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Gradient flow in perturbative QCD and lattice QCD

Inclusive quarkonium production: from perturbative to nonperturbative

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01 **Motivation**

Inclusive quarkonium production from first principles

02 **Gradient flow and perturbative matching**

A natural bridge between lattice and continuum schemes

03 **P-wave quarkonium decays from pNRQCD and lattice QCD**

A first application and validation of the methodology

04 **Heavy-quark expansion in gradient flow**

Including heavy-quark mass effects

05 **Summary and outlook**

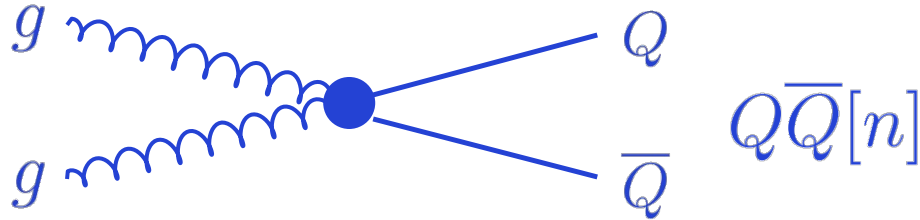
Nonrelativistic QCD (NRQCD) factorization

NRQCD factorization formula for inclusive quarkonium production

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X) \langle \mathcal{O}_n^H \rangle$$

Short-distance coefficients (SDCs)

Perturbative α_s expansion

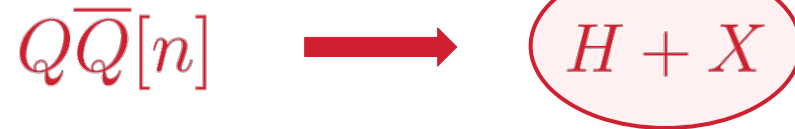


+ inclusive hard radiation

$$n = {}^{2S+1}L_J^{[1/8]}$$

Universal long-distance matrix elements (LDMEs)

v^2 expansion



Color-octet production LDMEs:

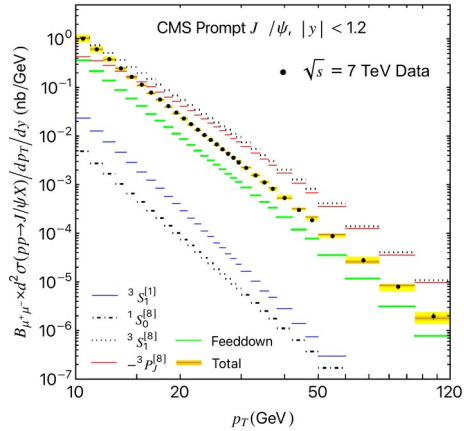
- Direct lattice calculation remains open
- Extracted from fits to experimental data

For $H = J/\psi, \psi(2S), \Upsilon(nS)$:

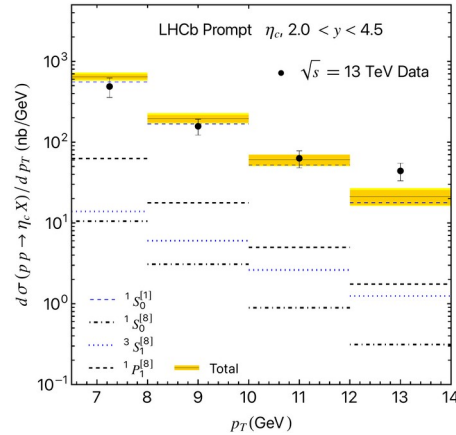
$$n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]}, \dots$$

Testing LDME universality

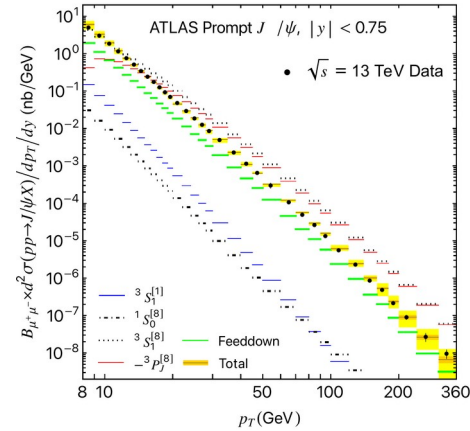
CMS: J/ψ yield (fit)



LHCb: η_c yield (fit)

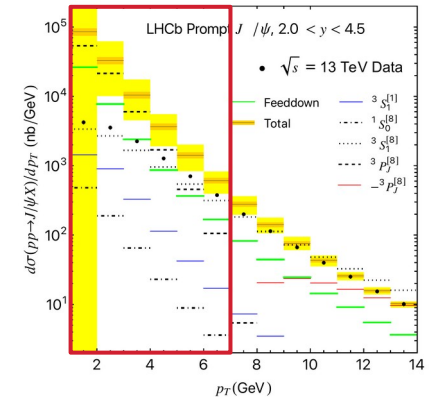


ATLAS: high- p_T yield

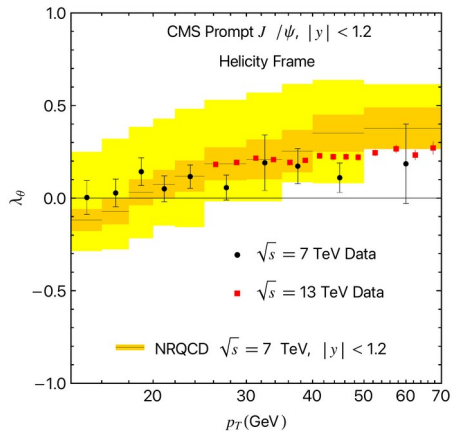


Kinematic tensions in end-point regions

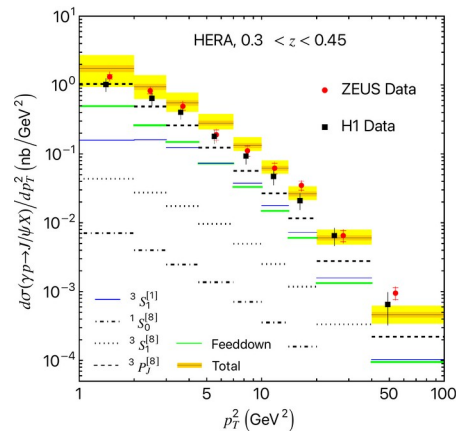
LHCb: $p_T < 7$ GeV



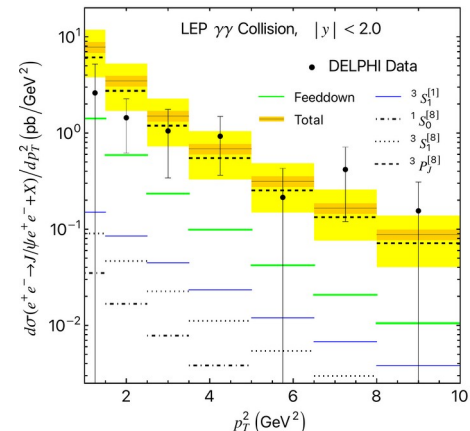
CMS: polarization



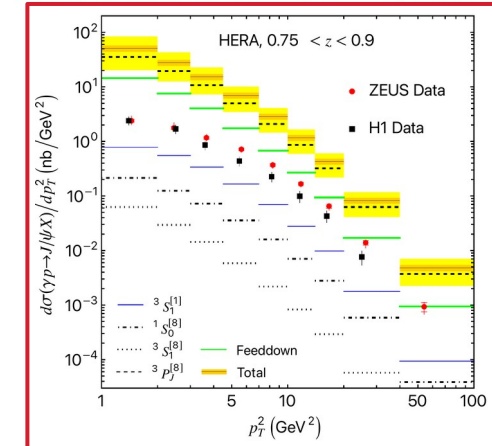
HERA: $0.30 < z < 0.45$



LEP: two-photon yield



HERA: $0.75 < z < 0.90$



Within strongly coupled potential NRQCD (pNRQCD), we have

Color-octet LDMEs

$$\langle \mathcal{O}^V(^3S_1^{[8]}) \rangle = \frac{1}{2N_c m^2} \frac{3 |R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{10;10},$$

$$\langle \mathcal{O}^V(^1S_0^{[8]}) \rangle = \frac{1}{6N_c m^2} \frac{3 |R_V^{(0)}(0)|^2}{4\pi} c_F^2 \mathcal{B}_{00},$$

$$\langle \mathcal{O}^V(^3P_J^{[8]}) \rangle = \frac{2J+1}{18N_c} \frac{3 |R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{00}.$$

Universal gluonic correlators

$$\mathcal{E}_{10;10} = \left\| d^{dac} \int_0^\infty dt_1 t_1 \int_{t_1}^\infty dt_2 g E^{b,i}(t_2) \Phi_0^{bc}(t_1; t_2) \right\|^2, \\ \times g E^{a,i}(t_1) \Phi_0^{df}(0; t_1) \Phi_\ell^{ef}(0) | \Omega \rangle$$

$$\mathcal{B}_{00} = \left\| \int_0^\infty dt g B^{a,i}(t) \Phi_0^{ac}(0; t) \Phi_\ell^{bc}(0) | \Omega \rangle \right\|^2,$$

$$\mathcal{E}_{00} = \left\| \int_0^\infty dt g E^{a,i}(t) \Phi_0^{ac}(0; t) \Phi_\ell^{bc}(0) | \Omega \rangle \right\|^2.$$

Φ_0 : temporal adjoint Wilson line

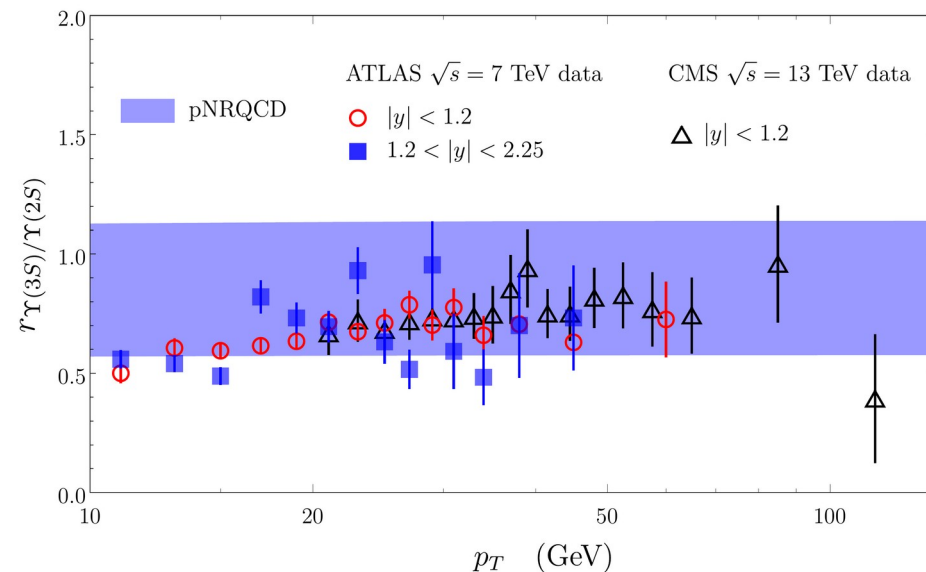
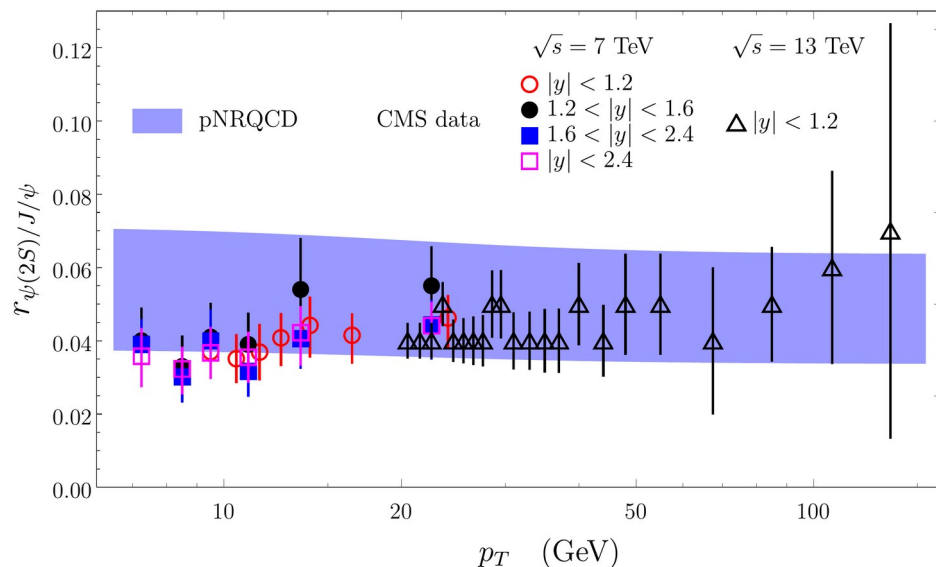
Φ_ℓ : adjoint gauge-completion Wilson line

Can be calculated on the lattice

Universality fixes the direct-production ratios

$$\frac{\sigma_{\psi(2S)}^{\text{direct}}}{\sigma_{J/\psi}^{\text{direct}}} = \frac{|R_{\psi(2S)}^{(0)}(0)|^2}{|R_{J/\psi}^{(0)}(0)|^2}$$

$$\frac{\sigma_{\Upsilon(3S)}^{\text{direct}}}{\sigma_{\Upsilon(2S)}^{\text{direct}}} = \frac{|R_{\Upsilon(3S)}^{(0)}(0)|^2}{|R_{\Upsilon(2S)}^{(0)}(0)|^2}$$



Data comparisons include feeddown and dimuon branching fractions

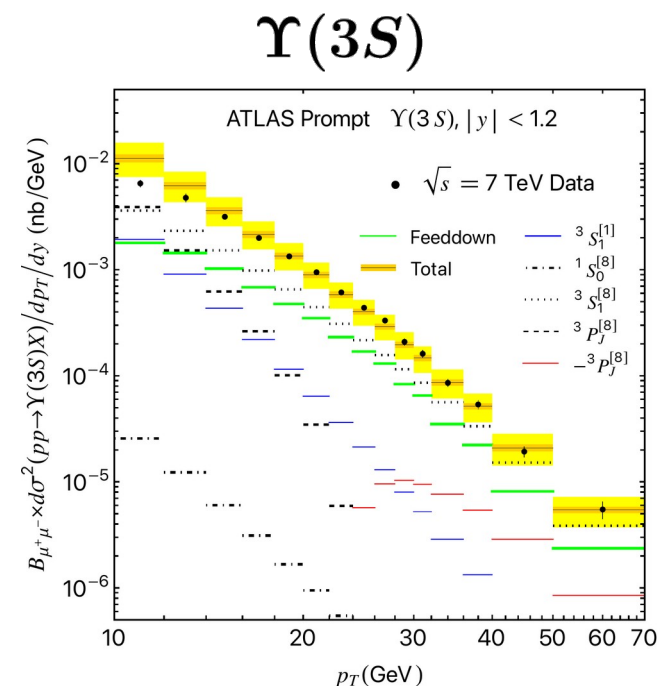
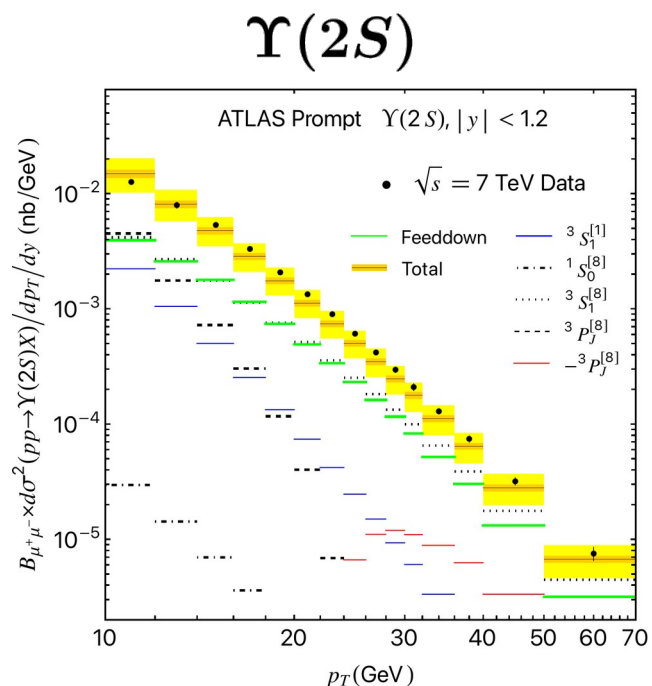
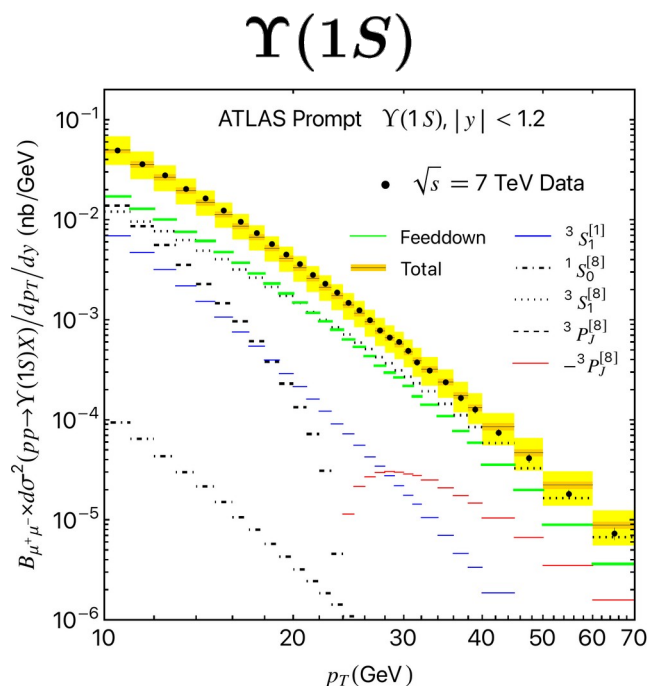
No explicit perturbative production calculation is needed

pNRQCD predictive power: example 2

Charmonium fit inputs $\longrightarrow$

Universal correlators
Scale evolution: charm $\rightarrow$ bottom

$\longrightarrow \Upsilon(nS)$ predictions



Wavefunctions and feeddown included. $\Upsilon(1S)$: extension of the strongly coupled hierarchy.

Challenge: determine the universal correlators independently from lattice QCD

Inclusive decay width

$$\Gamma(\chi_{QJ} \rightarrow \text{LH}) = \frac{3N_c |R'_{\chi_Q}(0)|^2}{\pi m_Q^4} \times \left[\text{Im } f_1(^3P_J) + \frac{2T_F}{9N_c} \text{Im } f_8(^3S_1) \mathcal{E}_3^{\overline{\text{MS}}}(\Lambda) \right].$$

Λ : NRQCD factorization scale

Chromoelectric correlator and moments

$$G_{EE}^{\text{latt}}(\tau) = -\left\langle 0 \left| gE_{i,a}^{\text{latt}}(\tau) \Phi_{ab}^{\text{latt}}(\tau, 0) gE_{i,b}^{\text{latt}}(0) \right| 0 \right\rangle$$

$$\mathcal{E}_3^{\overline{\text{MS}}}(\Lambda) = \left[\frac{T_F}{N_c} \int_0^\infty d\tau \tau^3 G_{EE}^{\overline{\text{MS}}}(\tau) \right]_{\overline{\text{MS}}, \Lambda}$$

$\Phi_{ab}^{\text{latt}}(\tau, 0)$: temporal adjoint Wilson line

Why the lattice calculation of $\mathcal{E}_3$ is difficult

1. Signal-to-noise ratio
2. Linear divergence
3. Conversion to $\overline{\text{MS}}$

Determine $\mathcal{E}_3$ on the lattice using **gradient flow**.

Gradient flow: a gauge invariant UV cutoff

Flow equations

$$\partial_t B_\mu = \mathcal{D}_\nu G_{\nu\mu} + \kappa \mathcal{D}_\mu \partial_\nu B_\nu$$

$$\partial_t \chi = [D_\mu D_\mu - \kappa(\partial_\mu B_\mu)] \chi$$

$$\partial_t \bar{\chi} = \bar{\chi} [\overleftarrow{D}_\mu \overleftarrow{D}_\mu + \kappa(\partial_\mu B_\mu)]$$

κ : gauge parameter, drops out in gauge-invariant observables.

Boundary conditions

$$B_\mu(0, x) = g_s A_{\mu,0}(x)$$

$$\chi(0, x) = \psi_0(x)$$

$$\bar{\chi}(0, x) = \bar{\psi}_0(x)$$

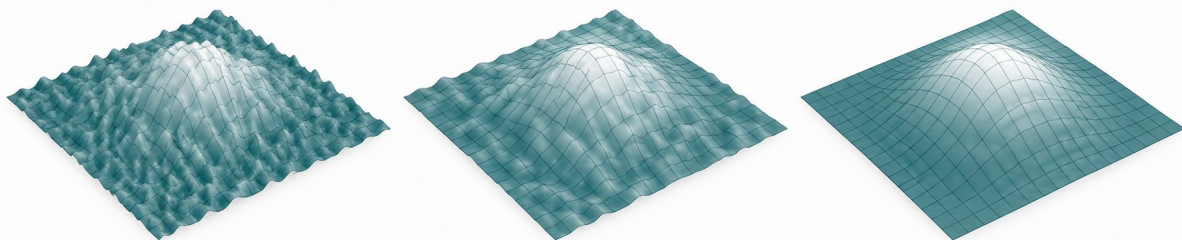
Flowed composite operators

Gauge composites

Finite at positive flow time after ordinary QCD parameter renormalizations

Flowed quarks

Require field normalization



**UV smoothing
can reduce noise**

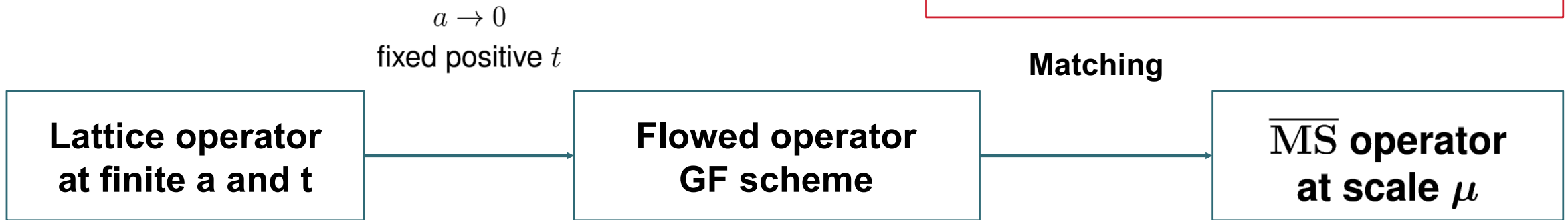
$$r_F = \sqrt{8t}$$

A natural bridge between lattice and continuum schemes

Heat kernels suppress UV: noise reduction

$$s, \beta, j \xrightarrow{p} t, \alpha, i = \delta_{ij} \frac{(-i\not{p} + m)_{\alpha\beta}}{p^2 + m^2} e^{-\underbrace{(t+s)p^2}_{\text{UV Suppr.}}}$$

Big advantage in matching:
Dimensional regularization
can be used in perturbative GF



$$O_i^{\text{GF}}(t) = \sum_j \zeta_{ij}(t, \mu) O_j^{\overline{\text{MS}}}(\mu) + \text{power corrections}$$

Matching coefficients are calculable in perturbation theory

G_{EE} from P-wave decay belongs to a widely used class of operators

Off-lightcone Wilson-line operators

$$\mathcal{O}_\Gamma(zv) = \bar{\psi}(zv) \Gamma W_F(zv, 0) \psi(0)$$

$$\mathcal{O}_g^{\mu\nu\alpha\beta}(zv) = g^2 F^{\mu\nu,a}(zv) W_A^{ab}(zv, 0) F^{\alpha\beta,b}(0)$$

Chromoelectric correlator

$$z = t, \quad v = (1, 0, 0, 0),$$

$$E^i = F^{i0}, \quad E^j = F^{j0},$$

$$G_{EE}(t) = - \sum_i \langle 0 | \mathcal{O}_g^{i0i0}(tv) | 0 \rangle$$

Euclidean convention

$$W_R(zv, 0) = \text{P exp} \left[ig \int_0^z ds v \cdot A^a(sv) T_R^a \right]$$

F: fundamental representation A: adjoint representation

Off-lightcone: $v^2 \neq 0$

**Off-lightcone
Wilson-line operators**

Physics applications

Quark bilocals

Space-like separation

Quark quasi-PDFs/DAs

Gluon bilocals

Space-like / time-like separation

Gluon quasi-PDFs/DAs

Quarkonium decay and production correlators

Quarkonium transport coefficients in the QGP

**Field-strength insertions
in Wilson loops**

Spin-dependent potentials

Auxiliary fields reduce nonlocal matching to local-current matching

One-loop matching provided; two-loop matching coming soon (X.-P. Wang, in preparation)

Matching in P-wave quarkonium decay

Continuum correlator: $a \rightarrow 0$ at fixed positive flow time

Lattice notation: τ_F = flow time; t = Euclidean time separation

$$G_{EE}^{\overline{\text{MS}}}(t) = \lim_{\tau_F \rightarrow 0} e^{dt/\sqrt{8\tau_F}} e^{m_0^{\overline{\text{MS}}}t} Z_{EE}^{\overline{\text{MS}}} G_{EE}(\tau_F, t)$$

d

**Power-divergence
subtraction**

From adjoint
Polyakov loops

$m_0^{\overline{\text{MS}}}$

**Finite matching
constant**

Short-distance matching
to the NLO correlator in $\overline{\text{MS}}$

$Z_{EE}^{\overline{\text{MS}}}$

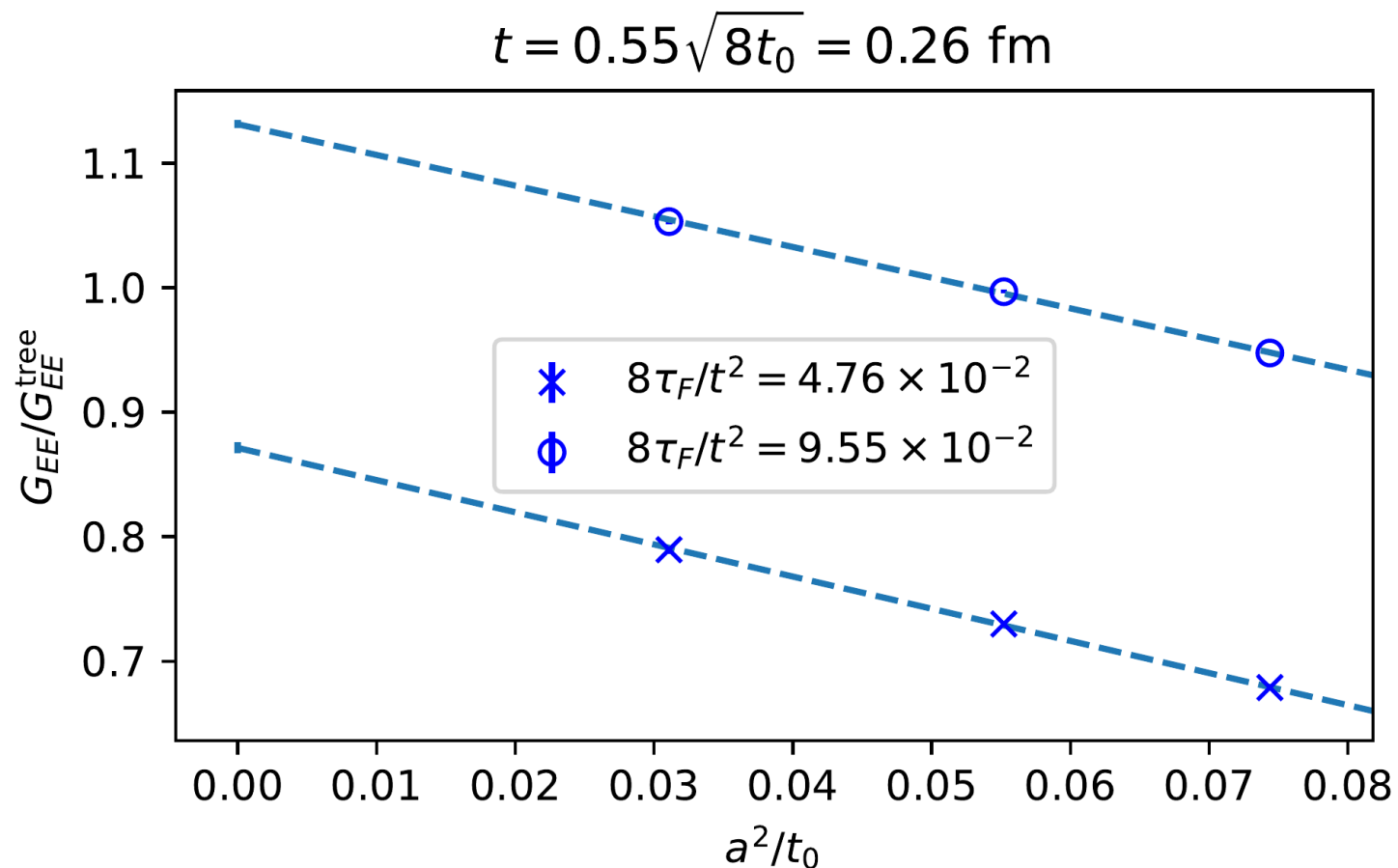
**Matching
coefficient**

Perturbative calculation
through one loop

$$Z_{EE}^{\overline{\text{MS}}} = 1 + \mathcal{O}(\alpha_s^2)$$

No decay-width data enter the determination of these factors

The continuum limit at fixed flow time



$$a \rightarrow 0$$

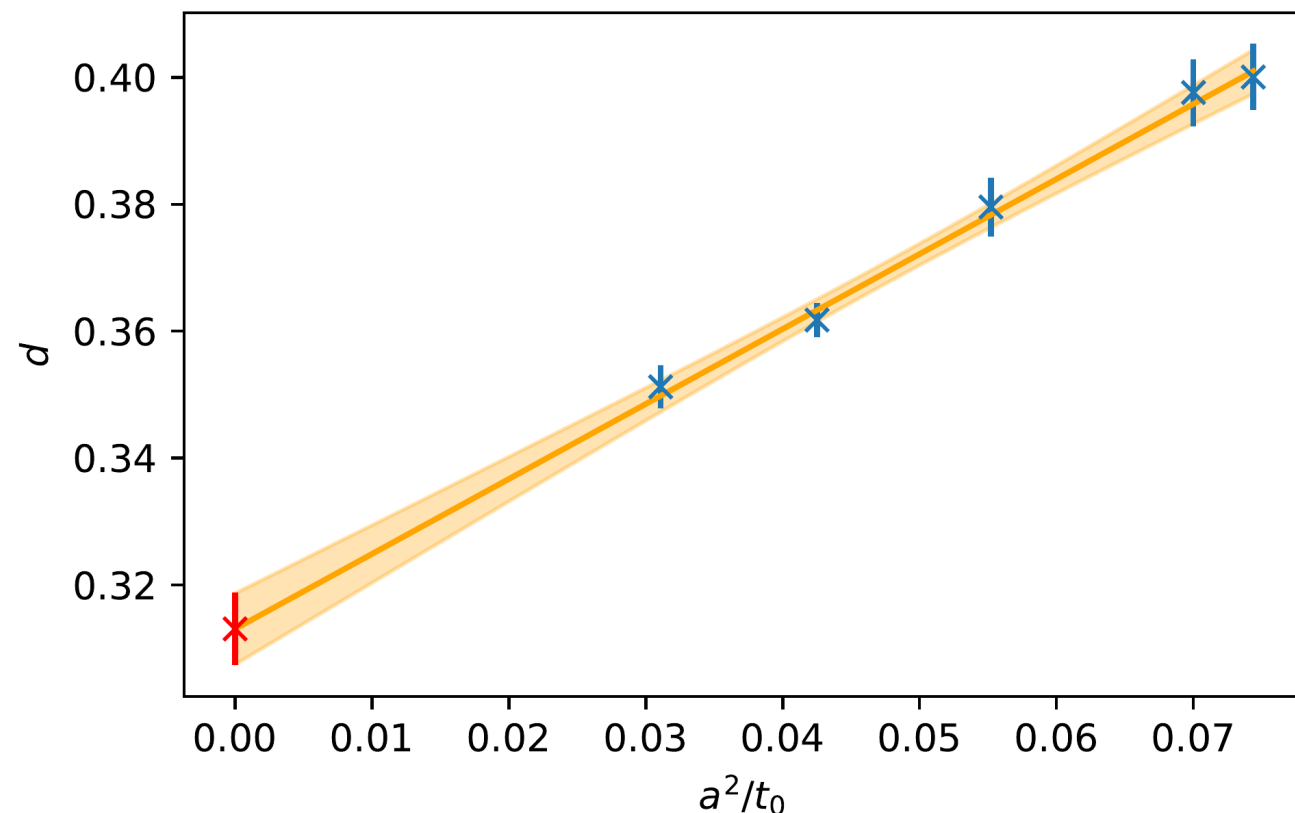
**Fixed positive
flow time**

Representative separation

$$t = 0.26 \text{ fm}$$

Taking the continuum limit before removing the flow controls discretization

Power-divergence subtraction



$$\exp \left[\frac{dt}{\sqrt{8\tau_F}} \right]$$

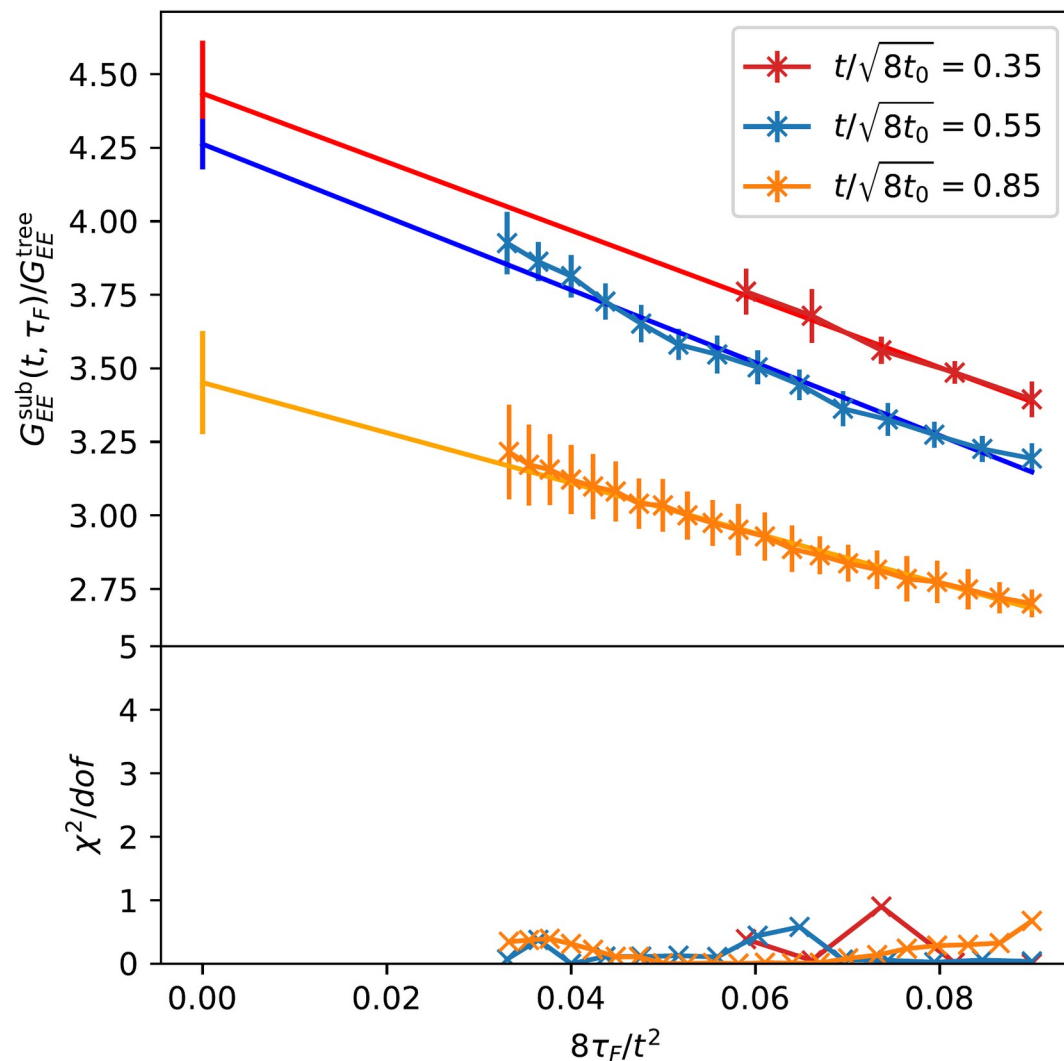
Multiplicative subtraction

$$d = 0.315(6)$$

Adjoint Polyakov loops
share the same Wilson-line
power divergence

Flow regulates the divergence. Its removal is an explicit analysis step.

The small-flow-time limit

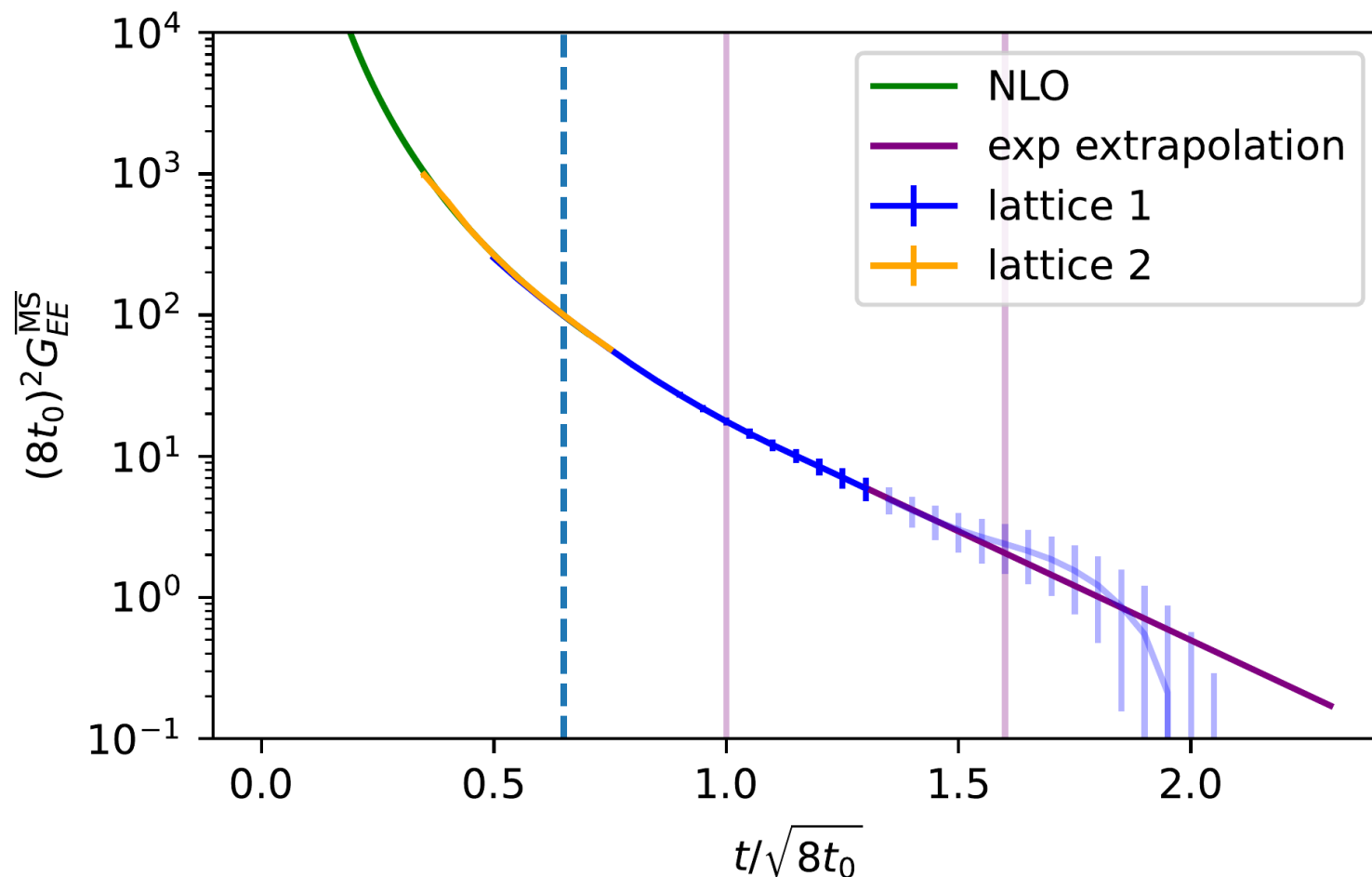


Short-distance data set

$$0.18 < \sqrt{8\tau_F}/t < 0.30$$

A finer lattice reaches
smaller Euclidean separations

Enables the short-distance
perturbative matching



A common $\overline{MS}$ definition

Short-distance data fix the finite matching constant

$$m_0^{\overline{MS}}$$

No decay-width data enter this matching

Extracting the universal moment

$$\mathcal{E}_3^{\overline{\text{MS}}}(\Lambda) = \mathcal{E}_3^{\text{pt}, \overline{\text{MS}}}(\Lambda, t_*) + \frac{T_F}{N_c} \int_{t_*}^{\infty} dt t^3 G_{EE}^{\overline{\text{MS}}}(t)$$

Decay widths require this renormalized moment



The t^3 weight leaves a UV logarithm: the moment also needs $\overline{\text{MS}}$ renormalization

The universal moment $\mathcal{E}_3$

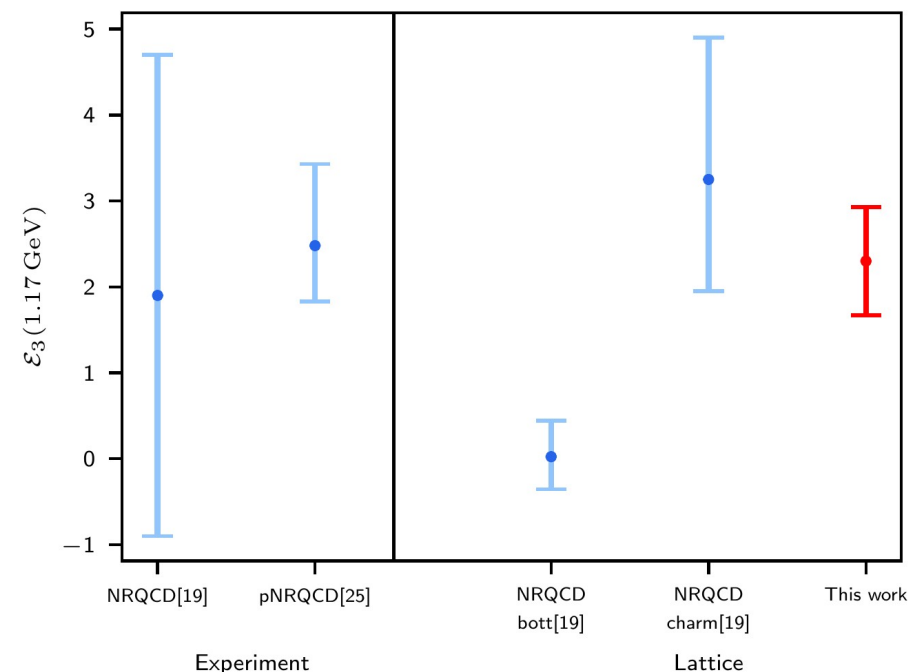
$$\mathcal{E}_3^{\overline{\text{MS}}}(1.17 \text{ GeV}) = 2.3$$

± 0.60 statistical

± 0.15 systematic

± 0.11 higher-order estimate

**Direct gluonic-correlator
determination (quenched)**



Dimensionless moment; the quoted errors do not include quenching

P-wave decay widths

Inclusive light-hadron partial widths [MeV]

Uncertainty components [MeV]

State	Exp.	Theory	trunc.	SDC	$\mathcal{E}_3$
$\chi_{c0}(1P)$	12.2 ± 0.5	13.19 ± 4.35	3.96	1.79	0.18
$\chi_{c1}(1P)$	0.552 ± 0.041	0.69 ± 0.28	0.21	0.03	0.18
$\chi_{c2}(1P)$	1.60 ± 0.06	1.66 ± 0.59	0.50	0.25	0.18
$\chi_{b0}(1P)$	—	1.11 ± 0.14	0.11	0.08	0.02
$\chi_{b1}(1P)$	—	0.13 ± 0.02	0.01	0.01	0.02
$\chi_{b2}(1P)$	—	0.25 ± 0.04	0.03	0.01	0.02
$\chi_{b0}(2P)$	—	1.42 ± 0.17	0.14	0.10	0.02
$\chi_{b1}(2P)$	—	0.17 ± 0.03	0.02	0.01	0.02
$\chi_{b2}(2P)$	—	0.32 ± 0.04	0.03	0.01	0.02
$\chi_{b0}(3P)$	—	1.64 ± 0.20	0.16	0.11	0.03
$\chi_{b1}(3P)$	—	0.19 ± 0.04	0.02	0.01	0.03
$\chi_{b2}(3P)$	—	0.36 ± 0.05	0.04	0.01	0.03

Trunc.: velocity expansion SDC: perturbative coefficients $\mathcal{E}_3$: universal moment

Quenched $\mathcal{E}_3$ + Cornell wavefunctions; quenching uncertainty is not included

Heavy-quark expansion in gradient flow

$$\mu_t = 1/\sqrt{8t}, \quad \mu_t, m_Q \gg \Lambda_{\text{QCD}}$$

Direct flowed QCD $\rightarrow$ HQET

$$O_i(t) = \sum_{\alpha} c_{i\alpha}(t, m_Q, \mu) O_{\alpha}^{\text{HQET}} + \dots$$

Finite-mass coefficient

Conventional QCD $\rightarrow$ HQET

$$O_j^{\overline{\text{MS}}} = \sum_{\alpha} C_{j\alpha}(\mu, m_Q) O_{\alpha}^{\text{HQET}} + \dots$$

Same HQET operator basis

Eliminate the same renormalized HQET operator basis

Flowed QCD $\rightarrow$ QCD conversion with heavy-quark mass dependence

$$O_i(t) = \sum_j \xi_{ij}(t, m_Q, \mu) O_j^{\overline{\text{MS}}} + \dots$$

$$\xi(t, m_Q, \mu) = \mathbf{c}(t, m_Q, \mu) \mathbf{C}^{-1}(\mu, m_Q)$$

Leading power; higher-power terms omitted. Matched bases with invertible C.

Example: heavy-light currents

Ringed flowed current

$$\dot{j}_\Gamma(t) = \dot{\bar{\chi}}_q(t) \Gamma \dot{\chi}_Q(t)$$

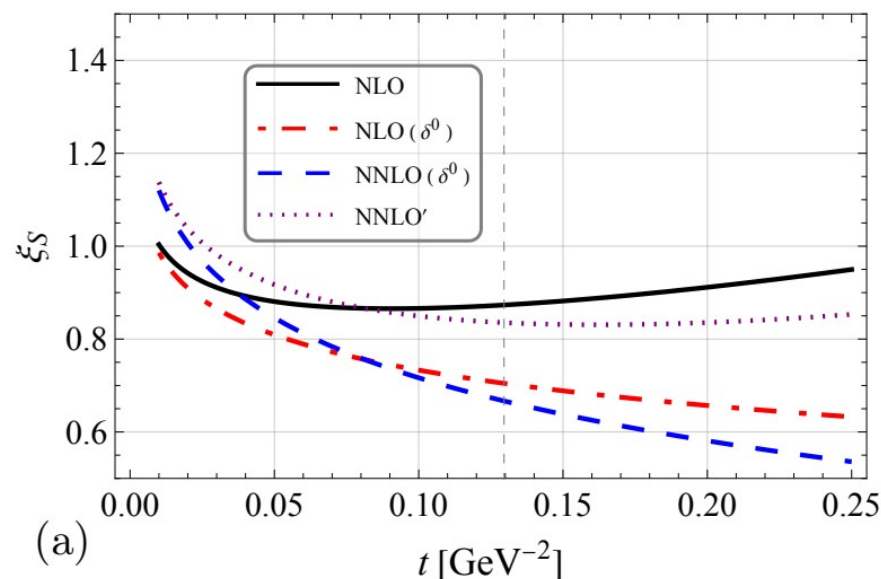
Projected channels at leading power:

$$\Gamma = S, P, V_\parallel, V_\perp, A_\parallel, A_\perp, T$$

$$\begin{aligned} \dot{j}_\Gamma(t) &= c_\Gamma J_\Gamma^{\text{HQET}} + \dots \\ J_\Gamma^{\text{QCD}} &= C_\Gamma J_\Gamma^{\text{HQET}} + \dots \end{aligned}$$

$$\dot{j}_\Gamma(t) = \xi_\Gamma J_\Gamma^{\text{QCD}} + \dots, \quad \xi_\Gamma = c_\Gamma / C_\Gamma$$

Scalar conversion at $\mu = m_c = 1.472 \text{ GeV}$



At the marked flow time, finite-mass effects exceed the massless NLO–NNLO change

NNLO': massless NNLO + NLO mass correction

Applications: finite-flow-time matrix elements and zero-flow extrapolations

Prospective: renormalon-free OPEs for heavy-flavor physics

Summary

pNRQCD relations

Universal quarkonium
production correlators

2022–2023

GF $\longrightarrow$ $\overline{\text{MS}}$ matching

Nonlocal off-lightcone
Wilson-line operators

2024

P-wave application HQE in gradient flow

Quenched $\mathcal{E}_3$ and decay widths
Heavy-quark mass effect
in GF matching

2026

Outlook

- Nonrelativistic QCD expansion in gradient flow
- Lattice determination of gluonic correlators in inclusive quarkonium production

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