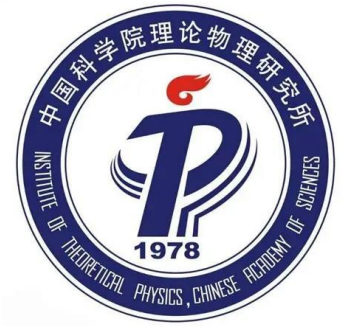
2026.10.10 *Lattice Workshop@Shenzhen*

Control of Excited-State Contamination in Nucleon Matrix Elements Using All-to-All Fermion Propagators

Speaker： Zhi-Cheng Hu(胡志成)



Institute of Theoretical Physics, CAS

Institute of Modern Physics, CAS



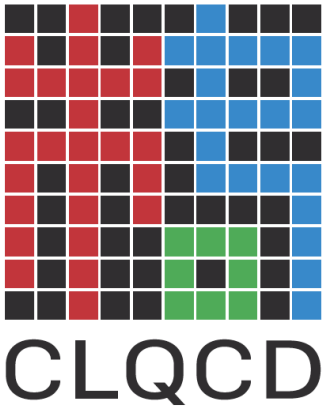
Collaborators: Liuming Liu, Peng Sun, Ji-hao Wang and Yi-Bo Yang

arXiv: 2505.01719

arXiv: 2511.02326

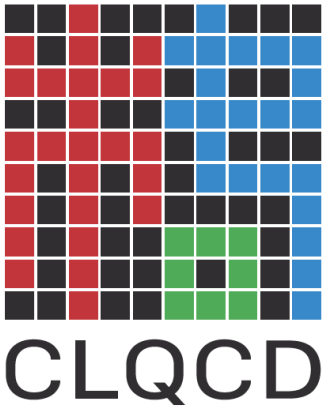
Context

- **Chapter 1: Introduction**
- **Chapter 2: Blending method -- All-to-All fermion propagator**
- **Chapter 3: Effects of $N\pi$ and $N\pi\pi$ to iso-vector charge**
- **Chapter 4: Current enhanced excited state contamination**
- **Chapter 5: Summary**



Context

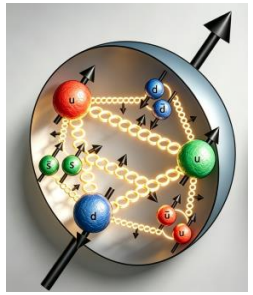
- **Chapter 1: Introduction**
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1. Introduction

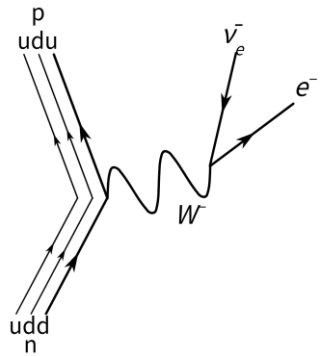
➢ **Hadron matrix elements contain rich information about the hadronic structure;**

- **Proton spin puzzle**

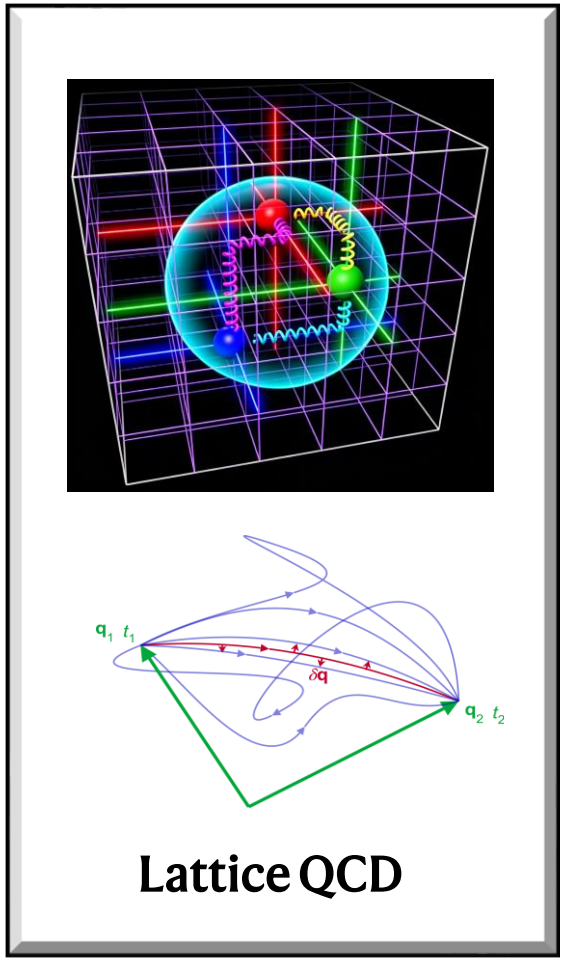


EMC, PLB ***206***,364(1988)
X.D. Ji, PRL ***78***, 610(1997)

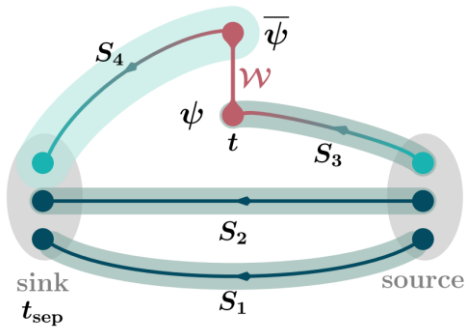
- **Neutron beta decay**



B. Markisch; et.al; PRL. ***122,*** 242501 (2019)
CalLAT, Nature ***558,*** 91(2018);



- **Parton Structure (PDFs)**



X.D. Ji, PRL ***110***, 262002(2013)
LPC, PRL ***131***.261901(2023)

✓ **Lattice QCD** provides **a first-principle, non-perturbative** description of the strong interaction.

K. G. Wilson, PRD. ***10***, 8 (1974);

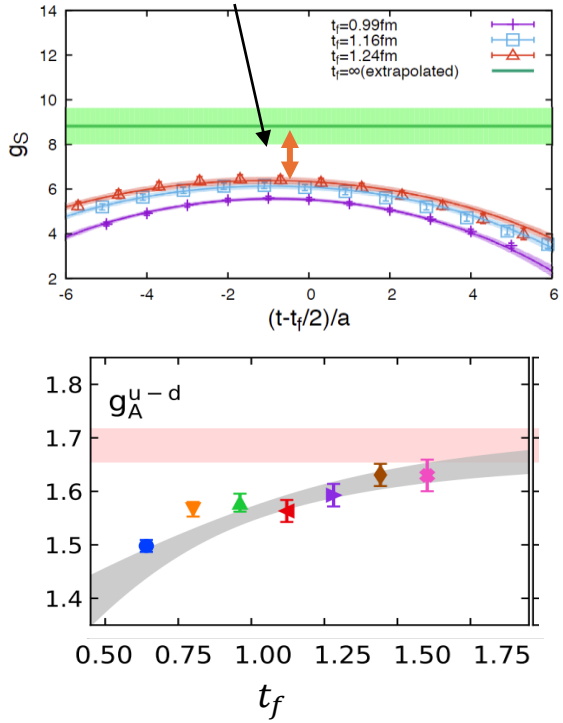
1. Introduction

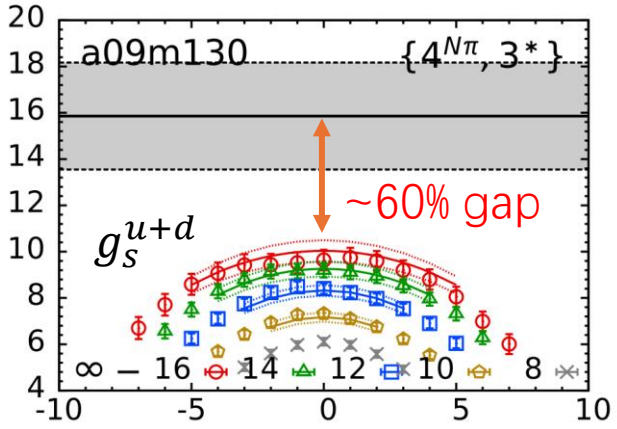
1. **Imaginary-time evolution presents a serious challenge to the signal-to-noise ratio;**

2. **The excited-state contamination of the baryon matrix elements is serious, which will introduce systematic errors;**

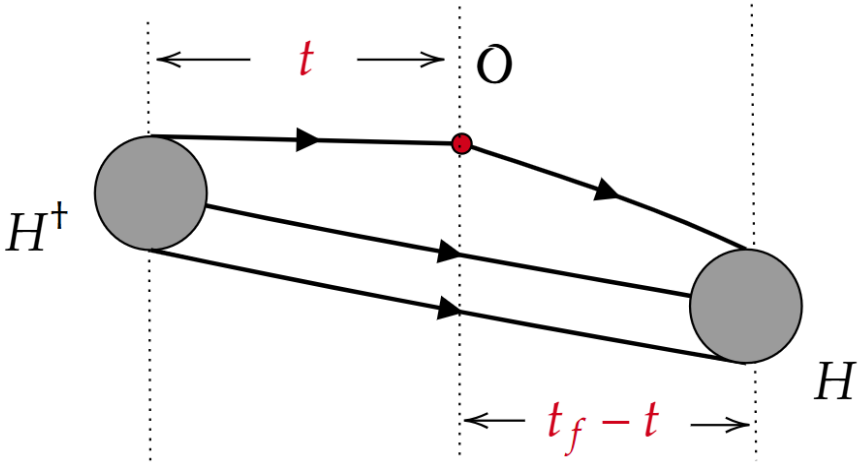
$$\langle H(t_f)\mathcal{O}(t)H^\dagger(t=0)\rangle = \exp(-E_0 t_f)\left(\langle 0|\mathcal{O}|0\rangle + \sum_{i>0}\langle i|\mathcal{O}|i\rangle e^{-\Delta_i t_f} + \sum_{i\neq 0;j\neq 0}\langle i|\mathcal{O}|j\rangle (e^{-\Delta_i(t_f-t)-\Delta_j t} + e^{-\Delta_j(t_f-t)-\Delta_i t})\right)$$

~20% gap





*Y.B. Yang, et al, PRD **94**(2016)5, 054503*
*ETMC, PRD **102**(2020)5, 054517*
*R. Gupta, PRL **127**(2021), 242002*

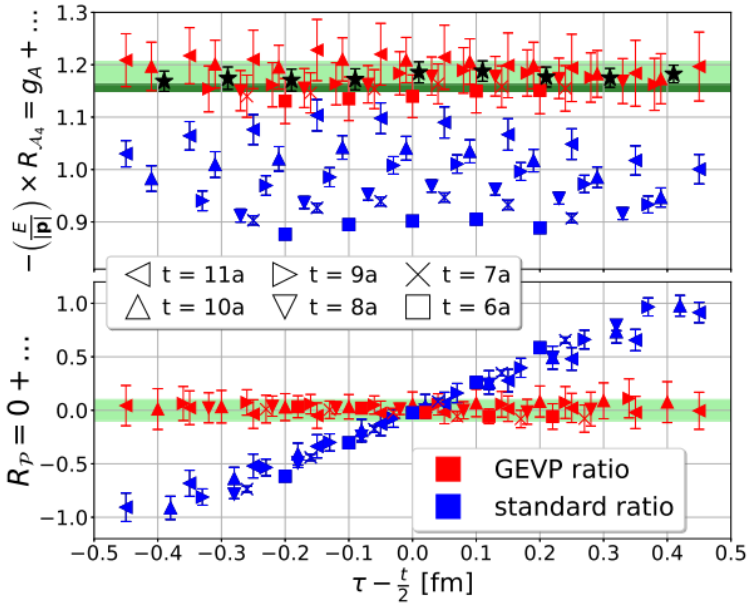


- **Signal-to-noise ratio decays exponentially with t_f;**

1. Introduction

- **Generalized Eigenvalue Problem(GEVP)** can enhance ground-state overlap;

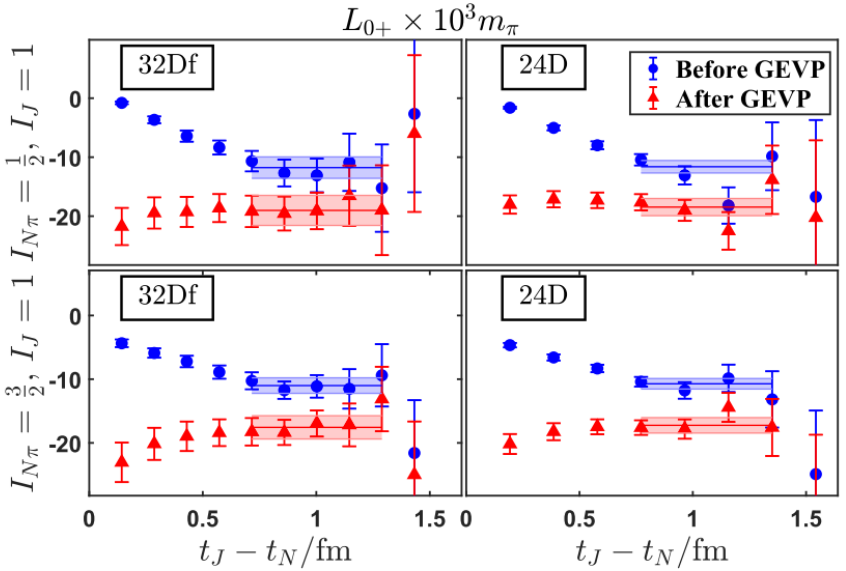
- **Advantage**: Convergent behaviors are improved;



- Inversion $\mathcal{O}(100\sim1000)$ per configuration

L. Barca, PRD 112(2025) 9, L091503

Disadvantage: $\mathcal{O}(100\sim1000)$ Wick contraction; cost is high;



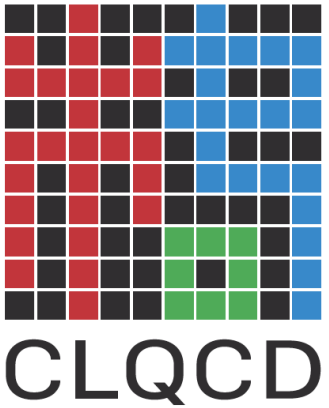
- No extra inversion with **all-to-all propagator (@Sparsening field)**;

*Y. Li, et al; PRD **103**(2021)1, 014514*
*Y.S. Gao, Z. L. Zhang et al; PRL **134**(2025)17, 171904*

✓ **All-to-all propagator** technique makes GEVP-analysis be efficient.

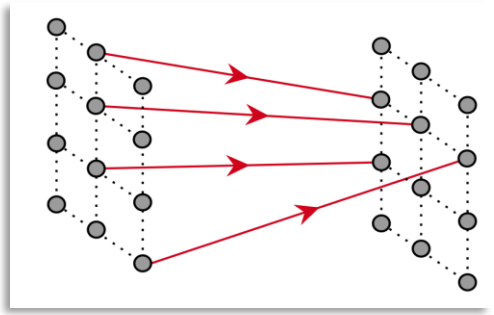
Context

- Chapter 1: Introduction
- **Chapter 2: Blending method -- All-to-All fermion propagator**
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2. Blending method

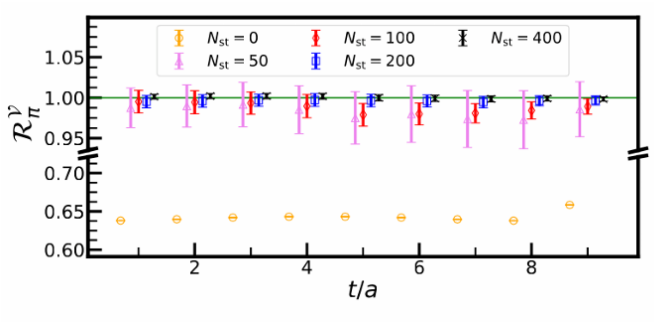
- All-to-all fermion propagator is extremely costly;

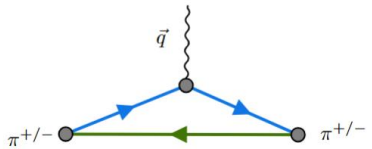


$$S_{ij} = \int \mathrm{d}^3x\mathrm{d}^3y\langle\phi_i(x)|S(x,y)|\phi(y)\rangle$$

- Distillation space $\mathcal{L}_1$ is not complete, resulting in a large bias of hadron matrix element calculation;

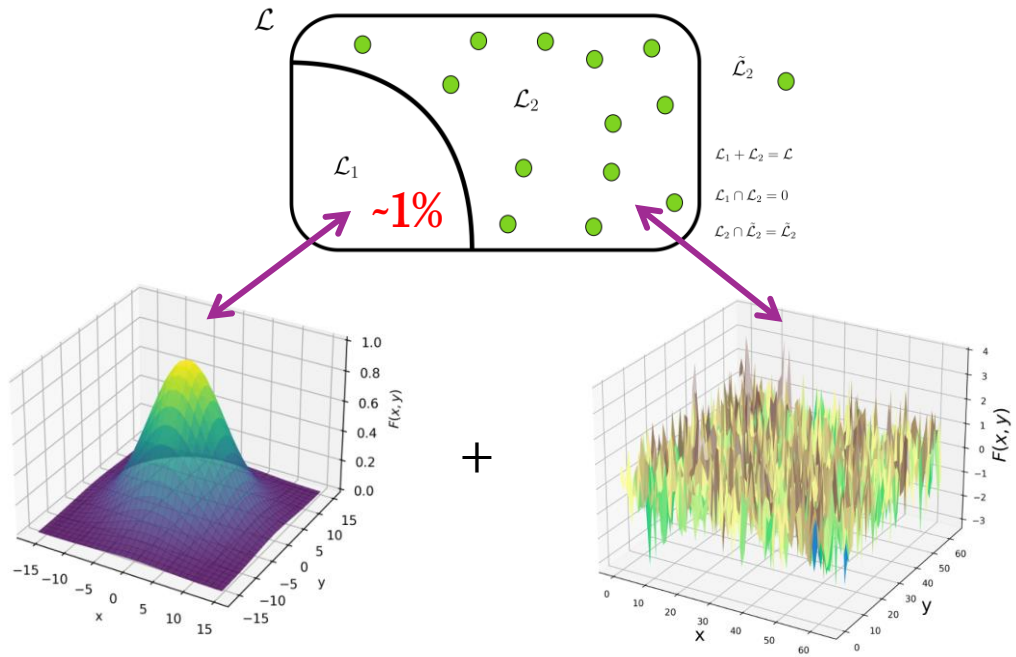
- The "Blending" method can provide an unbiased estimation of the identity matrix, based on samples in $\mathcal{L}_2$;





✓ 3-point function

- We proposed *"blending" method* to reconstruct all-to-all propagator;



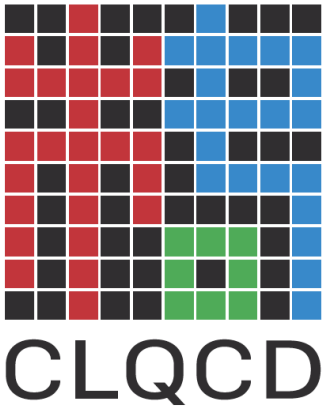
$$S_{ij} = \int \mathrm{d}^3x\mathrm{d}^3y\langle\phi_i(x)|S(x,y)|\phi(y)\rangle$$

Eigenvector: $\mathcal{L}_1$ Noise vector: $\mathcal{L}_2$

Z.C. Hu; Ji-Hao Wang; et. al; arXiv: 2505.01719

Context

- Chapter 1: Introduction
- Chapter 2: Blending method
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3. Effects of $N\pi$ and $N\pi\pi$ to iso-vector charge

- ChPT guides the very low-lying excited-states contamination (<300 MeV) from $N\pi$ and $N\pi\pi$ in 3-pts:

✓ **Pheo.**

- $N\pi$ ESC in N 2-pt and 3-pt(nucleon charge)

  B. Tiburzi, PRD 91 (2015) 094510

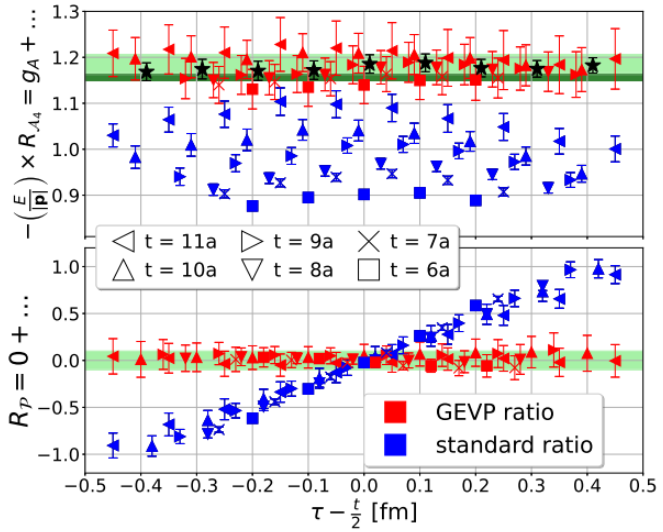
- $N\pi$ ESC in N 3-pt with $J^{\mu} = \bar{q}\gamma_5 q$ can be very large;

  O. Bär, PRD 100 (2019) 5, 054507

- $N\pi$ and $N\pi\pi$ in N 3-pts with $J^{\mu} = \bar{q}\, I\, q$ can be very large;

  *R. Gupta, PRL **127**(2021), 242002*

✓ **Lat.**



$$\langle \mathcal{N}|\bar{q}\gamma^5 q(Q^2)|\mathcal{N}\rangle \qquad \langle \mathcal{N}|\bar{q}\gamma^5\gamma^4 q(Q^2)|\mathcal{N}\rangle$$

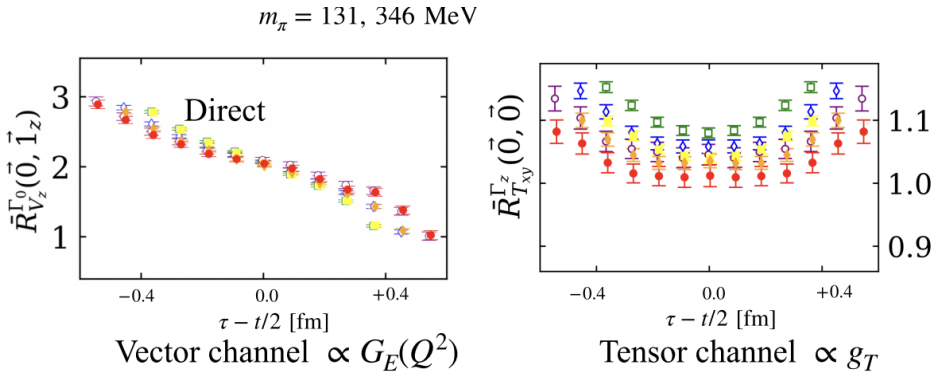
L. Barca, PRD 112(2025) 9, L091503

✓ After GEVP with $N\pi$, the convergence behaviors have been improved significantly for axial vector and pseudoscalar matrix element in LQCD;

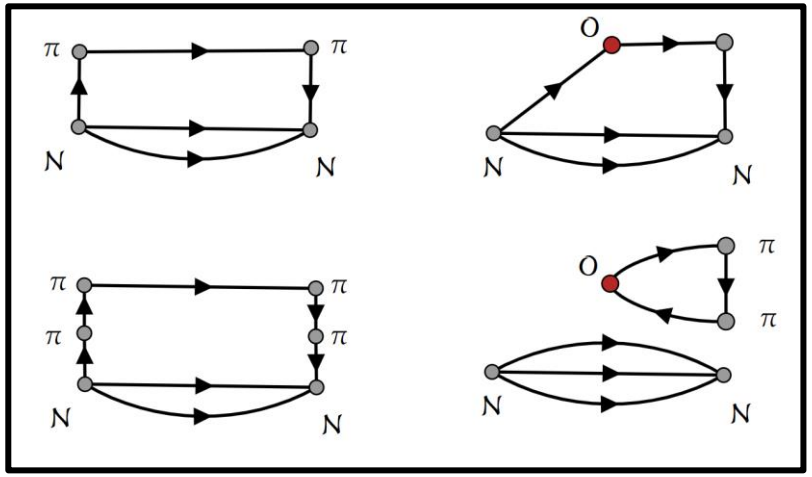
◆ **$N\pi$ is not general !!!**

3. Effects of $N\pi$ and $N\pi\pi$ to iso-vector charge

- $N\pi$ contributions are not necessarily significant for all ME;

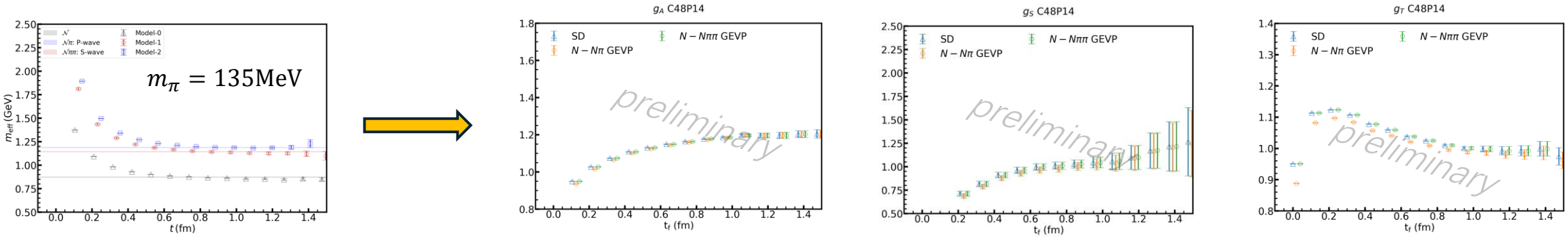


ETMC, *PRD 110 (2024) 9, 094514*



~10^3 wick contraction

- The "Blending" method works well at 2- and 3-body correlators, like $N\pi\pi$;

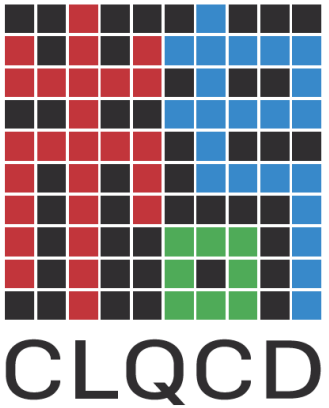


- The influence of $N\pi$ state is about 1%~2% at 1 fm for iso-vector charge， while $N\pi\pi$ less than 1%;

Ji-Hao Wang, Z.C. Hu, CLQCD, *arXiv:2511.02326*

Context

- Chapter 1: Introduction
- Chapter 2: Blending method—All-to-All
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4. Current enhanced Excited-State contaminations

➢ **Current enhanced Excited-State contamination**;

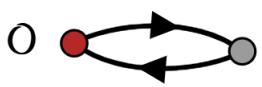
Lorenzo Barcal, PRD 112 (2025) 9, L091503

$$\mathcal{N}^{\text{pure}} = c^{(1)} N^{3q} + \sum_{i}^{2-\text{body}} c_i^{(2)} N^{3q} \chi_i^{2q} + \sum_{jk}^{3-\text{body}} c_{ij}^{(3)} N^{3q} \chi_j^{2q} \chi_k^{2q} + \cdots$$

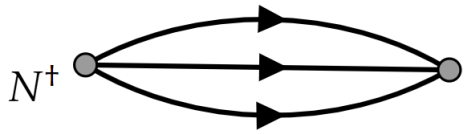
$$C_{ij}^{\text{GEVP}} \approx \begin{array}{c} \\ \text{1-body} \\ \text{2-body} \\ \text{3-body} \end{array} \begin{array}{c} \begin{array}{ccc} \text{1-body} & \text{2-body} & \text{3-body} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & V & V \\ 1 & V & V^2 \end{pmatrix} \end{array} \Rightarrow \begin{aligned} c^{(1)} &\propto 1 \\ c^{(2)} &\propto 1/V \\ c^{(3)} &\propto 1/V^2 \end{aligned}$$

➢ After volume enhancement, nucleon matrix elements get O(1) excited state contamination;

3-pt



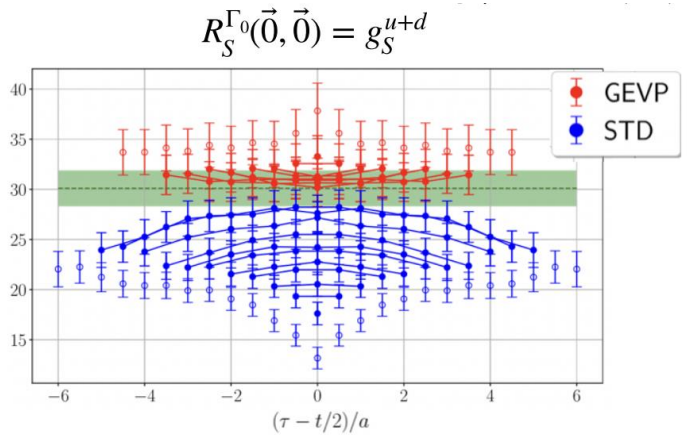
if $\langle \mathcal{O} | \pi, \sigma, \cdots > \neq 0$



$< \mathcal{N} | \mathcal{O} | \mathcal{N}\pi, \mathcal{N}\sigma \cdots > \sim V$

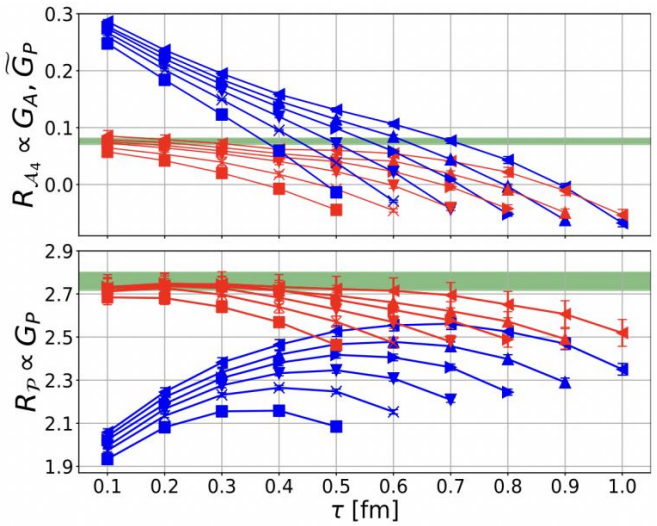
4. Current enhanced Excited-State contaminations

g_s v.s. $N\sigma$



Lorenzo Barcal, PRD 111 (2025) 3, L031505

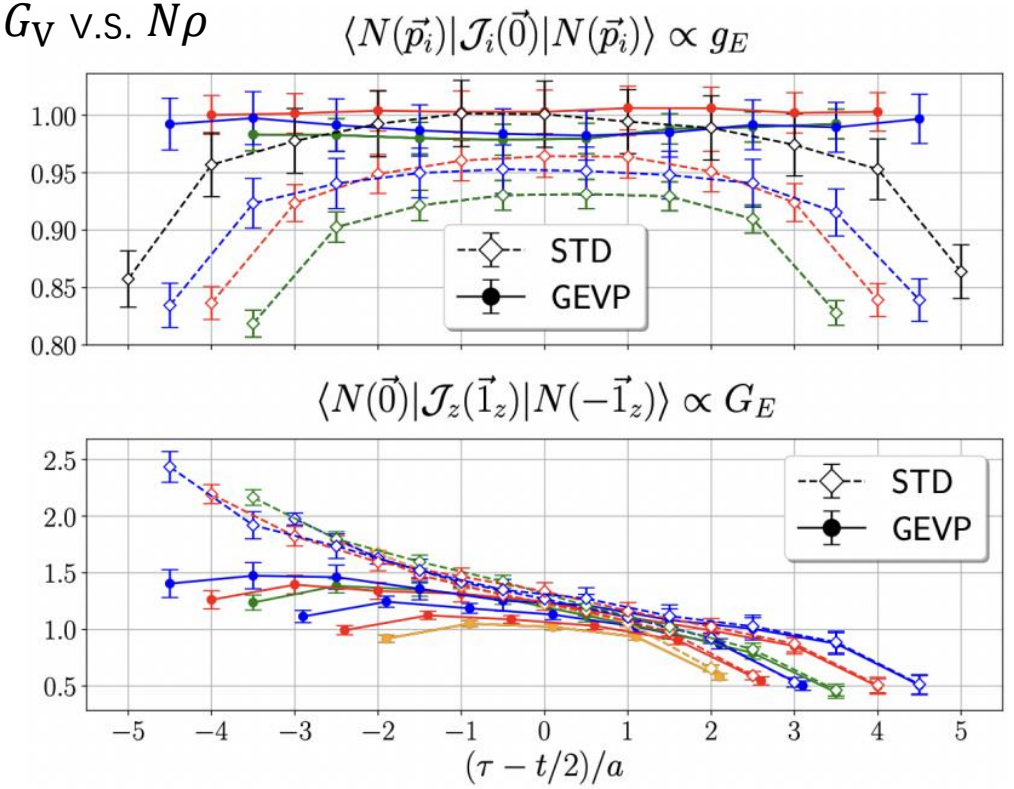
G_p v.s. $N\pi$



Lorenzo Barcal, PRD 107 (2023) 5, L051505

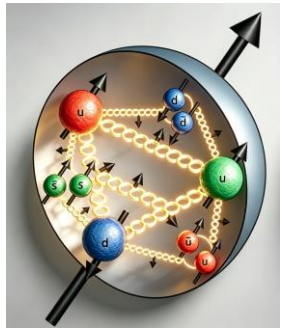
✓ GEVP removes leading order excited contaminations;

G_V v.s. $N\rho$



Lorenzo Barcal, PRD 112 (2025) 9, L091503

4. Current enhanced Excited-State contaminations



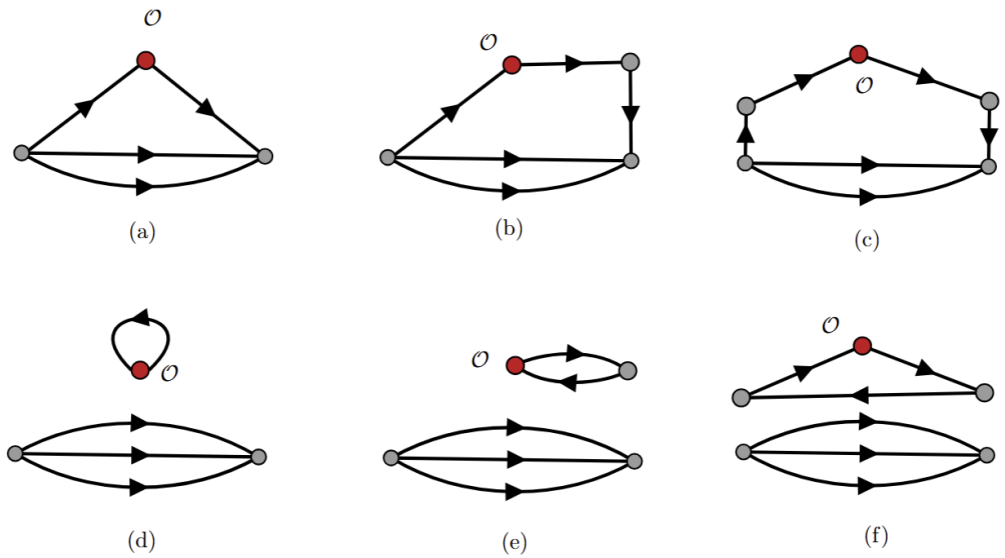
- Proton spin puzzle

$$\frac{\hbar}{2} = \frac{1}{2}\sum_q \Delta q + L_q + J_g$$

*X.D. Ji, PRL **78**, 610(1997)*

➢ Quark spin can be extracted from the local matrix element g_A^{u+d+s};

3x3 GEVP based on the operator basis $(\mathcal{N}, \mathcal{N}\mathrm{f}_1, \mathcal{N}\mathrm{f}'_1)$



✓ The Full GEVP analysis need **$\mathcal{O}(1000)$ Wick Construction**;

✓ The idea of "Current-enhanced excited state contamination" indicates the major excited contamination comes from the 5-quark state;

Barca L., arXiv: 2506.09006

✓ Major excited-state contamination comes from **$\mathcal{N}\mathrm{f}_1$ scattering state** for g_A^{u+d+s};

$\mathrm{f}_1(1285)$ width 23MeV

$$\mathcal{N} = P^+ u(uC\gamma^5 d)$$

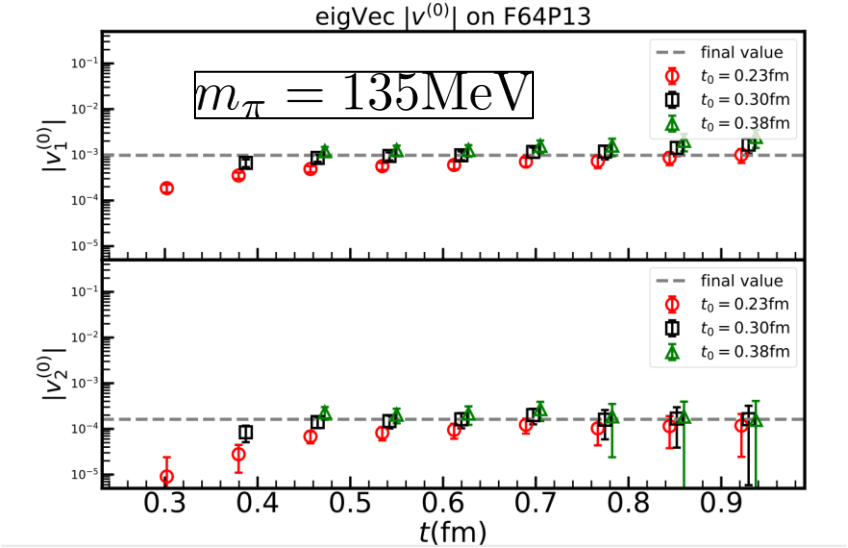
$$f_1 = \frac{1}{\sqrt{3}}\left(\bar{u}\gamma_5\gamma^i u + \bar{d}\gamma_5\gamma^i d + \bar{s}\gamma_5\gamma^i s\right)$$

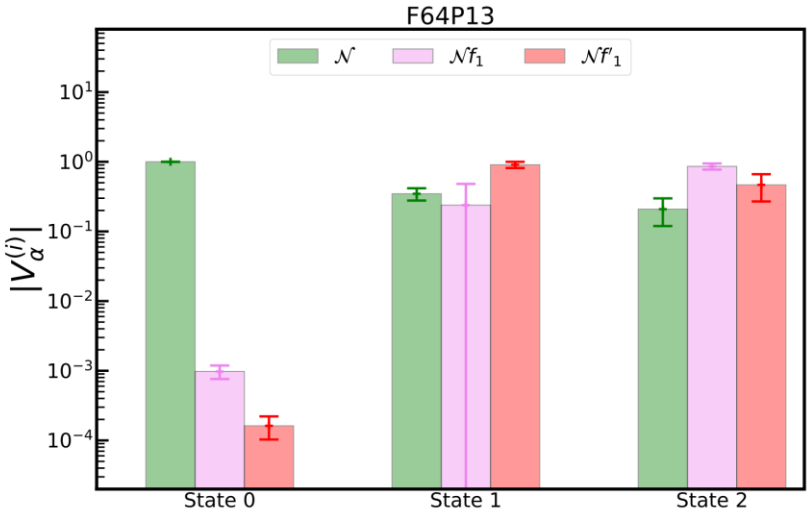
$$f_1' = \frac{1}{\sqrt{6}}\left(\bar{u}\gamma_5\gamma^i u + \bar{d}\gamma_5\gamma^i d - 2\bar{s}\gamma_5\gamma^i s\right)$$

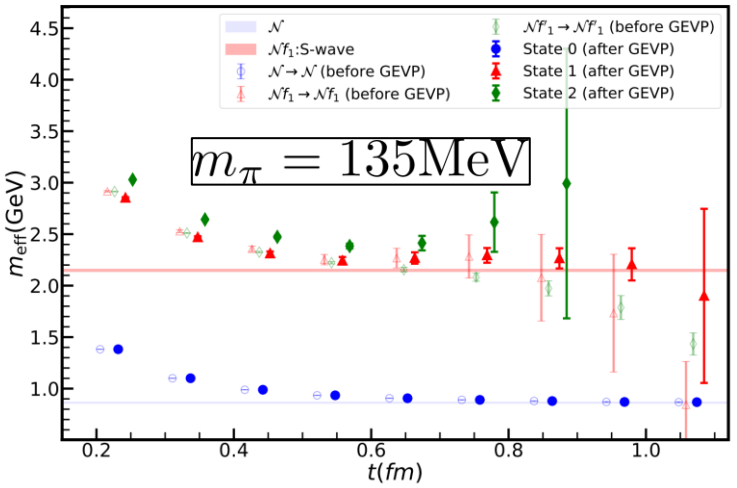
✓ The s-wave scattering of nucleons and $\mathrm{f}_1(1^+)$ mesons can couple to the nucleon $(1/2^+)$ representation.

4. Current enhanced Excited-State contaminations

$$C_{2,\alpha\beta}(t)V^n_\beta(t,t_0) = \lambda^n(t,t_0)C_{2,\alpha\beta}(t_0)V^n_\beta(t,t_0),$$







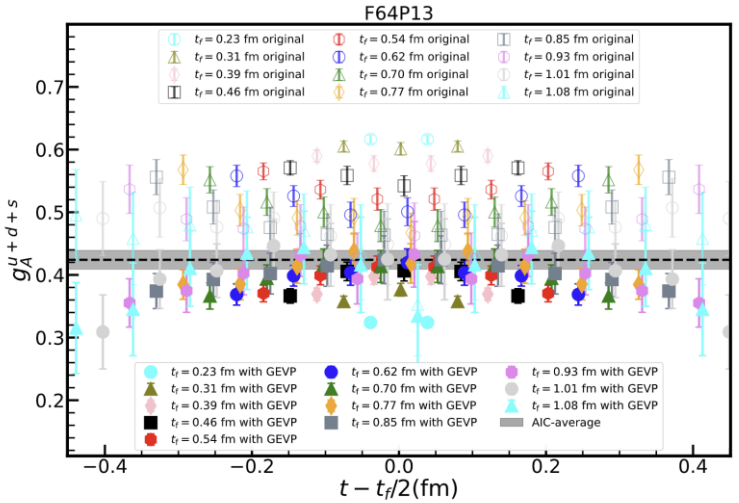
- ✓ GEVP vector can be uniquely determined by solving the GEVP equation and by the imaginary-time evolution;
- ✓ After applying the GEVP, a state with an energy near the no-interaction Nf_1 threshold is obtained;

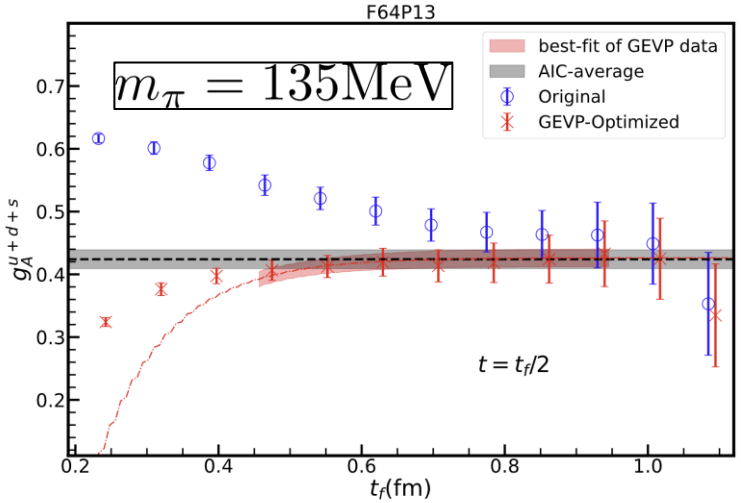
4. Current enhanced Excited-State contaminations

- ✓ Applying GEVP to 3-p function, Convergent behaviors are largely improved after GEVP;

$$C_{3,\alpha\beta}(t_f,t) \equiv \langle X_\alpha(t_f) A_\Sigma(t) X^\dagger_\beta(0)\rangle,$$

$$\tilde{C}_{3,00}(t_f,t) \equiv V^0_\alpha(t,t_0)\, C_{3,\alpha\beta}(t_f,t)\, V^0_\beta(t,t_0),$$





- ✓ Even though the overlap is only about 0.1%, it plays a very effective role in the 3-p process;

- ✓ The result with AIC on fit windows is $g_A^{u+d+s}(a = 0.077\text{fm}) = 0.417(17)$, achieveing world leading precision at physical point directly.

	Ensemble	L	T	a(fm)	m_π	n_{cfg}	g_A^s	g_A^{u+d+s}	Inversion	Inv.to 4% g_A^{u+d+s}
PNDME	a09m310	64	96	0.09	138	847	-0.047(27)	–	8.8 M	–
ETM17	cA2.09.48	48	96	0.09	135	2136	-0.021(5)	0.402(35)	7.2 M	34.1 M
ETM19	cB211.072.64	64	128	0.08	139	750	-0.046(7)	0.382(31)	1.37 M	5.63 M
This Work	F64P13	64	128	0.078	134	46	-0.0409(37)	0.416(17)	0.39 M	0.39 M

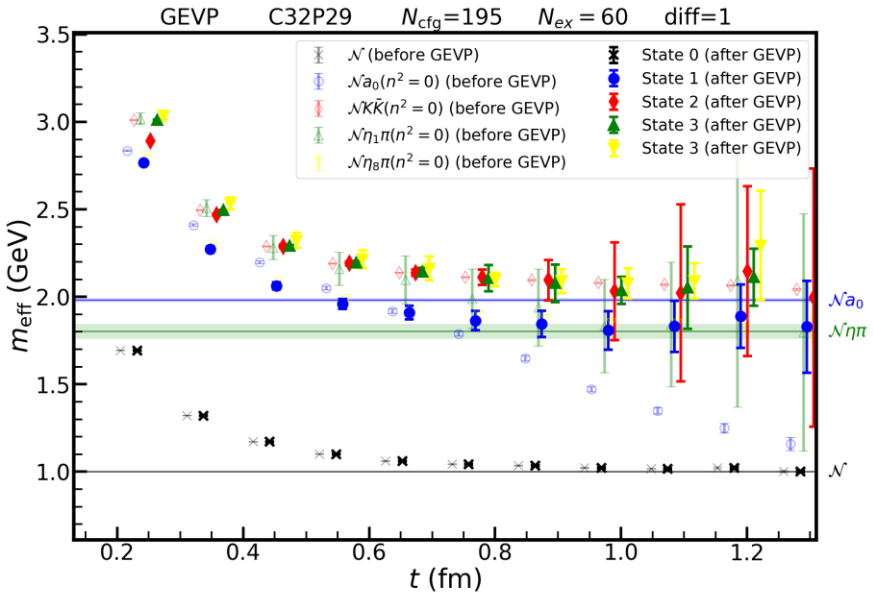
4. Current enhanced Excited-State contaminations

- Meson resonance:

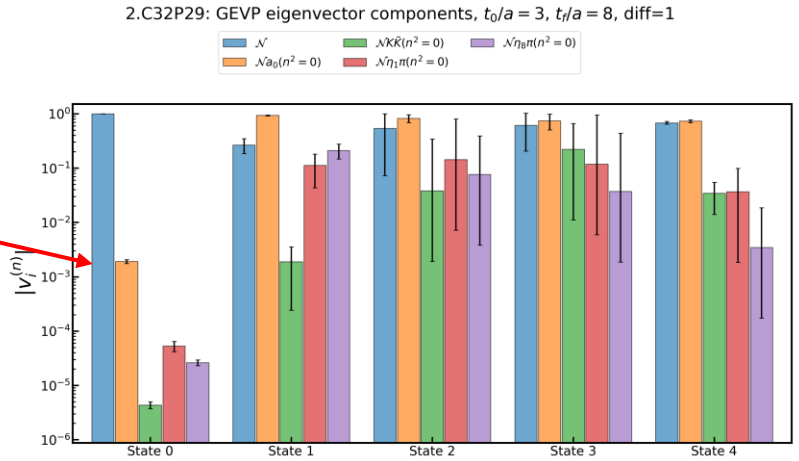
g_A^{u+d+s} -- $f_1 \to K^*K$; $f_1 \to a_0\pi$

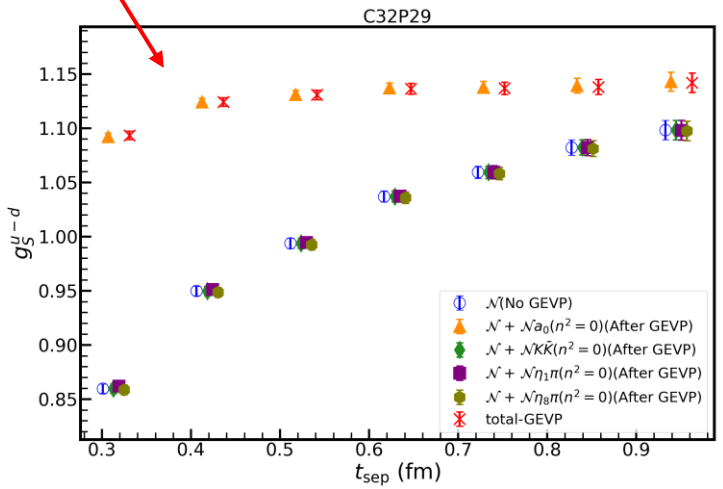
g_S^{u-d} -- $a_0 \to \eta\pi$; $a_0 \to K\bar{K}$

- Spectrum of Na_0 based on $\{N, Na_0, N\eta\pi, N\eta'\pi, N\bar{K}K\}$



- Ground-State: N

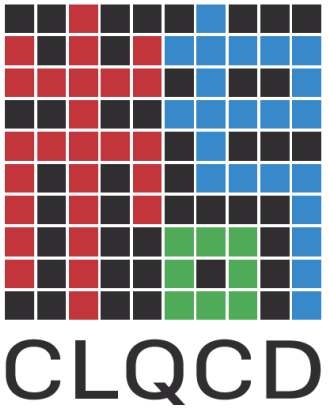




- Multi-hadron state is suppressed by volume;
- Excited-State contaminations are dominated by 2-body operator.

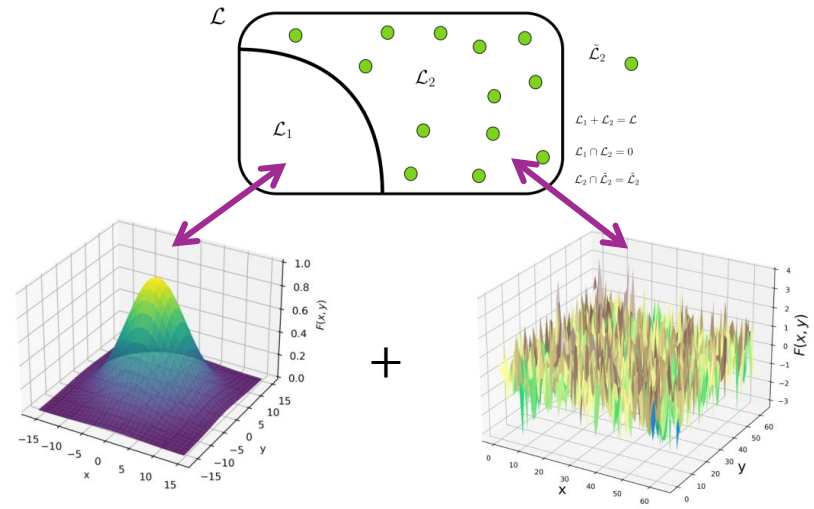
Context

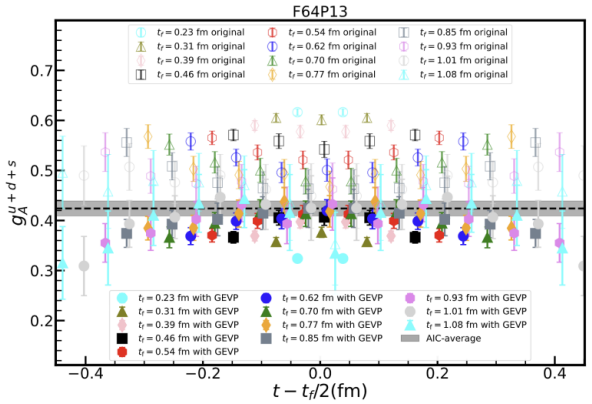
- Chapter 1: Introduction
- Chapter 2: Blending method—All-to-All
- Chapter 3: Effects of $N\pi$ and $N\pi\pi$ to iso-vector charge
- Chapter 4: Current enhanced excited state contamination
- **Chapter 5: Summary**



5. Summary

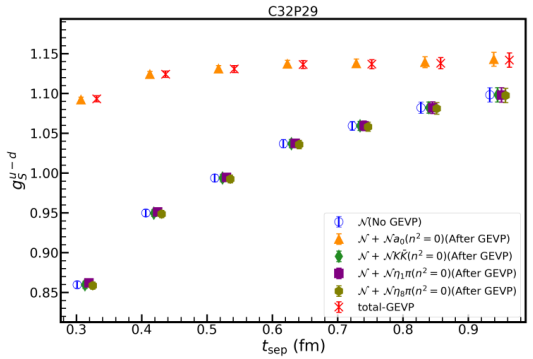
1. We proposed a novel *"Blending"* method to reconstruct the all-to-all propagator;





2. We achieve world leading precision of quark spin $\Delta\Sigma$ based on the "Blending" method and the GEVP analysis.

3. For the first time, We evaluate the effect of 3-body state(including $N\pi\pi$, $N\eta\pi$,$NK\bar{K}$) contamination on nucleon matrix elements.



Thank you!