Bottom baryon mass and sigma term contribution

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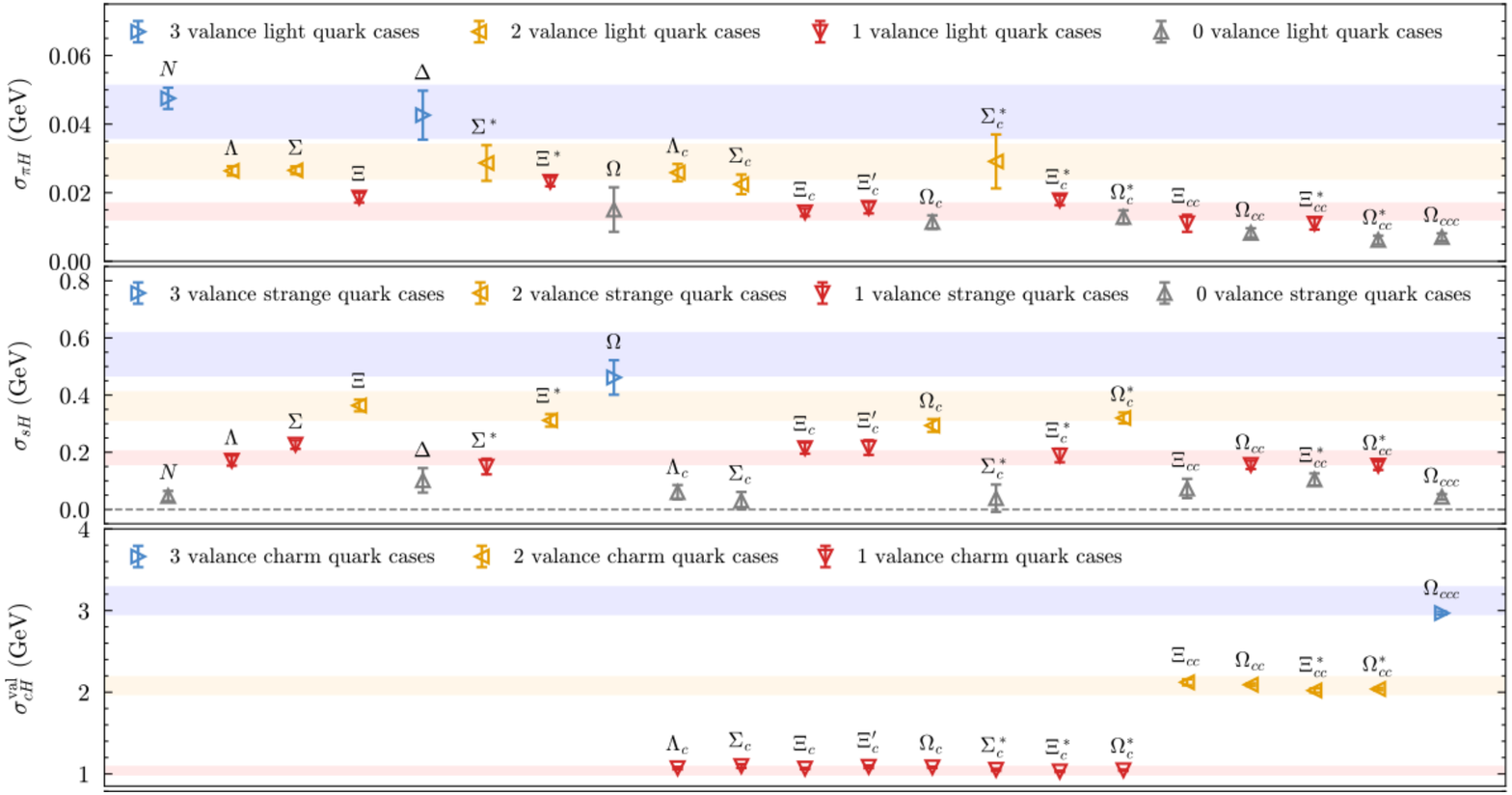
Hadron Mass Decomposition

- Hadron mass decomposition: $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$
- Sigma term: quark flavor contribution $\sigma_{q,H} \equiv m_q\langle\bar{q}q\rangle_H$, where $\langle\bar{q}q\rangle_H$ with $\langle O\rangle_H \equiv \langle H|O|H\rangle/\langle H|H\rangle$ can be understood as the condensate of quark and antiquark within the hadron H.
- Scale violation of quark mass dimension $\gamma_m = \frac{2}{\pi}\alpha_s + O(\alpha_s^2)$
- Gluon trace anomaly part: $\frac{\beta(\alpha_s)}{2\alpha_s} = (-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + O(\alpha_s^2)$
- Extract sigma term: Feynman-Hellman theorem $\sigma_{q,H}^{\mathrm{FH}} = m_q^{\mathrm{PC}}\frac{\partial m_H}{\partial m_q^{\mathrm{PC}}}$ or directly 3-pt calculation.

Hadron Mass Decomposition

Light and charmed baryon case

- Hadron mass decomposition:
$$m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$$
- Flavor-dependent enhancement for quark contribution: 4-8 for light quark, 2-3 for strange quark and 1.2-1.3 for charm quark.
- What about bottom case?



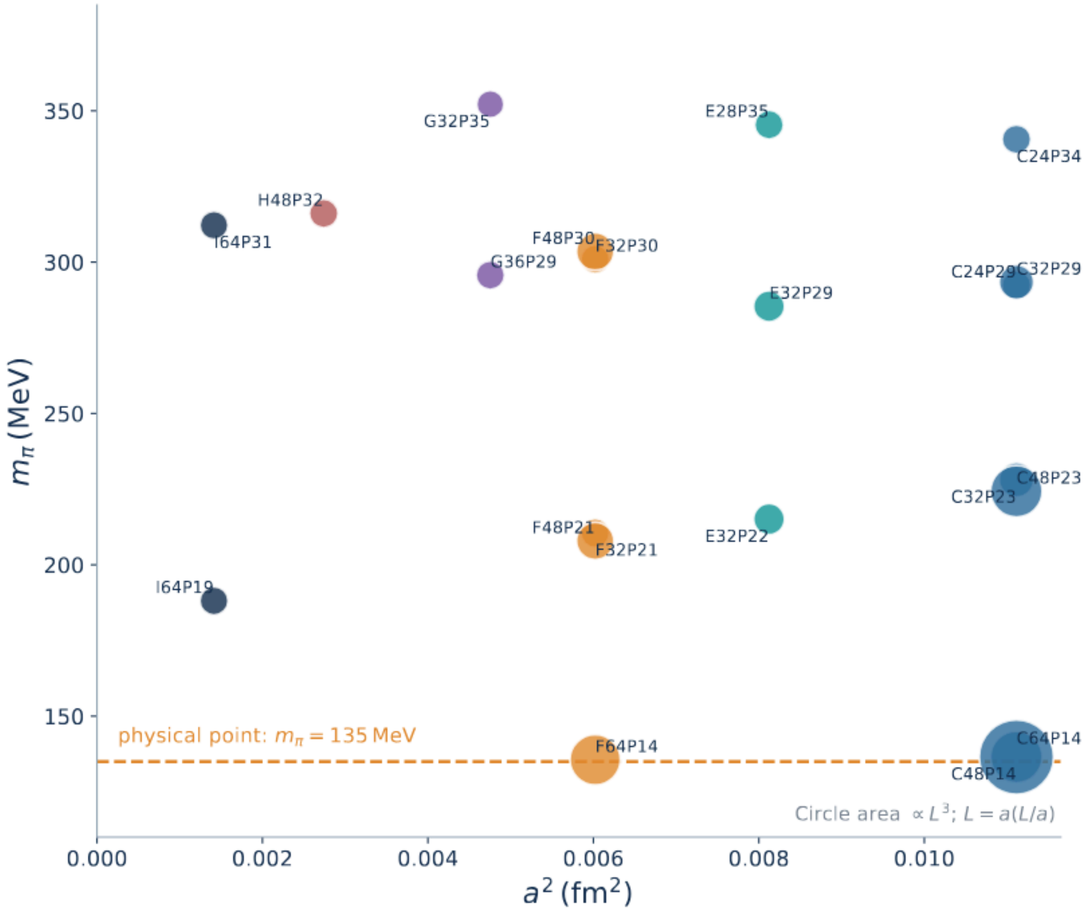
arXiv 2411.18402

Ensemble Information

CLQCD clover ensembles

- CLQCD Collaboration ensembles: 2+1 clover action, 6 lattice spacing, several sea π and η_s mass and different volume.
- More than 4000 configurations.
- Chiral and continuum extrapolation.
- Simulate light, strange and charm quark very well. Z.-C. Hu et al. (CLQCD), *Phys.Rev.D*, 109, 054507 (2024).

  H-Y. Du et al. (CLQCD), *Phys.Rev.D*, 111, 054504 (2025).



Bottom Baryon on each ensemble

Valence quark action and mass turning

- Physical bottom quark on lattice QCD: Anisotropic clover action for bottom quark

$$S_Q = a^4 \sum_x \bar{Q}[m_Q + \gamma_4 \nabla_4 - \frac{a}{2}\nabla_4^2 + \nu \sum_{i=1}^{3} (\gamma_i \nabla_i - \frac{a}{2}\nabla_i^2) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,j=1}^{3} \sigma_{ij} F_{ij}]Q$$

Z. S. Brown, et.al *Phys.Rev.D*, 90(9) 094507. L. Liu, et.al *Phys.Rev.D*, 81 094507

- Turning the values of m_Q and ν to satisfy dispersion relation and physical Υ (1S vector bottomonium) mass: $E_V(\vec{p})^2 = m_V^2 + c^2 p^2, c = 1, \quad m_V = m_\Upsilon = 9460.40(10)$ MeV.
- Physical charm quark: interpolate to pure QCD D_s mass 1966.7(1.5) MeV
- Physical strange quark: interpolate to "physical" η_s mass 689.89(49) MeV
- Valence light quark (i.e. pion mass) dependence, sea light and strange quark dependence: global fit

Bottom Baryon Mass

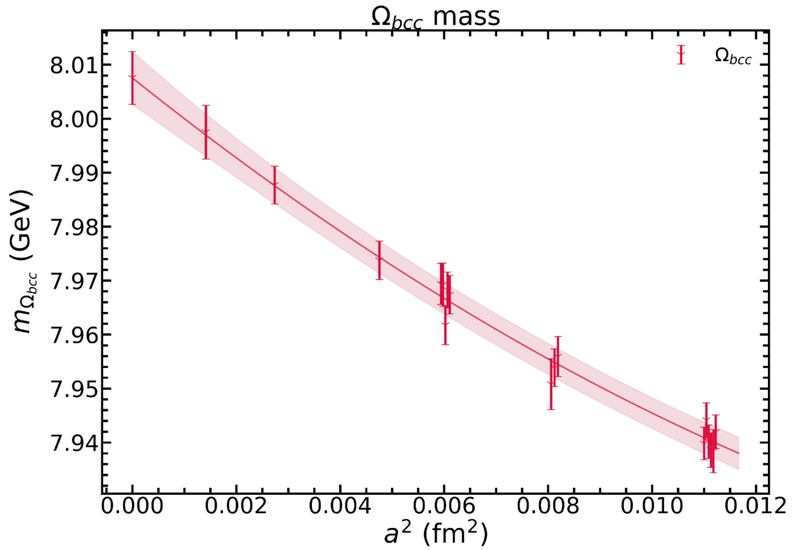
CLQCD preliminary

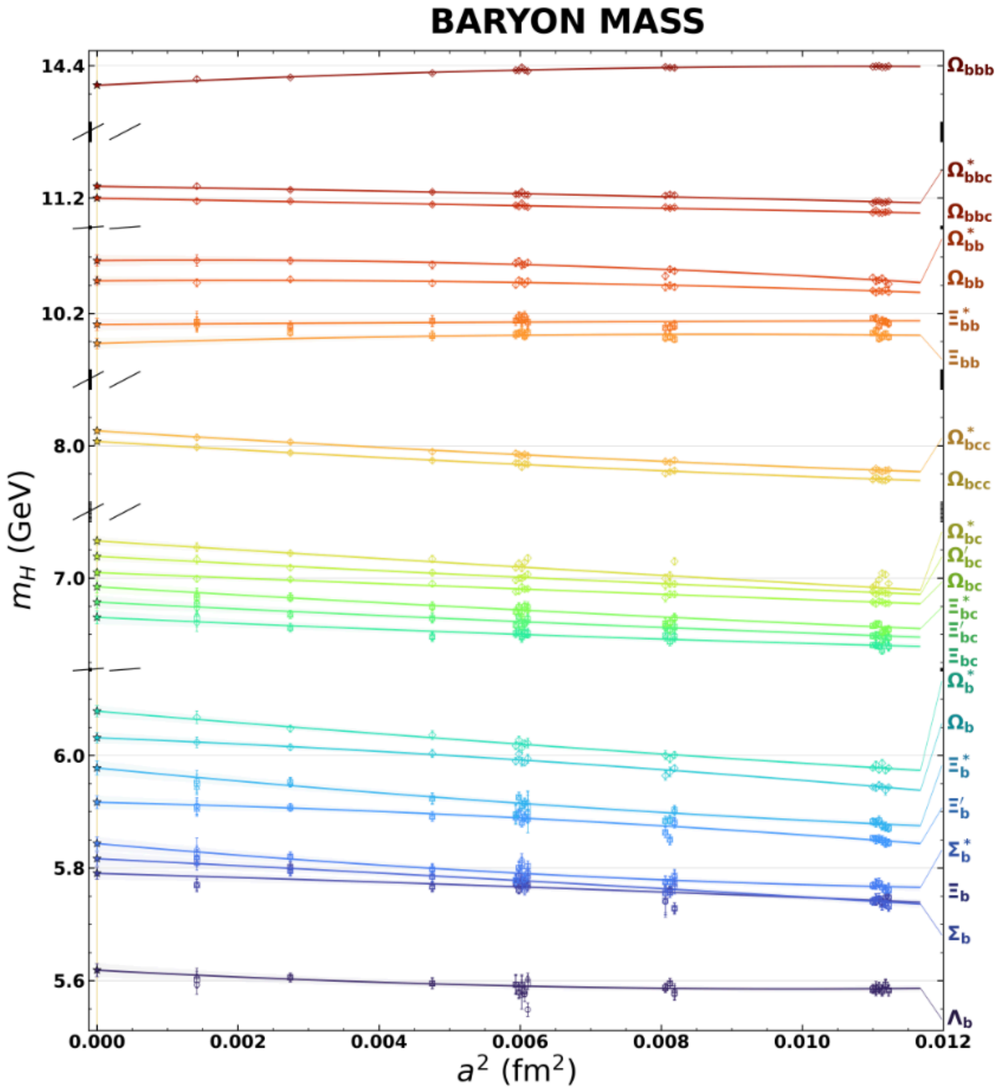
Global fit

- Global fit: sea pion and η_s mass dependence, finite volume effects

$$m_H(m_{\pi,\mathrm{val}}, m_{\pi,\mathrm{sea}}, m_{\eta_s,\mathrm{sea}}, a) = m_H^{\mathrm{phy}} + c_1(m_{\pi,\mathrm{val}}^2 + m_{\pi,\mathrm{sea}}^2 - m_{\pi,\mathrm{phy}}^2) + c_2(m_{\eta_s,\mathrm{sea}}^2 - m_{\eta_s,\mathrm{phy}}^2) + c_3 a^2 + c_4 a^4 + c_5 \frac{m_\pi}{L} e^{-m_\pi L}$$

- Smooth lattice-spacing dependence enables a controlled continuum extrapolation.
- Discretization uncertainty: less than 70 MeV





Bottom Baryon Mass

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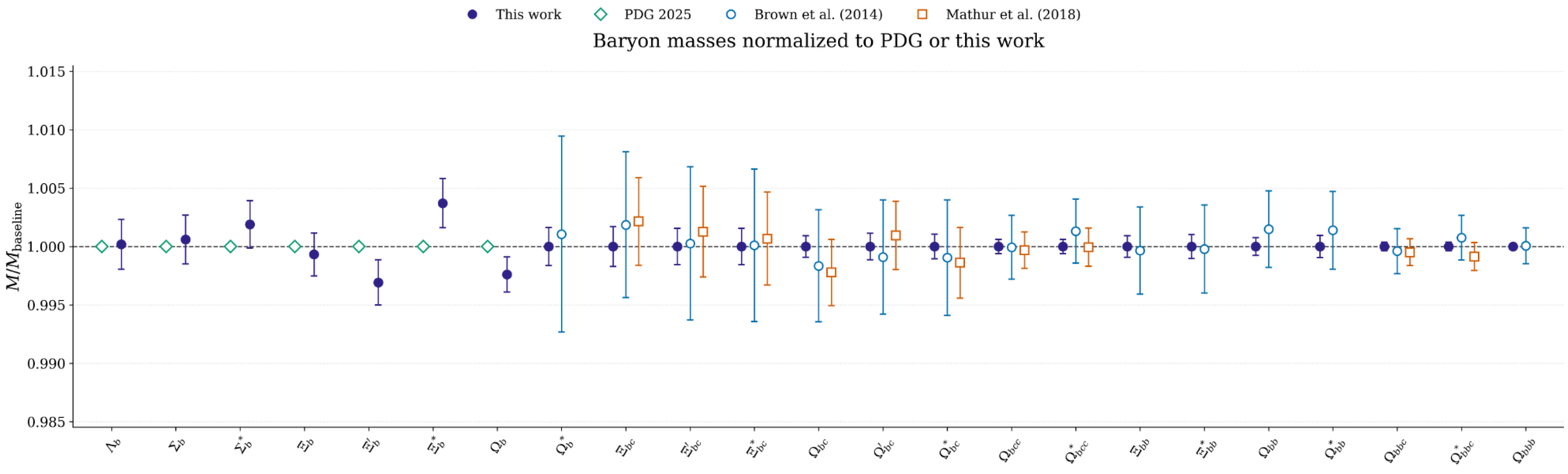
Results

- For experimentally known baryon states, this work is consistent with the PDG values within the quoted uncertainties.
- The results are also compatible with previous lattice calculations, and our results have higher precision.
- The agreement across single-, double- and mixed-heavy channels supports the control of the physical-bottom tuning and continuum effects. The unobserved states therefore provide useful lattice predictions.

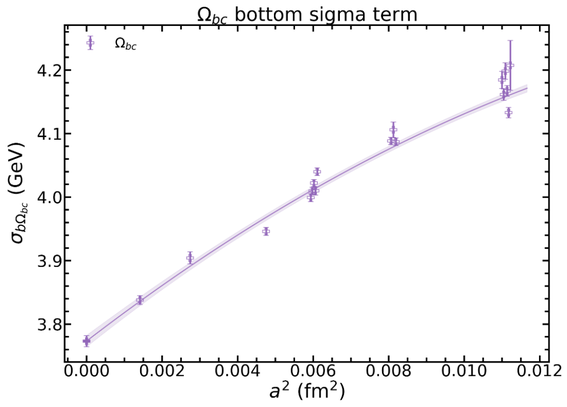


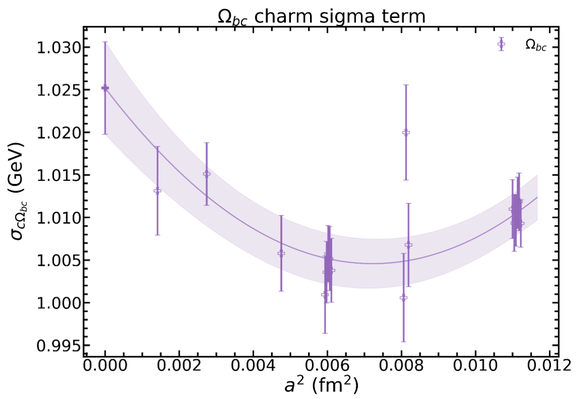
Bottom Baryon Sigma Term

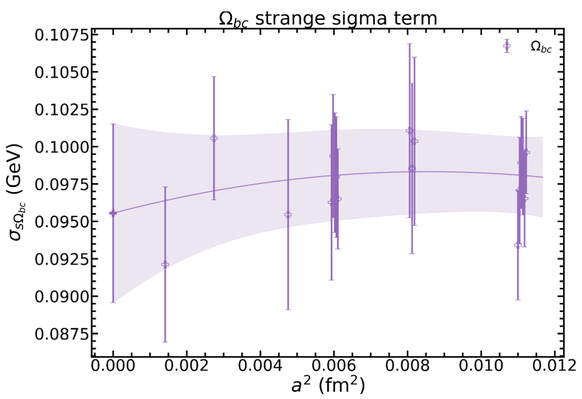
CLQCD preliminary

Valence bottom, charm and strange

- Extract $\sigma_{b,H}^{\rm val}$, $\sigma_{c,H}^{\rm val}$ and $\sigma_{s,H}^{\rm val}$: Feynman-Hellman theorem $\sigma_{q,H}^{\rm FH} = m_q^{\rm PC}\dfrac{\partial m_H}{\partial m_q^{\rm PC}}$ on each ensemble.
- Global fit: sea mass and lattice spacing dependence
$$\sigma_{q,H}(m_{\pi,\rm val}, m_{\pi,\rm sea}, m_{\eta_s,\rm sea}, a) = \sigma_{q,H}^{\rm phy} + c_1(m_{\pi,\rm sea}^2 - m_{\pi,\rm phy}^2) + c_2(m_{\eta_s,\rm sea}^2 - m_{\eta_s,\rm phy}^2) + c_3 a^2 + c_4 a^4 + c_5 e^{-m_\pi L}$$
- Discretization error for valence bottom, charm and strange: typical case Ω_{bc}
- High precision for heavy quark sigma term: high order correction.





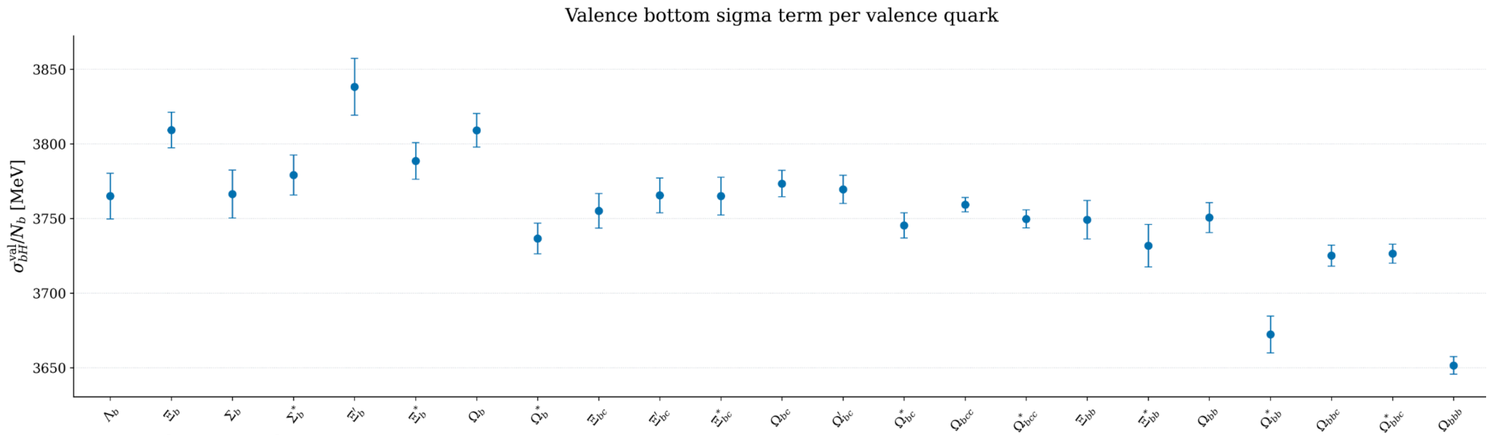


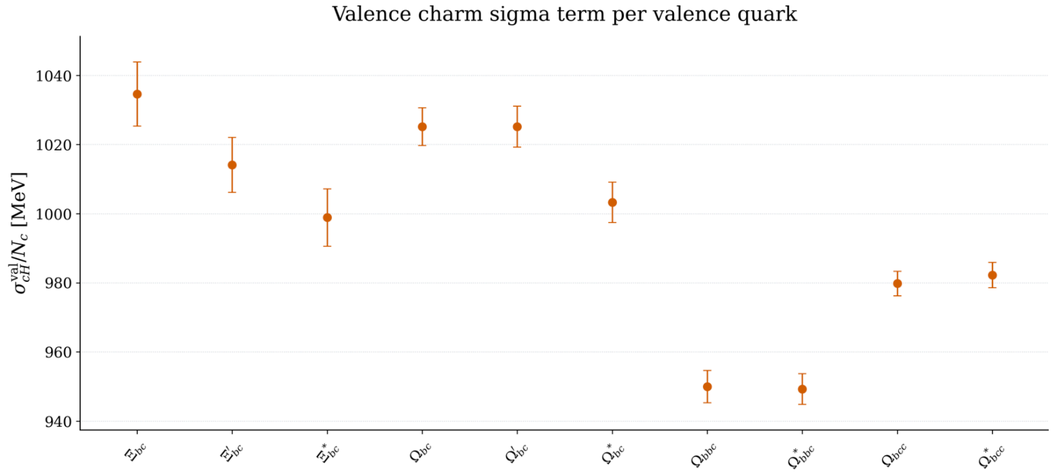
Bottom Baryon Sigma Term

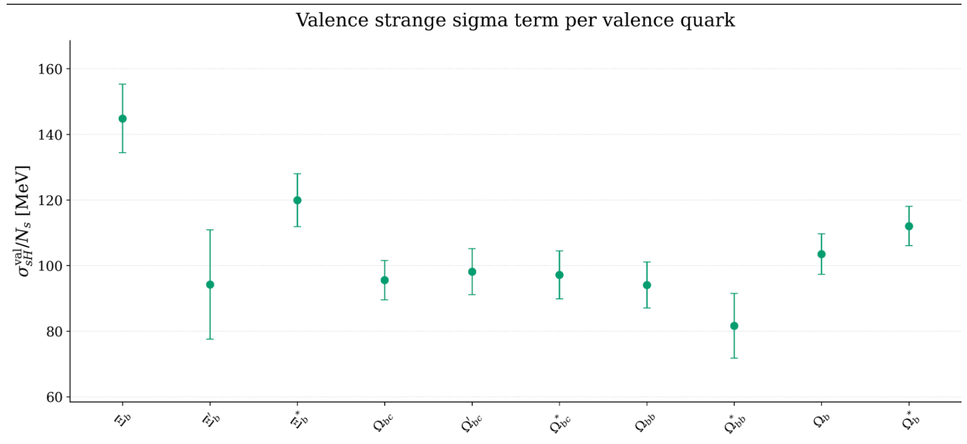
CLQCD preliminary

Valence bottom, charm and strange

- Valence bottom/charm/strange contribution: ~3.75 GeV per *b*, ~1 GeV per *c*, ~100 MeV per *s*.





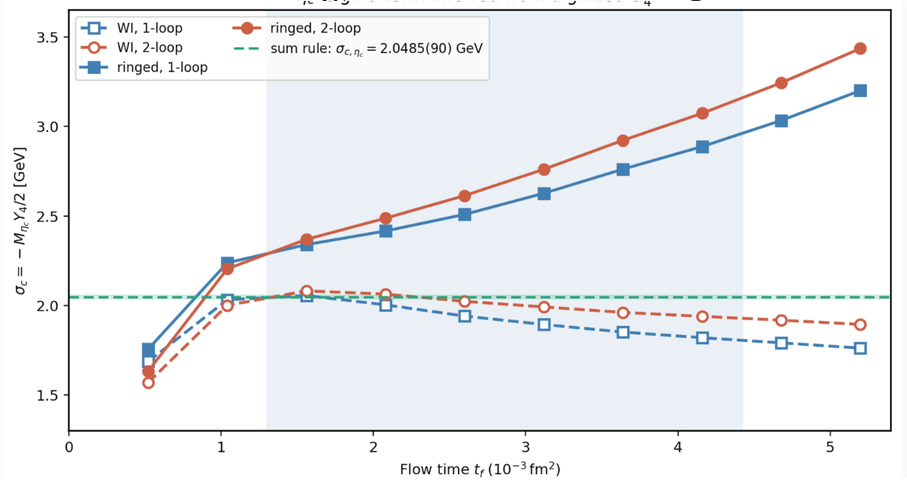


Bottom Baryon Sigma Term

CLQCD preliminary

Consistent check: Feynman-Hellman theorem & gradient flow

- Feynman-Hellman theorem $\sigma_{q,H}^{\mathrm{FH}} = m_q^{\mathrm{PC}} \dfrac{\partial m_H}{\partial m_q^{\mathrm{PC}}}$ and global fit.
- Gradient flow: operator $O_{4,\mu\nu} = \dfrac{1}{2} g_{\mu\nu} \bar{q} \gamma_\rho \overleftrightarrow{D}_\rho q, \qquad \overleftrightarrow{D}_\rho = \overrightarrow{D}_\rho - \overleftarrow{D}_\rho$. EoM $\left\langle O_{4,44}^{\overline{\mathrm{MS}}} \right\rangle_H = -\left\langle m_q \bar{q} q \right\rangle_H$
- Sigma term $\sigma_c = -\dfrac{M_{\eta_c}}{2} Y_4$ with dimensionless normalization: $Y_4 = \left\langle \eta_c \middle| O_{4,44}^{\overline{\mathrm{MS}}} \middle| \eta_c \right\rangle / (2M_{\eta_c}^2)$.
- Ward identity method & ringed fermion method: η_c valence charm sigma term consistent with sum rule (F-H).



Bottom Baryon Sigma Term

CLQCD preliminary

Sea light and strange, valence light

- Sea mass contribution from quark mass: sea mass dependence of $m_s^{\rm val}$, $m_c^{\rm val}$ and $m_b^{\rm val}$ lead to correction of sea light and sea strange sigma term.

$$\delta m_H = \delta m_l^{\rm sea}\langle\bar{l}l\rangle_H^{\rm sea} + \delta m_s^{\rm sea}\langle\bar{s}s\rangle_H^{\rm sea} + \delta m_s^{\rm val}\langle\bar{s}s\rangle_H^{\rm val} + \delta m_c^{\rm val}\langle\bar{c}c\rangle_H^{\rm val} + \delta m_b^{\rm val}\langle\bar{b}b\rangle_H^{\rm val}$$

$$= \delta m_l^{\rm sea}(\langle\bar{l}l\rangle_H^{\rm sea} + \frac{\partial m_s^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} + \frac{\partial m_c^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} + \frac{\partial m_b^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val}) + \delta m_s^{\rm sea}(\langle\bar{s}s\rangle_H^{\rm sea} + \frac{\partial m_s^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} + \frac{\partial m_c^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} + \frac{\partial m_b^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val})$$

$$\sigma_{\pi,H} = \sigma_{\pi,H}^{\rm val} + \sigma_{\pi,H}^{\rm sea}$$

$$\sigma_{\pi,H}^{\rm val} = m_l^{\rm phy}\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_l^{\rm val}}|_{m_l^{\rm val}=m_l^{\rm phy}} = (m_\pi^{\rm phy})^2\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_\pi^{\rm val})^2}|_{m_\pi^{\rm val}=m_\pi^{\rm phy}}$$

$$\sigma_{\pi,H}^{\rm sea} = m_l^{\rm phy}[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_l^{\rm sea}}|_{m_l^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val}]$$

$$= (m_\pi^{\rm phy})^2[\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_\pi^{\rm val})^2}|_{m_\pi^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{b}b\rangle_H^{\rm val}]$$

$$\sigma_{s,H} = \sigma_{s,H}^{\rm val} + \sigma_{s,H}^{\rm sea}$$

$$\sigma_{s,H}^{\rm val} = m_s^{\rm phy}\frac{\partial m_H(m_\pi^{\rm sea}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_s^{\rm val}}|_{m_s^{\rm val}=m_s^{\rm phy}} = (m_{\eta_s}^{\rm phy})^2\frac{\partial m_H(m_\pi^{\rm sea}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_{\eta_s}^{\rm val})^2}|_{m_{\eta_s}^{\rm val}=m_{\eta_s}^{\rm phy}}$$

$$\sigma_{s,H}^{\rm sea} = m_s^{\rm phy}[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm sea}, 0, 0)}{\partial m_s^{\rm sea}}|_{m_s^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val}]$$

$$= (m_{\eta_s}^{\rm phy})^2[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm sea}, 0, 0)}{\partial (m_{\eta_s}^{\rm val})^2}|_{m_{\eta_s}^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{b}b\rangle_H^{\rm val}]$$

Bottom Baryon Sigma Term

CLQCD preliminary

Sea light and strange, valence light

- Sea mass contribution from quark mass: ~3.5 MeV per valence light; ~100 MeV for sea strange sigma term; ~12 MeV for sea light sigma term.

$$\sigma_{\pi,H} = \sigma^{\rm val}_{\pi,H} + \sigma^{\rm sea}_{\pi,H}$$

$$\sigma^{\rm val}_{\pi,H} = m_l^{\rm phy}\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_l^{\rm val}}\Big|_{m_l^{\rm val}=m_l^{\rm phy}} = (m_\pi^{\rm phy})^2\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_\pi^{\rm val})^2}\Big|_{m_\pi^{\rm val}=m_\pi^{\rm phy}}$$

$$\sigma^{\rm sea}_{\pi,H} = m_l^{\rm phy}\Big[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_l^{\rm sea}}\Big|_{m_l^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial m_l^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val}\Big]$$

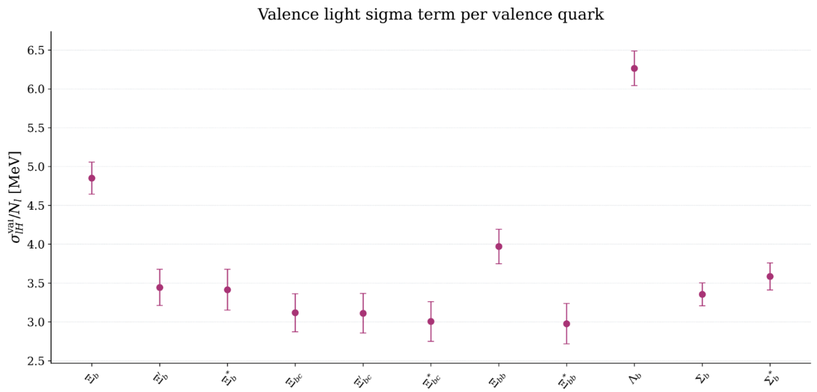
$$= (m_\pi^{\rm phy})^2\Big[\frac{\partial m_H(m_\pi^{\rm val}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_\pi^{\rm val})^2}\Big|_{m_\pi^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial (m_\pi^{\rm sea})^2}\langle\bar{b}b\rangle_H^{\rm val}\Big]$$

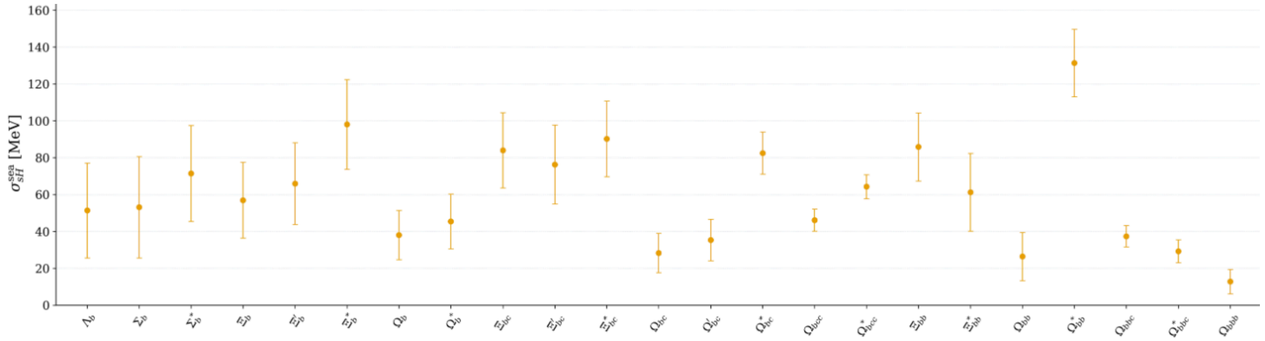
$$\sigma_{s,H} = \sigma^{\rm val}_{s,H} + \sigma^{\rm sea}_{s,H}$$

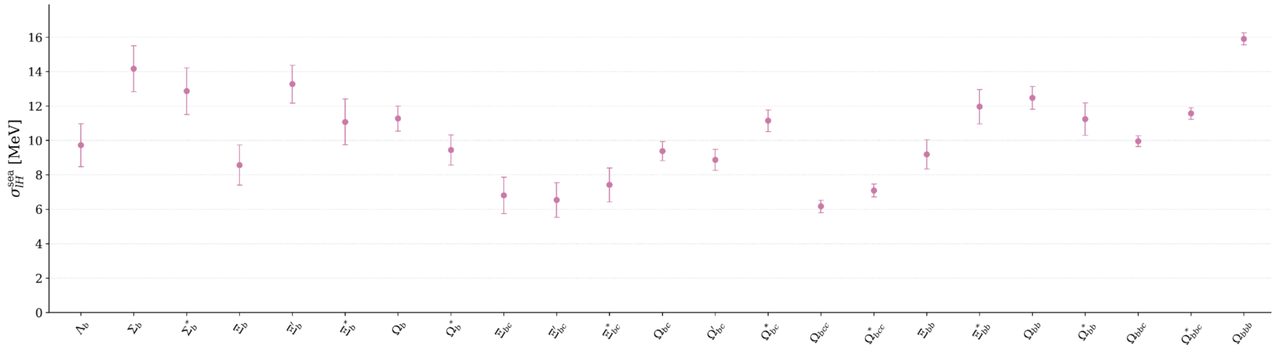
$$\sigma^{\rm val}_{s,H} = m_s^{\rm phy}\frac{\partial m_H(m_\pi^{\rm sea}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial m_s^{\rm val}}\Big|_{m_s^{\rm val}=m_s^{\rm phy}} = (m_{\eta_s}^{\rm phy})^2\frac{\partial m_H(m_\pi^{\rm sea}, m_\pi^{\rm sea}, m_{\eta_s}^{\rm phy}, 0, 0)}{\partial (m_{\eta_s}^{\rm val})^2}\Big|_{m_{\eta_s}^{\rm val}=m_{\eta_s}^{\rm phy}}$$

$$\sigma^{\rm sea}_{s,H} = m_s^{\rm phy}\Big[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm sea}, 0, 0)}{\partial m_s^{\rm sea}}\Big|_{m_s^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial m_s^{\rm sea}}\langle\bar{b}b\rangle_H^{\rm val}\Big]$$

$$= (m_{\eta_s}^{\rm phy})^2\Big[\frac{\partial m_H(m_\pi^{\rm phy}, m_\pi^{\rm phy}, m_{\eta_s}^{\rm sea}, 0, 0)}{\partial (m_{\eta_s}^{\rm val})^2}\Big|_{m_{\eta_s}^{\rm sea}} - \frac{\partial m_s^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{s}s\rangle_H^{\rm val} - \frac{\partial m_c^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{c}c\rangle_H^{\rm val} - \frac{\partial m_b^{\rm val}}{\partial (m_{\eta_s}^{\rm sea})^2}\langle\bar{b}b\rangle_H^{\rm val}\Big]$$





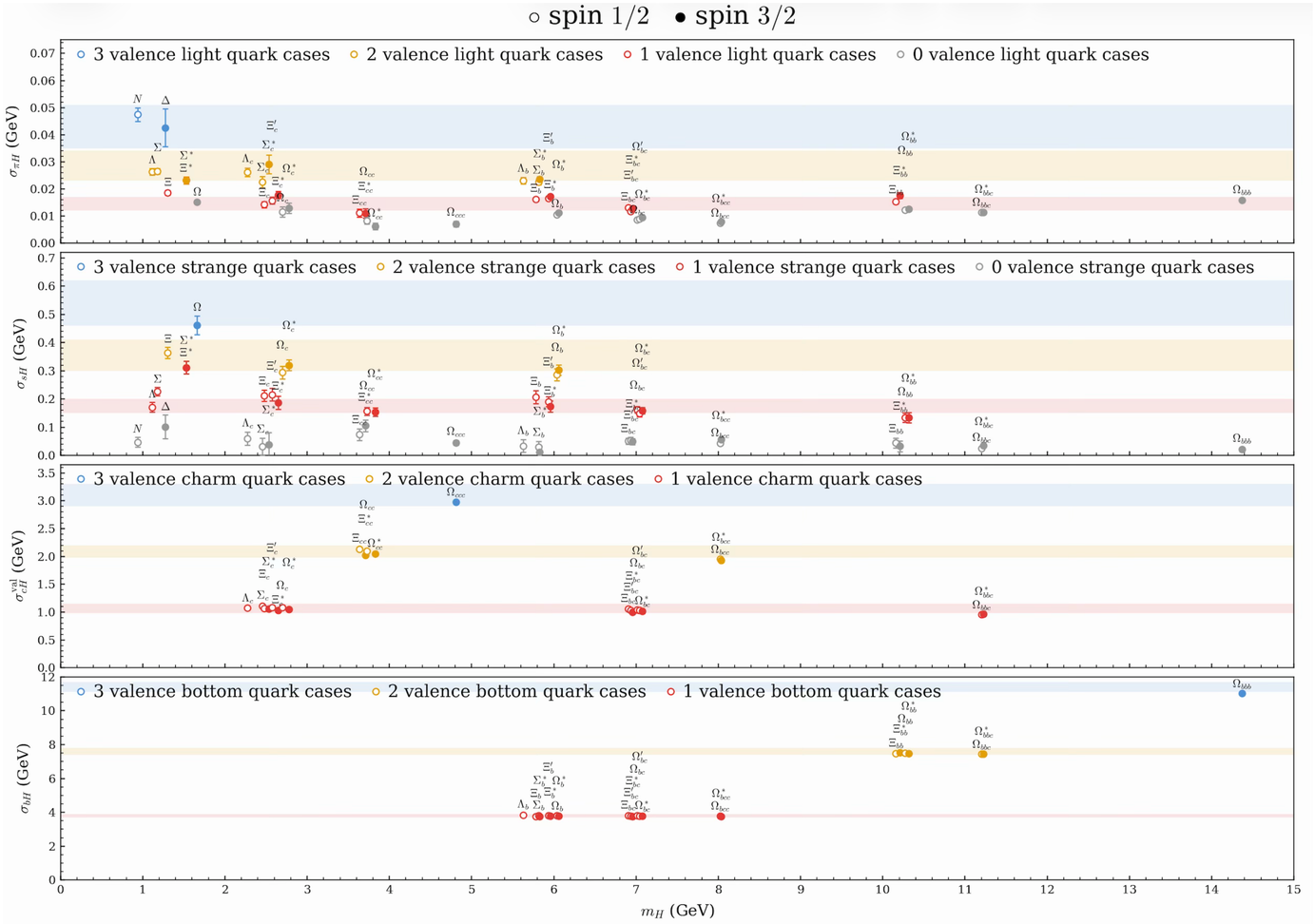


Bottom Baryon Sigma Term

CLQCD preliminary

Sigma term contribution to baryon mass

- Total sigma term: sum over all flavor contributions, including valence and sea.
- Valence bottom, charm and strange quark sigma term approximately proportional to the number of quark in bottom baryon.
- Sigma term is the domain part of the bottom baryon mass.
- Remain part: trace anomaly.

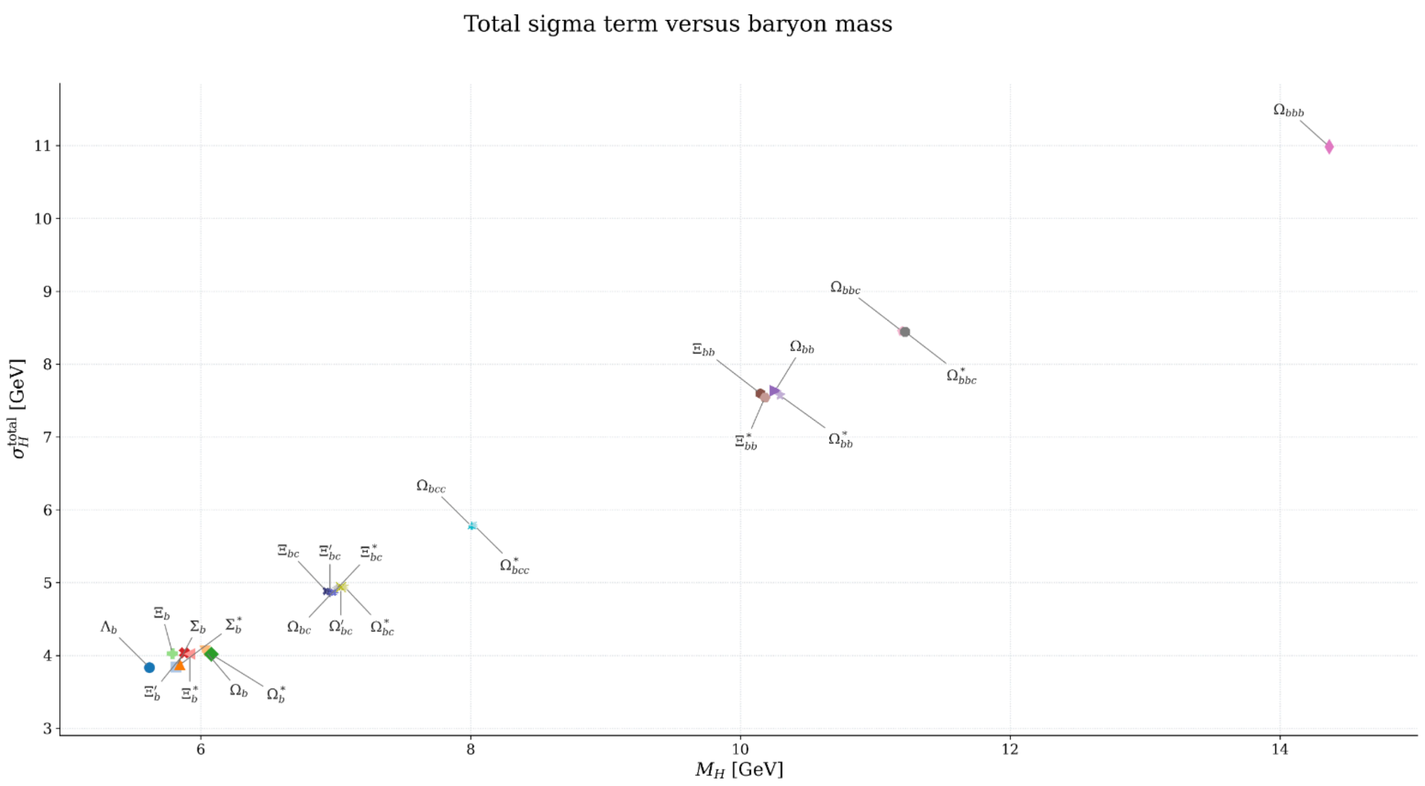


Bottom Baryon Trace Anomaly

CLQCD preliminary

Trace anomaly contribution

- Hadron mass decomposition:
$$m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$$
- Total sigma term is the main component of the bottom baryon mass.
- Trace anomaly: $\langle H_a\rangle_H = m_H^{\text{phy}} - \sigma_H$, $\langle H_a^g\rangle_H = m_H^{\text{phy}} - (1+\gamma_m)\sigma_H$. Scale dependence for both quark and gluon part.

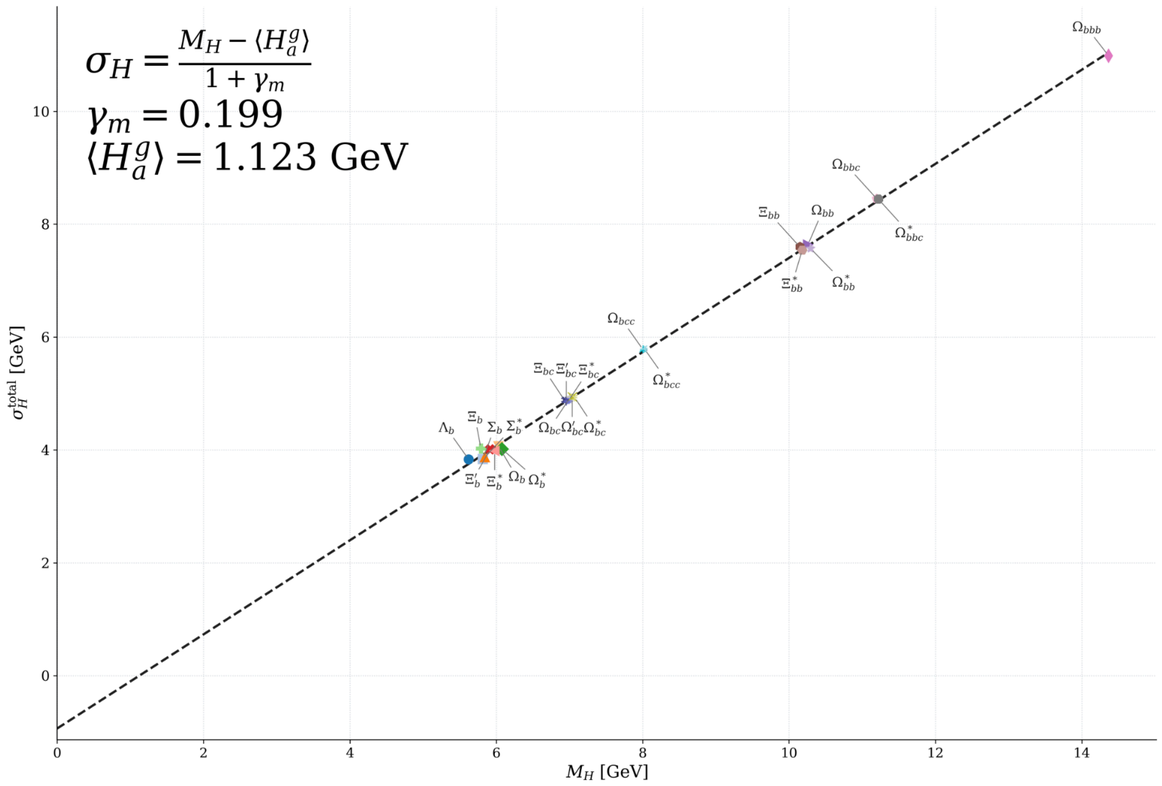


Bottom Baryon Trace Anomaly

CLQCD preliminary

Scale dependence of trace anomaly

- $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$,
  $\langle H_a\rangle_H = m_H^{\text{phy}} - \sigma_H,$
  $\langle H_a^g\rangle_H = m_H^{\text{phy}} - (1+\gamma_m)\sigma_H.$
- $\langle H_a^g\rangle_H \sim 1.13$ GeV when $\gamma_m \sim 0.2$: bottom baryon seems have a typical scale.
- $\gamma_m \sim 0.2$: about $\overline{\text{MS}}$ 4 GeV
- Is $\overline{\text{MS}}$ 4 GeV general?

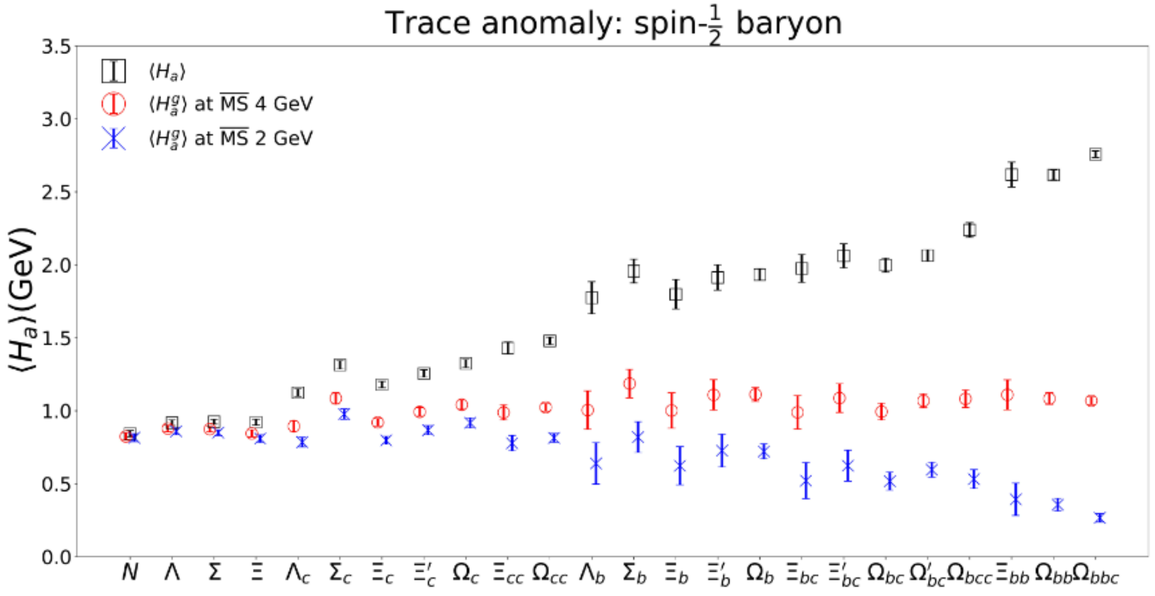


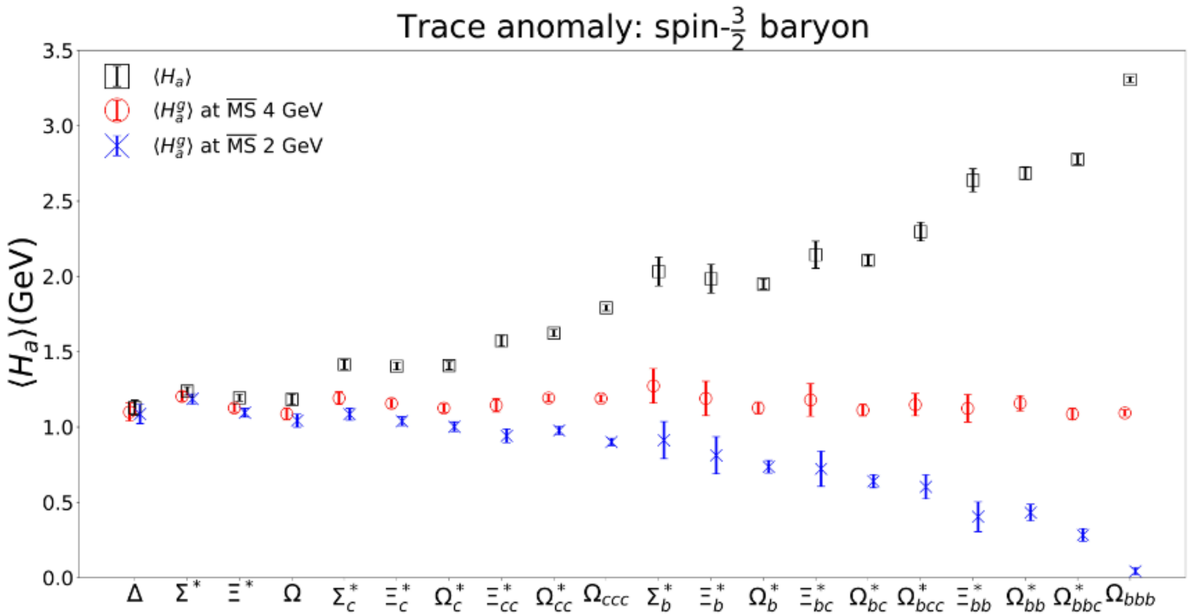
Bottom Baryon Trace Anomaly

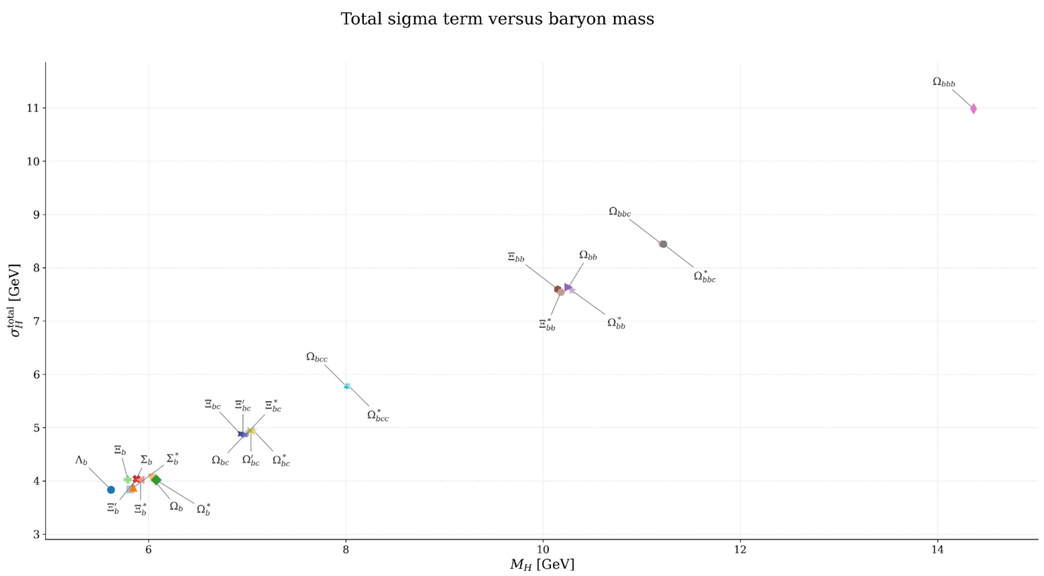
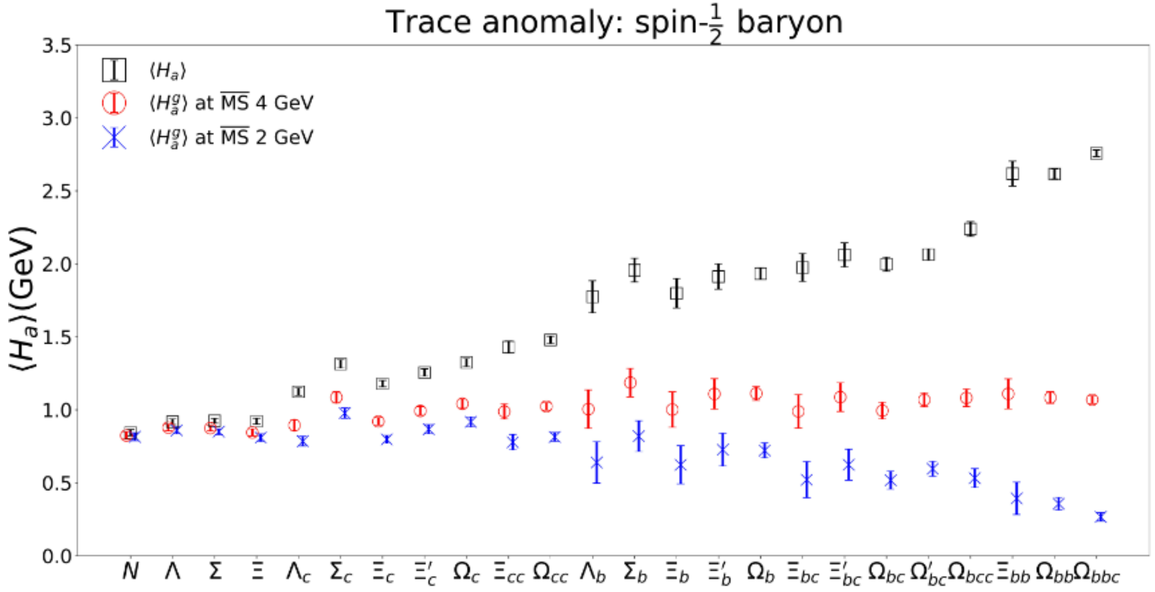
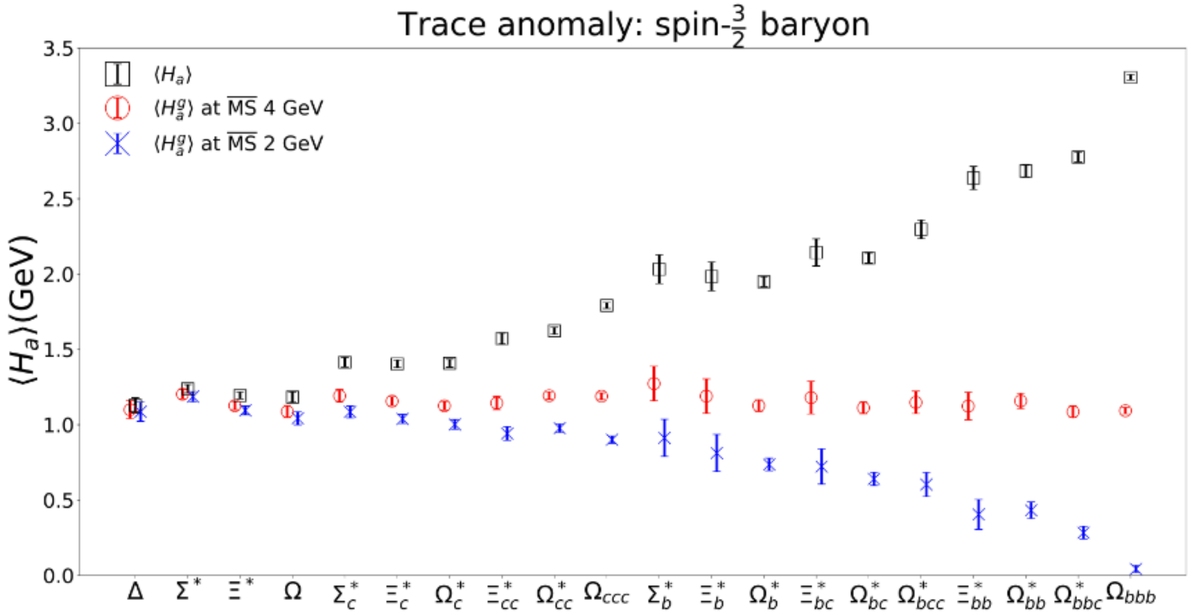
CLQCD preliminary

Scale dependence of trace anomaly

- $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$, $\langle H_a\rangle_H = m_H^{\text{phy}} - \sigma_H$, $\langle H_a^g\rangle_H = m_H^{\text{phy}} - (1+\gamma_m)\sigma_H$.
- $\overline{\text{MS}}$ 4 GeV & $\overline{\text{MS}}$ 2 GeV:





Summary

- High precision lattice results on bottom baryon, and predicted the baryon mass with experimentally undiscovered.
- Sigma term contributions to bottom baryon.
- Trace anomaly contributions to bottom baryon mass, and its scale dependence.