



中国科学院大学
University of Chinese Academy of Sciences

Vector-meson light-cone distribution amplitude moments from lattice QCD with gradient flow

Speaker: Tong-Wei Lin

Advisor: Yi-Bo Yang

ICTP-AP



A leading-twist LCDA describes the valence quark–antiquark momentum sharing in an energetic meson. It is a nonperturbative input to hard exclusive processes, including B decays involving vector mesons.

1. Compute a quasi-DA using spatial nonlocal operators at large momentum, then apply LaMET matching to obtain the LCDA.
2. Compute moments using local operators and reconstruct the LCDA.

Hua et al., Phys. Rev. Lett. 127, 062002 (2021), arXiv:2011.09788

LaMET calculations provide longitudinal and transverse K^* and ϕ LCDAs. Our goal is a precise comparison of their moments with local-operator results in a common renormalization scheme and at a common scale.



Light-ray definitions

Leading-twist vector-meson LCDAs are defined by vacuum-to-meson matrix elements of renormalized nonlocal quark–antiquark light-ray operators:

$$\langle 0 | \bar{d}(z_1 n) \not{n} [z_1 n, z_2 n] u(z_2 n) | V(p, \lambda) \rangle$$

$$= m_V f_V (e^{(\lambda)} \cdot n) \int_0^1 dx e^{-i(p \cdot n)(z_1(1-x) + z_2 x)} \varphi_V^{\parallel}(x, \mu)$$

$$e_{\perp, \mu}^{(\lambda')} n_\nu \langle 0 | \bar{d}(z_1 n) \sigma^{\mu\nu} [z_1 n, z_2 n] u(z_2 n) | V(p, \lambda) \rangle$$

$$= i f_V^T (e_{\perp}^{(\lambda')} \cdot e_{\perp}^{(\lambda)}) (p \cdot n) \int_0^1 dx e^{-i(p \cdot n)(z_1(1-x) + z_2 x)} \varphi_V^{\perp}(x, \mu)$$

$n^2 = 0$. $[z_1 n, z_2 n]$ is a straight Wilson line. $\xi = 2x - 1$, $\alpha = \parallel$ or $\perp$. Braun et al., 1612.02955

Moments and reconstruction

Mellin (ξ) moments

$$\langle \xi^n \rangle_V^\alpha = \int_0^1 dx (2x - 1)^n \varphi_V^\alpha(x, \mu)$$

Gegenbauer moments

$$a_n^\alpha(\mu) = \frac{2(2n + 3)}{3(n + 1)(n + 2)} \times \int_0^1 dx C_n^{3/2}(2x - 1) \varphi_V^\alpha(x, \mu)$$

Gegenbauer expansion

$$\varphi_V^\alpha = 6x(1 - x) \left[1 + \sum_{n=1}^{\infty} a_n^\alpha C_n^{3/2}(\xi) \right]$$



Longitudinal operator basis

$$O_{(\mu\nu\rho)}^- = \bar{q}_2 \gamma_{(\mu} \vec{D}_{\nu} \vec{D}_{\rho)} q_1$$

$$O_{(\mu\nu\rho)}^+ = \bar{q}_2 \gamma_{(\mu} [\vec{D}_{\nu} \vec{D}_{\rho} + \vec{D}_{\nu} \vec{D}_{\rho} + 2 \vec{D}_{\nu} \vec{D}_{\rho}] q_1$$

$$\langle 0 | O_{(\mu\nu\rho)}^- | V \rangle = N_{(\mu\nu\rho)} \langle \xi^2 \rangle_V^{\parallel, bare}$$

$$\langle 0 | O_{(\mu\nu\rho)}^+ | V \rangle = N_{(\mu\nu\rho)} \langle 1^2 \rangle^{bare}$$

Braun et al., Eqs. (2.22–2.26), (3.9)

Two-point functions and ratios

$$C_{\mu\nu}(\tau, p) = a^3 \sum_x e^{-ip \cdot x} \langle J_{\mu}(\tau, x) \tilde{J}_{\nu}^{\dagger}(0) \rangle$$

$$C_{\mu_1\mu_2\mu_3\nu}^{\pm}(\tau, p) = a^3 \sum_x e^{-ip \cdot x} \langle O_{\mu_1\mu_2\mu_3}^{\pm}(\tau, x) \tilde{J}_{\nu}^{\dagger}(0) \rangle$$

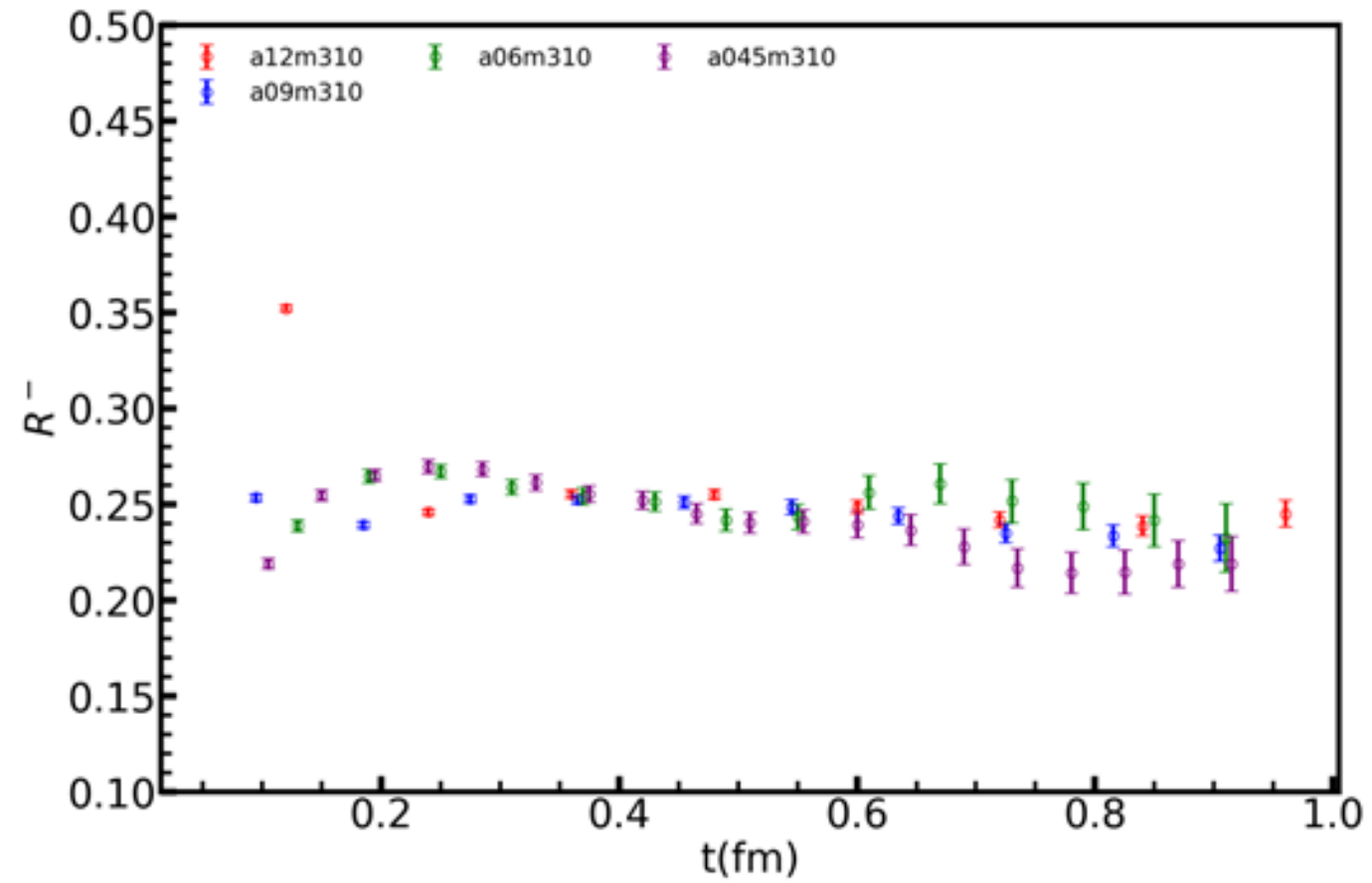
$p = (2\pi/L) e_j, j \neq k$. Extract the single-state plateau:

$$R^{\pm} = -\frac{3}{i p_j E} \frac{\hat{p}^- \hat{t}^- C_{\{4jk\}k}^{\pm}}{\hat{p}^+ \hat{t}^+ C_{kk}}$$

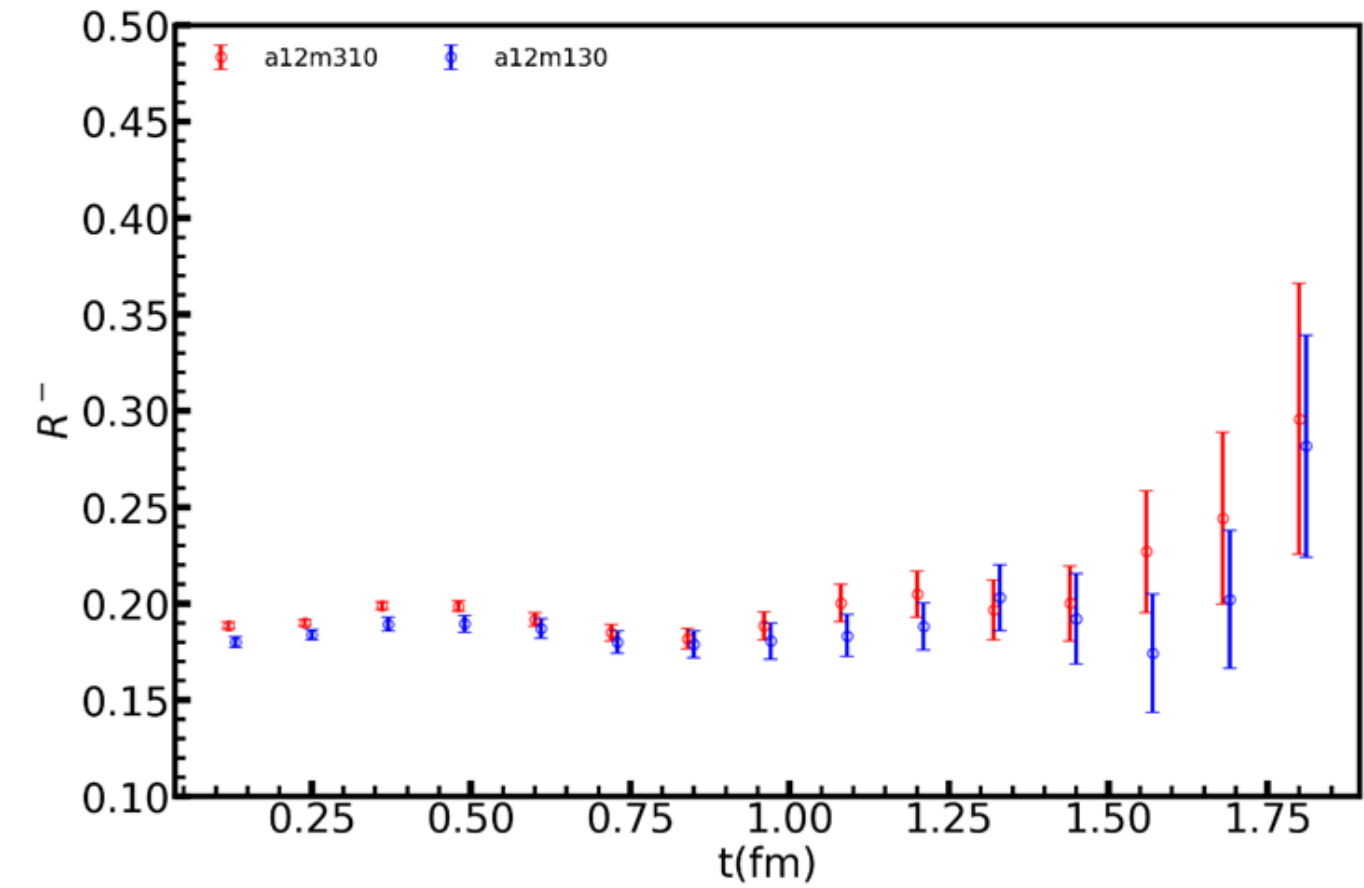
$$R^- = \langle \xi^2 \rangle^{\parallel, bare} \quad R^+ = \langle 1^2 \rangle^{bare}$$



03 Transverse φ : conventional moments



Small lattice-spacing dependence



Weak pion-mass dependence within the current uncertainties

Transverse φ LCDA moments from the conventional local-operator method



Gradient flow evolves gauge and fermion fields in an auxiliary time and smooths ultraviolet fluctuations.

It introduces a smoothing scale for composite operators and supports renormalization through continuum matching.

Shindler, arXiv:2311.18704

Proposed arbitrary-order non-singlet PDF moments with one-loop matching, addressing power-divergent lattice mixing.

Francis et al., arXiv:2510.26738

Computed pion PDF ratios through $\langle x^5 \rangle / \langle x \rangle$ at four lattice spacings, with continuum extrapolation and NNLO matching.

Continuum strategy

At fixed flow time, take the continuum limit to restore $O(4)$ symmetry and avoid lattice power-divergent subtractions.

PDF evidence and the DA target

Pion PDFs: temporal indices allow zero spatial momentum. Ratios reach $\langle x^5 \rangle / \langle x \rangle$.

DA target: compute moments with gradient flow and compare with conventional numerical results.



Generate flowed fields at positive auxiliary flow time t_f .

$$\partial_{t_f} B_\mu = D_\nu G_{\nu\mu} \quad B_\mu(0, x) = A_\mu(x)$$

$$\partial_{t_f} \chi = D^2 \chi \quad \partial_{t_f} \bar{\chi} = \bar{\chi} \overleftarrow{D}^2$$

$$\chi(0, x) = \psi(x) \quad \bar{\chi}(0, x) = \bar{\psi}(x) \quad r_f = \sqrt{(8t_f)}$$

The flow supplies a physical smoothing scale for the lattice operators.

D and G use the flowed gauge field B. Flow time is distinct from Euclidean source/sink time.



Use vector-current normalization to calibrate the common flowed-field factor.

$$\chi_R = Z_\chi^{1/2} \chi \quad \bar{\chi}_R = Z_\chi^{1/2} \bar{\chi} \quad O_{n,R} = Z_\chi O_n \text{ for all } n$$

Vector channel: $\Gamma = \gamma_i$, with $i = 1, 2, 3$

$$C_{\gamma_i}(t_f; \tau) = \sum_x \langle 0 | \mathcal{O}_{\gamma_i}(t_f; x, \tau) \mathcal{O}_{\gamma_i}^\dagger(0; 0, 0) | 0 \rangle$$

$$\frac{C_{\gamma_i}(t_f; \tau)}{C_{\gamma_i}(0; \tau)} \rightarrow \frac{\langle 0 | \mathcal{O}_{\gamma_i}(t_f) | h_0 \rangle}{\langle 0 | \mathcal{O}_{\gamma_i}(0) | h_0 \rangle}$$

$$\mathring{Z}_\chi^V(t_f, a) \equiv \frac{\zeta_V(t_f) Z_V(a)}{\langle R_{V,i}^L(t_f, a) \rangle}$$

Gao et al., Eqs. (8), (10), (12), (13)



Two-point ratios in the bilinear channels

$$C_{\Gamma}(t_f; \tau) = \sum_x \langle 0 | \mathcal{O}_{\Gamma}(t_f; x, \tau) \mathcal{O}_{\Gamma}^{\dagger}(0; 0, 0) | 0 \rangle$$

$$\frac{C_{\Gamma}(t_f; \tau)}{C_{\Gamma}(0; \tau)} \rightarrow \frac{\langle 0 | \mathcal{O}_{\Gamma}(t_f) | h_0 \rangle}{\langle 0 | \mathcal{O}_{\Gamma}(0) | h_0 \rangle}$$

$$\langle O_{\Gamma}^R(\mu) \rangle = \lim_{t_f \rightarrow 0} \lim_{a \rightarrow 0} \frac{\mathring{Z}_{\chi}^V(t_f, a) \langle O_{\Gamma}^L(t_f, a) \rangle}{\zeta_{\Gamma}(\mu, t_f)}$$

First $a \rightarrow 0$ at fixed physical t_f , then $t_f \rightarrow 0$ after matching.

Gao et al., arXiv:2609.23379, Eqs. (10), (12), (16)



PDF: forward matrix element

$$\langle h(p) | O | h(p) \rangle$$

DA: vacuum-to-meson matrix element

$$\langle 0 | O | \varphi(p, \lambda) \rangle$$

$$\langle p' | \partial_\alpha O | p \rangle = i(p'_\alpha - p_\alpha) \langle p' | O | p \rangle$$

Total derivatives vanish in forward matrix elements.

For DAs, time derivatives can contribute even for a meson at rest.

DA matching in the $\{V^-, V^+\}$ operator basis

$$\hat{Z}_\chi^V M^-(t_f) = \zeta_{--} M_R^-(\mu) + \zeta_{-+} M_R^+(\mu) + O(t_f)$$

After $a \rightarrow 0$ at fixed t_f . $M^\pm = \langle 0 | V^\pm | \varphi \rangle$. $V^\pm \equiv O^\pm$ on the local-operator slide. ζ_{--} and ζ_{-+} depend on (t_f, μ) .



Longitudinal ϕ second moment: conventional $\{4jk\}$ versus flowed V_{i44} .

Conventional: distinct indices $\{4jk\}$

$$V_{\{4jk\}}^-(0) = \bar{s} \gamma_{\{4} \overleftrightarrow{D}_j \overleftrightarrow{D}_k s$$

Choose distinct indices 4, j, k ($j \neq k$)
to avoid additional lattice mixing.
The conventional ratio uses nonzero
spatial momentum.

Gradient flow: V_{i44}

$$V_{i44}^-(t_f) = ST[\bar{\chi}_s \gamma_i \overleftrightarrow{D}_4 \overleftrightarrow{D}_4 \chi_s]$$

Use one spatial index i and two
Euclidean-time indices 4.
 V_{i44} has a nonzero matrix element
for a meson at zero spatial momentum.

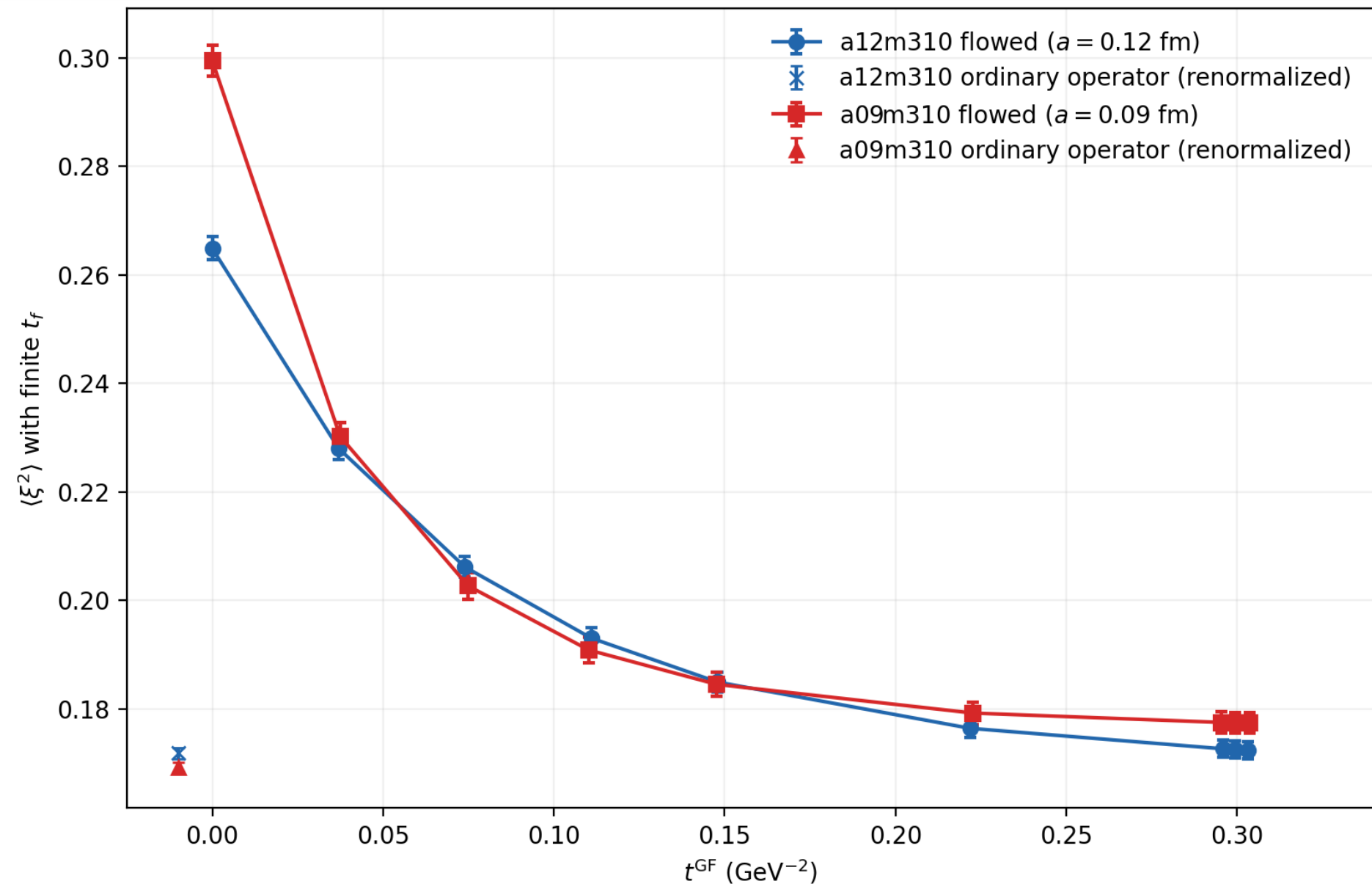
i, j, k = 1, 2, 3. Index 4 denotes Euclidean time. ST: symmetrization and trace subtraction.



Planned ensembles

Ensemble	a (fm)	$L^3 \times T$	m_π^v (MeV)	$m_{\eta_s}^v$ (MeV)	m_π^s (MeV)	$m_{\eta_s}^s$ (MeV)
a04m310	0.0425(5)	$64^3 \times 192$	300	636	315	693
a06m310	0.0574(5)	$48^3 \times 144$	322	656	329	725
			162	718		
a09m310	0.0882(7)	$32^3 \times 96$	313	647	316	697
			152	690		
a12m310	0.1213(9)	$24^3 \times 64$	310	612	305	676
			190	659		
a12m220S	0.1213(9)	$24^3 \times 64$	222		220	675
a12m220	0.1213(9)	$32^3 \times 64$	225		220	675
a12m130	0.1213(9)	$48^3 \times 64$	131	679	132	675





Ensemble	a12m310	a09m310
a(fm)	0.12	0.09
$m_{\eta_s}^v$ (MeV)	659	690
conf	199	129
src	8	12

Small flow time

Large separation, consistent with strong UV sensitivity and flow-enhanced cutoff effects.

Larger flow time

The two lattice spacings give closer values.

Current result: longitudinal ϕ at two lattice spacings, before matching.



Conventional local operators require carefully chosen index structures and correlators at nonzero momentum to avoid unwanted lattice mixing.

Gradient flow permits V_{i44} at zero spatial momentum and alleviates lattice mixing constraints through the continuum construction.

DA calculations retain mixing with total-derivative operators, so matching remains a matrix problem.

Next steps: compute flowed moments at more lattice spacings to study cutoff effects and support a continuum extrapolation.
Calculate the DA matching matrix to determine the operator mixing.

References: arXiv:1612.02955, 2311.18704, 2510.26738 and 2609.23379

