

Constraining DVCS Compton Form Factors Using Lattice QCD informed Neural Network



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Outline

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GPDs and DVCS

To provide a proper description of exclusive processes, we need to introduce a more general type of parton distribution, known as generalized parton distributions (GPDs).

GPDs are defined through the Fourier transforms of bilocal quark and gluon correlation functions on the light cone, followed by a Lorentz decomposition.

- unpolarized: H, E
$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \left| \bar{\psi}^q \left(-\frac{z}{2} \right) \gamma^+ \psi^q \left(\frac{z}{2} \right) \right| p_1 \right\rangle_{z_{\perp}=0, z^+=0} = \frac{1}{2P^+} \left(H^q(x, \xi, t) \bar{u}(p_2) \gamma^+ u(p_1) + E^q(x, \xi, t) \bar{u}(p_2) \frac{i\sigma^{+\mu} \Delta_\mu}{2M} u(p_1) \right)$$
- polarized : $\tilde{H}, \tilde{E}$
$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \left| \bar{\psi}^q \left(-\frac{z}{2} \right) \gamma^+ \gamma_5 \psi^q \left(\frac{z}{2} \right) \right| p_1 \right\rangle_{z_{\perp}=0, z^+=0} = \frac{1}{2P^+} \left(\tilde{H}^q(x, \xi, t) \bar{u}(p_2) \gamma^+ \gamma_5 u(p_1) + \tilde{E}^q(x, \xi, t) \bar{u}(p_2) \frac{\gamma_5 \Delta^+}{2M} u(p_1) \right)$$

GPD Variables

average momentum: $P = \frac{p_1 + p_2}{2},$

momentum transfer: $\Delta = p_2 - p_1,$

momentum fraction: x



skewness : $\xi = -\frac{\Delta^+}{P^+},$

four-momentum transfer squared: $t = \Delta^2,$

momentum fraction : x

GPDs and DVCS

GPDs unify the connection between the nucleon's spatial structure and longitudinal momentum fraction, providing a powerful tool for exploring its three-dimensional structure.

- zero-momentum-transfer limit ($t \rightarrow 0$):

- GPDs $\rightarrow$ PDFs

$$H^q(x, 0, 0) = q_q(x), \quad \tilde{H}^q(x, 0, 0) = \Delta q$$

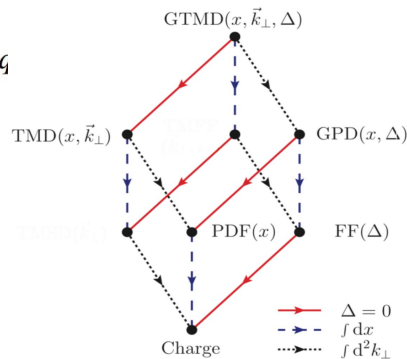
- integrate over the momentum fraction ($\int dx$):

- GPDs $\rightarrow$ FFs

$$\int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t)$$

$$\int_{-1}^1 dx E^q(x, \xi, t) = F_2^q(t)$$

Parton Distribution Family

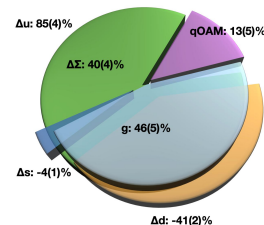


GPDs describe the spin structure of the nucleon.

- Nucleon Spin Sum Rule:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2} \Delta \Sigma_q + L_q + J_g$$

$$J_{q,g} = \frac{1}{2} \int_{-1}^1 dx [H_{q,g}(x, \xi, 0) + E_{q,g}(x, \xi, 0)]$$

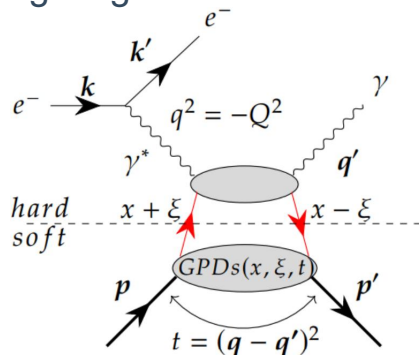


GPDs contain rich information about the nucleon structure and can be indirectly extracted from exclusive processes.

GPDs and DVCS

DVCS is one of the cleanest channels for probing GPDs

handbag diagram



Factorization

$$T \sim C(x, \xi, t) \otimes F(x, \xi, t)$$

hard process

QCD Wilson Coefficients (LO) :

$$C(x, \xi, t) \sim \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon}$$

soft process

GPDs:

$$F(x, \xi, t) \sim \begin{matrix} H(x, \xi, t), E(x, \xi, t) \\ \tilde{H}(x, \xi, t), \tilde{E}(x, \xi, t) \end{matrix}$$

Compton Form Factors:

$$\mathcal{H}(\xi, t, Q^2) = C(x, \xi, Q^2) \otimes H(x, \xi, t)$$

$$\mathcal{E}(\xi, t, Q^2) = C(x, \xi, Q^2) \otimes E(x, \xi, t)$$

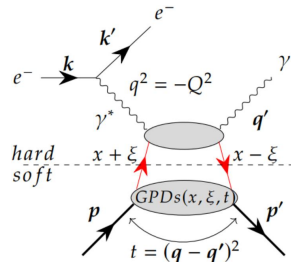
$$\tilde{\mathcal{H}}(\xi, t, Q^2) = C(x, \xi, Q^2) \otimes \tilde{H}(x, \xi, t)$$

$$\tilde{\mathcal{E}}(\xi, t, Q^2) = C(x, \xi, Q^2) \otimes \tilde{E}(x, \xi, t)$$

GPDs and DVCS

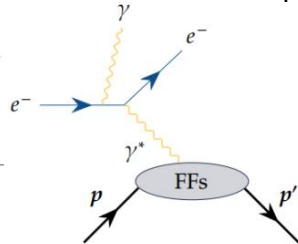
The observed photon can arise from two mechanisms: DVCS and Bethe–Heitler (BH).

DVCS process



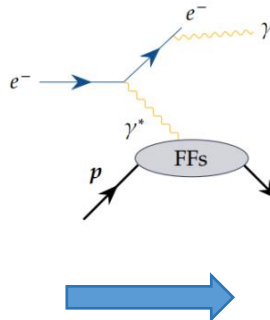
Bethe–Heitler process

can be calculated precisely within QED.



experimental observables

data from JLab、HERMES、ZEUS、H1



$$A_{LU,I}^{\sin \phi} \propto \text{Im} \left[F_1 \mathcal{H} + \xi(F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4m^2} F_2 \mathcal{E} \right],$$

$$A_{UL,I}^{\sin \phi} \propto \text{Im} \left[\xi(F_1 + F_2) \left(\mathcal{H} + \frac{\xi}{1+\xi} \mathcal{E} \right) + F_1 \tilde{\mathcal{H}} - \xi \left(\frac{\xi}{1+\xi} F_1 + \frac{t}{4M^2} F_2 \right) \tilde{\mathcal{E}} \right]$$

$$A_{LL,I}^{\cos \phi} \propto \text{Re} \left[\xi(F_1 + F_2) \left(\mathcal{H} + \frac{\xi}{1+\xi} \mathcal{E} \right) + F_1 \tilde{\mathcal{H}} - \xi \left(\frac{\xi}{1+\xi} F_1 + \frac{t}{4M^2} F_2 \right) \tilde{\mathcal{E}} \right]$$

$$A_{UT,I}^{\sin(\phi-\phi_s) \cos \phi} \propto \text{Im} \left[-\frac{t}{4M^2} (F_2 \mathcal{H} - F_1 \mathcal{E}) + \xi^2 \left(F_1 + \frac{t}{4M^2} F_2 \right) (\mathcal{H} + \mathcal{E}) \right]$$

$$- \xi^2 (F_1 + F_2) \left(\tilde{\mathcal{H}} + \frac{t}{4M^2} \tilde{\mathcal{E}} \right),$$

$$A_{UT,I}^{\cos(\phi-\phi_s) \sin \phi} \propto \text{Im} (F_2 \tilde{\mathcal{H}} - F_1 \xi \tilde{\mathcal{E}}),$$

$$A_{LT,I}^{\sin(\phi-\phi_s) \sin \phi} \propto \text{Re} (F_2 \mathcal{H} - F_1 \mathcal{E}),$$

$$A_{LT,I}^{\cos(\phi-\phi_s) \cos \phi} \propto \text{Re} (F_2 \tilde{\mathcal{H}} - F_1 \xi \tilde{\mathcal{E}}).$$

Amplitude Decomposition:

$$|\mathcal{T}|^2 = |\mathcal{T}_{BH}|^2 + |\mathcal{T}_{DVCS}|^2 + \mathcal{I},$$

Lattice constraints and Neural Network

Dispersion relations with arbitrary subtractions

$$\sum_{j \text{ even}}^n h_j \xi^{-j} = \Re \mathcal{H}(\xi) - \frac{1}{\pi} \text{P.V.} \int_0^1 \Im \mathcal{H}(x) \left(\frac{x}{\xi} \right)^n \left[\frac{1}{\xi - x} - (-1)^n \frac{1}{\xi + x} \right] dx$$

$$\text{for } j = 0: \quad \Re \mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi} \text{P.V.} \int_0^1 dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) \Im \mathcal{H}(x, t)$$

$$\text{for } j > 0: \quad h_{2j} = \frac{2}{\pi} \int_0^1 \Im \mathcal{H}(\xi') (\xi')^{2j-1} d\xi'$$

$$e_0 = -\Delta(t) \text{ for CFF E}$$

$$e_{2j} \text{ for CFF E}$$

Lattice constraints and Neural Network

Subtraction constants are related to generalized form factors

$$h_0^q(t) \simeq 5\bar{D}_1^q(t) \left(1 - \frac{t}{3Q^2}\right) - \frac{4M^2}{Q^2} c_0^{11} \left[\left(1 - \frac{t}{4M^2}\right) A^q(t) + \frac{t}{2M^2} J^q(t) \right]$$

$$h_2^q(t) \simeq c_0^{01} A^q(t) - \frac{4t}{Q^2} c_0^{11} [A^q(t) - J^q(t)] ,$$

$$h_2^q(t) + e_2^q(t) \simeq 4 \left(c_0^{01} - \frac{2t}{Q^2} c_0^{11} \right) J^q(t) ,$$

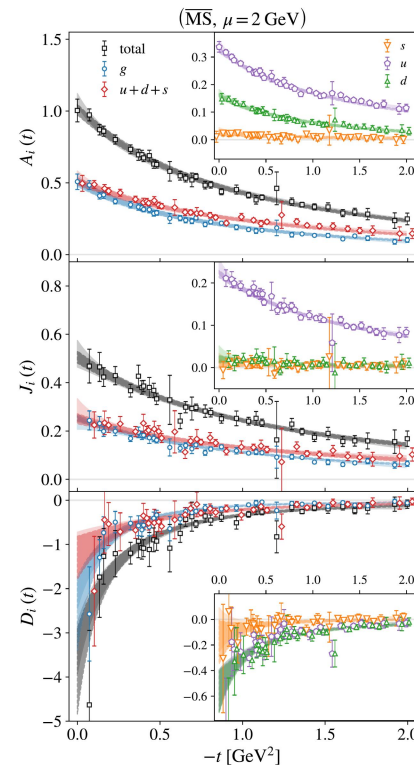
$$h_4^q(t) \simeq \left(\frac{1}{6} c_0^{03} - \frac{3t}{2Q^2} c_0^{13} \right) A_{3,0} + \frac{2t}{3Q^2} c_0^{13} B_{3,0}$$

$$h_4^q(t) + e_4^q(t) \simeq \frac{1}{3} \left(c_0^{03} - \frac{t}{Q^2} c_0^{13} \right) (A_{3,0} + B_{3,0}) ,$$

V. Martinez-Fernandez and C. Mezrag, Phys. Rev. D 113, 094004 (2026).

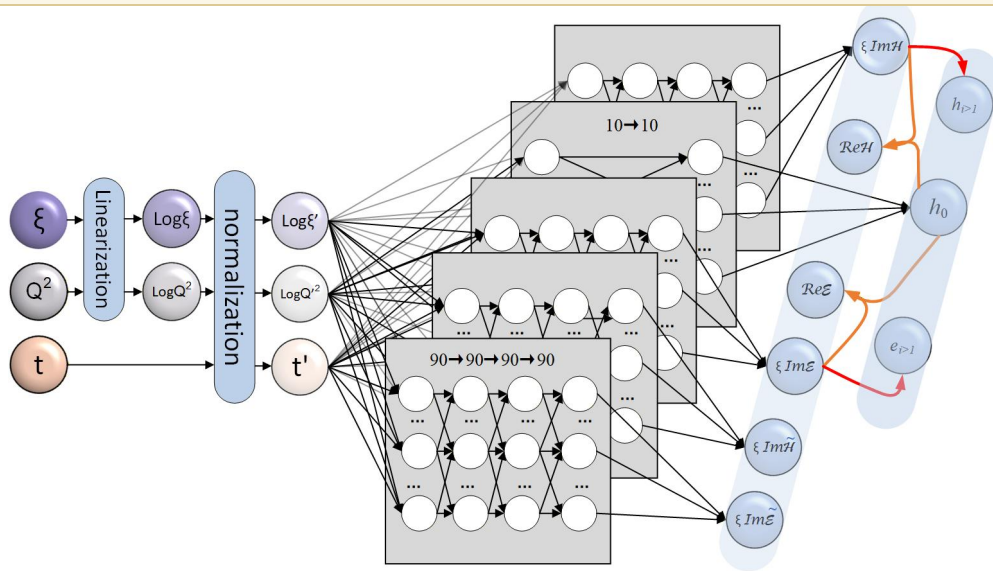
D. C. Hackett, D. A. Pefkou, and P. E. Shanahan, Phys. Rev. Lett. 132, 251904 (2024).

S. Bhattacharya et al., Phys. Rev. D 108, 014507 (2023).



Lattice constraints and Neural Network

Physics-informed neural-network architecture



CFF network

Inputs: (ξ, t, Q^2)

4 layers $\times$ 90 neurons

Outputs: $\Im m \mathcal{H}$, $\Im m \mathcal{E}$, ...

Subtraction network

Inputs: (t, Q^2)

2 layers $\times$ 10 neurons

Output: $h_0(t, Q^2)$

Dispersion relations reconstruct the real parts, while constraints on higher-order subtraction terms provide additional constraints on the imaginary parts.

Lattice constraints and Neural Network

Physics-informed neural-network architecture

Loss function

$$Loss = Huber(\sigma_{DVCS}) + \lambda Huber(\sigma_{LQCD})$$

Preprocessing

Log transforms of ξ and Q^2
Standardized kinematics

Validation

Training/test = 70/30
Early stopping

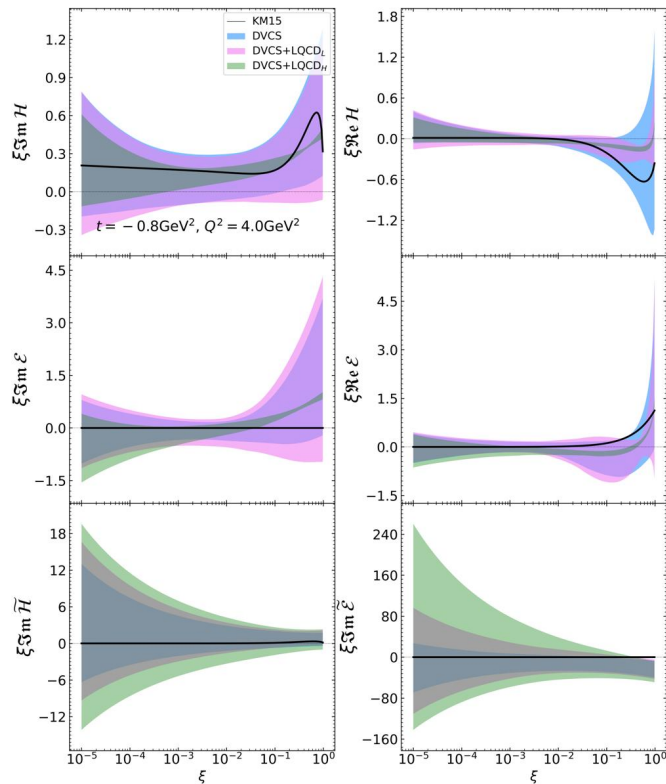
Uncertainty

100 replicas
Propagate data and
lattice errors

Implementation

PyTorch + Gepard
Compute DVCS
observables

Main results



Blue: DVCS only

The dispersion relation imposes constraints on the CFF. However, the error bars in areas where data are insufficient remain relatively large.

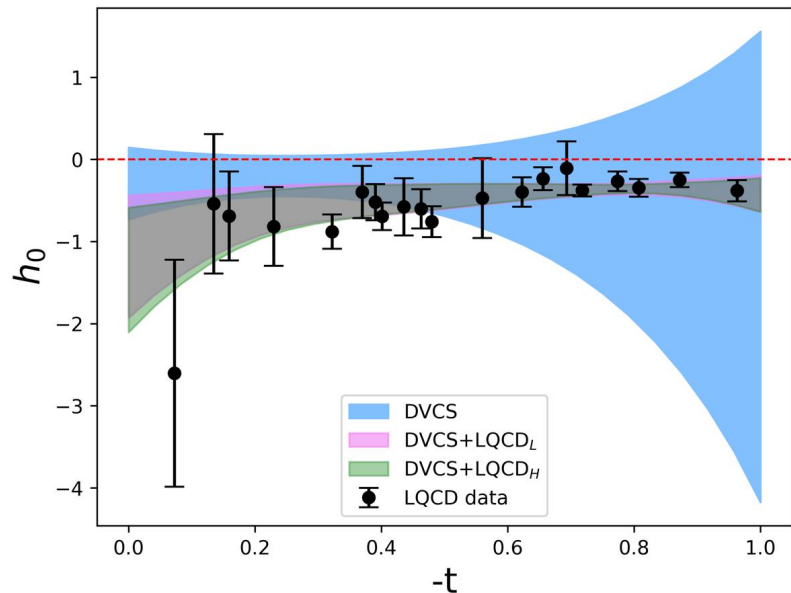
Magenta: leading lattice input

LO subtraction tightens Re H and Re E at large ξ . Central values remain compatible with the DVCS-only fit.

Green: higher lattice moments

Higher subtractions use lattice moments to narrow both real and imaginary CFF bands further.

Main results



DVCS only

The constraints on h_0 remain limited. The error band can extend to 0.

$$h_0(0) = -0.40 \pm 0.50$$

+ LQCD low

Extrapolated to $t = 0$, it remains negative within the uncertainty band.

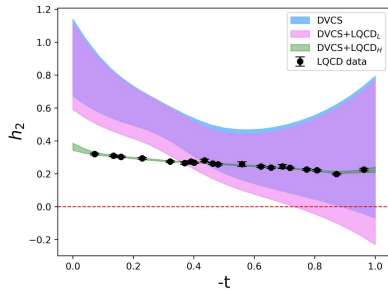
$$h_0(0) = -1.28 \pm 0.70$$

+ LQCD high

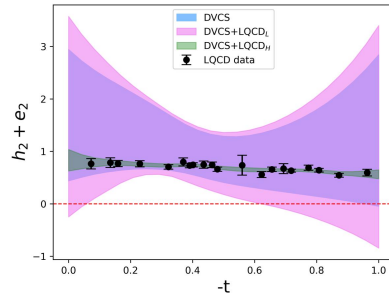
Consistent within uncertainties.

$$h_0(0) = -1.07 \pm 0.72$$

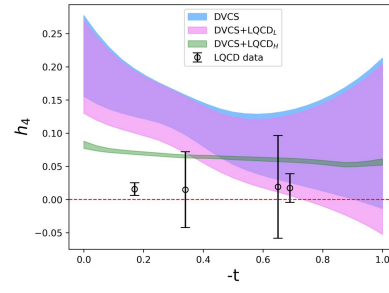
Main results



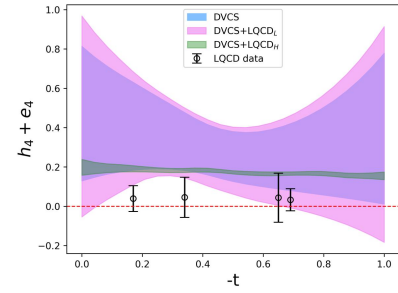
h_2



$h_2 + e_2$



h_4



$h_4 + e_4$

- h_2 and h_4 are positive and decrease overall as $-t$ grows

- $h_2 + e_2$ and $h_4 + e_4$ show weak t dependence but remain imprecise

Conclusions

- 1 Dispersion relations provide a common interface between experimental CFFs and lattice GFFs
- 2 Leading lattice inputs substantially reduce the uncertainties in $\Re\mathcal{H}$ and $\Re\mathcal{E}$
- 3 Higher subtractions further constrain both real and imaginary parts and test higher Mellin moments

Lattice inputs and DVCS data are compatible at the current precision

Thank you

Constraints on the t and Q^2 dependence

