

# Computing hadron structure with ancilla-free quantum circuits

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collaborate with

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# Quantum computation in High energy physics

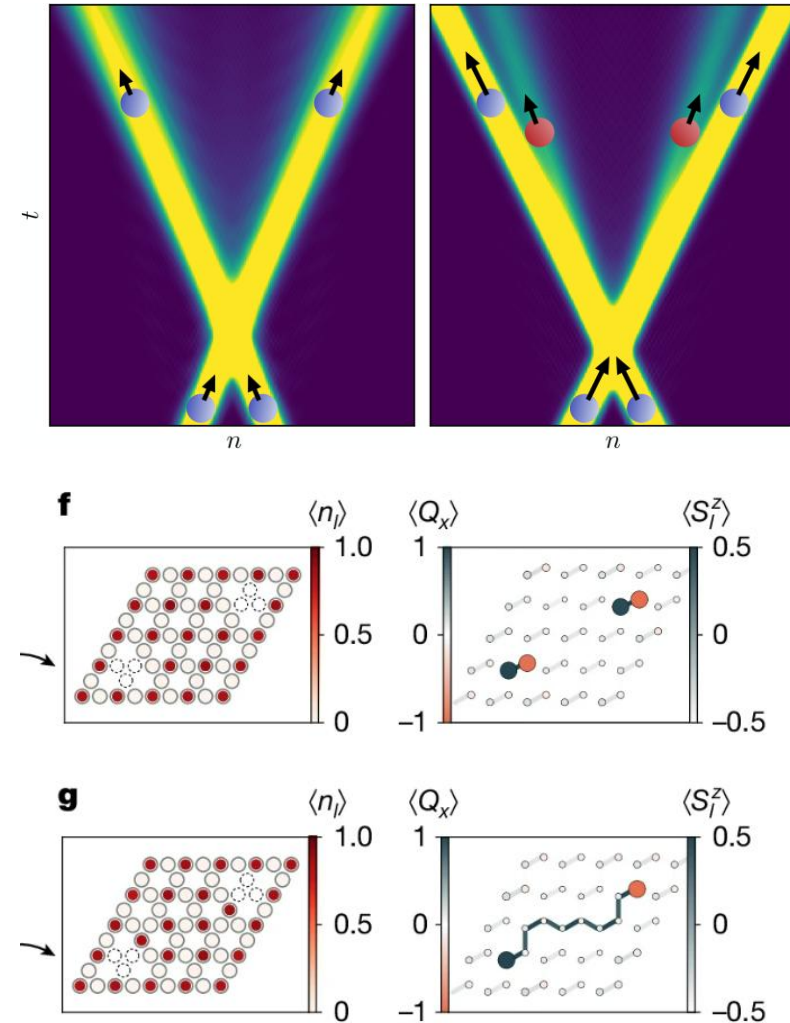
Simulate quantum system with quantum computers

## Quantum advantage

- Exponential dimensions Hilbert space (computation resource reduce)
- Real-time evolution (no sign problem, Minkowski spacetime)

[J. Preskill, PoS (2018)]

- Particle scattering [S. P. Jordan, et al Science(2012).]
- Dynamics, string breaking [D. González-Cuadra et al. Nature(2025)]



# Simulation of quantum field theory

$n$ -point correlation function

$$G(x_1, x_2, \dots, x_n) = \langle \Omega | \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \cdots \mathcal{O}_n(x_n) | \Omega \rangle$$

Classical simulation  
(Lagrangian, imaginary-time)

- Wick rotation
- **Feynman path integral**
- Monte Carlo sampling
- Error analysis

Quantum simulation  
(Hamiltonian, real-time)

*Stephen P. Jordan et al. Science(2012).*

- Initial state preparation (VQE, QAOA)
- **Hamiltonian evolution** (Trotter-Suzuki)
- Measure correlations
- Error mitigation

**Sign problem, Critical slowing down**

$$G = \frac{1}{Z} \int D\phi D\bar{\psi} DU(\dots) e^{-S}$$

$$|\Omega\rangle \rightarrow O_1(x_1) O_2(x_2) |\Omega\rangle \rightarrow |G|^2 = |\langle \Omega | O_1(x_1) O_2(x_2) | \Omega \rangle|^2$$

# Parton distribution function

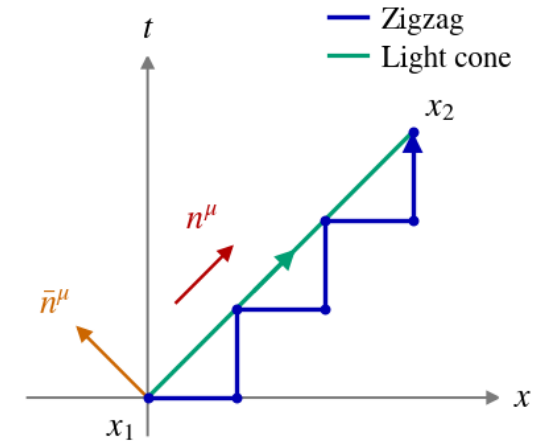
Light-cone correlation function in quark PDF

$$\mathbb{D}(z) = \langle h | e^{iHt} \bar{\psi}(0, -z) e^{-iHt} \gamma^+ W(-z, 0) \psi(0, 0) | h \rangle$$

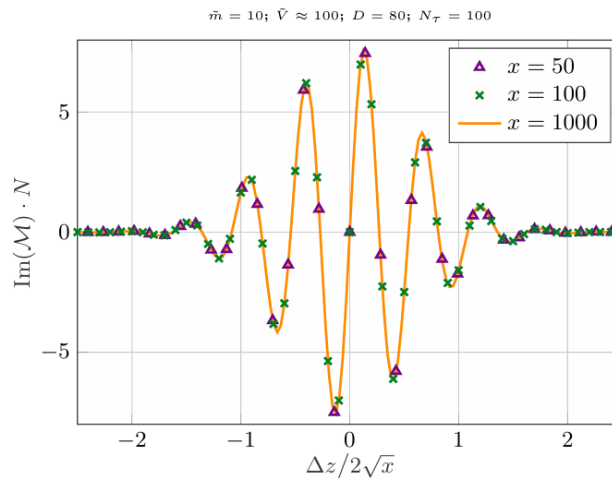


staggered fermion

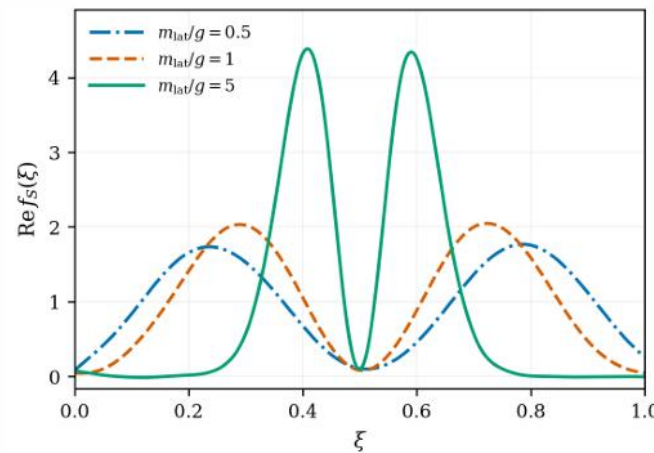
$$D(z) = \langle h | e^{iHt_1} \phi(n_1) \prod_{n=n_1}^{n_2-1} [e^{-iH} W(n)] \phi^\dagger(n_2) e^{-iHt_2} | h \rangle$$



Tensor network

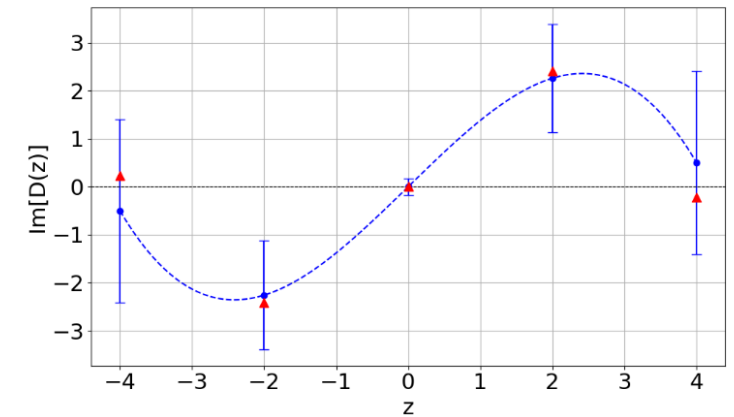


[M. C. Bañuls et al., PRD(2026).]



Yahui Chai, et al. Talk on Oct. 9

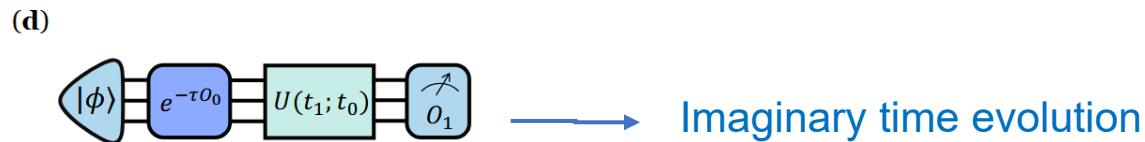
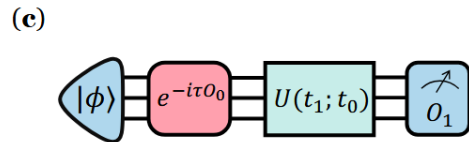
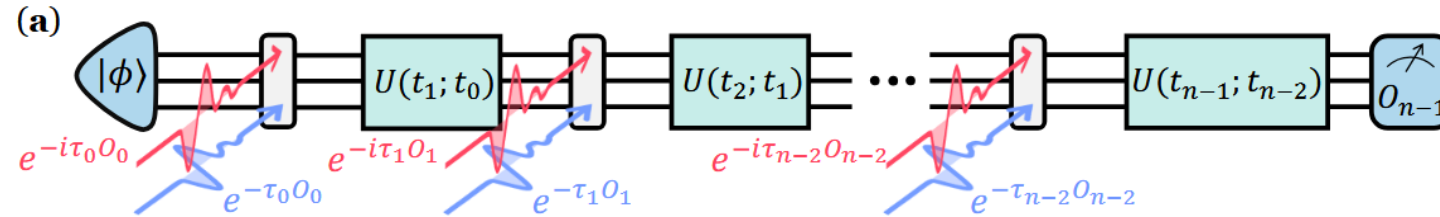
Quantum simulation



[J.-W. Chen, et al.(2025).]

# Ancilla-free algorithm for correlation function

Expectation value method [X. Wang et al. PRL(2025).]



Ancilla free, efficient for NISQ

$$\partial_\tau \langle \phi | e^{i\tau O_0} O_1(t_1) e^{-i\tau O_0} | \phi \rangle \Big|_{\tau=0} = -i \langle \phi | [O_1(t_1), O_0]_- | \phi \rangle$$

$$\partial_\tau \langle \phi | e^{-\tau O_0} O_1(t_1) e^{-\tau O_0} | \phi \rangle \Big|_{\tau=0} = - \langle \phi | [O_1(t_1), O_0]_+ | \phi \rangle$$

- Hard for  $n$ -point function (Circuits  $\sim O(2^n)$ )
- QITE error

$$\langle \phi | [O_1(t_1), O_0]_- | \phi \rangle + \langle \phi | [O_1(t_1), O_0]_+ | \phi \rangle = 2 \langle \phi | O_1(t_1) O_0 | \phi \rangle$$

# Iterative algorithm for correlation function

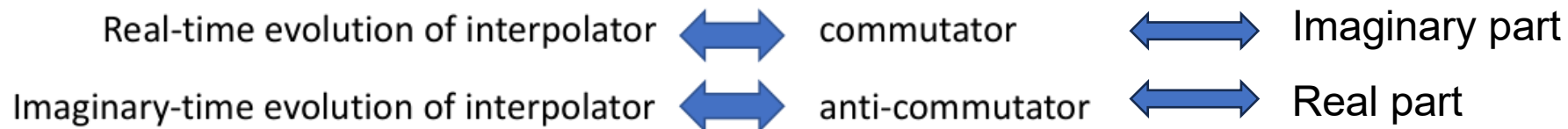
Derive 2-point function from 1-point function

$$C_1 \equiv \langle \psi | \sigma_1(t_1) | \phi \rangle \quad C_2 \equiv \langle \psi | \sigma_1(t_1) \sigma_2(t_2) | \phi \rangle$$

Real and imaginary rotation

$$\begin{aligned} P_{\tau}^{-} &= \langle \phi | e^{-i\tau\sigma_2(t_2)} \sigma_1^{\dagger} | \psi \rangle \times \langle \psi | \sigma_1(t_1) e^{i\tau\sigma_2(t_2)} | \phi \rangle, \\ P_{\tau}^{+} &= \langle \phi | e^{-\tau\sigma_2(t_2)} \sigma_1^{\dagger}(t_1) | \psi \rangle \times \langle \psi | \sigma_1(t_1) e^{-\tau\sigma_2(t_2)} | \phi \rangle. \end{aligned} \quad \longrightarrow \quad \begin{aligned} \left. \frac{\partial P_{\tau}^{-}}{\partial \tau} \right|_{\tau=0} &= -2\text{Im}(C_2 C_1^*), \\ \left. \frac{\partial P_{\tau}^{+}}{\partial \tau} \right|_{\tau=0} &= -2\text{Re}(C_2 C_1^*) \end{aligned}$$

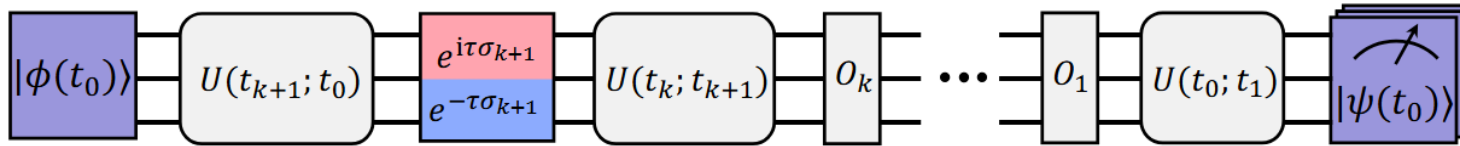
$$C_2 = -i \frac{1}{2C_1^*} \left( \frac{\partial P_{\tau}^{+}}{\partial \tau} + i \frac{\partial P_{\tau}^{-}}{\partial \tau} \right) \Big|_{\tau=0}.$$



# Iterative algorithm for correlation function

Assume  $C_0 = \langle \psi | \phi \rangle$  is already known

$$C_{k+1} = \cos(a_{k+1})C_k - i \frac{\sin(a_{k+1})}{2C_k^*} \left( \frac{\partial P_\tau^+}{\partial \tau} + i \frac{\partial P_\tau^-}{\partial \tau} \right) \Big|_{\tau=0}.$$



$$C_0 = \langle \psi | \phi \rangle \xrightarrow{\text{insert } n \text{ operators}} C_n \equiv \langle \psi | O_1(t_1) O_2(t_2) \dots O_n(t_n) | \phi \rangle.$$

Statistical error  $\mathbb{E}|\hat{C}_n - C_n|^2 \leq \frac{4}{M_{\text{amp}}} + \frac{64p_n n}{M p_{\min}} + 2\delta_T^2$

Sampling complexity  $N_{\text{sample}} \sim O\left(\frac{n^2}{\epsilon_C^2 p_{\min}}\right) \longrightarrow \text{For PDF with gauge-link } n \sim O(L), \text{ sampling } N \sim O(L^2)$

- Ancilla free (efficient for limited connectivity quantum computer)
- Imaginary time evolution free (Parameter-shift rule & mid-circuit measurement, no QITE error)
- Efficient for  $n$ -point function (number of circuits  $2^n \rightarrow n^2$ ) **Scalable**

# Schwinger model on lattice

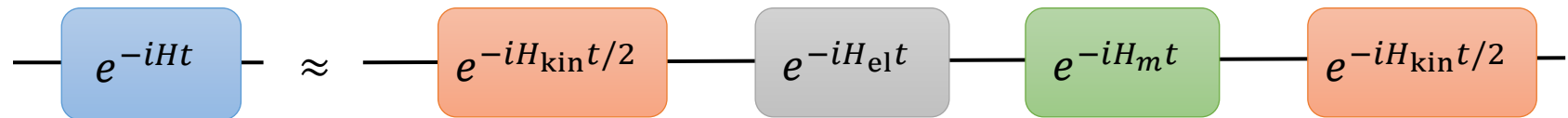
Schwinger model on 2L staggered lattices

$$\hat{H} = \hat{H}_m + \hat{H}_{kin} + \hat{H}_{el} = \underbrace{\frac{m}{2} \sum_{j=0}^{2L-1} \left[ (-1)^j \hat{Z}_j + \hat{I} \right]}_{\text{mass term}} + \underbrace{\frac{1}{2} \sum_{j=0}^{2L-2} (\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{h.c.})}_{\text{kinetic term}} + \frac{g^2}{2} \sum_{j=0}^{2L-2} \left( \sum_{k \leq j} \hat{Q}_k \right)^2$$

electric term

diagonalizable

Trotter-Suzuki



Initial states

Hadron state  
 $P = C = -1$

Vacuum

Approximated in weak coupling

$$|P\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^N |0\rangle^{\otimes(i-1)} |1\rangle |0\rangle^{\otimes(N-i)}$$

$$|\Omega\rangle = \overbrace{|0101 \cdots 01\rangle}^N$$

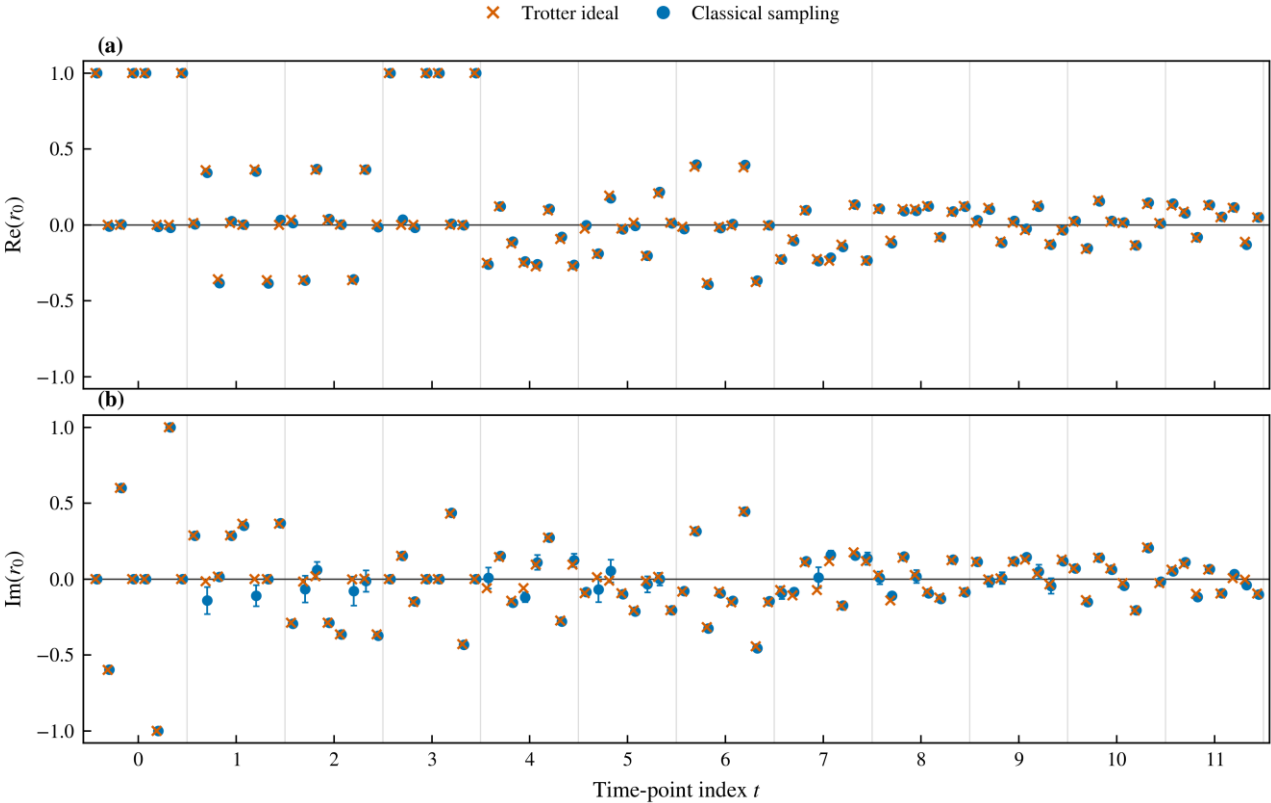
VQE

$$|\psi_{lk}\rangle = U_{kl} |\psi_{lk}\rangle_{\text{ref}}$$

$$U(\theta) = \prod_{i=1}^p \prod_{j=1}^n \exp(i\theta_{ij} H_j)$$

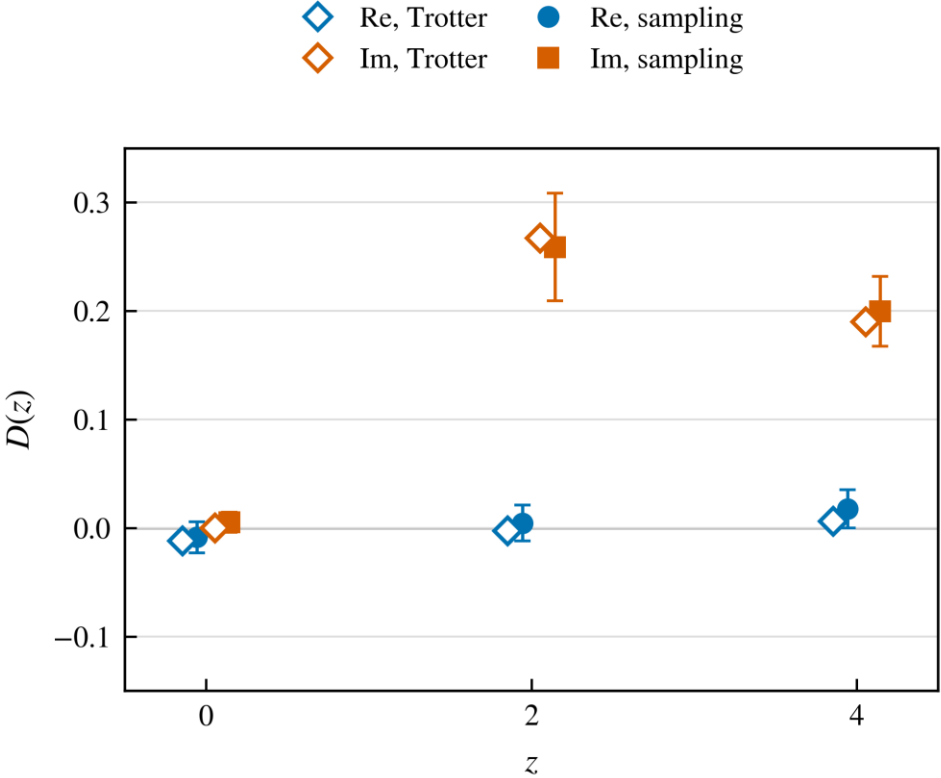
# Classical simulation

$$D(z) = \langle \psi | e^{iHt_1} \phi(n_1) \prod_{n=n_1}^{n_2-1} [e^{-iH} W(n)] \phi^\dagger(n_2) e^{-iHt_2} | \psi \rangle$$

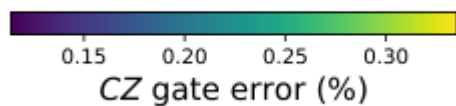
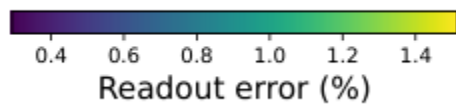
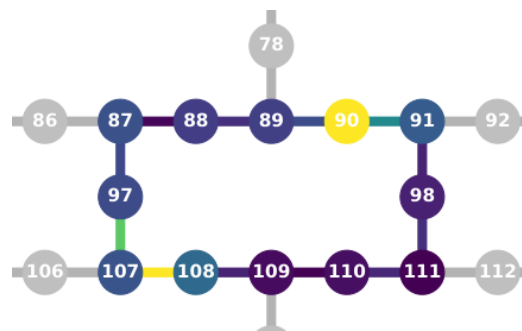
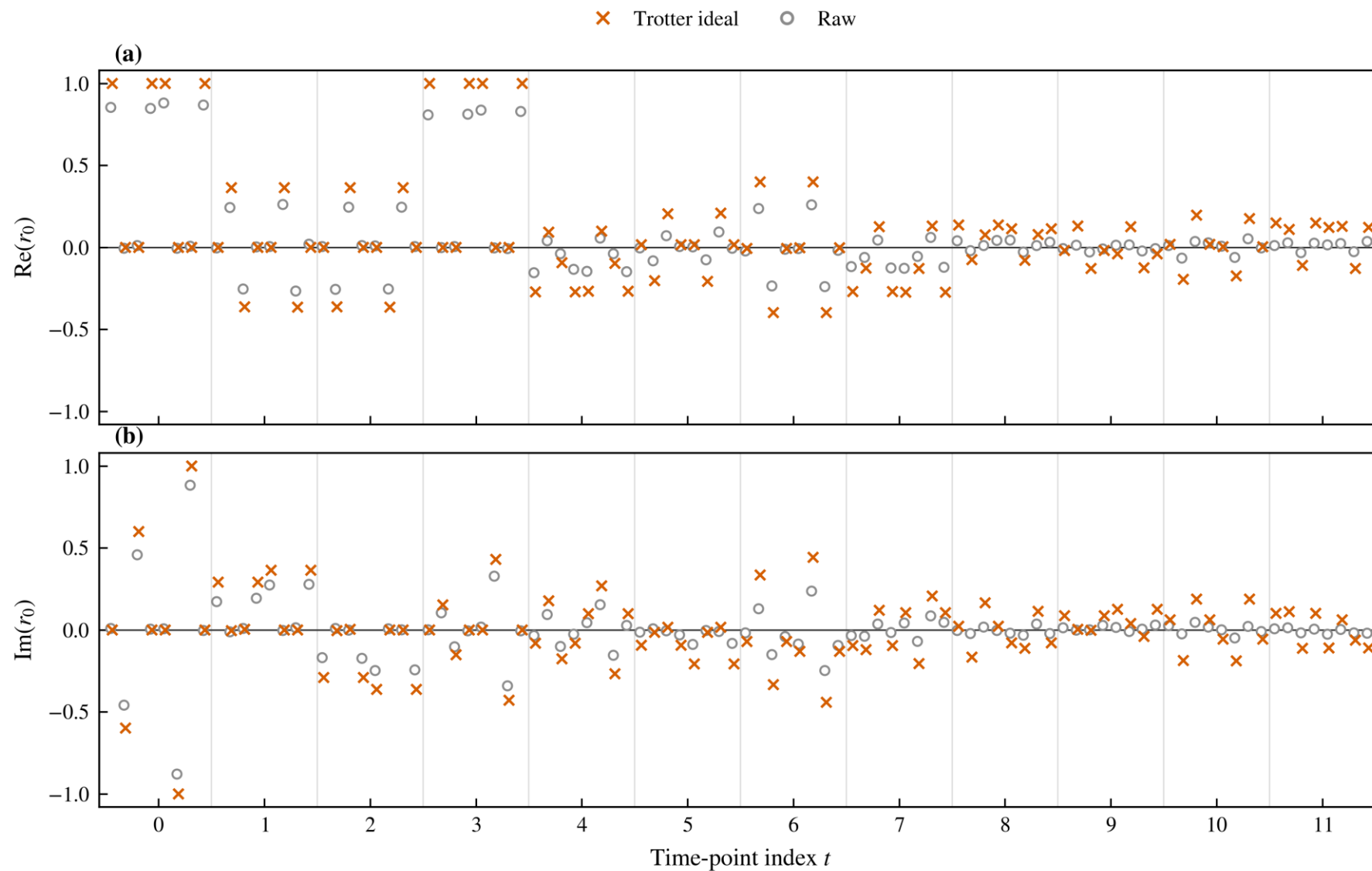


Qiskit sampling with shots=5000

| z            | 0   | 1   | 2   |
|--------------|-----|-----|-----|
| 2-gate depth | 132 | 234 | 334 |



# Quantum simulation

12-qubits with circular topology  
on ibm\_pittsburgh

# Error mitigation

Quantum circuits are noisy, signal decays with the growth of circuits depth

## ODR & CDR

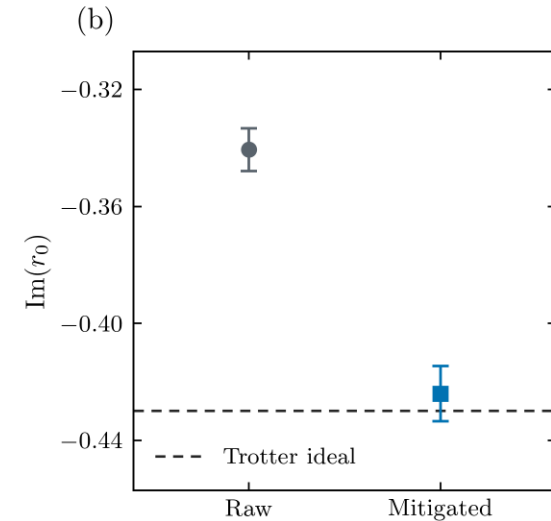
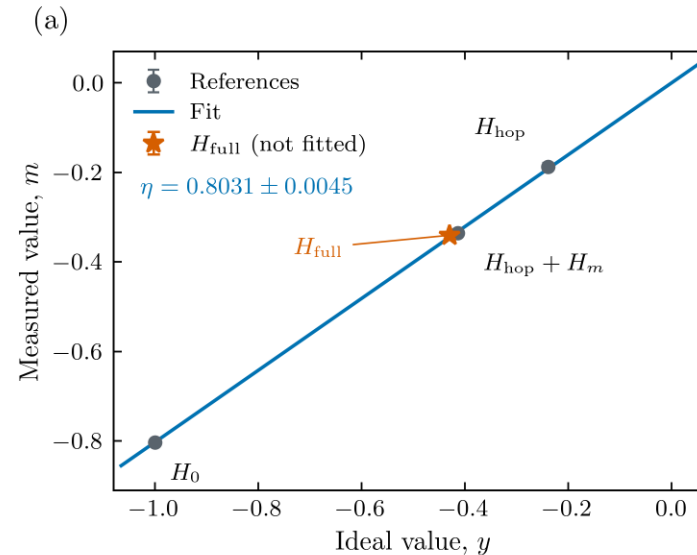
Depolarizing channel noise [\[M. Urbanek et al., PRL\(2021\).\]](#)

$$\overline{\langle O \rangle} = (1 - p)\langle O \rangle + \frac{p}{2^n} \text{tr}(O).$$

- Free Hamiltonian evolution is easy to simulate classically
- Free Hamiltonian evolution has a circuit structure similar to that of the full Hamiltonian

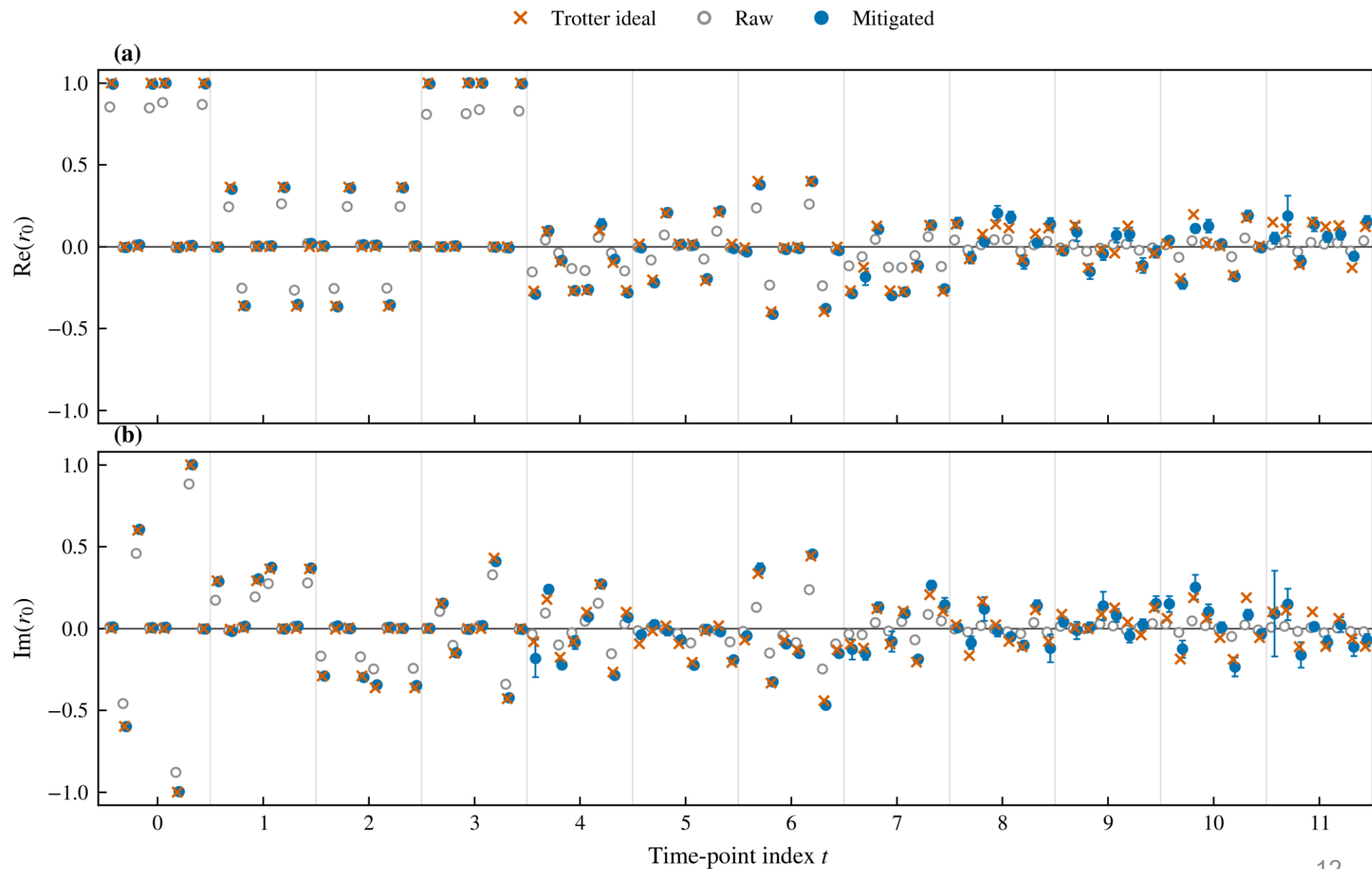
free Hamiltonian + few Clifford gates

Linearly fit the attenuation factor  $p$

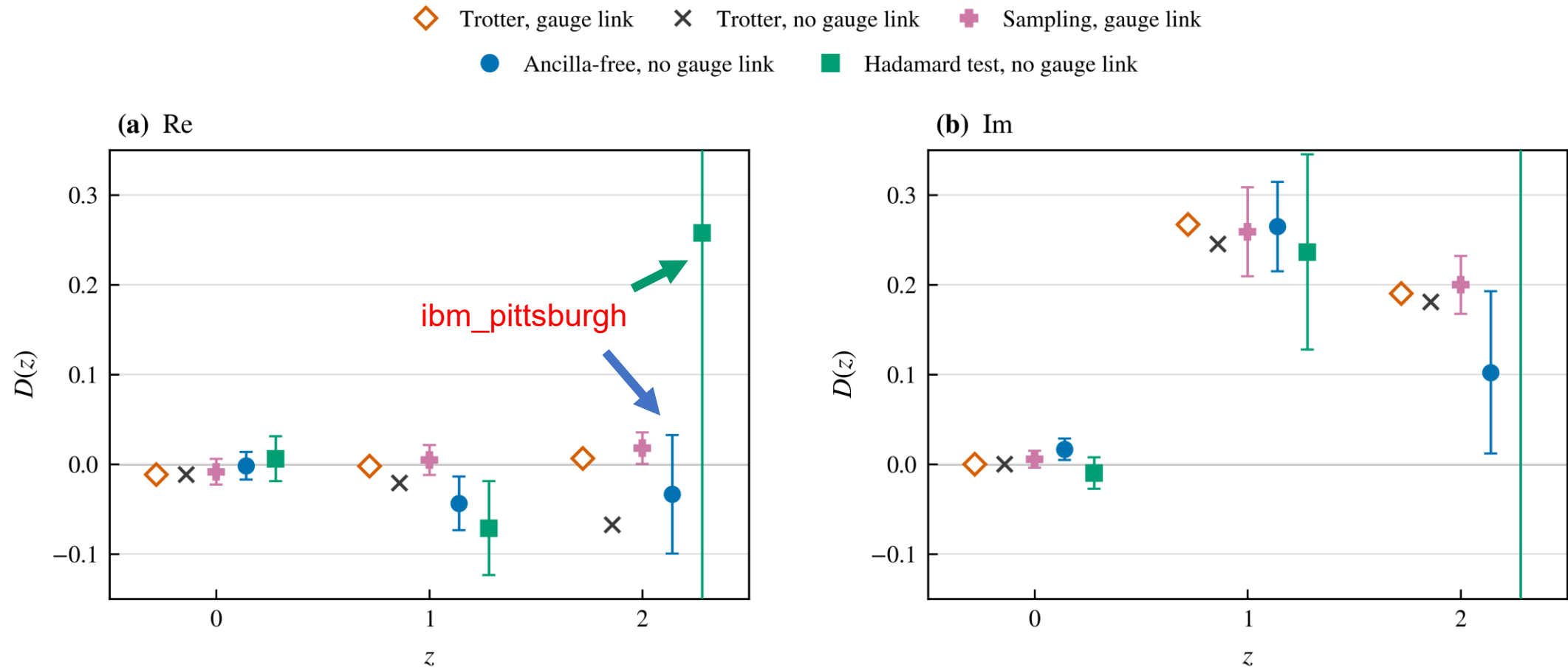


# Error mitigation

Before the signals decay and are obscured by noise, our method can recover most of them to the ideal value



# Results on quantum computer

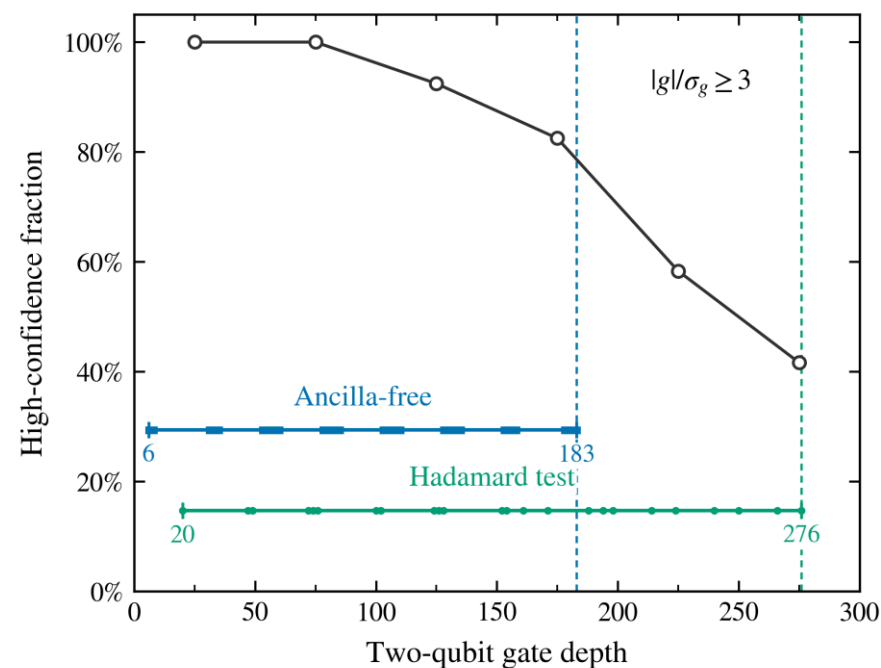
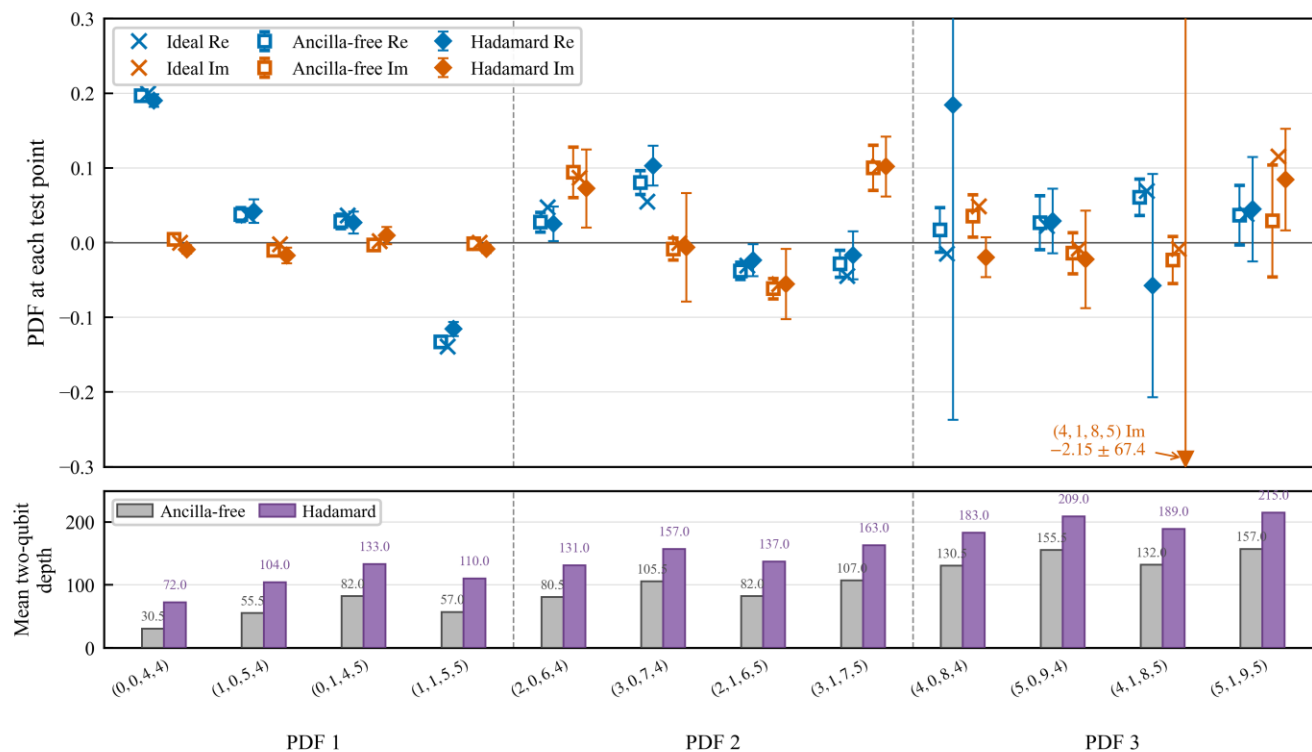


- For 2 operators, linear connectivity. Hadamard test – Ancilla free  $\approx 55$  layers (in 2-depth)
- Error mitigation consist with noiseless results within the error of statistics

# Hardware implementation

To contract effective signal on quantum computer, we simplify the circuits and reduce the circuit depths

- Signals decay with the growth of circuit depth.
- Current IBM quantum chips can only execute circuits with 2-depth<250, or the signals are buried in noise



# Summary & Outlook

- The ancilla-free algorithm **reduce** the number of gates and depth on NISQ hardware, making it **efficient and applicable** on practical quantum computer
- Using the ancilla-free algorithm, the quantum algorithm derives reasonable results.
- In the next few years, we may
  1. add gauge link (2-depth  $\sim 400$ )
  2. simulate (2+1), (3+1)-dimensional model and develop more efficient algorithms
  3. simulate QCD hadron structure and Standard model

## IBM roadmap

