

η and η' mesons from lattice QCD

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Outline

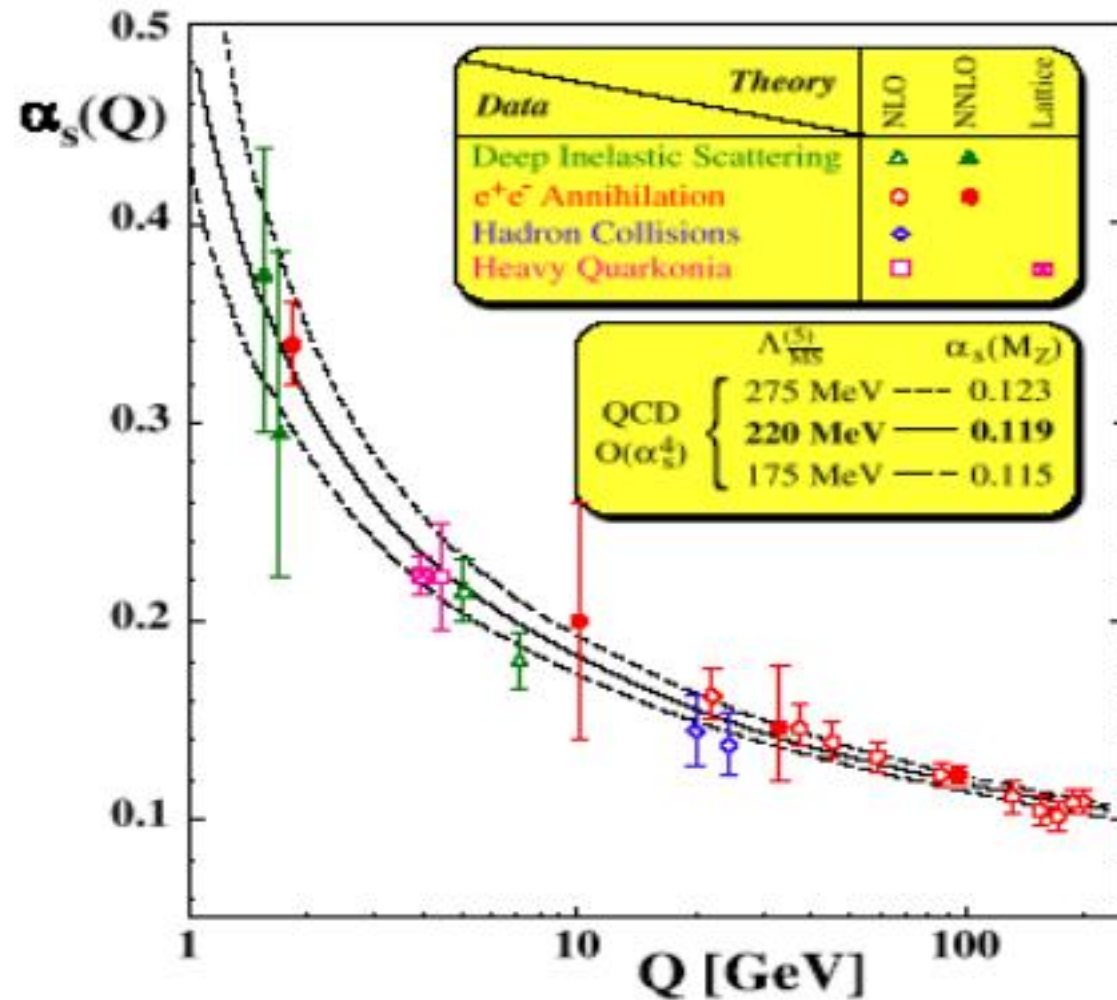
01 Introduction

02 Computing methods

03 Results presentation

04 Conclusions

Chiral Symmetry Breaking in QCD



High-energy region: asymptotic freedom

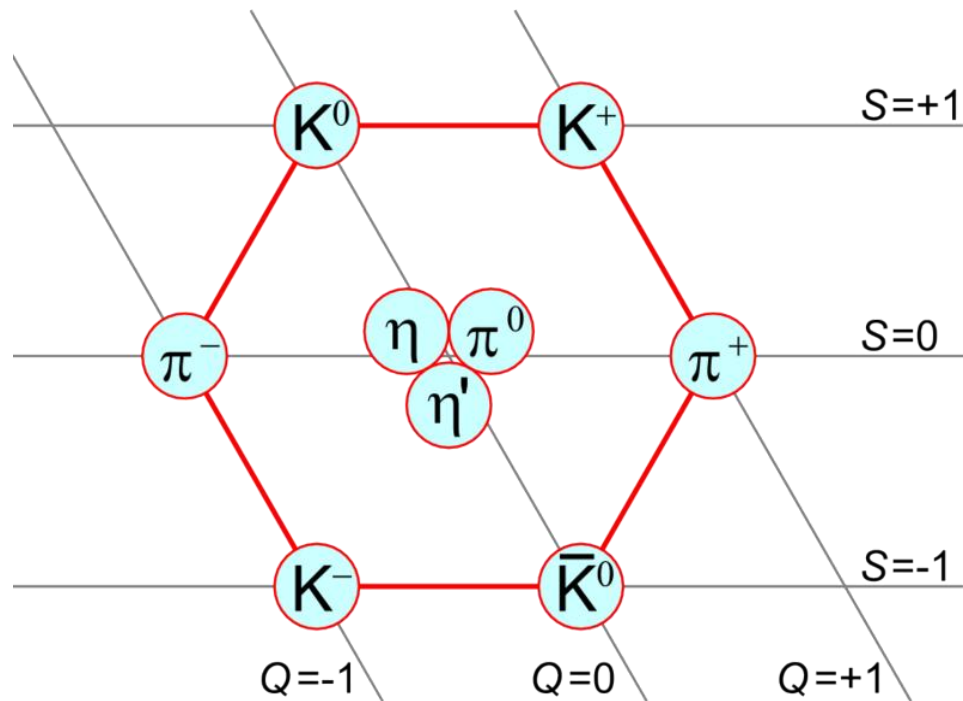
Low-energy region: quark confinement,

chiral symmetry breaking



Eight Goldstone Bosons

SU(3) Classify and $U(1)_A$ Anomaly



$$\text{SU}(3) \quad 3 \otimes \bar{3} = 8 \oplus 1$$

Flavor-Octet: π , K , η_8

Flavor-Singlet: $\eta_1(\eta_0)$

A quantum anomaly explicitly
breaks
the $U(1)_A$ symmetry

$$\Delta M_{\eta_1}^2 = \frac{2N_f}{f_\pi^2} \chi_{YM}$$

The vacuum is not empty

Edward Witten, Nuclear Physics B, 156 (1979) 269-283

The mixture of states

$$O_8=\frac{1}{\sqrt{6}}(\bar{u}\gamma_5u+\bar{d}\gamma_5d-2\bar{s}\gamma_5s)$$

$$O_1=\frac{1}{\sqrt{3}}(\bar{u}\gamma_5u+\bar{d}\gamma_5d+\bar{s}\gamma_5s)$$

described by a 2×2
correlation function matrix

$$C_{8-1}(t)=\begin{pmatrix} O_8(t+t')O_8^+(t') & O_8(t+t')O_1^+(t') \\ O_1(t+t')O_8^+(t') & O_1(t+t')O_1^+(t') \end{pmatrix}$$

Pseudoscalar Mesons (u,d,s)	Q	S	I	I ₃
π^+	1	0	1	1
π^0	-1	0	1	-1
π^-	0	0	1	0
K^+	1	1	1/2	1/2
K^-	-1	-1	1/2	-1/2
K^0	0	1	1/2	-1/2
$\bar{K}^0$	0	-1	1/2	1/2
η_8	0	0	0	0
η_1/η_0	0	0	0	0

Generalized Eigenvalue Problem (GEVP)

GEVP

$$C(t)v(t, t_0) = \lambda(t, t_0)C(t_0)v(t, t_0)$$



$$C_{l-s}(t) = \begin{pmatrix} O_l(t+t')O_l^+(t') & O_l(t+t')O_s^+(t') \\ O_s(t+t')O_l^+(t') & O_s(t+t')O_s^+(t') \end{pmatrix}$$

$$= \begin{pmatrix} C_{ll} + 2D_{ll} & \sqrt{2}D_{ls} \\ \sqrt{2}D_{sl} & C_{ss} + D_{ss} \end{pmatrix}$$

Basis vectors transformation

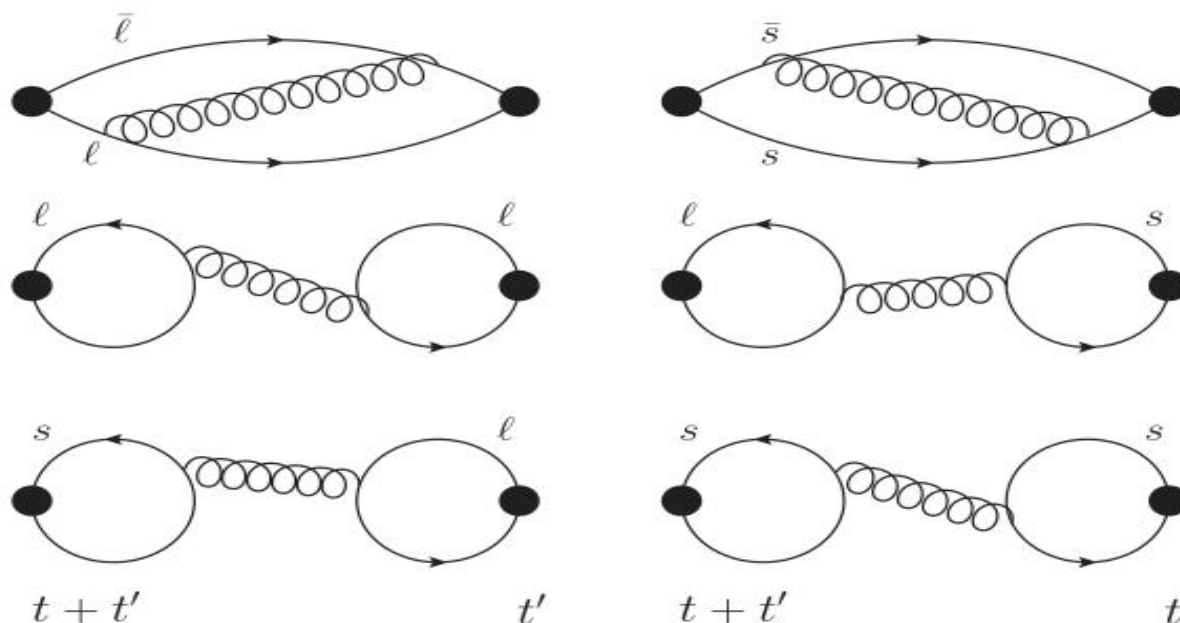
$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(|\bar{u}\gamma_5 u\rangle + |\bar{d}\gamma_5 d\rangle - 2|\bar{s}\gamma_5 s\rangle)$$

$$|\eta_1\rangle = \frac{1}{\sqrt{3}}(|\bar{u}\gamma_5 u\rangle + |\bar{d}\gamma_5 d\rangle + |\bar{s}\gamma_5 s\rangle)$$



$$|\eta_l\rangle = \frac{1}{\sqrt{2}}(|\bar{u}\gamma_5 u\rangle + |\bar{d}\gamma_5 d\rangle)$$

$$|\eta_s\rangle = |\bar{s}\gamma_5 s\rangle$$



Calculation of the Mixing Angle

overlap factor

$$C_{ij}(t) = \sum_n \frac{1}{2E_n} \langle 0|O_i|n\rangle \langle n|O_j^+|0\rangle e^{-E_n t}$$

$$Z_n^i(t) = \langle 0|O_i(t)|n\rangle = \sqrt{2E_n} e^{E_n t_0/2} \sum_j C_{ij}(t_0) v_n^j(t, t_0)$$

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} |\eta_l\rangle \\ |\eta_s\rangle \end{pmatrix}$$



$$\begin{aligned} |\eta\rangle &= \cos \phi |\eta_l\rangle - \sin \phi |\eta_s\rangle \\ |\eta'\rangle &= \sin \phi |\eta_l\rangle + \cos \phi |\eta_s\rangle \end{aligned}$$

$$Z_\eta^l = \langle 0|O_l|\eta\rangle = \cos \phi \langle 0|O_l|l\rangle - \sin \phi \langle 0|O_l|s\rangle = a_l \cos \phi$$

$$Z_{\eta'}^l = \langle 0|O_l|\eta'\rangle = \sin \phi \langle 0|O_l|l\rangle + \cos \phi \langle 0|O_l|s\rangle = a_l \sin \phi$$

$$Z_\eta^s = \langle 0|O_s|\eta\rangle = \cos \phi \langle 0|O_s|l\rangle - \sin \phi \langle 0|O_s|s\rangle = -a_s \sin \phi$$

$$Z_{\eta'}^s = \langle 0|O_s|\eta'\rangle = \sin \phi \langle 0|O_s|l\rangle + \cos \phi \langle 0|O_s|s\rangle = a_s \cos \phi$$

$$\begin{pmatrix} Z_\eta^l & Z_\eta^s \\ Z_{\eta'}^l & Z_{\eta'}^s \end{pmatrix} = \begin{pmatrix} a_l \cos \phi & -a_s \sin \phi \\ a_l \sin \phi & a_s \cos \phi \end{pmatrix}$$



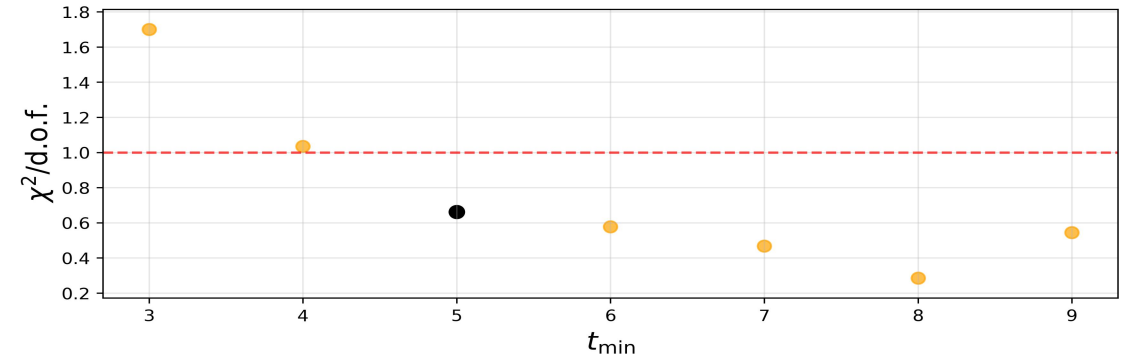
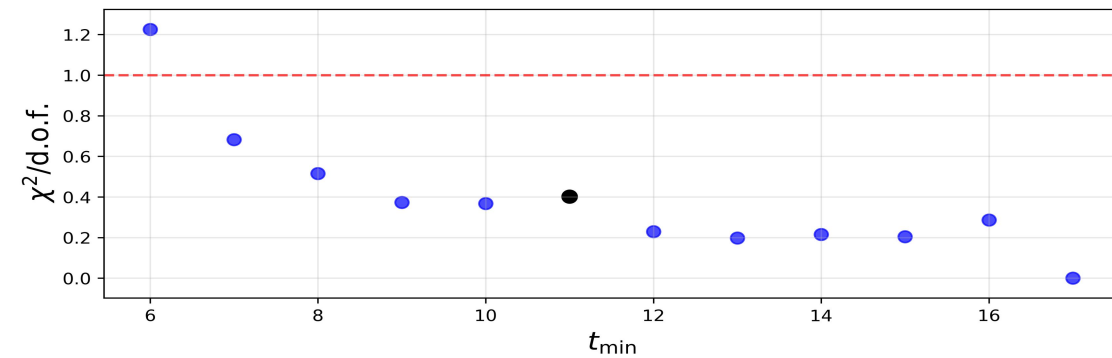
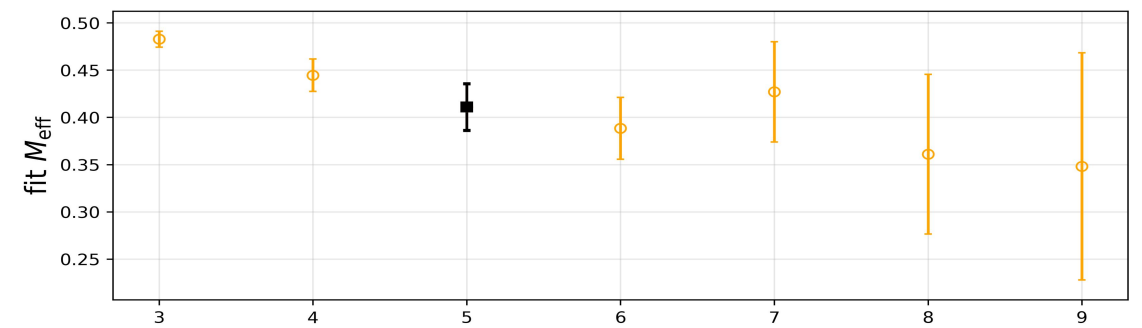
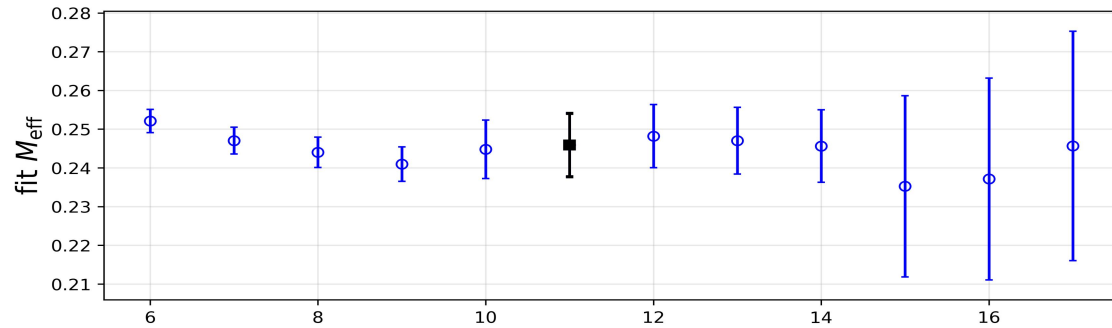
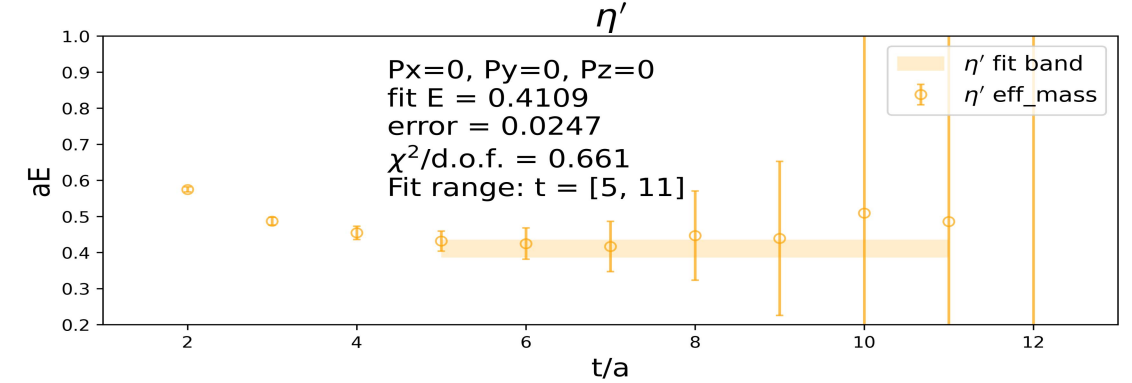
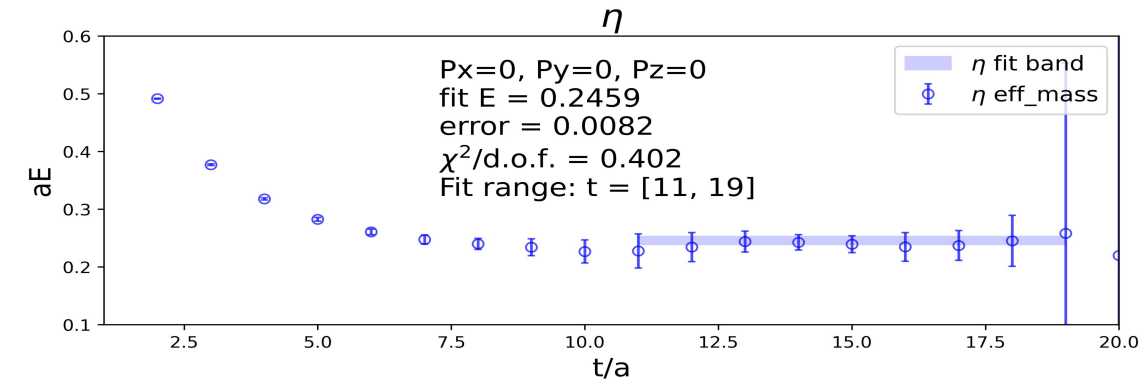
$$\tan^2 \phi = -\frac{Z_\eta^l Z_{\eta'}^s}{Z_\eta^s Z_{\eta'}^l}$$

Lattice Ensembles and Parameters

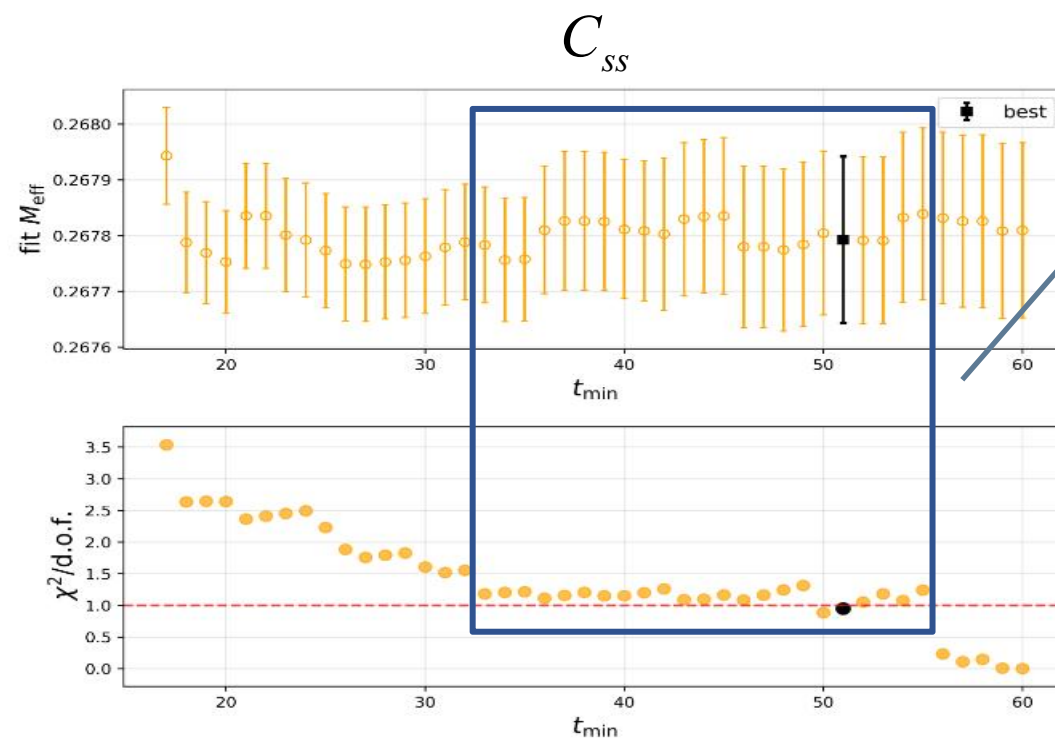
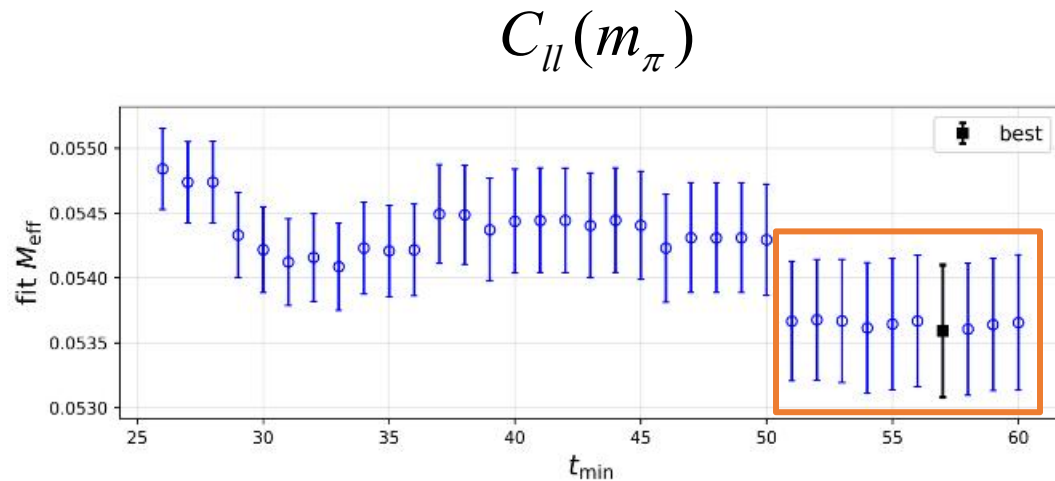
ID	β (a/fm)	$(L/a)^3 \times T/a$	M_π / MeV	M_K / MeV	$M_\pi L$	Nconf
C48P14	6.20 (0.1053)	$48^3 \times 96$	136	510	3.47	259
C48P23		$48^3 \times 96$	226	484	5.78	265
C32P23		$32^3 \times 64$	228	484	3.89	451
C32P29		$32^3 \times 64$	292	509	4.99	984
C48P29		$48^3 \times 96$	293	510	7.50	747
C24P29	6.308 (0.0897)	$24^3 \times 72$	293	509	3.74	879
E32P29		$32^3 \times 64$	293	539	4.26	300
F64P14		$64^3 \times 128$	137	492	3.47	74
F48P21	6.41 (0.0775)	$48^3 \times 96$	207	493	3.90	267
F32P21		$32^3 \times 64$	211	492	2.65	459
F48P30		$48^3 \times 96$	303	524	5.72	359
F32P30		$32^3 \times 96$	303	525	3.81	771
H48P32		$48^3 \times 144$	317	536	4.00	453

η and η' masses from a physical point ensemble

F64P14 M_{eff} of η and η'



Fit of the ground-state correlation function



A coincidence:

$$\eta(1295) \sim \pi(1300)$$

fit each sample using the
best platform area data

$$\begin{pmatrix} C_{ll} + 2D_{ll} & \sqrt{2}D_{ls} \\ \sqrt{2}D_{sl} & C_{ss} + D_{ss} \end{pmatrix}$$

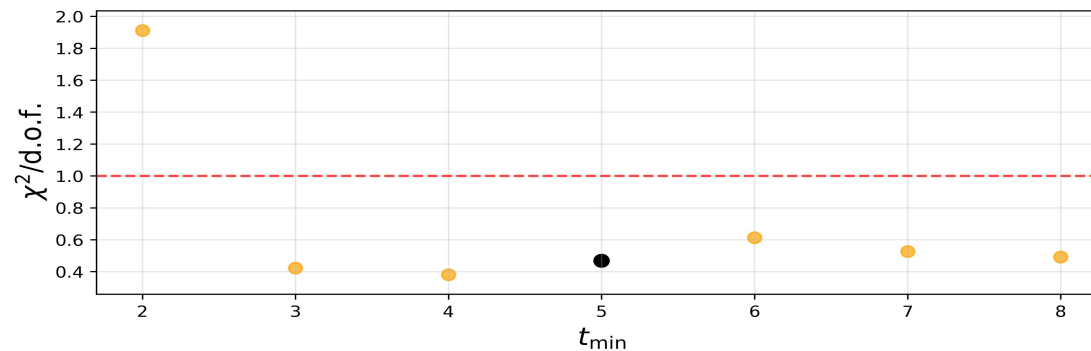
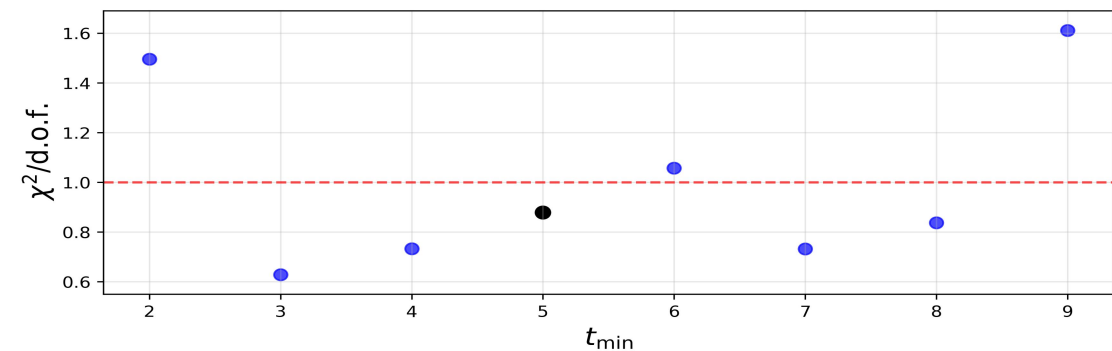
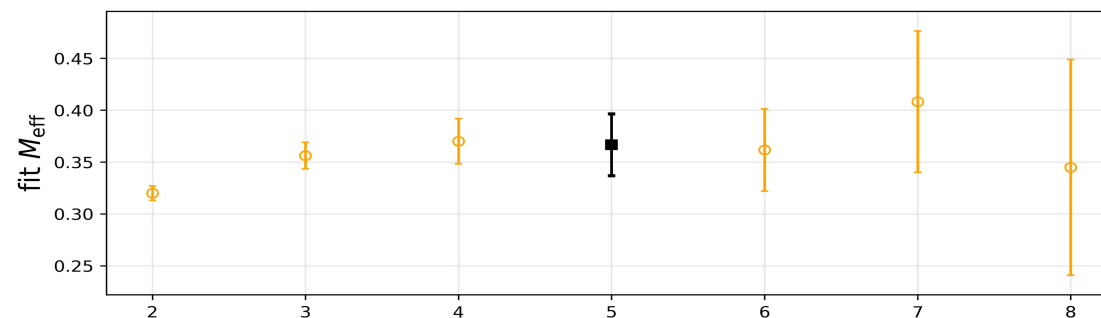
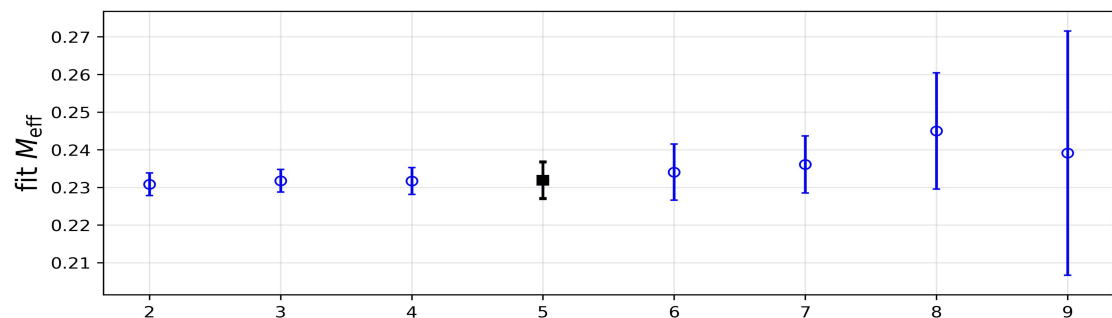
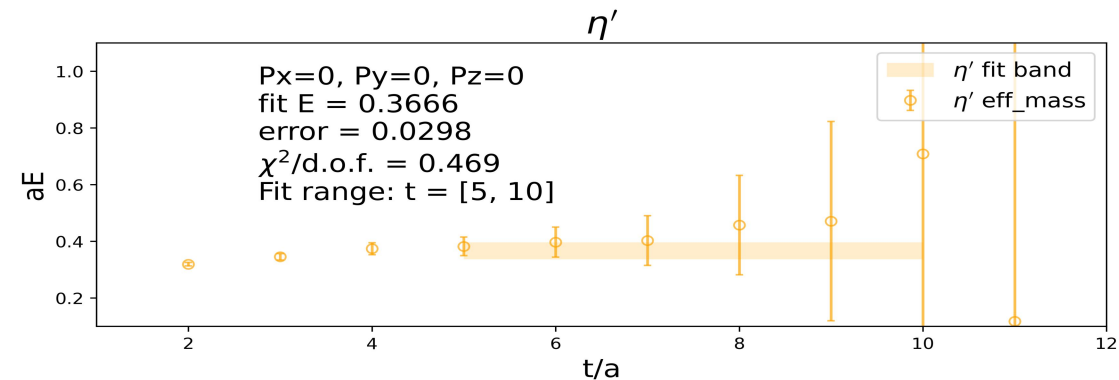
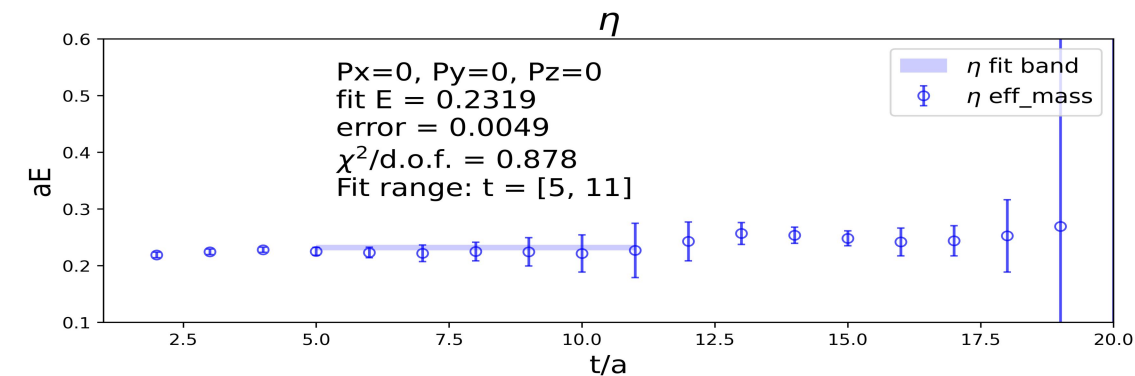
$$\begin{pmatrix} C_{ll}^g + 2D_{ll} & \sqrt{2}D_{ls} \\ \sqrt{2}D_{sl} & C_{ss}^g + D_{ss} \end{pmatrix}$$

C. McNeile, C. Michael, Phys. Lett. B 491,123 (2000)

K. Jansen et al., Eur. Phys. J. C 58,261 (2008)

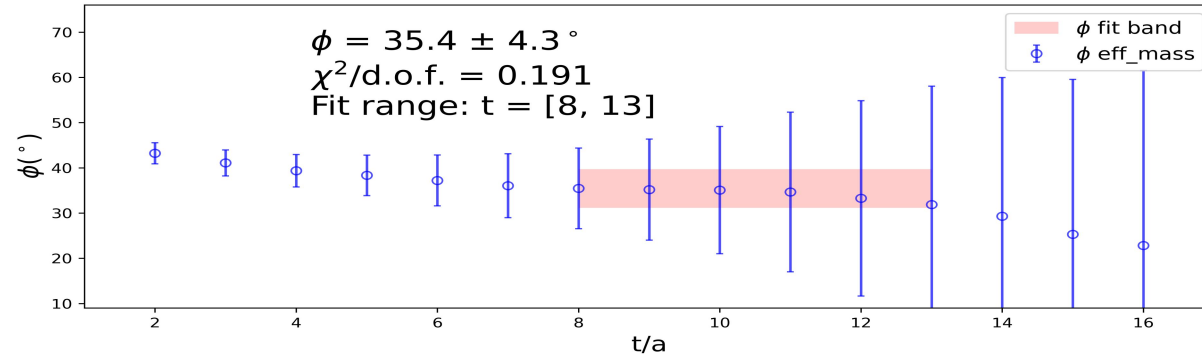
The fitting results after replacement

F64P14 M_{eff} of η and η'



Mixing angle from another physical point ensemble

C48P14 Mixing angle of η_l and η_s

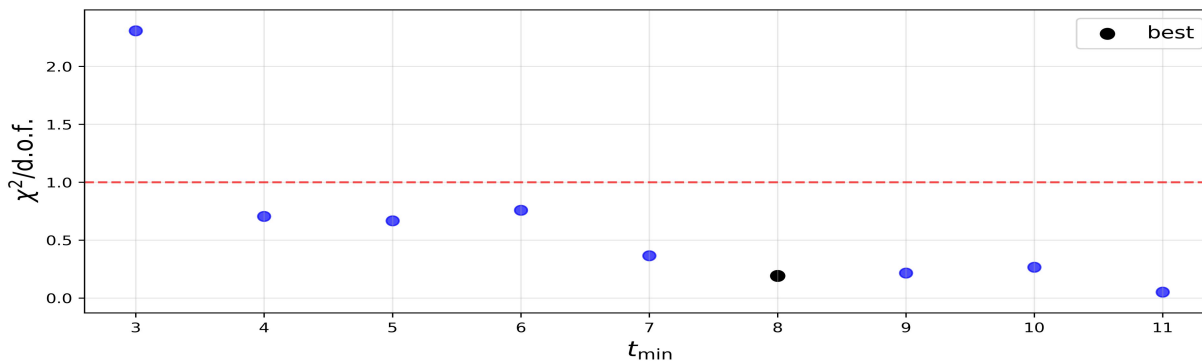
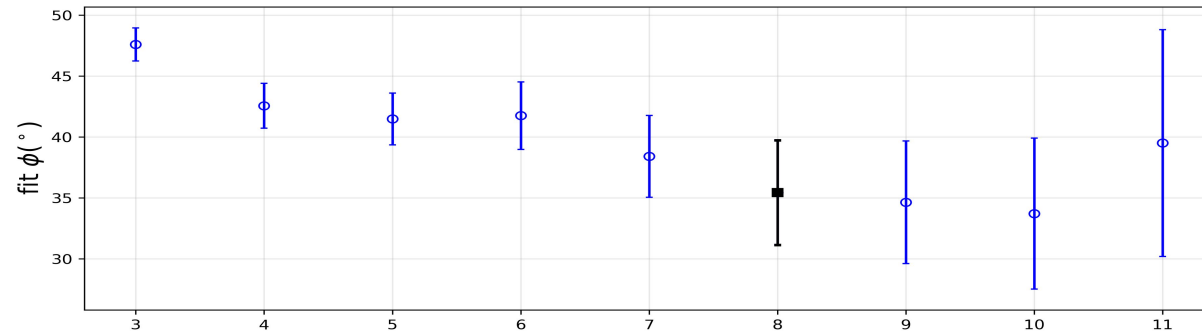


C48P14 $m_\pi L = 3.47$

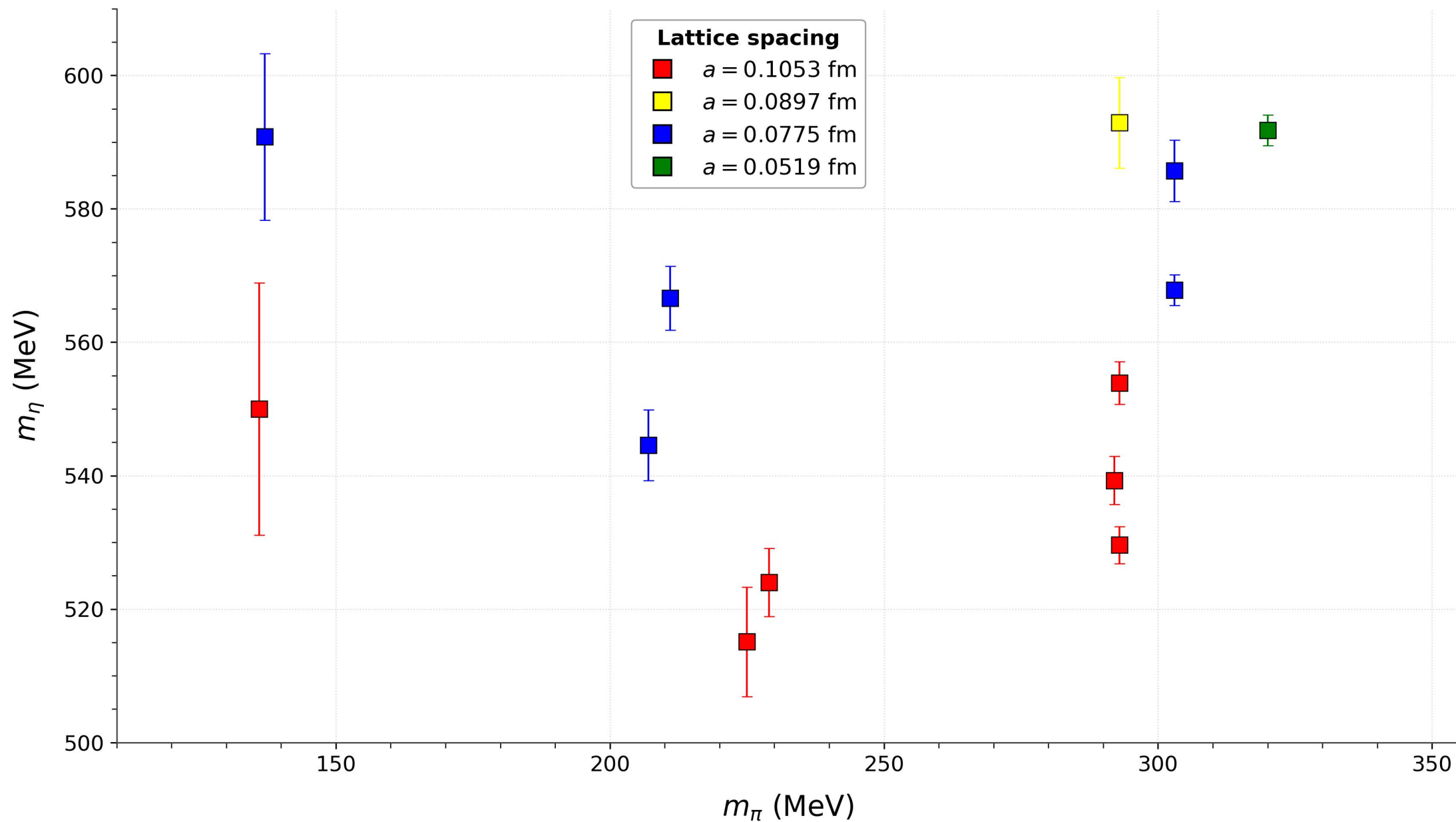
FVE Correction:

$$\phi^{phy} \sim \phi + c \frac{M_\pi^2}{\sqrt{M_\pi} L} e^{-M_\pi L}$$

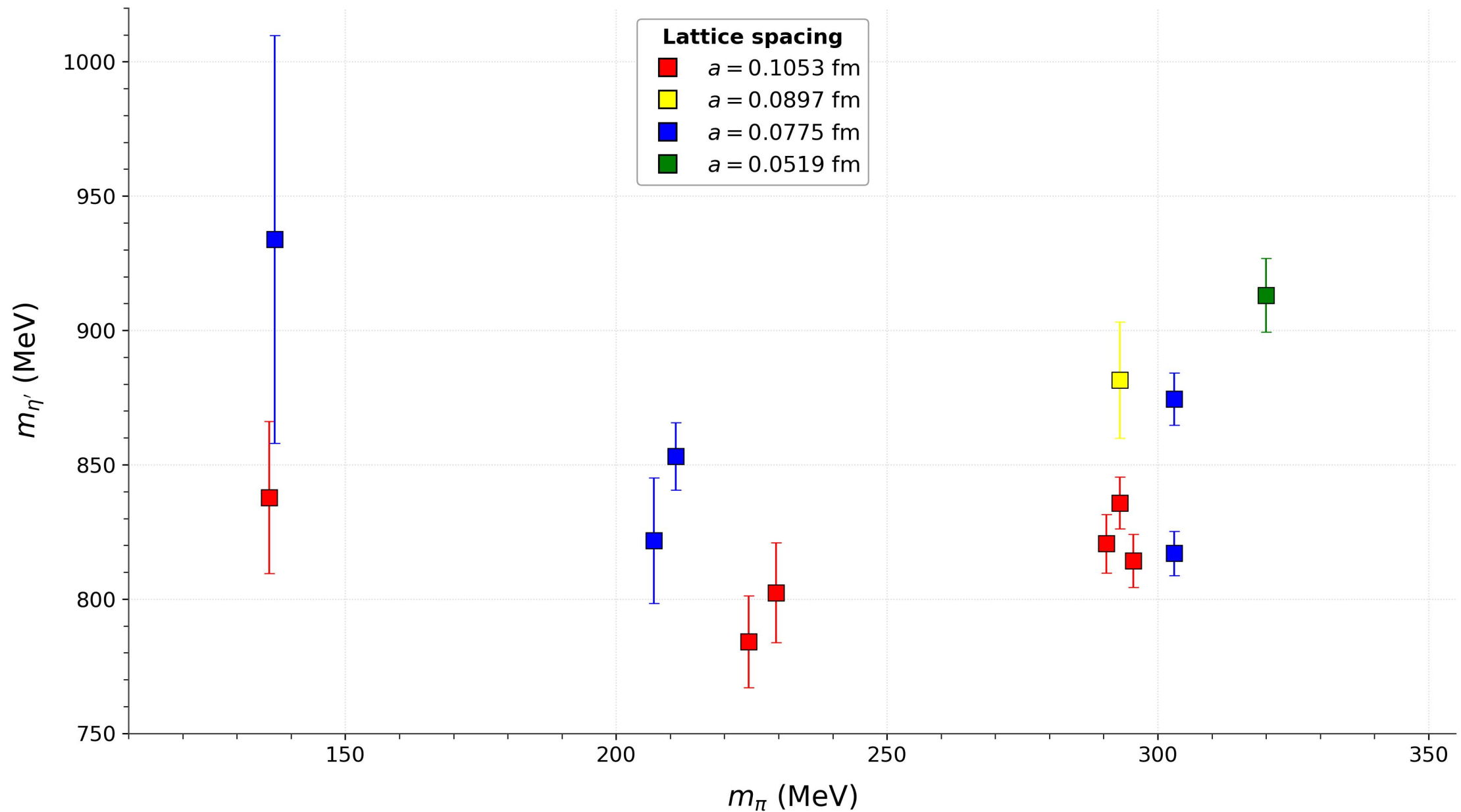
Konstantin Ottnad et al. Eur. Phys. J. A 61,169 (2025)



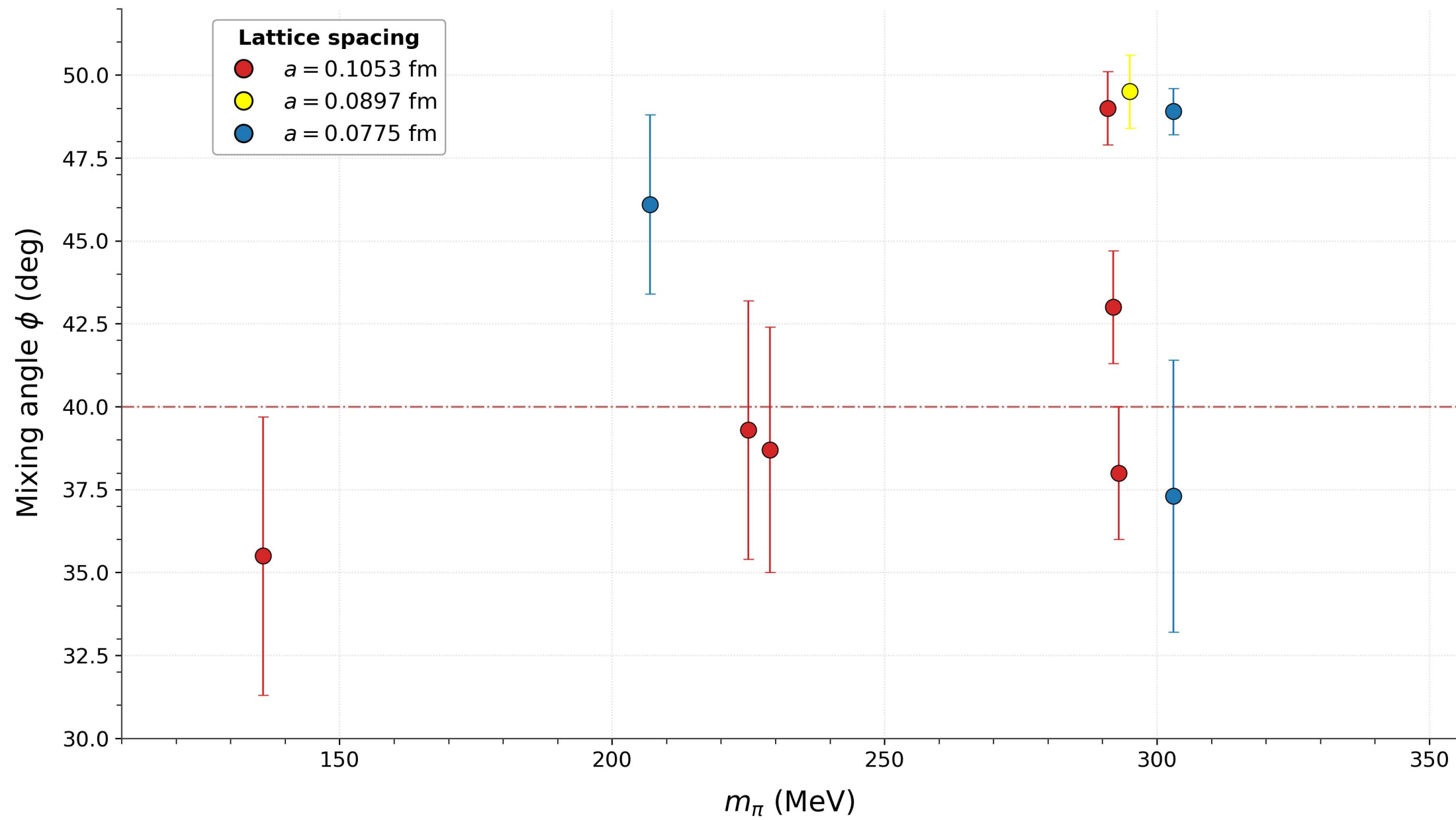
η Masses across All Ensembles



η' Masses across All Ensembles



Mixing angles across Ensembles with good signal



Conclusions

- We study the properties of η and η' mesons in lattice QCD, addressing chiral symmetry breaking and the $U(1)_A$ quantum anomaly.
- By employing the GEVP method, we successfully extract the η and η' masses and the mixing angles across multiple lattice ensembles.
- Multi-ensemble joint physical extrapolation is currently in progress.

Thank you!

Questions & Discussion