



上海交通大学
SHANGHAI JIAO TONG UNIVERSITY



Lattice Parton
Collaboration

Heavy meson light-cone distribution amplitudes from lattice QCD

高浩飞

2026.10.10 第六届中国格点量子色动力学研讨会, 深圳

In collaboration with : Xue-ying Han(SJTU), Wei Wang(SJTU) and Qi-an Zhang(BUAA)

Based on : 2604.25802; 2605.10946

- **Motivation**
- **Theory Framework**
- **Lattice QCD Calculation**
- **Summary**

Why heavy meson LCDAs

3

➤ Weak decays of heavy mesons play a crucial role in

- Precise tests of Standard Model

$$F_i^{B \rightarrow K^*}(E)$$

- Searching for New Physics

$$= C_i^I(E) \xi_a(E) + \int_0^\infty d\omega \int_0^1 du T_i^{II}(E, \omega, u) \varphi_B^+(\omega) \phi_{K^*}^{(a)}(u)$$

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

$$\mathcal{V}_{B \rightarrow K^*}(0) = 0.359^{+0.141}_{-0.085} \Big|_{\lambda_B} \begin{matrix} +0.019 \\ -0.019 \end{matrix} \Big|_{\sigma_1} \begin{matrix} +0.001 \\ -0.062 \end{matrix} \Big|_{\mu} \\ +0.010 \Big|_{M^2} \begin{matrix} +0.016 \\ -0.017 \end{matrix} \Big|_{s_0} \begin{matrix} +0.153 \\ -0.079 \end{matrix} \Big|_{\varphi_{\pm}(\omega)},$$

λ_B, σ_1 : Inverse moment of heavy meson LCDA

$\varphi_{\pm}$: systematic error from model uncertainty

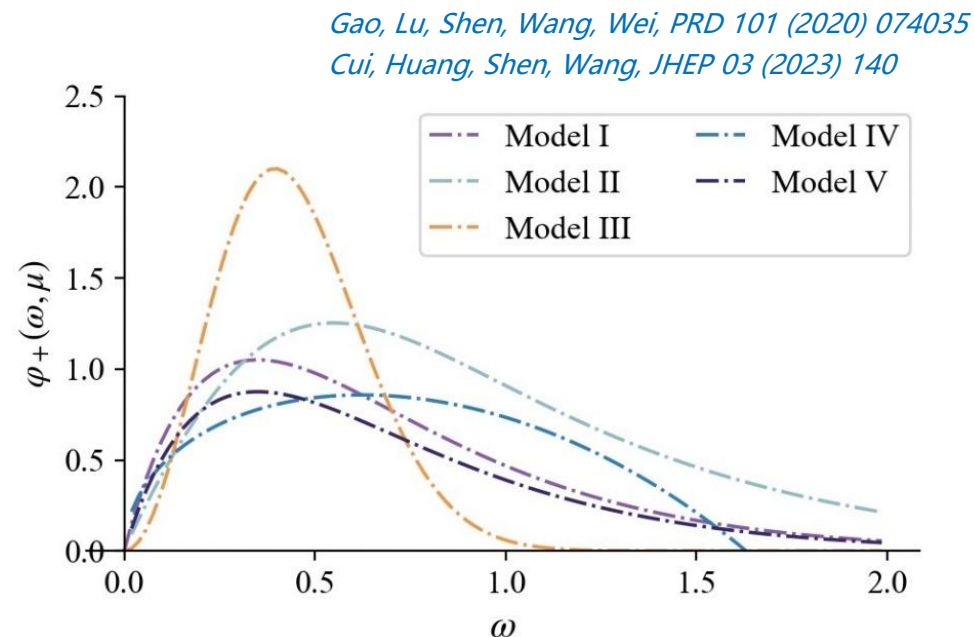
Without reliable heavy meson LCDA, it is impossible to discuss precision calculation!

- the uncertainty originating from model selection play an important role:

$$\mathcal{V}_{B \rightarrow K^*}(0) = 0.359^{+0.141}_{-0.085} \Big|_{\lambda_B}^{+0.019}_{-0.019} \Big|_{\sigma_1}^{+0.001}_{-0.062} \Big|_{\mu}^{+0.010}_{-0.004} \Big|_{M^2}^{+0.016}_{-0.017} \Big|_{s_0}^{+0.153}_{-0.079} \Big|_{\varphi_{\pm}(\omega)}$$

λ_B, σ_1 : Inverse moment of heavy meson LCDA

$\varphi_{\pm}$: systematic error from model uncertainty



Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Calculating heavy meson LCDAs from first principles significantly improves the precision of theoretical predictions.

- LCDAs contribute to the exclusive decays:

$$\varphi^+(\omega, \mu) = \frac{1}{i \tilde{f}_{H_Q} m_{H_Q} \mathbf{n}_+ \cdot \mathbf{v}} \int \frac{dt}{2\pi} e^{-i\omega t \mathbf{n}_+ \cdot \mathbf{v}} \langle 0 | \bar{q}(tn_+) \not{n}_+ \gamma_5 W_c(tn_+, 0) \mathbf{h}_v(0) | H_Q(v) \rangle$$

- Direct calculation of HQET LCDAs faces the following issues:

- Light-like correlators
- Cusp divergence

Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

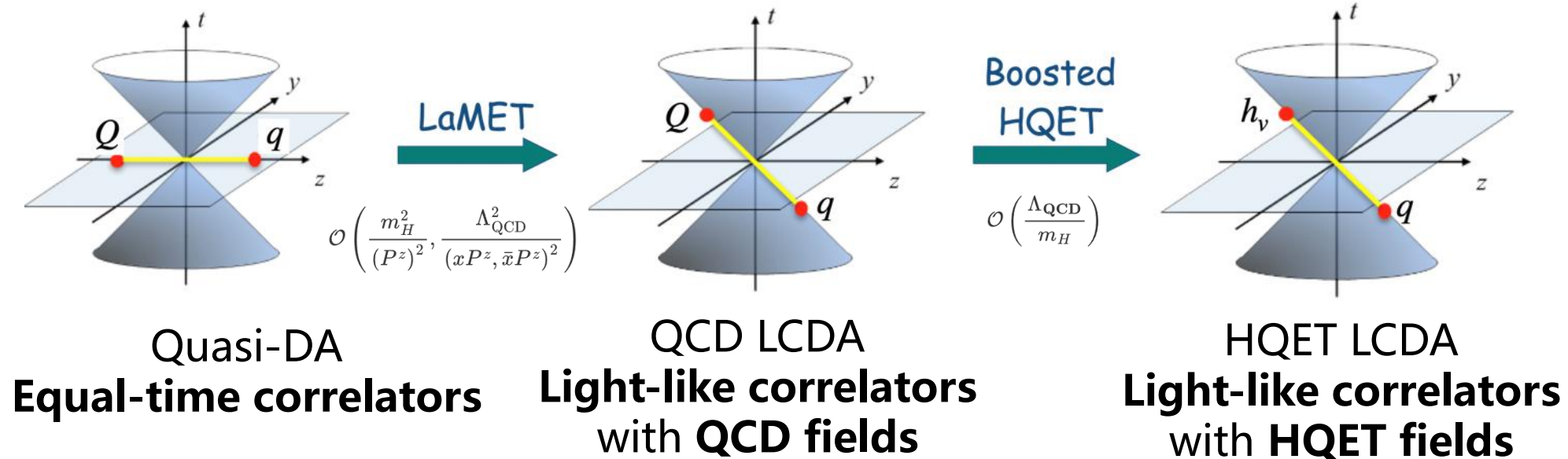
Direct calculation ×

OPE ×

The HQET LCDA cannot be calculated directly on the lattice.

Sequential effective theory —HQLaMET

6



- **Scales:** $\Lambda_{\text{QCD}}, m_Q, P^z$
- **LQCD calculable**

Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)
Han, QAZ, et.al., PRD111, 034503, (2025)

LaMET:

- $\Lambda_{\text{QCD}}, m_Q \ll P^z$
- **Integrate out P^z**

Hierarchy:

$$\Lambda_{\text{QCD}} \ll m_Q \ll P^z$$

bHQET:

- $\Lambda_{\text{QCD}} \ll m_Q$
- **Integrate out m_Q**

Ishaq, Jia, Xiong, Yang, PRL125, 132001 (2020)
Zhao, PRD 101, 071503 (2020)
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lattice setup

7

- **continuum extrapolation.**
- **infinite momentum extrapolation.**
- **physical mass extrapolation.**

Ensemble	a (fm)	$L^3 \times T$	m_π (MeV)	m_D (MeV)	$n_{\text{cfg}} \times n_{\text{meas}}$		
					LaMET	OPE moment $\langle \xi \rangle$	OPE moment $\langle \xi^2 \rangle$
C24P29	0.1053	$24^3 \times 72$	292.7(1.2)	1885.7(3.6)	—	50×16	440×16
C48P14		$48^3 \times 96$	135.5(1.6)	1864.8(3.2)	—		304×48
F32P30	0.0775	$32^3 \times 96$	303.2(1.3)	1887.9(1.3)	900×102	50×16	231×16
F32P21		$32^3 \times 64$	210.9(2.2)	1869.3(3.1)	459×128	—	—
G36P29	0.0683	$36^3 \times 108$	295.1(1.2)	1873.1(1.0)	656×86	50×16	117×16
H48P32	0.0519	$48^3 \times 144$	317.2(0.9)	1882.4(0.8)	550×108	50×16	111×16

Data processing workflow

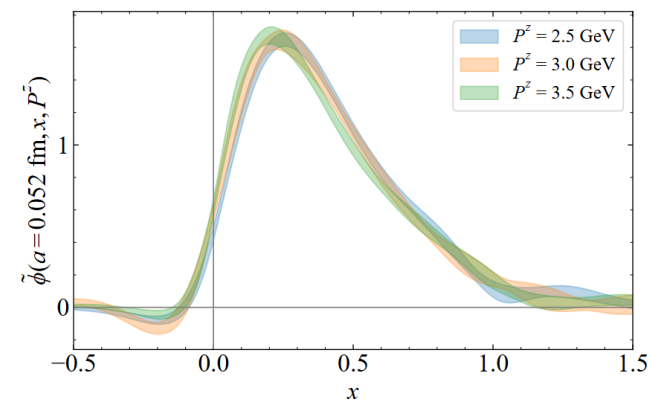
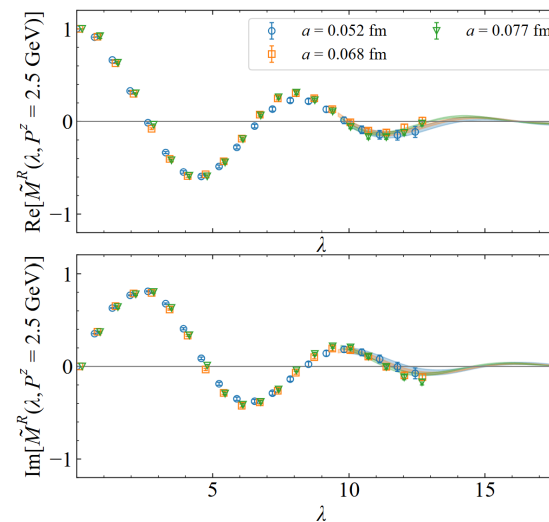
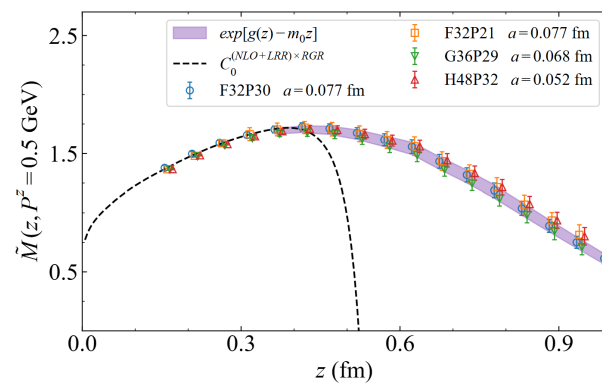
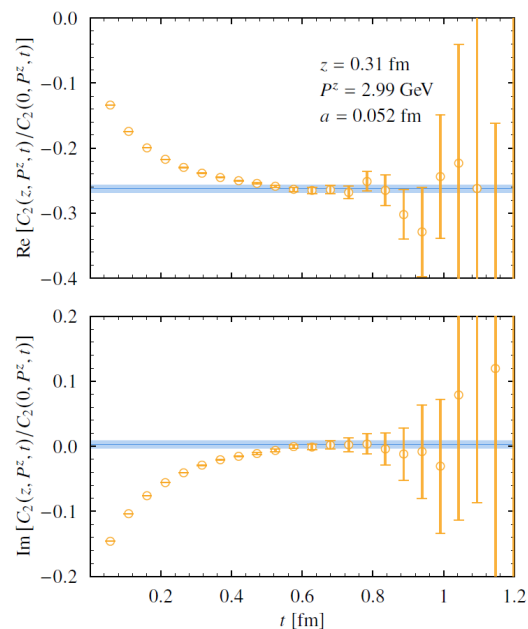
8

Bare matrix element

Renormalization

λ extrapolation

Fourier transformation



Matching from Quasi-DA to QCD LCDA

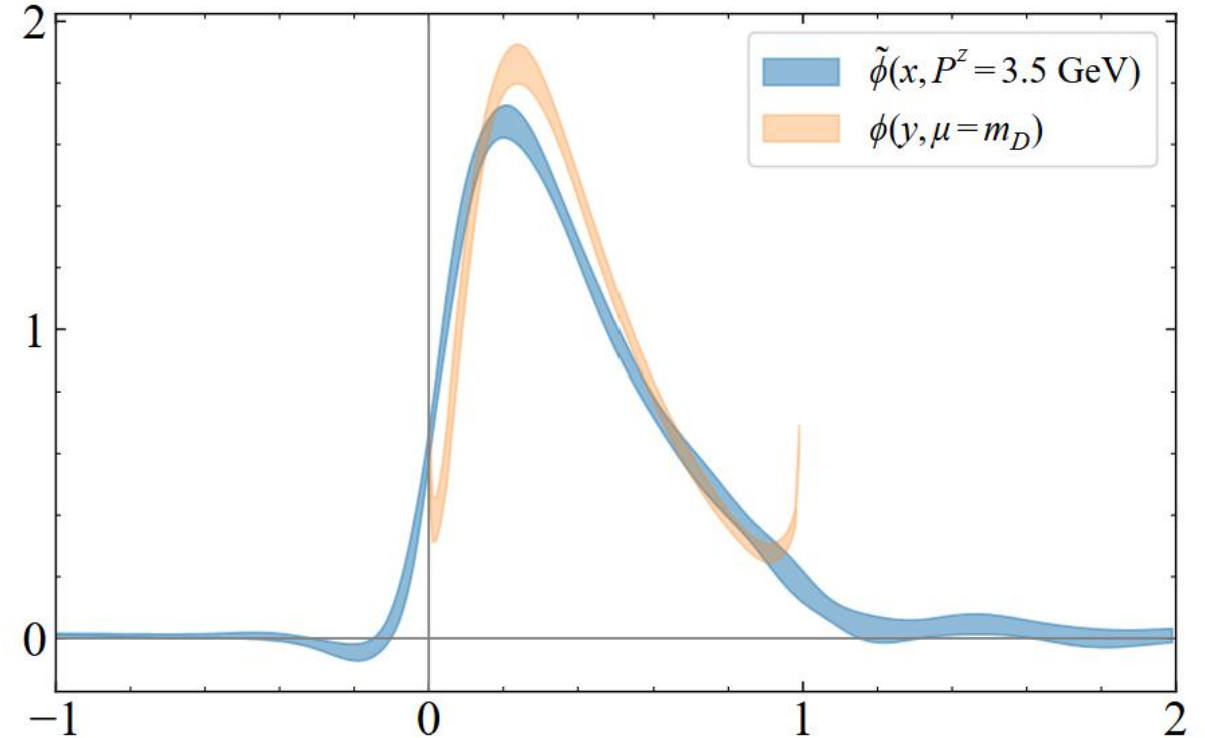
9

➤ Matching from quasi-DA to QCD LCDA:

$$f(y, \mu) = \int \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{f}(x, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(yP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-y)P^z)^2}, \frac{M_H^2}{(P^z)^2}\right)$$

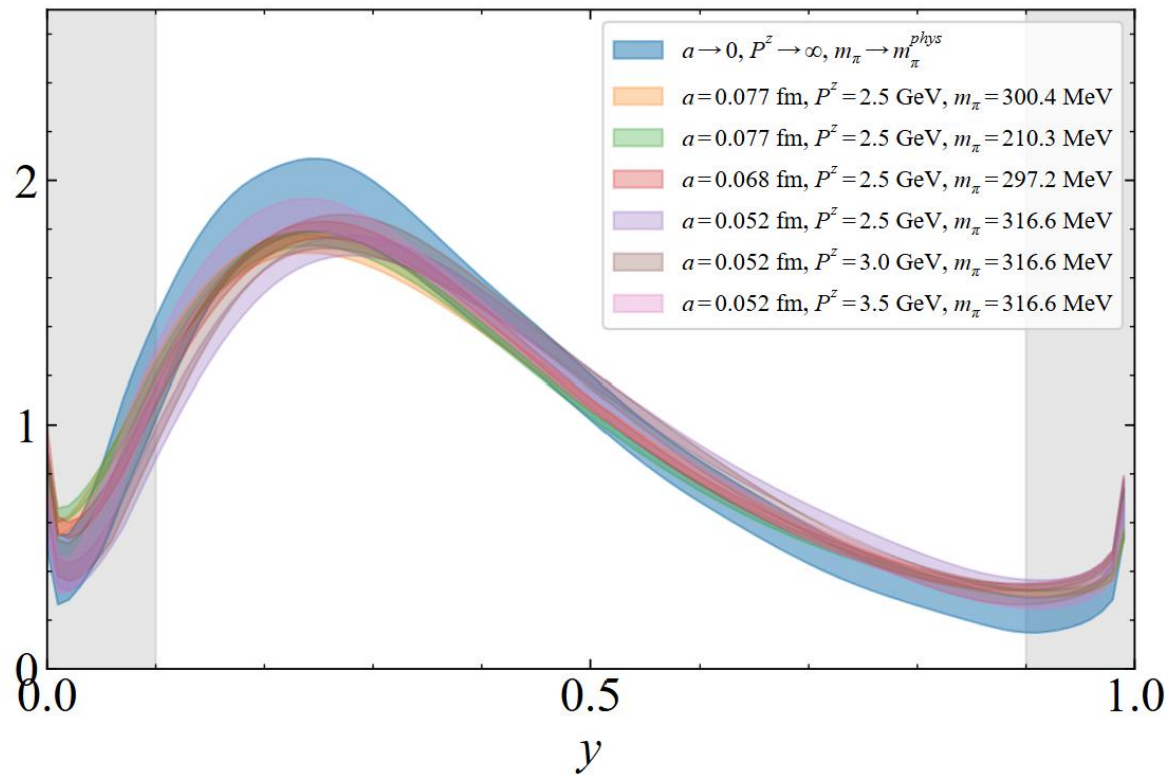
Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)

Han, QAZ, et.al., PRD111, 034503, (2025)



Quasi-DA and QCD LCDA on H48P32

- Full distribution of D meson QCD LCDA from LaMET after Continuum, infinite-momentum and physical mass extrapolation:



$$\phi(x, \mu; a, P^z, m_\pi) = \phi(x, \mu; m_\pi^{phys}) [1 + c_1(x)a^2] \left[1 + \frac{c_2(x)}{(P^z)^2} \right] \times [1 + c_3(x)(m_\pi^2 - (m_\pi^{phys})^2)]$$

- Peak region: $y \sim O(\Lambda_{QCD}/m_H)$
- Tail region: $y \sim O(1)$
- End-point region: $yP^z \leq \Lambda_{QCD}$

$$O\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}, \frac{M^2}{(P^z)^2}\right)$$

LaMET vs. Lattice OPE for QCD LCDA

11

- The moments of the D-meson LCDA can be **calculated directly** using the OPE method.
- Only **the lowest moments** can be accessed.
- The power correction $O(m_h^2/(P^z)^2)$ can be benchmarked by the OPE moments.

	LaMET	OPE
Output	x -dependent partonic distributions of LCDAs	A finite set of moments of LCDAs
Pros	1) Direct access to the full x -dependence of partonic distributions 2) Direct comparison with global fits and phenomenology	1) No requirement for large hadron momentum 2) Precise determination of lowest-order moments 3) QCD sum-rule constraints for consistency checks
Cons	1) Requirement for large hadron momentum P^z 2) Difficulties with discretization effects, signal-to-noise degradation, and excited-state contamination	1) Difficulties with higher moments (larger noise, operator mixing, complex renormalization) 2) Inability to directly determine full partonic distributions

➤ Moments from local operators:

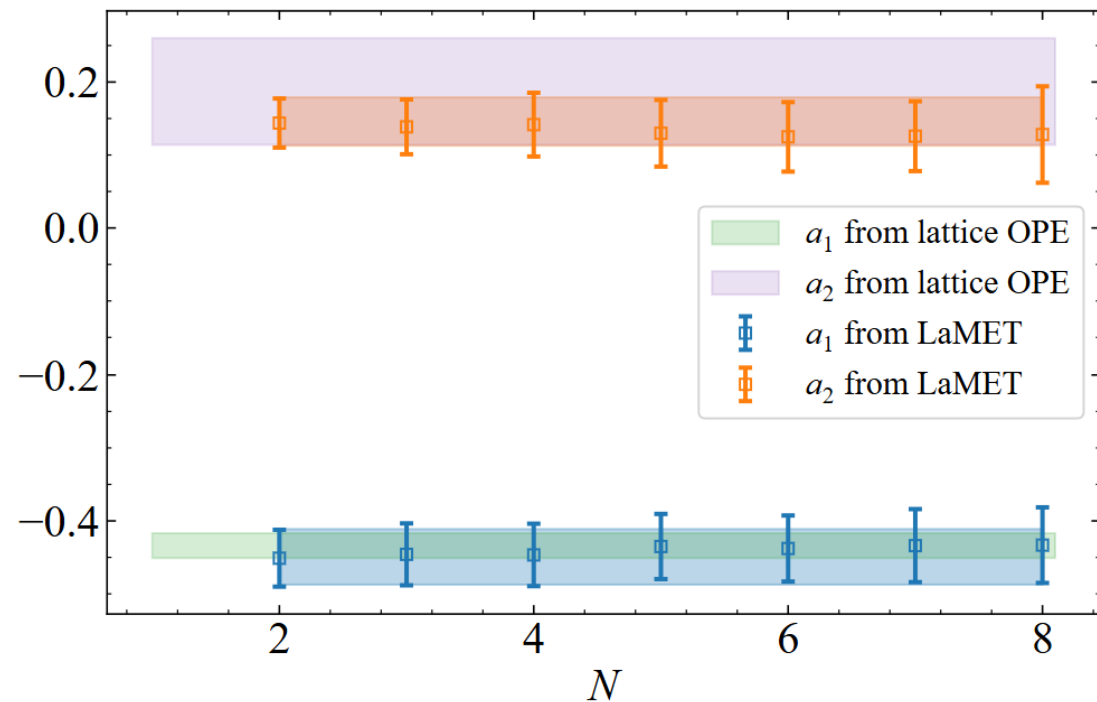
$$\bar{q}(z_2 n) \not{n} \gamma_5 W_c(z_2 n, z_1 n) q'(z_1 n)$$

$$= \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \dots n^{k+l} \mathcal{M}_{\rho\mu_1 \dots \mu_{k+l}}^{(k,l)} + \dots,$$

$$\mathcal{M}_{\rho\mu_1 \dots \mu_{k+l}}^{(k,l)} = \bar{q}(0) \overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \dots \overrightarrow{D}_{\mu_{k+l}} \gamma_\rho \gamma_5 q'(0).$$

➤ Gegenbauer moments from LaMET:

$$\phi(x, \mu^2) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2x-1) \right],$$



LaMET

$$a_1^{LaMET} = -0.450(38)$$

$$a_2^{LaMET} = 0.146(33)$$

OPE

$$a_1^{OPE} = -0.434(17)$$

$$a_2^{OPE} = 0.183(73)$$

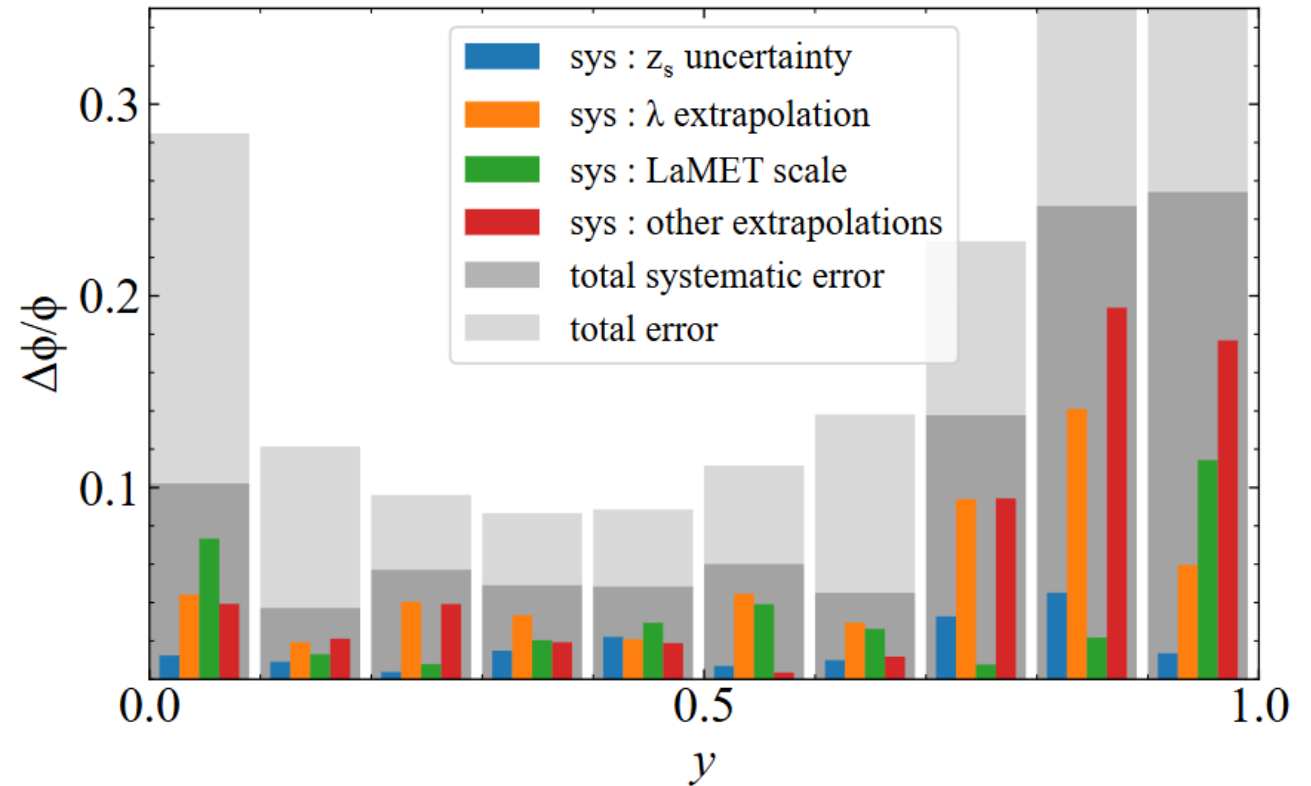
$O(m_D^2/(P^z)^2)$ is under control.

Analysis of systematic errors

13

➤ Sources of systematic error:

- ✓ Hybrid renormalization
- ✓ Lambda extrapolation
- ✓ LaMET scale
- ✓ Extrapolation



Even after accounting for systematic uncertainties, the total uncertainty in the peak region remains small (~10%).

Prediction for HQET LCDA

14

- Leading twist HQET LCDA:

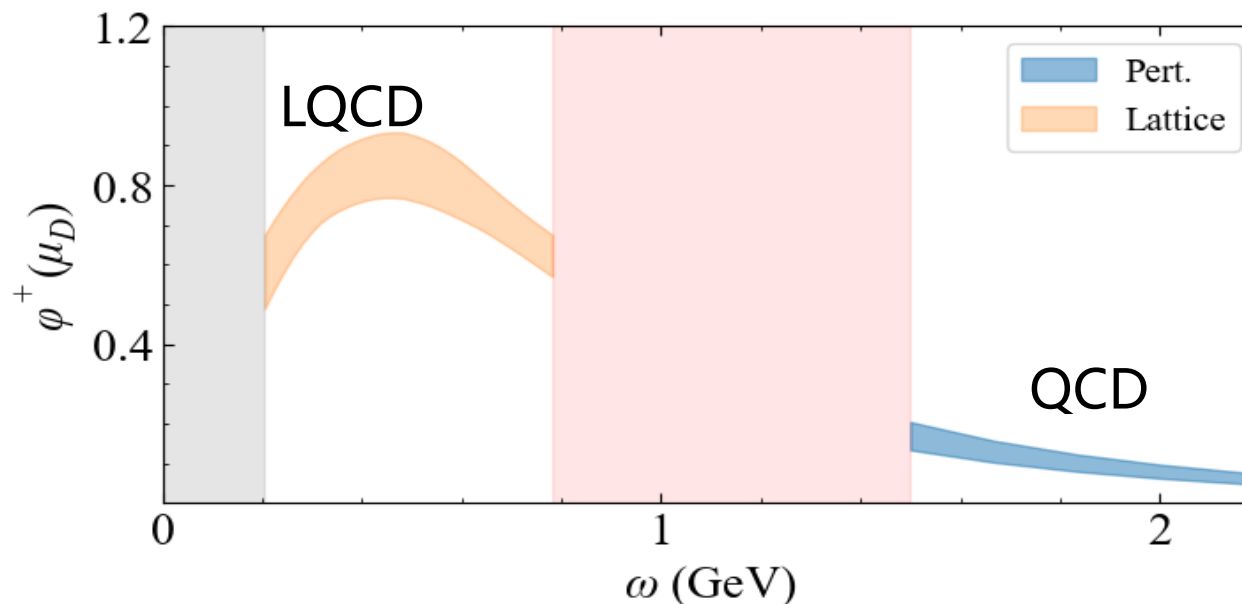
Lee, Neubert, et al. PRD 72 (2005) 094028

QCD LCDA, nonperturbative

$$\varphi^+(\omega, \mu) = \begin{cases} \frac{1}{m_H} \left[1 - \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{2} \ln^2 \frac{\mu^2}{m_H^2} + \frac{1}{2} \ln \frac{\mu^2}{m_H^2} + \frac{3}{2} \ln \frac{\mu^2}{m_Q^2} + \frac{\pi^2}{12} + 4 \right) + \mathcal{O}(\alpha_s^2) \right] \phi\left(\frac{\omega}{m_H}, \mu\right), & \omega \sim \Lambda_{\text{QCD}} \\ \frac{\alpha_s C_F}{\pi \omega} \left[\left(\frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}}{3\omega} \left(2 - \ln \frac{\omega}{\mu} \right) \right] + \mathcal{O}(\alpha_s^2), & \omega \sim m_H \end{cases}$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Prediction for HQET LCDA from first-principle:



14

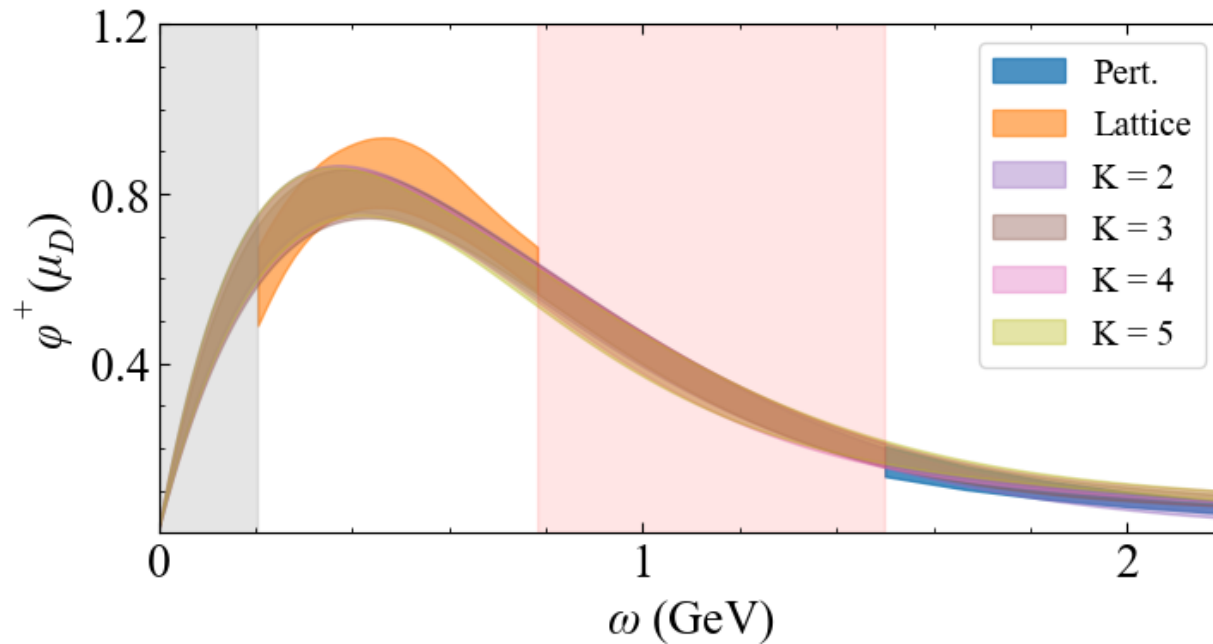
Prediction for HQET LCDA

15

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization**:

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

$$\varphi^+(\omega, \mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^K \frac{a_k(\mu)}{1+k} L_k^{(1)}\left(\frac{2\omega}{\omega_0}\right)$$



- Agree well with the data.
- Excellent consistency is observed across different orders.
- With full distributions, further phenomenological discussions can be preformed.

Inverse and inverse-logarithmic moments

16

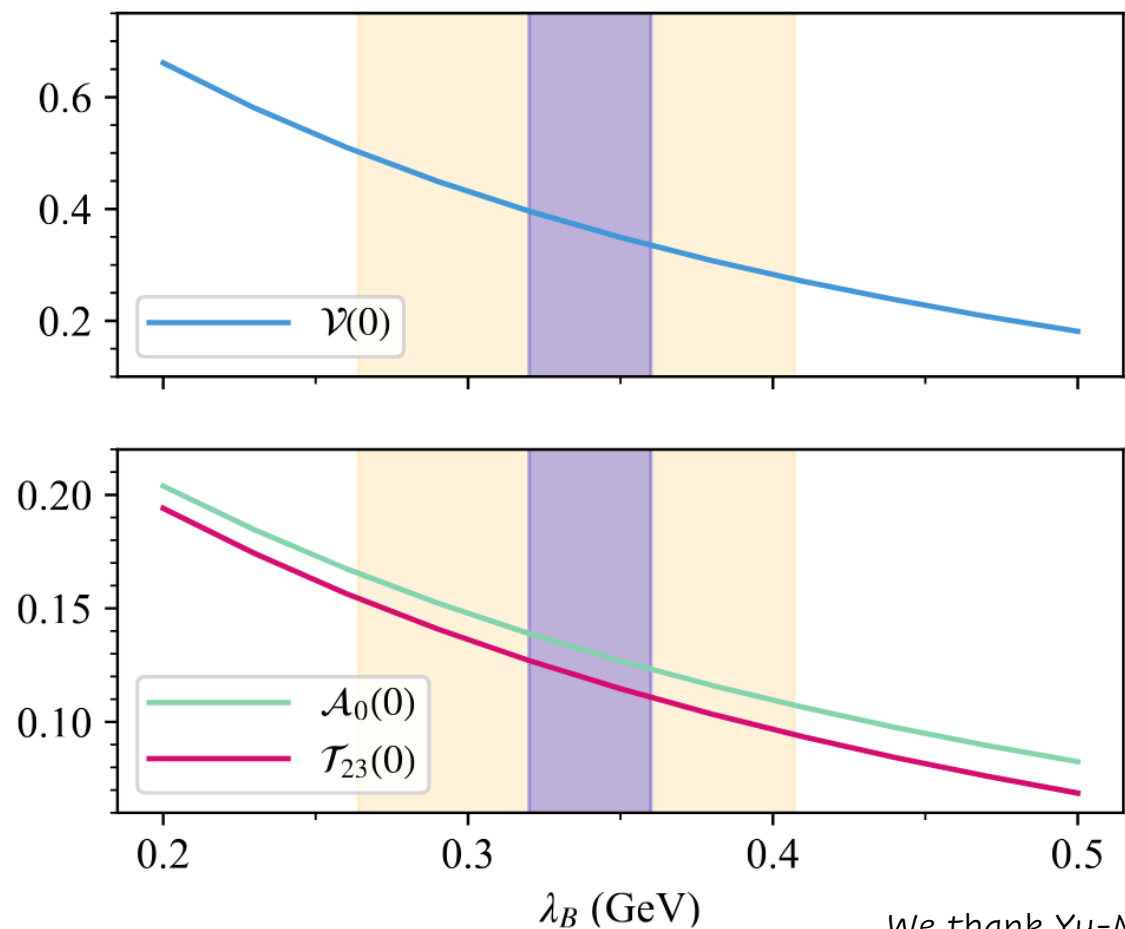
- Predictions of inverse and inverse-logarithmic moments from the full distribution of

HQET LCDA:

μ	Reference (Method)	λ_B (GeV)	$\sigma_B^{(1)}$	$\sigma_B^{(2)}$	Leading power!
m_D	This work	0.423(28)	2.041(62)	5.50(26)	
	Ref. [39] (LQCD)	0.420(71)	2.17(16)	6.33(80)	PRD111 (2025), 034503
1 GeV	This work	0.340(20)	1.685(63)	—	
	Ref. [39] (LQCD)	0.376(63)	1.66(13)	—	
	Ref. [19] (Experiment)	> 0.24	—	—	PRD98 (2018), 112016
	Ref. [13] (QCD sum rule)	$0.343^{+0.064}_{-0.079}$	1.4(4)	—	PRD101 (2020), 074035
	Ref. [23] (QCD sum rule)	0.46(11)	1.4(4)	—	PRD69 (2004), 034014
	Ref. [21] (QCD sum rule)	0.383(153)	—	—	JHEP10 (2020), 043
	Ref. [22] (OPE)	0.48(11)	1.6(2)	—	PRD72 (2005), 094028
	Ref. [18] (Asymptotic behavior)	0.35(15)	—	—	PRD55 (1997), 272290
	Ref. [83] (Global Fit)	0.338(68)	—	—	PLB848 (2024), 138345

$B \rightarrow K^*$ Form Factors in LCSR

17



- In PRD101(2020), 074035: (orange band)

$$V(0) = 0.359^{+0.141}_{-0.085}$$

$$A(0) = 0.129^{+0.035}_{-0.021}$$

$$T_{23}(0) = 0.116^{+0.036}_{-0.022}$$

- With our result λ_B : (purple band)

$$V(0) = 0.362^{+0.034}_{-0.029}$$

Leading power!

$$A(0) = 0.131^{+0.008}_{-0.008}$$

$$T_{23}(0) = 0.119^{+0.008}_{-0.008}$$

We thank Yu-Ming Wang for his code to calculate the $B \rightarrow V$ form factors in LCSRs @ PRD 101 (2020) 074035

- Heavy-meson HQET LCDAs are **essential nonperturbative inputs** for precision studies of exclusive weak B-meson decays, but have long suffered from model dependence.
- We provide a precision lattice QCD determination of the heavy-meson HQET LCDA within HQLaMET:
 - Continuum limit
 - Infinite momentum limit
 - Physical mass
 - Systematic uncertainties
- We are also collaborating with Wei to **estimate power corrections**, aiming to obtain more precise results.

Thanks for your attention!

Back up

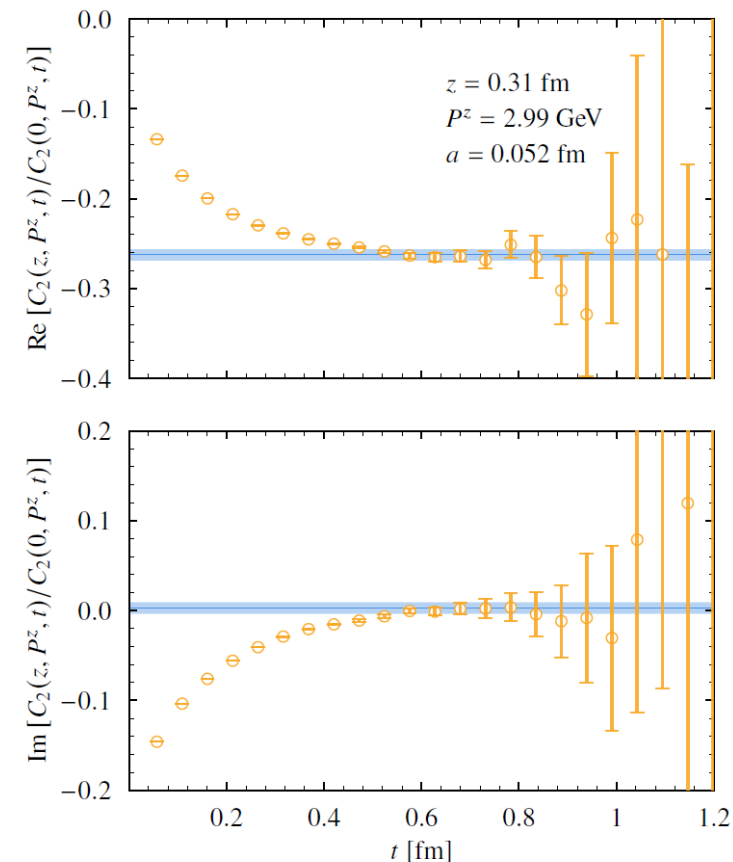
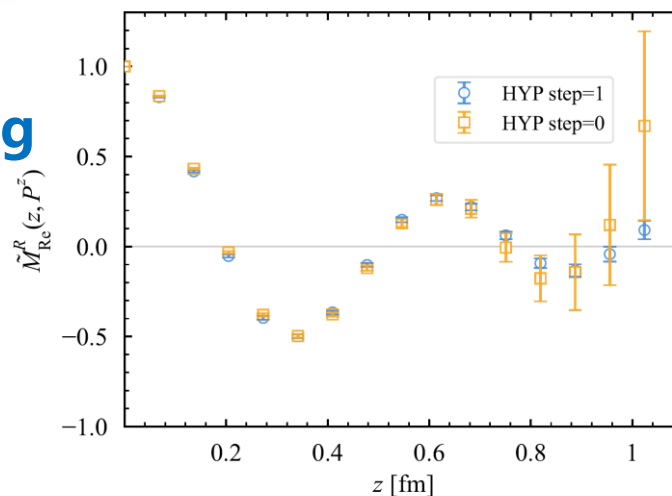
- Nonlocal 2-point correlation function:

$$C_2(z, P^z, t) = \sum_{x^3} e^{-iP^z x^3} \langle \gamma_5 G_q^\dagger(x^3, t, 0) \gamma_5 \Gamma_{sink} \\ \times W_c(x^3, x^3 + z) \Gamma_{source} G_Q(x^3 + z, t, 0) \rangle$$

- Beside high statistics, **HYP smearing** is used to improve the signal.

- Bare matrix elements $\tilde{M}^B(z, P^z)$:

$$\frac{C_2(z, P^z, t)}{C_2(0, P^z, t)} = \tilde{M}^B(z, P^z) e^{-E_0 t} \\ \times \left[\sum_{n=0}^{N_{stat}-1} A_n e^{-(E_n - E_0)t} \right].$$



Hybrid renormalization

21

- $\tilde{M}^B(z, P^z)$ contains both **linear** and **logarithmic** divergences, can be **multiplicatively renormalized**.

- **Hybrid renormalization scheme:**

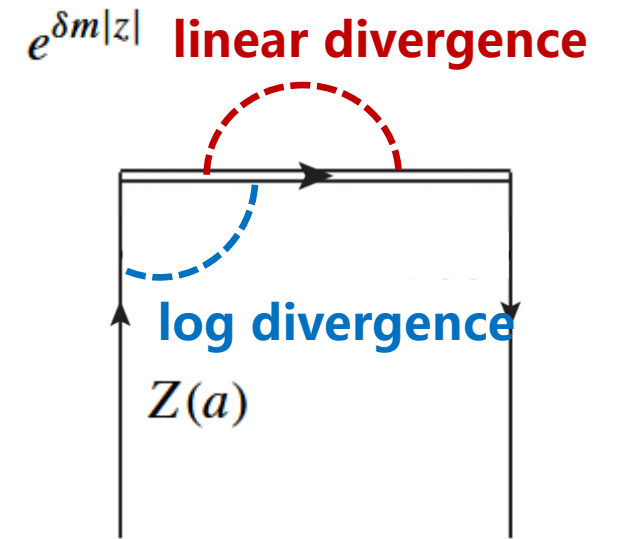
$$\tilde{M}^R(z, P^z) = \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{\tilde{M}^{(0)}(z, P_{\text{ref}}^z, 1/a)} \theta(z_s - |z|) + \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{Z_R(z, 1/a)} \theta(|z| - z_s)$$

- Short distance (perturbative region): **ratio scheme**

$$\langle 0 | \bar{q}(z) \gamma^z \gamma_5 W_c(z, 0) Q(0) | H(P^z) \rangle \propto P^z \quad \Rightarrow \quad P_{\text{ref}}^z = 0.5 \text{ GeV}$$

- Long distance (nonperturbative region): **self renormalization scheme**

$$Z_R(z, 1/a) = \exp \left\{ \frac{kz}{a \ln[a\Lambda_{\text{QCD}}]} + m_0 z + f(z) a^2 + \frac{3C_F}{b_0} \ln \left[\frac{\ln[1/(a\Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\text{QCD}}^{\text{MS}}]} \right] + \ln \left[1 + \frac{d}{\ln[a\Lambda_{\text{QCD}}]} \right] \right\}$$



Hybrid renormalization

22

➤ Self renormalization scheme:

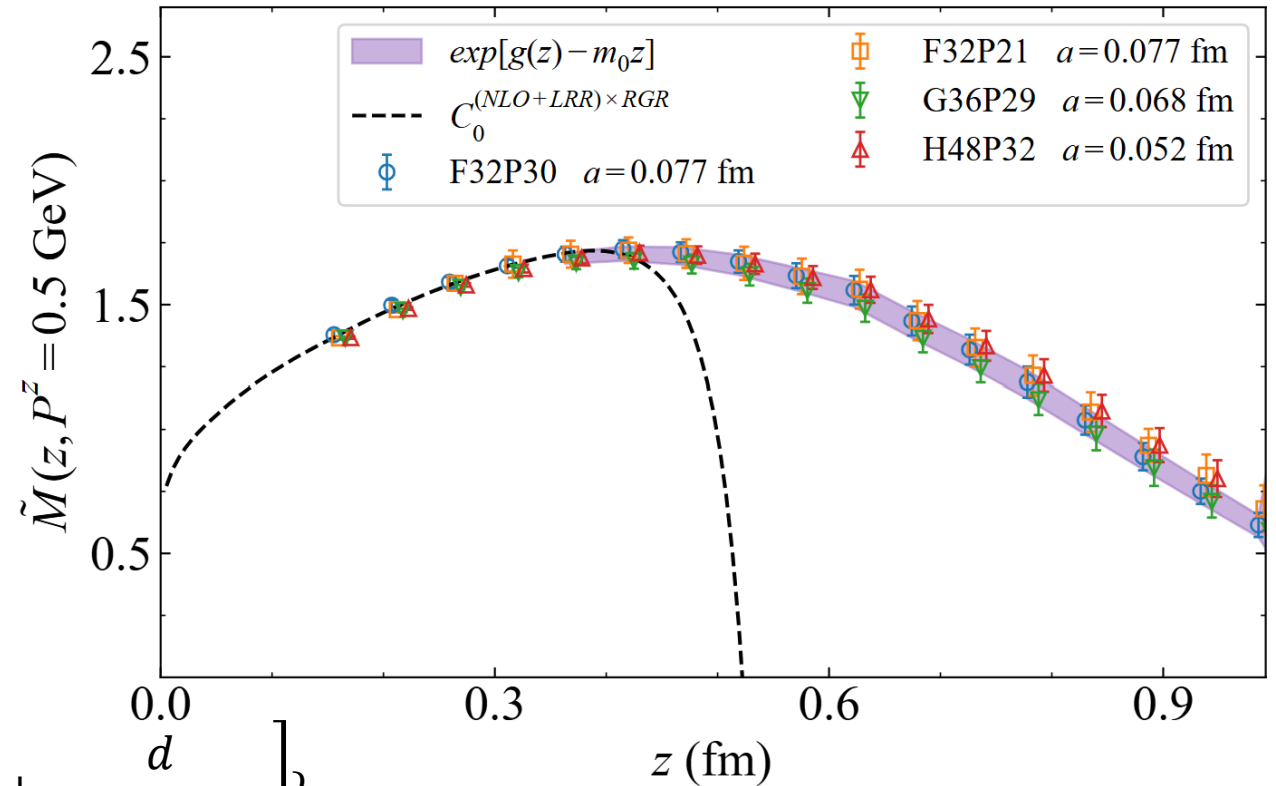
• Short distance:

Lattice **agree well** with perturbation results

• Long distance:

$$Z_R(z, 1/a) = \exp\left\{\frac{kz}{a \ln[a\Lambda_{\text{QCD}}]} + m_0 z + f(z)a^2 + \frac{3C_F}{b_0} \ln\left[\frac{\ln[1/(a\Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\text{QCD}}^{\overline{\text{MS}}}]}\right] + \ln\left[1 + \frac{d}{\ln[a\Lambda_{\text{QCD}}]}\right]\right\}.$$

Fit results: ($\chi^2/d.o.f = 0.42$)

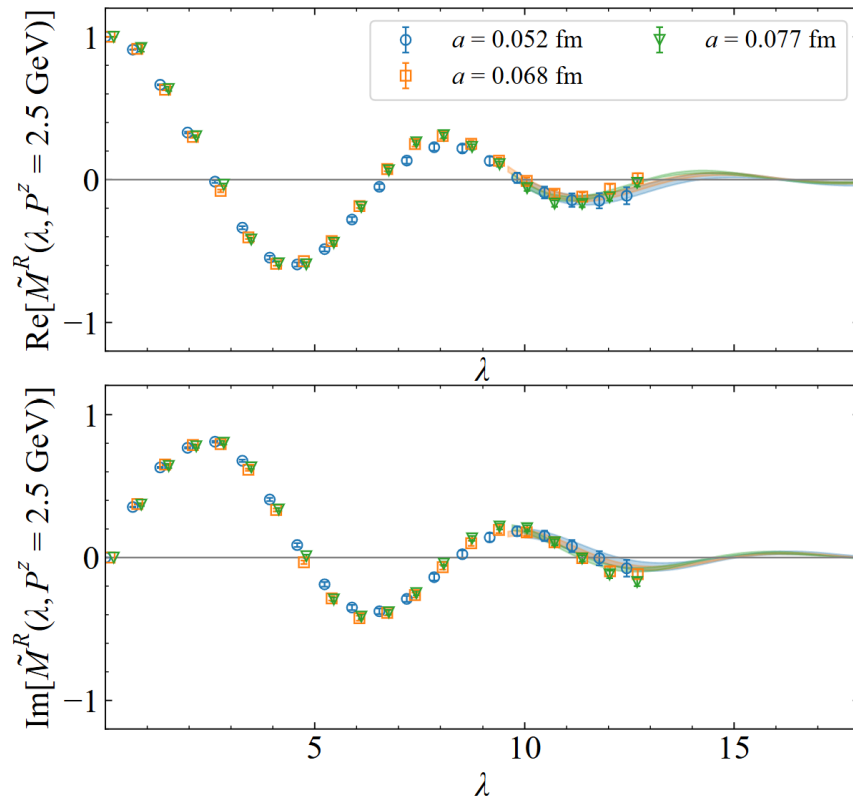


$$k = 1.03(36), \Lambda_{\text{QCD}} = 0.05(11)\text{GeV},$$

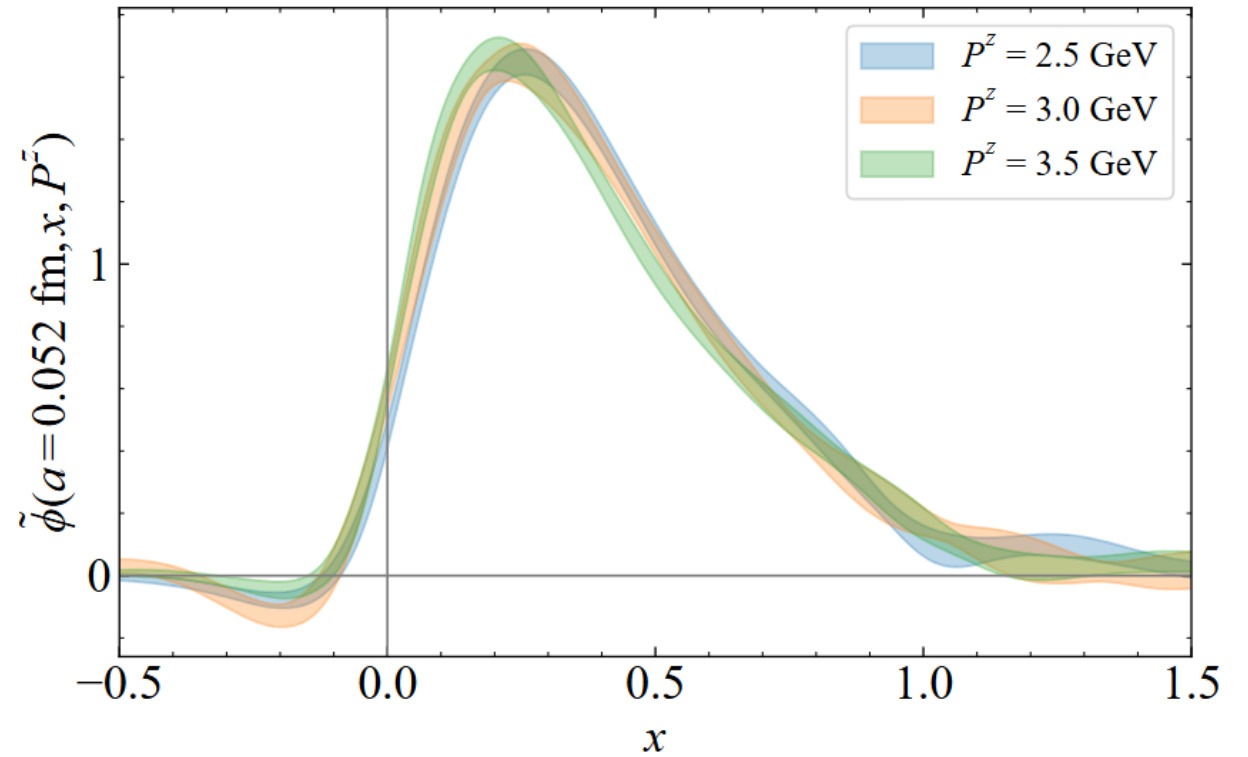
$$m_0 = 0.13(15)\text{GeV}, d = 0.9(1.2)$$

- a-dependence, $P^z = 2.5 \text{ GeV}$

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda} \frac{c_2}{(i\lambda)^{n_2}} \right] e^{-\lambda/\lambda_0}.$$



Quasi-DA: $\tilde{\phi}(x, P^z) = \int_{-\infty}^{+\infty} \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}^R(z, P^z).$



A brute-force truncation at large λ will lead to strong oscillation after FT.

Matching from Quasi-DA to QCD LCDA

24

➤ Matching from quasi-DA to QCD LCDA:

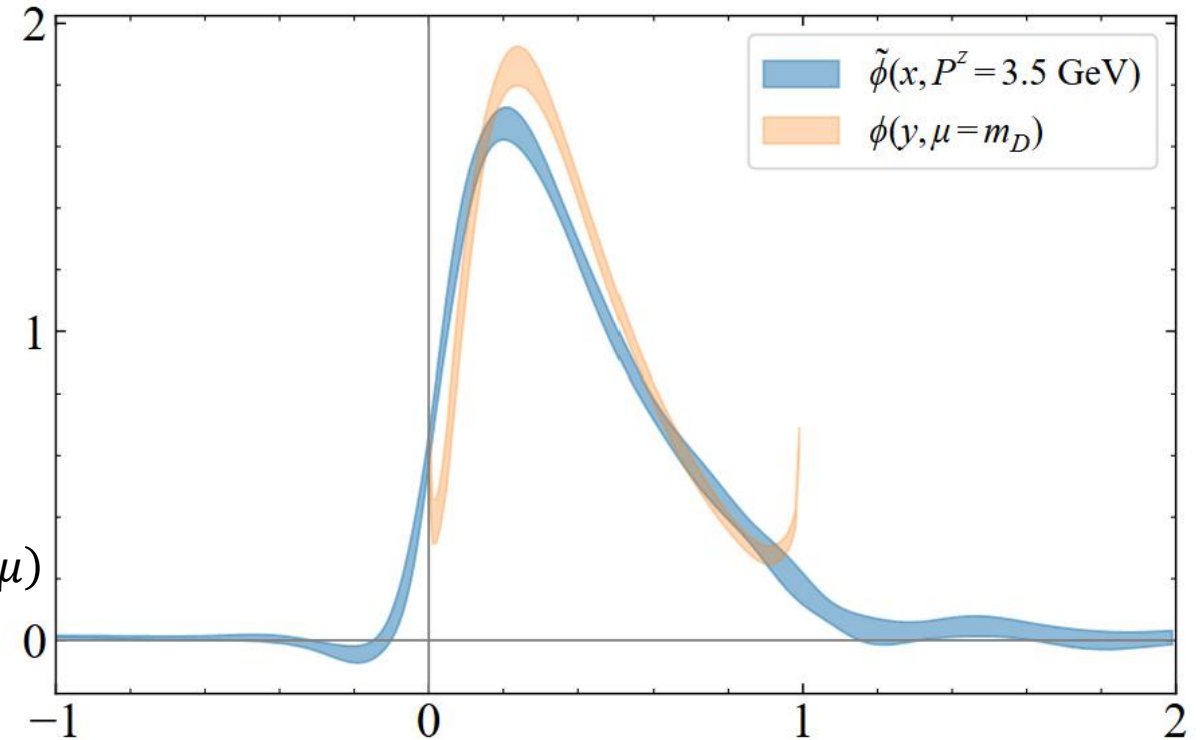
$$f(y, \mu) = \int \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{f}(x, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(yP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-y)P^z)^2}, \frac{M_H^2}{(P^z)^2}\right)$$

Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)

Han, QAZ, et.al., PRD111, 034503, (2025)

$$\frac{dC^{-1}(x, y, P^z, \mu)}{d \ln \mu^2} = - \int_0^1 d\xi V[\xi, y, \alpha_s(\mu)] C^{-1}(x, \xi, P^z, \mu)$$

$$\phi(y, 2yP^z) \xrightarrow{\text{ERBL}} \phi(y, m_D)$$



Quasi-DA and QCD LCDA on H48P32

➤ Moments from local operators:

$$\bar{q}(z_2 n) \not{n} \gamma_5 W_c(z_2 n, z_1 n) q'(z_1 n)$$

$$= \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \dots n^{k+l} \mathcal{M}_{\rho\mu_1 \dots \mu_{k+l}}^{(k,l)} + \dots,$$

$$\mathcal{M}_{\rho\mu_1 \dots \mu_{k+l}}^{(k,l)} = \bar{q}(0) \overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \dots \overrightarrow{D}_{\mu_{k+l}} \gamma_\rho) \gamma_5 q'(0).$$

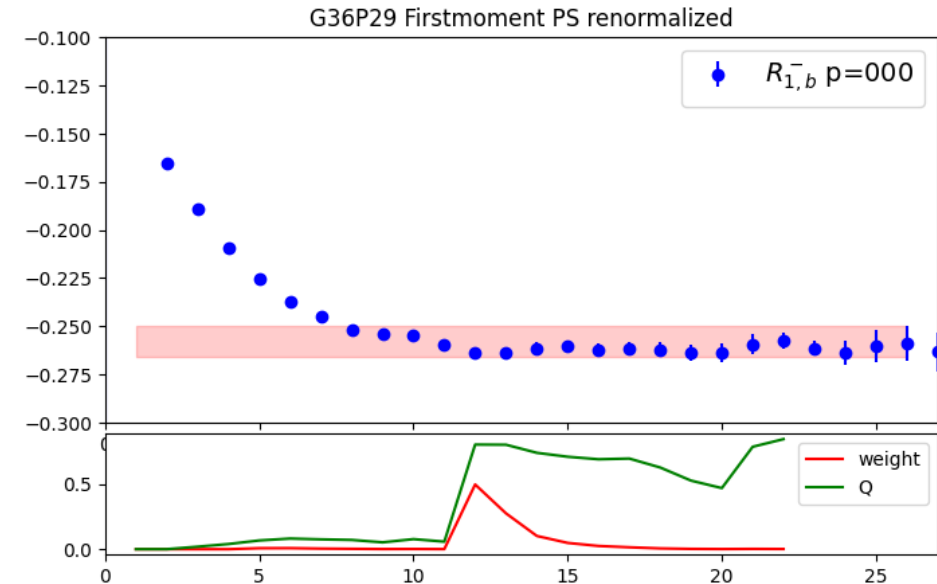
➤ Definition of operator

$$P(x) = \bar{\psi}_1(x) \gamma_5 \psi_2(x)$$

$$O_{\mu\nu}^-(x) = \bar{\psi}_1(x) \gamma_{(\mu} \Gamma[\overrightarrow{D}_{\nu)} - \overleftarrow{D}_{\nu)}] \psi_2(x)$$

➤ Mellin moment

$$R_{1,a}^- = \frac{i}{3} \sum_{j=1}^3 \frac{1}{p_j} \frac{C_{O_{4j}P}(t)}{C_{A_4P}(t)} = \langle \xi \rangle \quad R_{1,b}^- = \frac{4E}{3E^2 + \vec{p}^2} \frac{C_{O_{44}P}(t)}{C_{A_4P}(t)} = \langle \xi \rangle$$



$$a_1 = -0.434(17)$$

$$a_2 = 0.183(73)$$

LaMET vs. Lattice OPE

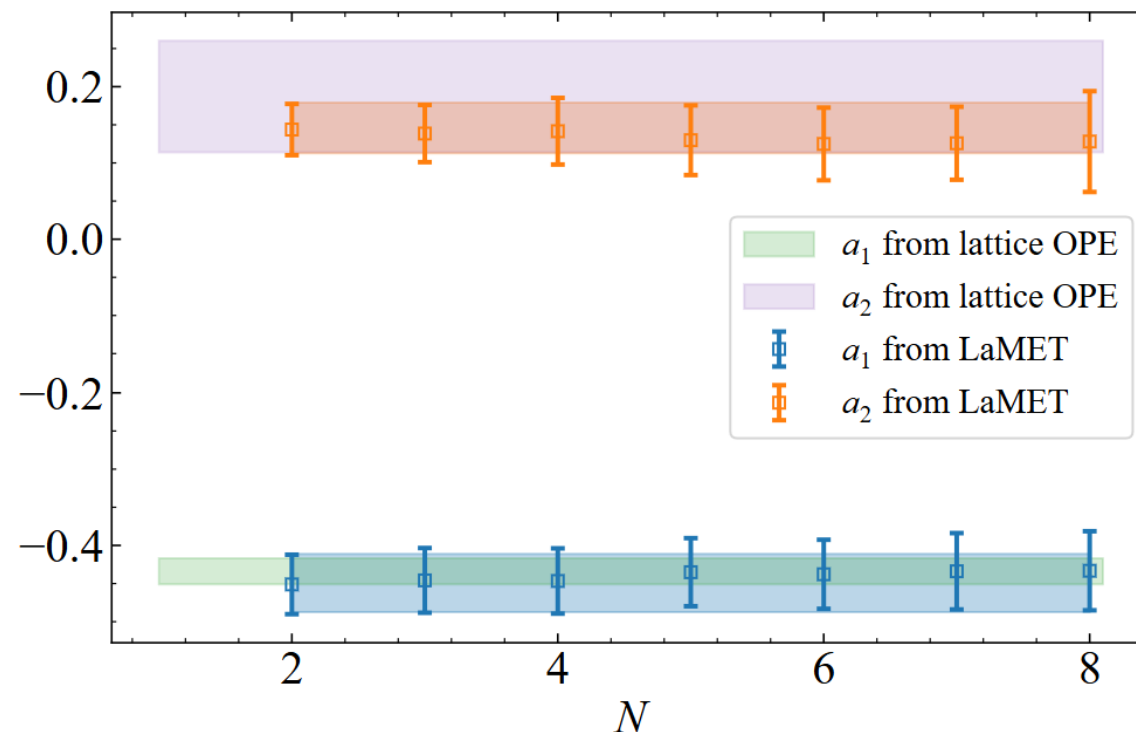
26

➤ Gegenbauer moments from LaMET:

$$\phi(x, \mu^2) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2x-1) \right],$$

Fitting from data excluding the end-point region:

n	a_1	a_2	a_3	a_4
2	-0.451(39)	0.144(34)		
4	-0.447(43)	0.142(44)	0.006(38)	0.003(23)
6	-0.438(45)	0.125(48)	0.028(46)	-0.025(37)
8	-0.433(52)	0.128(66)	0.033(53)	-0.021(64)



LaMET

$$a_1^{LaMET} = -0.450(38)$$

$$a_2^{LaMET} = 0.146(33)$$

OPE

$$a_1^{OPE} = -0.434(17)$$

$$a_2^{OPE} = 0.183(73)$$

$O(m_D^2/(P^z)^2)$ is under control.