

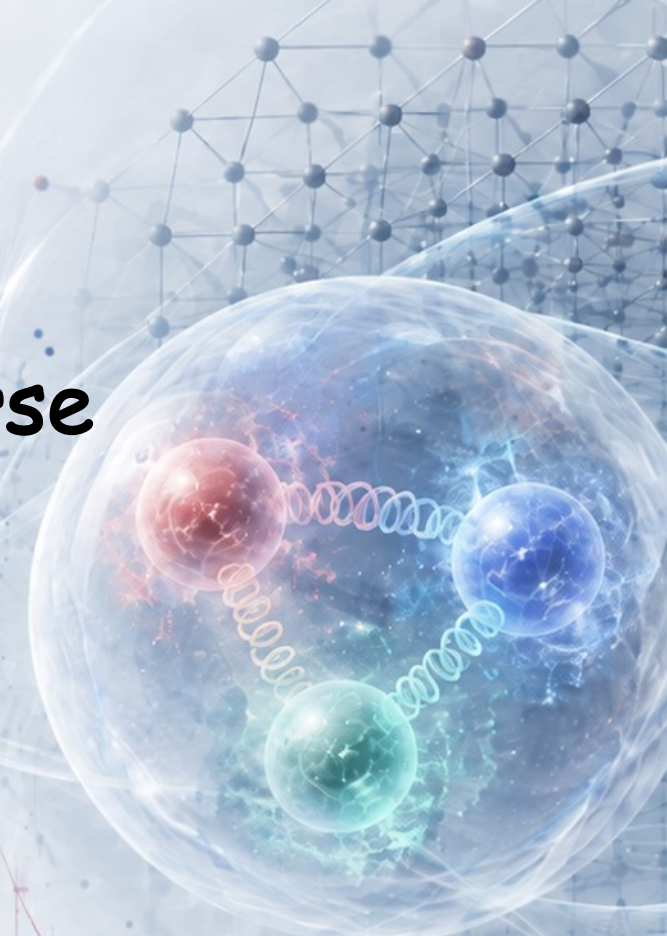


Lattice Parton
Collaboration

From Spin Structure to Transverse Momentum Distributions: A Lattice View of the Nucleon

Jin-Xin Tan (SJTU)

CLQCD2026, Oct.10,2026 @ Shenzhen

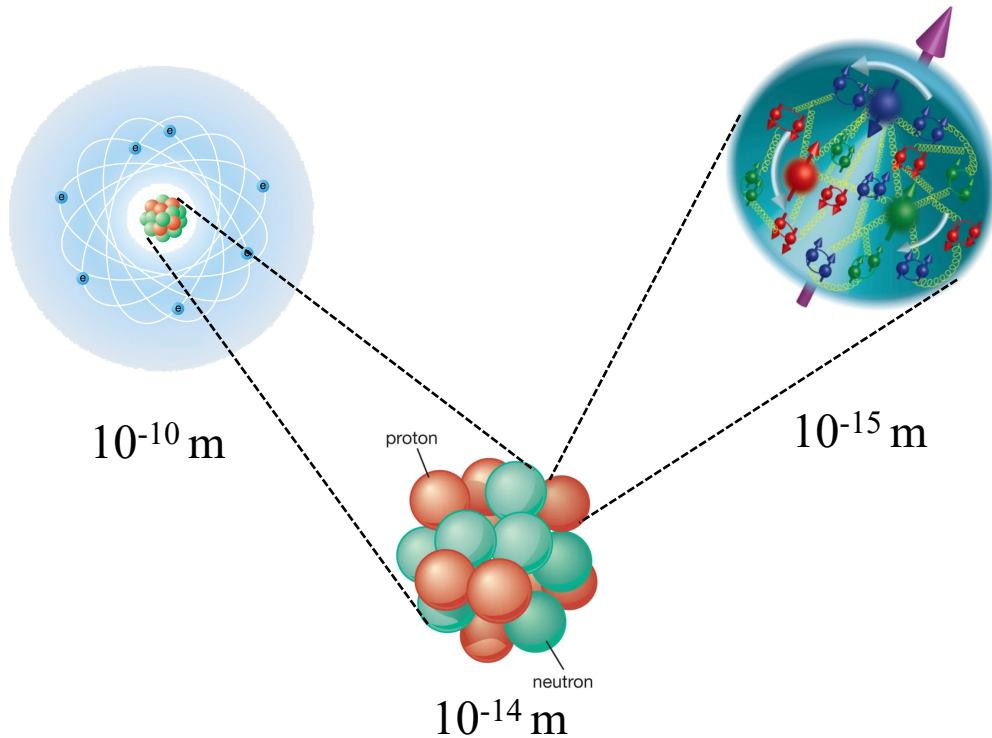


OUTLINE

- **Motivation**
- Unpolarized TMDPDF and jet-SIDIS in the future EIC
- Helicity TMDPDF and Spin Alignment
- Boer-Mulders Function and its Asymmetry
- Summary and Outlook

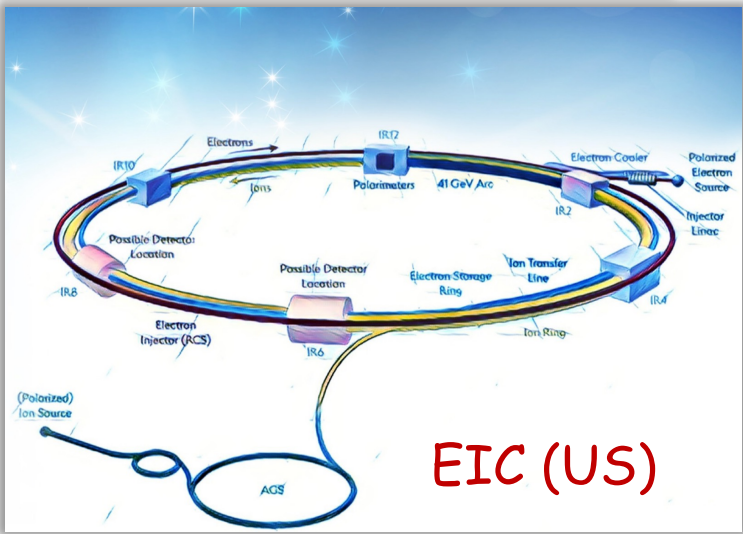
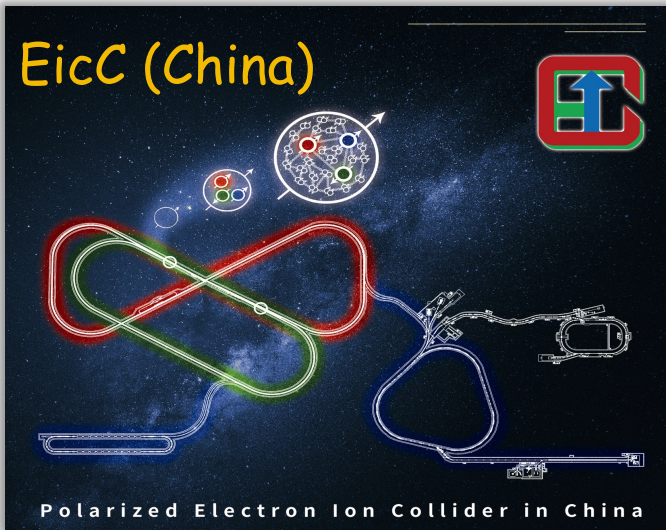
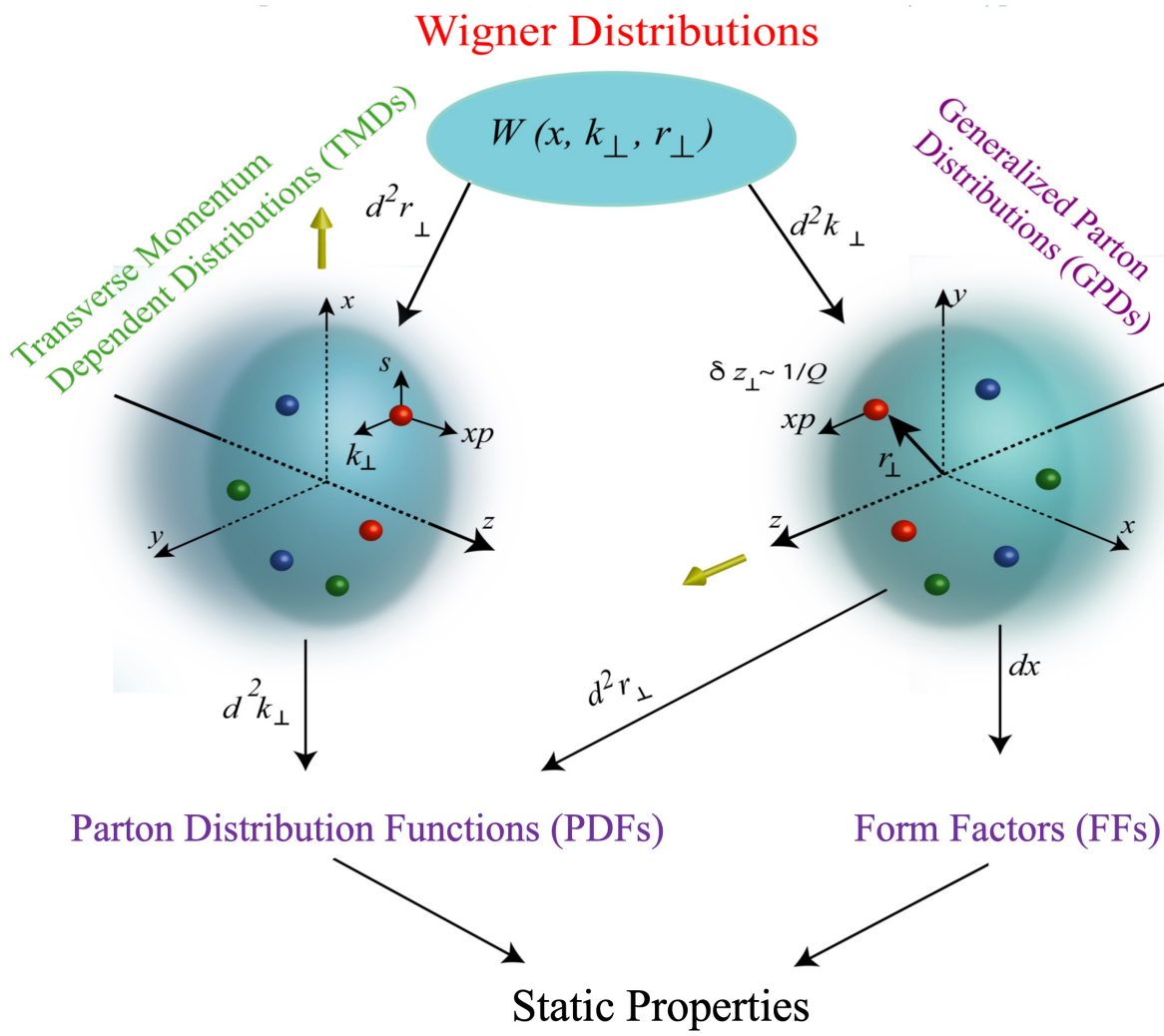
Nucleons: Building Blocks of the Visible Universe

Despite more than half a century of exploration, the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe

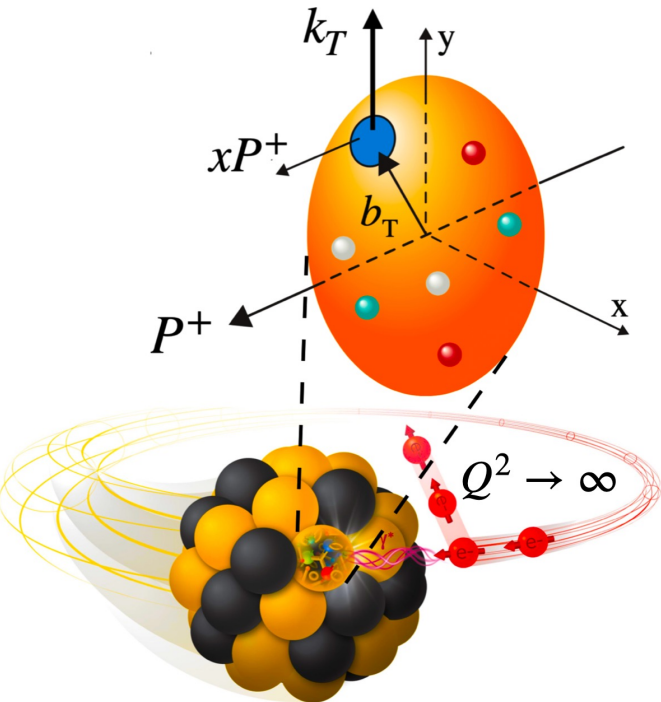


- How do quarks & gluons form nucleon, how do they distribute?
- How does nucleon mass & spin emerge from strong interactions?

The Landscape of Nucleon Structure



TMDs: 3D Momentum Tomography of Nucleon



TMDs: Spin-dependent 3D momentum space images from semi-inclusive scattering

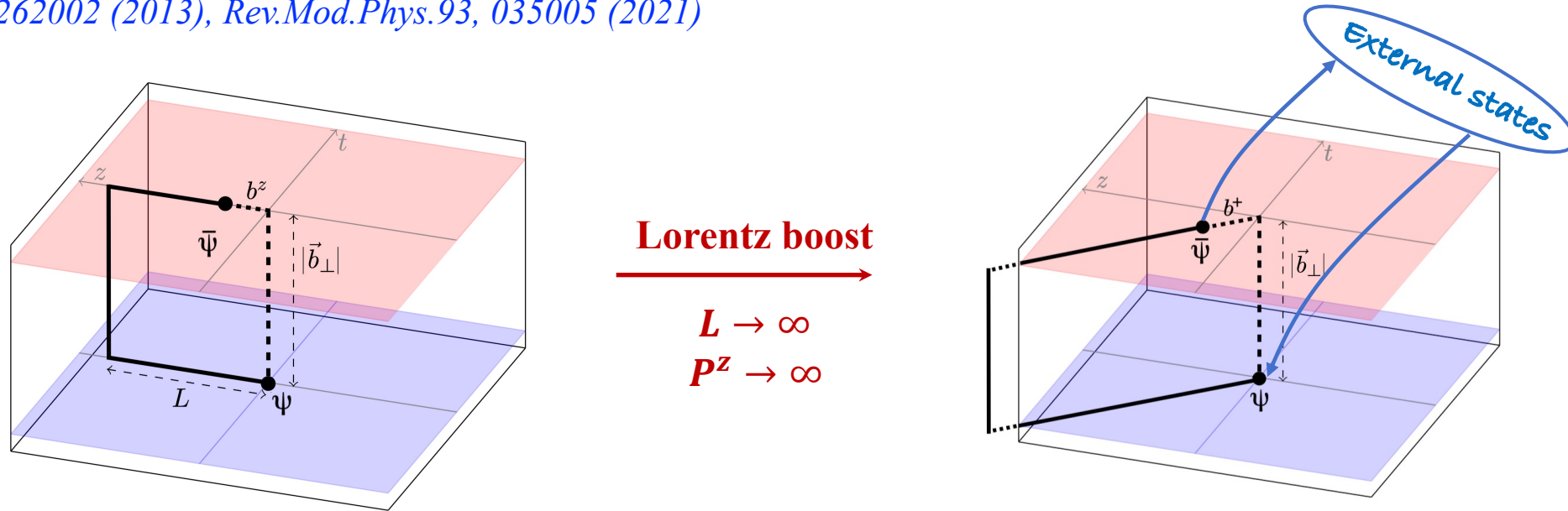
Leading Quark TMDPDFs ○ → Nucleon Spin ○ → Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{○} \cdot$ Unpolarized		$h_1^\perp = \text{○} \downarrow - \text{○} \uparrow$ Boer-Mulders
	L		$g_1 = \text{○} \rightarrow - \text{○} \rightarrow$ Helicity	$h_{1L}^\perp = \text{○} \nearrow - \text{○} \searrow$ Worm-gear
	T	$f_{1T}^\perp = \text{○} \uparrow - \text{○} \downarrow$ Sivers	$g_{1T}^\perp = \text{○} \rightarrow - \text{○} \rightarrow$ Worm-gear	$h_1 = \text{○} \uparrow - \text{○} \downarrow$ Transversity $h_{1T}^\perp = \text{○} \nearrow - \text{○} \searrow$ Pretzelosity

TMDs from Lattice QCD within LaMET

Large-momentum effective theory (LaMET): connecting Euclidean lattice and light-cone observables

Ji, PRL110, 262002 (2013), Rev.Mod.Phys.93, 035005 (2021)



TMD factorization in LaMET: *PLB811, 135946 (2020); JHEP04, 178, (2022)*

Perturbative matching kernel

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z) \sqrt{S_I(b_\perp, \mu)} = H_\Gamma(\zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\zeta^z}{\zeta}\right] f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta^z}\right)$$

Quasi TMDs

Intrinsic Soft Function

See Zhi-Chao's talk

Collins-Soper kernel

LPC, PRD 113 (2026) 5, 054505

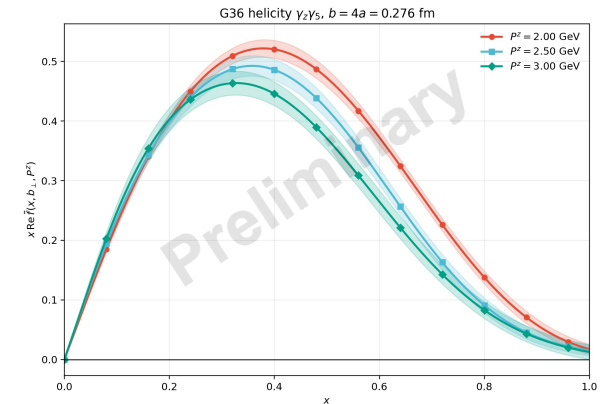
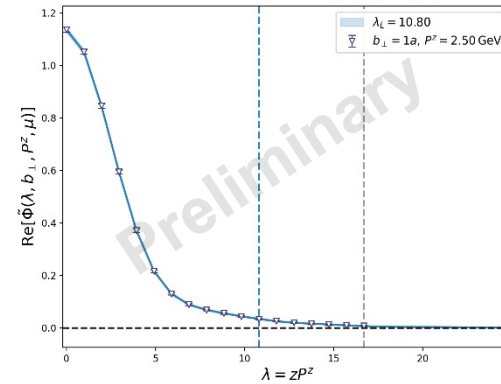
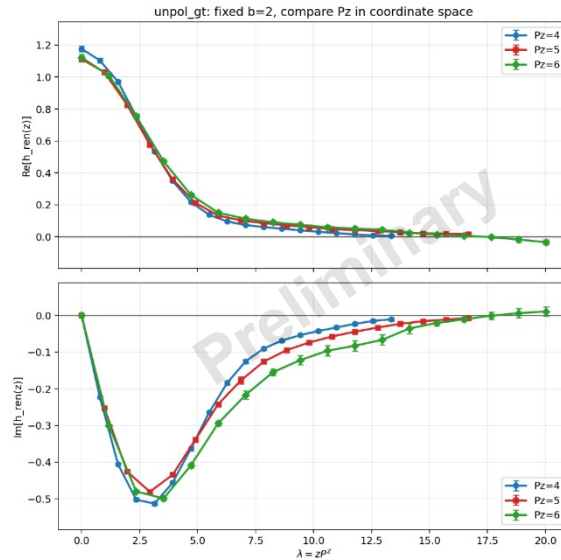
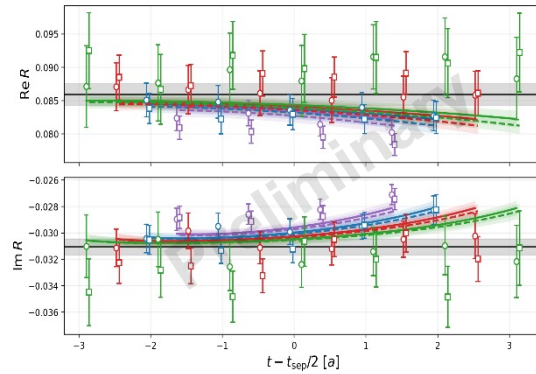
Light-cone TMDs

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Precise Calculation for Leading-Twist Quark TMDPDFs

- E.g.: Unpolarized TMDPDF



Ground-state fit
the ratio of 3pt/2pt

Nonlocal renormalization
to get coordinate-space
distribution

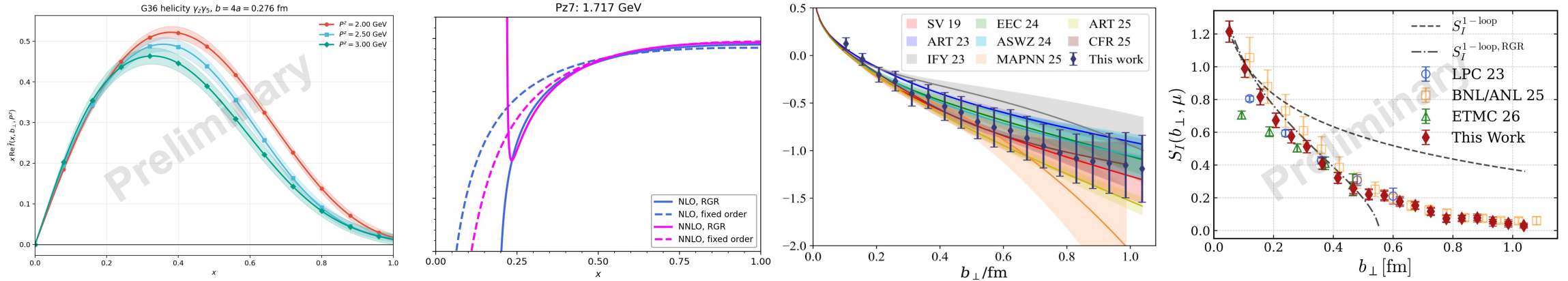
Large- λ
extrapolation

Fourier transform to get
momentum-space
quasi-TMDPDF

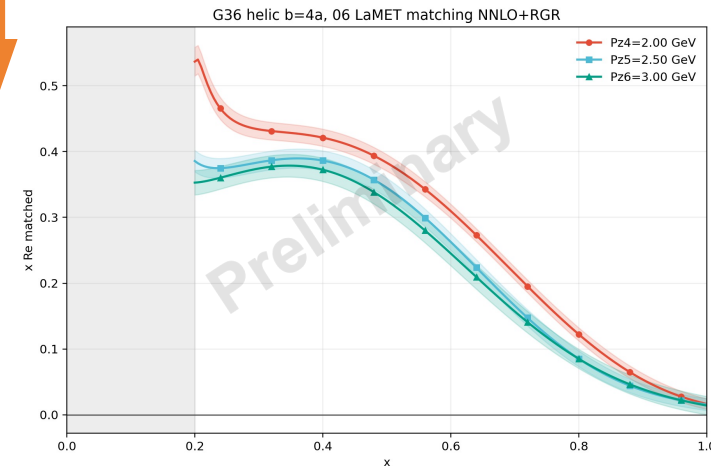
Precise Calculation for Leading-Twist Quark TMDPDFs

- E.g.: Unpolarized TMDPDF

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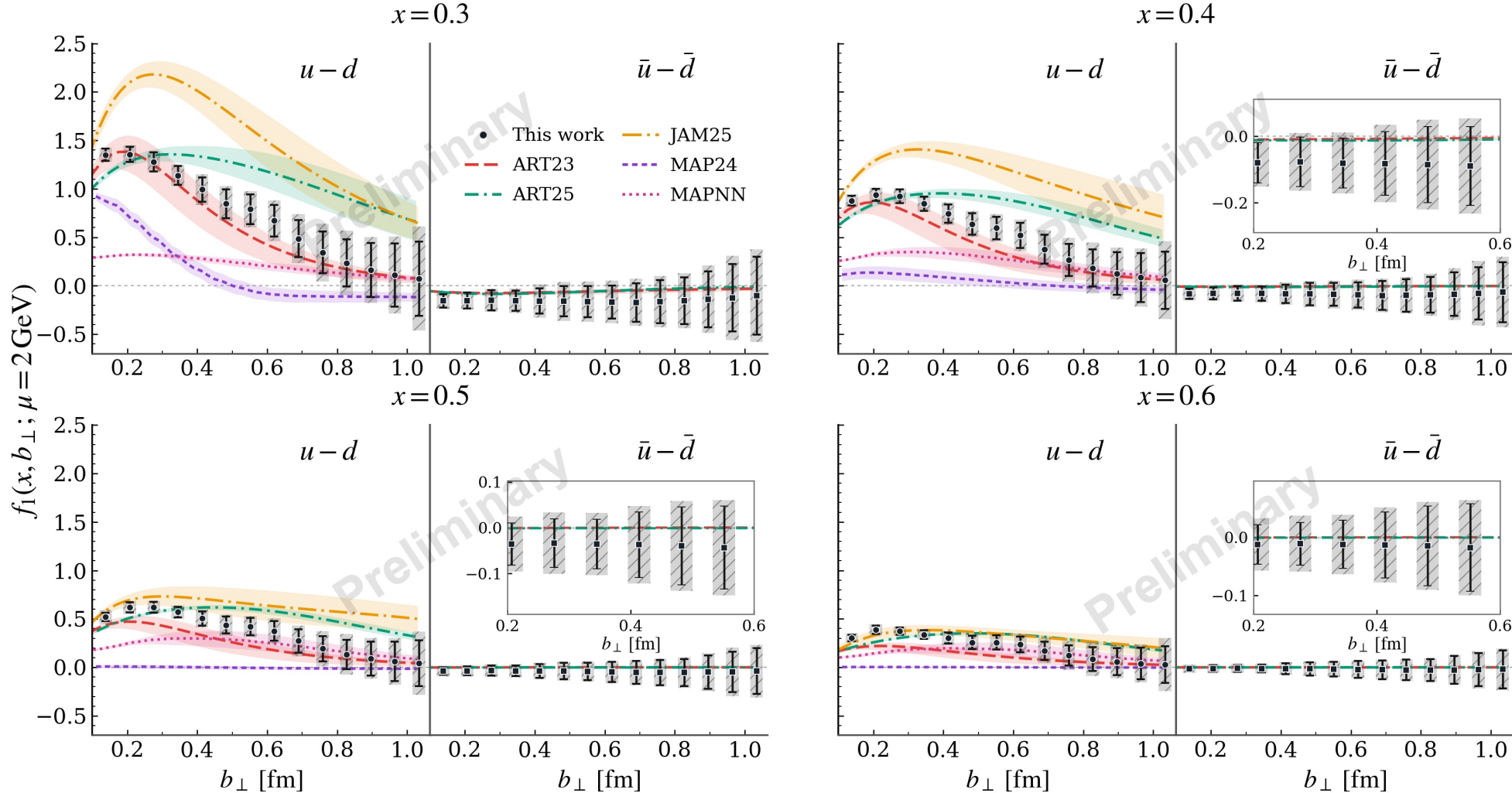
LaMET matching and repeat it
at different $b_{\perp}$, a , m_{π} .



Unpolarized TMDPDFs from Lattice QCD

✓ Continuum limit ✓ Physical mass limit ✓ $b_T \sim 1\text{fm}$

LPC 2026, In preparation



MAP24, JHEP 08 (2024) 232
MAPNN, PRL 135 (2025) 2, 021904
ART23, JHEP 05 (2024) 036
ART25, JHEP 11 (2025) 134
JAM25, PRL (2026), accepted

Unpolarized TMDPDFs to EIC jet-SIDIS observable

- The jet-SIDIS process will provide **a more clean platform** for extracting the TMDPDFs:

$$e + p \rightarrow e' + \text{jet} + X, \quad e + n \rightarrow e' + \text{jet} + X.$$

- Factorization of the cross-section difference

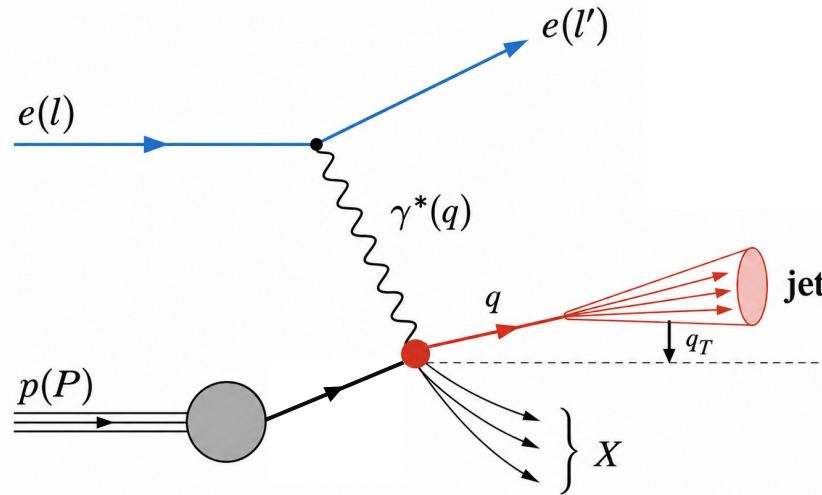
$$\frac{d\Delta\sigma}{dQ^2 dx dz dq_T} = 2\pi q_T \frac{2\pi\alpha_{em}^2}{Q^4} [1 + (1-y)^2] \frac{1}{3} H_{DIS}(Q^2, \mu) \int_0^\infty \frac{b db}{2\pi} J_0(bq_T) f^{u-d}(x, b, \mu, \zeta) J_q\left(z, b, \frac{QR}{2}, \mu, \zeta\right)$$

where inelasticity $y = \frac{Q^2}{xs}$, $\frac{1}{3} = e_u^2 - e_d^2$.

Pert. QCD

Lattice

Pert. QCD

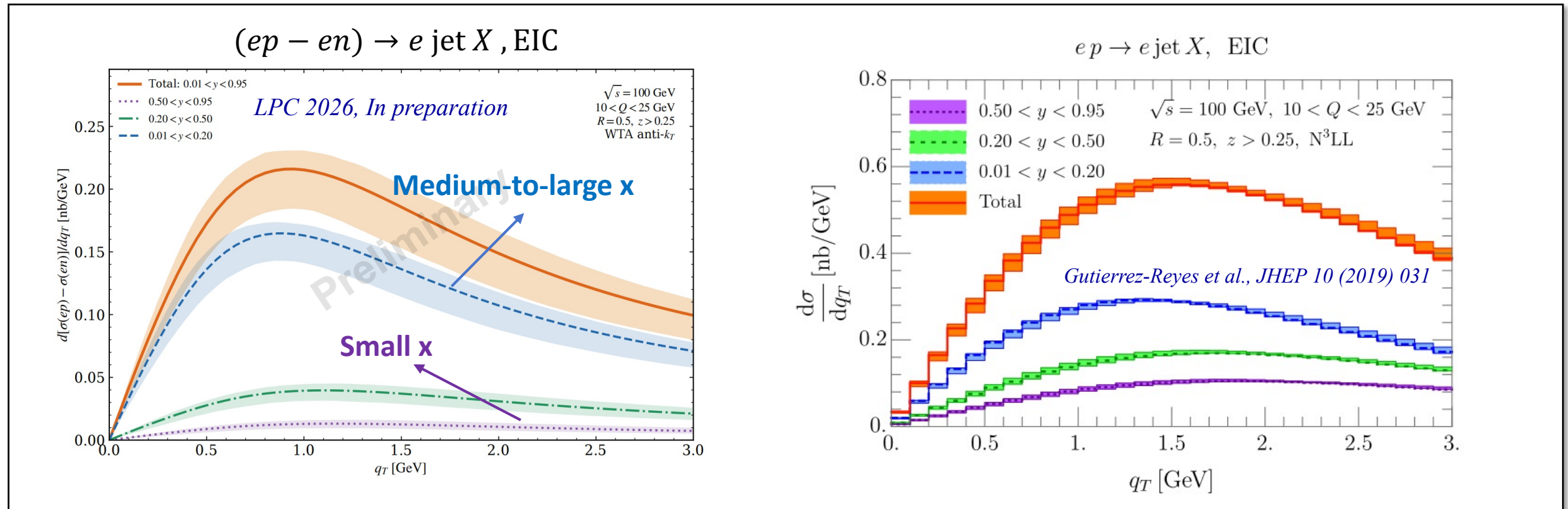


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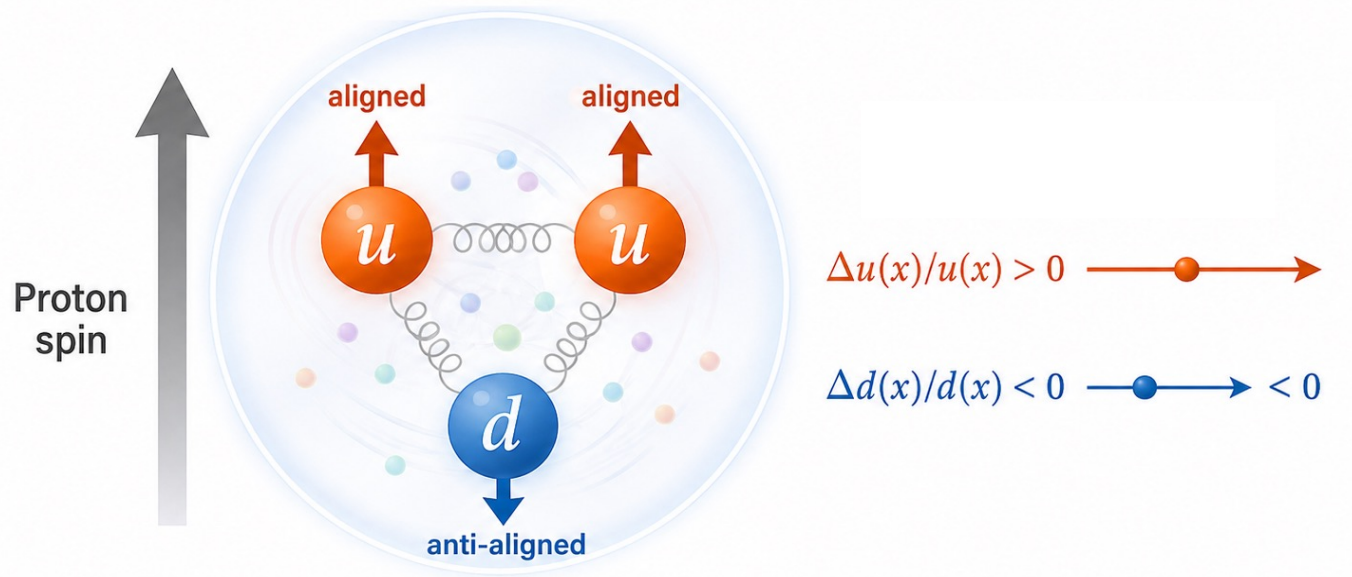
- Factorization of the cross-section difference



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Why Helicity?



- Quark helicity PDF:

$$\Delta q(x) = q^\uparrow(x) - q^\downarrow(x)$$

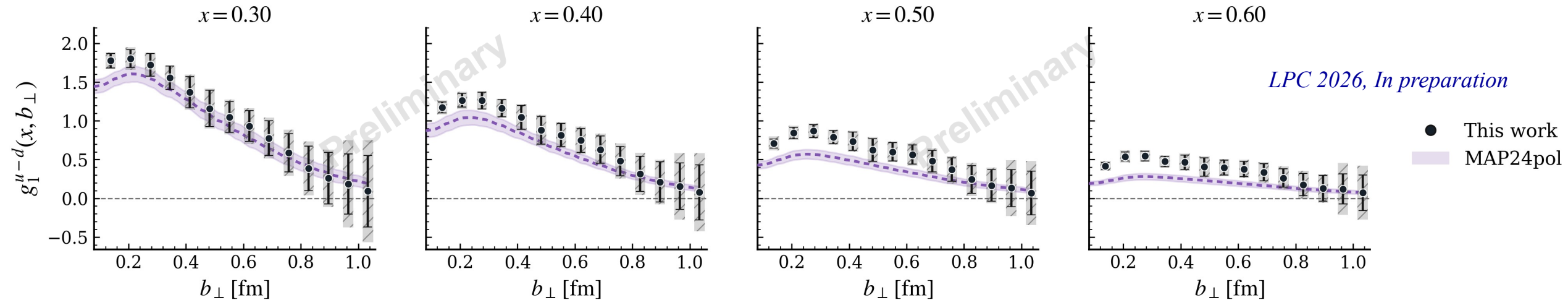
- Quark unpolarized PDF:

$$q(x) = q^\uparrow(x) + q^\downarrow(x)$$

$\Delta q(x)/q(x)$ encodes the longitudinal spin alignment of quarks inside the proton, providing a clean probe of its spin-flavor structure

Helicity TMDPDFs from Lattice QCD

- ✓ Continuum limit ✓ Physical mass limit ✓ $b_T \sim 1\text{fm}$
- Overall good agreement with MAP24pol.
- Lattice QCD provides first-principles constraints in the moderate-to-large- x and nonperturbative- $b_\perp$ region.

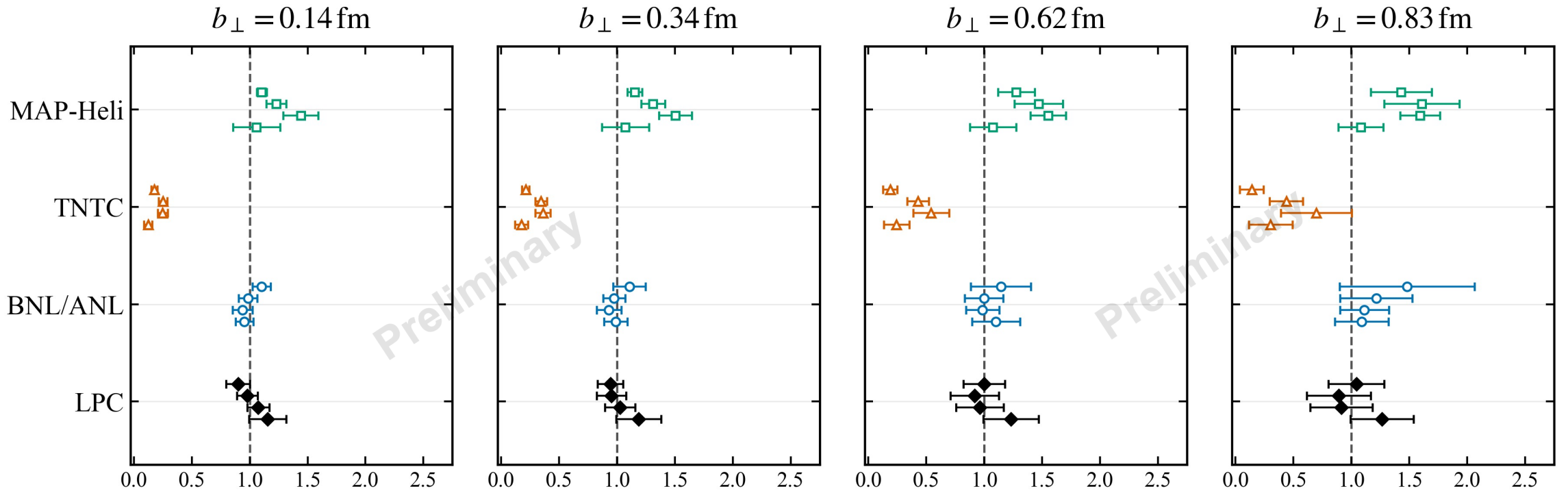


MAP, PRL. 134 (2025) 12, 121901

Spin Alignment

- Normalized Helicity-to-Unpolarized TMD Ratio:

$$R_{g_{1L}/f_1}^{u-d}(x, b_T) \cdot g_A \equiv \frac{g_{1L}^{u-d}(x, b_T; \zeta, \mu)}{f_1^{u-d}(x, b_T; \zeta, \mu)} = \frac{\tilde{g}_{1L}^{u-d}(x, b_T, P_z, \mu)}{\tilde{f}_1^{u-d}(x, b_T, P_z, \mu)}.$$



(The points from top to bottom correspond to $x=0.2, 0.3, 0.4, 0.6$)

BNL/ANL (LQCD): PRL135, 20 (2025)
 MAP (Global fit): PRL134, 121901 (2025)
 TNTC (Global fit): PRL134, 121902 (2025)

OUTLINE

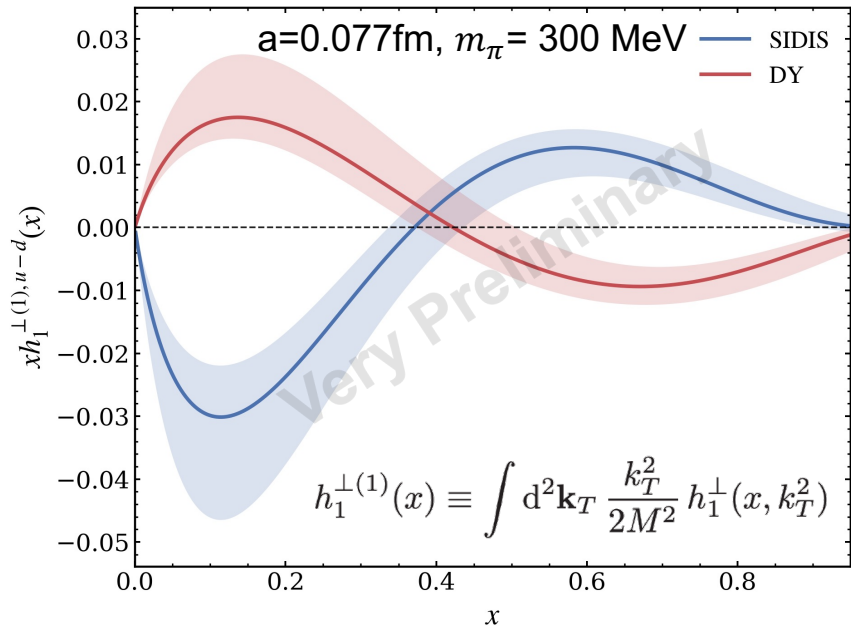
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Quark Spin-transverse-momentum Correlation (Boer-Mulders function)

- First moment of quasi Boer-Mulders function and its matching

$$\tilde{h}_1^{\perp(1)}(x, b_\perp^2, P^z, \mu) = \frac{1}{iM\epsilon_{\perp}^{ij}b_{\perp j}} \frac{E}{P^z} \int \frac{dz}{2\pi} e^{ixP^z z} \frac{\sum_S \text{Tr} \left[P_+ u(P, S) \langle N(P, S) | O^{i\sigma^{\perp z} \gamma_5}(z, b_\perp) | N(P, S) \rangle \bar{u}(P, S) \right]_R}{\sum_S \text{Tr} [P_+ u(P, S) \bar{u}(P, S)]_R}$$

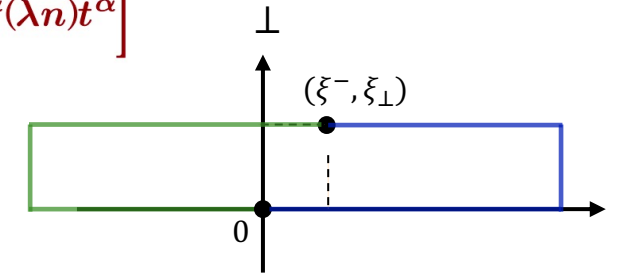
$$\tilde{h}_1^{\perp(1)}(x, b_\perp^2, P^z, \mu) \sqrt{S_I(b_\perp, \mu)} = \exp \left[\frac{1}{2} \ln \frac{\zeta_z}{\zeta} K(b_\perp, \mu) \right] H \left(\frac{x}{y}, \frac{\mu^2}{\zeta_z} \right) h_1^{\perp(1)}(y, b_\perp^2, \mu, \zeta) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z} \right)$$



- Gauge links generate initial and final state interactions and distinguish the SIDIS & Drell-Yan processes

$$\mathcal{U}_n(a, b) = \exp \left[-ig \int_a^b d\lambda n \cdot A^\alpha(\lambda n) t^\alpha \right]$$

LPC (2026), in preparation

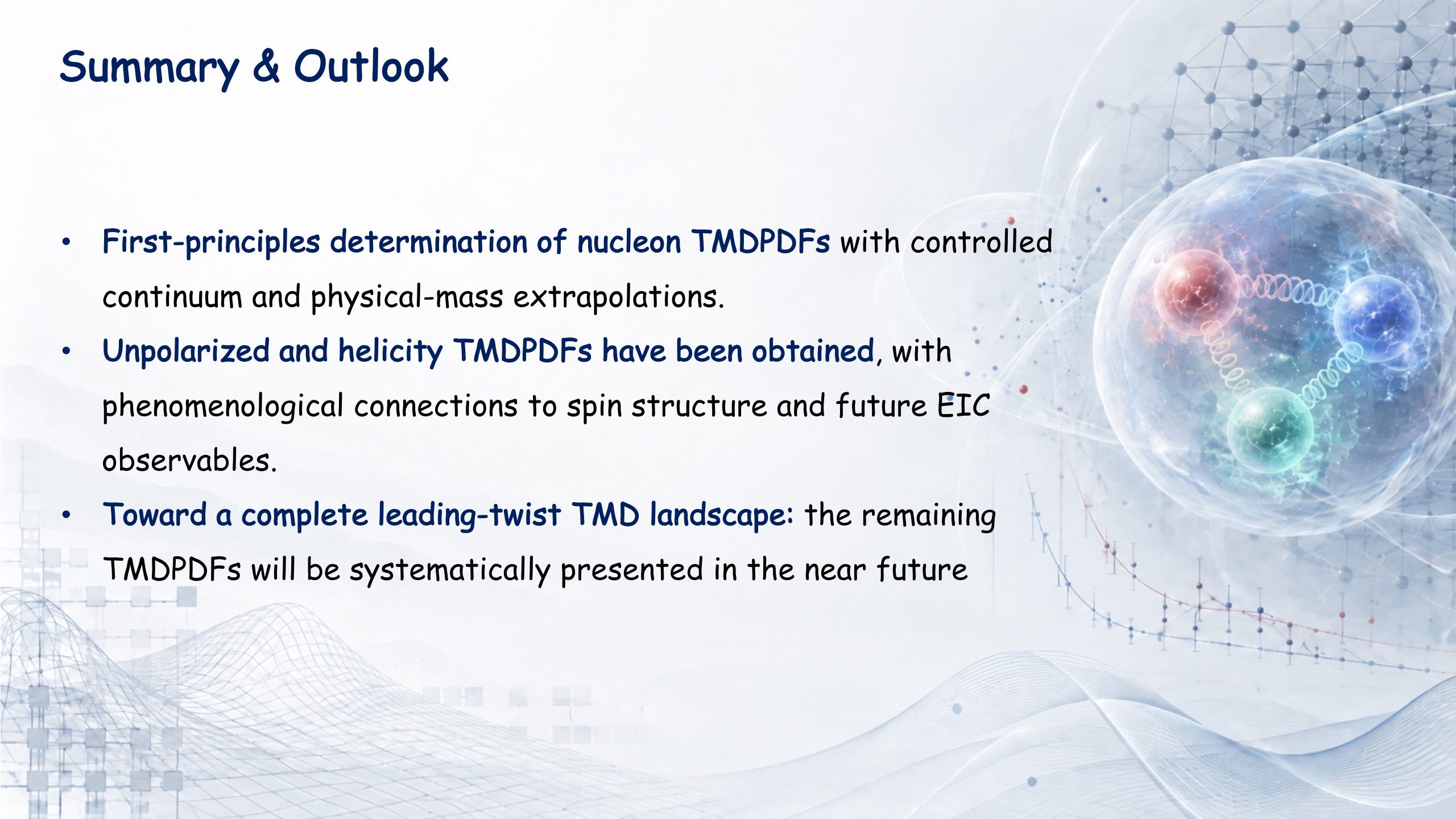


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Summary & Outlook

- **First-principles determination of nucleon TMDPDFs** with controlled continuum and physical-mass extrapolations.
- **Unpolarized and helicity TMDPDFs have been obtained**, with phenomenological connections to spin structure and future EIC observables.
- **Toward a complete leading-twist TMD landscape:** the remaining TMDPDFs will be systematically presented in the near future



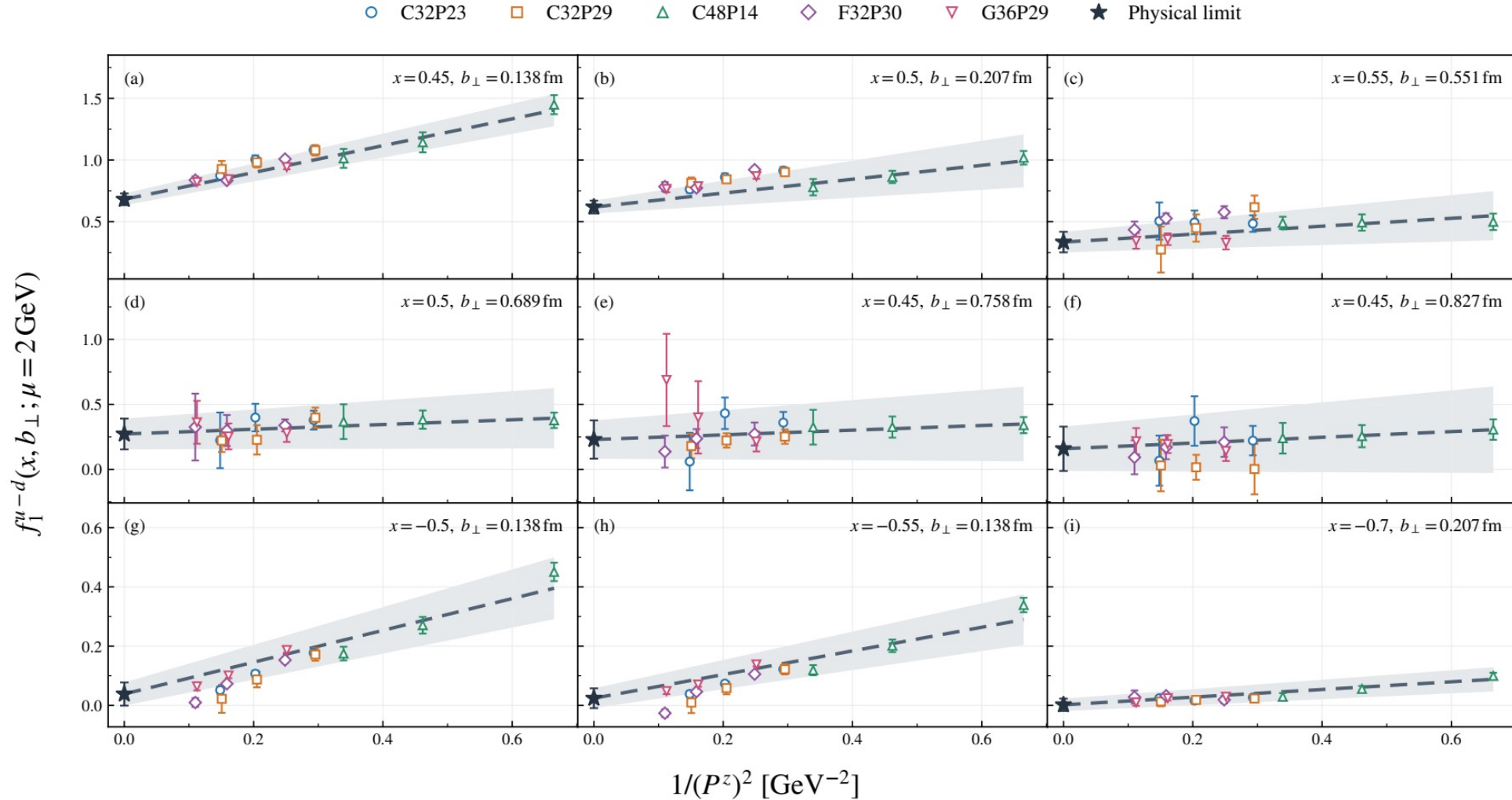
Backup

Lattice Setup

Ensemble	a (fm)	m_π (MeV)	P^z (GeV)	$N_{\text{cfg}} \times N_{\text{meas}}$
C32P29	0.1053	292	1.84, 2.21, 2.58	1000×4
C32P23	0.1053	228	1.84, 2.21, 2.58	1000×4
C48P14	0.1053	136	1.22, 1.47, 1.71	300×8
F32P30	0.0775	303	2.00, 2.50, 3.00	1000×4
G36P29	0.0689	295	1.99, 2.49, 2.99	500×4

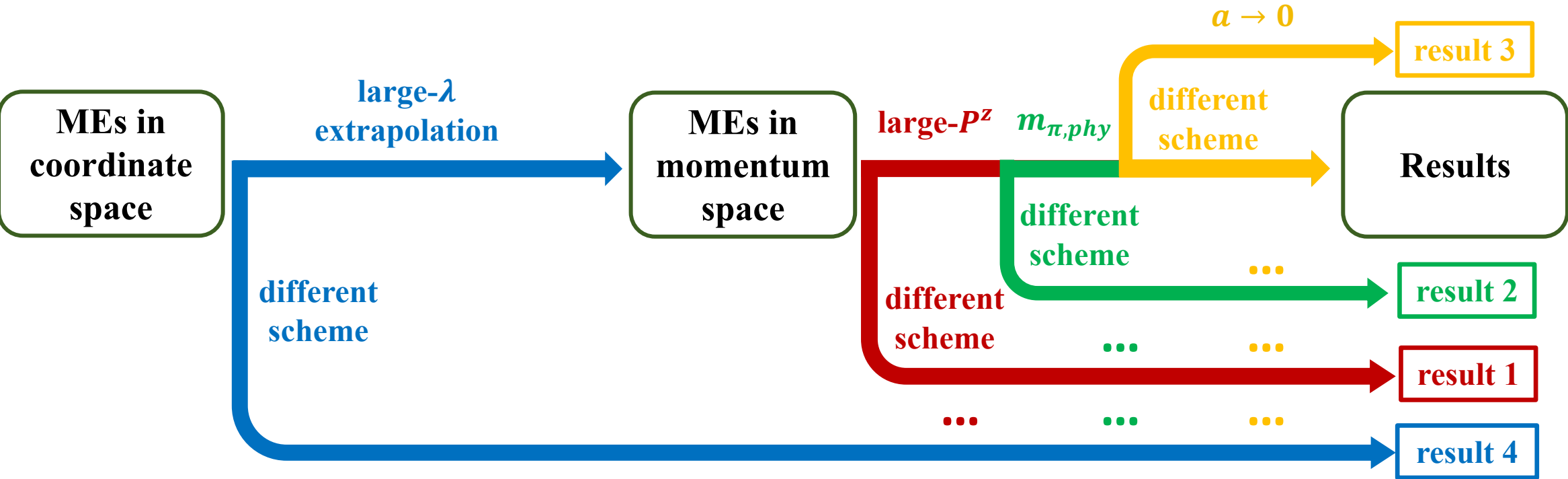
Physical limit extrapolation

$$f_1^{u-d}(x, b_\perp; a, m_\pi, P^z) = f_{1,\text{phys}}^{u-d}(x, b_\perp) + \frac{A(x, b_\perp)}{(P^z)^2} + B(x, b_\perp)a^2 + C(x, b_\perp)(m_\pi^2 - m_{\pi,\text{phys}}^2)$$



Systematic error analysis process

The core idea is to change a small part of the approach in a certain analysis step, while keeping everything else the same, and then compare the differences in the results.

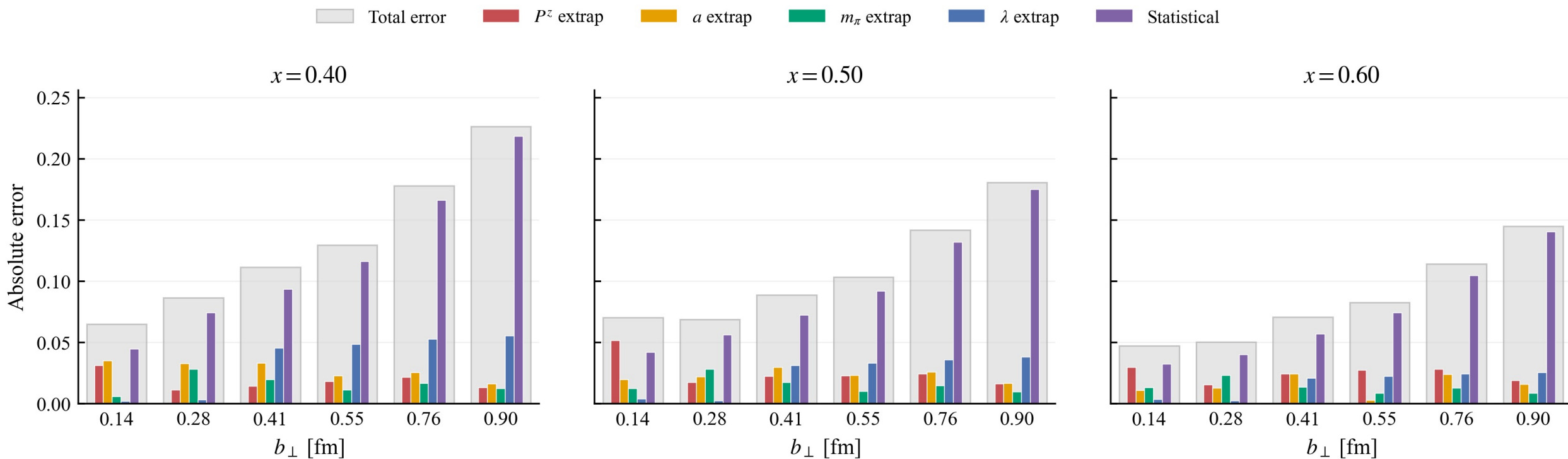


Analysis scheme

Term	Scheme	
Discretization effect	$+(aP^z)^2$ term	$f^{G36P29} \Big _{P^z \rightarrow \infty}$
Power correction	Cases with P_{min}^z on each ensemble removed	Replace large- P^z term with the case of P_{max}^z
Chiral effect	$f^{C48P14} \Big _{P^z \rightarrow \infty}$	
Large- λ extrapolation	Fit region $z_{min} - 1$ and $z_{min} + 1$	

Systematic error contribution

The error contribution from **the power correction at small b** is large, while the error contribution from **the λ extrapolation at large b** is large.



Unpolarized TMDPDFs to EIC jet-SIDIS observable

- Jet-based TMD measurements are explicitly included in the EIC Yellow Report as part of the future 3D imaging program.

- The cross-section prediction is dominated by the small- y region, inelasticity $y = \frac{Q^2}{xs}$, corresponding to large x , precisely where **lattice calculations can provide strong constraints** while phenomenological determinations remain less precise.

Jet-based TMD studies: electron-jet Sivers, hadron-in-jet Collins, and TMD evolution with substructure

Over the last few years, various studies [28, 29, 126, 517, 553, 592–607] showed that jets offer a novel way to probe quark TMDs and TMD evolution at the EIC — this possibility was not discussed in the INT Proceedings [544] and the EIC White Paper [2]. Jets are excellent proxies for partons, cleanly separate current from target fragmentation [28, 126, 592], and can deconvolve TMD PDFs from FFs [29, 592–599]. Moreover, jet substructure observables can probe TMD-

EIC yellow report, Nucl. Phys. A 1026 (2022) 122447

