

Unpolarized Gluon PDF from Lattice QCD



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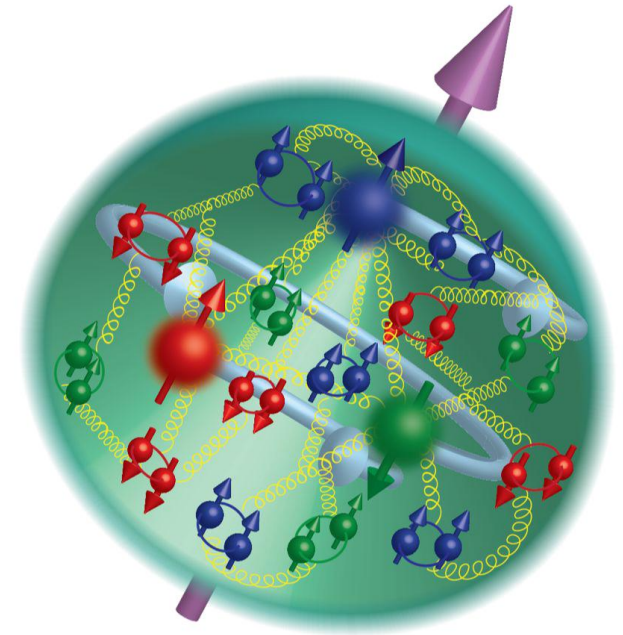
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in collaboration with

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- Nucleons: complicated dynamical systems of quarks and gluons
- PDFs: describe the probability density of finding a parton with momentum fraction x inside a nucleon
- Phenomenological studies : model dependent



Lattice QCD study of gluon PDFs :

- Previous studies: mostly based on pseudo-PDF method

[1] T. Khan et al., Unpolarized gluon distribution in the nucleon from lattice quantum chromodynamics, Phys. Rev. D 104, 094516 (2021)

[2] Z. Fan, W. Good, and H.-W. Lin, Gluon parton distribution of the nucleon from (2+1+1)-flavor lattice QCD in the physical-continuum limit, Phys. Rev. D 108, 014508 (2023).

[3] J. Delmar et al., Gluon PDF of the proton using twisted mass fermions, Phys. Rev. D 108, 094515 (2023).

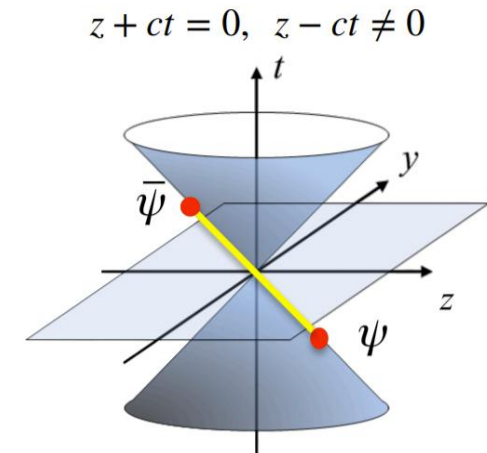
- Recent LaMET calculations: unphysical pion mass

[4] William Good, et al., First Nucleon Gluon PDF from Large Momentum Effective Theory, Phys. Lett. B 872, 140067 (2026).

[5] Alex NieMiera et al., Systematic study of the self-renormalized nucleon gluon PDF in large-momentum effective theory, Phys.Rev.D 114 (2026) 5, 054509

The light-cone PDF of gluon is defined as:

$$xg(x, \mu) = \int \frac{dz^-}{2\pi P^+} e^{-ixP^+ z^-} \langle P | F_{+\mu}^a(z^-) U^{ab}(z^-, 0) F^{b\mu}_{-}(0) | P \rangle$$

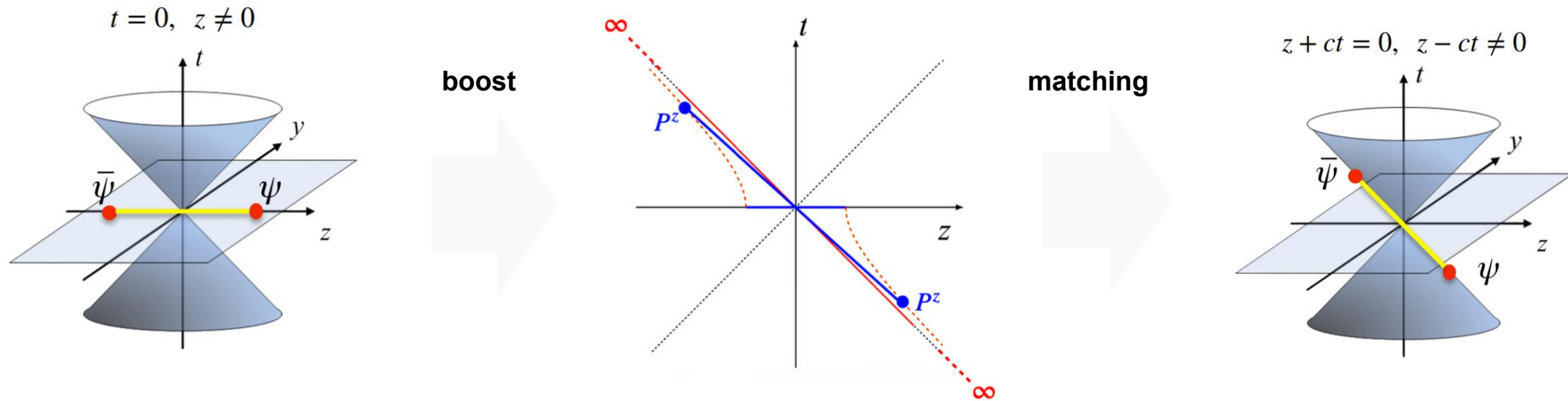


- Light-cone quantities cannot be calculated directly using Lattice QCD approach.

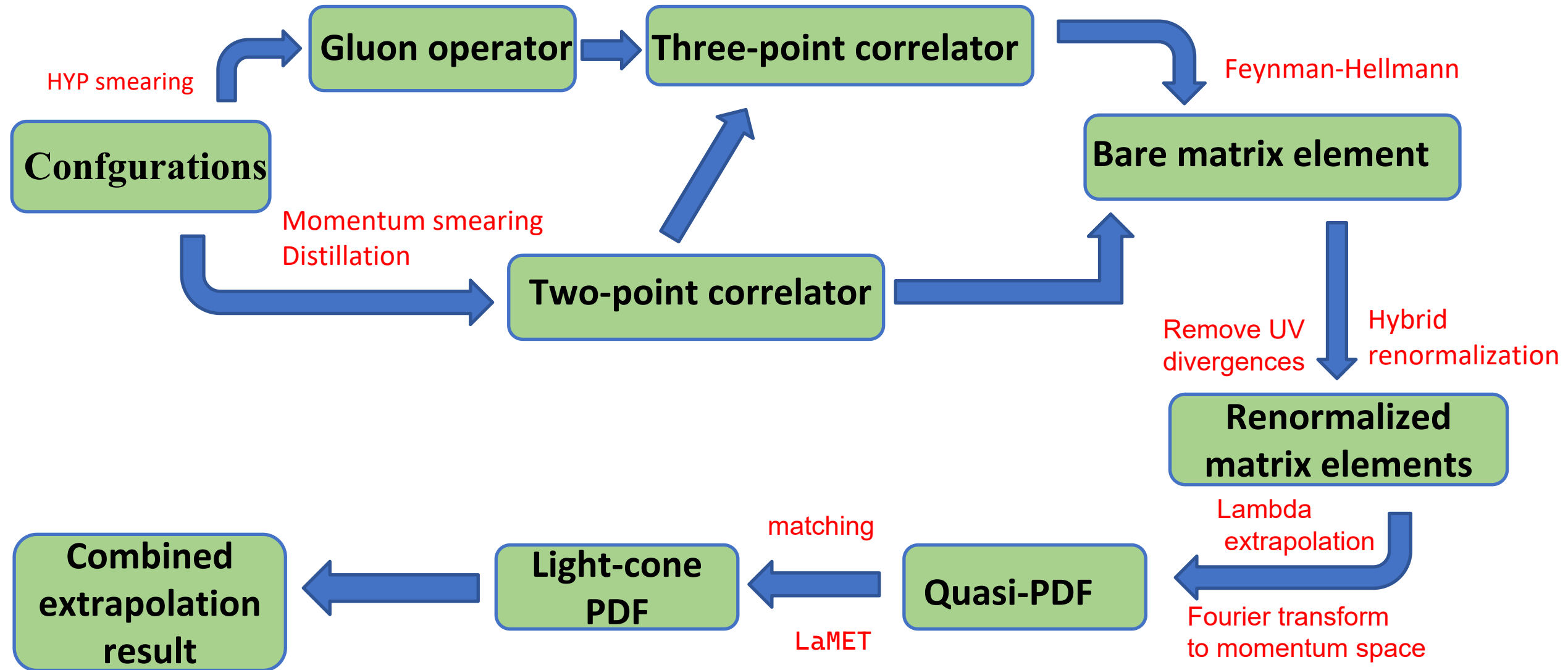
Introduction

Large momentum effective theory(LaMET)

Phys.Rev.Lett.
110 (2013) 262002



- By boosting the nucleon, the quasi-PDF recovers leading-twist light-cone PDF in the infinite momentum limit.
- The difference is in UV physics, perturbative matching can connect the quasi-PDFs (lattice) and the light-cone PDFs.



Lattice Setup

Ensemble	a (fm)	m_π (MeV)	P_z (GeV)	$N_{\text{conf.}}$
C24P29	0.105	292	0, 1.95, 4.46, 2.95	878
C32P29	0.105	292	1.84, 2.21	980
C32P23	0.105	228	1.84, 2.21	597
C48P14	0.105	136	1.72, 1.96, 2.21, 2.46	347
E32P29	0.0897	287	0, 1.72, 2.16, 2.59	1700
F32P30	0.0775	300	0, 2.0, 2.5, 2.99	777
G36P29	0.0688	297	0, 2.0, 2.5, 3.0	349
H48P32	0.0519	317	0, 1.99, 2.48, 2.98	650



PRD109(2024)5,054507
PRD111(2025)5,054504

♦ Continuum limit

Five different lattice spacings

♦ Physical pion mass

Pion mass: 136 - 317MeV

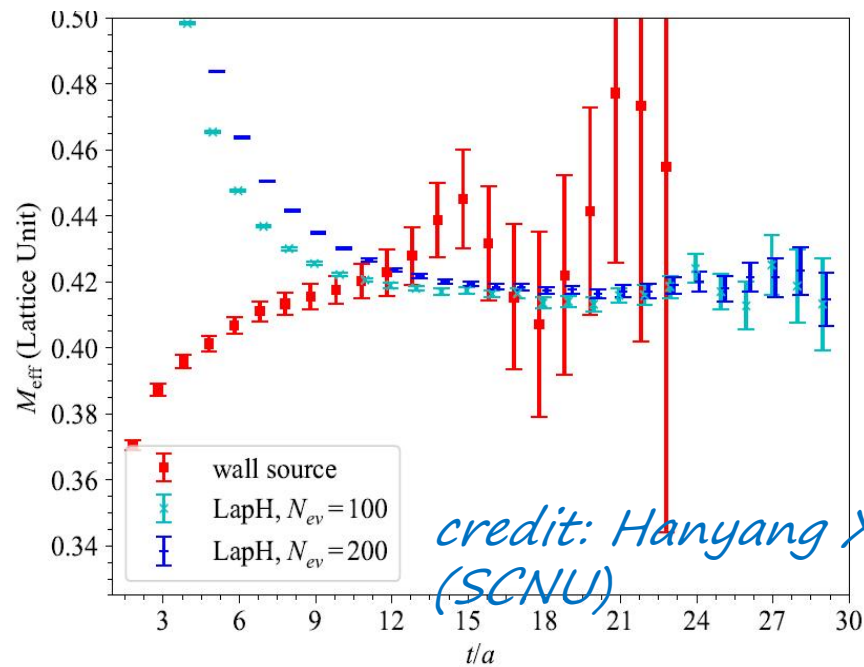
♦ Infinite-momentum limit

Various momenta up to 3GeV

Noise reduction technique

- Distillation technique: $\psi \rightarrow VV^\dagger\psi$

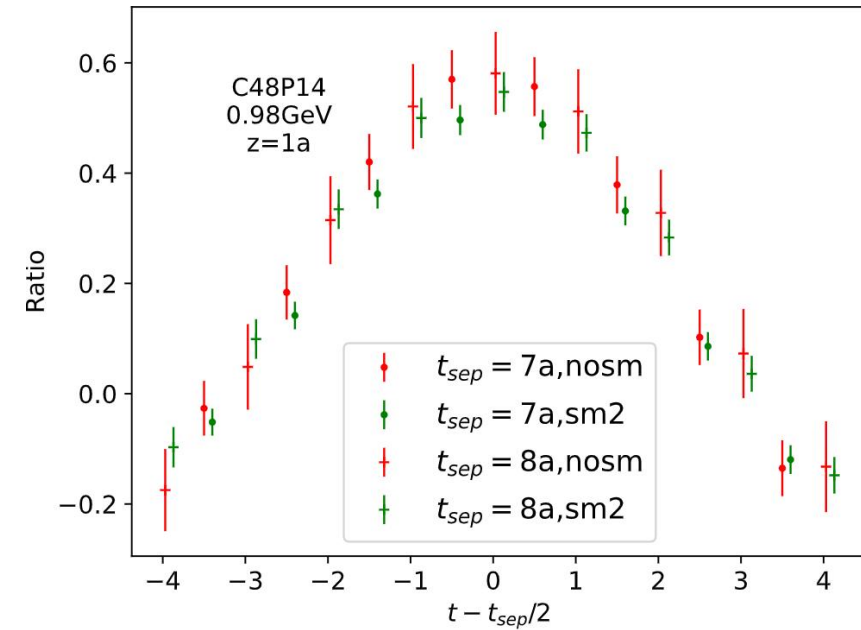
V is made up of a small number (N_{ev}) of eigenvector of Laplacian operator ∇^2



Effective mass of nucleon of ensemble F48P30 at $P_z = 0$ (Nconf=100)

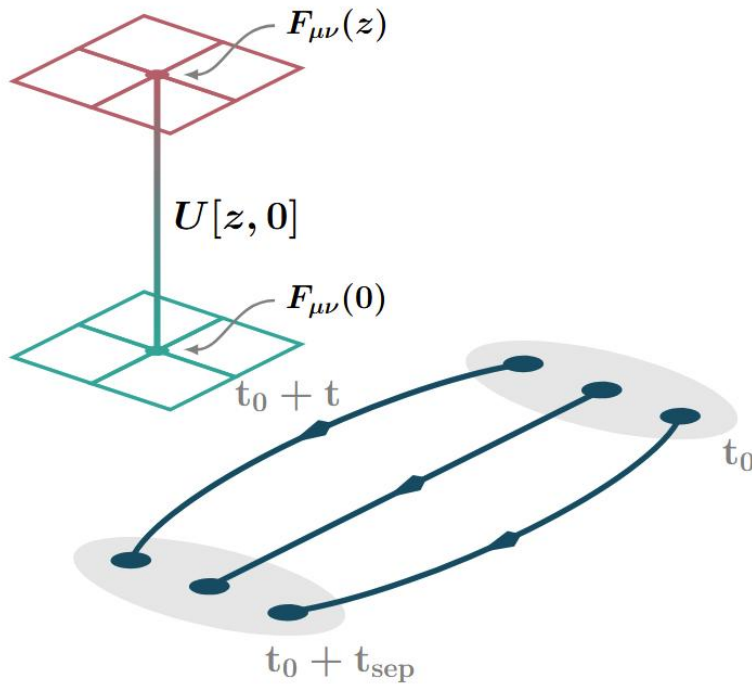
- Momentum smearing technique

$$\tilde{V}(\vec{x}, t) = e^{i\vec{\zeta} \cdot \vec{x}} V(\vec{x}, t), \quad \vec{\zeta} = \left(0, 0, \frac{2 \times 2\pi}{L}\right)$$



- 10 step HYP smearing on gluon operator
- Use $N = P_+(u^T C \gamma_5 \gamma_t d)u$ to enhance ground state overlap at large P_z

Quasi-matrix element



An illustration of gluon's three-point correlator

Two-point correlator is defined as

$$C_{2\text{pt}}(P_z; t_{\text{sep}}) = \sum_{t_0} \langle 0 | N(P_z; t_0 + t_{\text{sep}}) N^\dagger(P_z; t_0) | 0 \rangle$$

with

$$N(\vec{P}; t) = \sum_{\vec{x}} e^{-i\vec{x} \cdot \vec{P}} P_+ (u^T C \gamma_5 \gamma_t d) u(\vec{x}; t)$$

Three-point correlator is defined as

$$C_{3\text{pt}}(P_z; z; t_{\text{sep}}, t) = \sum_{t_0} \langle 0 | N(P_z; t_0 + t_{\text{sep}}) O(z; t_0 + t) N^\dagger(P_z; t_0) | 0 \rangle$$

where $O(z; t)$ is gluon operator

$$O(z; t) = M_{tx;tx}(z; t) + M_{ty;ty}(z; t) - 2M_{xy;xy}(z; t)$$

with

$$M_{\mu\lambda;\nu\rho}(z; t) = \sum_{\vec{x}} \text{Tr} [F_{\mu\lambda}(\vec{x} + z\hat{z}; t) U(\vec{x} + z\hat{z}, \vec{x}; t) F_{\nu\rho}(\vec{x}; t) U(\vec{x}, \vec{x} + z\hat{z}; t)]$$

Bare matrix element

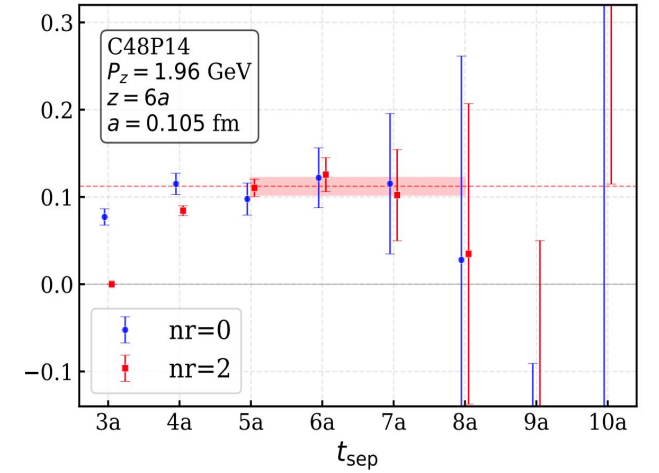
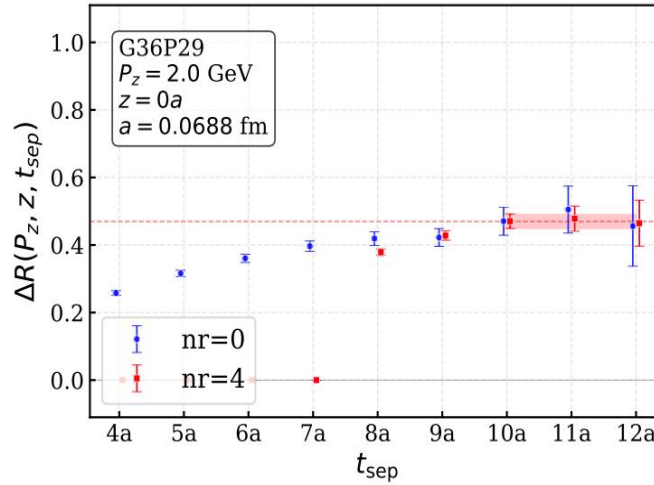
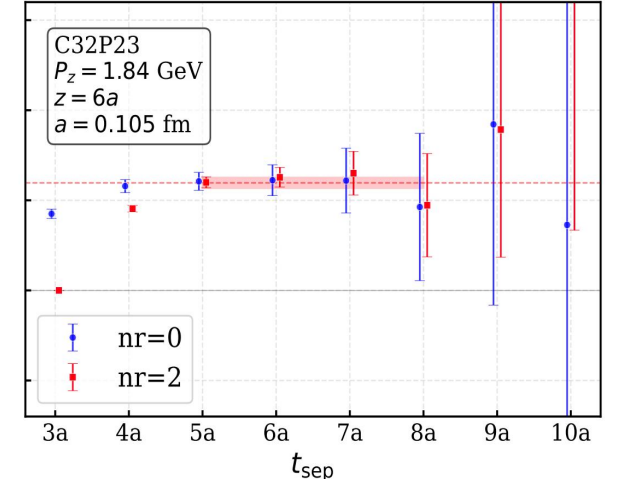
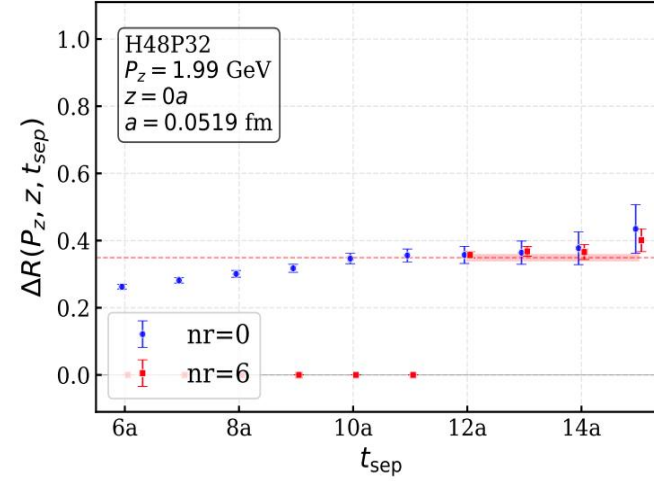


$$R(t, t_{\text{sep}}) = \frac{C^{3\text{pt}}}{C^{2\text{pt}}} = h_B + c_1 e^{-\Delta E(t_{\text{sep}} - t)} + c_1 e^{-\Delta E t} + c_2 e^{-\Delta E t_{\text{sep}}} + \dots,$$

- Feynman-Hellmann theory:

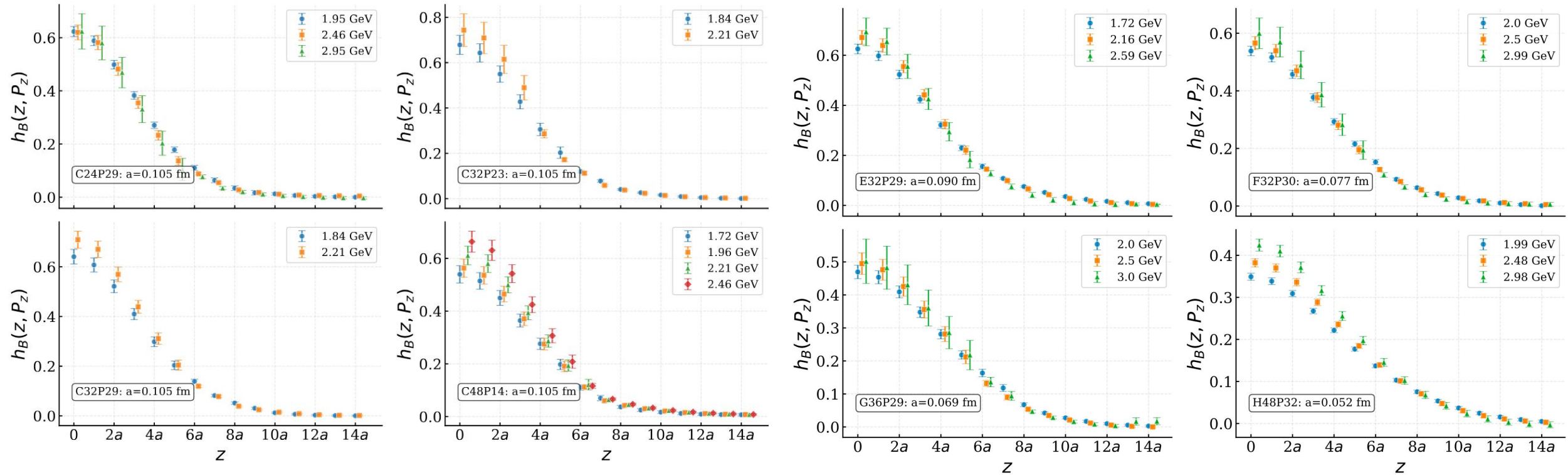
$$S(t_{\text{sep}}) = \sum_{t_i = n_r a}^{t_{\text{sep}} - n_r a} R(t_i, t_{\text{sep}}) = t_{\text{sep}} h_B + \dots,$$

$$\Delta R(t_{\text{sep}}) = S(t_{\text{sep}}) - S(t_{\text{sep}} - 1) \xrightarrow{t_{\text{sep}} \rightarrow \infty} h_B.$$



Effective matrix element ΔR

Bare matrix element



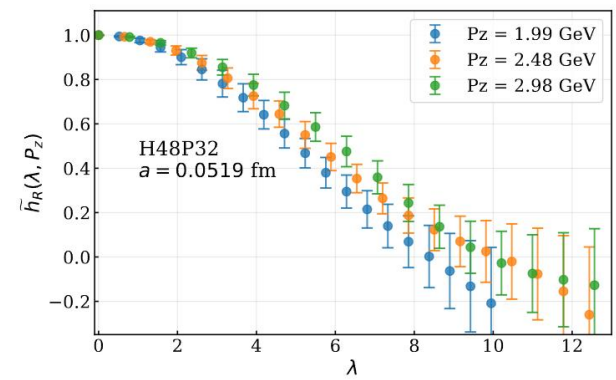
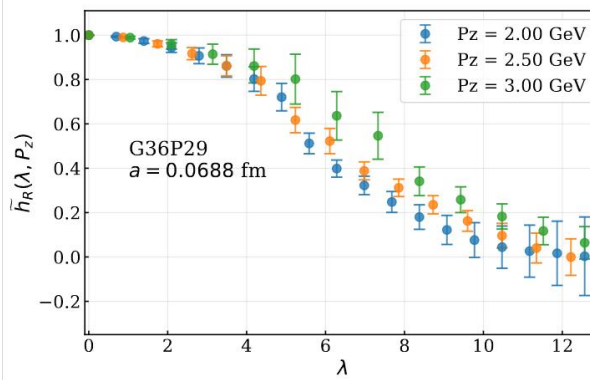
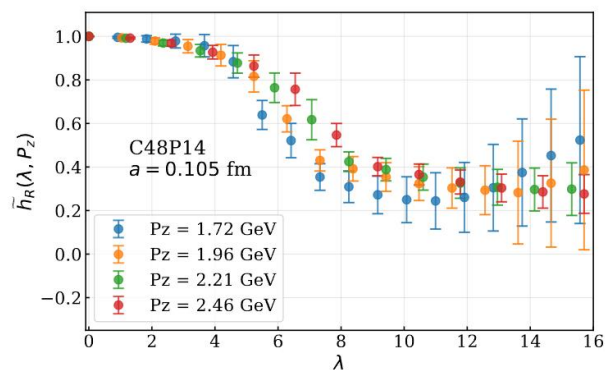
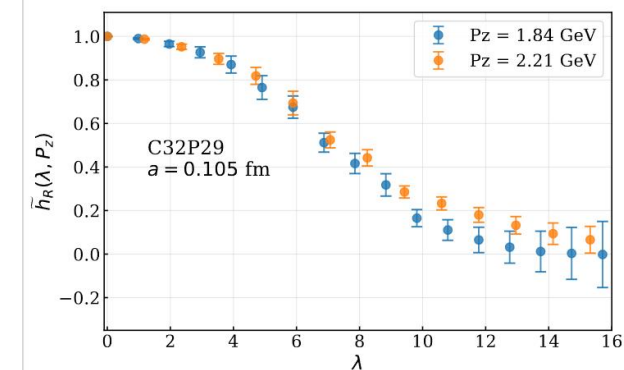
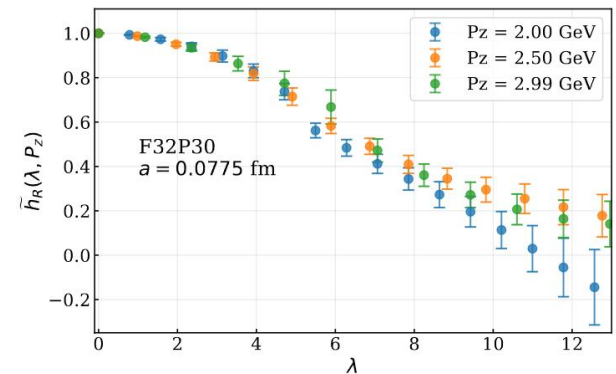
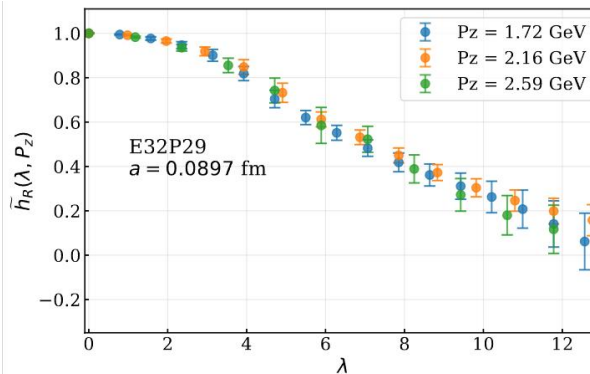
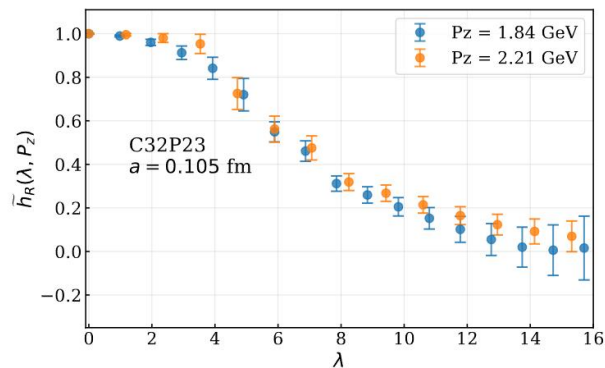
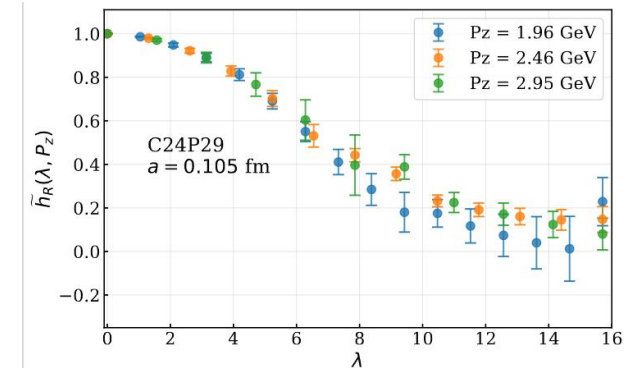
Renormalization

Hybrid renormalization:

$$\tilde{h}_R(z, P_z) = \frac{\tilde{h}_B^n(z, P_z, 1/a)}{\tilde{h}_B^n(z, P_z, 0, 1/a)} \theta(z_s - |z|) + \eta_s \frac{\tilde{h}_B^n(z, P_z, 1/a)}{Z_R(z, 1/a)} \theta(|z| - z_s), z_s = 0.3 \text{ fm}$$

ratio scheme

self renormalization scheme

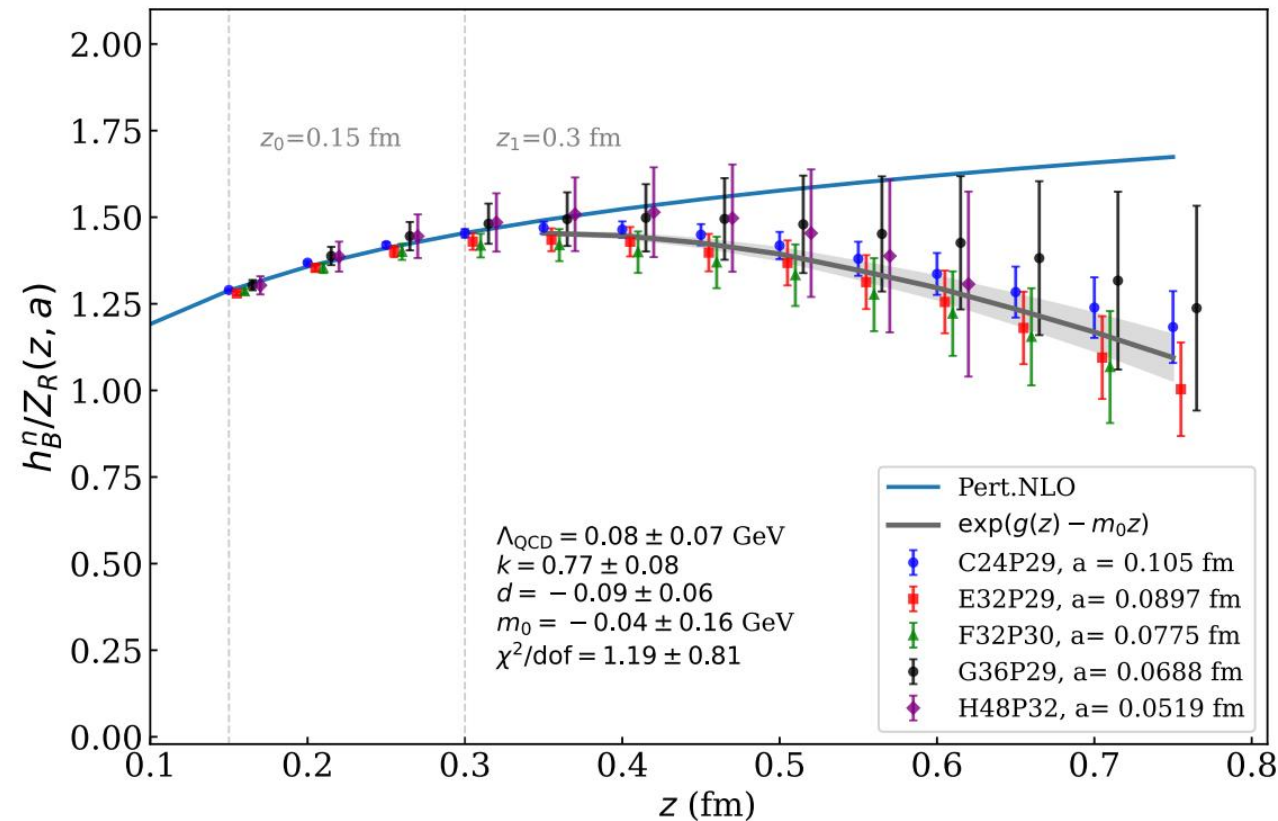


$$\lambda = zP_z$$

Renormalized matrix elements

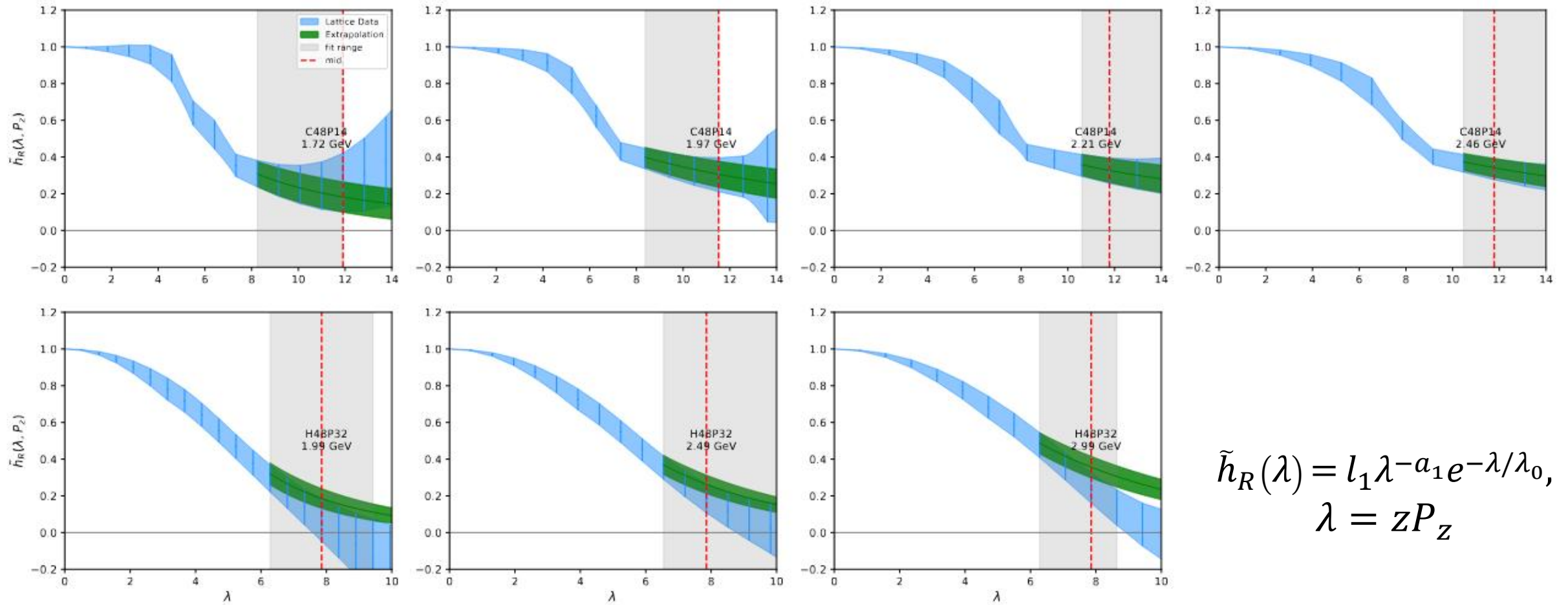
$P_z=0$ data at five lattice spacings are used to extract self-renormalization factor

$$Z_R(z, 1/a, \mu) = \exp \left\{ \frac{kz}{a \ln[a\Lambda_{\text{QCD}}]} + m_0 z \right. \quad \text{linear divergence} \\ \left. + \frac{5C_A}{3b_0} \ln \left[\frac{\ln(1/a\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})} \right] + \frac{1}{2} \ln \left[1 + \frac{d}{\ln[a\Lambda_{\text{QCD}}]} \right]^2 \right\} \quad \text{logarithmic UV divergence}$$



Self-renormalized matrix elements at $P_z = 0$

Large Lambda extrapolation



$$\tilde{h}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0},$$

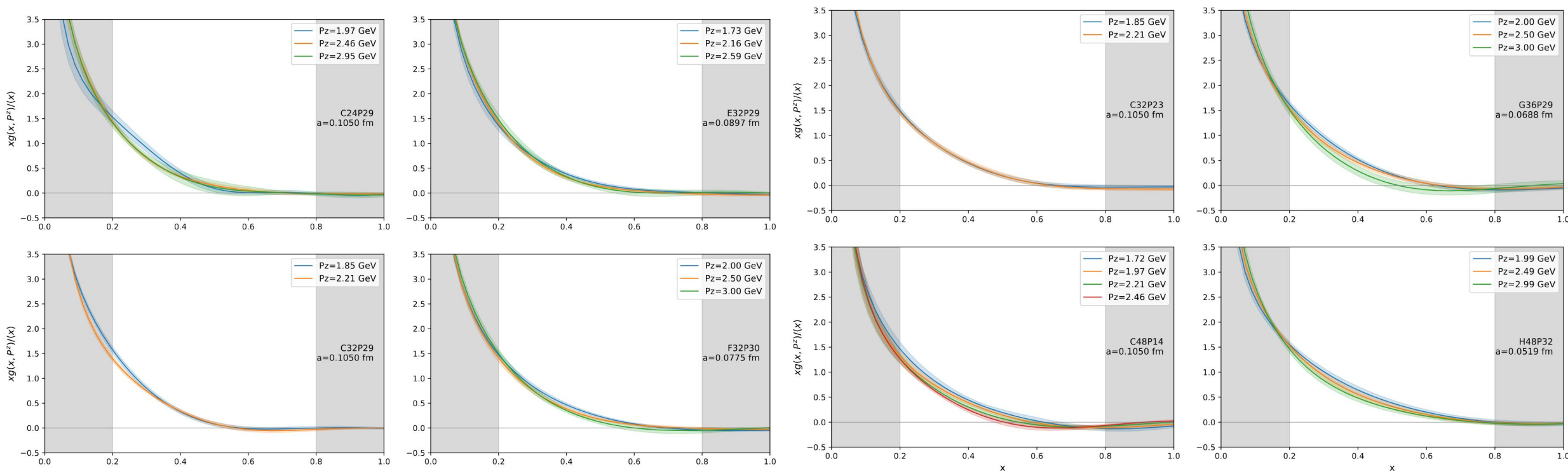
$$\lambda = zP_z$$

Perturbative matching



$$x\tilde{g}(x, P_z) = \int_{-1}^1 \frac{dy}{|y|} C_{\text{hybrid}}\left(\frac{x}{y}, \lambda_s, \frac{\mu}{yP_z}\right) yg(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(xP_z)^2}, \frac{\Lambda_{QCD}^2}{((1-x)P_z)^2}\right)$$

Quasi-PDF (Lattice) Matching kernel (Perturbative) light-cone PDF



Light-cone PDFs

$$\langle x \rangle = \int_0^1 x g(x) dx$$

Combined extrapolation



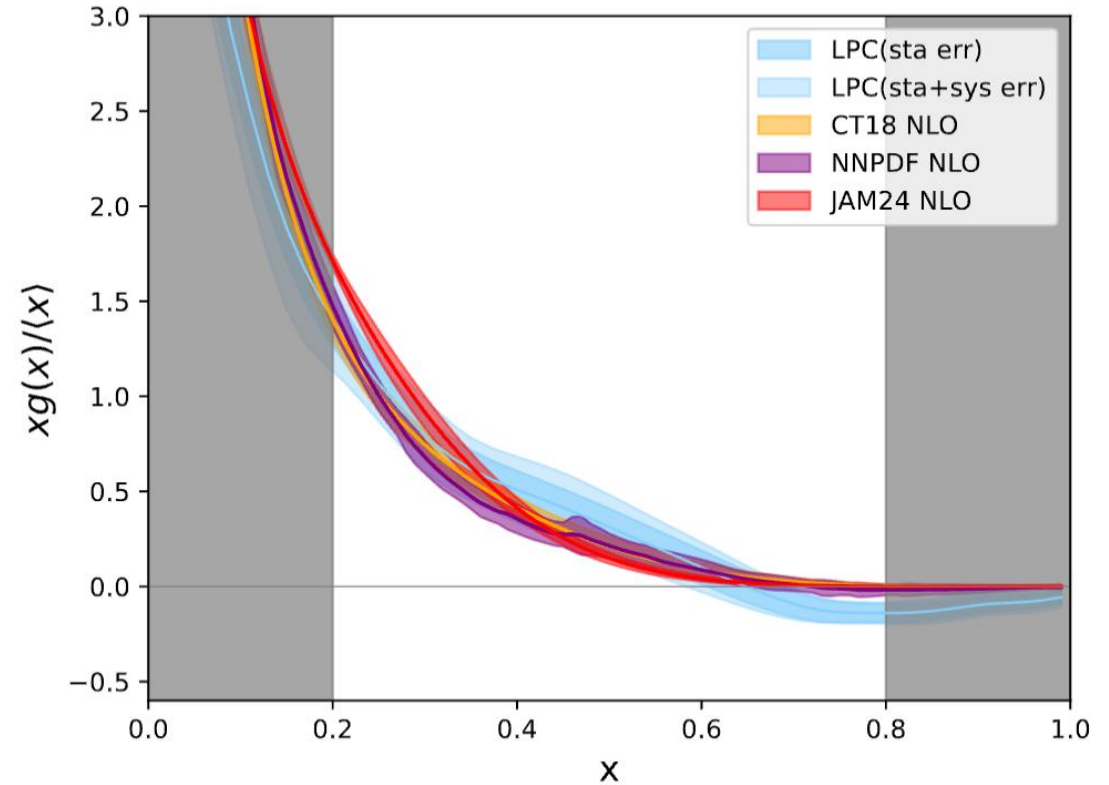
- infinite momentum limit ☒ continuum limit ☒ physical pion mass ☒

$$xg(x, P_z, a) = xg_0(x) + a^2 f(x) + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$

$$xg(x, P_z, a) = xg_0(x) + \textcolor{red}{a} \textcolor{red}{f(x)} + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$

$$xg(x, P_z, a) = xg_0(x) + a^2 f(x) + \textcolor{red}{a} \textcolor{red}{P_z^2 h(x)} + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$

the results are
combined
together by AIC
model average



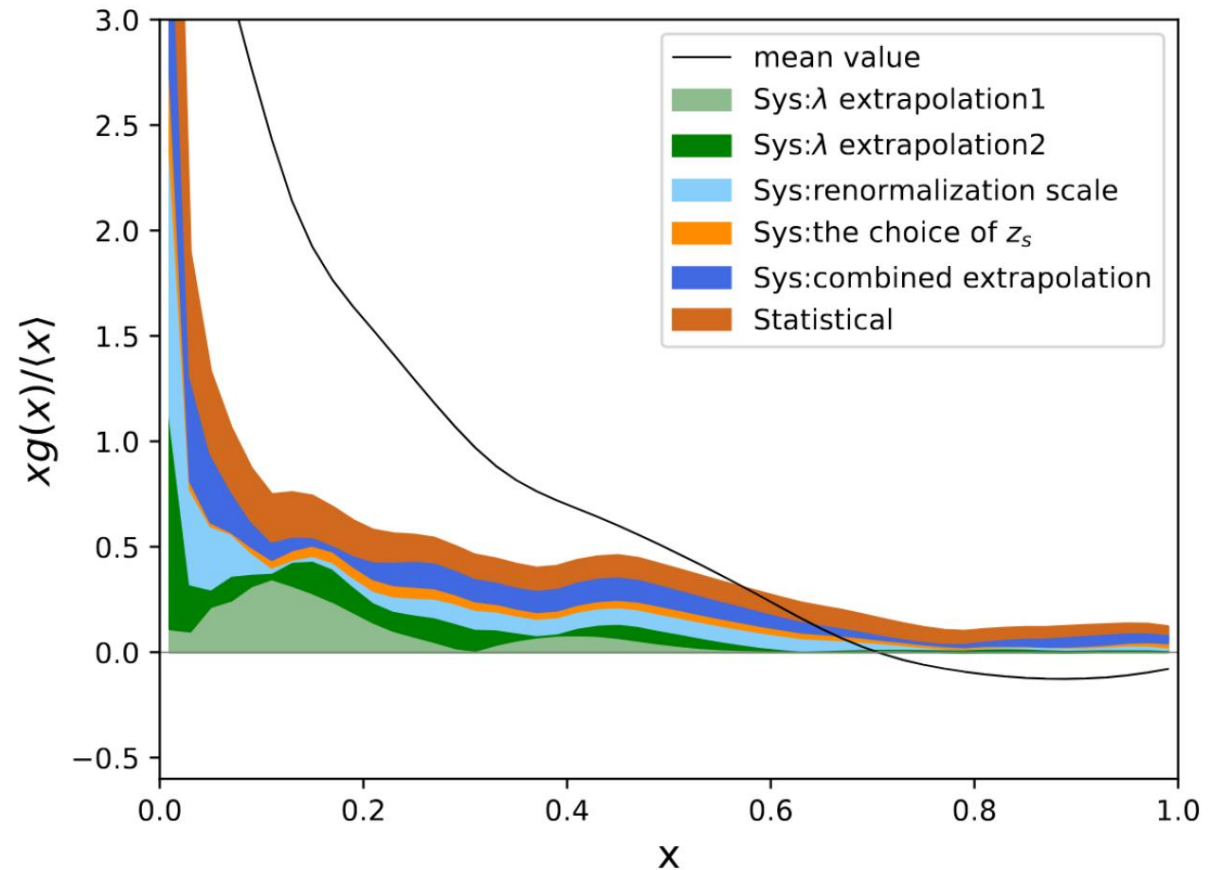
Lattice results in comparison with global fit results

Systematic uncertainties



Systematic uncertainties:

- λ extrapolation
- Renormalization scale μ dependence
- Combined extrapolation
- Hybrid scheme z_s dependence



Estimation of statistical & systematic uncertainties

Summary and outlook

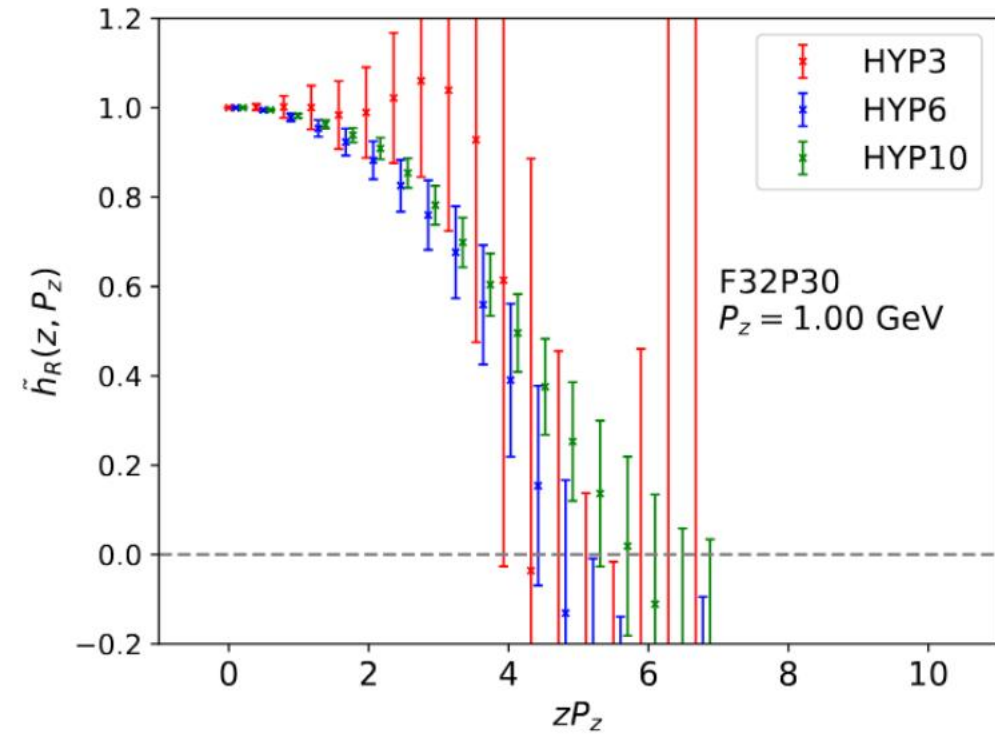
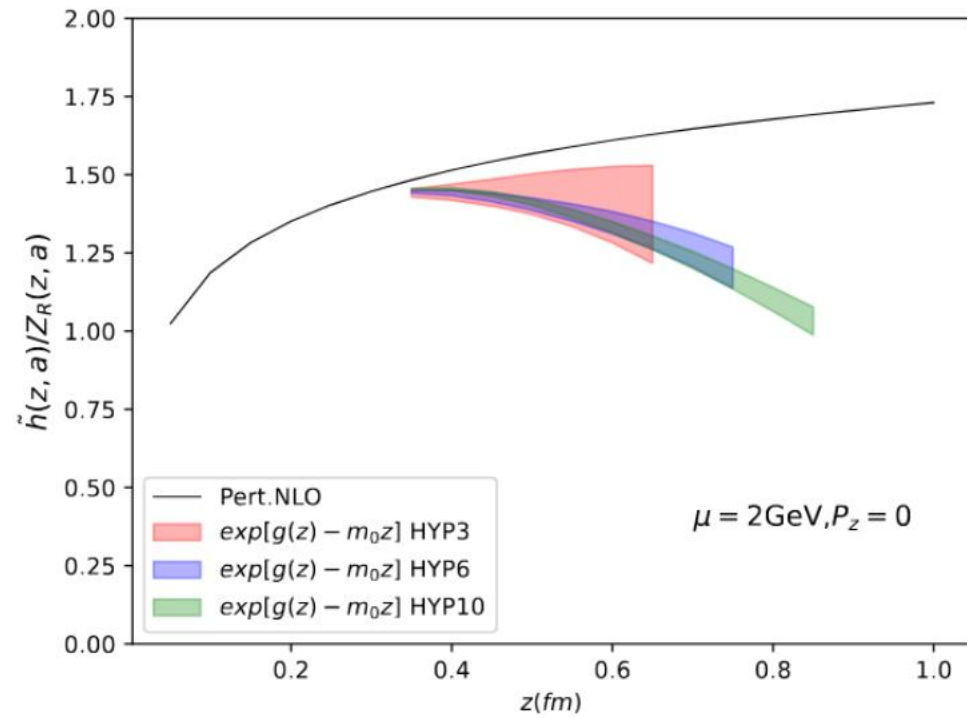


- We calculated the unpolarized gluon PDF of the nucleon in LaMET framework.
- Extrapolated to infinite momentum-limit, continuum limit and physical pion mass.
- Our result agrees with the global fit result within 2σ .

Thankyou!

Backups

Hype smearing



Renormalized matrix elements under different HYP smearing times

Matching kernel

$$C_{\text{ratio}}\left(\xi, \frac{\mu}{yP_z}\right) = \delta(1 - \xi) + \frac{\alpha_s C_A}{2\pi} \begin{cases} \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi} \ln \frac{\xi}{\xi-1} + \frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)} \right]_+ & \xi > 1 \\ \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi} \left(-\ln \frac{\mu^2}{4y^2 P_z^2} + \ln(\xi(1-\xi)) \right) - \frac{15-56\xi+102\xi^2-96\xi^3+48\xi^4}{6(1-\xi)} \right]_+ & 0 < \xi < 1 \\ \left[\frac{-2(1-\xi+\xi^2)^2}{1-\xi} \ln \frac{\xi}{\xi-1} - \frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)} \right]_+ & \xi < 0, \end{cases}$$

$$C_{\text{hybrid}}(\xi, \lambda_s, \frac{\mu}{yP_z}) = C_{\text{ratio}}\left(\xi, \frac{\mu}{yP_z}\right) + \frac{\alpha_s C_A}{2\pi} \frac{5}{6} \left(-\frac{1}{|1-\xi|} + \frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)} \right)_+$$

AIC model average

The normalized AIC weight is defined as

$$\omega_i = \frac{\exp \left[-\frac{1}{2} (\chi_i^2 + 2n_{i,\text{par}} - n_{i,\text{data}}) \right]}{\sum_j \exp \left[-\frac{1}{2} (\chi_j^2 + 2n_{j,\text{par}} - n_{j,\text{data}}) \right]},$$

The cumulative distribution weight is defined as:

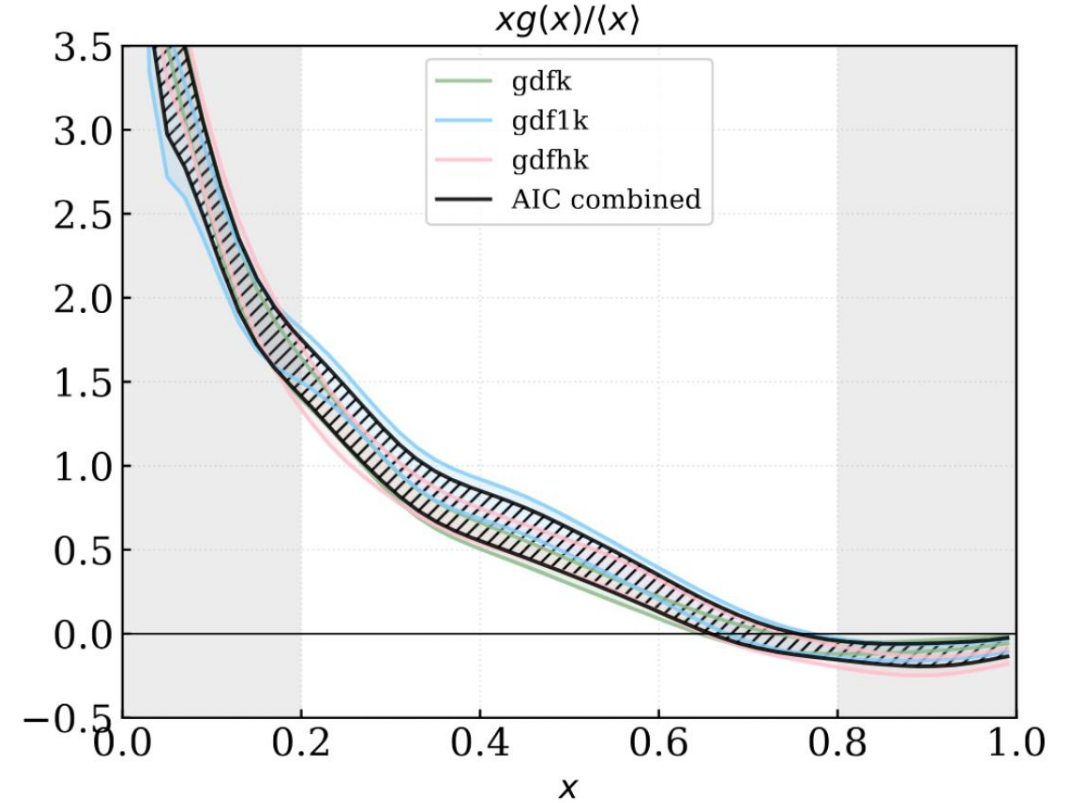
$$P(y; \lambda) = \int_{-\infty}^y dx \sum_i \omega_i \frac{1}{\sigma_i \sqrt{\lambda} \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - m_i}{\sigma_i \sqrt{\lambda}} \right)^2 \right],$$

The quantiles $y_q(\lambda)$ is defined by:

$$P(y_q(\lambda); \lambda) = q.$$

The AIC combined central value and uncertainties can be estimated by:

$$m_{\text{AIC}} = \frac{y_{0.84}(1) + y_{0.16}(1)}{2}, \quad \sigma_{\text{tot}}(\lambda) = \frac{y_{0.84}(\lambda) - y_{0.16}(\lambda)}{2}$$



light-cone PDF