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Flowed quark field renormalization in lattice QCD: A Ward-identity approach and its validation using quark bilinears

based on 2609.23379

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2026.10.11, ShenZhen

Gradient flow: a gauge-covariant UV filter

- Gradient flow provides a **smooth** and **gauge-covariant** way to suppress short-distance fluctuations.

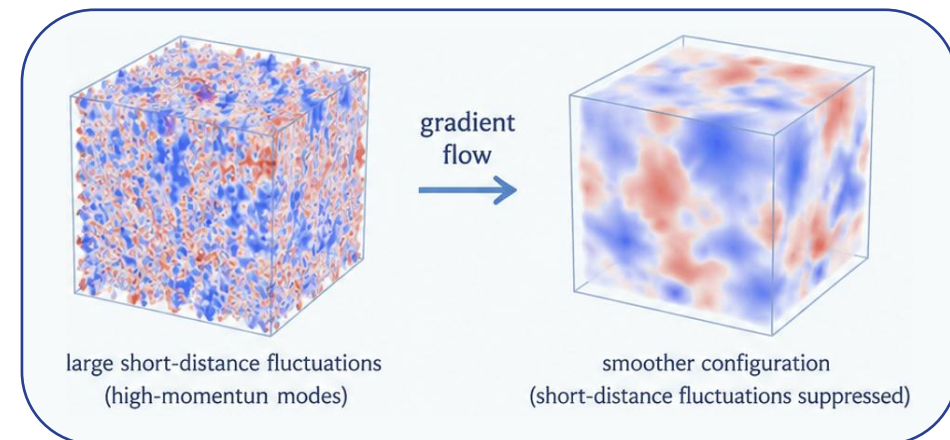
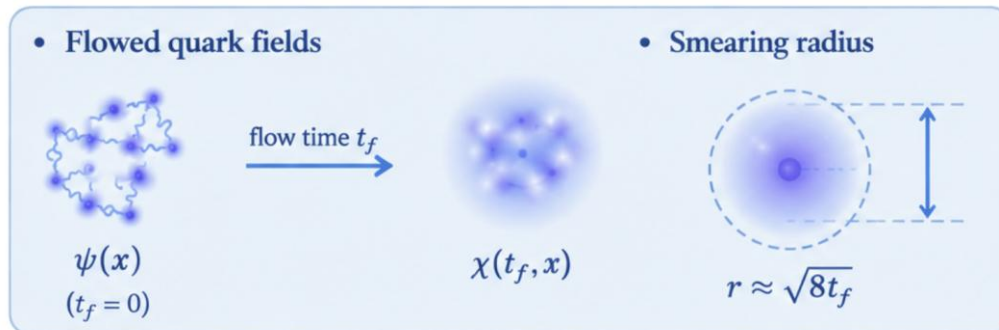
$$\partial_{t_f} \chi(t_f, x) = \Delta \chi(t_f, x), \quad \partial_{t_f} \bar{\chi}(t_f, x) = \bar{\chi}(t_f, x) \overleftarrow{\Delta} \quad \Delta = D_\mu D_\mu \quad \overleftarrow{\Delta} = \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

with $\chi(0, x) = \psi(x) \quad \bar{\chi}(0, x) = \bar{\psi}(x)$

Martin Lüscher and Peter Weisz. JHEP, 02:051, 2011.

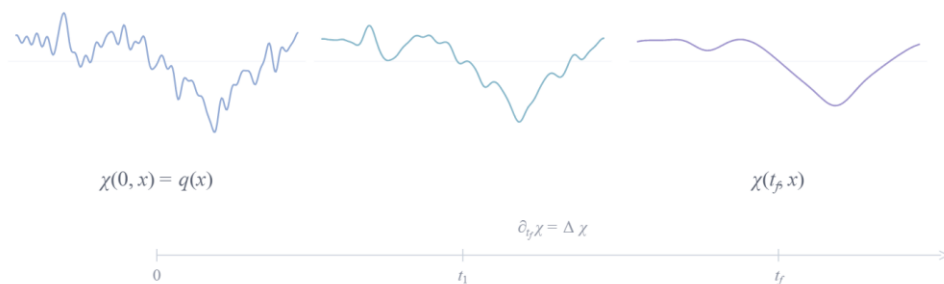
- By evolving the gauge and fermion fields along a fictitious flow time t_f , the high-momentum modes are exponentially suppressed, effectively smearing the fields over a physical radius.

- Flow time t_f provides a new scale.



Flowed quark fields need a renormalization

- Flowed quark fields $\chi(t_f)$ need a renormalization constant Z_χ . $\chi_R(t_f) = \mathring{Z}_\chi^{1/2} \chi(t_f)$



Schematic of boundary coupling

The gauge sector is protected by gauge invariance, whereas the flowed quark fields are **not**.

Martin Lüscher. JHEP, 04:123, 2013.

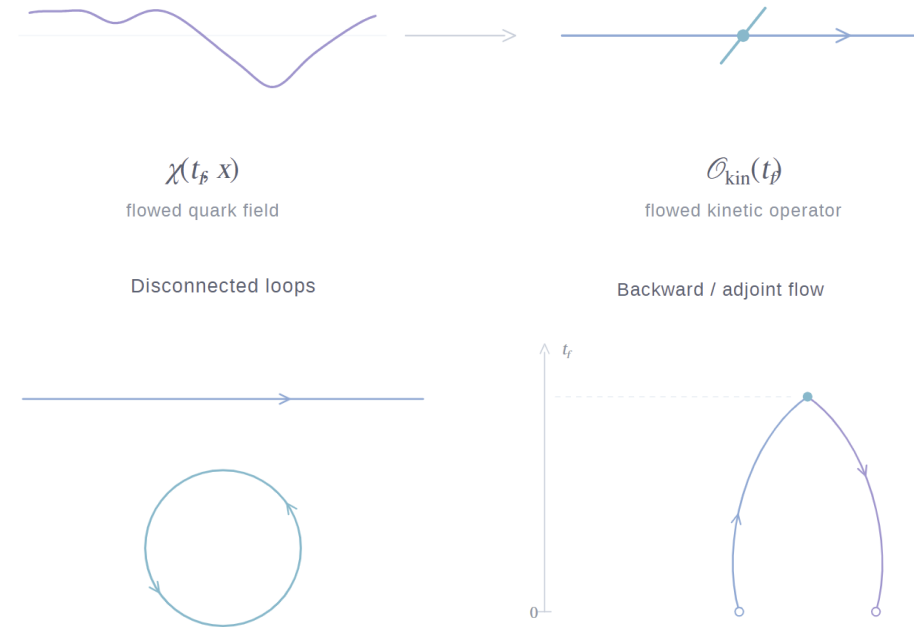
- Z_χ is essential for physical matrix elements.
- If we want to use flowed operators to calculate physical matrix elements, we need a reliable determination of Z_χ .

Conventional route: a clean definition, a costly evaluation.

- Ringed prescription: normalise the renormalized kinetic operator to its free-field value.

[Based on flowed kinetic operator]

$$\mathring{Z}_\chi(t_f) = \frac{-2N_c}{(4\pi t_f)^2 \langle \bar{\chi}(t_f, x) \overleftrightarrow{D} \chi(t_f, x) \rangle}.$$



- Numerically demanding:
 - Disconnected quark loops
 - Backward parts with adjoint fermion flow

Hiroshi Suzuki. Progress of Theoretical and Experimental Physics, 2013(8):083B03, 2013.

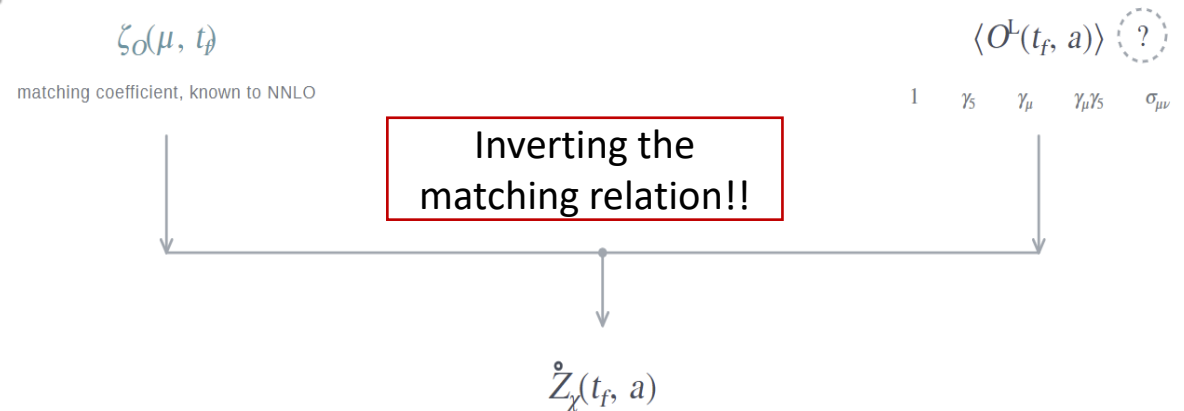
Can we do it differently?



Borgulat, Harlander, Kohnen, Lange, JHEP 05 (2024) 179

$$\lim_{a \rightarrow 0} \left[\mathring{Z}_\chi(t_f, a) \langle O^L(t_f, a) \rangle \right] = \zeta_O(\mu, t_f) \langle O^R(\mu) \rangle + \mathcal{O}(t_f)$$

- *We want Z_χ without the kinetic operator.*
- Inverting the matching relation:
No disconnected contribution
No adjoint flow
- BUT which operator to choose??



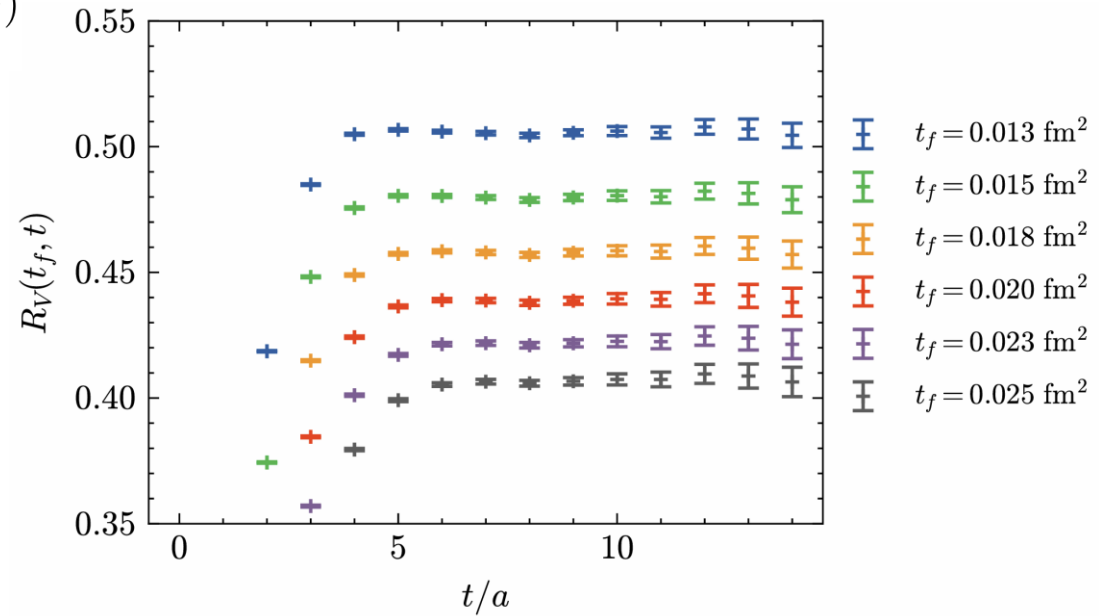
Ward identity gives us a direct route

$$\lim_{a \rightarrow 0} \left[\dot{Z}_\chi(t_f, a) \langle V_\mu^L(t_f, a) \rangle \right] = \zeta_V(t_f) \langle V_\mu^R \rangle + \mathcal{O}(t_f)$$

$$\lim_{a \rightarrow 0} \left[Z_V(a) \langle V_\mu^L(0, a) \rangle \right] = \langle V_\mu^R \rangle$$

$$\dot{Z}_\chi^V(t_f, a) \equiv \frac{\zeta_V(t_f) Z_V(a)}{\langle R_{V,\mu}^L(t_f, a) \rangle}$$

$$R_{V,i}^L(t_f, a) \equiv \frac{C_{V,i}^L(t_f; t)}{C_{V,i}^L(0; t)} \longrightarrow \frac{\langle V_i^L(t_f, a) \rangle}{\langle V_i^L(0, a) \rangle}$$



- Conceptually attractive: the normalization is obtained directly from a symmetry relation.
- Cheap — it avoids the disconnected contribution — and the signal is excellent.

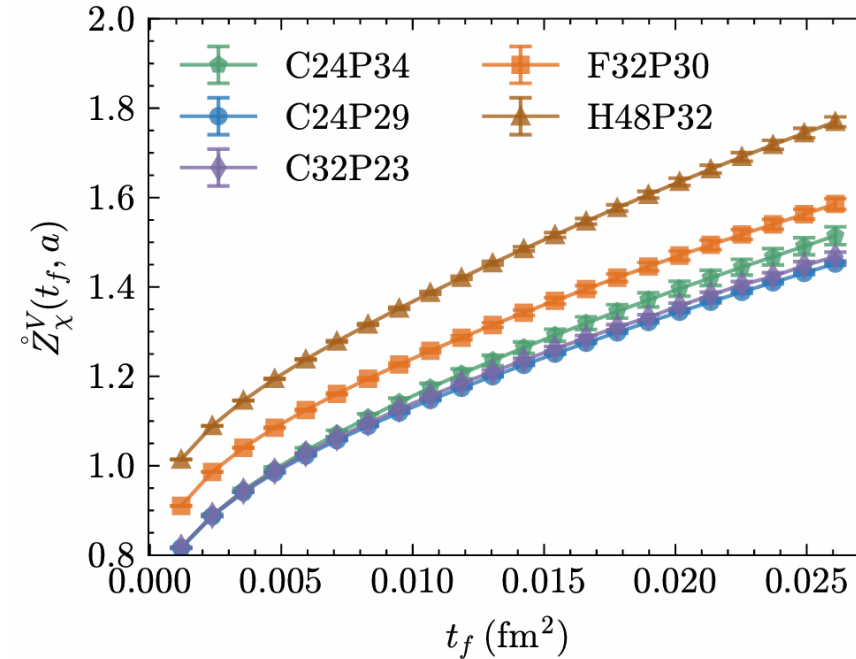
The numerical determination of Z_χ

- Flavor-nonsinglet vector bilinear
- Runge-Kutta flow $\epsilon_f = 0.01$ (lattice units)
- Flow-time range:
 $t_f = 0.0012\text{--}0.026 \text{ fm}^2$, $\Delta t_f = 0.0012 \text{ fm}^2$

$$C_\Gamma(t_f; t) = \sum_{\vec{x}} \langle 0 | \mathcal{O}_\Gamma(t_f; \vec{x}, t) \mathcal{O}_\Gamma^\dagger(0; \vec{0}, 0) | 0 \rangle$$

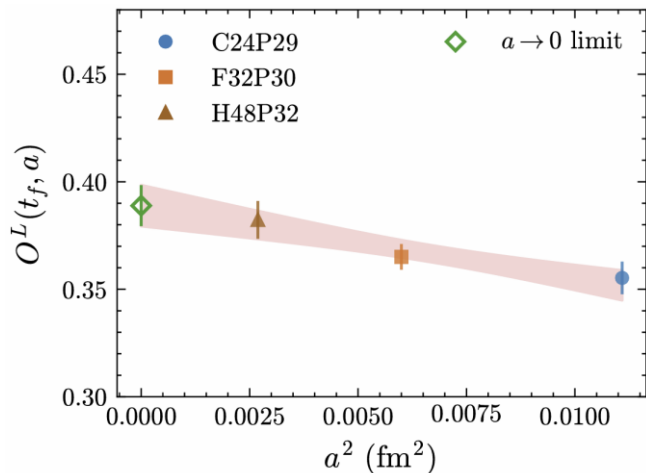
$$\mathcal{O}_\Gamma(t_f; \vec{x}, t) = \bar{\chi}(t_f; \vec{x}, t) \Gamma \chi(t_f; \vec{x}, t)$$

Ensemble	a (fm)	m_π (MeV)	n_{cfg}	$L^3 \times T$
C24P34	0.10530(18)	341.1(1.8)	247	$24^3 \times 64$
C24P29	0.10530(18)	292.7(1.2)	962	$24^3 \times 72$
C32P23	0.10530(18)	228.0(1.2)	499	$32^3 \times 64$
F32P30	0.07746(18)	303.2(1.3)	787	$32^3 \times 96$
H48P32	0.05187(26)	317.2(0.9)	610	$48^3 \times 144$

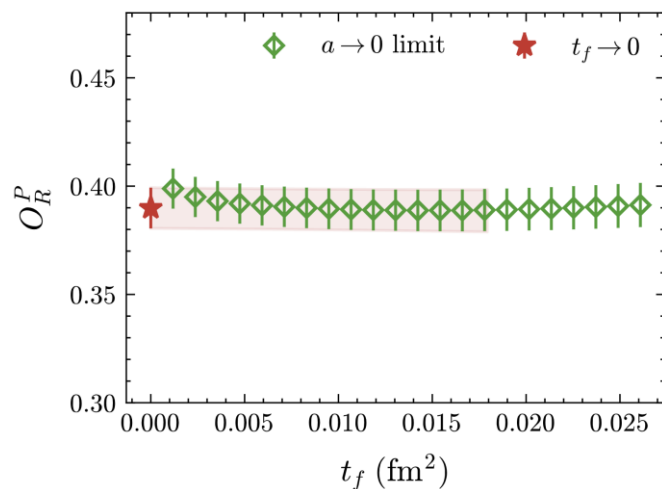


- Smooth and monotonic in t_f .
- A clear lattice-spacing dependence, as expected.
- Weak pion-mass dependence.

Universality of Z_χ^V across bilinear channels

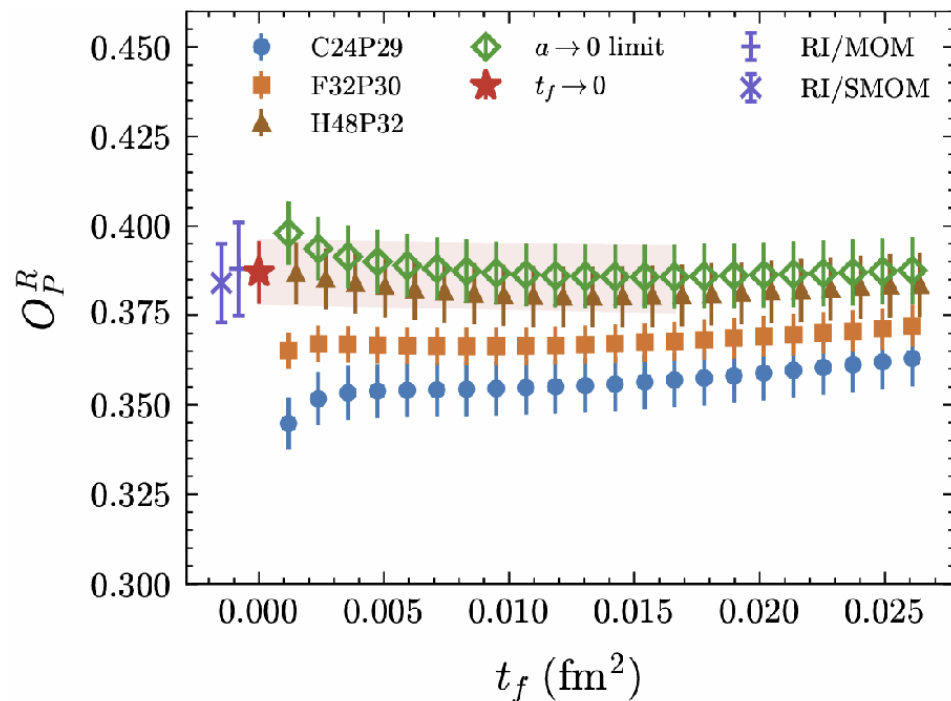


$$O_{\text{lat}}(a, t_f; \mu) = O_{\text{cont}}(t_f; \mu) + c_a(t_f; \mu)a^2$$



$$O_{\text{cont}}(t_f; \mu) = \langle O^R(\mu) \rangle + c_{t_f}(\mu)t_f$$

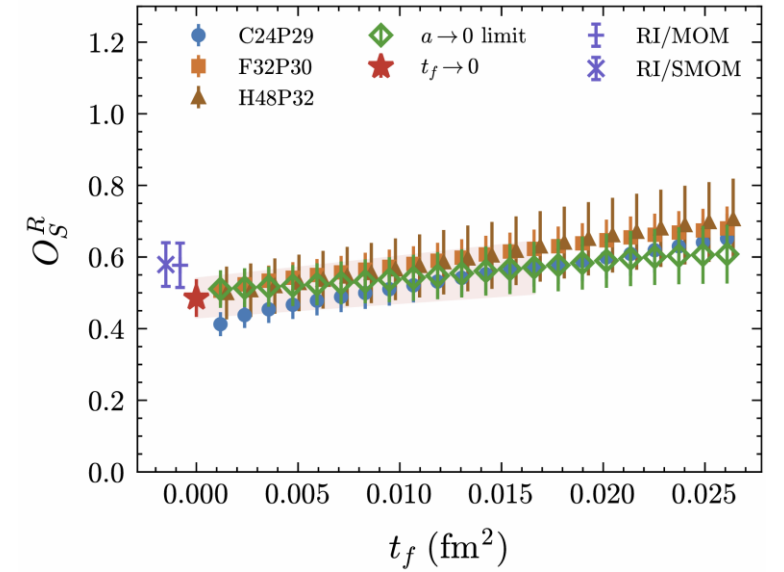
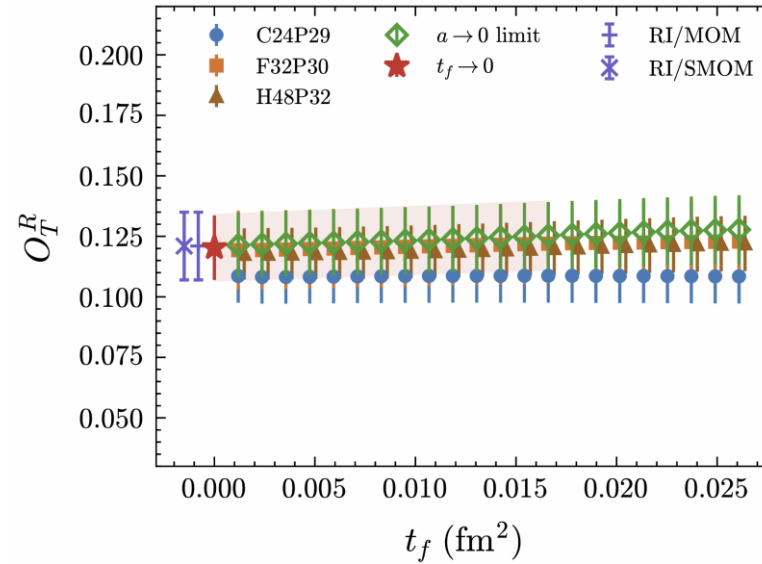
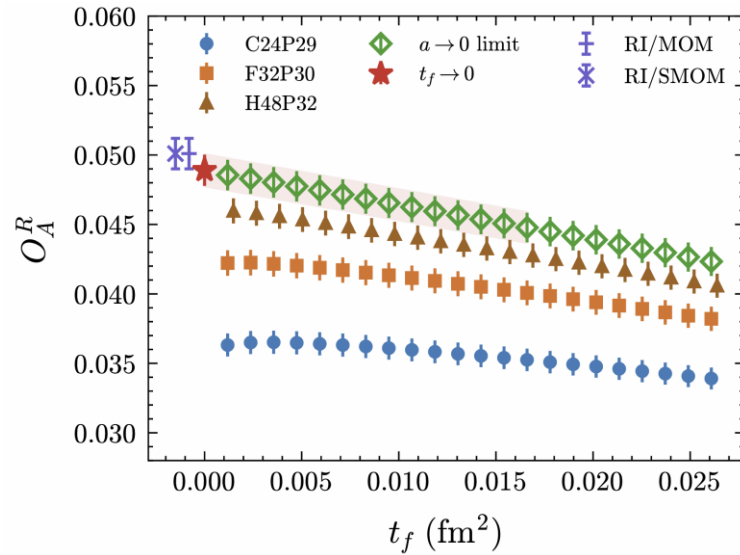
$$\langle O^R(\mu) \rangle = \lim_{t_f \rightarrow 0} \lim_{a \rightarrow 0} \left[\frac{\dot{Z}_\chi^V(t_f, a) \langle O^L(t_f, a) \rangle}{\zeta_O(\mu, t_f)} \right]$$



Z.-C. Hu et al., *Phys. Rev. D* 109, 054507 (2024).
H.-Y. Du et al., *Phys. Rev. D* 111, 054504 (2025).

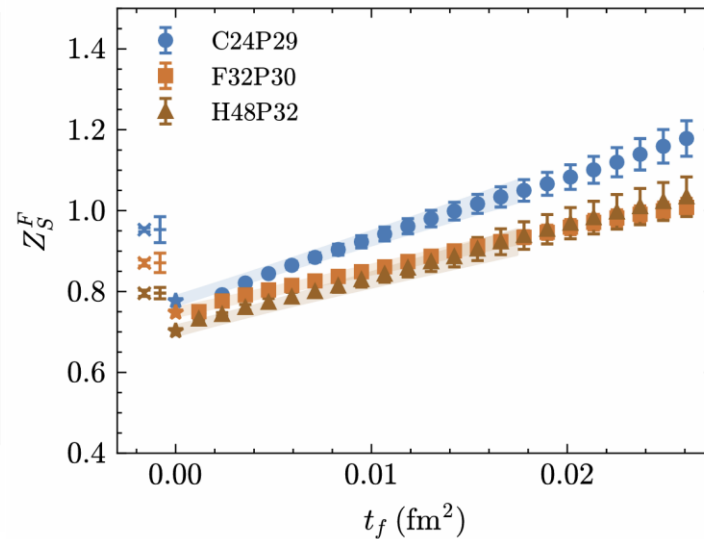
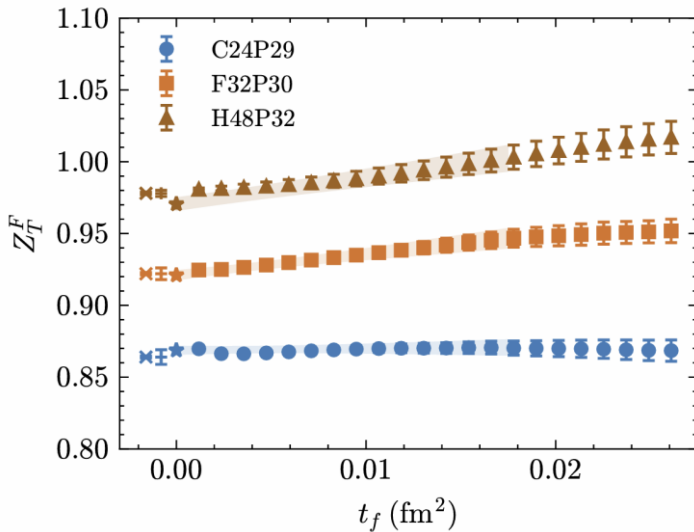
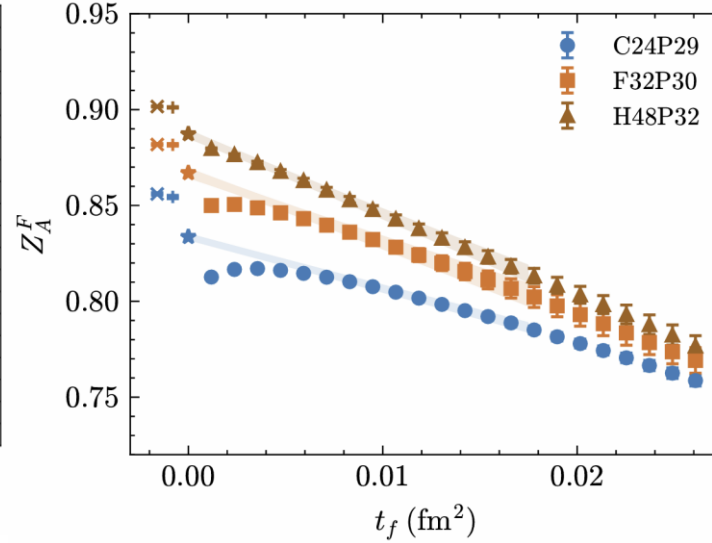
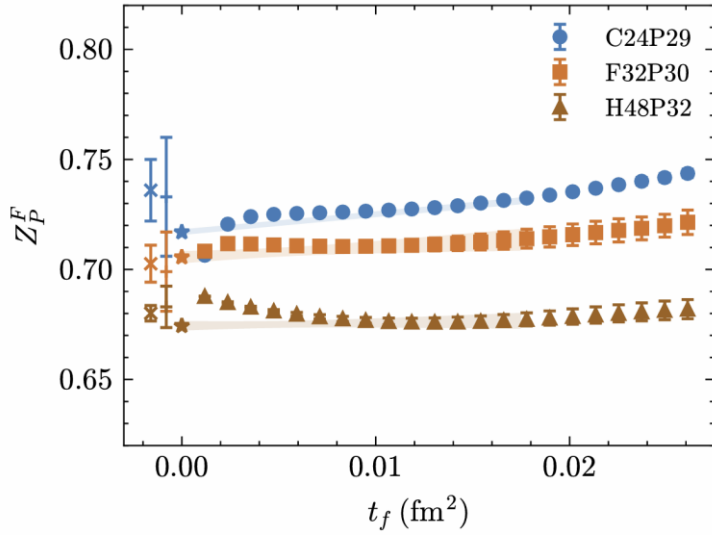
- Z_χ^V is fixed from the vector current, then used for all bilinears P, S, A, T , with channel dependent ζ_O .
- RI/MOM, RI/SMOM, and flow give the same physical quantity, and agree within uncertainties.

Universality of Z_χ^V across bilinear channels



- P, A : stronger a -dependence and larger continuum extrapolation.
- S, T : milder lattice-spacing dependence within current uncertainties.
- Small- t_f deviations are most visible in P, A , especially on coarse lattices.

Flow-based bilinear renormalization constant



$$Z_O^F(a, \mu) \equiv \frac{\langle O^R(\mu) \rangle}{\langle O^L(0, a) \rangle}$$

$$Z_O^F(a, \mu) \equiv \lim_{t_f \rightarrow 0} \left[\frac{\dot{Z}_\chi^V(t_f, a) \langle O^L(t_f, a) \rangle}{\zeta_O(\mu, t_f) \langle O^L(0, a) \rangle} \right]$$

$$= \lim_{t_f \rightarrow 0} \left[\frac{\dot{Z}_\chi^V(t_f, a) R_O^L(t_f, a)}{\zeta_O(\mu, t_f)} \right].$$

- Z_O^F is broadly consistent with RI/MOM [“+”] and RI/SMOM [“x”].
- Agreement tends to improve toward finer lattices.
- Perturbative truncation and fit-window dependence need further study.

Where does this lead?

- ✓ Bilinears provide a clean test of flowed-field renormalization, and are themselves important probes of hadron structure.
- With a reliable Z_χ , this opens the door to the more complicated quantities: such as parton distributions and the energy momentum tensor form factors and

Moments and parton distributions:

$$\begin{aligned} C_q^m &= \int_0^1 dx x^{m-1} f_q(x) \\ &= \frac{1}{\pi} \int_0^1 dx x^{m-1} \sum_{n=2,4,\dots} x^{-n} \text{Im } A_q^n \\ &\sim \text{Im} \frac{1}{\pi} \int_1^\infty d\omega \sum_n \omega^{n-m-1} A_q^n \\ &= \sum_n \frac{1}{2\pi i} \int \frac{dw}{w^{m-1}} w^{n-2} A_q^n \\ &= A_q^m. \end{aligned}$$

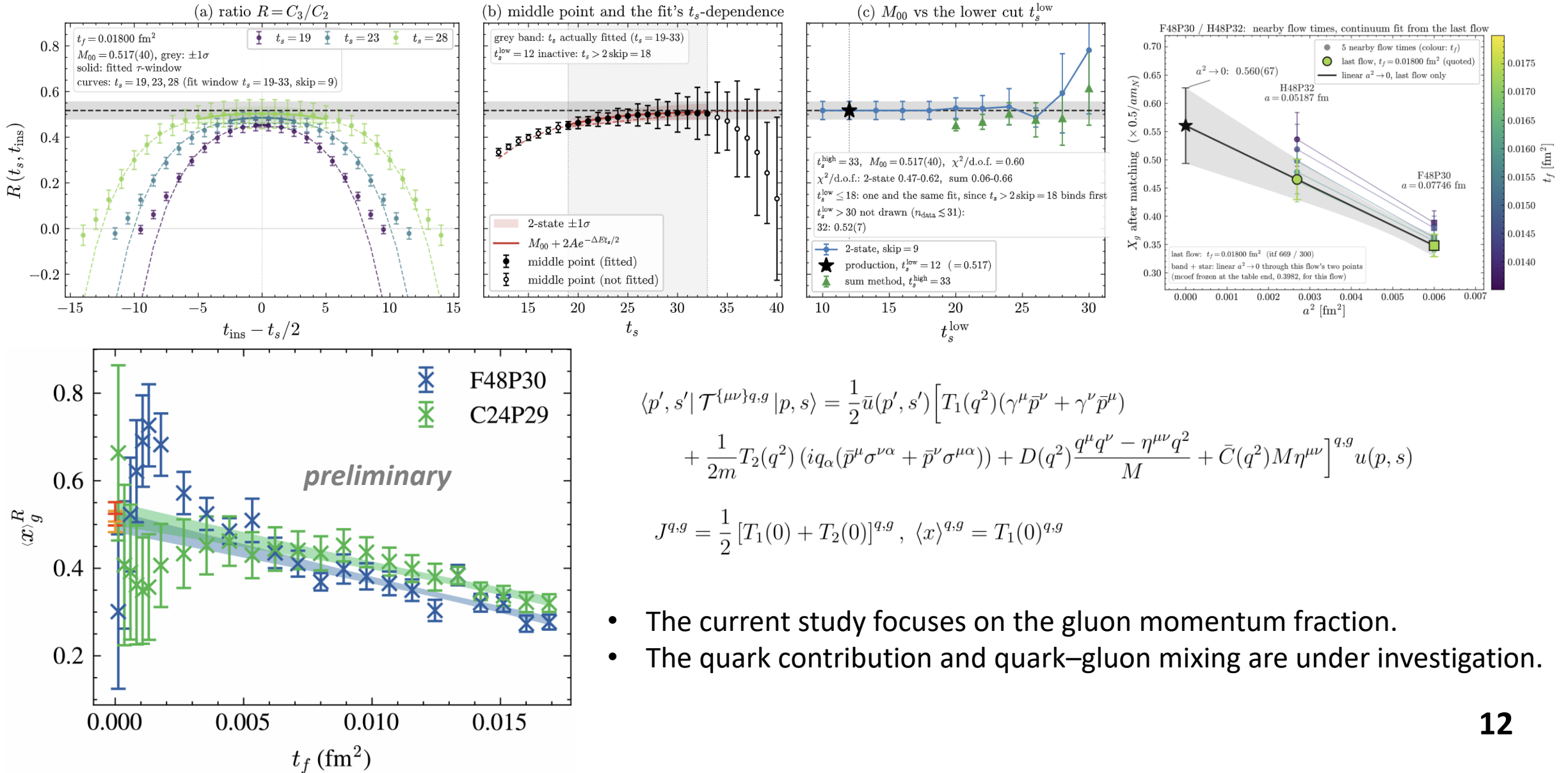
*Shindler, Phys. Rev. D 110, L051503 (2024);
A. Francis et al., Phys. Rev. Lett. 136, 171903 (2026).*

Energy–momentum tensor:

$$\begin{aligned} \mathcal{O}_{1,\mu\nu}(x) &\equiv F_{\mu\rho}^a(x) F_{\nu\rho}^a(x), \\ \mathcal{O}_{2,\mu\nu}(x) &\equiv \delta_{\mu\nu} F_{\rho\sigma}^a(x) F_{\rho\sigma}^a(x), \\ \mathcal{O}_{3_f,\mu\nu}(x) &\equiv \bar{\psi}_f(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi_f(x), \\ \mathcal{O}_{4_f,\mu\nu}(x) &\equiv \delta_{\mu\nu} \bar{\psi}_f(x) \overleftrightarrow{D} \psi_f(x), \\ \mathcal{O}_{5_f,\mu\nu}(x) &\equiv \delta_{\mu\nu} m_{f,0} \bar{\psi}_f(x) \psi_f(x), \\ \mathcal{O}_{i,\mu\nu}(x) &= \sum_{f=1}^{n_F} \mathcal{O}_{i_f,\mu\nu}(x), \quad i \in \{3, 4, 5\}, \end{aligned}$$

*R. V. Harlander, et al., Eur. Phys. J. C 78, 944 (2018);
Edwards et al., Phys. Rev. D 114, 014516 (2026);
D. Bollweg, et al., Phys. Rev. Lett. 137, 061901 (2026).*

A look at the proton gluon momentum fraction



What is done —and what is still open?

- Ward-identity determination of Z_χ^V : no disconnected quark loop, no adjoint fermion flow.
- Validated in P, S, A, T : the reconstructed matrix elements and flow-based renormalization factor agree with RI/MOM and RI/SMOM within uncertainties.
- Proton gluon momentum fraction is preliminary; the quark part, quark-gluon mixing are in progress.
- We do not claim higher precision than RI/MOM and RI/SMOM, but gradient flow provides a clean and promising route to renormalization.

Thank you!