



# Hyperons Gluon Helicity from Lattice QCD :

Probing SU(3) Flavor symmetry breaking

The 6th Chinese Lattice QCD Workshop

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# Background : The Origin of Proton Spin to The latest lattice computation results

J. Ashman et al. (EMC), DOI: [10.1016/0370-2693\(88\)91523-7](https://doi.org/10.1016/0370-2693(88)91523-7)

the EMC deep-inelastic scattering (DIS) experiment :

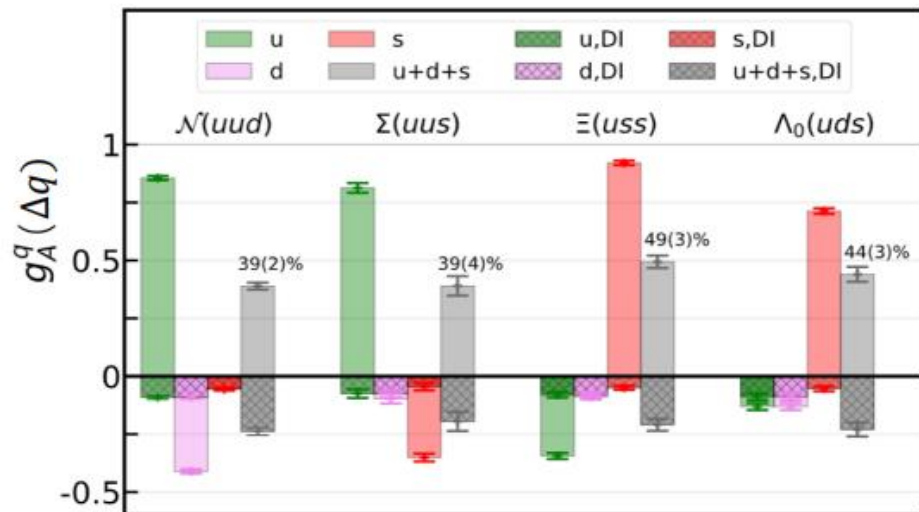
$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14$$

**Jaffe-Manohar :**  $J = \frac{1}{2} \Delta\Sigma + L_q^{JM} + \Delta G + L_G$

R. L. Jaffe and A. Manohar, DOI: [10.1016/0550-3213\(90\)90506-9](https://doi.org/10.1016/0550-3213(90)90506-9)

Quark helicity in lattice QCD : by **Zhi-Chen Hu**(preliminary)

$$\Delta q = \langle PS | \bar{q} \gamma_5 \vec{\gamma} \cdot \vec{S} q | PS \rangle$$



$$\Delta\Sigma_N = 0.39(2)$$

$$\Delta\Sigma_{\Sigma^+, \Sigma^0, \Sigma^-} = 0.39(4)$$

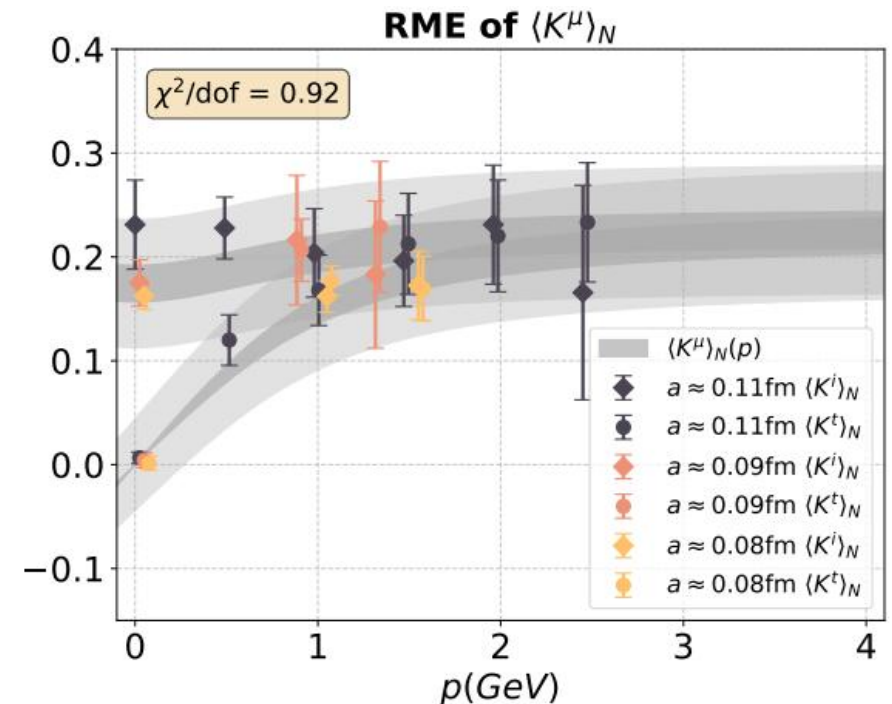
$$\Delta\Sigma_{\Lambda^0} = 0.44(3)$$

$$\Delta\Sigma_{\Xi^0, \Xi^-} = 0.49(3)$$

**Gluon Helicity from Lattice : by Zhao-Dian Jun**

D.-J. Zhao, et al., DOI: [10.48550/arXiv.2512.24315](https://doi.org/10.48550/arXiv.2512.24315)

$$\langle P' S_{\text{Proj.Z}} | K^{t/z} | P S_{\text{Proj.Z}} \rangle_{\text{C.G.}} \propto \Delta G + \text{h.t.}$$



$$N : \Delta G = 0.231(17)^{\text{sta.}} (44)^{\text{sym}}$$

# Hyperons Gluon Helicity from Lattice QCD

## bare $\Delta G$ matrix element

In the Coulomb gauge, the proton matrix element of the topological current is proportional to the gluon helicity:

$$\langle P' S_{\text{Proj.Z}} | K^{t/z} | P S_{\text{Proj.Z}} \rangle_{\text{C.G.}} \propto \Delta G + \text{h.t} \quad \text{Zhuo-Yi Pang et al. 10.1007/JHEP07(2024)222}$$

**Topological Current:**  $K^\mu = \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left[ A_\nu F_{\rho\sigma} - \frac{2}{3} i g_s A_\nu A_\rho A_\sigma \right]$

**Disconnect:**  $C_{3\text{pt}}(P_z, t, t_0, z, t_g) = \langle (\mathcal{O}_g(z, t_g) - \langle \mathcal{O}_g(z, t_g) \rangle) (C_{2\text{pt}}(P_z, t, t_0) - \langle C_{2\text{pt}}(P_z, t, t_0) \rangle) \rangle$

**Ratio method:**  $R_{K^{t/z}}(t_f, t) = \frac{C_3(t_f, t)}{C_2(t_f)} = \mathcal{K}_{00} + A_1 e^{-\Delta E(t_f - t)} + A_2 e^{-\Delta E t}$

**F-H / sum ratio fit:**  $\Delta SR(t_f) = SR(t_f) - SR(t_f - 1) \approx \Delta G_{\text{bare}} + c_1 t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}$

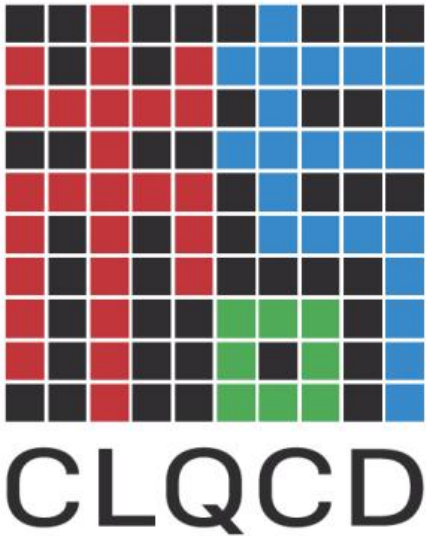
**AIC-weighted model averaging :**  $\bar{c}_0 = \sum_i w_i c_{0,i} \quad w_i = \frac{\exp [-(\text{AIC}_i - \text{AIC}_{\text{min}})/2]}{\sum_j \exp [-(\text{AIC}_j - \text{AIC}_{\text{min}})/2]}$

**LaMET:**  $\Delta G_{z/t}^{\text{bare}} = k \left( \Delta G + \frac{C_{\text{h.t.}}}{E^2} \right) + C_a \cdot a^2 + C_m (m_{\pi, \text{sea}}^2 - m_{\pi, \text{phys}}^2)$

# Lattice Setup

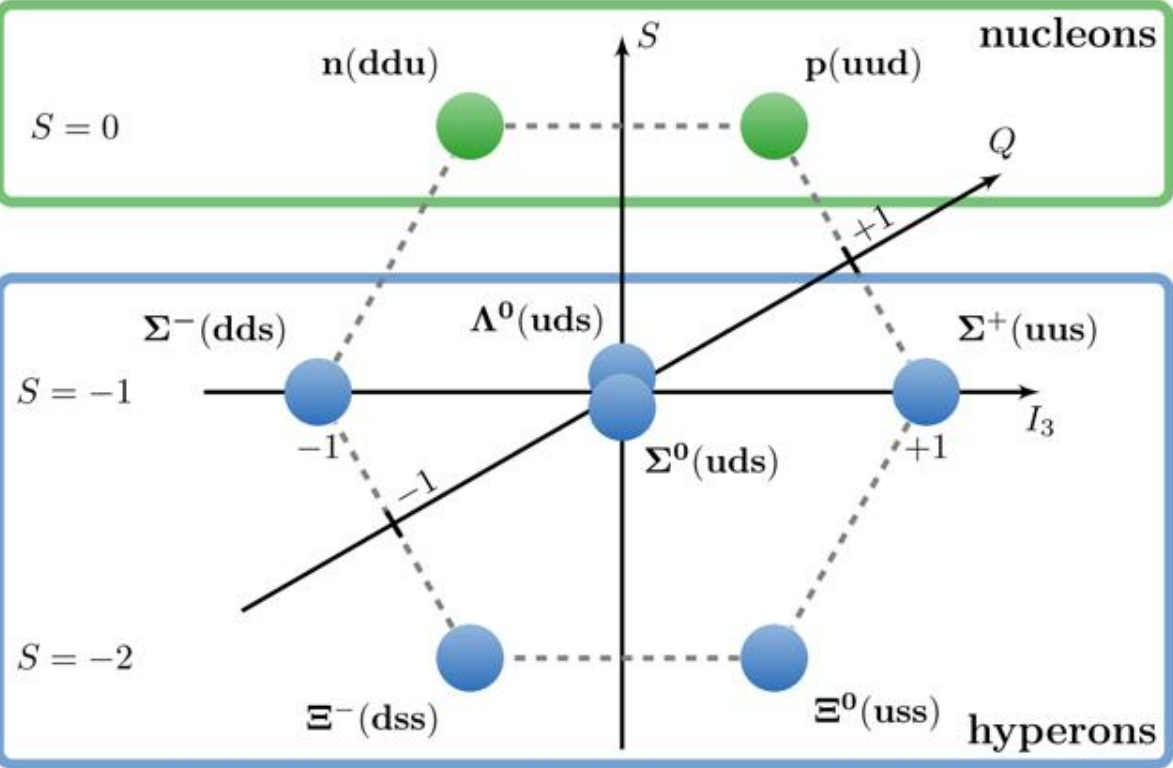
➤ 2+1 flavor tadpole-improved Clover Fermion with Symanzik gauge action;

| Ensemble | a (fm)          | m $\pi$ (MeV) | Lx $\times$ Lt               | Ncfg |
|----------|-----------------|---------------|------------------------------|------|
| C24P29   | 0.10524(05)(62) | 292.3(1.0)    | 24 <sup>3</sup> $\times$ 72  | 880  |
| C32P29   | 0.105           | 292           | 32 <sup>3</sup> $\times$ 64  | 987  |
| F32P30   | 0.07753(03)(45) | 300.4(1.2)    | 32 <sup>3</sup> $\times$ 96  | 800  |
| H48P32   | 0.05199(08)(31) | 316.6(1.0)    | 48 <sup>3</sup> $\times$ 144 | 453  |



Z.-C.Hu et al., (CLQCD), PRD109(2024)5,054507

# Two-point correlation function



$$n = P^+ [u(u^T C \gamma^5 d)]$$
$$\Xi^+ = P^+ [s(s^T C \gamma^5 u)]$$
$$\Sigma^+ = P^+ [u(u^T C \gamma^5 s)]$$
$$\Sigma^0 = P^+ [d(u^T C \gamma^5 s) + u(d^T C \gamma^5 s)]$$
$$\Lambda_0 = P^+ [2s(u^T C \gamma^5 d) + d(u^T C \gamma^5 s) - u(d^T C \gamma^5 s)]$$

# smear to quark field

1)distillation [M. Peardon, et al., DOI: 10.1103/PhysRevD.80.054506](#)

$$\square(\vec{x},\vec{y},t) = \sum_{k=1}^{R_{\mathcal{D}}} \xi^{(k)}(\vec{x},t) \xi^{(k)\dagger}(\vec{y},t)$$
$$P_{\alpha\beta}(t,t_0) = V^\dagger(t) G_{\alpha\beta} V(t_0)$$

$$\psi^{\text{dist.}}(x) = \sum_{i=1}^{N_{\text{vec}}} v_i \langle v_i | \psi(x) \rangle$$

2)momentumsmear ud

$$\square(\vec{x},\vec{y},t) = \sum_{k=1}^{R_{\mathcal{D}}} \xi^{(k)}(\vec{x},t) \xi^{(k)\dagger}(\vec{y},t)$$
$$\tilde{\xi}^{(k)}(\vec{z},t) = e^{i\vec{\zeta}\cdot\vec{z}} \xi^{(k)}(\vec{z},t)$$

$$\vec{\zeta} = \frac{2\pi}{L} \cdot n_z \cdot \hat{z} \qquad z : n_z = -2:$$

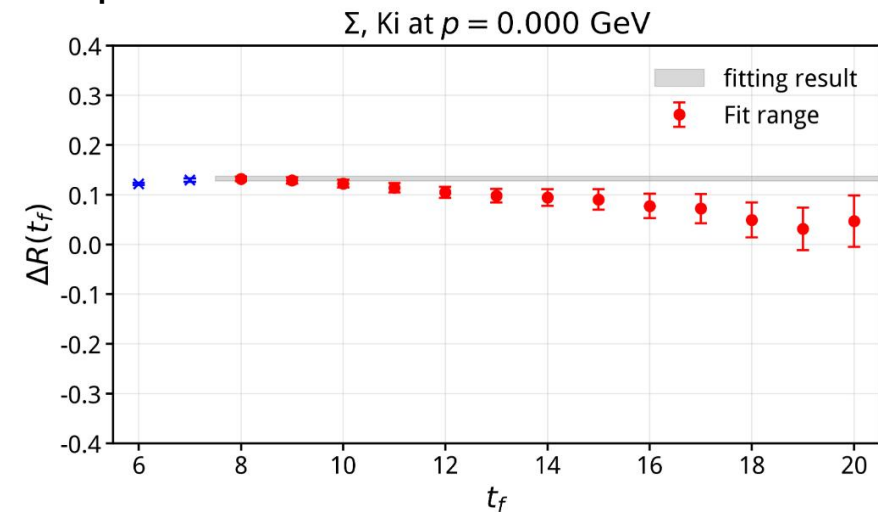
$$\psi^{\text{MSmear.}}(x) = e^{-ip_{\text{MSmear.}}x} \psi(x)$$

**F32P30**  $\Xi^0(\text{uss})$ ,  $\Xi^-(\text{dss})$  **Ki (z+y+x)**

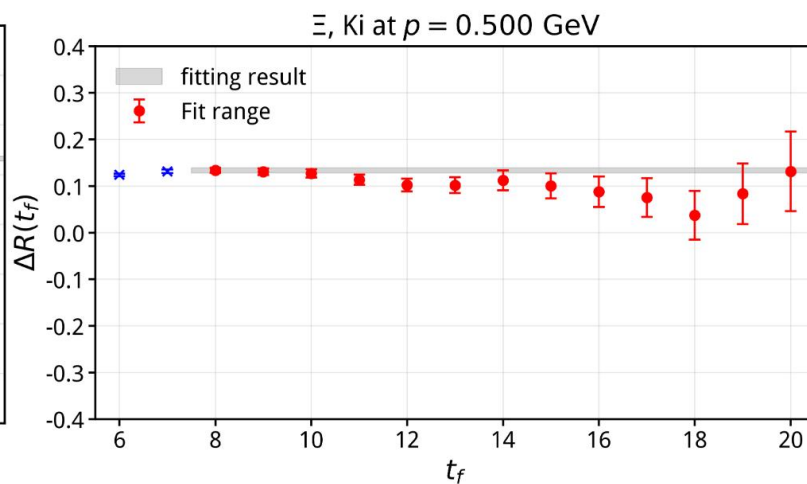
$$DR = \boxed{c_0} + c_1 t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}$$

$\langle P' S_{\text{Proj.Z}} | K^i | P S_{\text{Proj.Z}} \rangle_{\text{C.G.}}$

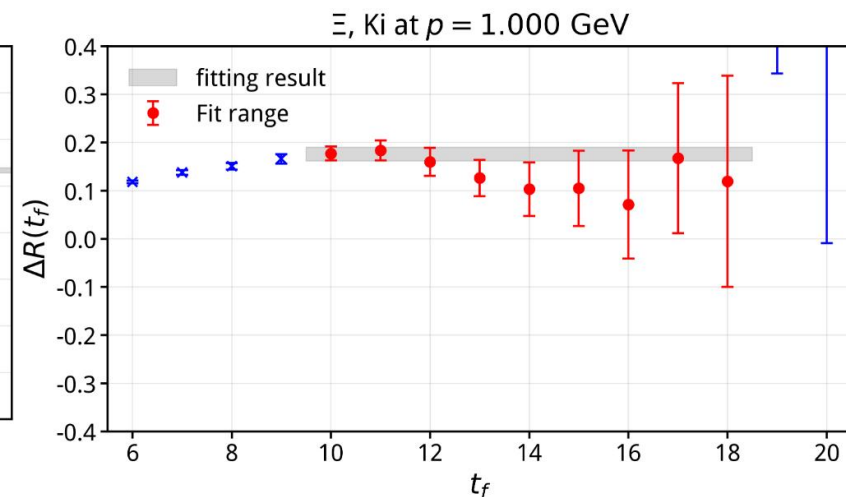
$p = 0$



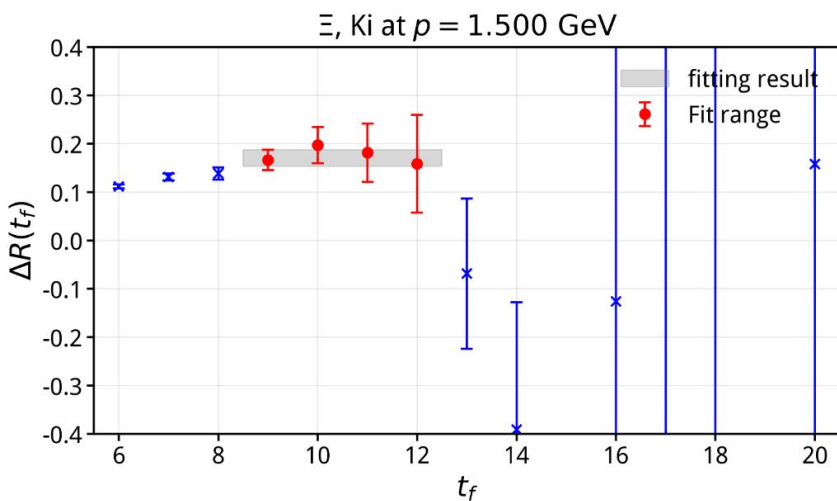
$p = 0.5\text{GeV}$



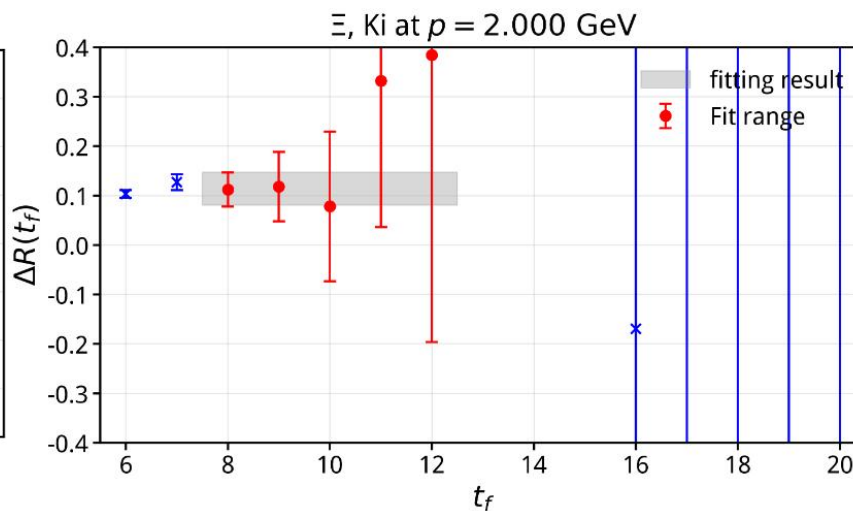
$p = 1\text{GeV}$



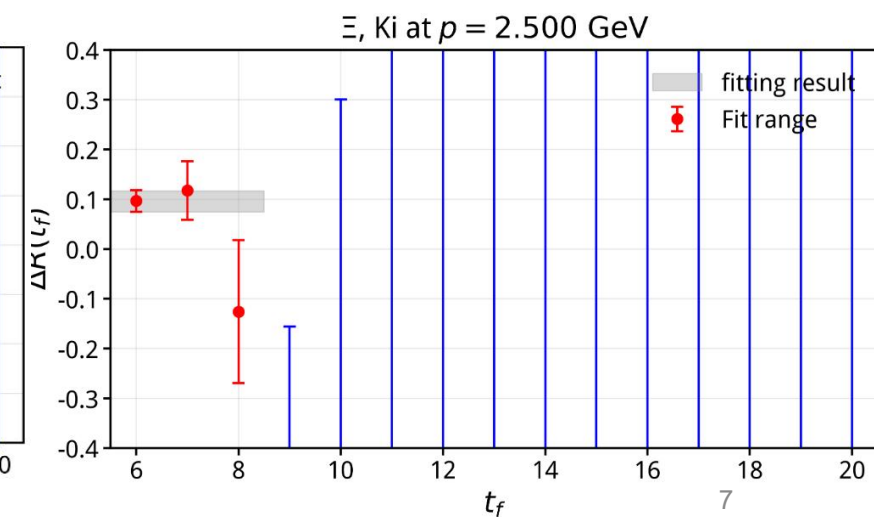
$p = 1.5\text{GeV}$



$p = 2\text{GeV}$



$p = 2.5\text{GeV}$

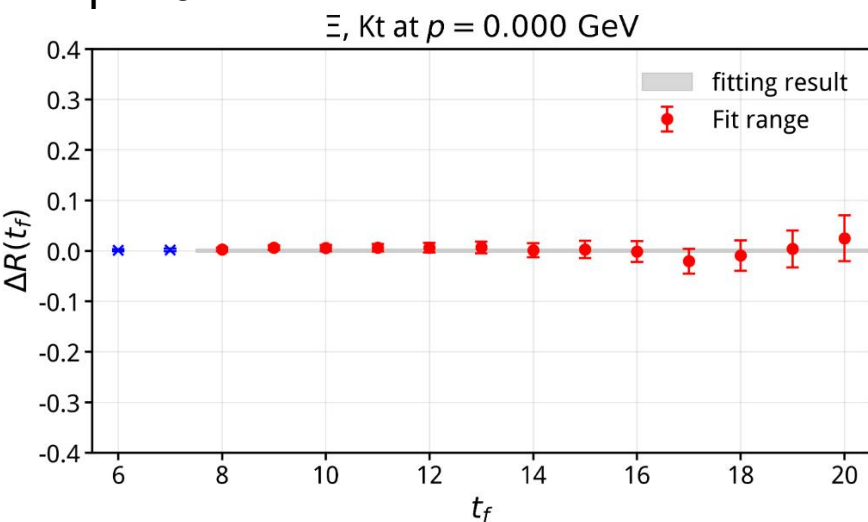


**F32P30**     $E^0(\text{uss})$ 、 $E^-(\text{dss})$     **Kt (z+y+x)**

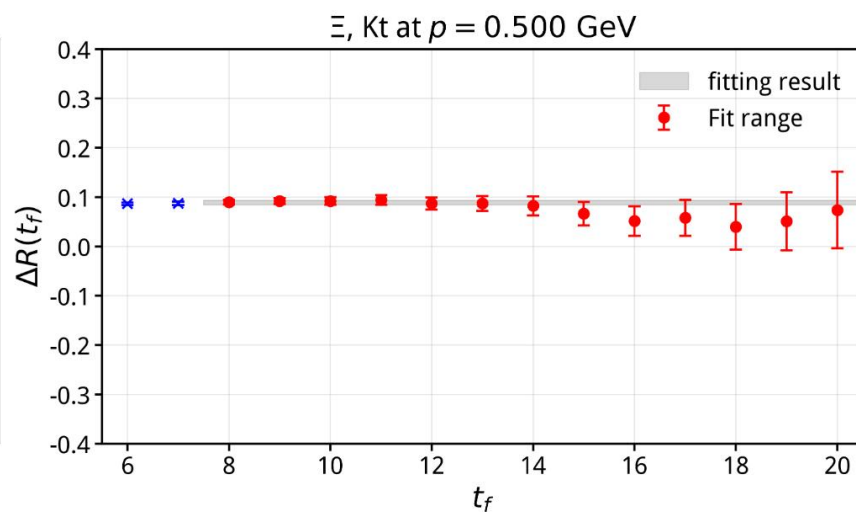
$$\langle P' S_{\text{Proj.Z}} | K^i | P S_{\text{Proj.Z}} \rangle_{\text{C.G.}}$$

$$DR = \boxed{c_0} + c_1 t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}$$

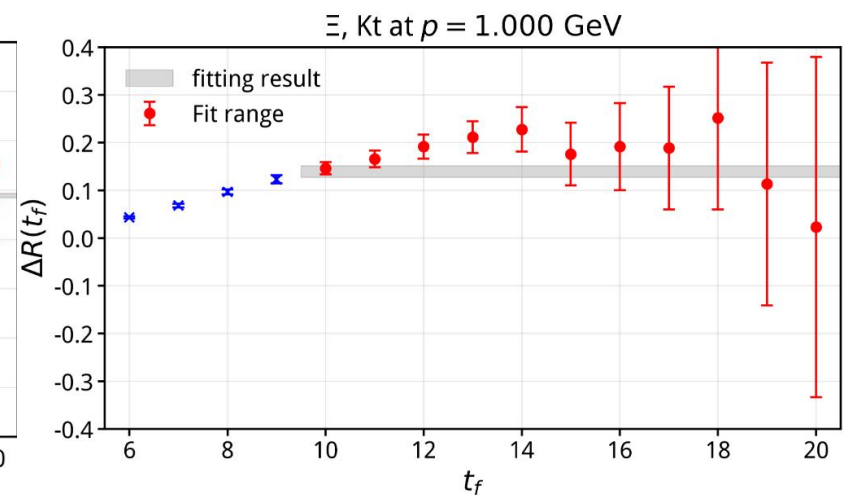
p = 0



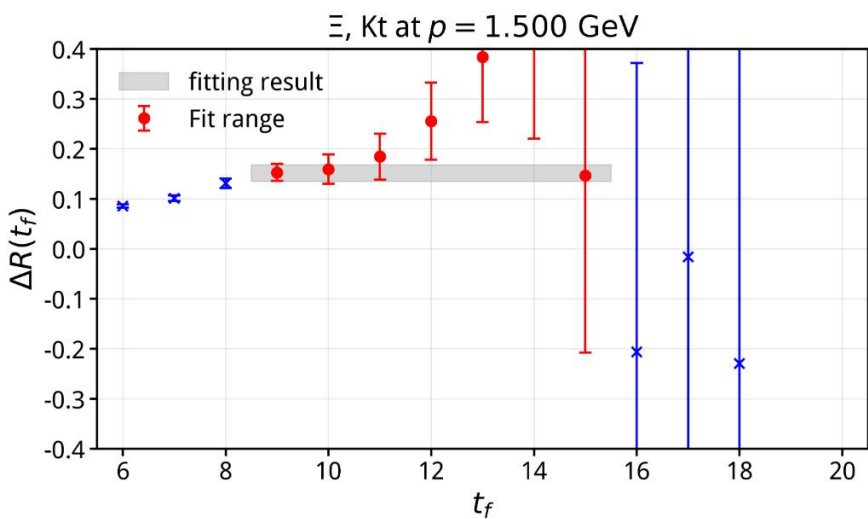
p = 0.5GeV



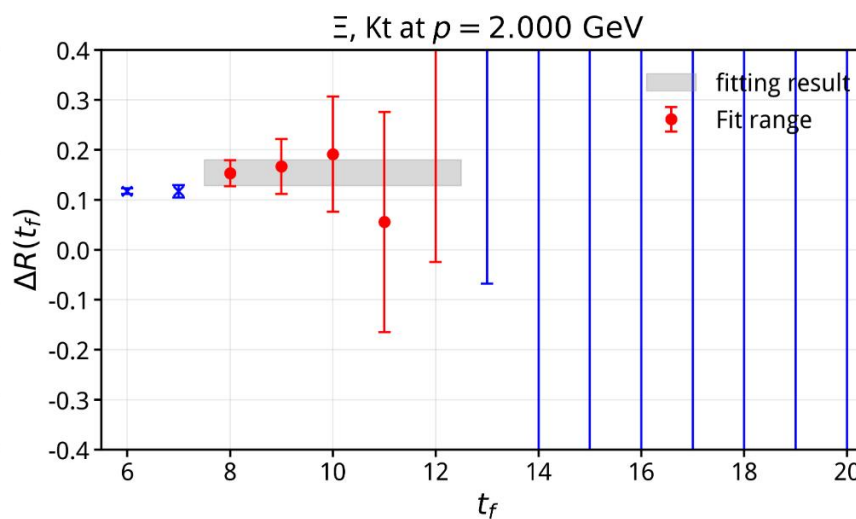
p = 1GeV



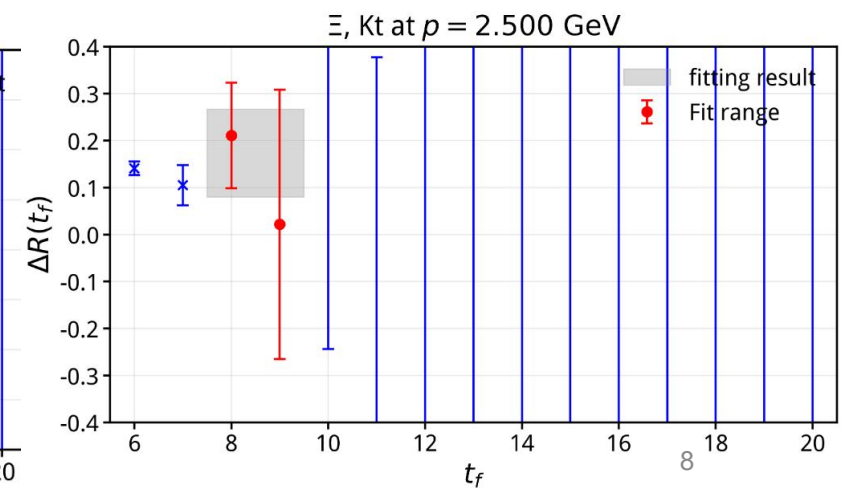
p = 1.5GeV



p = 2GeV



p = 2.5GeV



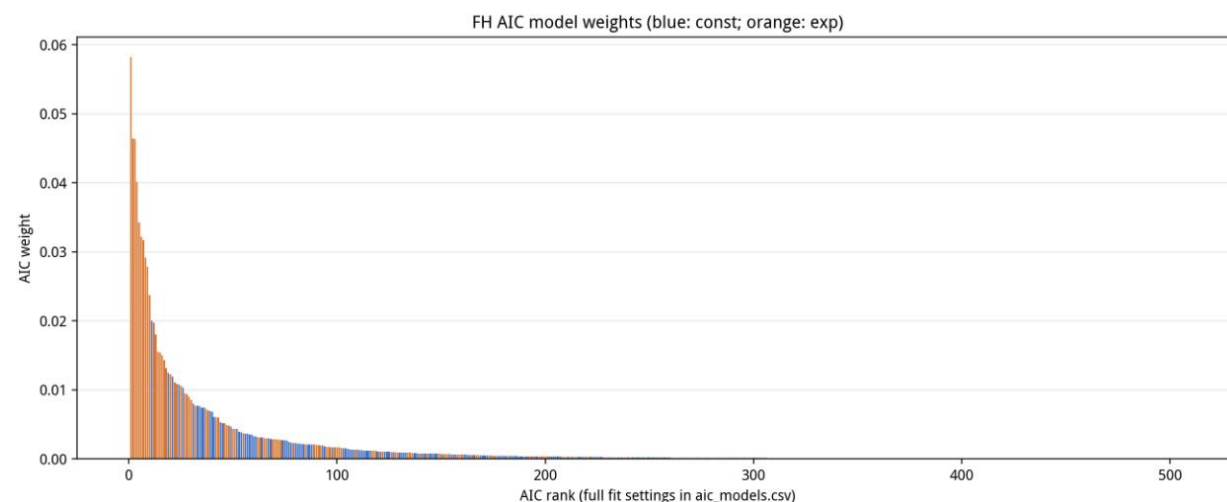
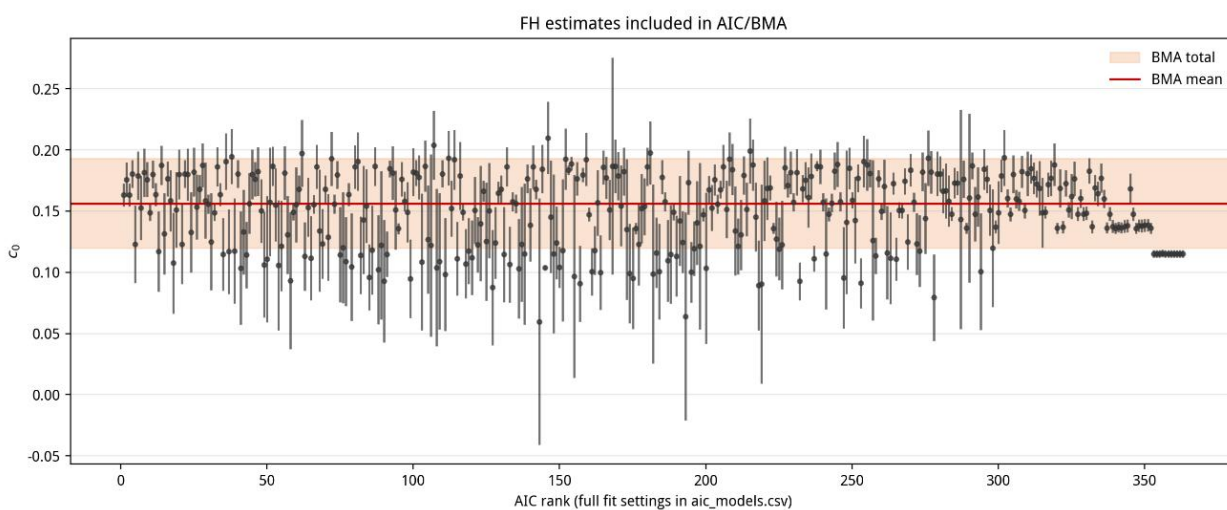
# AIC-weighted Bayesian model averaging

$$\Delta SR(t_f) \approx \Delta G_{\text{bare}} + c_1 t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}$$

$M = (n_{\text{cut}}, \text{fit\_mode}, \text{fit\_range})$

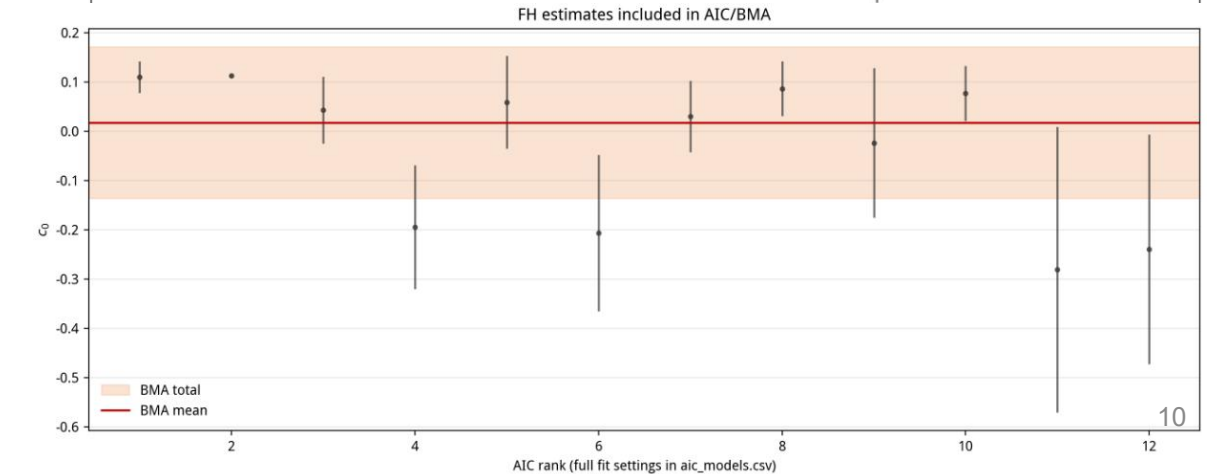
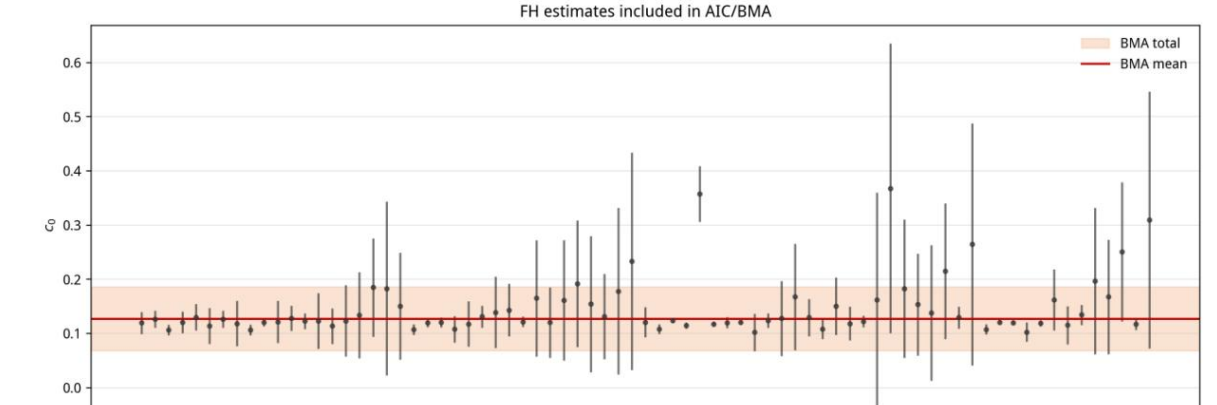
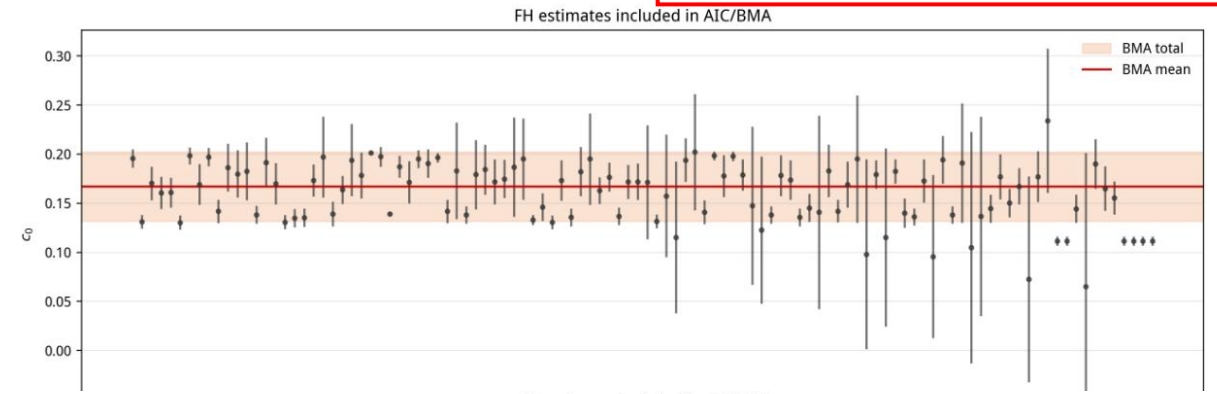
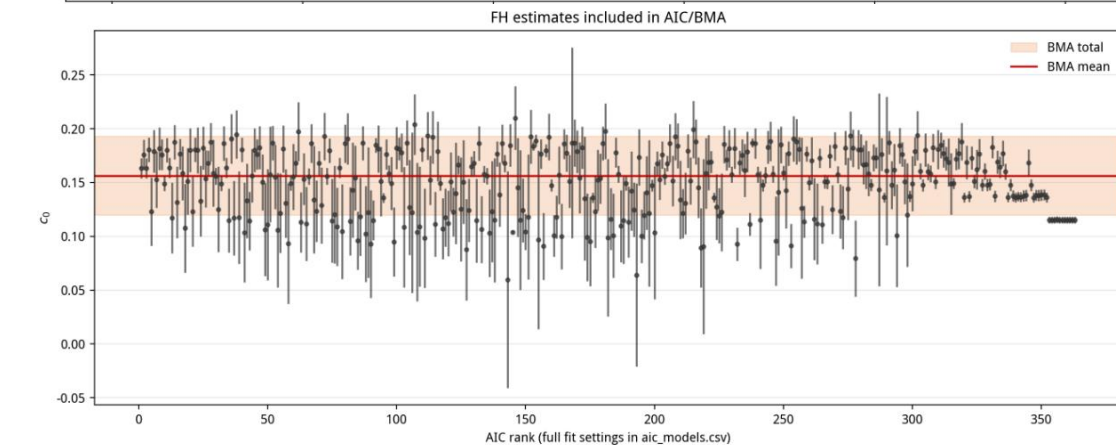
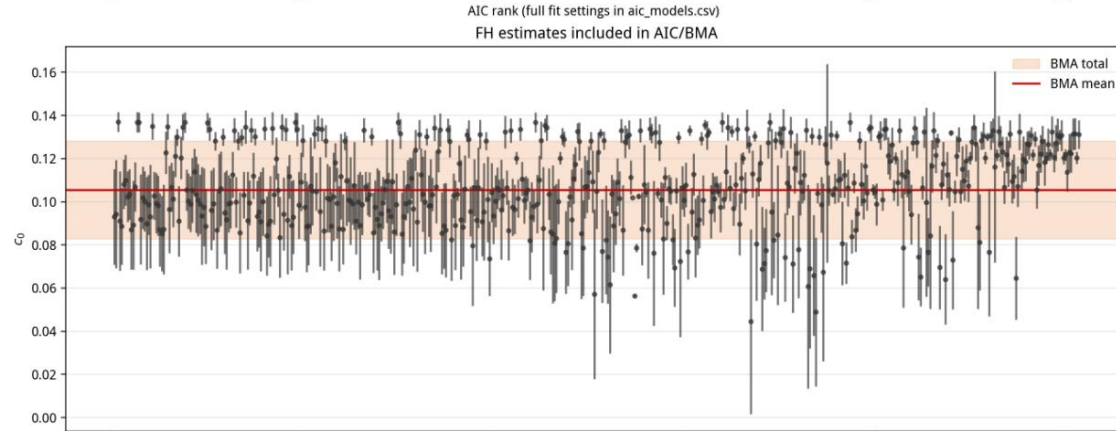
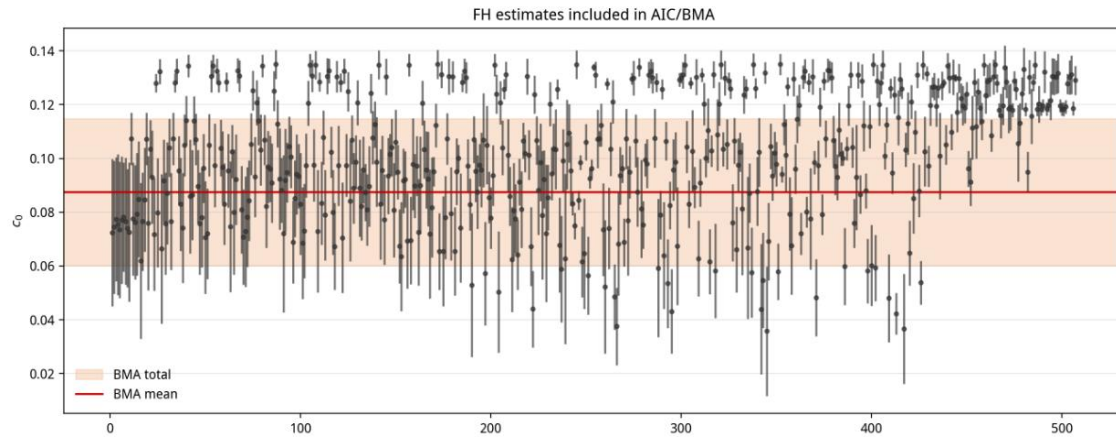
$$\text{AIC}_i = \chi_{\text{aug},i}^2 + 2k_i + 2N_{\text{cut},i} \quad w_i = \frac{\exp[-(\text{AIC}_i - \text{AIC}_{\min})/2]}{\sum_j \exp[-(\text{AIC}_j - \text{AIC}_{\min})/2]}$$

$$\bar{c}_0 = \sum_i w_i c_{0,i} \quad , \quad \sigma_{\text{total}}^2 = \sum_i w_i \sigma_i^2 + \sum_i w_i (c_{0,i} - \bar{c}_0)^2$$



**F32P30**     $E^0(\text{uss})$ 、 $E^-(\text{dss})$     **Ki**    **(z+y+x)**

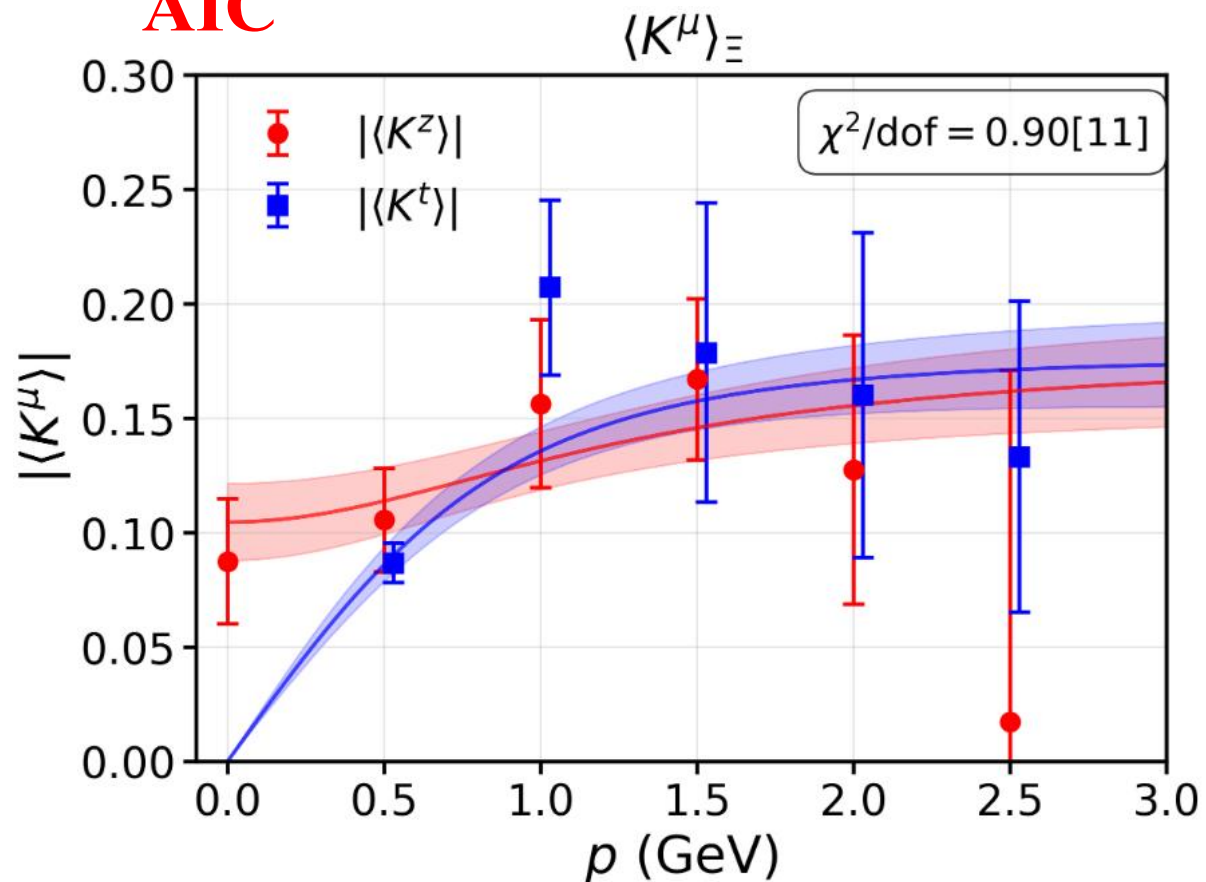
$$DR = c_0 + c_1 t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}$$



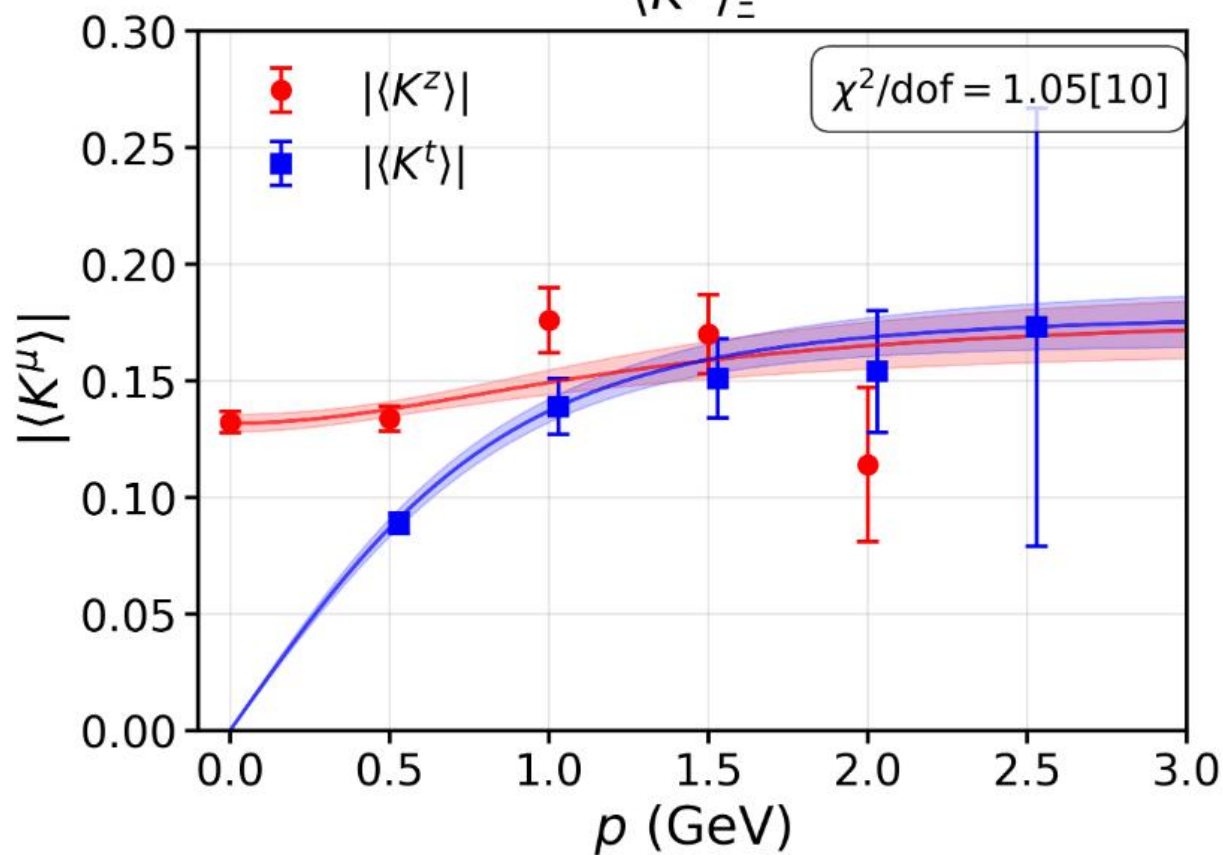
# bare $\Delta G$ matrix element

$$\Delta G_i^{\text{bare}} = \Delta G + \frac{C_{h,t}}{m^2 + p^2} \quad \Delta G_t^{\text{bare}} = \frac{|\mathbf{p}|}{\sqrt{m^2 + p^2}} \left( \Delta G + \frac{C_{h,t}}{m^2 + p^2} \right)$$

AIC



$$\Delta G^{\text{bare}} = 0.178(24)$$



$$\Delta G^{\text{bare}} = 0.179(15)$$

# Renormalization of topological current $K\mu$

**RI/MOM** : non-perturbative renormalization scheme on LQCD

Zhuo-Yi Pang et al. 10.1007/JHEP07(2024)222

$$\langle PS | \{K^\rho, J^\rho\} | PS \rangle^{\text{tree.}} = Z_{\{1,2\}1}^{\text{RI}} \langle PS | K^\rho | PS \rangle^{\text{lat.}} + Z_{\{1,2\}2}^{\text{RI}} \langle PS | J^\rho | PS \rangle^{\text{lat.}}$$

$$J_\rho = \bar{q} \gamma_5 \gamma_\rho q:$$

$$K^\rho = \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left[ A_\nu F_{\mu\sigma} - \frac{2}{3} i g_s A_\nu A_\mu A_\sigma \right]$$

$$\begin{pmatrix} \langle PS | K^{0/z} | PS \rangle_{\text{tree}} \\ \langle PS | J_5^{0/z} | PS \rangle_{\text{tree}} \end{pmatrix} = \begin{pmatrix} Z_{11}^{\text{lattice/DR}} & Z_{12}^{\text{lattice/DR}} \\ Z_{21}^{\text{lattice/DR}} & Z_{22}^{\text{lattice/DR}} \end{pmatrix} \begin{pmatrix} \langle PS | K^{0/z} | PS \rangle_{\text{covariant}, P^2=-\mu_R^2, P^z=P_R^z}^{\text{lattice/DR}} \\ \langle PS | J_5^{0/z} | PS \rangle_{\text{covariant}, P^2=-\mu_R^2, P^z=P_R^z}^{\text{lattice/DR}} \end{pmatrix}$$

$$\begin{aligned} Z_{11}^{\text{lat.}} &= \frac{J^{\text{lat.,q}} K^{\text{tree.,g}} - J^{\text{lat.,g}} K^{\text{tree.,q}}}{K^{\text{lat.,g}} J^{\text{lat.,q}} - J^{\text{lat.,g}} K^{\text{lat.,q}}}, & Z_{12}^{\text{lat.}} &= \frac{K^{\text{lat.,q}} K^{\text{tree.,g}} - K^{\text{lat.,g}} K^{\text{tree.,q}}}{K^{\text{lat.,q}} J^{\text{lat.,g}} - J^{\text{lat.,q}} K^{\text{lat.,g}}} \\ Z_{21}^{\text{lat.}} &= \frac{J^{\text{lat.,q}} J^{\text{tree.,g}} - J^{\text{lat.,g}} J^{\text{tree.,q}}}{K^{\text{lat.,g}} J^{\text{lat.,q}} - J^{\text{lat.,g}} K^{\text{lat.,q}}}, & Z_{22}^{\text{lat.}} &= \frac{K^{\text{lat.,q}} J^{\text{tree.,g}} - K^{\text{lat.,g}} J^{\text{tree.,q}}}{K^{\text{lat.,q}} J^{\text{lat.,g}} - J^{\text{lat.,q}} K^{\text{lat.,g}}} \end{aligned}$$

$$\Delta \Sigma^{\text{RI}} = Z_A^{\text{RI}} \Delta \Sigma^{\text{B}}$$

$$\Delta G^{\text{RI}} = Z_{11}^{\text{RI}} \left( \Delta G^{\text{B}} - \frac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}} \Delta \Sigma^{\text{B}} \right)$$

$$K^{\text{tree.,q}} = 0$$

$$J^{\text{tree.,g}} = 0$$

$$J^{\text{lat.,g}} \rightarrow 0$$

$$\begin{aligned} Z_{11}^{\text{RI}} &= \frac{K^{\text{tree.,g}}}{K^{\text{lat.,g}}} & Z_{12}^{\text{RI}} &= \frac{K^{\text{tree.,q}}}{J^{\text{lat.,q}}} - \frac{K^{\text{lat.,q}} K^{\text{tree.,g}}}{J^{\text{lat.,q}} K^{\text{lat.,g}}} = - \frac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}} Z_{11}^{\text{lat.}} \propto \alpha_s Z_{11}^{\text{lat.}} \\ Z_{21}^{\text{RI}} &= 0 & Z_{22}^{\text{RI}} &= \frac{J^{\text{tree.,q}}}{J^{\text{lat.,q}}} \end{aligned}$$

D.-J. Zhao, et al., DOI: 10.48550/arXiv.2512.24315

Zhi-Cheng Hu et al. 10.1103/PhysRevD.109.054507

# Renormalization of topological current $K_\mu$

**RI/MOM** : non-perturbative renormalization scheme on LQCD

Zhuo-Yi Pang et al. 10.1007/JHEP07(2024)222

$$Z_{11}^{\text{RI}} = \frac{K^{\text{tree},g}}{K^{\text{lat},g}}$$

$$\langle A_\mu | K^\rho | A_\nu \rangle^{\text{tree}} = i \epsilon^{\rho\mu\sigma\nu} \epsilon_\mu^* \hat{p}_\sigma \epsilon_\nu$$

RI/MOM scheme: 
$$Z_{11}^{\text{RI}}(\mu^2) = \frac{Z_{g, \text{diag/off}}(\mu^2)}{\text{Tr} \left[ S_g^{-1}(p_1) G_K(p_1, p_2) S_g^{-1}(p_2) (K^{\text{tree},g})^{-1} \right]}$$

$$G_K^{\mu\rho\nu}(p_1, p_2) = \sum_{x,y} e^{-i(p_1 \cdot x - p_2 \cdot y)} \langle A^\mu(x) K^\rho(0) A^\nu(y) \rangle$$

$$S_g^{\mu\nu}(p) = \text{CV} \langle \text{Tr} [A^\mu(p) A^\nu(-p)] \rangle \quad \text{CV} = 2 / [(N_c^2 - 1) \tilde{V}].$$

final : Z11

$$\begin{aligned} Z_{11}^{-1,\text{RI}}(\mu^2) &= \sum_{\mu,\rho,\nu,\sigma} \frac{2\text{CV} \hat{p}^2 G_K^{\mu\rho\nu}(p,p)}{i \epsilon^{\rho\mu\sigma\nu} \hat{p}_\sigma (S_g^{\mu\mu} + S_g^{\nu\nu})(p^2)} \Bigg|_{\mu^2=p^2} \\ &= \sum_{\mu,\rho,\nu,\sigma} \frac{\text{CV} \hat{p}^4 G_K^{\mu\rho\nu}(p,p)}{i \epsilon^{\rho\mu\sigma\nu} \hat{p}_\sigma \hat{p}_\mu \hat{p}_\nu S_g^{\mu\nu}(p^2)} \Bigg|_{\mu^2=p^2} \end{aligned}$$

$$G_K^{\mu\rho\nu}(p,p)$$

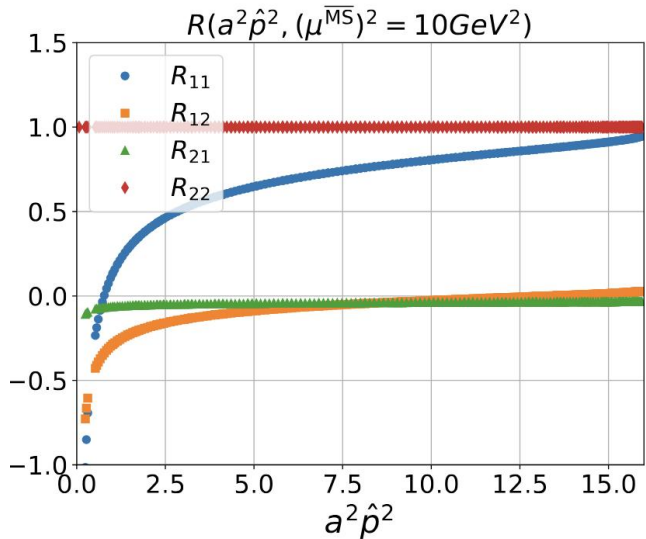
**CDER** : Cluster decomposition error reduction

- 1)  $K_{r'}^\rho(x) = \int_{|r'| < r_2} d^4 r' K^\rho(x + r')$
- 2)  $B_{\mu,r'}^\rho(x) = A_\mu(x) K_{r'}^\rho(x)$
- 3)  $\int d^4 x d^4 y e^{ip \cdot (x-y)} B_{\mu,r'}^\rho(x) A_\nu(y)$

$$G_K^{\mu\rho\nu,\text{CDER}}(p,p) = \left\langle \int_{|r| < r_1} d^4 r \int_{|r'| < r_2} d^4 r' \int d^4 x e^{ip \cdot r'} K^\rho(x + r') \text{Tr} [A_\mu(x) A_\nu(x + r)] \right\rangle$$

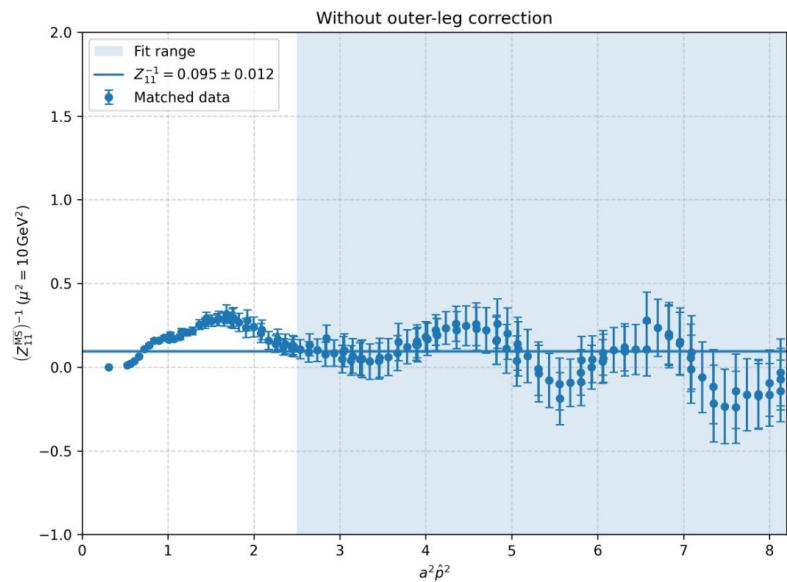
**RI/MOM**  $\rightarrow$   **$\overline{MS}$**

$$\Delta G^{\overline{MS}} = Z^{\text{RI}} \cdot R^{\overline{MS},\text{RI}} \cdot \Delta G^{\text{bare}}$$

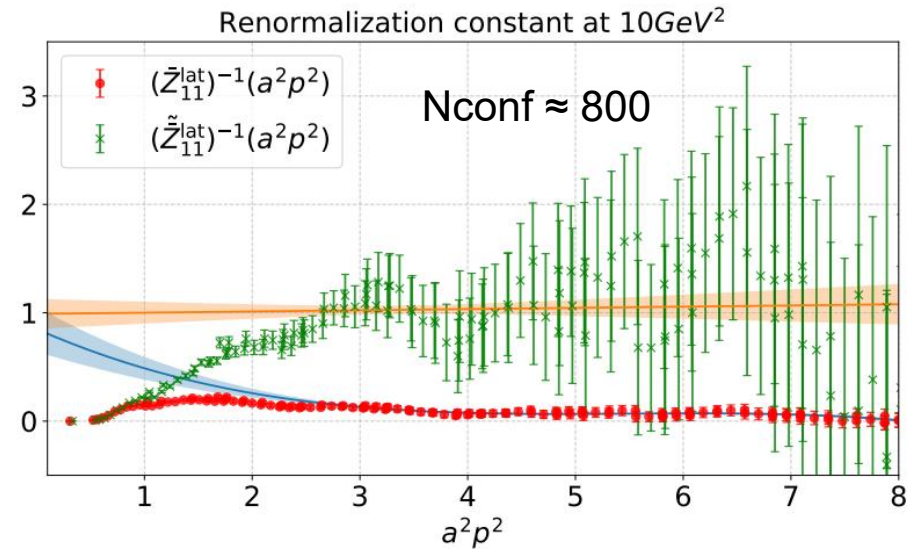
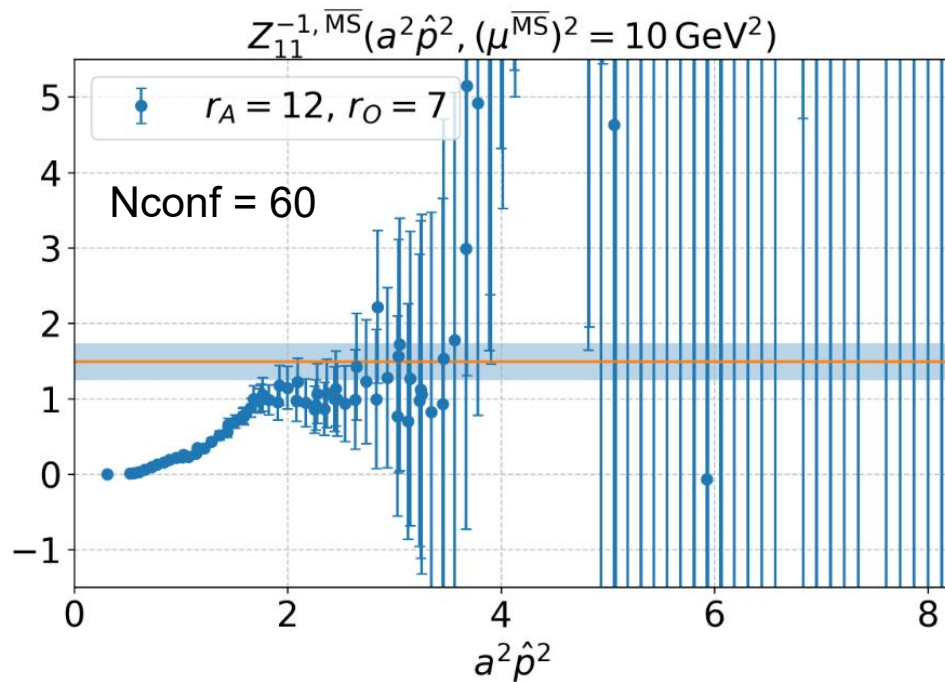
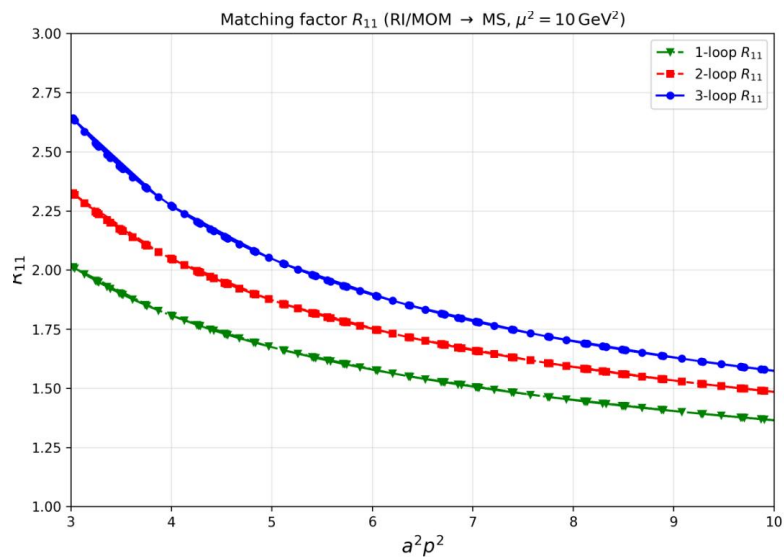


Yi-Bo Yang et al. 10.1103/PhysRevD.98.074506

$$Z_{11}^{-1,RI}(\mu^2) = \sum_{\mu,\rho,\nu,\sigma} \frac{CV\hat{p}^4 G_K^{\mu\rho\nu}(p,p)}{i\epsilon^{\rho\mu\sigma\nu} \hat{p}_\sigma \hat{p}_\mu \hat{p}_\nu S_g^{\mu\nu}(p^2)} \Big|_{\mu^2=p^2}$$



matching kernel :R11



D.-J. Zhao, et al., DOI: [10.48550/arXiv.2512.24315](https://arxiv.org/abs/2512.24315)

Zhao dianjun :  $R = Z^{RI} \cdot R^{\overline{MS},RI} = 1.09(12)(04)$

**C24P29 :  $\Delta G^{\text{bare}}$**

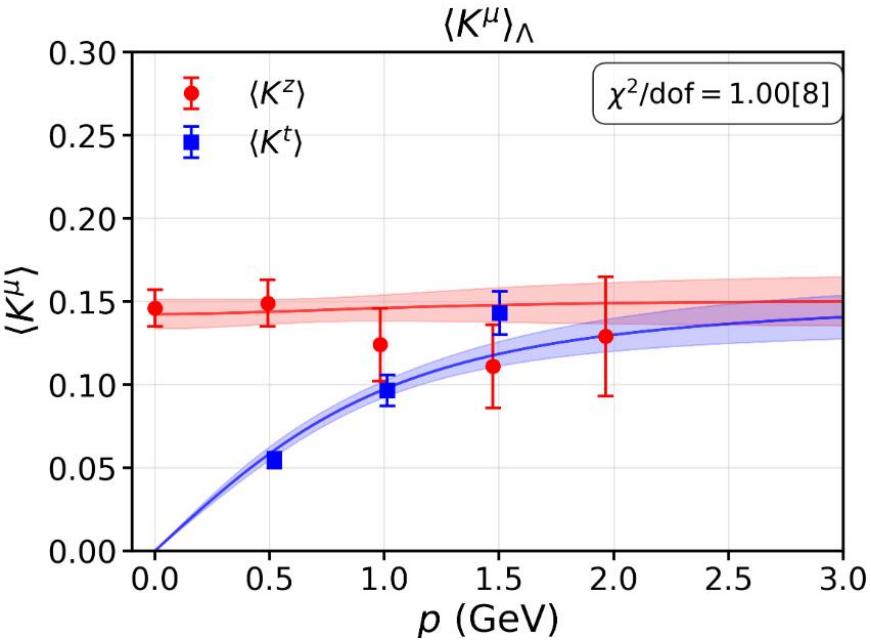
$$\Delta G_i^{\text{bare}} = \Delta G + \frac{C_{h,t}}{m^2 + p^2}$$

$$\Delta G_t^{\text{bare}} = \frac{|\mathbf{p}|}{\sqrt{m^2 + p^2}} \left( \Delta G + \frac{C_{h,t}}{m^2 + p^2} \right)$$

Zhao dianjun :  $R = Z^{RI} \cdot R^{\overline{MS},RI} = 1.09(12)(04)$

$$\Delta G = R * \Delta G^{\text{bare}}$$

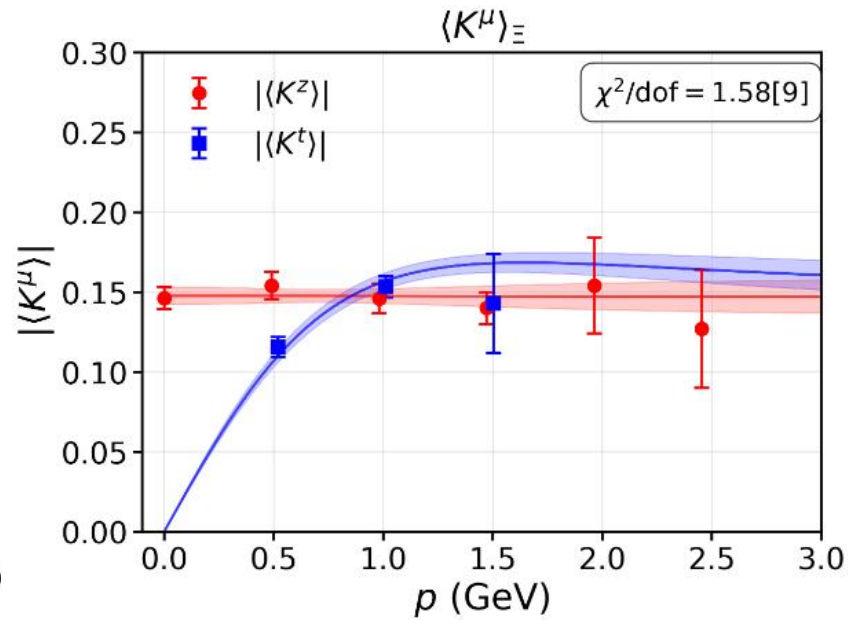
$\Lambda^0(\text{uds})$ :



$$\Delta G^{\text{bare}} = 0.166(18)$$

$$\Delta G_{\Lambda^0} = 0.165(19)\text{stat}(19)\text{ren}$$

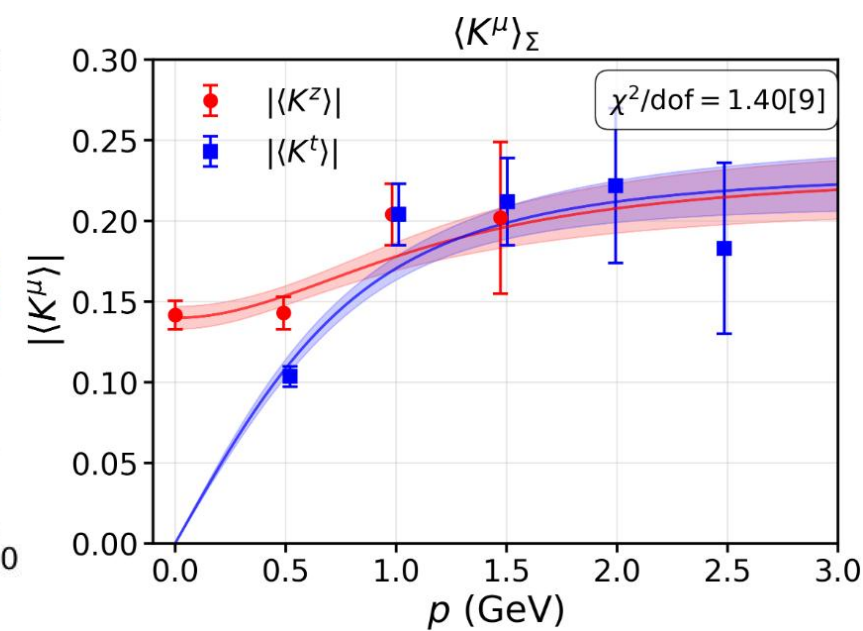
$\Xi^0(\text{uss})$ ,  $\Xi^-(\text{dss})$



$$\Delta G^{\text{bare}} = 0.147(13)$$

$$\Delta G_{\Xi^0, \Xi^-} = 0.160(14)\text{stat}(19)\text{ren}$$

$\Sigma(\text{dds})$ ,  $\Sigma^+(\text{uus})$ ,  $\Sigma^0(\text{uds})$ :



$$\Delta G^{\text{bare}} = 0.232(21)$$

$$\Delta G_{\Sigma, \Sigma^+, \Sigma^0} = 0.253(24)\text{stat}(28)\text{ren}$$

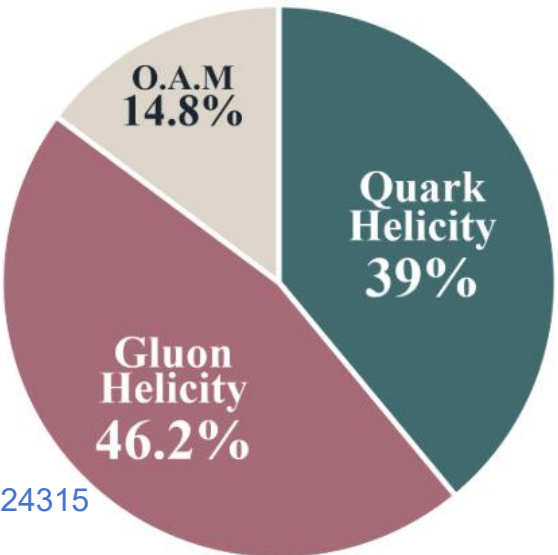
# Results Summary

Quark Helicity

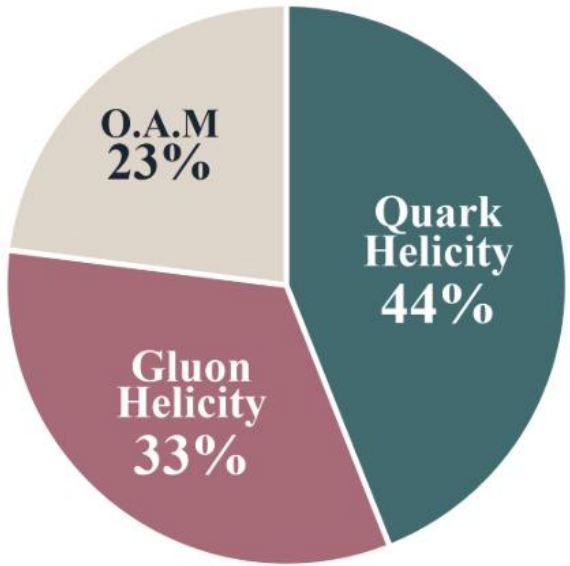
Gluon Helicity

O.A.M

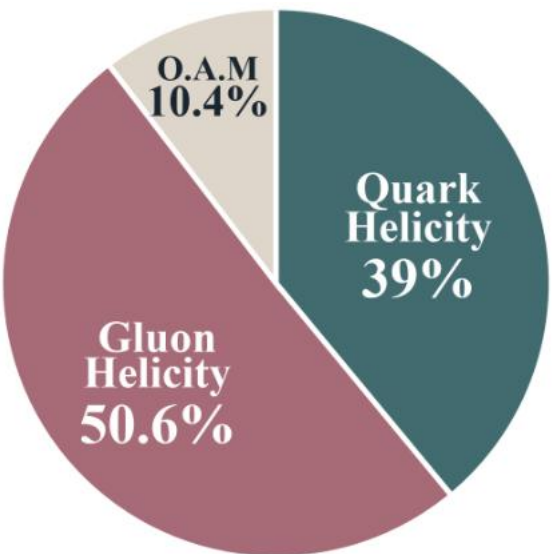
$n(ppu)$  ,  $p(uud)$ :



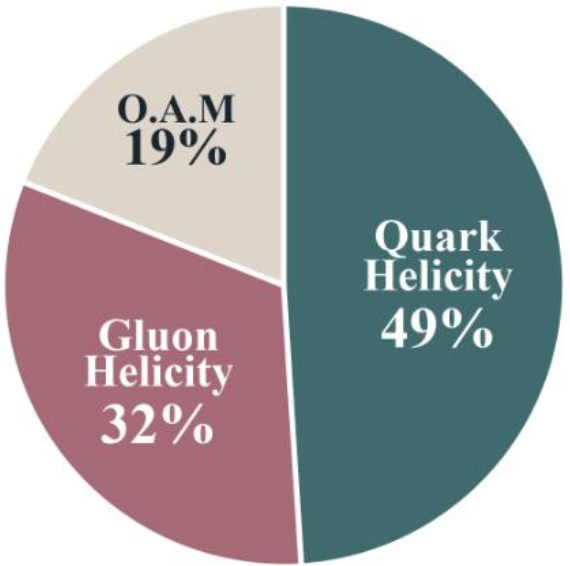
$\Lambda^0(uds)$ :



$\Sigma(dds)$ 、 $\Sigma^+(uus)$ 、 $\Sigma^0(uds)$ :



$\Xi^0(uss)$ 、 $\Xi^-(dss)$ :



D.-J. Zhao, et al., DOI: 10.48550/arXiv.2512.24315

# Summary

1. **Distillation + Momentum smear** to Calculate the **two-point correlation function** and **bare  $\Delta G$  matrix element** in **lancet framework**
2. **CDER**(Cluster decomposition error reduction) to **RI/MOM** renormalization then matching to  $\overline{MS}$
3. Preliminary results are given for a single configuration :

$$\Delta G_{\Lambda^0} = 0.165(19) \quad \Delta G_{\Xi^0, \Xi^-} = 0.160(14) \quad \Delta G_{\Sigma, \Sigma^+, \Sigma^0} = 0.253(24)$$

# Outlook

1. Continuous extrapolation and More rigorous systematic error analysis is needed
2. Exploring the effect of SU(3) flavor symmetry breaking on spin



# THANKS FOR LISTENING!

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