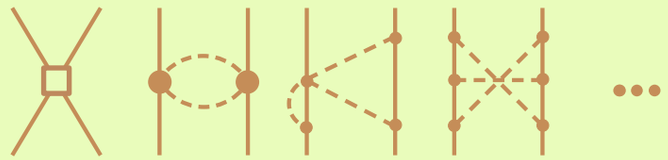
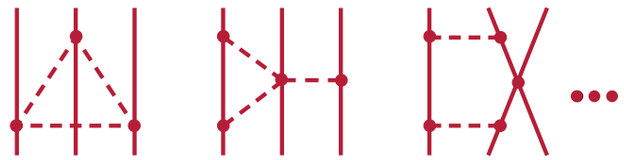
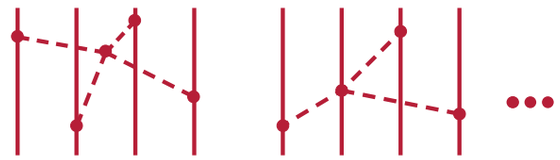
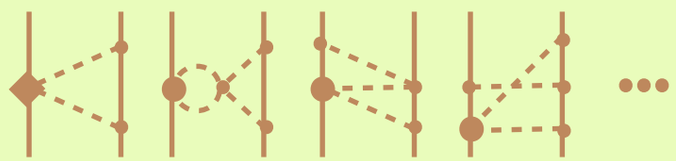
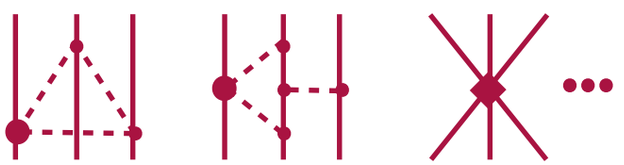
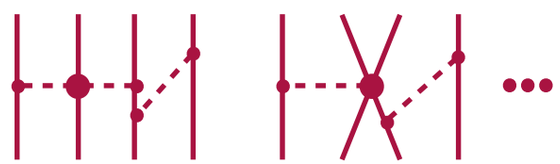
  → Overprediction of GT by 4…9 %

GT seems to be fine-tuned: $\delta\text{GT}_{\text{MEC,long-range}} \sim 10 \times 10^{-2}$ → one needs strong negative contribution from c_D to reproduce GT_{emp}

Slow convergence of chiral expansion of GT seen in explorative works on 2N current at N³LO:

$\delta\text{GT}^{\text{dim-reg}}_{\text{N3LO,long-range}} = (-3.64 \ldots -5.43) \times 10^{-2}$ **Baroni et al. PRC98 (2018) 044003,**

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Methods for Derivation of 3NF beyond N²LO

HK, Epelbaum, PRC110 (2024) 4, 044003

HK, Epelbaum, PRC 110 (2024) 4, 044004

Path-Integral Framework for Derivation of Nuclear Forces

HK, Epelbaum, PRC110 (2024) 4, 044003

Path-integral Approach

We start with generating functional:

$$Z[\eta^\dagger,\eta]=\int[DN^\dagger][DN][D\pi]\exp\left(i\int d^4x\left(\mathscr{L}_\pi+\mathscr{L}_{\pi N}+\mathscr{L}_{NN}+\mathscr{L}_{NNN}+\eta^\dagger(x)N(x)+N^\dagger(x)\eta(x)\right)\right)$$

- Integrate over pion fields via loop-expansion of the action

  → expansion of the action around the classical pion solution

- Perform instant decomposition of the remaining interactions between nucleons

- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction

- Calculate functional determinant to get one-loop corrections to few-nucleon forces

Checks in dimensional regularization

Unitary transformation (Okubo) & path-integral approaches lead to the same chiral EFT nuclear forces up to N^4LO

Symmetry Preserving Regulator

HK, Epelbaum, PRC 110 (2024) 4, 044004

Gradient-Flow Equation (GFE)

Balitsky, Yung, PL168B (1986) 113; Irwin, Manton, PLB 385 (1996) 187

Yang-Mills gradient flow in QCD: **Lüscher, JHEP 04 (2013) 123**

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \quad \text{with} \quad B_\mu|_{\tau=0} = A_\mu \quad \& \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT: **HK, Epelbaum, PRC 110 (2024) 4, 044004**

(Proposed in various talks by D. Kaplan for nuclear forces)

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \text{ with } w = \sqrt{W} \text{ and } \mathrm{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\tau\cdot\phi(1-\alpha\phi^2) - \frac{\phi^2}{2}\left[1+\left(\frac{1}{4}-2\alpha\right)\phi^2\right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty}\frac{1}{F^n}\phi_b^{(n)}$$

In the absence of external sources we have

$$\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\phi_b^{(1)}(x,\tau) = 0, \quad \phi_b^{(1)}(x,0) = \pi_b(x)$$

$$\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\phi_b^{(3)}(x,\tau) = (1-2\alpha)\partial_\mu\phi^{(1)}\cdot\partial_\mu\phi^{(1)}\phi_b^{(1)} - 4\alpha\partial_\mu\phi^{(1)}\cdot\phi^{(1)}\partial_\mu\phi_b^{(1)}$$
$$+\frac{M^2}{2}(1-4\alpha)\phi^{(1)}\cdot\phi^{(1)}\phi_b^{(1)}, \quad \phi_b^{(3)}(x,0) = 0$$

Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q,\tau) = \int d^4x\, e^{iq\cdot x}\phi_b^{(n)}(x,\tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2+M^2)}\tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}(2\pi)^4\delta(q-q_1-q_2-q_3)\int_0^\tau ds\, e^{-(\tau-s)(q^2+M^2)}e^{-s\sum_{j=1}^3(q_j^2+M^2)}$$
$$\times\left[4\alpha\, q_1\cdot q_3 - (1-2\alpha)q_1\cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\pi}(q_1)\cdot\tilde{\pi}(q_2)\tilde{\pi}_b(q_3)$$

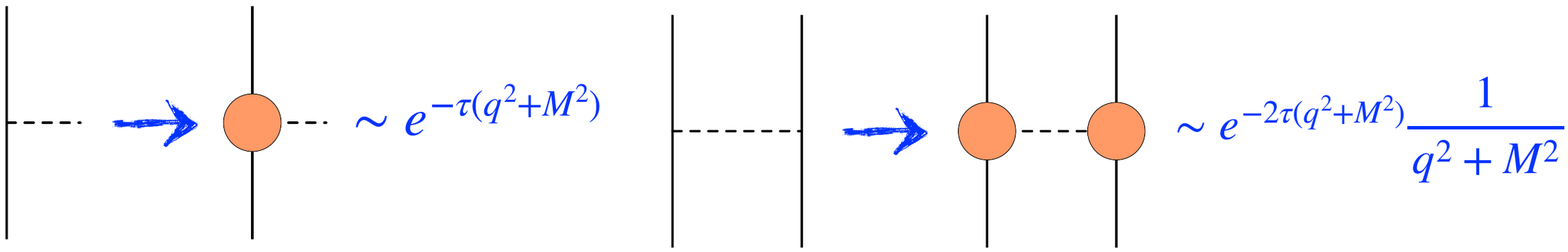
Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Regularization for Nuclear Forces

To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathscr{L}_{\pi}^{(2)}$ & $\mathscr{L}_{\pi}^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

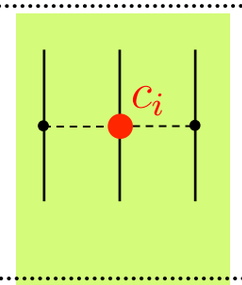
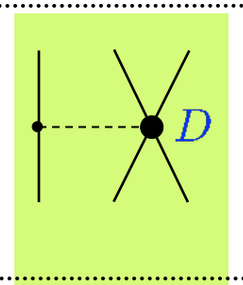
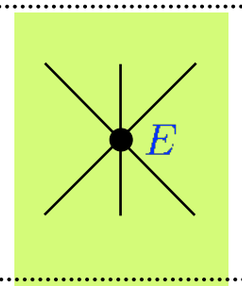
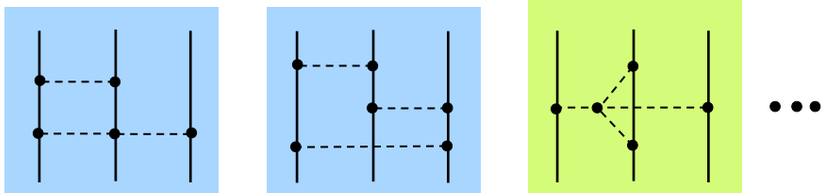
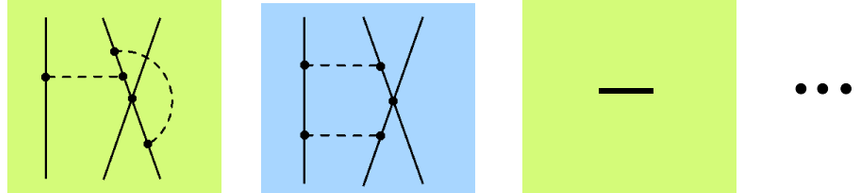
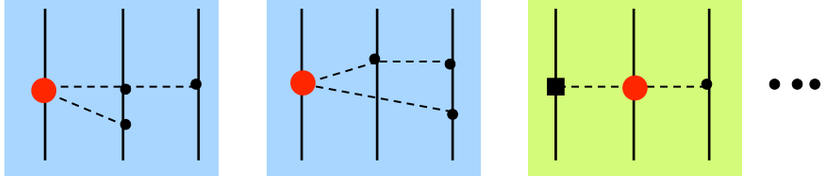
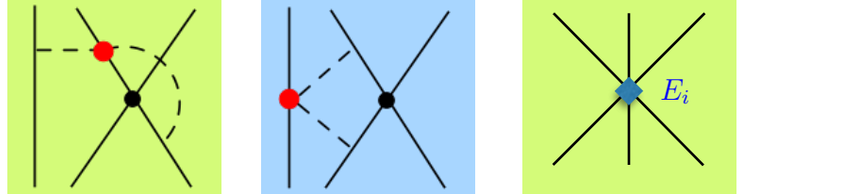
$$\mathscr{L}_{\pi N}^{(1)} = N^{\dagger}\Big(D^0 + g\, u \cdot S\Big)N \to N^{\dagger}\Big(D_w^0 + g\, w \cdot S\Big)N$$

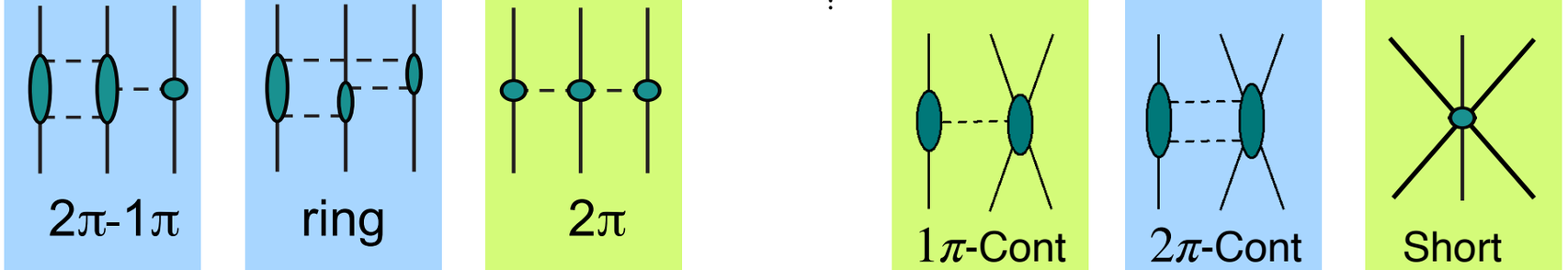


For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Status Report on 3NF

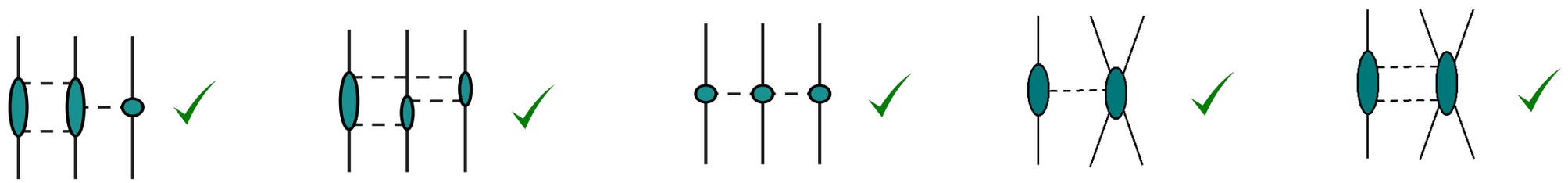
3NF up to N⁴LO

	Long - range	Short - range
NLO	—	—
N²LO	 van Kolck '94, Epelbaum et al. '02	 
N³LO	 Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)	 Bernard, Epelbaum, HK, Meißner, PRC84 (11)
N⁴LO	 HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)	 Work in progress Girlanda, Kievsky, Viviani, PRC84 (11)



Status Report on 3N at N³LO

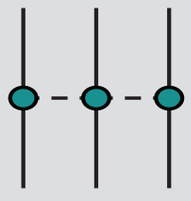
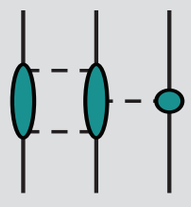
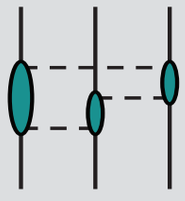
- We calculated all long- and short-range contributions to 3NF & 4NF at N³LO



3NF's are given in terms of integrals over Schwinger parameters

$$V_{3N}^{2\pi-1\pi} = \tau_1 \cdot \tau_2 \times \tau_3 \vec{q}_1 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3 \frac{e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{q_3^2+M_\pi^2} \left(-\frac{g_A^4}{F_\pi^6} \frac{q_1}{2048\pi} \int_0^\infty d\lambda \, \mathrm{erfi}\left(\frac{q_1 \lambda}{2\Lambda\sqrt{2+\lambda}} \right) \frac{\exp\left(-\frac{q_1^2+4M_\pi^2}{4\Lambda^2}(2+\lambda) \right)}{2+\lambda} + \ldots \right) + \ldots$$

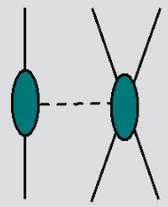
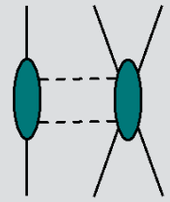
Dimension of integrals over Schwinger parameters depends on topology

Space			
Momentum	2	1	3
Coordinate	4	1	0

Short Range 3NF at N^{3}LO

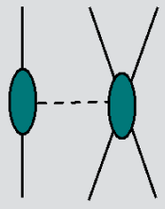
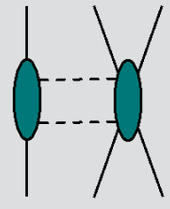
Two versions of 3NF

Version 1: Non-local short-range 3NF which can be used with SMS potential

Space		
Momentum	1	1

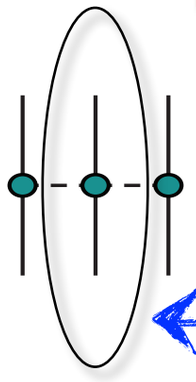
2.4 MB

Version 2: Local short-range 3NF to be used with the new NN potential

Space		
Momentum	1	1
Coordinate	0	0

0.4 MB

Pion-Nucleon Scattering up to Q^3

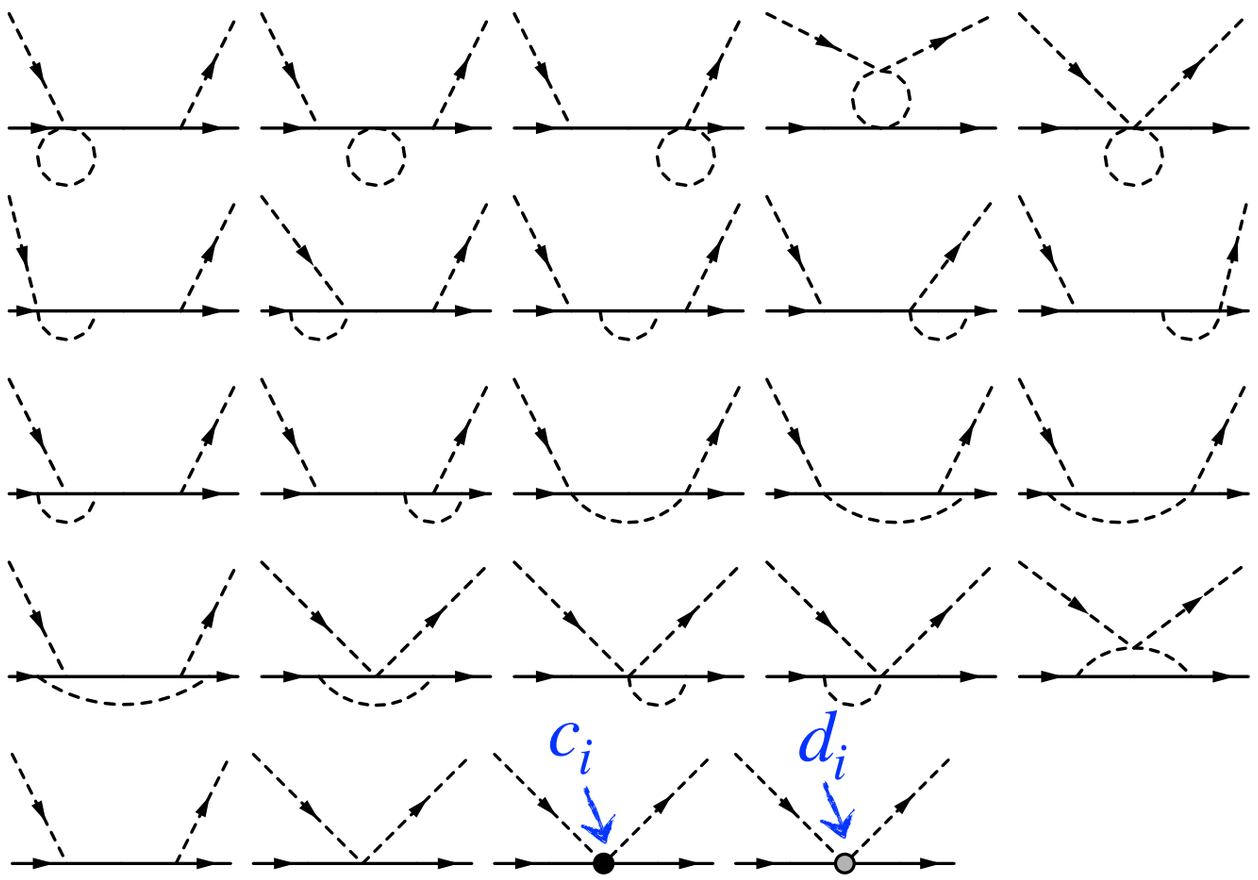


TPE topology includes pion-nucleon amplitude as a subprocess

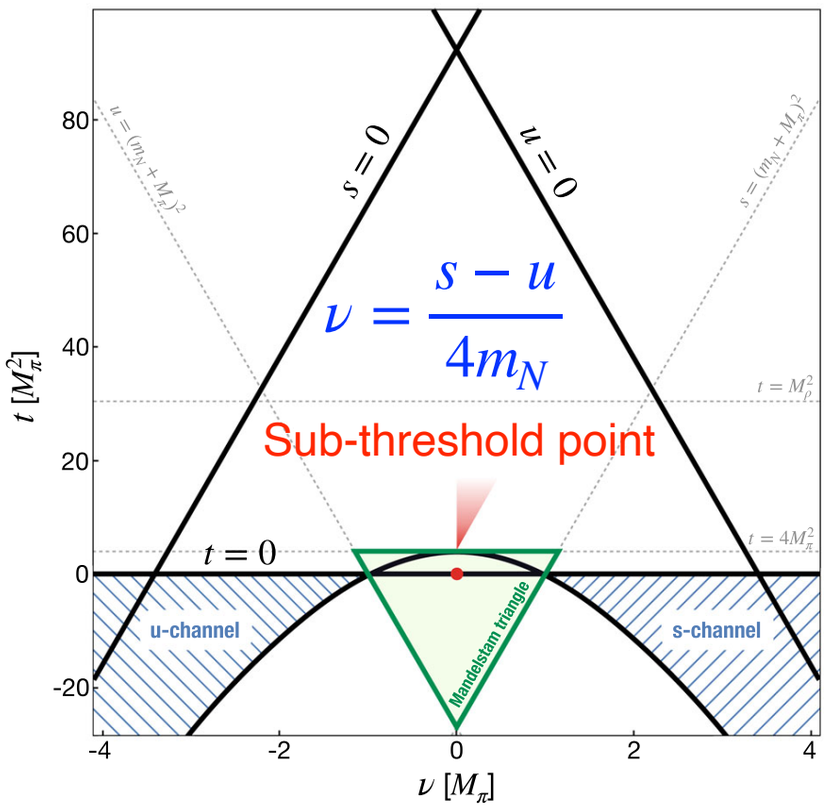
Pion-nucleon amplitude with gradient-flow regulator depends on c_i's

Calculation of pion-nucleon scattering with gradient-flow regulator required

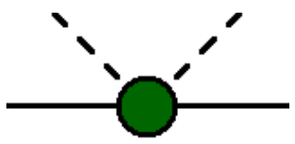
Patrick Walkowiak's master thesis



Fix LECs from pion-nucleon sub-threshold coefficients which are determined from Roy-Steiner equation



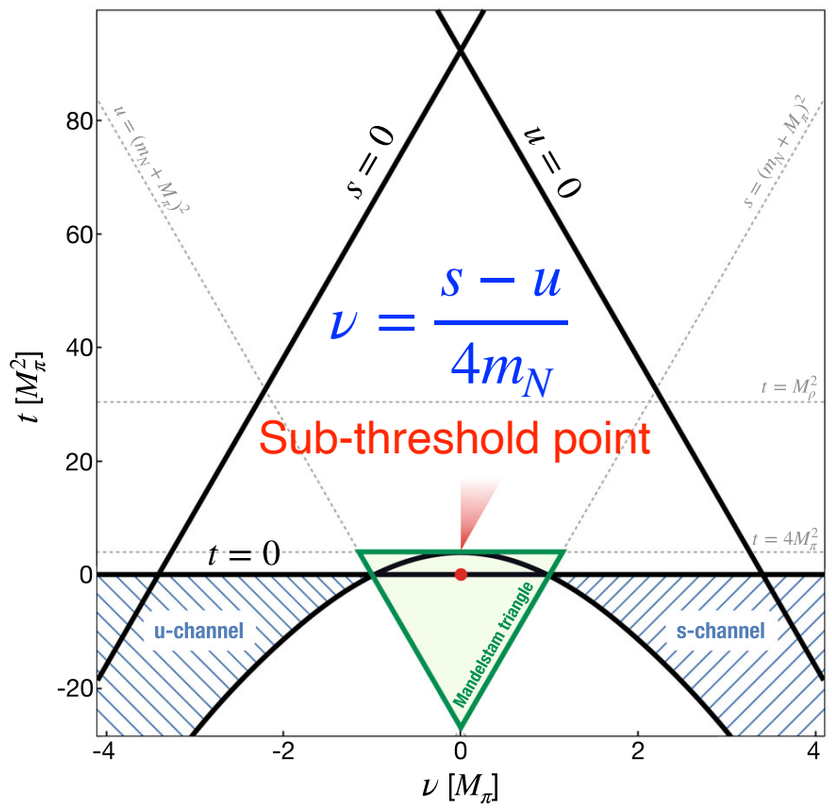
Pion-Nucleon Scattering up to Q^4



In 3NF pion-nucleon sub-amplitude appears with pion-energies $\omega_\pi = 0$

Pion-energy at sub-threshold point is close to $\omega_\pi = 0$

$$\omega_{\rm sth} = M_\pi \sqrt{\frac{M_\pi^2}{M_\pi^2 + m_N^2}} \simeq 20\,{\rm MeV} \text{ in CMS}$$



Sub-threshold expansion of pion-nucleon amplitude

$$T_{a',a} = \delta_{a',a} T^+ + \frac{1}{2}[\tau_{a'}, \tau_a] T^-, \quad T^\pm = \bar{u}' \left[D^\pm - \frac{1}{4m_N} [\not{q}', \not{q}] B^\pm \right] u$$

$$D_p^+(\nu,t) = d_{00}^+ + d_{10}^+ \nu^2 + d_{01}^+ t + d_{20}^+ \nu^4 + d_{11}^+ \nu^2 t + d_{02}^+ t^2, \quad B_p^+(\nu,t) = b_{00}^+ \nu,$$

$$D_p^-(\nu,t) = d_{00}^- \nu + d_{10}^- \nu^3 + d_{01}^- \nu t, \quad B_p^-(\nu,t) = b_{00}^- + b_{10}^- \nu^2 + b_{01}^- t$$

- Express LECs c_i, d_i, e_i in terms of sub-threshold coefficients $d_{ij}^\pm, b_{ij}^\pm$ which are determined from Roy-Steiner equation

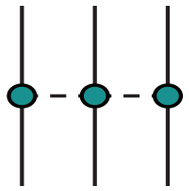
Hoferichter, Ruiz de Elvira, Kubis, Meißner, Phys. Rep. 625 (2016) 1

Partial Wave Decomposition of 3NF

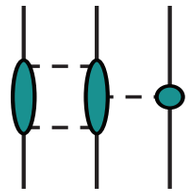
- PWD all N^{3}LO contributions to 3NF finished **Kai Hebeler, Andreas Nogga**



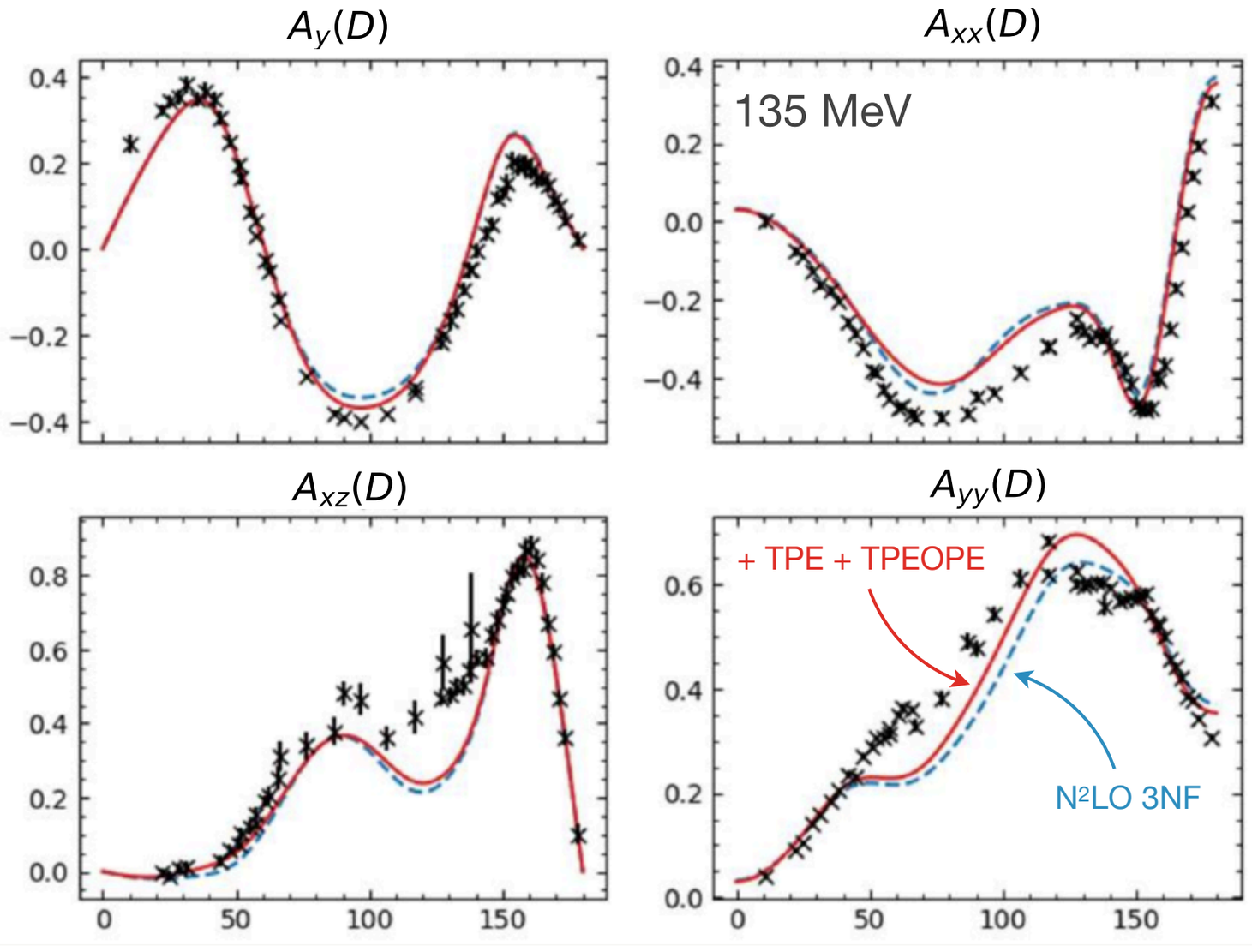
- first experience with TPE & TPE-OPE: sizable, but cancelations



$\delta B_{^3\mathrm{H}} = -315\,\mathrm{keV}$ (repulsive)

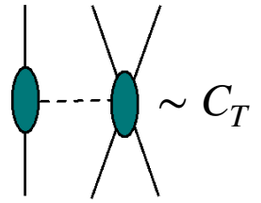


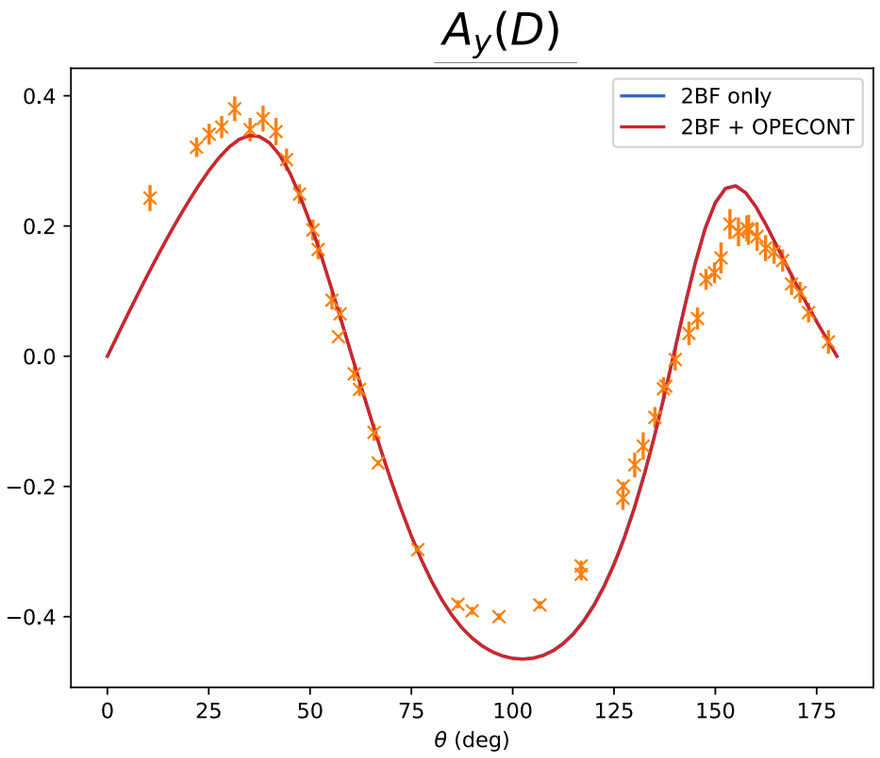
$\delta B_{^3\mathrm{H}} = 308\,\mathrm{keV}$ (attractive)

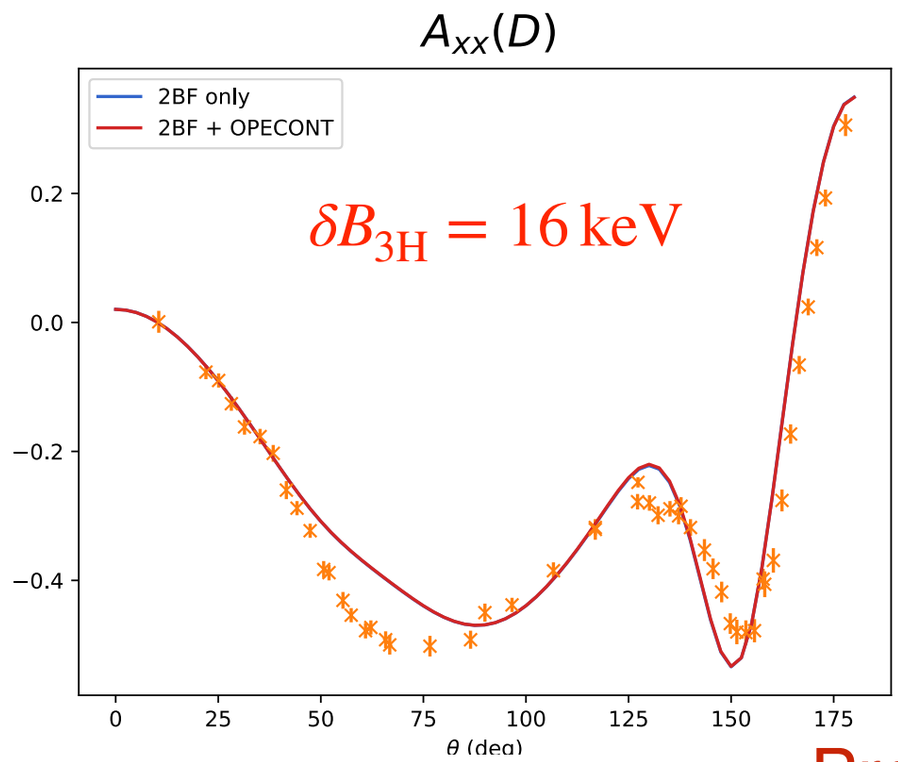


Preliminary

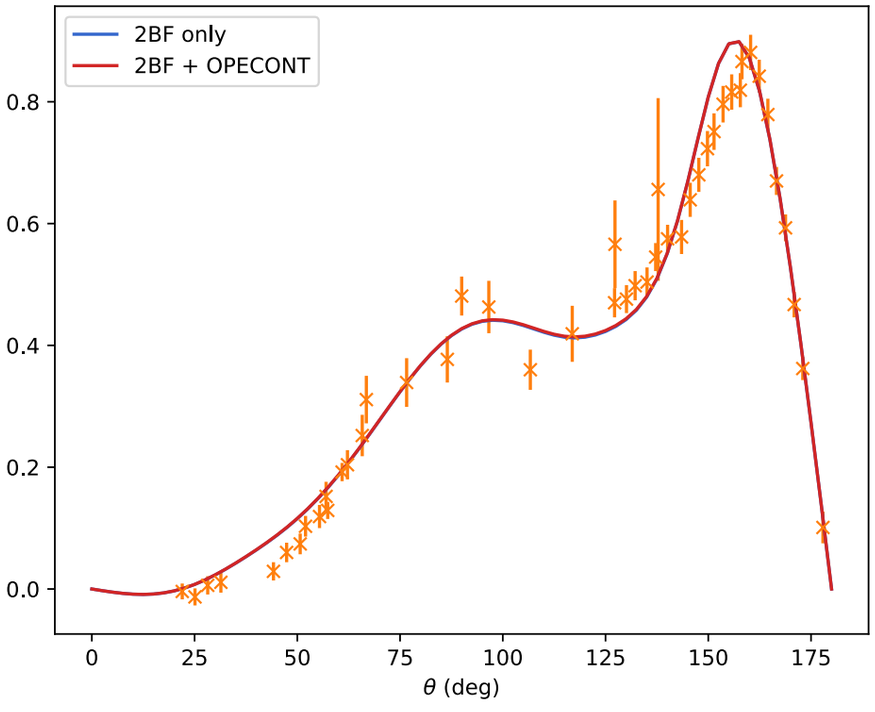
Partial Wave Decomposition of 3NF

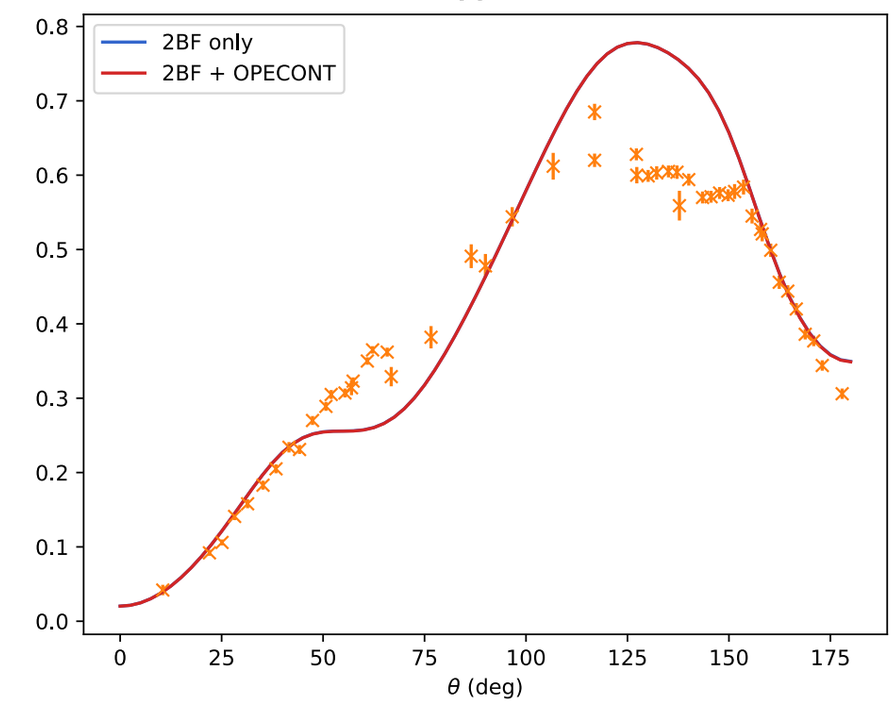
- Small contribution to ND-scattering at $E = 135\,\mathrm{MeV}$ from short-range part 





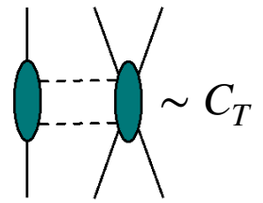
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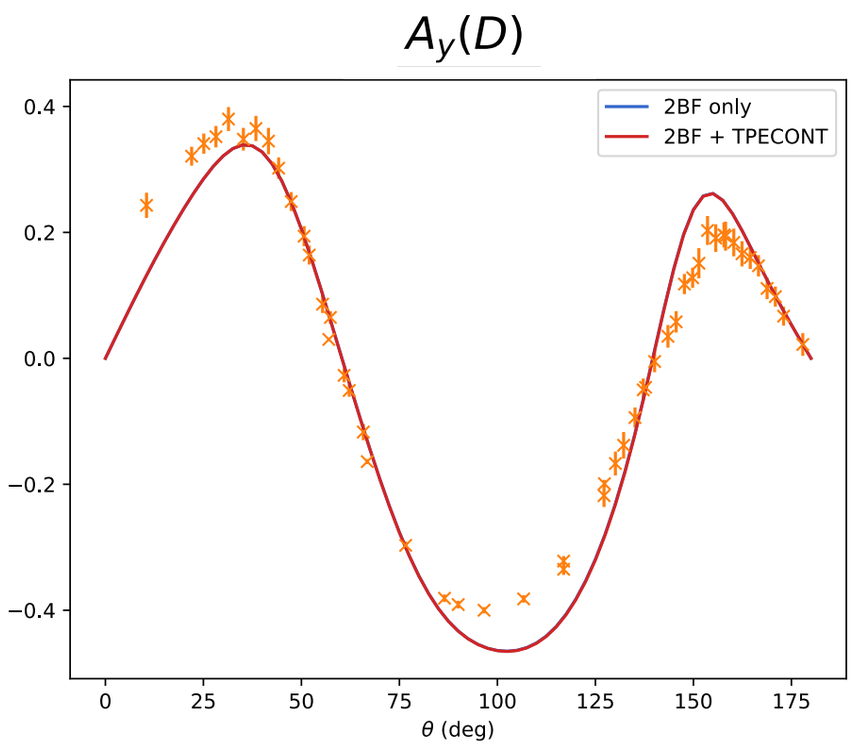


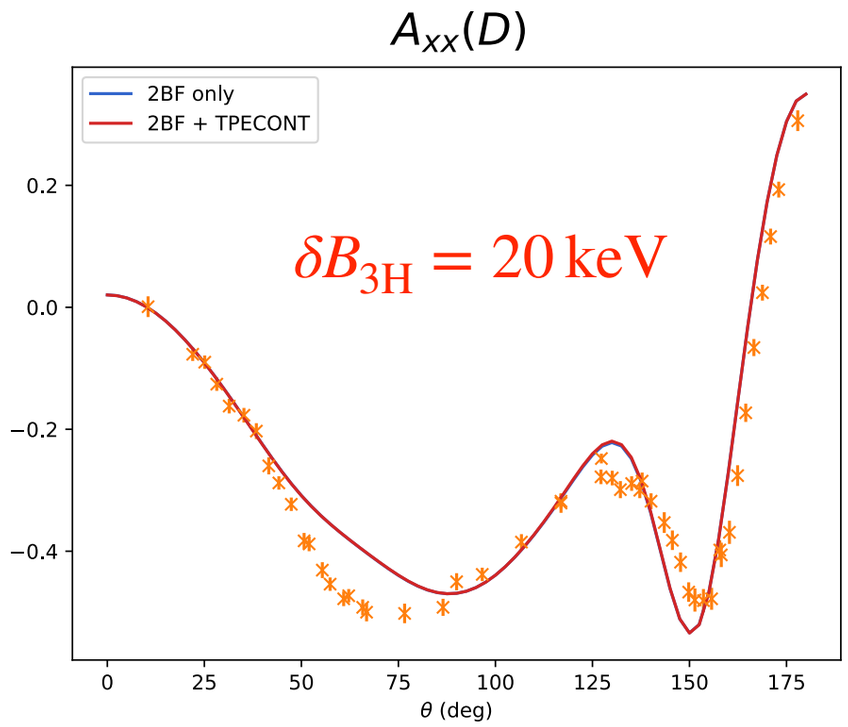


Partial Wave Decomposition of 3NF

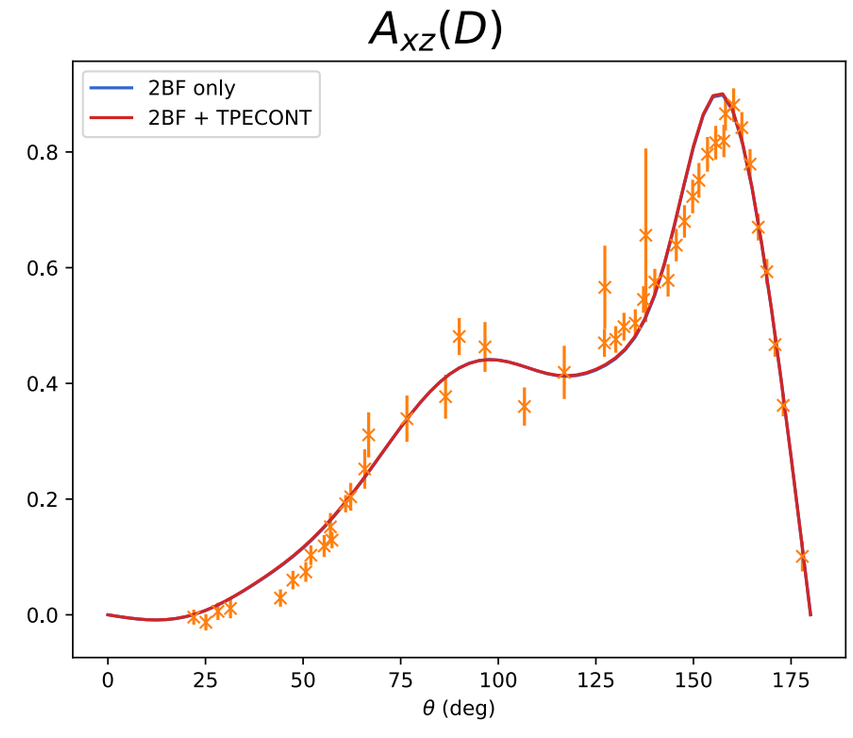
- Small contribution to ND-scattering at $E = 135\,\mathrm{MeV}$ from short-range part

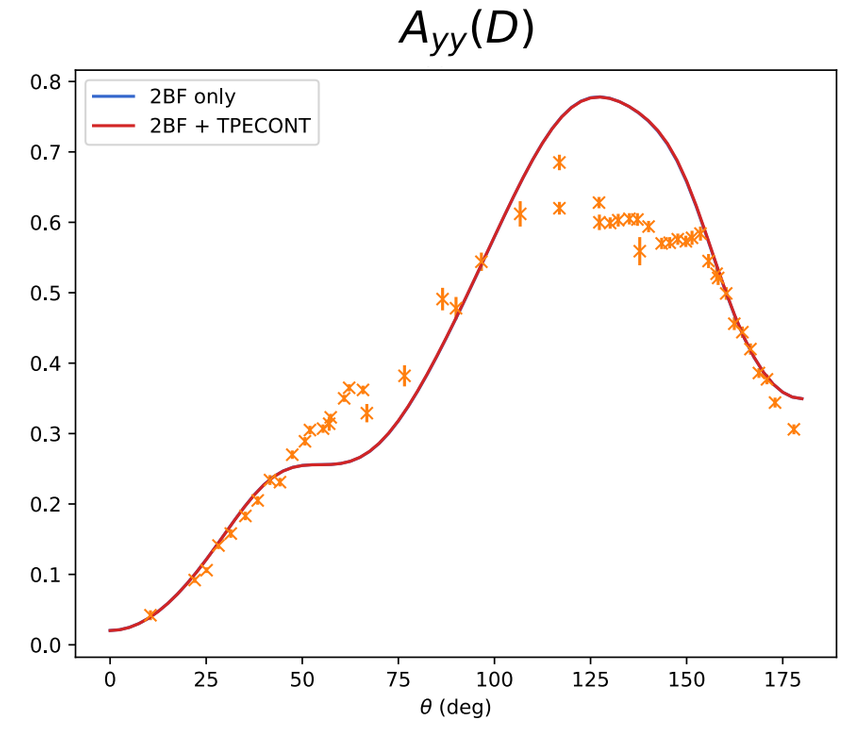




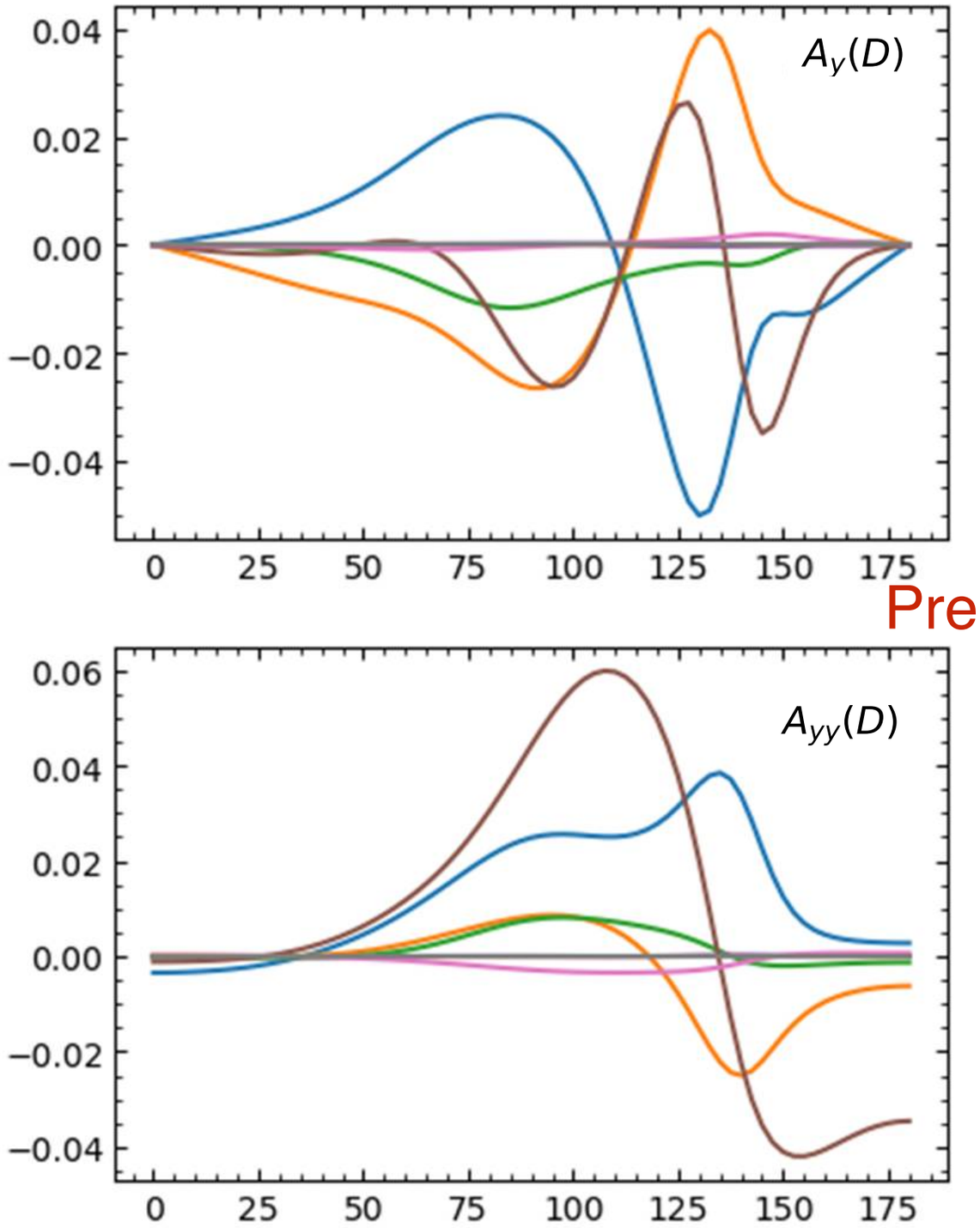


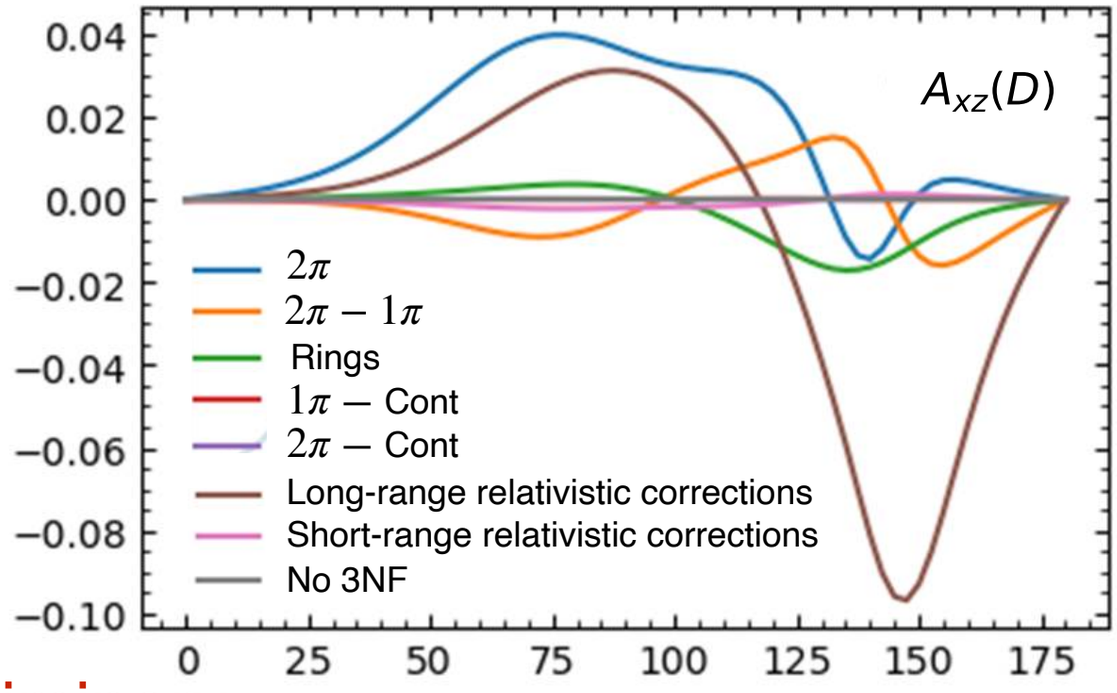
Preliminary



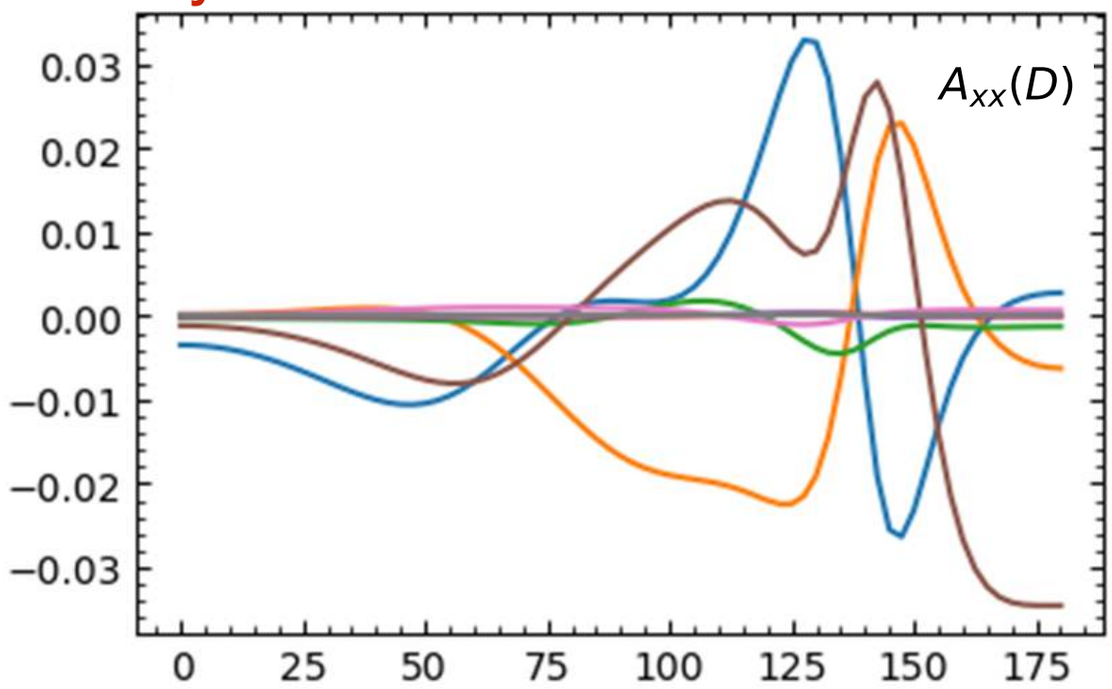


Various Contributions of N3LO 3NF





Preliminary



Three-Nucleon Forces

Counter Terms at N^{4}LO

Induced 3NF from Short-Range NN UT

Girlanda, Kievsky, Viviani, PRC84 (2011) 014001; PRC102 (2020) 019903(E)

$$U = \exp\Big(-\sum_j \beta_j T_j\Big) \;\rightarrow\; \delta H = U^\dagger H U - H_0 \simeq \sum_j \Big[(H_0 + V^{\rm LO}_{\rm short-range} + H^{\rm LO}_{1\pi}), \beta_j T_j\Big]$$

$$\langle \vec{p}\,'_1 \vec{p}\,'_2 \,|\, T_1 \,|\, \vec{p}_1 \vec{p}_2 \rangle = \vec{k}\cdot\vec{q}, \quad \langle \vec{p}\,'_1 \vec{p}\,'_2 \,|\, T_2 \,|\, \vec{p}_1 \vec{p}_2 \rangle = \vec{k}\cdot\vec{q}\,\vec{\sigma}_1\cdot\vec{\sigma}_2, \quad \langle \vec{p}\,'_1 \vec{p}\,'_2 \,|\, T_3 \,|\, \vec{p}_1 \vec{p}_2 \rangle = \vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{q} + \vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{k}$$

$$\langle \vec{p}\,'_1 \vec{p}\,'_2 \,|\, T_4 \,|\, \vec{p}_1 \vec{p}_2 \rangle = i\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{P}\times\vec{k}), \quad \langle \vec{p}\,'_1 \vec{p}\,'_2 \,|\, T_5 \,|\, \vec{p}_1 \vec{p}_2 \rangle = \frac{1}{2}\big(\vec{\sigma}_1\cdot\vec{P}\,\vec{\sigma}_2\cdot\vec{q} - \vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{P}\big)$$

lead to $O(m^2)$ enhanced boost operators:
incompatible with assumed non-relativistic limit for interacting nucleons
→ $\beta_{4,5} = O(m^{-1} F_\pi^{-2} \Lambda_b^{-2})$ → Induced 3NF is N^{5}LO

δH has the form of N^{4}LO 3NF, however $\beta_i = O(m\, F_\pi^{-2} \Lambda_b^{-4})$

$$m/\Lambda_b = (m/Q)(Q/\Lambda_b) \sim (Q/\Lambda_b)^{-2}(Q/\Lambda_b) = (Q/\Lambda_b)^{-1}$$

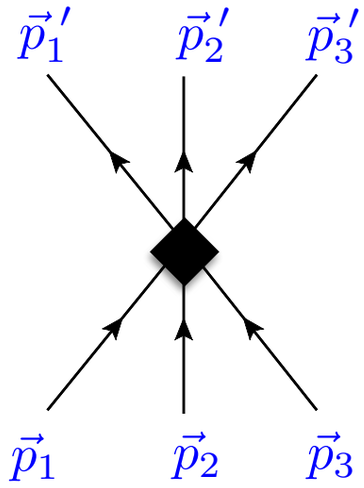
→ Induced 3NFs contribute already at N^{3}LO

E-Like 3NF LECs at N^{4}LO

Girlanda, Kievsky, Viviani, PRC84 (2011) 014001; PRC102 (2020) 019903(E)

Complete set of independent operators

$$
\begin{aligned}
\mathcal{O}_1 &= -\,\vec{q}_1^{\,2}, \\
\mathcal{O}_2 &= -\,\vec{q}_1^{\,2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
\mathcal{O}_3 &= -\,\vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2, \\
\mathcal{O}_4 &= -\,\vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
\mathcal{O}_5 &= -\,(3\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{\sigma}_2 - \vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2), \\
\mathcal{O}_6 &= -\,(3\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{\sigma}_2 - \vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
\mathcal{O}_7 &= -\,i\,\big((\vec{k}_1-\vec{k}_2)\times\vec{q}_1\big)\cdot(\vec{\sigma}_1+\vec{\sigma}_2)/2, \\
\mathcal{O}_8 &= -\,i\,\big((\vec{k}_1-\vec{k}_2)\times\vec{q}_1\big)\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3/2, \\
\mathcal{O}_9 &= -\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_2\cdot\vec{\sigma}_2, \\
\mathcal{O}_{10} &= -\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_2\cdot\vec{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
\mathcal{O}_{11} &= -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1, \\
\mathcal{O}_{12} &= -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
\mathcal{O}_{13} &= -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,.
\end{aligned}
$$



$$\vec{q}_j = \vec{p}_j' - \vec{p}_j$$

$$\vec{k}_j = (\vec{p}_j' + \vec{p}_j)/2$$

$$V = \sum_{j=1}^{13} E_j \mathcal{O}_j + 5\ \text{perm}$$

Most of E_j's can be determined from elastic nucleon-deuteron scattering

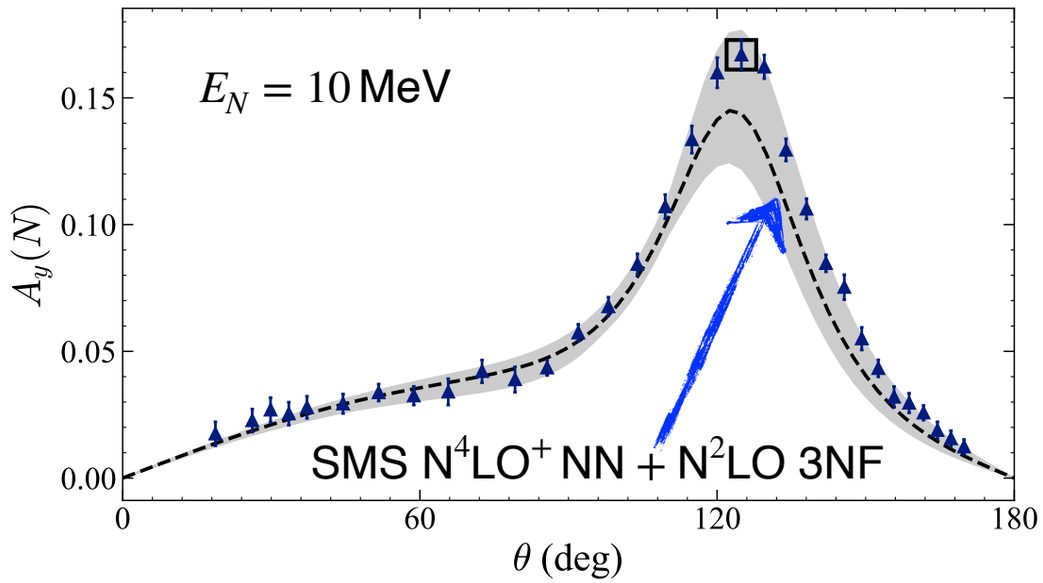
Spectroscopic E-like 3NF LECs

Cava, Filin, Heihoff, Huessmann, Epelbaum, ArXiv: 2607.26958

$V = \sum_{j=1}^{13} E_j \mathcal{O}_j + 5\ \text{perm}$: Each E_j contributes to several $J^P(T)$ channels

- Introduce E-like LECs S_j which contribute to one specific $J^P(T)$ channel

LEC	Isospin	J^P	Nd transition
S_1	$T = 1/2$	$1/2^+$	${}^2S \leftrightarrow {}^2S$
S_2			${}^2S \leftrightarrow {}^2S$
S_3			${}^2S \leftrightarrow {}^4D$
S_4			${}^2S \leftrightarrow$ breakup
S_5		$1/2^-$	${}^2P \leftrightarrow {}^2P$
S_6			${}^4P \leftrightarrow {}^4P$
S_7			${}^2P \leftrightarrow {}^4P$
S_8		$3/2^-$	${}^2P \leftrightarrow {}^2P$
S_9			${}^4P \leftrightarrow {}^4P$
S_{10}			${}^2P \leftrightarrow {}^4P$
S_{11}		$5/2^-$	${}^4P \leftrightarrow {}^4P$
S_{12}	$T = 3/2$	$1/2^-$	—
S_{13}		$3/2^-$	—



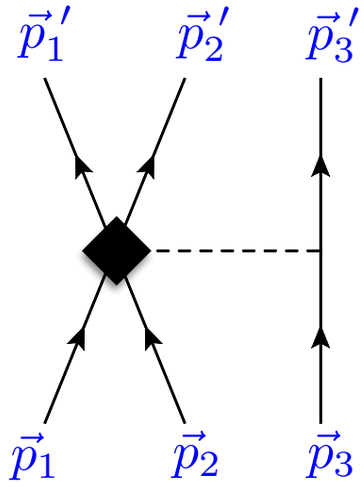
Disagreement for $A_y(N)$ at low energy can be removed by adjustment of S_{10} & S_{11}

D-Like 3NF LECs at N⁴LO

Huesmann, HK, Epelbaum, arXiv: 2602.12879v1

Complete set of independent operators

$$\mathcal{O}_1 = i\,(\vec{q}\times\vec{k})\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_2 = i\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_3 = i\,\vec{k}\cdot\vec{q}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_4 = i\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,\vec{k}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_5 = q^2(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_6 = (\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_7 = q^2(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_8 = q^2(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_9 = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{10} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{11} = i\,\vec{\sigma}_2\cdot\vec{q}\,(\vec{\sigma}_1\times\vec{k})\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{12} = i\,\vec{\sigma}_1\cdot\vec{q}_3\,(\vec{q}\times\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{13} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}_3\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{14} = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{15} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}\,q_3^2\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{16} = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,q_3^2\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\,.$$



$$\vec{q}_3 = \vec{p}_3{}' - \vec{p}_3$$
$$\vec{q} = \vec{p}\,' - \vec{p}$$
$$\vec{k} = (\vec{p}\,' + \vec{p})/2$$
$$\vec{p}\,' = (\vec{p}_1{}' - \vec{p}_2{}')/2$$
$$\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$$

$$V = \frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\boldsymbol{\tau}_3\cdot\sum_{j=1}^{16}F_j\mathcal{O}_j + 5\ \text{perm}$$

Constraints:

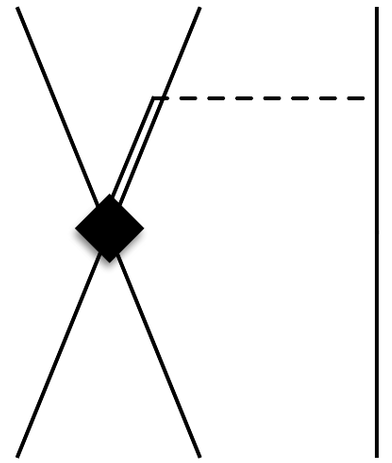
- Isospin symmetry
- Chiral symmetry
- Parity and time - reversal invariance
- Rotation - invariance in 3-dim
- Galilei - invariance

Resonance Saturation of LECs

Huesmann, HK, Epelbaum, arXiv: 2602.12879v1

Complete set of independent $NN \to N\Delta$ transition operators

$$
\begin{aligned}
\mathcal{O}_1^\Delta &= \left[i\,(\vec{q}_3 \cdot \vec{S}_2\, \vec{S}_2^\dagger \cdot ((\vec{q} - \vec{q}_3/2) \times (\vec{k} - \vec{q}_3/4))\, \mathcal{T}_2\, \mathcal{T}_2^\dagger \cdot \boldsymbol{\tau}_1 + 1 \leftrightarrow 2 \right] - \text{h.c.}, \\
\mathcal{O}_2^\Delta &= \left[\vec{q}_3 \cdot \vec{S}_2\, \vec{S}_2^\dagger \cdot \vec{\sigma}_1\, (\vec{q} - \vec{q}_3/2)^2\, \mathcal{T}_2\, \mathcal{T}_2^\dagger \cdot \boldsymbol{\tau}_1 + 1 \leftrightarrow 2 \right] - \text{h.c.}, \\
\mathcal{O}_3^\Delta &= \left[\vec{q}_3 \cdot \vec{S}_2\, \vec{S}_2^\dagger \cdot (\vec{q} - \vec{q}_3/2)\, \vec{\sigma}_1 \cdot (\vec{q} - \vec{q}_3/2)\, \mathcal{T}_2\, \mathcal{T}_2^\dagger \cdot \boldsymbol{\tau}_1 + 1 \leftrightarrow 2 \right] - \text{h.c.}, \\
\mathcal{O}_4^\Delta &= \left[\vec{q}_3 \cdot \vec{S}_2\, \vec{S}_2^\dagger \cdot (k - \vec{q}_3/4)\, \vec{\sigma}_1 \cdot (\vec{k} - \vec{q}_3/4) \mathcal{T}_2\, \mathcal{T}_2^\dagger \cdot \boldsymbol{\tau}_1 + 1 \leftrightarrow 2 \right] - \text{h.c.}\,.
\end{aligned}
$$



$\mathcal{S}$ & $\mathcal{T}$ spin & isospin $1/2 \to 3/2$ transition matrices

$$
V = \frac{g_A^2}{4F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_3 \cdot \sum_{j=1}^{4} \alpha_j \mathcal{O}_j^\Delta + 5 \text{ perm} = \frac{g_A^2}{4F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_3 \cdot \sum_{j=1}^{16} F_j^\Delta \mathcal{O}_j + 5 \text{ perm}
$$

$$
\begin{aligned}
F_1^\Delta &= -4F_2^\Delta = 4F_4^\Delta = \frac{8\alpha_1}{9\Delta}, \\
F_3^\Delta &= F_{11}^\Delta = 0, \\
F_5^\Delta &= -\frac{4\alpha_2 + 2\alpha_3}{9\Delta}, \\
F_6^\Delta &= \frac{2\alpha_3 + \alpha_4}{9\Delta}, \\
F_7^\Delta &= \frac{8\alpha_2 - \alpha_4}{18\Delta}, \\
F_8^\Delta &= \frac{4\alpha_2}{9\Delta}, \\
F_9^\Delta &= \frac{8\alpha_3 + \alpha_4}{18\Delta}, \\
F_{10}^\Delta &= 4F_{12}^\Delta = \frac{4\alpha_3 - \alpha_4}{9\Delta}, \\
F_{13}^\Delta &= \frac{2\alpha_1 - 8\alpha_2 - 4\alpha_3 + \alpha_4}{18\Delta}, \\
F_{14}^\Delta &= \frac{-2\alpha_1 - 16\alpha_2 - 8\alpha_3 + \alpha_4}{36\Delta}, \\
F_{15}^\Delta &= \frac{-2\alpha_1 - 4\alpha_3 + \alpha_4}{18\Delta}, \\
F_{16}^\Delta &= \frac{2\alpha_1 - 8\alpha_3 - \alpha_4}{36\Delta},
\end{aligned}
$$

Woodbury Emulator for 3N

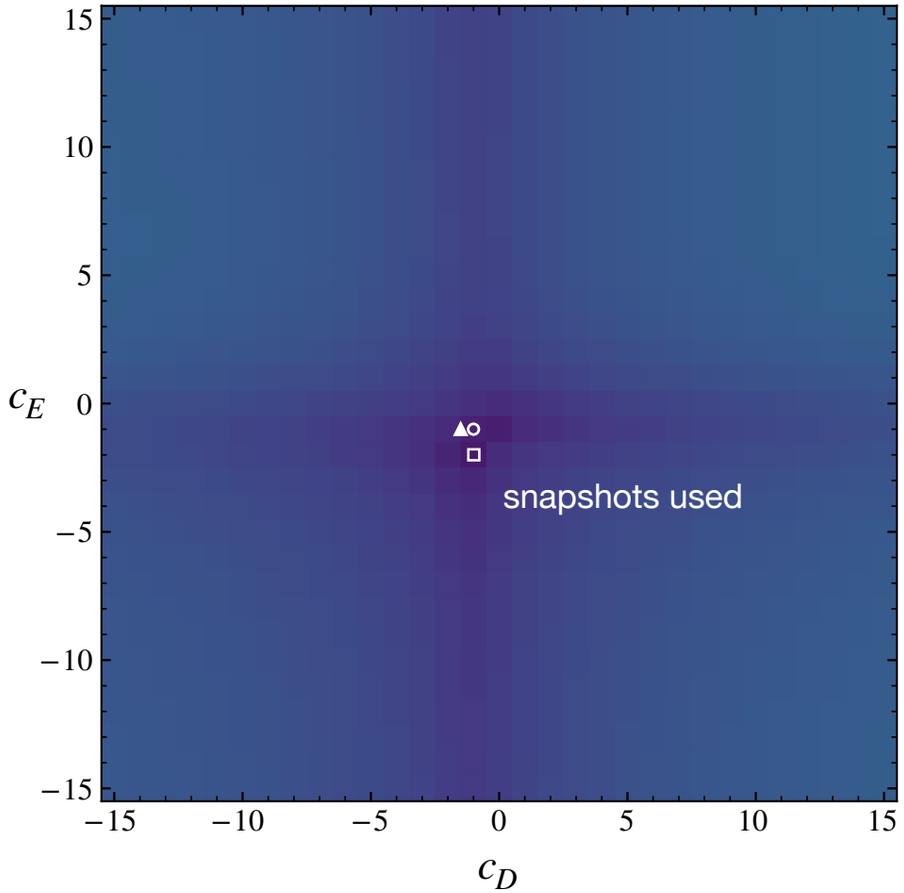
Sven Heihoff, Arseniy Filin, Evgeny Epelbaum ArXiv:2604.25792

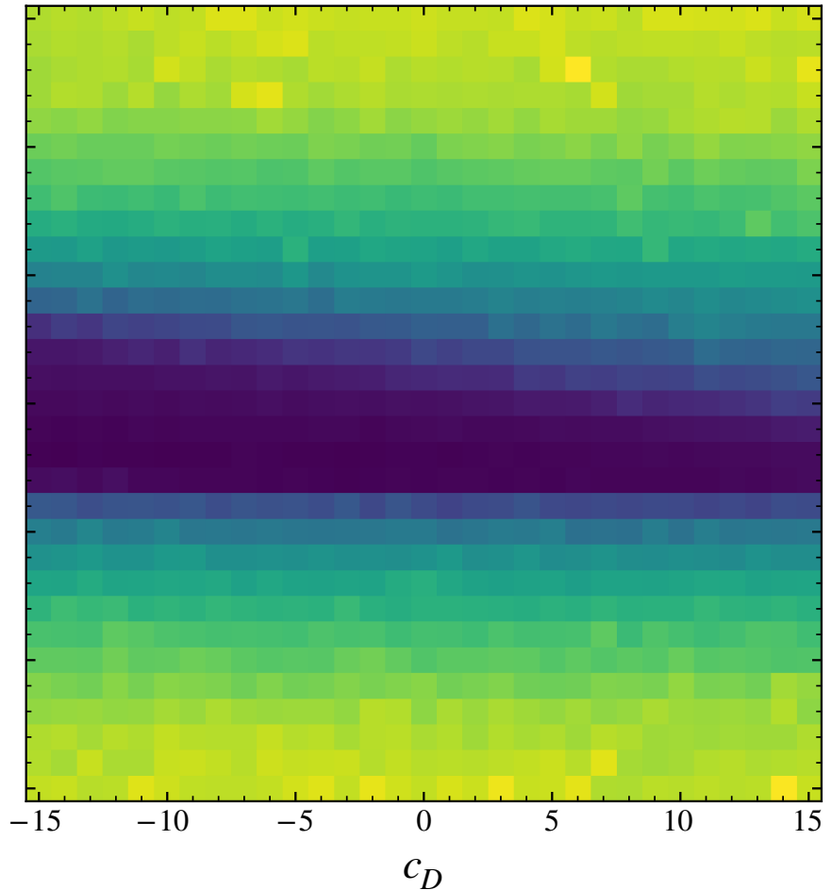
In a discretized Faddeev Eq. Interaction $V = V_0 + \sum_i c_i V_i$ is an $n \times n$ matrix

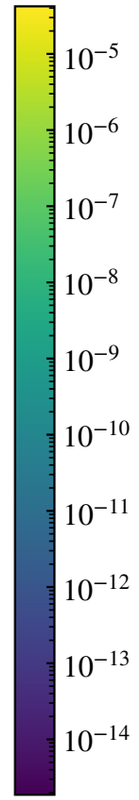
$$T^{3N}(c) = T_0^{3N} + \sum_{a=1}^{r} f_a(c) T_a^{3N} \text{ with } r = \text{rank}(V_i)$$

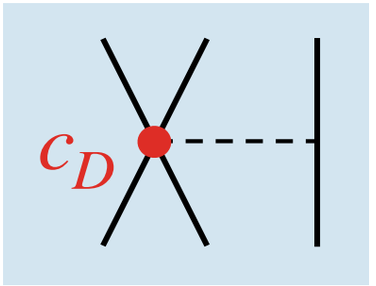
- $n \times n$ matrix Faddeev Eq. can be reduced to $r \times r$ matrix Eq. Via Woodbury identities

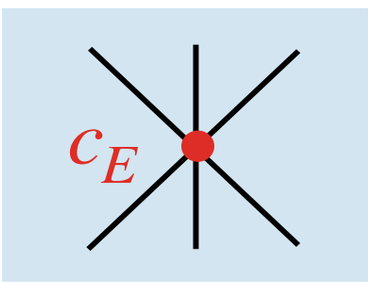
$$T^{3N}(c) = T_0^{3N} + \sum_{a=1}^{r} f_a(c) T_a^{3N} \text{ : one needs only } r+1 \text{ snapshots to determine } T_j^{3N}$$











Promising method for determination of 3N LECs

Summary I

- SMS NN up to N^4LO^+ and 3N up to N^2LO
  - Reproduce ground states of light and medium mass nuclei but underestimate point-radii
  - Consistent weak current at N^2LO overestimates GT (fine-tuning)

- Method for derivation of the regularized nuclear forces in chiral EFT
  - Path-integral approach for derivation of nuclear forces
  - Gradient flow regularization preserves chiral symmetry

Summary II

- At N^{3}LO: calculation of 3NF at N^{3}LO is finished
  - PWD of 3NF at N^{3}LO is finished
  - From 3H expectation values we see sizable TPE and TPE-OPE contributions that almost cancel each other
  - Short-range contributions seem to be small (due to $\sim C_T$)

- At N^{4}LO: 16 D-like counter terms worked out → 16 D-like + 13 E-like = 29 LECs
  - Spectroscopic basis for E-like LECs
  - One can reduce the number of D-like terms by using Δ - resonance saturation
    - → 4 D-like LECs ($\alpha_{1,2,3,4}$)
  - Promising Woodbury emulator for determination of 3N LECs at N^{4}LO