

Neural-network extraction of collinear transversity distributions and tensor charges with distribution-level lattice-QCD input

The 6th Chinese Lattice QCD Workshop



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The collinear transversity PDF

The transversity distribution has collinear definitions

Observables

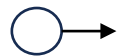


Asymmetry



PDF $\otimes$ FF

PDFs		Quark polarization		
		Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon polarization	U	f_1 Unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_{1L} Helicity	$h_{1L}^\perp$ Longi-transversity
	T	$f_{1T}^\perp$ Sivers	g_{1T} Trans-helicity	h_1 Transversity $h_{1T}^\perp$ Pretzelosity



Nucleon spin



Parton spin

The extraction of transversity PDF via Di-hadron mechanism

1° Dihadron Production in SIDIS

$$l(k) + N(P) \rightarrow l(k') + h_1(P_1) + h_2(P_2) + X$$

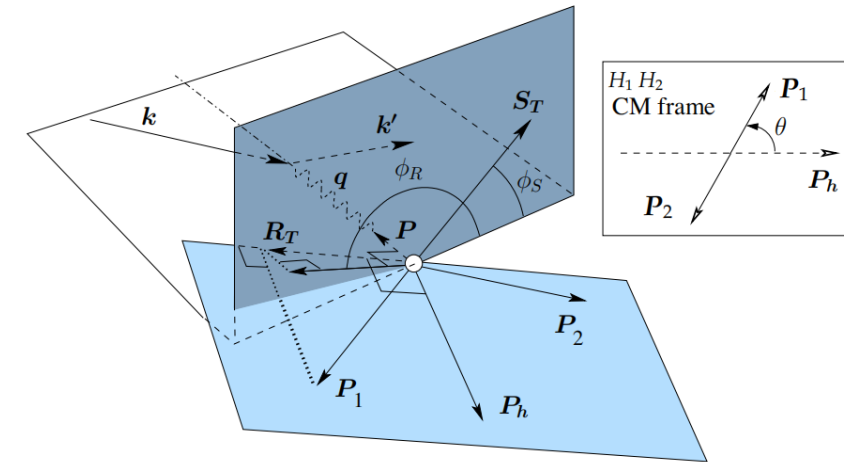
$$\vec{P}_h = \vec{P}_1 + \vec{P}_2 \quad \vec{R} = (\vec{P}_1 - \vec{P}_2)/2 \quad \vec{R}_T = R \sin\theta$$

$$\frac{d\sigma}{dx dy d\psi dz d\phi_R dM_h^2 d\cos\theta} = \frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU} + |\mathbf{S}_T| \varepsilon \sin(\phi_R + \phi_S) F_{UT}^{\sin(\phi_R + \phi_S)} \right\}$$

$$A_{UT}^{SIDIS} = c(y) \frac{\sum_q e_q^2 h_1^q(x) H_1^{\star,q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)} = c(y) \frac{H_1^{\star,q}(z, M_h) \frac{1}{9} \left[4 \left(h_1^u(x) - h_1^{\bar{u}}(x) \right) - \left(h_1^d(x) - h_1^{\bar{d}}(x) \right) \right]}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)}$$

D_1 : DiFF for unpolarized quarks

$H_1^{\star}$: DiFF for transversely polarized quarks



Alessandro Bacchetta et. al.
10.1007/JHEP03(2013)119

The extraction of transversity PDF via Di-hadron mechanism

2° Dihadron Production in Electron-Positron Annihilation

$$e^+ + e^- \rightarrow (\pi^+ \pi^-) + X$$

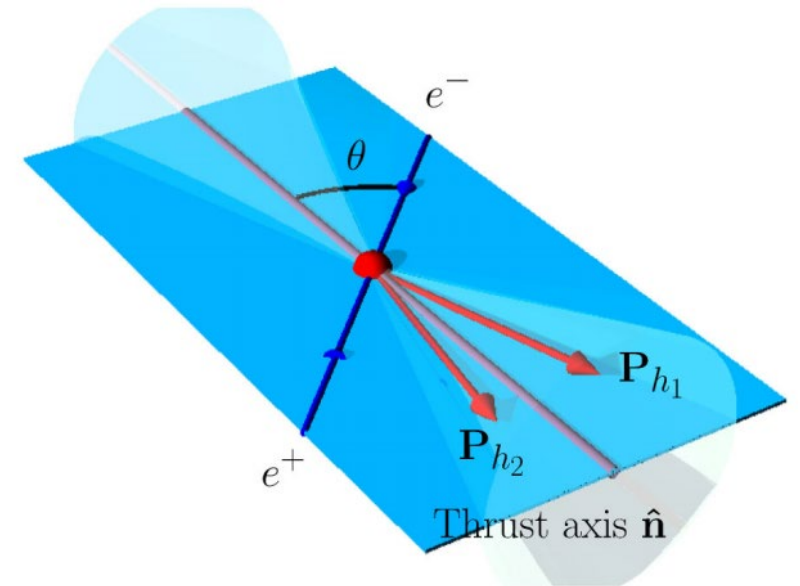
Cross section:

$$\frac{d\sigma}{dzdM_h} = 4\pi N_c \alpha_{em}^2 \sum_q \frac{e_q^2 D_1^q(z, M_h)}{3s}$$

$$e^+ + e^- \rightarrow (\pi^+ \pi^-) + (\pi^+ \pi^-) + X$$

Artru-Collins asymmetry:

$$A_{e^+e^-} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \sin \theta \sin \bar{\theta} \frac{|\mathbf{R}|}{M_h} \frac{|\bar{\mathbf{R}}|}{\bar{M}_h} \frac{\sum_q e_q^2 H_1^{\triangleleft q}(z, M_h; Q^2) H_1^{\triangleleft \bar{q}}(\bar{z}, \bar{M}_h; Q^2)}{\sum_q e_q^2 D_1^q(z, M_h; Q^2) D_1^{\bar{q}}(\bar{z}, \bar{M}_h; Q^2)}$$



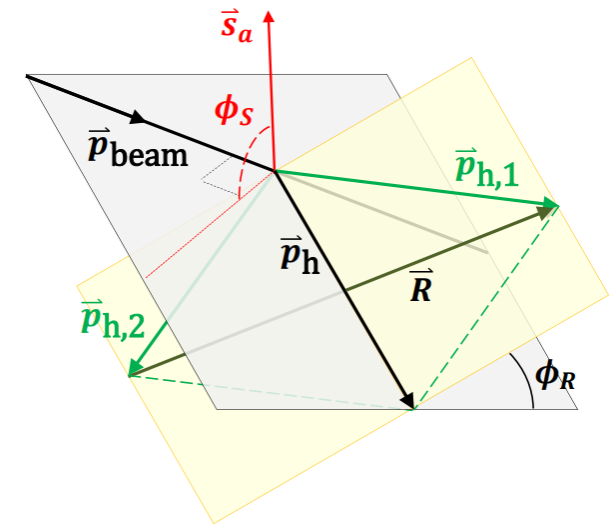
PHYSICAL REVIEW D 96, 032005 (2017)

The extraction of transversity PDF via Di-hadron mechanism

3° Dihadron Production in Proton-Proton Collisions

- $p + p^\uparrow \rightarrow (\pi^+\pi^-) + X$

$$A_{UT}^{pp} = \frac{\mathcal{H}(M_h, P_{hT}, \eta)}{\mathcal{D}(M_h, P_{hT}, \eta)}$$

**Phys.Rev.Lett. 115 (2015) 242501**

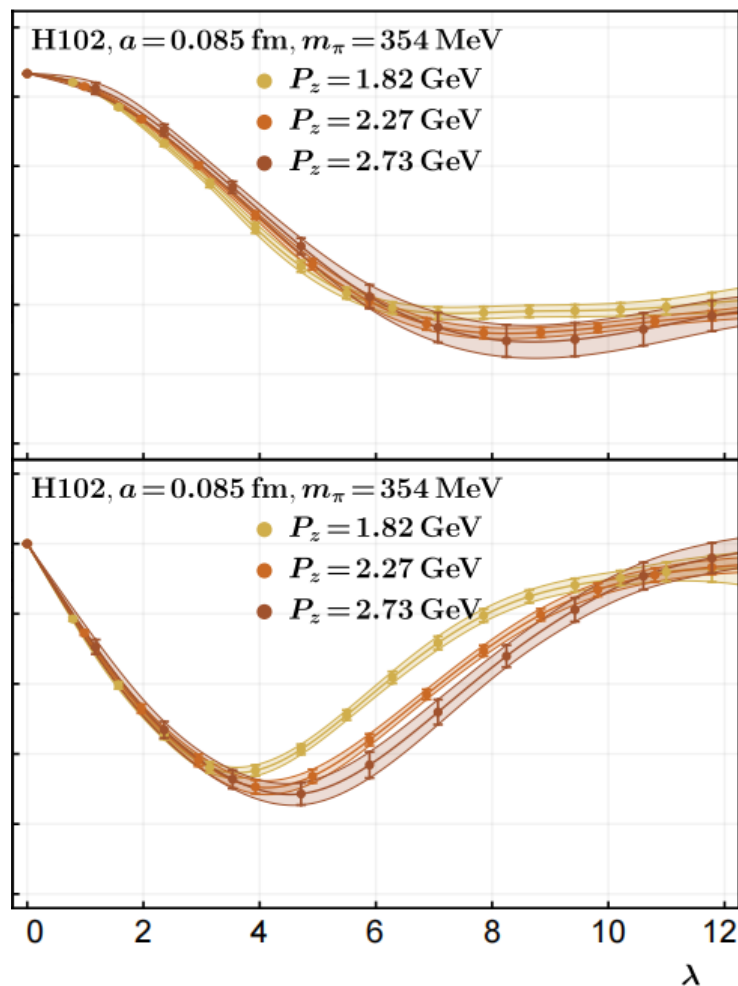
$$\mathcal{H}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{min}}^1 dx_a \int_{x_b^{min}}^1 \frac{dx_b}{z} h_1^a(x_a) f_1^b(x_b) \frac{d\Delta\hat{\sigma}_{a\uparrow b \rightarrow c\uparrow d}}{d\hat{t}} H_1^{\not{c},c}(z, M_h)$$

$$\mathcal{D}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{min}}^1 dx_a \int_{x_b^{min}}^1 \frac{dx_b}{z} f_1^a(x_a) f_1^b(x_b) \frac{d\Delta\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1^c(z, M_h)$$

Transversity distribution from Lattice QCD

Lattice → Phenomenon

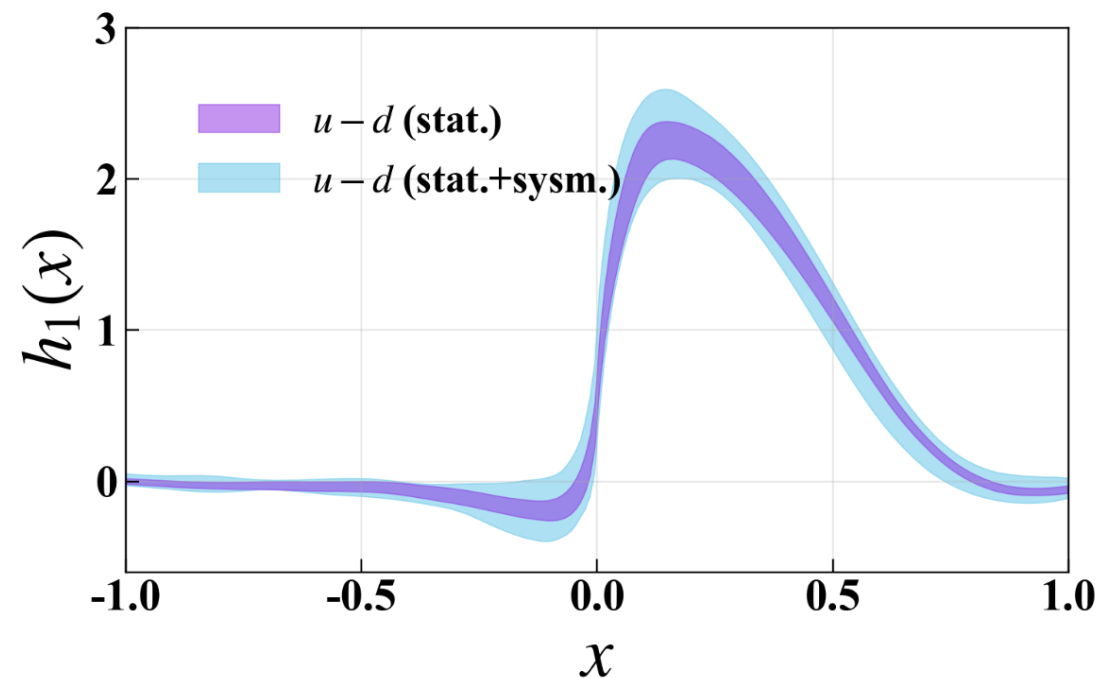
The renormalized matrix elements



1° Fourier transform

2° Matching

3° Extrapolation

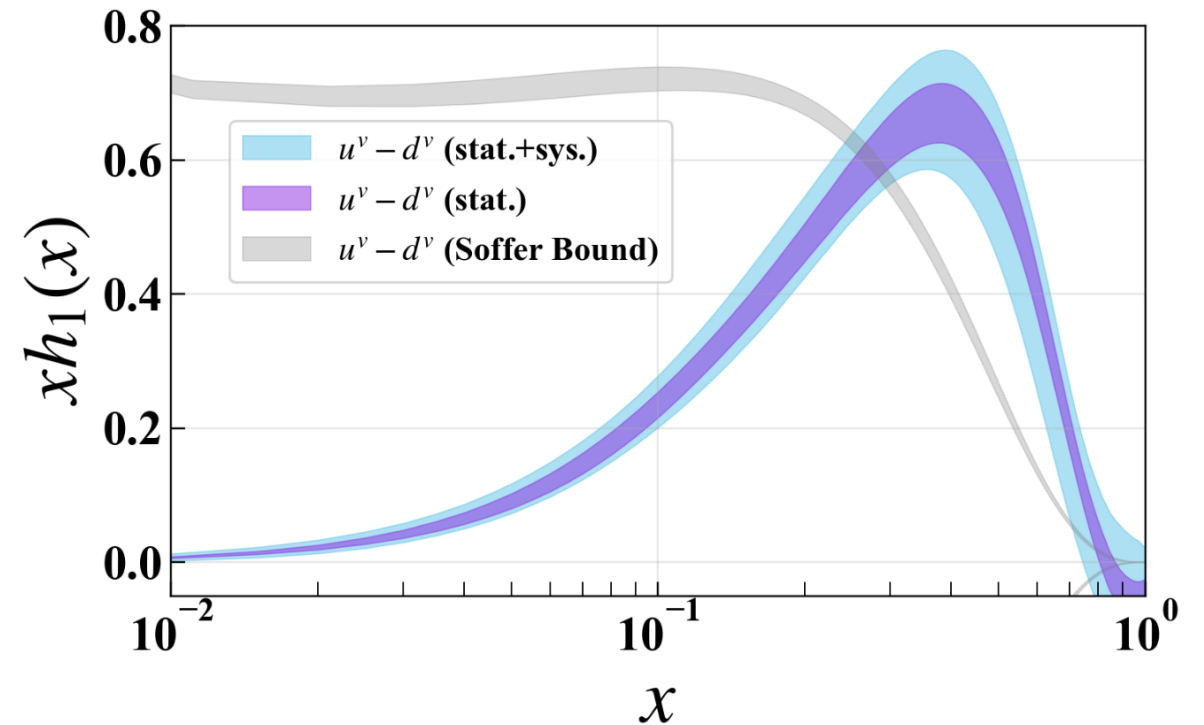
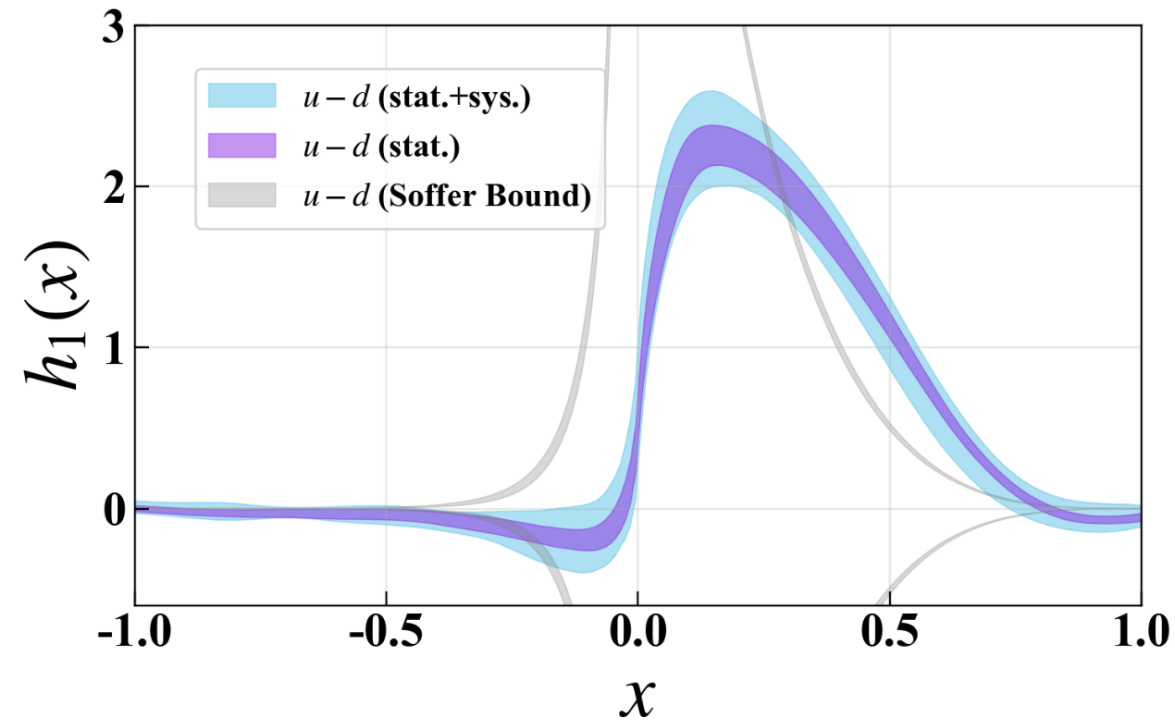


Transversity distribution from Lattice QCD

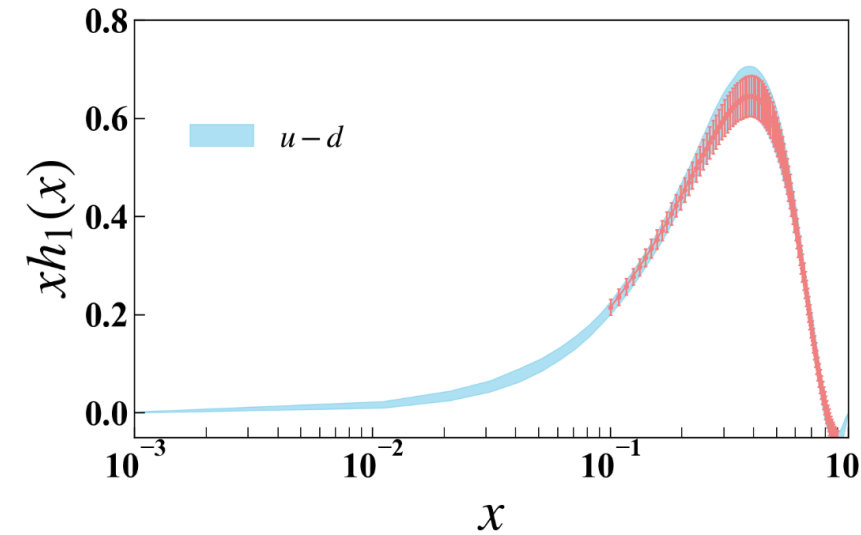
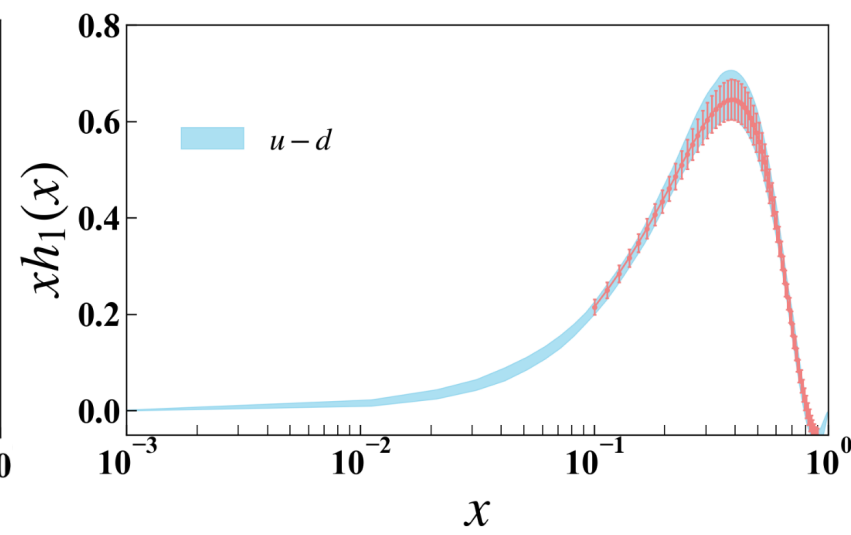
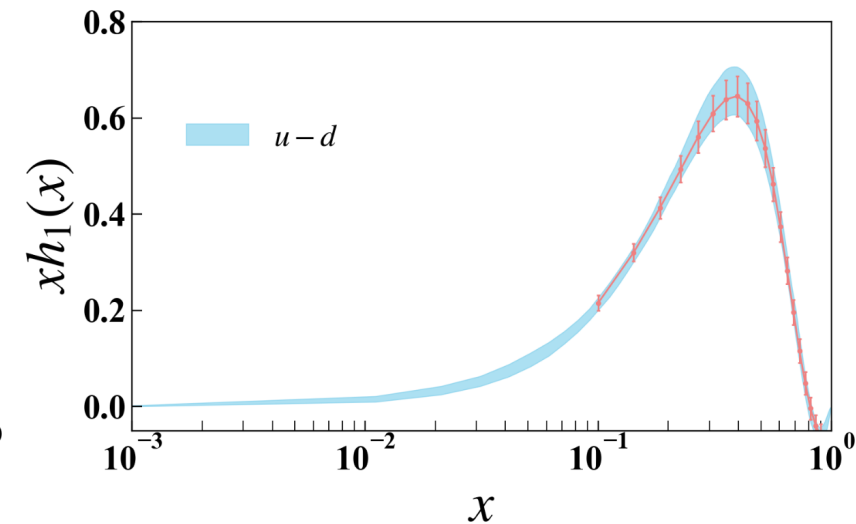
Lattice transversity distribution and Soffer bound

Soffer bound : $|h_1^i(x)| \leq \frac{1}{2} [f_1^i(x) + g_1^i(x)] \quad i = u, d, \bar{u}, \bar{d}$

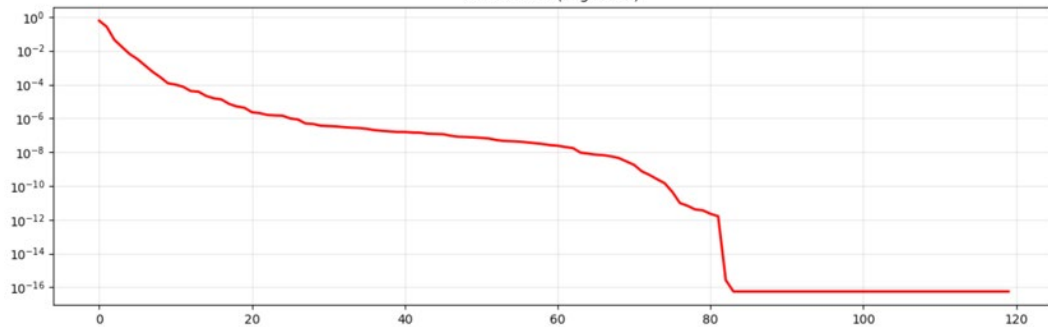
$$h_1^u(-x) = -h_1^{\bar{u}}(x) \quad h_1^d(-x) = -h_1^{\bar{d}}(x)$$



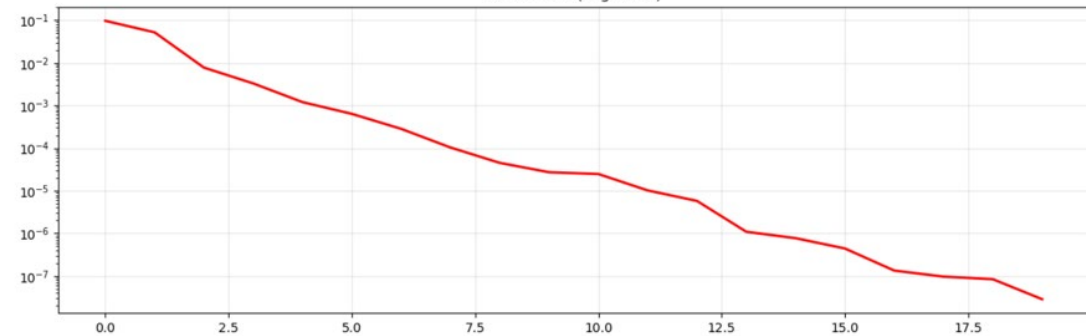
Transversity distribution from Lattice QCD

 $N_{data} = 120$  $N_{data} = 60$  $N_{data} = 20$  $N_{data} = 120$

SVD Values (Log Scale)

 $N_{data} = 20$

SVD Values (Log Scale)



Evolution equation for transversity PDFs & DiFFs

DGLAP evolution :

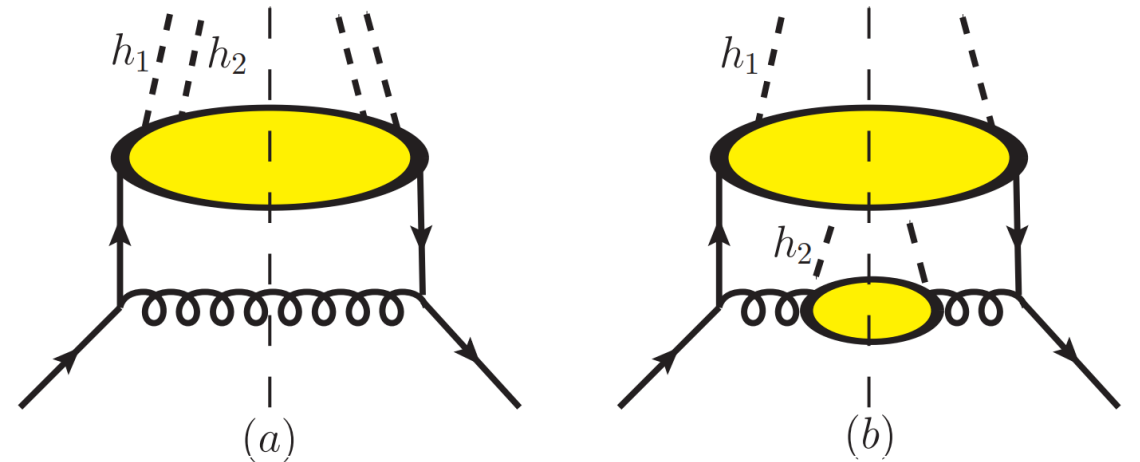
$$\frac{dh_1^i(x, \mu)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{ij}(y, \alpha_s) h_1^j\left(\frac{x}{y}, \mu\right)$$

$$h_1 : P_{qq'}^{h_1}(y) = C_F \left[\frac{2y}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right] \delta q q'$$

$$\frac{dD_i^{h_1 h_2}(z, \mu)}{d \ln \mu^2} = \sum_j \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} P_{ij}(y, \alpha_s) D_j^{h_1 h_2}\left(\frac{z}{y}, \mu\right)$$

$$D_1 : P_{qq'}^{D_1}(y) = C_F \left[\frac{(1+y^2)}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right] \delta q q'$$

$$H_1^\star : P_{qq'}^{H_1^\star}(y) = C_F \left[\frac{2y}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right] \delta q q'$$



Example diagrams of the (a) homogeneous and (b) inhomogeneous terms for the evolution of the DiFF

Phys.Rev.Lett. 132 (2024) 1, 011902

World datasets for the global fit

$$0.25 \leq z$$

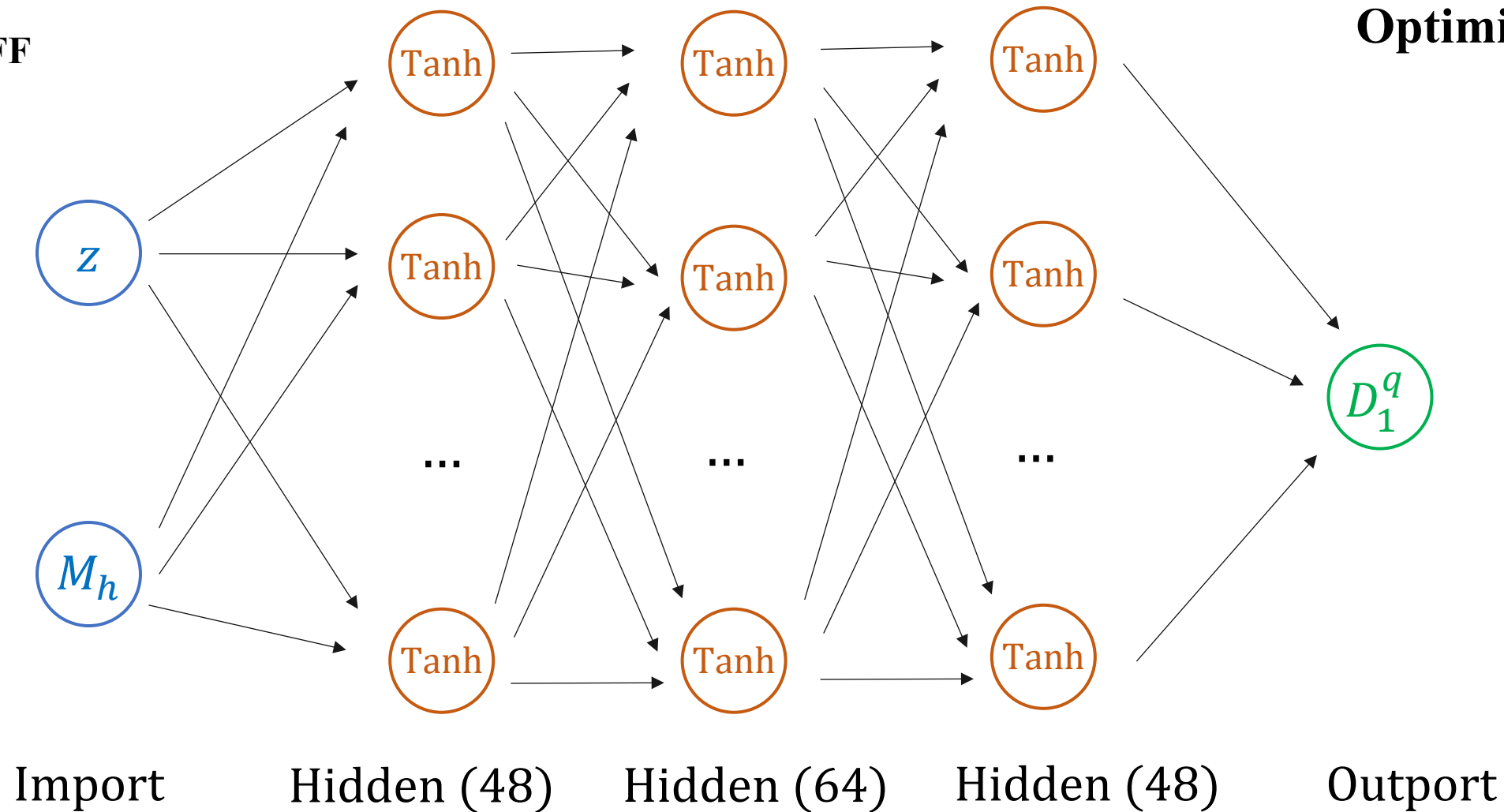
$$2m_\pi \leq M_h \leq 1.2 \text{ GeV}$$

Collaboration	Observable	Process	Functions	N_{data}
Belle PRD. 96 (2017) 3, 032005	$d\sigma/(dz dM_h)$	$e^+e^- \rightarrow (\pi^+\pi^-)X$	D_1	651
Belle PRL. 107 (2011) 072004	$A^{e^+e^-}$	$e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)X$	D_1, H_1^π	145
HERMES JHEP. 06 (2008) 017	$A_{UT}^{(SIDIS)}$	$ep^\uparrow \rightarrow e'(\pi^+\pi^-)X$	D_1, H_1^π, h_1	12
COMPASS PLB. 736 (2014) 124-131	$A_{UT}^{(SIDIS)}$	$\mu p^\uparrow \rightarrow \mu'(\pi^+\pi^-)X$	D_1, H_1^π, h_1	24
COMPASS PLB. 845 (2023) 138155	$A_{UT}^{(SIDIS)}$	$\mu D^\uparrow \rightarrow \mu'(\pi^+\pi^-)X$	D_1, H_1^π, h_1	24
STAR PRL. 115 (2015) 242501 PLB. 780 (2018) 332-339	$A_{UT}^{(pp)}$	$pp^\uparrow \rightarrow (\pi^+\pi^-)X$	D_1, H_1^π, h_1	130
LPC PRL. 131 (2023) 26, 261901	$h_1^{u^v-d^v}$		h_1	80

Neural Network

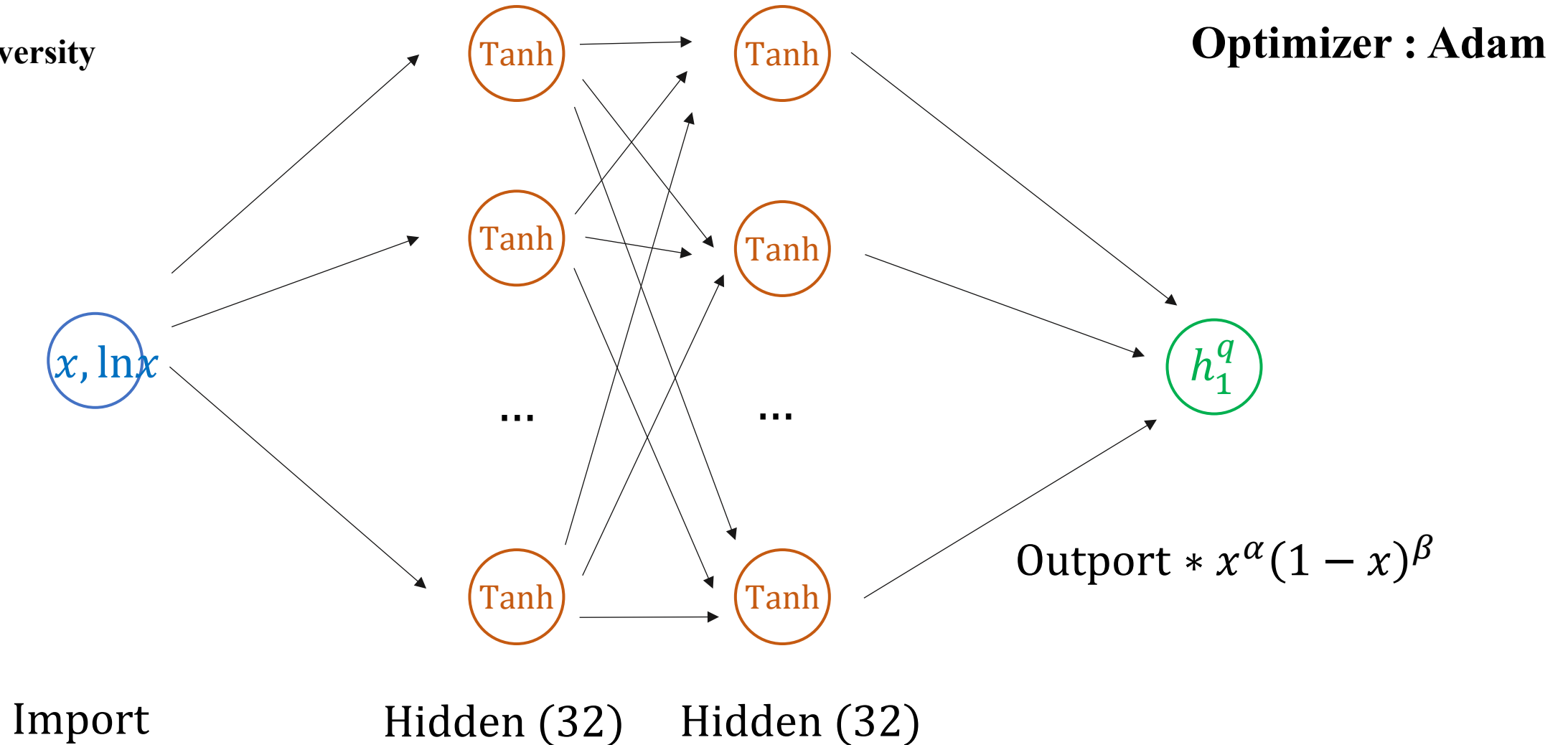
- DiFF

Optimizer : Adam

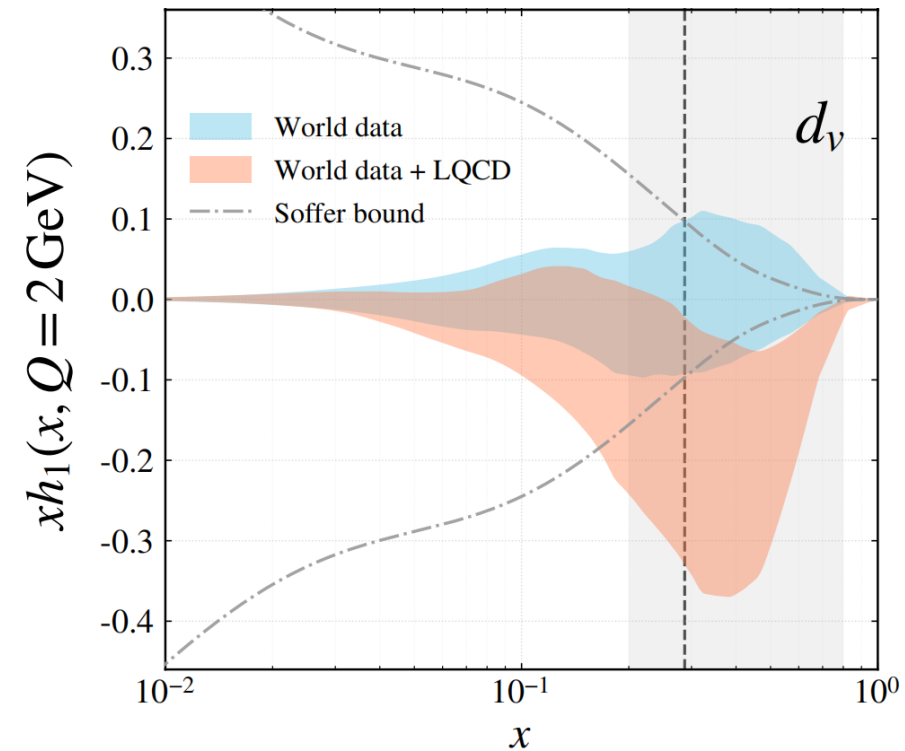
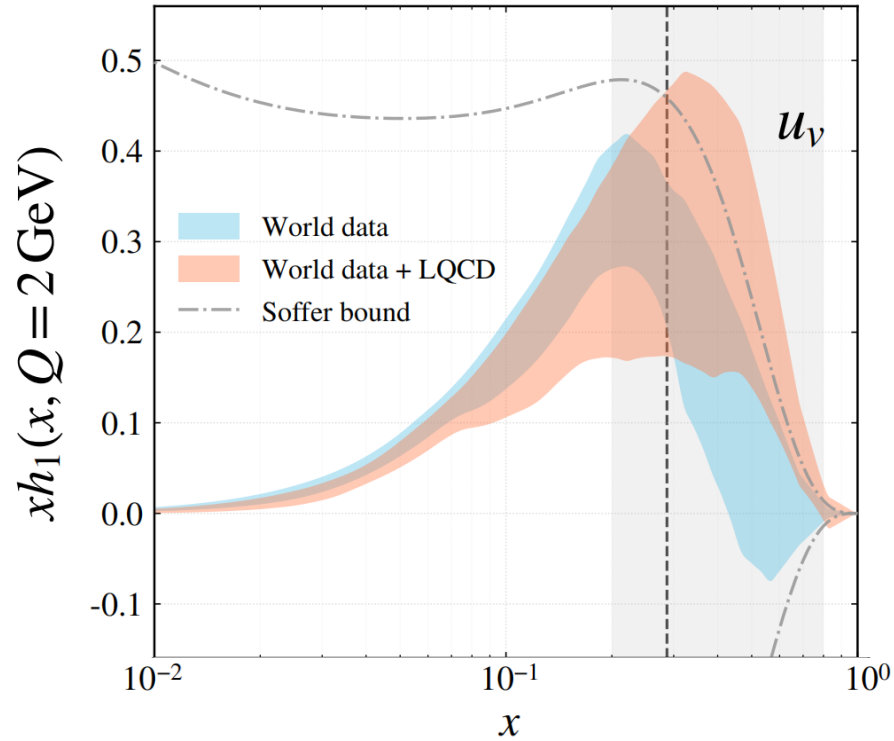


Neural Network

- Transversity



Neural network results for transversity

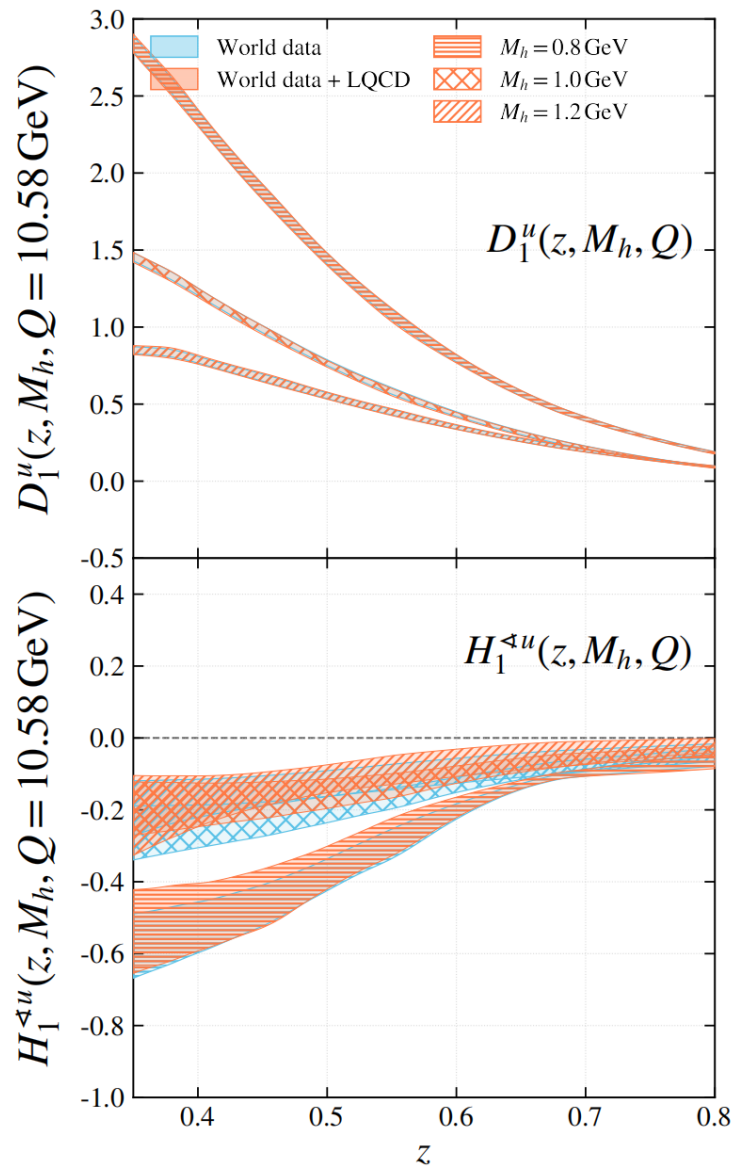


$$\alpha = 0.8, 1.2, 1.6$$

$$\beta = 4, 5, 6$$

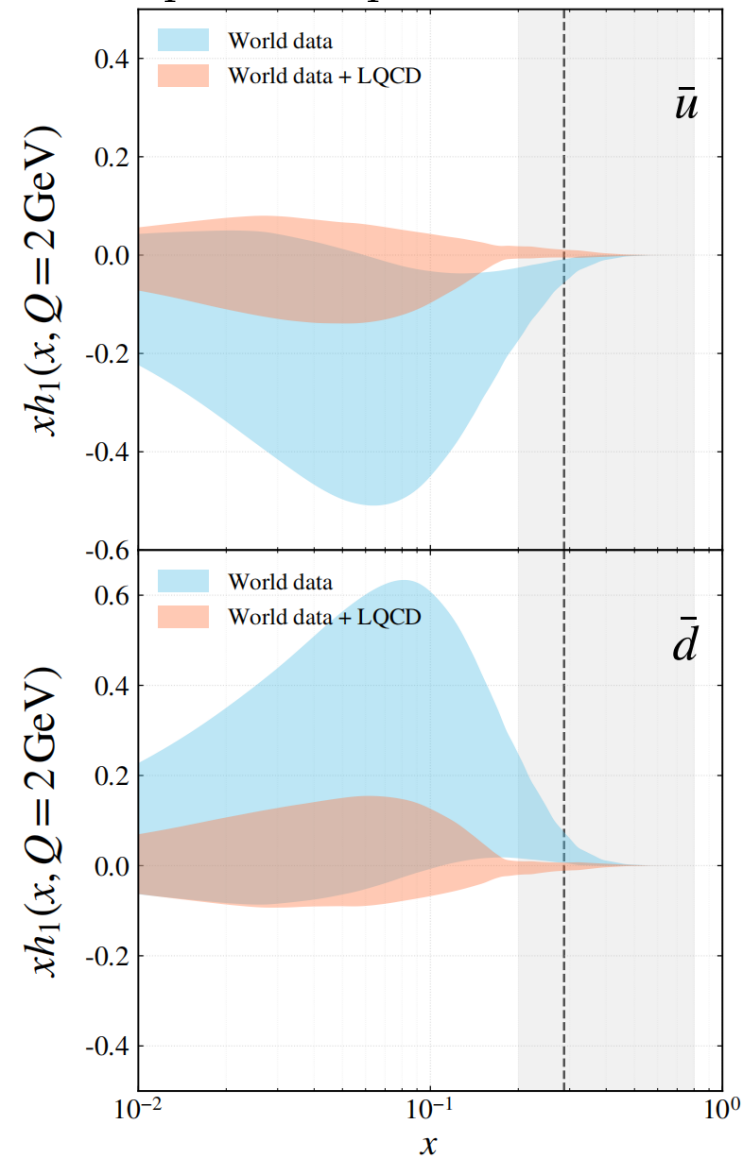
χ^2/N	Belle $d\sigma/(dz dM_h)$	Belle $A^{e^+e^-}$	HERMES $A_{UT}^{\text{SIDIS}}(p)$	COMPASS $A_{UT}^{\text{SIDIS}}(p)$	COMPASS $A_{UT}^{\text{SIDIS}}(D)$	STAR A_{UT}^{pp}	Lattice $h_1^{u^v-d^v}$	ALL
World data	0.143	0.444	0.939	0.780	0.810	1.188		0.327
World data + LQCD	0.131	0.400	0.894	1.012	0.787	1.209	0.909	0.316

Neural network results for DiFFs



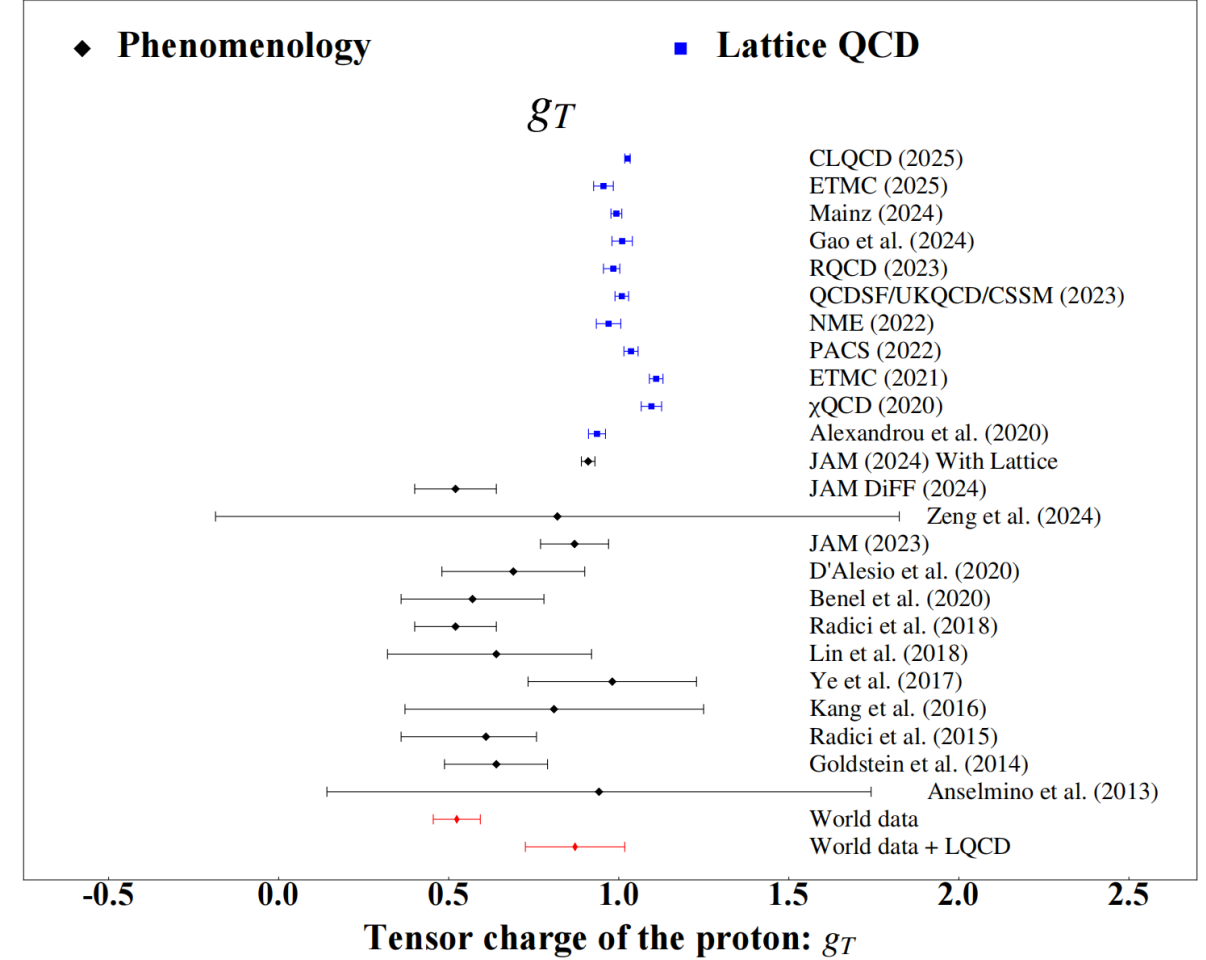
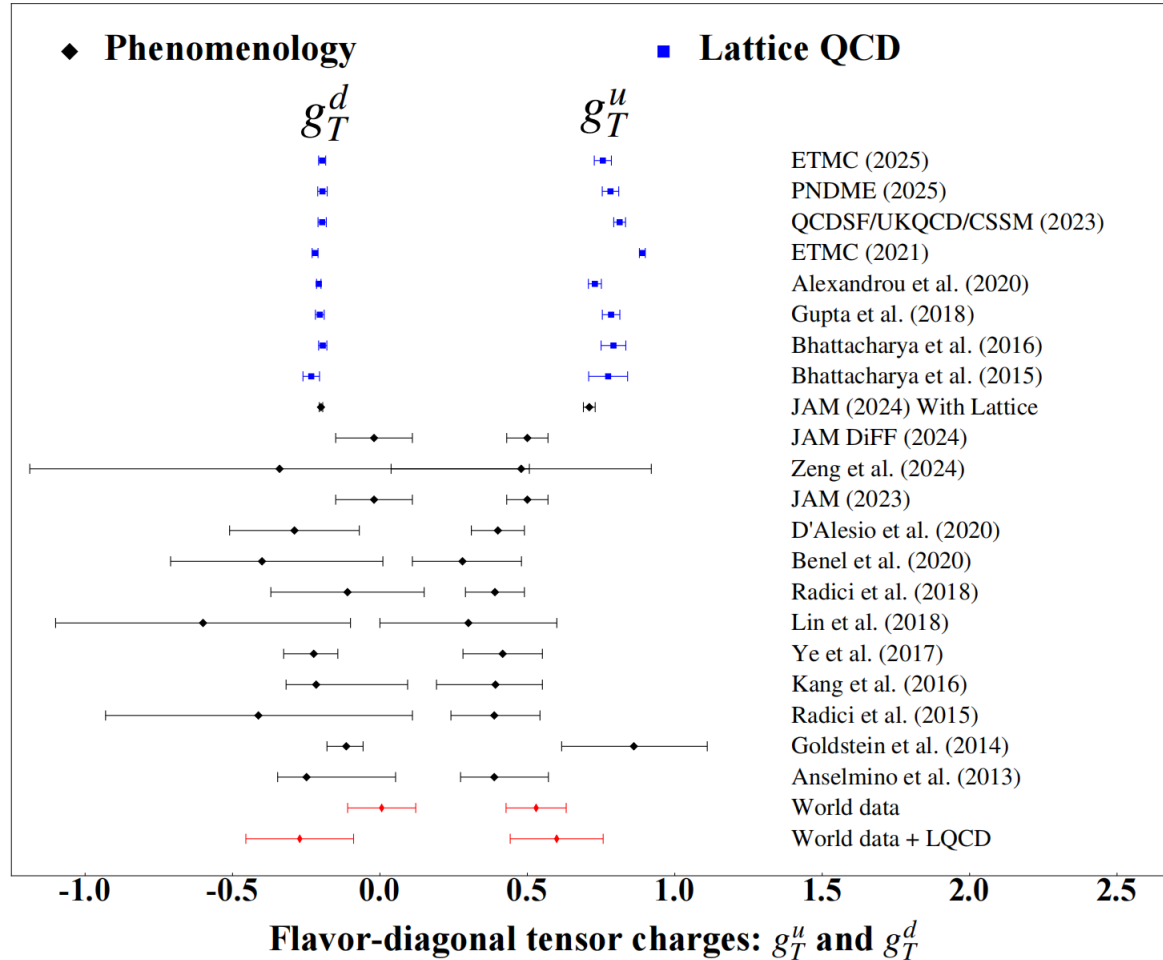
$$D_1^{\pi^+\pi^-/u} = D_1^{\pi^+\pi^-/d} = D_1^{\pi^+\pi^-/\bar{u}} = D_1^{\pi^+\pi^-/\bar{d}}$$

$$H_1^{\pi^+\pi^-/u} = -H_1^{\pi^+\pi^-/d} = -H_1^{\pi^+\pi^-/\bar{u}} = H_1^{\pi^+\pi^-/\bar{d}}$$



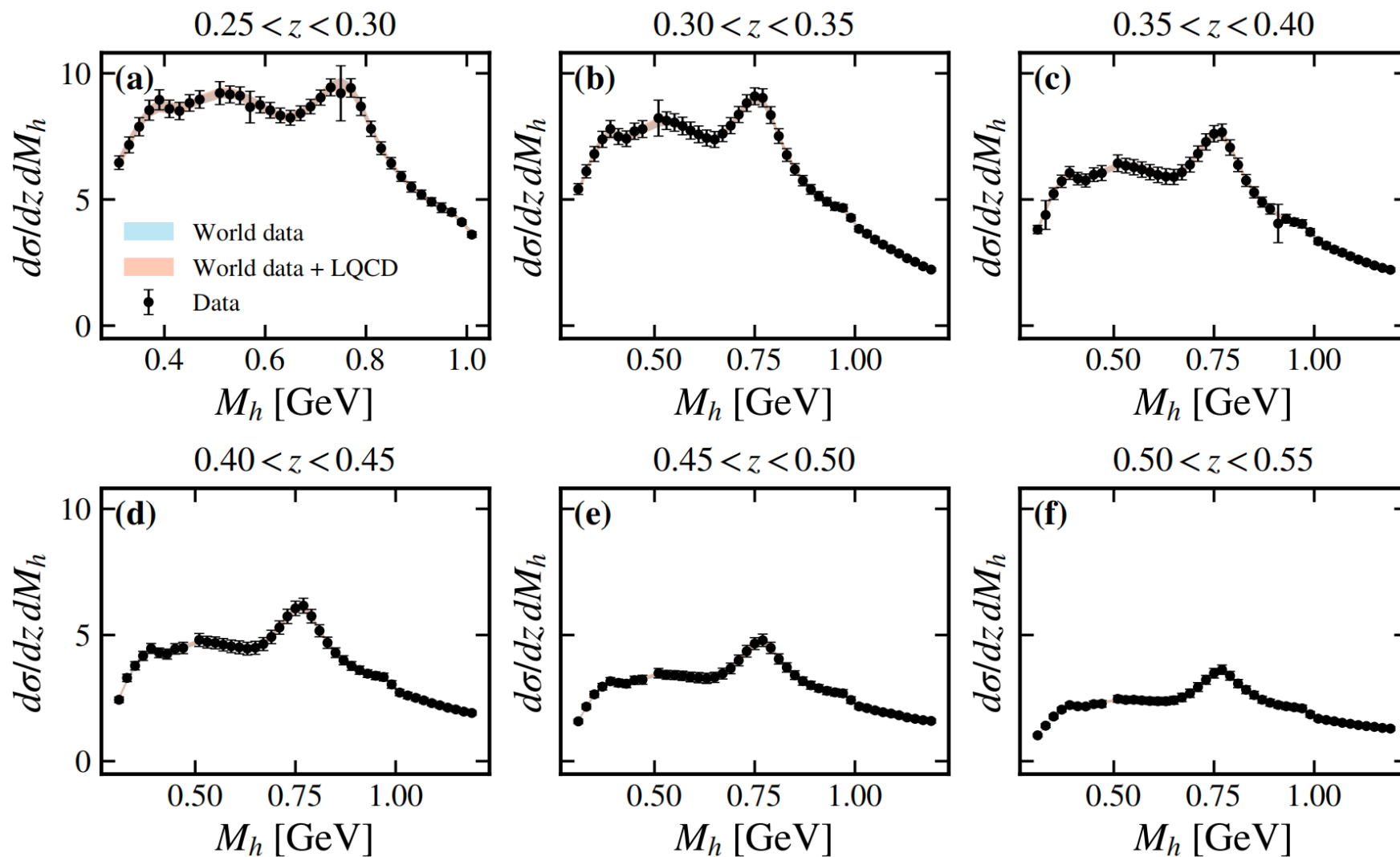
Neural network results for tensor charges

$$g_T^q(\mu) = \int_0^1 dx [h_1^q(x, \mu) - h_1^{\bar{q}}(x, \mu)], \quad g_T = g_T^u - g_T^d$$



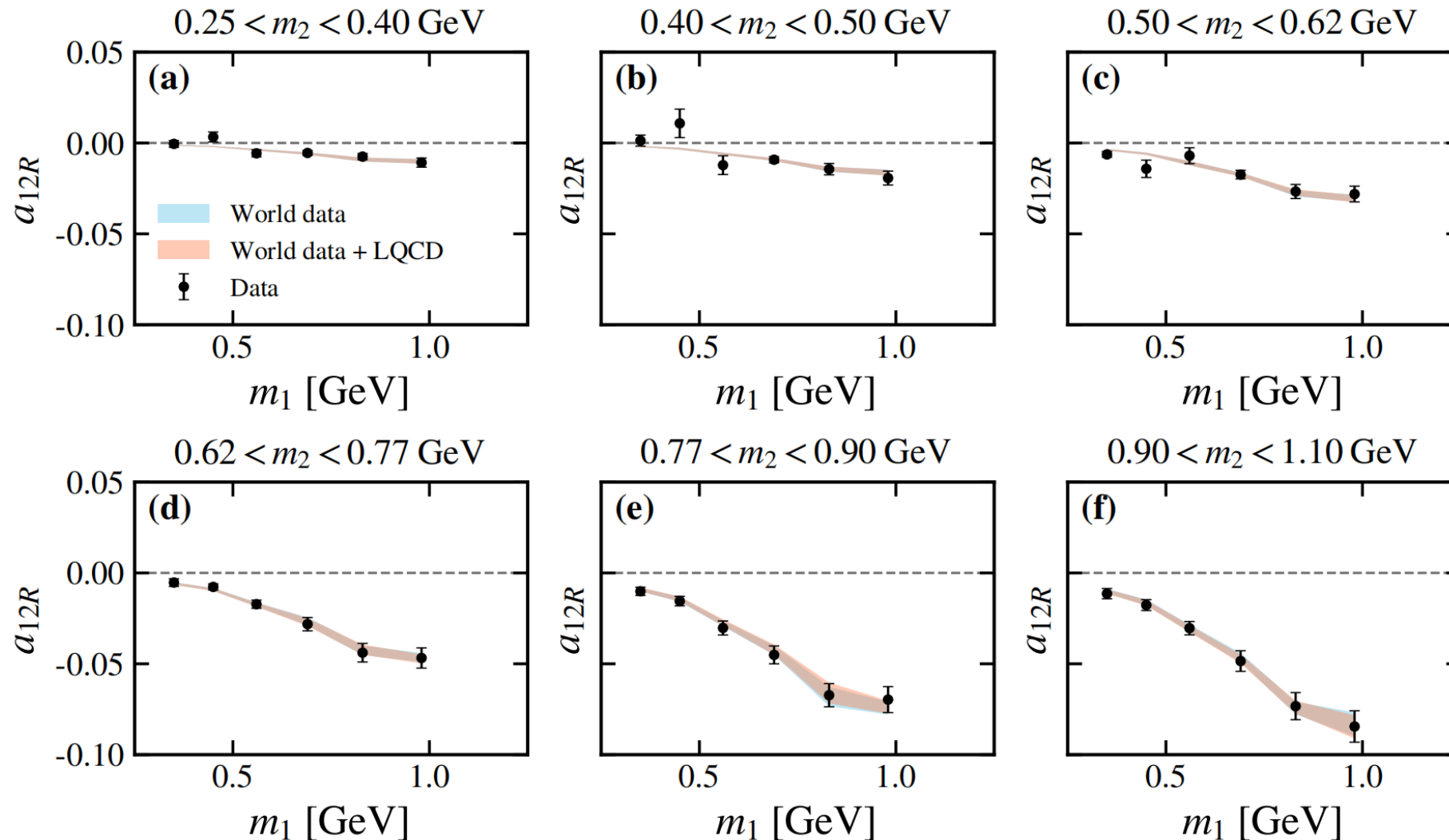
Compare with world datasets

Belle cross section



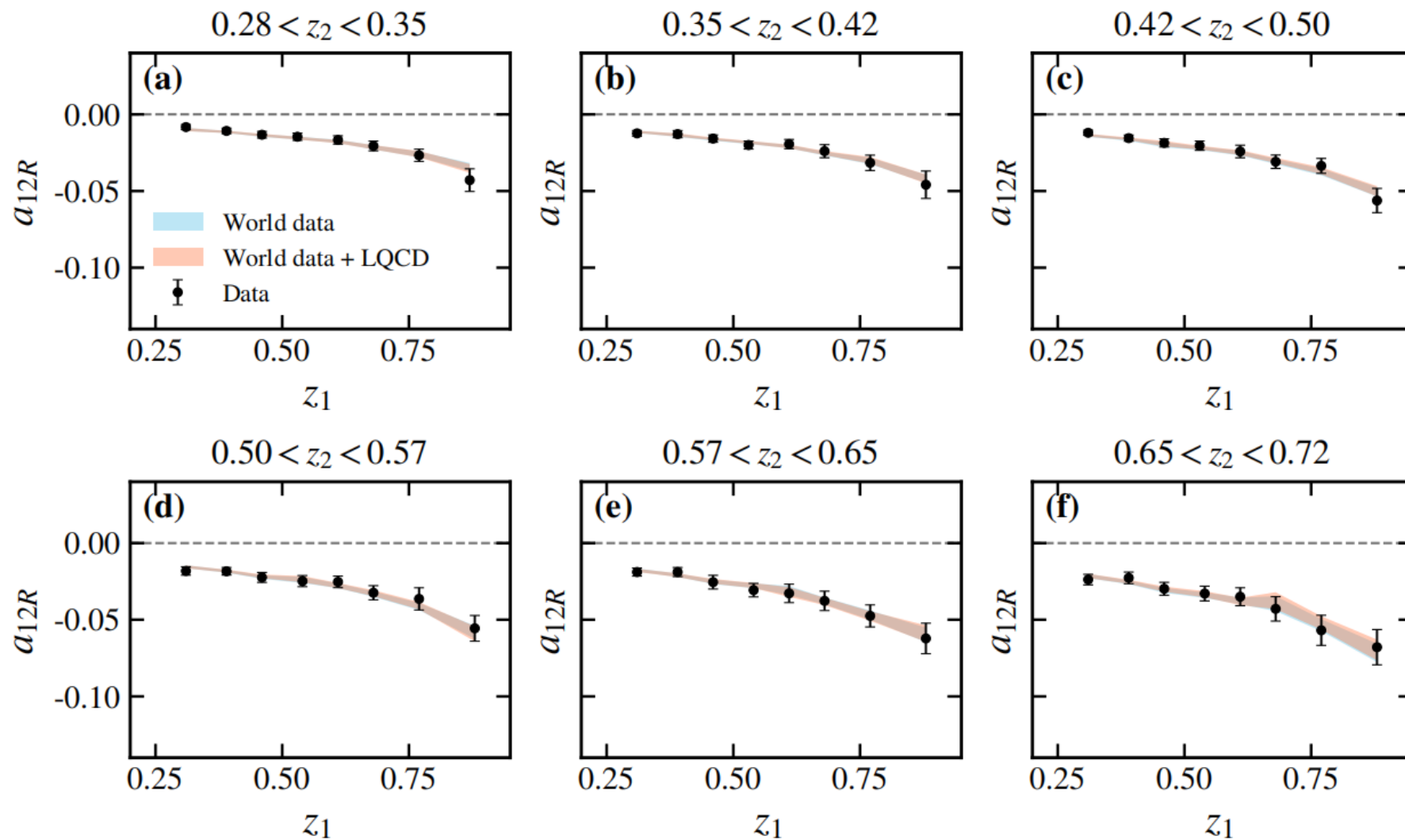
Compare with world datasets

Belle asymmetry $m_1 m_2$ binning



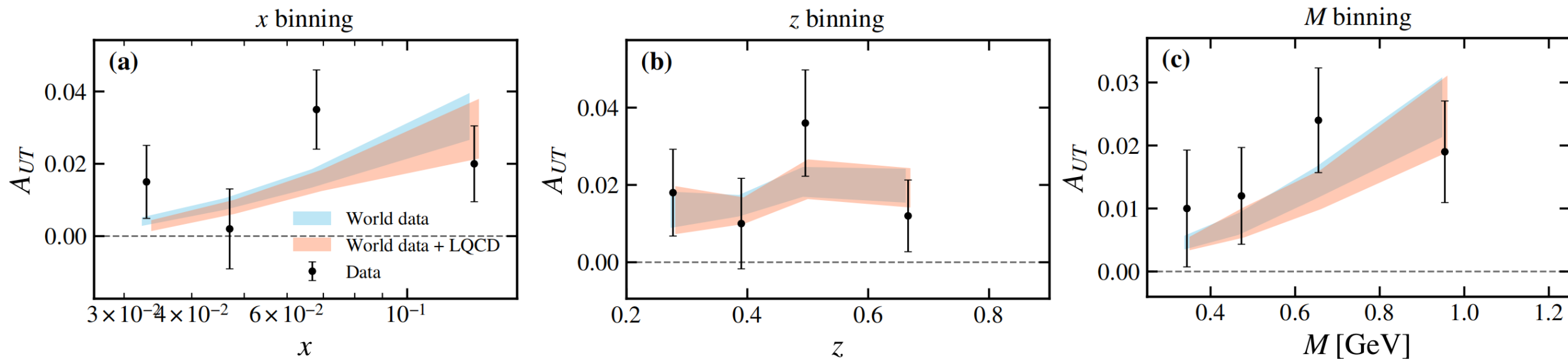
Compare with world datasets

Belle asymmetry $z_1 z_2$ binning



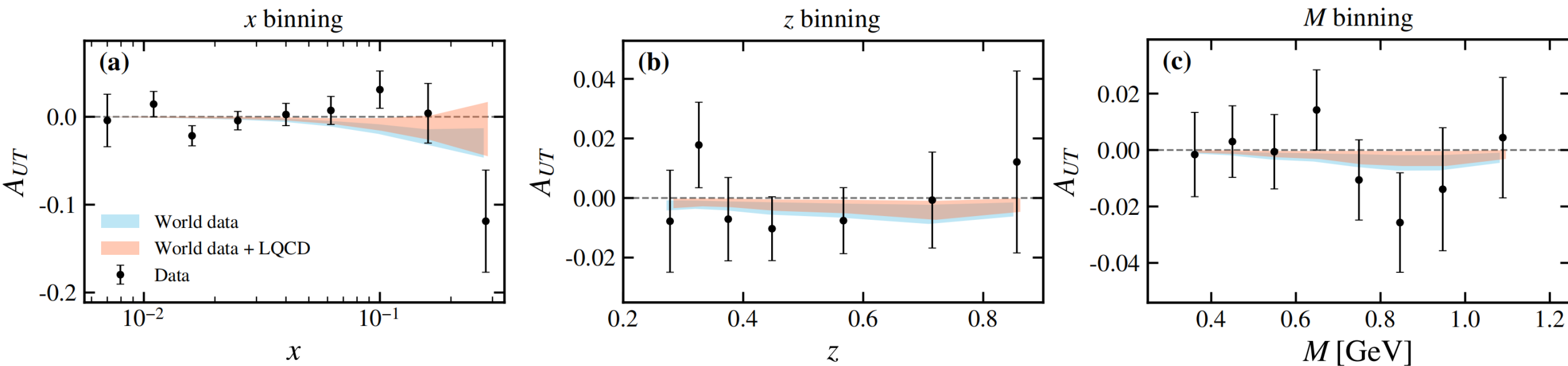
Compare with world datasets

HERMES Proton target



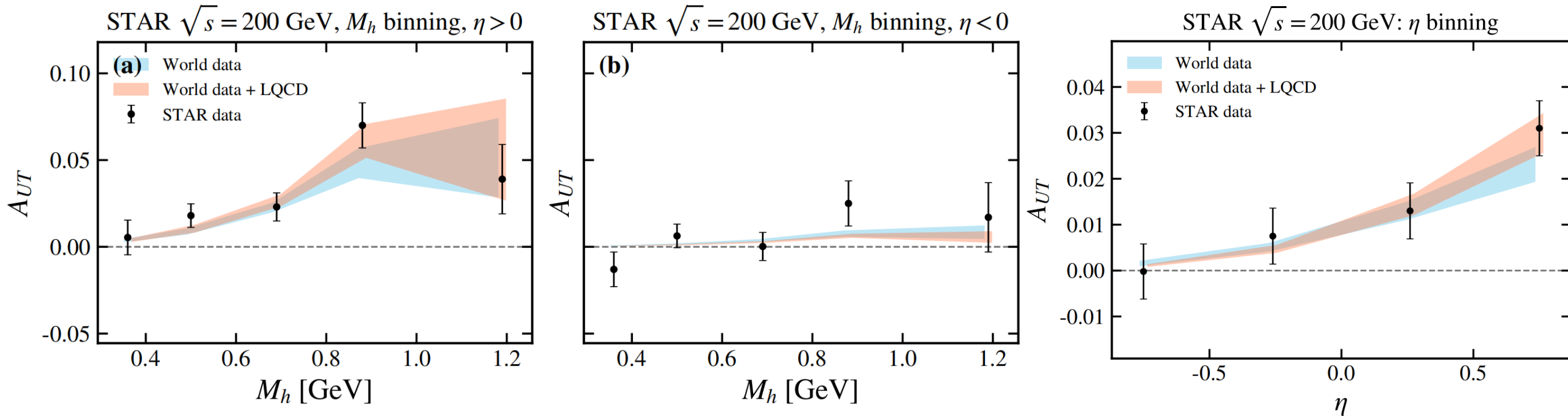
Compare with world datasets

COMPASS Deuteron target



Compare with world datasets

STAR $\sqrt{s} = 200\text{GeV}$



Summary

1. We have performed an NN analysis that simultaneously determines transversity and DiFFs from Belle, HERMES, COMPASS, STAR measurements, and lattice distributions.
2. The lattice input extends the distribution-level constraints to large x while preserving the description of experimental observables.
3. More precise measurements with flavor separation and broader kinematic coverage, together with improved lattice distributions, will further clarify the nucleon's transverse spin structure.