



中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

The 6th China Lattice QCD Workshop

Portraits of Charmoniumlike States from Lattice QCD

Speaker: Qingyang Liu

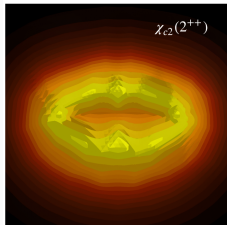
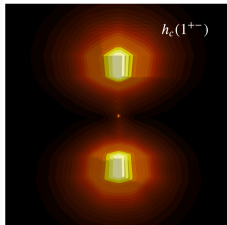
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Based on: *arXiv:2505.21193*



1. What does a meson (charmonium $c\bar{c}$) look like inside?
2. How important are relativistic effects in charmonium?
3. What is the $c\bar{c}$ distribution inside a hybrid $c\bar{c}g$?

Density–density correlation

Meson interpolators: $O_H^{(i)} = \bar{c} \Gamma_H^{(i)} c, \quad i = 3$

$$\begin{array}{lll} \eta_c : \gamma_5 & J/\psi : \gamma_i & h_c : \epsilon_{ijk} \sigma_{jk} \\ \chi_{c0} : \mathbf{1} & \chi_{c1} : \gamma_5 \gamma_i & \chi_{c2} : |\epsilon_{ijk}| \gamma_j D_k \end{array}$$

Density operator: normal-ordered charm-number density

$$\rho(\vec{x}) = [: \bar{c} \gamma_4 c :](\vec{x}) = [: c^\dagger c :](\vec{x}).$$

Density–density correlation

For a hadron at rest, probe the equal-time relative separation:

$$C_H(\vec{r}) = \langle H | \rho(\vec{0}) \rho(\vec{r}) | H \rangle.$$

Wave-function interpretation

Neglect $c\bar{c}$ annihilation and treat the pair as $c\bar{c}'$:

$$C_H(\vec{r}) = \int d^3\vec{x} |\Phi_c(\vec{x})|^2 |\Phi_{c'}(\vec{x} - \vec{r})|^2.$$

Lattice calculation

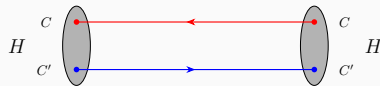
$$C_H(\vec{r}) \approx \frac{C_H^{(4)}(t_1, t; \vec{r})}{C_H^{(2)}(t_1)}$$

source $\rightarrow$ insertion $\rightarrow$ sink: $0 \rightarrow t \rightarrow t_1$

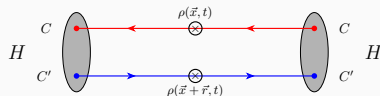
$$t = t_1/2, \quad t_1/a_t = 24, 48$$

$$N_f = 2, \sim 400 \text{ cfgs}, N_S^3 \times N_T = 16^3 \times 128$$

$$a_s/a_t \simeq 5, a_s = 0.136(2) \text{ fm}, m_\pi \simeq 420 \text{ MeV}$$

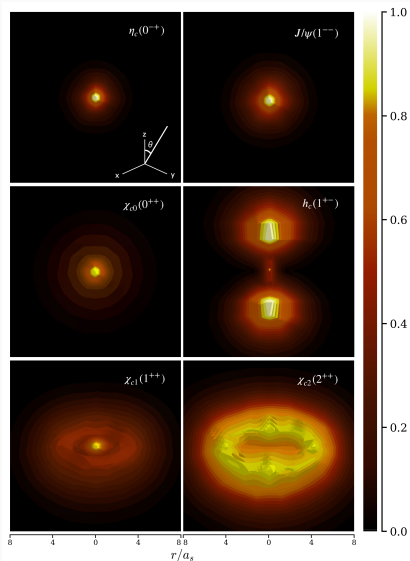


two-point function $C_H^{(2)}$



four-point function $C_H^{(4)}$

Charmonium spatial distributions



spherical · spindle-shaped · toroidal
(doughnut-shaped)

Nonrelativistic L - S coupling $\Rightarrow J^{PC}$

$$\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

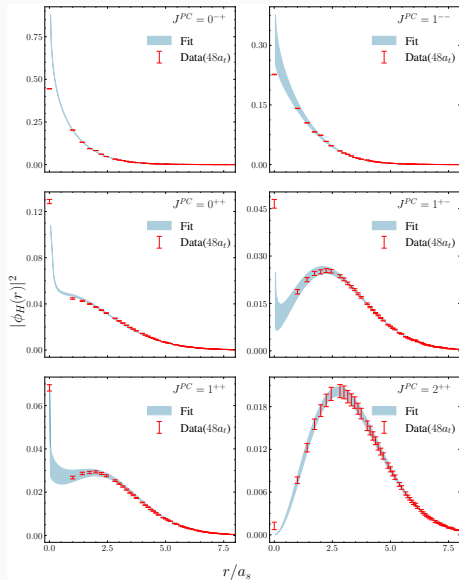
$L \backslash S$	$S = 0$	$S = 1$
$L = 0$	$^1S_0 : \eta_c(0^{-+})$	$^3S_1 : J/\psi(1^{--})$
$L = 1$	$^1P_1 : h_c(1^{+-})$	$^3P_J : \chi_{cJ}(J^{++}), J = 0, 1, 2$

Total distribution: radial part $\times$ angular part

$$C_H(\vec{r}) = |\phi_H(r)|^2 |\xi_H(\theta, \phi)|^2,$$

Angular part ξ_H is determined by Clebsch–Gordan coefficients and spherical harmonics Y_{Lm_L} .

$H(J J_z\rangle)$	$ \xi_H(\theta, \phi) ^2$
$\eta_c(00\rangle), J/\psi(10\rangle)$	$ Y_{00} ^2 = \frac{1}{4\pi} \propto \text{const.}$
$\chi_{c0}(00\rangle)$	$\frac{1}{3} \sum_{m=-1}^1 Y_{1m} ^2 = \frac{1}{4\pi} \propto \text{const.}$
$h_c(10\rangle)$	$ Y_{10} ^2 \propto \cos^2 \theta$
$\chi_{c1}(10\rangle)$	$ Y_{1\pm 1} ^2 \propto \sin^2 \theta$
$\chi_{c2}((22\rangle - 2, -2\rangle)/\sqrt{2})$	$ Y_{1\pm 1} ^2 \propto \sin^2 \theta$



Panels (top left $\rightarrow$ bottom right): η_c , J/ψ , χ_{c0} , h_c , χ_{c1} , χ_{c2} .

Nonrelativistic expectation

- Radial normalization ($d^3\mathbf{r} = r^2 dr d\Omega$):

$$1 = 4\pi \int_0^\infty dr r^2 |\phi_H(r)|^2.$$
- NRQM bound-state form:

$$\phi_{nL}(r) = r^L f_{nL}(r), \quad f_{nL}(0) \neq 0, \quad f_{nL}(r \rightarrow \infty) \rightarrow 0.$$

- r^L controls the origin, $f_{nL}(r)$ the large- r falloff

$$S \text{ wave : } |\phi_{1S}(r)|^2 \xrightarrow{r \rightarrow 0} \text{const.}, \quad |\phi_{1S}(0)|^2 \neq 0,$$

$$P \text{ wave : } |\phi_{1P}(r)|^2 \xrightarrow{r \rightarrow 0} \propto r^2, \quad |\phi_{1P}(0)|^2 = 0.$$

- Lattice:** $|\phi_{h_c, \chi_{c0}, \chi_{c1}}(0)|^2 \neq 0$, while $|\phi_{\chi_{c2}}(0)|^2 \approx 0$.
Pure NRQM fails—relativistic effects are required.
- Relativistic fit:** Blue bands are best-fit solutions of the Cornell–Dirac equation.

1. Solve the single-particle Dirac equation in an isotropic Cornell potential model.

$$[\vec{\alpha} \cdot \hat{p} + \beta(\bar{m} + \sigma r)] \Psi = \left(E + \frac{\alpha_c}{r}\right) \Psi, \quad \Psi_{\kappa, m}(\vec{r}) = \begin{pmatrix} F_{\kappa}(r) \chi_{jm}^{(l)}(\theta, \phi)/r \\ iG_{\kappa}(r) \chi_{jm}^{(l')}(\theta, \phi)/r \end{pmatrix}.$$

$l = j - \frac{1}{2}$ ($\kappa < 0$), $l = j + \frac{1}{2}$ ($\kappa > 0$), $l' = 2j - l$: upper/lower orbital angular momenta,
 $\kappa = -(l + 1)$ (with $j = l + \frac{1}{2}$), $\kappa = l$ (with $j = l - \frac{1}{2}$) : Dirac spin-orbit quantum number.

2. Upper and lower orbital components of each charmonium.

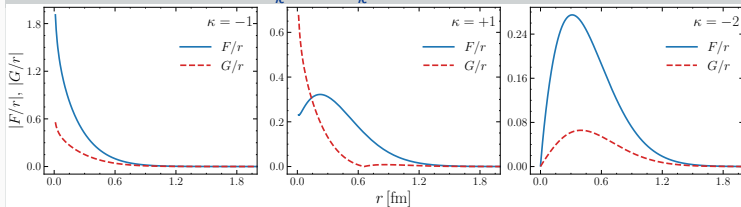
$H(J^{PC})$	κ	j_c	F_{κ} : upper (l)	G_{κ} : lower (l')
$\eta_c(0^{-+})$	-1	$\frac{1}{2}$	S ($l = 0$)	P ($l' = 1$)
$J/\psi(1^{--})$	-1	$\frac{1}{2}$	S ($l = 0$)	P ($l' = 1$)
$\chi_{c0}(0^{++})$	1	$\frac{1}{2}$	P ($l = 1$)	S ($l' = 0$)
$h_c(1^{+-})$	1, -2	$\frac{1}{2}, \frac{3}{2}$	P ($l = 1$)	S ($l' = 0$) $\oplus$ D ($l' = 2$)
$\chi_{c1}(1^{++})$	1, -2	$\frac{1}{2}, \frac{3}{2}$	P ($l = 1$)	S ($l' = 0$) $\oplus$ D ($l' = 2$)
$\chi_{c2}(2^{++})$	-2	$\frac{3}{2}$	P ($l = 1$)	D ($l' = 2$)

3. The separated radial equation is solved numerically and fits the lattice distributions very well.

$$\begin{pmatrix} \bar{m} + \sigma r - \frac{\alpha_c}{r} & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & -\bar{m} - \sigma r - \frac{\alpha_c}{r} \end{pmatrix} \begin{pmatrix} F_{\kappa} \\ G_{\kappa} \end{pmatrix} = E \begin{pmatrix} F_{\kappa} \\ G_{\kappa} \end{pmatrix}.$$

Radial effects of the lower Dirac component

Numerical solutions for F_κ and G_κ



- F_κ/r : upper component.
- G_κ/r : lower component.
- The lower component is appreciable over the relevant radial range.

Short-distance behavior in the central-field model

$$F_\kappa, G_\kappa \sim r^s, \quad s = \sqrt{\kappa^2 - \alpha_c^2}, \quad (F_\kappa^2 + G_\kappa^2)/r^2 \sim r^{2s-2}$$

- $|\kappa| = 1$: short-distance enhancement for $0 < \alpha_c < 1$.
- $\chi_{c2}, \kappa = -2$: suppression at the origin for the fitted couplings.

The continuum asymptotics are not directly resolved at finite lattice spacing.

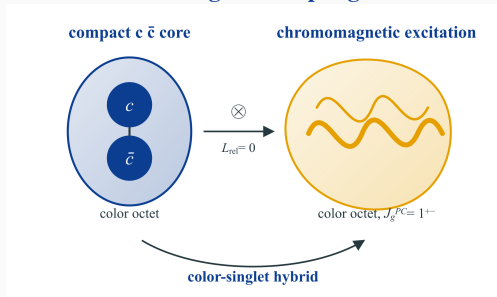
1. Exotic quantum numbers require gluonic degrees of freedom.

A pure $q\bar{q}$ meson obeys $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$; therefore 1^{-+} and 0^{+-} are forbidden.

A charmonium hybrid couples $c\bar{c}$ to an excited gluonic field, schematically $|c\bar{c}g\rangle$.

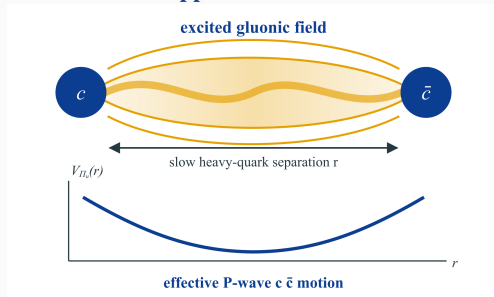
2. Two representative pictures of hybrid dynamics (schematics by OpenAI Codex).

Core-gluon coupling



Compact $c\bar{c}(8_c)$ core $\otimes$ chromomagnetic $1^{+-}(8_c)$ excitation $\rightarrow 1_c$.

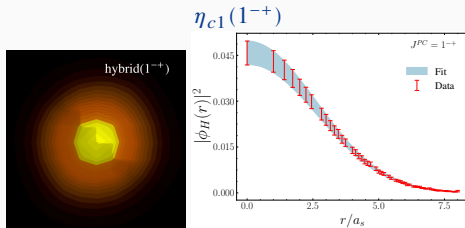
Born-Oppenheimer / flux tube



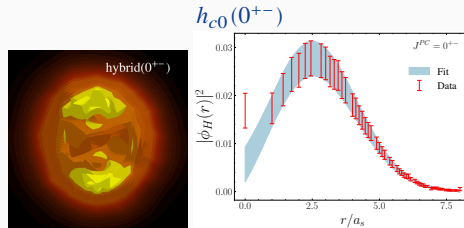
Slow $c\bar{c}$ motion in an excited gluonic BO potential; the simplest 1^{-+} portrait has effective P-wave motion.

Which picture does Nature prefer?

$C_H(\vec{r})$ measures the $c\text{--}\bar{c}$ separation, not the full $c\bar{c}g$ size



$$\sqrt{\langle r^2 \rangle} = 0.58(7) \text{ fm}$$



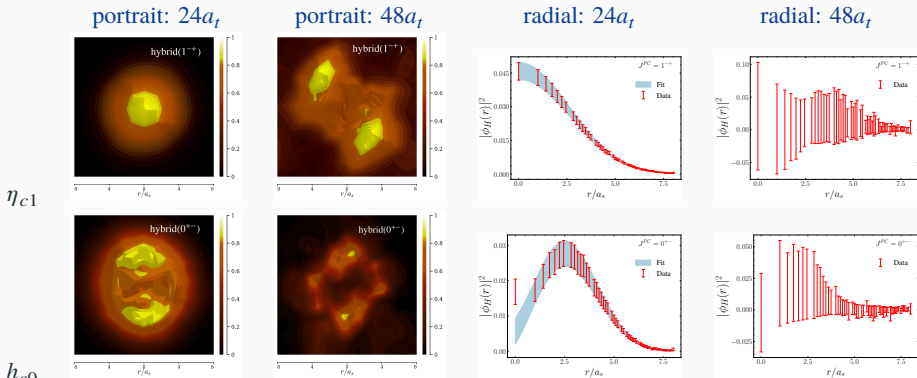
$$\sqrt{\langle r^2 \rangle} = 0.59(8) \text{ fm}$$

Qualitative interpretation

- **Scale comparison:** $1S$: 0.43–0.47 fm | exotic $c\bar{c}$: 0.58–0.59 fm | $1P$: 0.63–0.72 fm.
- **Core–gluon picture:** η_{c1} : $[c\bar{c}(1^{--})] \otimes [g_{\text{mag}}(1^{+-})]_{L_{\text{rel}}=0}$, h_{c0} : $[c\bar{c}(1^{++})] \otimes [g_{\text{mag}}(1^{+-})]_{L_{\text{rel}}=0}$.
The portraits are compatible with relative S-wave coupling; the h_{c0} assignment is more tentative.
- **Uncertainty:** Higher-state contamination may remain; Affected by statistical fluctuations.
- **Within uncertainties, the core–gluon assignments are favored over the simplest adiabatic BO picture.**

Exotic channels: time separation and radial fits

At $48a_t$ the exotic radial errors exceed the central values; only $t_1 = 24a_t$ retains qualitative spatial information.



The $24a_t$ profiles admit a common empirical fit.

$$\phi_H(r) = A_H(1 + a_H r)e^{-b_H r^{\omega_H}}.$$

	η_{c1}	h_{c0}
A_H	0.213(9)	0.075(23)
$a_H a_t$	0	0.8(4)
$b_H [\text{GeV}^2]$	0.074(13)	0.082(46)
ω_H	2.05(9)	2.20(27)

1. What does a meson (charmonium $c\bar{c}$) look like inside?

The first lattice-QCD spatial portraits reveal the expected angular geometries.

2. How important are relativistic effects in charmonium?

The Dirac description accounts for the short-distance contrast among the conventional states.

3. What is the $c\bar{c}$ distribution inside a hybrid $c\bar{c}g$?

The exotic channels contain compact $c\bar{c}$ subsystems, favoring an S-wave core–gluon picture.

4. What is the $c\bar{c}$ distribution inside a fully charmed tetraquark $T_{cc\bar{c}\bar{c}}$? *Coming soon...*

Previously, in $J/\psi J/\psi$ scattering, we identified an $X(6200) 0^{++}$ virtual-state candidate and an $X(6600) 2^{++}$ resonance candidate (arXiv:2505.23220; arXiv:2505.24213).

Thank you

Supplemental Material: nonrelativistic angular derivation

Polarization choices used for the angular portraits $J/\psi, h_c, \chi_{c1} : |JJ_z\rangle = |10\rangle, \quad \chi_{c2} : (|22\rangle - |2, -2\rangle)/\sqrt{2}.$

Clebsch–Gordan coupling maps each $^{2S+1}L_J$ assignment to an angular density.

$$\xi_{JM}^{LS}(\hat{r}) = \sum_{m_L, m_S} \langle Lm_L, Sm_S | JM \rangle Y_{Lm_L}(\hat{r}) \chi_{Sm_S}, \quad W_H(\hat{r}) = \sum_{m_S} |\xi_{H, m_S}(\hat{r})|^2.$$

The resulting projections reproduce the leading lattice shapes.

$$\begin{aligned} \eta_c(|00\rangle), J/\psi(|10\rangle) : \quad \xi &= Y_{00}\chi_{00}, Y_{00}\chi_{10}, & W &= \frac{1}{4\pi}; \\ h_c(|10\rangle) : \quad \xi &= Y_{10}\chi_{00}, & W &= \frac{3}{4\pi} \cos^2 \theta; \\ \chi_{c0}(|00\rangle) : \quad \xi &= \frac{1}{\sqrt{3}} (Y_{11}\chi_{1,-1} - Y_{10}\chi_{10} + Y_{1,-1}\chi_{11}), & W &= \frac{1}{4\pi}; \\ \chi_{c1}(|10\rangle) : \quad \xi &= \frac{1}{\sqrt{2}} (Y_{11}\chi_{1,-1} - Y_{1,-1}\chi_{11}), & W &= \frac{3}{8\pi} \sin^2 \theta; \\ \chi_{c2}((|22\rangle - |2, -2\rangle)/\sqrt{2}) : \quad \xi &= \frac{1}{\sqrt{2}} (Y_{11}\chi_{11} - Y_{1,-1}\chi_{1,-1}), & W &= \frac{3}{8\pi} \sin^2 \theta. \end{aligned}$$

Thus χ_{c0} is spherical, while the polarized h_c, χ_{c1} , and χ_{c2} retain characteristic anisotropies.

$$|Y_{00}|^2 = \frac{1}{4\pi}, \quad |Y_{10}|^2 = \frac{3}{4\pi} \cos^2 \theta, \quad |Y_{1,\pm 1}|^2 = \frac{3}{8\pi} \sin^2 \theta.$$

The Coulomb term fixes the short-distance power law.

$$\begin{aligned} \frac{dF_\kappa}{dr} &= -\frac{\kappa}{r}F_\kappa + \frac{\alpha_c}{r}G_\kappa, & \frac{dG_\kappa}{dr} &= +\frac{\kappa}{r}G_\kappa - \frac{\alpha_c}{r}F_\kappa, \\ F_\kappa &= ar^s, \quad G_\kappa = br^s : & \begin{pmatrix} s + \kappa & -\alpha_c \\ \alpha_c & s - \kappa \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} &= 0 \implies s = \sqrt{\kappa^2 - \alpha_c^2}. \end{aligned}$$

For $|\kappa| = 1$ the density diverges weakly, but remains integrable.

$$|\Psi_\kappa(\vec{r})|^2 \sim r^{2(s-1)}, \quad \int_0^\epsilon dr r^2 |\Psi_\kappa(r)|^2 \sim \int_0^\epsilon dr r^{2s} < \infty.$$

$$|\kappa| = 1 : s < 1 \Rightarrow \text{divergent at } r = 0, \quad |\kappa| = 2 : s > 1 \Rightarrow \text{vanishing at } r = 0.$$

The scalar confining term produces a normalizable Gaussian tail.

$$F'_\kappa \simeq \sigma r G_\kappa, \quad G'_\kappa \simeq \sigma r F_\kappa \implies F''_\kappa, G''_\kappa \simeq \sigma^2 r^2 (F_\kappa, G_\kappa) \implies F_\kappa, G_\kappa \propto e^{-\sigma r^2/2}.$$

Couple the charm-quark Dirac spinor to the antiquark spin, then sum over spin components.

$$\Psi_{\kappa m}(\vec{r}) = \left(\begin{array}{c} F_{\kappa}(r) \chi_{jm}^{(l)}/r \\ iG_{\kappa}(r) \chi_{jm}^{(l')}/r \end{array} \right) \xrightarrow{\langle jm, \frac{1}{2} m_{\bar{c}} | JM \rangle} \Psi_{JM} \xrightarrow{\sum_{s_c, s_{\bar{c}}} |\cdot|^2} |\Psi_H(\vec{r})|^2,$$

$$\left(\begin{array}{c} \Psi_{h_c} \\ \Psi_{\chi_{c1}} \end{array} \right) = \left(\begin{array}{cc} 1/\sqrt{3} & -\sqrt{2/3} \\ \sqrt{2/3} & 1/\sqrt{3} \end{array} \right) \left(\begin{array}{c} \Xi^{(1/2)} \\ \Xi^{(3/2)} \end{array} \right).$$

Densities after summing over both constituent spins.

$$\mathcal{N}(r) \equiv \frac{1}{4\pi r^2}, \quad F_{\kappa} \equiv F_{\kappa}(r), \quad G_{\kappa} \equiv G_{\kappa}(r), \quad |\Psi_{\eta_c}(\vec{r})|^2 = |\Psi_{J/\psi}(\vec{r})|^2 = \mathcal{N} (F_{-1}^2 + G_{-1}^2),$$

$$|\Psi_{\chi_{c0}}(\vec{r})|^2 = \mathcal{N} (F_1^2 + G_1^2),$$

$$|\Psi_{h_c}(\vec{r})|^2 = \mathcal{N} \left[(F_1 - \sqrt{2}F_{-2})^2 \cos^2 \theta + \frac{1}{3} \{ G_1^2 + G_{-2}^2 (3 \cos^2 \theta + 1) + 2G_1 G_{-2} (3 \cos^2 \theta - 1) \} \right],$$

$$|\Psi_{\chi_{c1}}(\vec{r})|^2 = \mathcal{N} \left[(\sqrt{2}F_1 + F_{-2})^2 \frac{\sin^2 \theta}{2} + \frac{1}{6} \{ 4G_1^2 + G_{-2}^2 (3 \cos^2 \theta + 1) - 4G_1 G_{-2} (3 \cos^2 \theta - 1) \} \right],$$

$$|\Psi_{\chi_{c2,xy}}(\vec{r})|^2 = \mathcal{N} \left[\frac{3}{2} (F_{-2}^2 + G_{-2}^2) \sin^2 \theta \right].$$

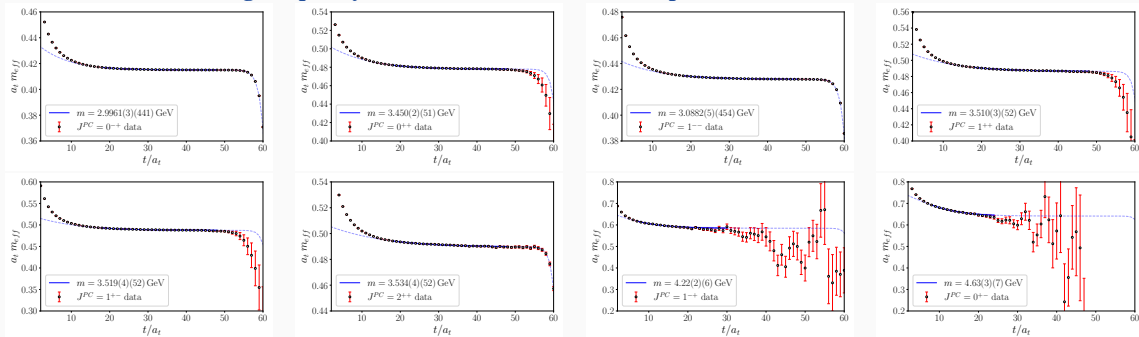
Channel-by-channel Cornell–Dirac fits describe the conventional radial distributions.

$$\langle r^2 \rangle = \frac{\int d^3\vec{r} r^2 C_H(\vec{r})}{\int d^3\vec{r} C_H(\vec{r})}.$$

$H(J^{PC})$	$\bar{m}$ [GeV]	α_c	σ [GeV ²]	$\sqrt{\langle r^2 \rangle}$ [fm]	κ
$\eta_c(0^{-+})$	0.81(2)	0.473(1)	0.083(2)	0.430(6)	−1
$J/\psi(1^{--})$	0.95(8)	0.278(41)	0.087(4)	0.469(8)	−1
$h_c(1^{+-})$	0.50(46)	0.59(10)	0.108(5)	0.666(12)	1, −2
$\chi_{c0}(0^{++})$	0.83(2)	0.688(2)	0.061(2)	0.627(9)	1
$\chi_{c1}(1^{++})$	1.23(16)	0.244(35)	0.089(10)	0.644(10)	1, −2
$\chi_{c2}(2^{++})$	0.73(16)	0.334(65)	0.096(19)	0.717(11)	−2

The RMS radii separate compact $1S$ states from the broader $1P$ states. Dimensional errors include the uncertainty in $a_s = 0.136(2)$ fm.

Signal quality sets the usable source–sink separation in each channel.

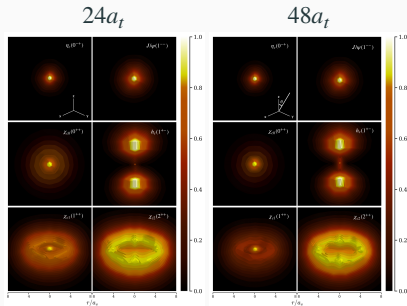


Conventional: use $t_1 = 48a_t$ to reduce excited-state contamination.

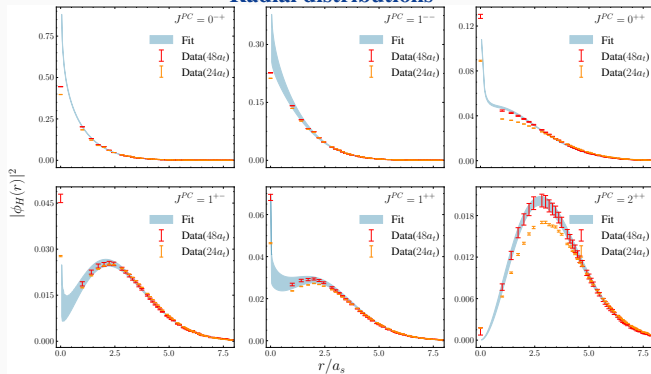
Exotic: use $t_1 = 24a_t$ before the signal deteriorates.

Conventional distributions are precise and qualitatively consistent at $t_1 = 24a_t$ and $48a_t$.

Angular portraits



Radial distributions



We therefore quote the $48a_t$ results, where excited-state contamination is expected to be smaller.