



中国科学院大学
University of Chinese Academy of Sciences

Lattice QCD Study of $\bar{D}N$ Scattering

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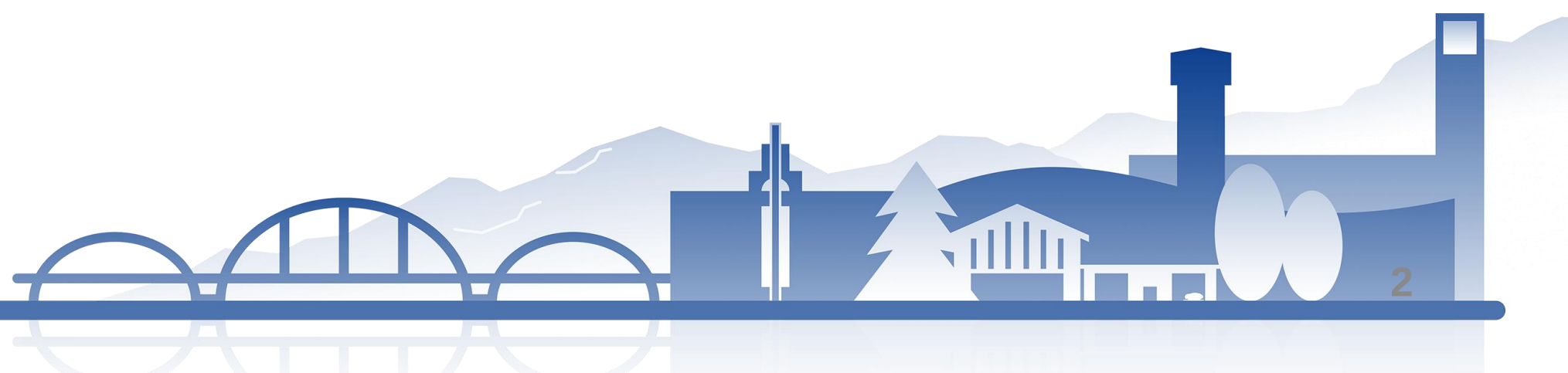
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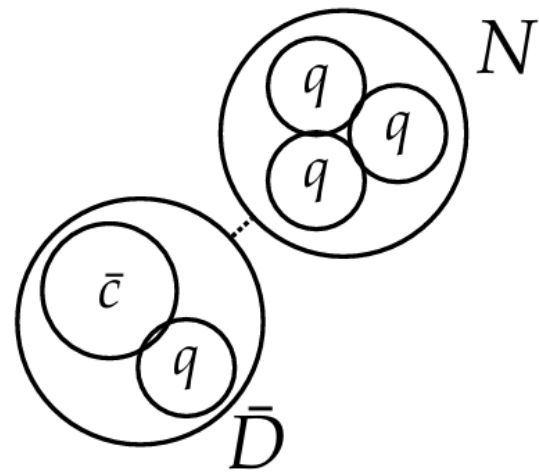


01

Background

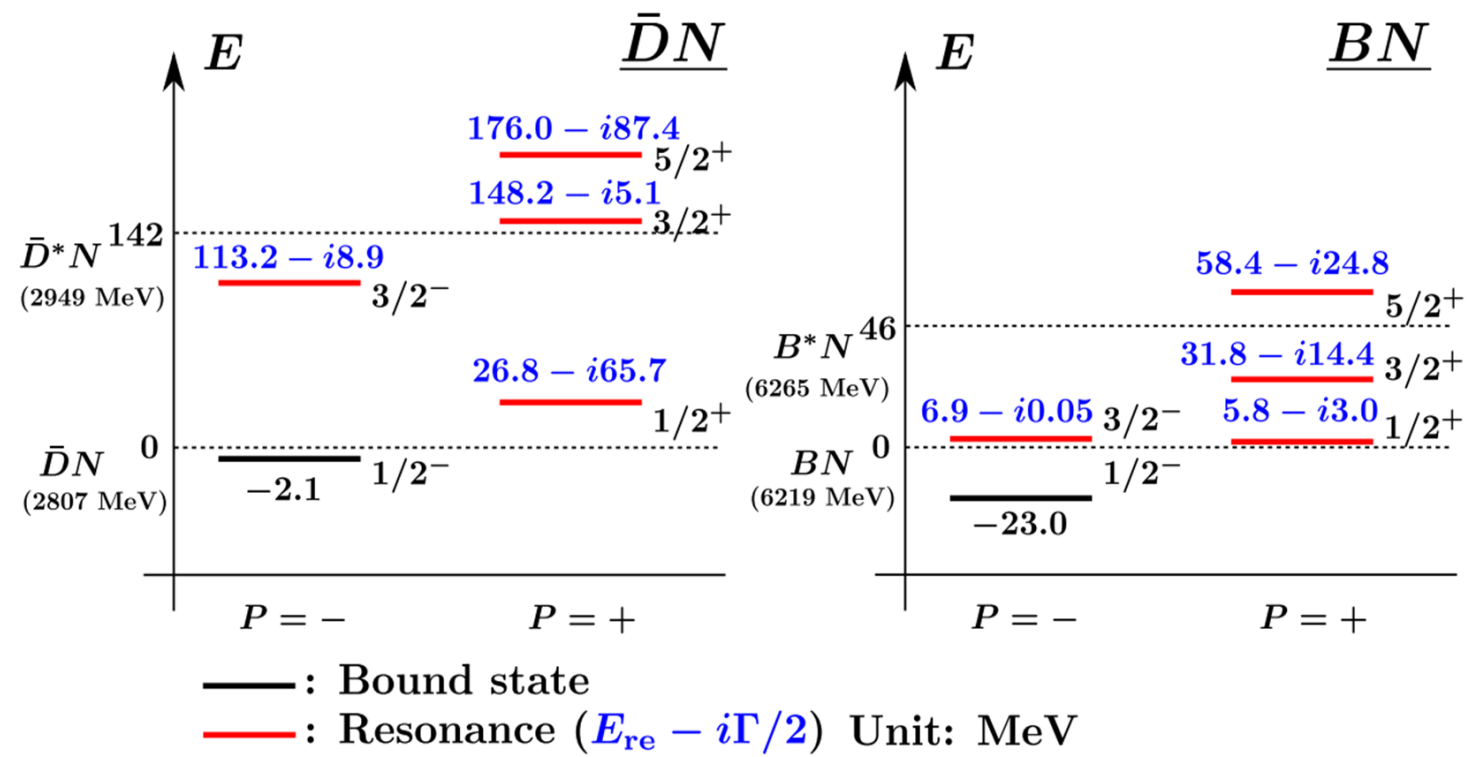


Exotic hadron state



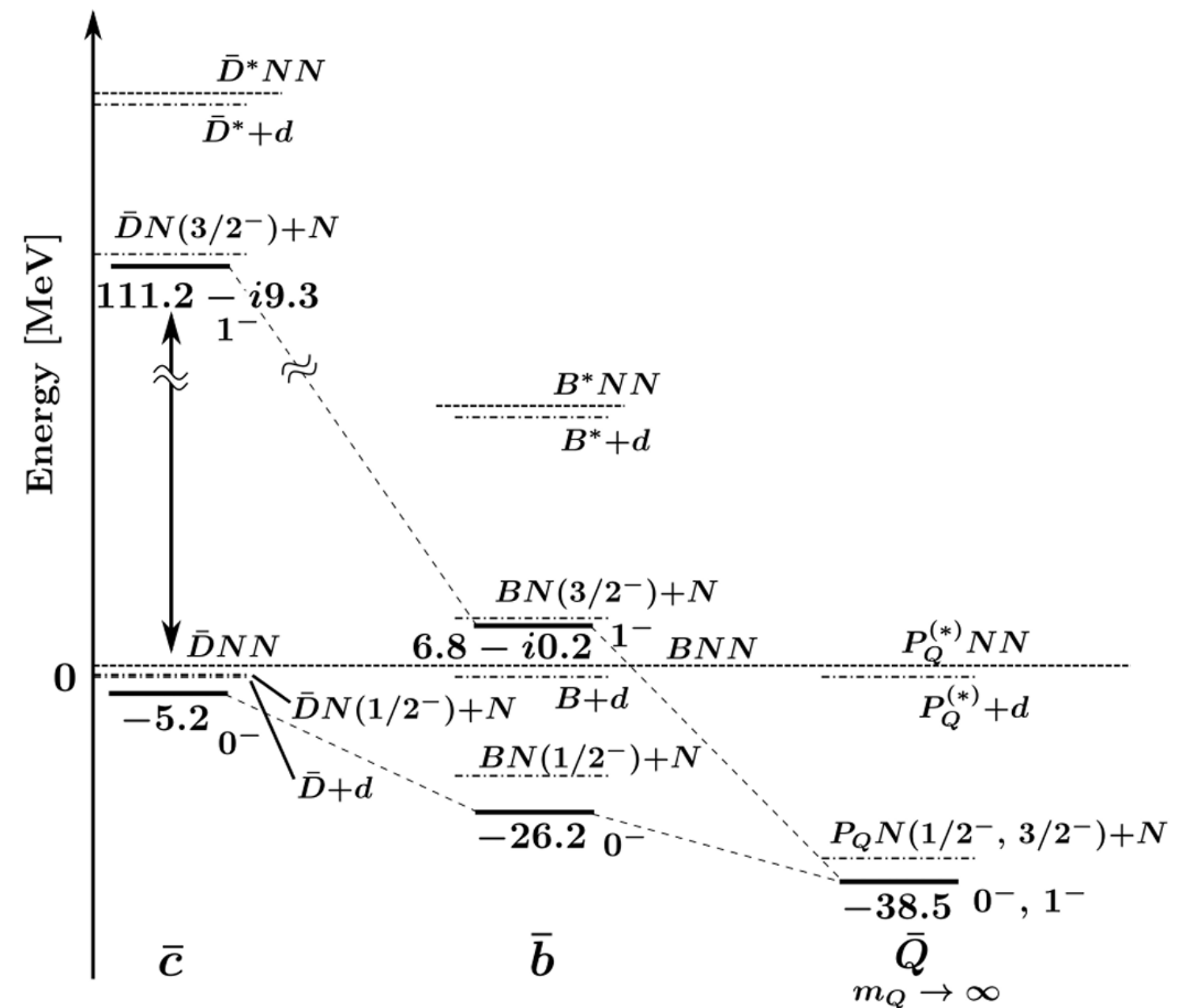
■ One of the simplest systems for studying the charm hadron system

■ No $q\bar{q}$ annihilation



Yamaguchi et al. arXiv:1209.0187

A. Hosaka et al. / Progress in Particle and Nuclear Physics 96 (2017) 88–153



Ensemble setup

LQCD:

- Unphysical light quark mass
- Finite lattice spacing
- Finite volume

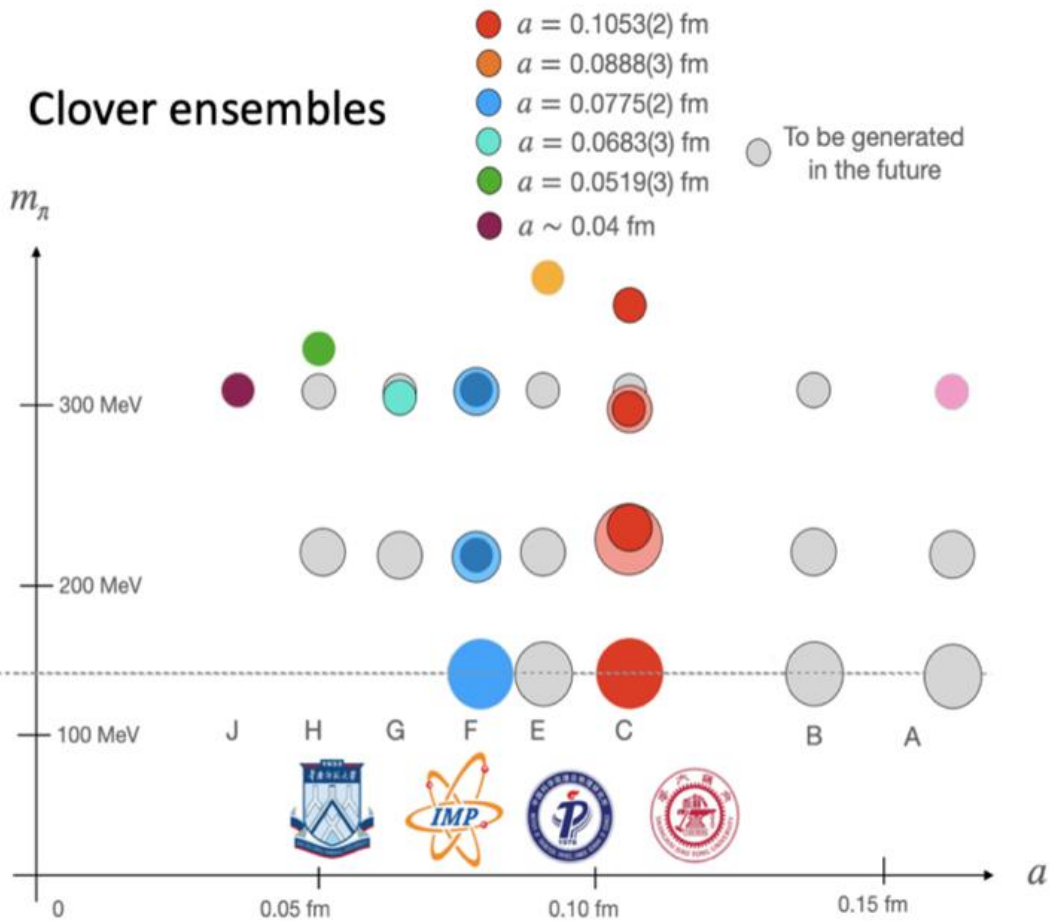
CLQCD ensemble:

- 2+1 flavor
- Stout-smeared clover fermion
- Symanzik gauge actions



组态	β	a (fm)	$(L/a)^3 \times T/a$	m_π (MeV)	m_K (MeV)
F32P30	6.41	0.07746(18)	$32^3 \times 96$	303.2(1.3)	524.6(1.8)
F48P30	6.41	0.07746(18)	$48^3 \times 96$	303.4(0.9)	523.6(1.4)

Z.-C.Hu et al., (CLQCD), Phys.Rev.D 109(2024)5,054507





02

Lattice Calculation



The steps of spectrum calculation

- Build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \dots\}$ with desired quantum numbers

- Construct the matrix of correlation function

$$C_{ij}(t, t_0) = \langle \mathcal{O}_i(t) \bar{\mathcal{O}}_j(t_0) \rangle = \sum_n Z_{in} Z_{jn}^* e^{-E_n t}$$

- Solve the generalized eigenvalue problem (GEVP)
Determine the corresponding energy

$$C(t) \cdot v_n(t) = \lambda_n(t, t_0) C(t_0) \cdot v_n(t)$$

$$\lambda_n(t, t_0) \sim e^{-E_n(t-t_0)} + \dots \rightarrow E_n$$

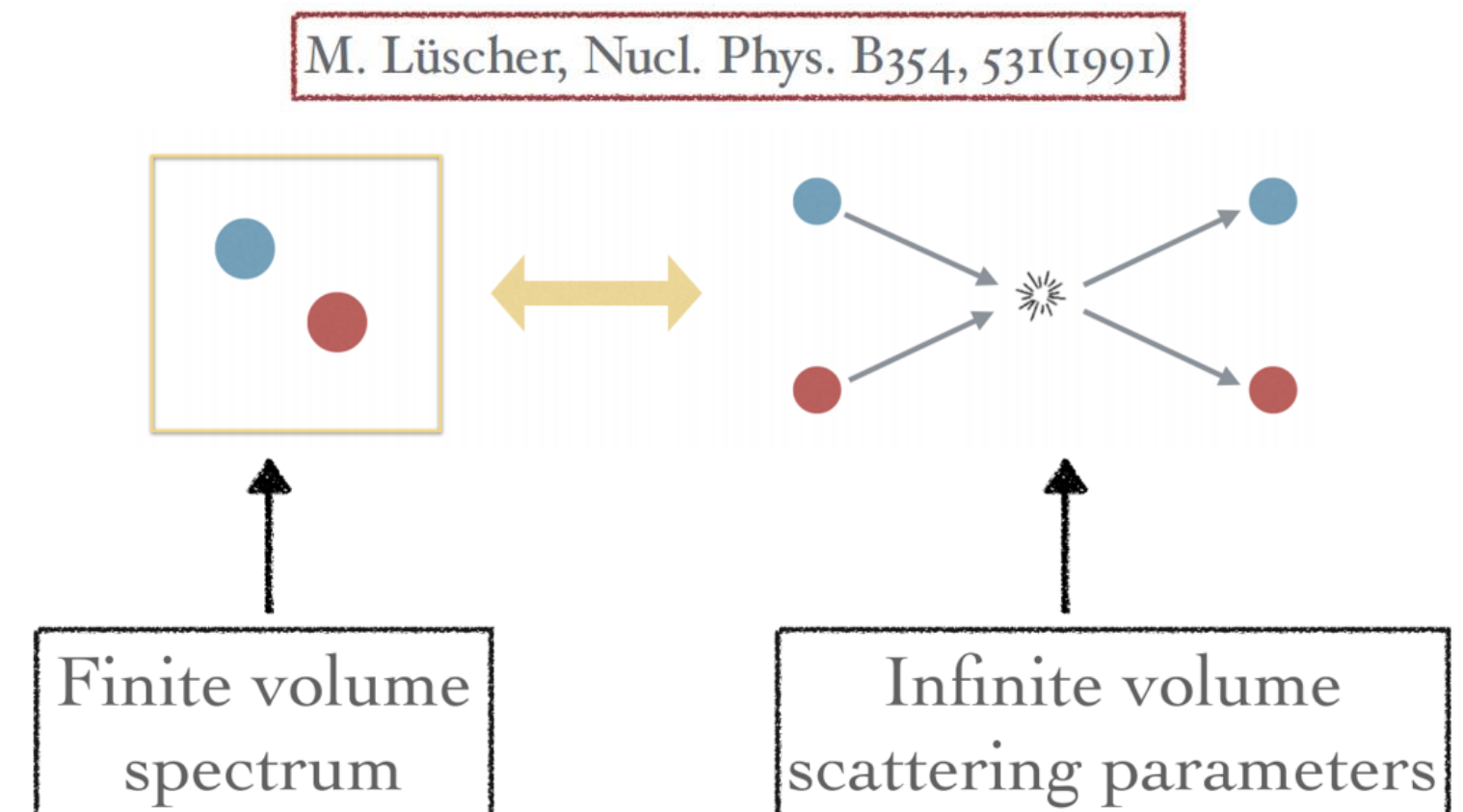
- Further numerical analysis of the obtained energy

$$\det[\mathbf{1} + i\rho \cdot t \cdot (1 + i\mathbf{M})] = 0$$

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$

$$\mathbf{P}_{\text{tot}} = [0, 0, 0]$$

channels	Quantum number	I(J ^P)	irrep
$\bar{D}N (S = 1/2)$	$0^- \otimes 1/2^+$	$0(1/2^-), 1(1/2^-)$	G_1^-
$\bar{D}^*N (S = 1/2)$	$1^- \otimes 1/2^+$	$0(1/2^-), 1(1/2^-)$	G_1^-



Operator construction

Single-particle operators

$$\begin{aligned} P(\mathbf{p}) &= \sum_{\mathbf{x}} \bar{q}(\mathbf{x}) \gamma_5 q(\mathbf{x}) e^{i\mathbf{p} \cdot \mathbf{x}}, \\ V_i(\mathbf{p}) &= \sum_{\mathbf{x}} \bar{q}(\mathbf{x}) \gamma_i q(\mathbf{x}) e^{i\mathbf{p} \cdot \mathbf{x}}, \quad i = x, y, z, \\ N_\mu(\mathbf{p}) &= \sum_{\mathbf{x}} \epsilon_{abc} [q^{aT}(\mathbf{x}) C \gamma_5 q^b(\mathbf{x})] q_\mu^c(\mathbf{x}) e^{i\mathbf{p} \cdot \mathbf{x}}. \end{aligned}$$

$$\begin{aligned} O_{D^-}(t, x) &= \bar{c}(t, x)_d \gamma_5 d(t, x)_d & O_p(t, y) &= \epsilon_{abc} u(t, y)_a [u(t, y)_b C \gamma_5 d(t, y)_c] \\ O_{\bar{D}^0}(t, x) &= \bar{c}(t, x)_d \gamma_5 u(t, x)_d & O_n(t, y) &= \epsilon_{abc} d(t, y)_a [u(t, y)_b C \gamma_5 d(t, y)_c] \end{aligned}$$

$$O(t, p) = \sum_x e^{-ipx} O(t, x)$$

J	irrep $\Gamma(\text{dim})$
0	$A_1(1)$
1/2	$G_1(2)$
1	$T_1(3)$
3/2	$H(4)$
2	$E(2) \oplus T_2(3)$
5/2	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

Two-particle operators for **S-wave** scattering $J^P = \left(\frac{1}{2}\right)^- \rightarrow G_1^-$

$$\mathbf{p}^2 = 0, 1, 2, 3, 4 \quad \mathbf{p}_2 = -\mathbf{p}_1$$

$$\begin{aligned} \mathcal{O}_{\bar{D}N; I=0}^{\Gamma, r} &= \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2}^{\Gamma, r} \left[\frac{1}{\sqrt{2}} \mathcal{O}_{D^-}(\mathbf{p}_1) \bar{\mathcal{O}}_p(\mathbf{p}_2) - \frac{1}{\sqrt{2}} \mathcal{O}_{\bar{D}^0}(\mathbf{p}_1) \bar{\mathcal{O}}_n(\mathbf{p}_2) \right] \\ \mathcal{O}_{\bar{D}N; I=1}^{\Gamma, r} &= \sum_{\alpha, \mathbf{p}_1, \mathbf{p}_2} C_{\alpha, \mathbf{p}_1, \mathbf{p}_2}^{\Gamma, r} \left[\frac{1}{\sqrt{2}} \mathcal{O}_{D^-}(\mathbf{p}_1) \bar{\mathcal{O}}_p(\mathbf{p}_2) + \frac{1}{\sqrt{2}} \mathcal{O}_{\bar{D}^0}(\mathbf{p}_1) \bar{\mathcal{O}}_n(\mathbf{p}_2) \right] \end{aligned}$$

<< OpTion`

OpTion: package for Operator construction in lattice QCD.

Authors: Haobo Yan

Contact: haobo@stu.pku.edu.cn

H. Yan et al., JHEP 10 (2025) 210

LapH smearing and distillation

Distillation projects the quark fields onto a low-dimensional subspace spanned by the **low-lying eigenvectors** of the gauge-covariant spatial Laplacian

replacing explicit spatial and color indices with distillation-space indices while retaining the spin indices.

$$\square(t) = V(t)V^\dagger(t) \implies \square_{xy}(t) = \sum_{k=1}^N v_x^{(k)}(t)v_y^{(k)\dagger}(t)$$

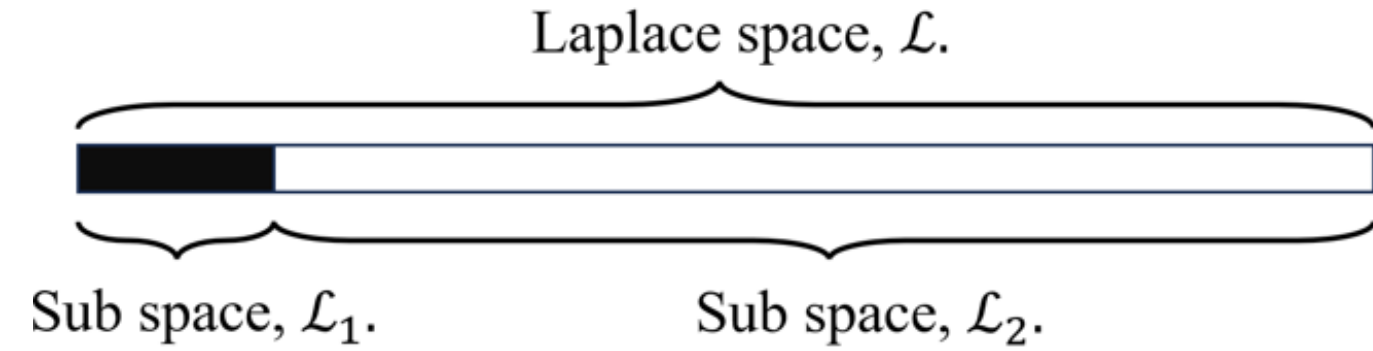
$$q_x^s(t) = \sum_{k=1}^{N_v} v_x^{(k)}(t)v_y^{(k)\dagger}(t)q_y(t) \equiv \square_{xy}(t)q_y(t)$$

$$\nabla^2(t)v^{(k)}(t) = \lambda^{(k)}(t)v^{(k)}(t)$$

$$-\nabla_{xy}^2(t) = 6\delta_{xy} - \sum_{j=1}^3 (\tilde{U}_j(x,t)\delta_{x+\hat{j},y} + \tilde{U}_j^\dagger(x-\hat{j},t)\delta_{x-\hat{j},y})$$

$$\pi^-(\vec{p},t) = \bar{u}_x(t)\square_{xy}(t) \cdot e^{-ip \cdot y}\Gamma_{yz}(t) \cdot \square_{zw}(t)d_w(t)$$

$$P_{\alpha\beta}(t,t_0) = V^\dagger(t)G_{\alpha\beta}V(t_0)$$



define

$$VdV = V^\dagger e^{-ipx}V \quad VVV = e^{-ipz}\varepsilon_{abc}V(t)_{za}V(t)_{zb}V(t)_{zc}$$

$$(\Gamma_{\text{sink}})^{\delta'_1\epsilon_1} = \gamma_5^{\delta'_1\delta_1}\gamma_5^{\delta_1\epsilon_1}, (\Gamma_{\text{source}})^{\epsilon_2\delta'_2} = \gamma_5^{\epsilon_2\delta'_2}\gamma_5^{\delta_2\delta'_2} \quad \gamma_7^{\beta_1\gamma_1} = (C\gamma_5)^{\beta_1\gamma_1}$$

The two-point correlation function of mesons and protons can be written as

$$C_{\bar{D}^0}(t,t_0) = VdV(t,-p_1)_{\mathbf{m}_1\mathbf{n}_1}(\Gamma_{\text{sink}})^{\delta'_1\epsilon_1}(\Gamma_{\text{source}})^{\epsilon_2\delta'_2}VdV(t_0,-p_2)^*_{\mathbf{m}_2\mathbf{n}_2}$$

$$P_d(t,t_0)_{\mathbf{n}_1\mathbf{n}_2}^{\epsilon_1\epsilon_2}P_c^*(t,t_0)_{\mathbf{m}_1\mathbf{m}_2}^{\delta'_1\delta'_2}$$

$$C_p(t,t_0)^{\alpha_1\alpha_2} = VVV(t,p_1)_{i_1j_1k_1}\gamma_7^{\beta_1\gamma_1}\gamma_7^{\beta_2\gamma_2}VVV^*(t_0,p_2)_{i_2j_2k_2} \\ \left[\left(P_u(t,t_0)_{i_1i_2}^{\alpha_1\alpha_2}P_u(t,t_0)_{j_1j_2}^{\beta_1\beta_2} - P_u(t,t_0)_{i_1j_2}^{\alpha_1\beta_2}P_u(t,t_0)_{j_1i_2}^{\beta_1\alpha_2} \right) P_d(t,t_0)_{k_1k_2}^{\gamma_1\gamma_2} \right]$$

Multiquark Wick contractions

$$|I = 0, I_3 = 0\rangle = \frac{1}{\sqrt{2}} (|D^- p\rangle - |\bar{D}^0 n\rangle)$$

$$C_{|0,0\rangle}(t, t_0) = \langle O(t) \bar{O}(t_0) \rangle$$

$$= \frac{1}{2} [\langle O_{D^- p}(t) \bar{O}_{D^- p}(t_0) \rangle - \langle O_{D^- p}(t) \bar{O}_{\bar{D}^0 n}(t_0) \rangle$$

$$|I = 1, I_3 = +1\rangle = |\bar{D}^0 p\rangle$$

$$|I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}} (|D^- p\rangle + |\bar{D}^0 n\rangle)$$

$$|I = 1, I_3 = -1\rangle = |D^- n\rangle$$

$$- \langle O_{\bar{D}^0 n}(t) \bar{O}_{D^- p}(t_0) \rangle + \langle O(t) \bar{O}_{\bar{D}^0 n}(t_0) \rangle]$$

$$= \frac{1}{2} [C_{11} - C_{12} - C_{21} + C_{22}]$$

$$C_{11} = C_{22}, \quad C_{12} = C_{21}$$

$$C_{11}(t, t_0)^{\alpha_1 \alpha_2} = \langle O_{D^- p}(t) \bar{O}_{D^- p}(t_0) \rangle$$

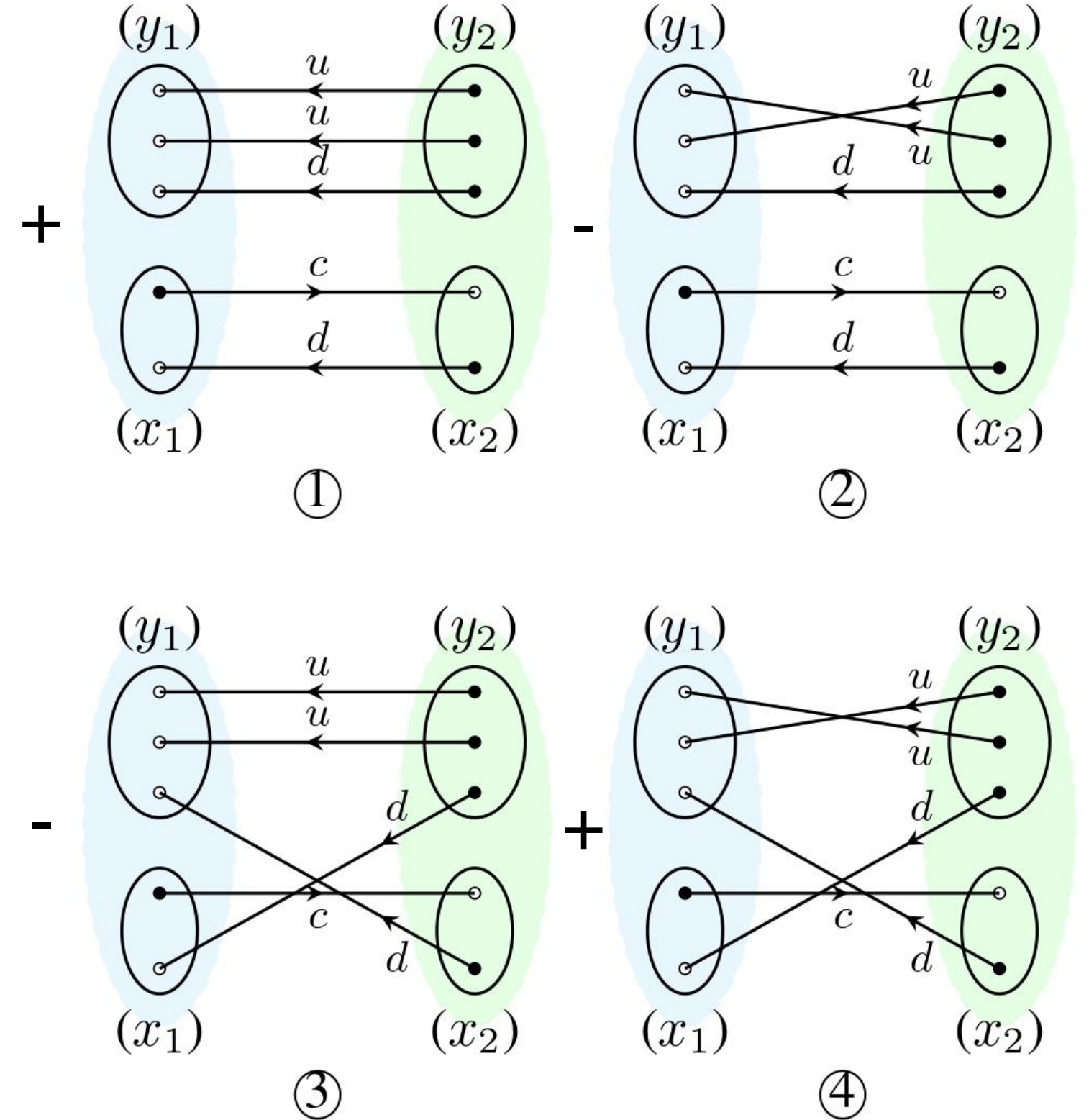
$$= \left\langle \left\{ e^{ip_1 x_1} \bar{c}(t)_{x_1 d_1}^{\delta_1} \gamma_5^{\delta_1 \epsilon_1} d(t)_{x_1 d_1}^{\epsilon_1} \right\} \left\{ e^{-ip_1 y_1} \epsilon_{a_1 b_1 c_1} u(t)_{y_1 a_1}^{\alpha_1} \left[u(t)_{y_1 b_1}^{\beta_1} C \gamma_5^{\beta_1 \gamma_1} d(t)_{y_1 c_1}^{\gamma_1} \right] \right\} \right. \\ \left. \left\{ e^{ip_2 y_2} \epsilon_{b_2 c_2 a_2} \left[\bar{u}(t)_{y_2 b_2}^{\beta_2} C \gamma_5^{\beta_2 \gamma_2} \bar{d}(t)_{y_2 c_2}^{\gamma_2} \right] \bar{u}(t)_{y_2 a_2}^{\alpha_2} \right\} \left\{ e^{-ip_2 x_2} \bar{d}(t)_{x_2 d_2}^{\epsilon_2} \gamma_5^{\delta_2 \epsilon_2} c(t)_{x_2 d_2}^{\delta_1} \right\} \right\rangle \\ = VVV(t)_{i_1 j_1 k_1} VdV(t)_{m_1 n_1} \left(\gamma_7^{\beta_1 \gamma_1} \gamma_7^{\beta_1 \gamma_1} \Gamma_{sink}^{\delta_1' \epsilon_1} \Gamma_{source}^{\epsilon_2 \delta_2'} \right) VdV^*(t_0)_{m_2 n_2} VVV^*(t_0)_{i_2 j_2 k_2}$$

$$\left\{ + P_u(t, t_0)_{i_1 i_2}^{\alpha_1 \alpha_2} P_u(t, t_0)_{j_1 j_2}^{\beta_1 \beta_2} P_d(t, t_0)_{k_1 k_2}^{\gamma_1 \gamma_2} P_c^*(t, t_0)_{m_1 m_2}^{\delta_1' \delta_2'} P_d(t, t_0)_{n_1 n_2}^{\epsilon_1 \epsilon_2} \right. \quad \textcircled{1}$$

$$- P_u(t, t_0)_{i_1 j_2}^{\alpha_1 \beta_2} P_u(t, t_0)_{j_1 i_2}^{\beta_1 \alpha_2} P_d(t, t_0)_{k_1 k_2}^{\gamma_1 \gamma_2} P_c^*(t, t_0)_{m_1 m_2}^{\delta_1' \delta_2'} P_d(t, t_0)_{n_1 n_2}^{\epsilon_1 \epsilon_2} \quad \textcircled{2}$$

$$- P_u(t, t_0)_{i_1 i_2}^{\alpha_1 \alpha_2} P_u(t, t_0)_{j_1 j_2}^{\beta_1 \beta_2} P_d(t, t_0)_{k_1 n_2}^{\gamma_1 \epsilon_2} P_c^*(t, t_0)_{m_1 m_2}^{\delta_1' \delta_2'} P_d(t, t_0)_{n_1 k_2}^{\epsilon_1 \gamma_2} \quad \textcircled{3}$$

$$+ P_u(t, t_0)_{i_1 j_2}^{\alpha_1 \beta_2} P_u(t, t_0)_{j_1 i_2}^{\beta_1 \alpha_2} P_d(t, t_0)_{k_1 n_2}^{\gamma_1 \epsilon_2} P_c^*(t, t_0)_{m_1 m_2}^{\delta_1' \delta_2'} P_d(t, t_0)_{n_1 k_2}^{\epsilon_1 \gamma_2} \left. \right\} \quad \textcircled{4}$$



GEVP (generalized eigenvalue problem)

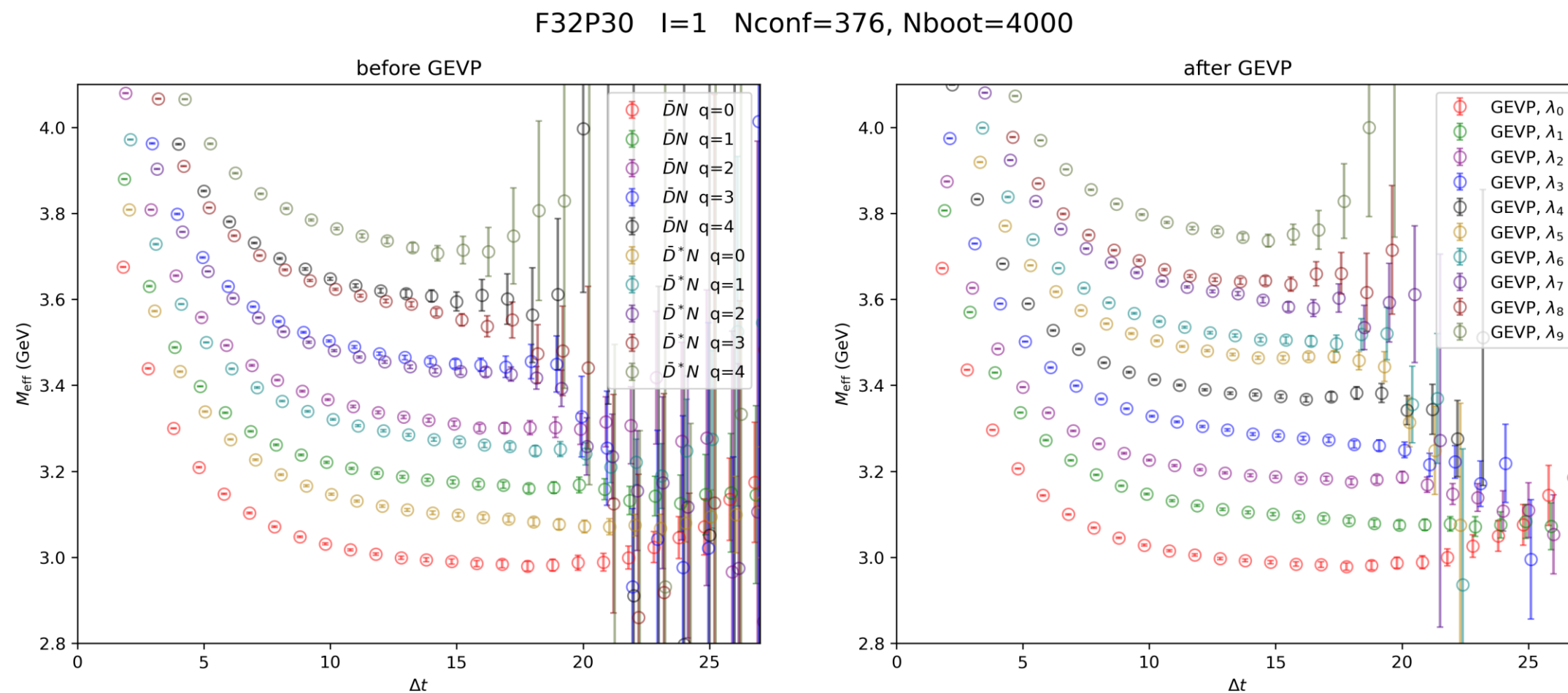
Within a given operator basis, the GEVP employs a calculus of variations to find **optimal linear combinations of operators** that maximize their overlap with individual energy eigenstates.

The resulting generalized eigenvectors satisfy an **orthogonality relation** with respect to the metric defined by $C(t_0)$

$$C_{ij}(t) \cdot v_{jn}(t) = \lambda_n(t, t_0) C_{ij}(t_0) \cdot v_{jn}(t)$$

$$\Omega_m = \sum_i v_{im}^* O_i, \quad \bar{\Omega}_n = \sum_j O_j^\dagger v_{jn}$$

$$\sum_{ij} v_{im}^* C_{ij}(t_0) v_{jn} = \delta_{mn}$$



Energy levels

single-particle effective mass

$$\text{meson: } C_0 e^{-\frac{T}{2} E_0} \cosh \left[E_0 \left(\frac{T}{2} - t \right) \right]$$

$$\text{baryon: } C_0 e^{-E_0 t}$$

two-particle effective mass

- eigenvalue:

$$C(t) \cdot v_n(t) = \lambda_n(t, t_0) C(t_0) \cdot v_n(t_0)$$

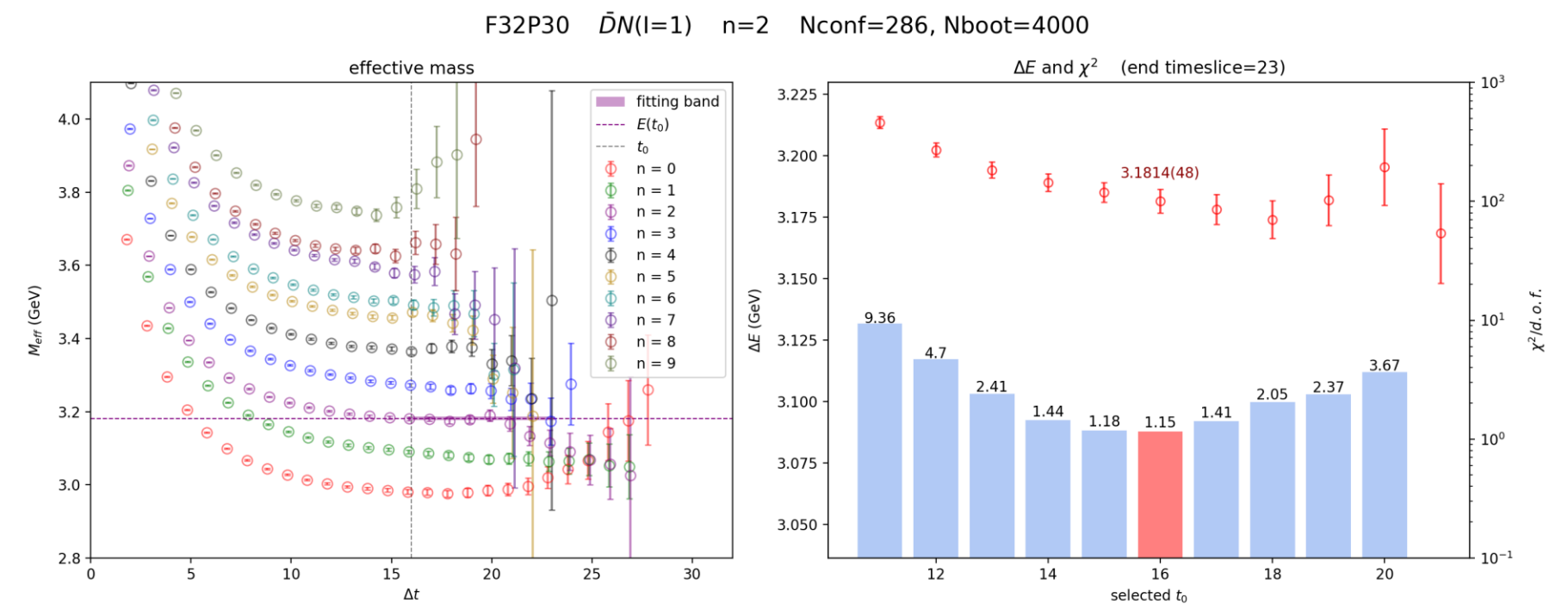
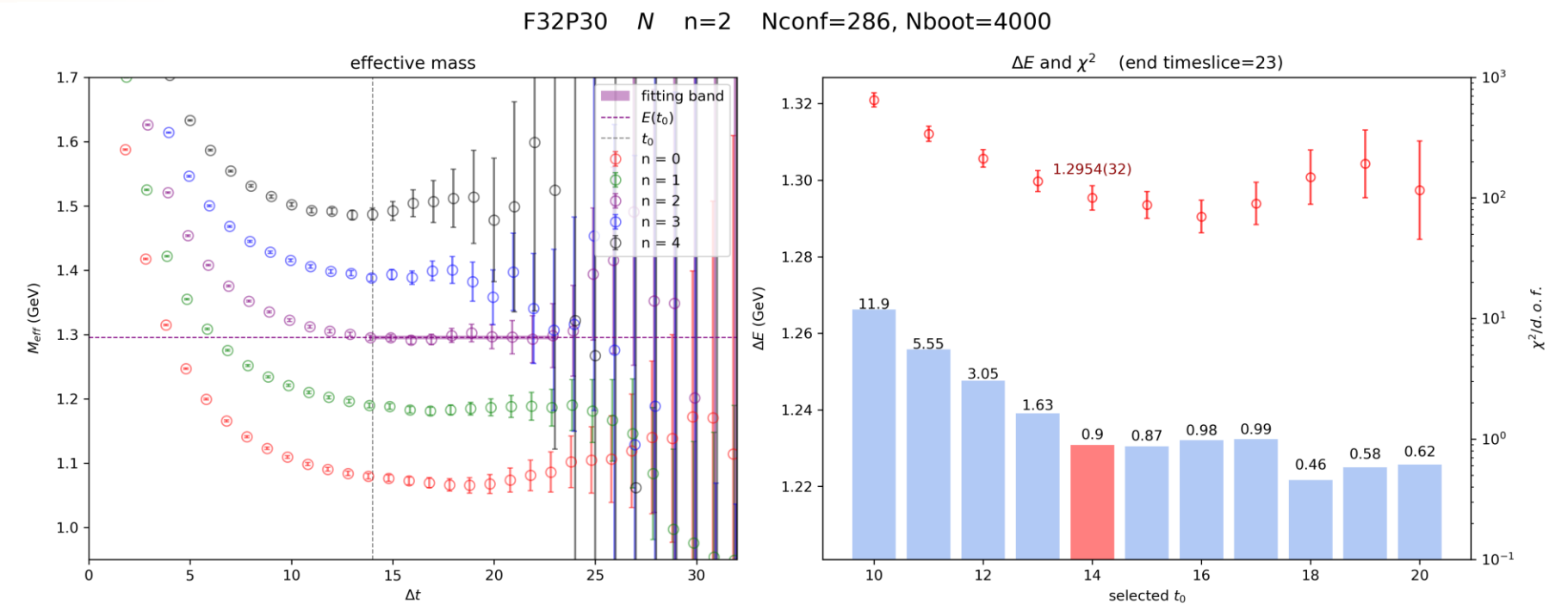
$$\lambda_n(t, t_0) \sim e^{-E_n(t-t_0)} + \dots \rightarrow E_n$$

- ratio correlation function:

$$R_n(t, t_0) = \frac{\lambda_n(t, t_0)}{C_n^{(1)}(t) \cdot C_n^{(2)}(t)}$$

$$R_n(t, t_0) \sim e^{-\Delta E_n(t-t_0)} + \dots \rightarrow \Delta E_n$$

$$E_n = E_n^{(1)} + E_n^{(2)} + \Delta E_n$$



Scattering analysis

Lüscher's formula for two-body scattering

$$\det[1 + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of
phase-space factors
 $\rho_{ij} = \delta_{ij} \frac{2k_i}{E_{cm}}$

Infinite-volume
scattering matrix

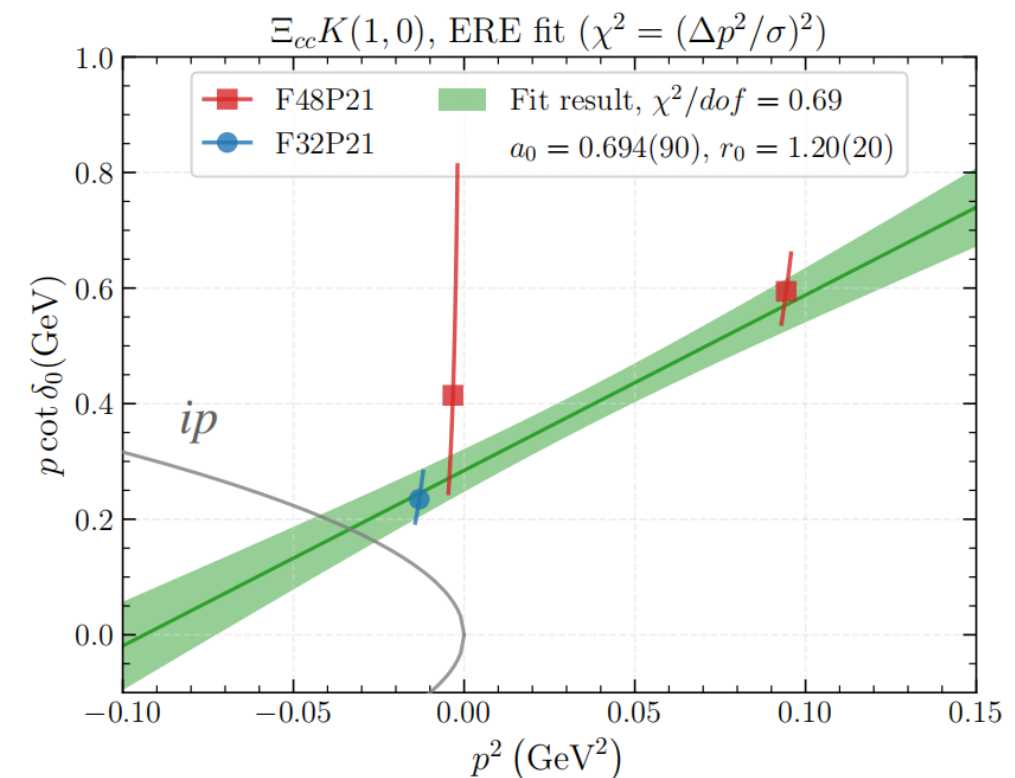
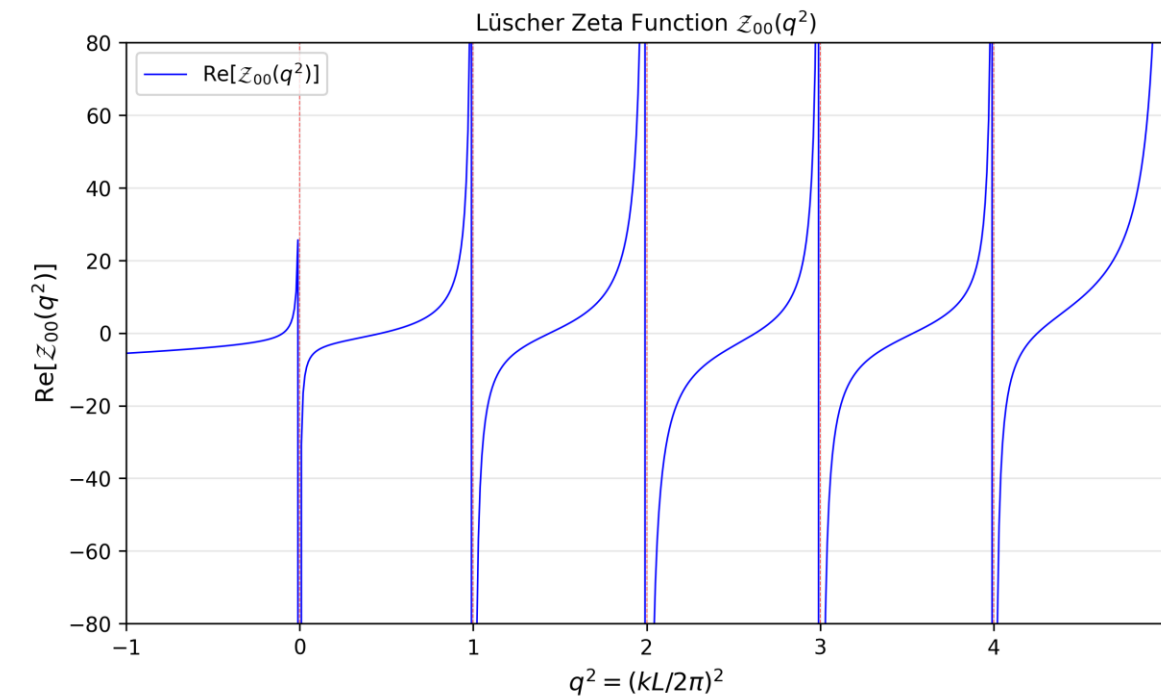
Finite volume
information
 $M(E_{cm}, L)$

For single-channel S-wave scattering

$$k \cot \delta_0(k) = \frac{2}{L\sqrt{\pi}} \mathcal{Z}_{00}(1; q^2)$$

Effective range expansion

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$



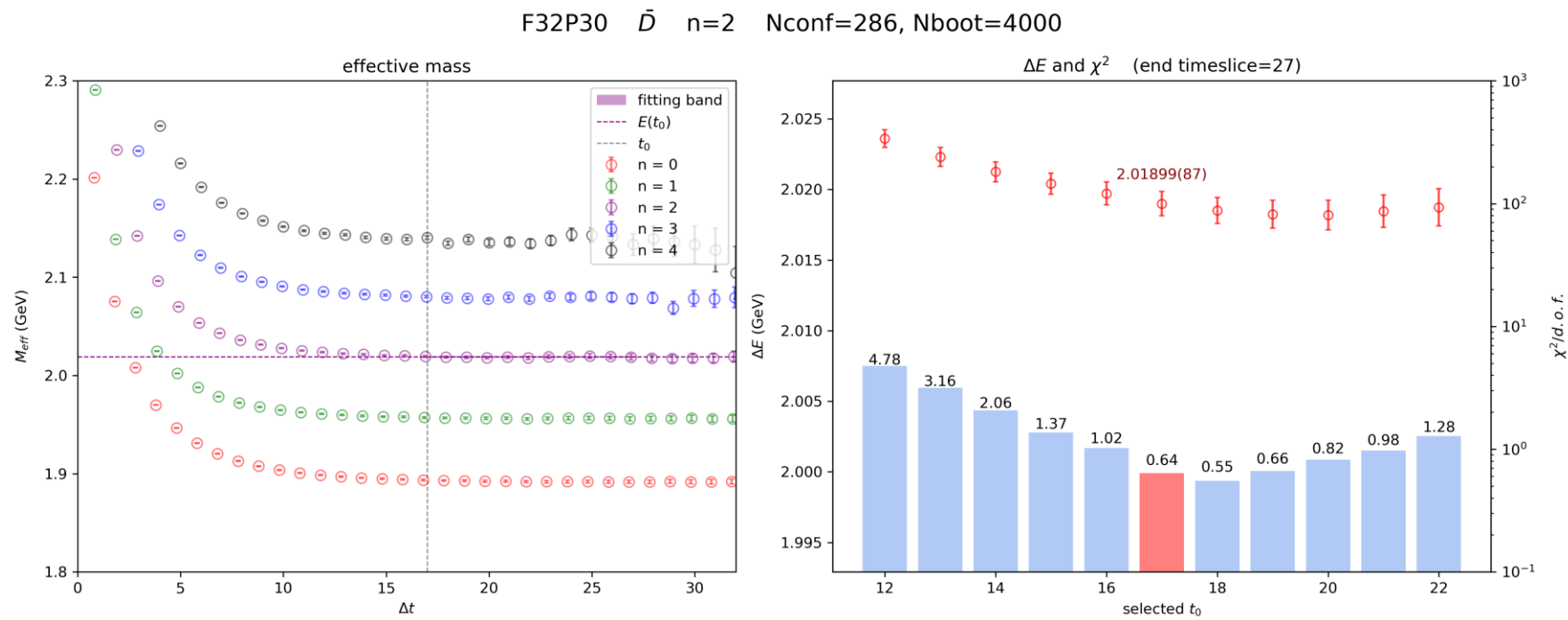


03

Data Analysis



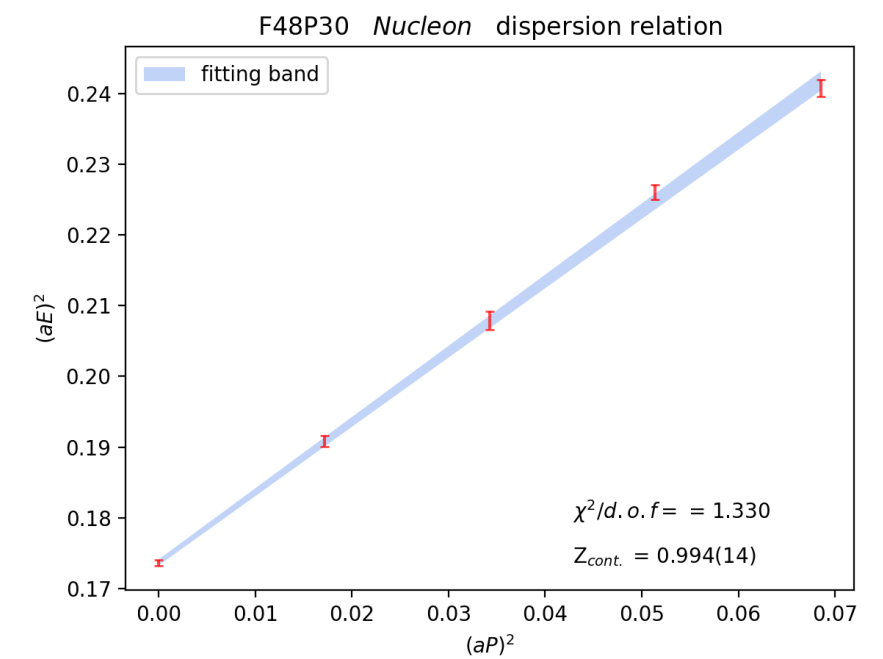
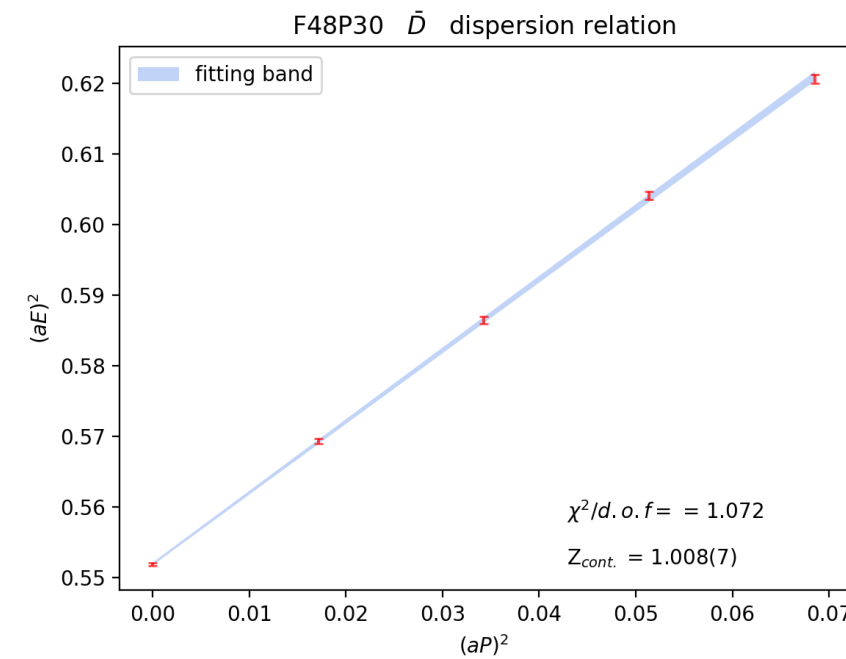
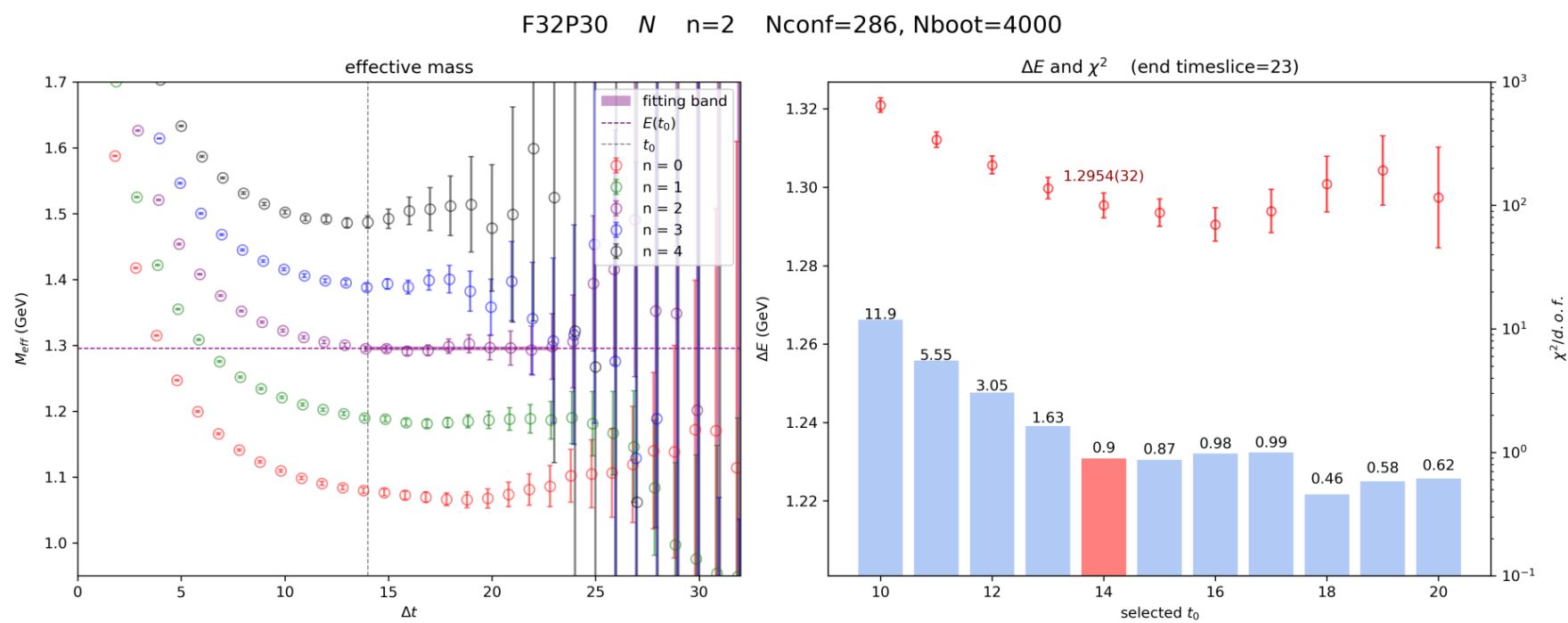
Caculation of single-particle



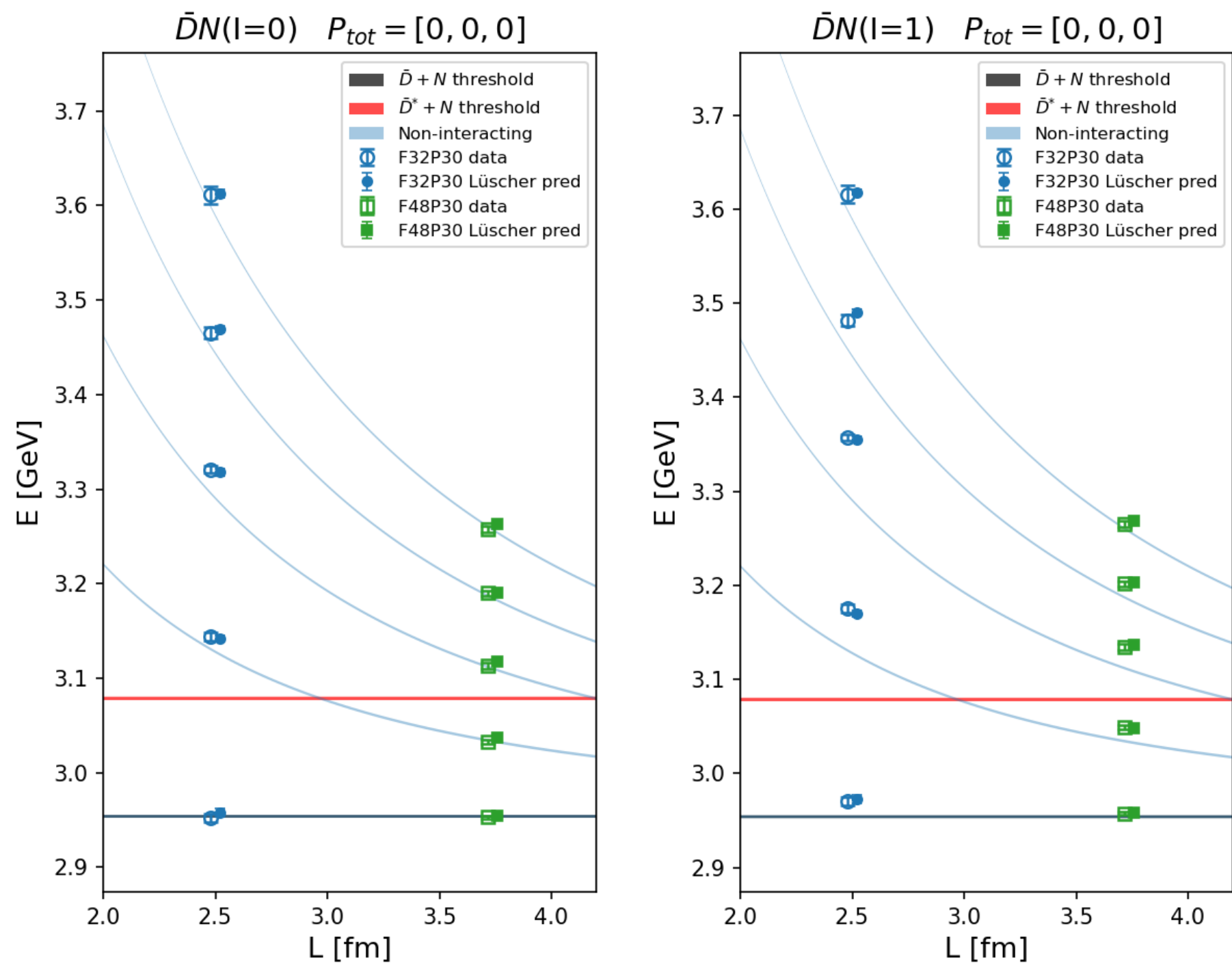
		$E(\text{GeV})$				
ensemble	particle	$p^2 = 0$	$p^2 = 1$	$p^2 = 2$	$p^2 = 3$	$p^2 = 4$
F32P30	$\bar{D}$	1.8912(06)	1.9567(06)	2.0190(10)	2.0810(10)	2.1388(10)
	N	1.0644(40)	1.1830(33)	1.2868(39)	1.3824(60)	1.4776(64)
F48P30	$\bar{D}$	1.8921(03)	1.9218(06)	1.9506(09)	1.9797(10)	2.0065(10)
	N	1.0614(13)	1.1127(22)	1.1614(36)	1.2111(28)	1.2499(32)

Single-particle dispersion relation

$$E^2 = m_0^2 + Z \cdot p^2, \quad Z = c^2$$

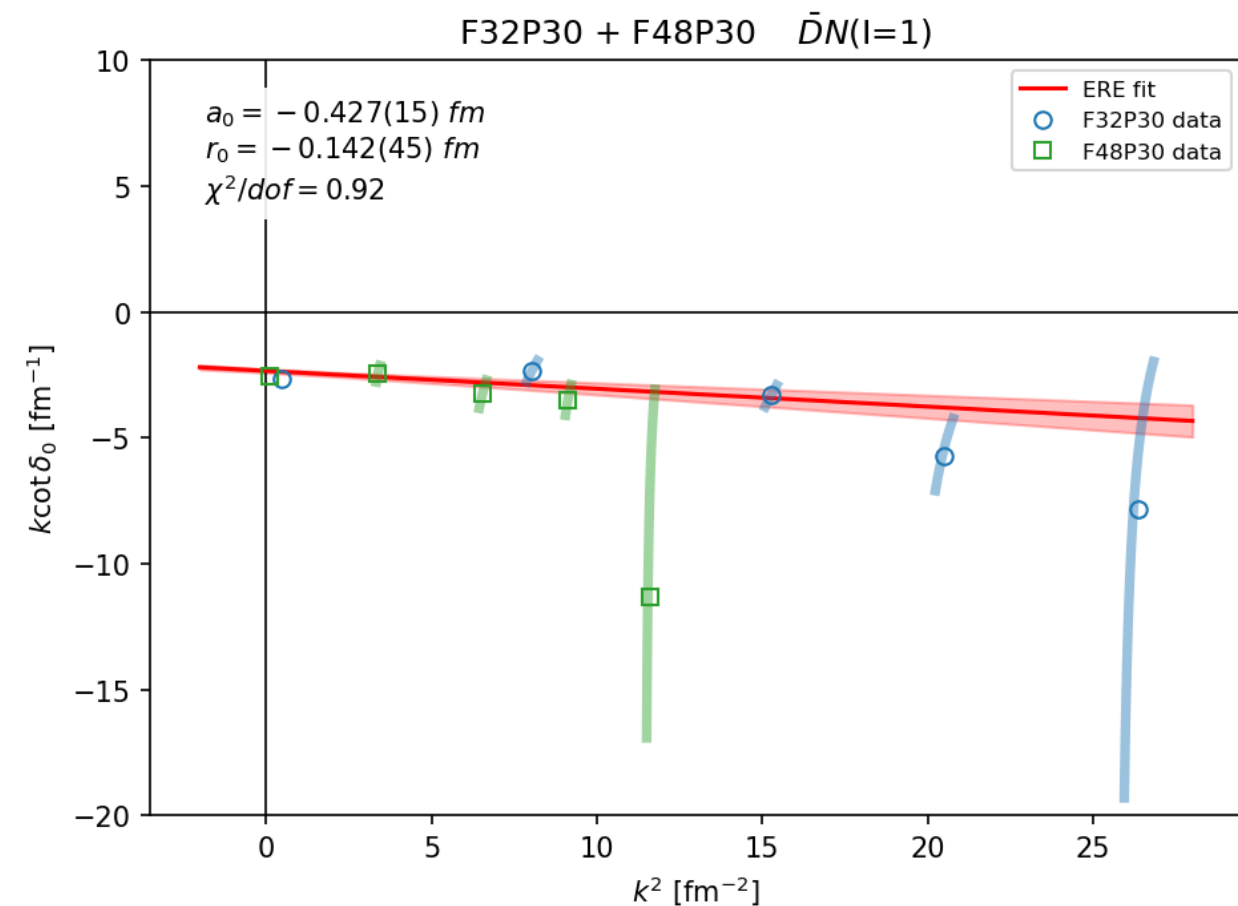


Single-channel results



$l=0 \quad E_{int}(\text{GeV})$ from ratio

n	0	1	2	3	4
F32P30	2.9517(37)	3.1444(36)	3.3210(35)	3.4669(48)	3.6142(77)
F48P30	2.9527(15)	3.0325(20)	3.1126(37)	3.1897(29)	3.2568(33)

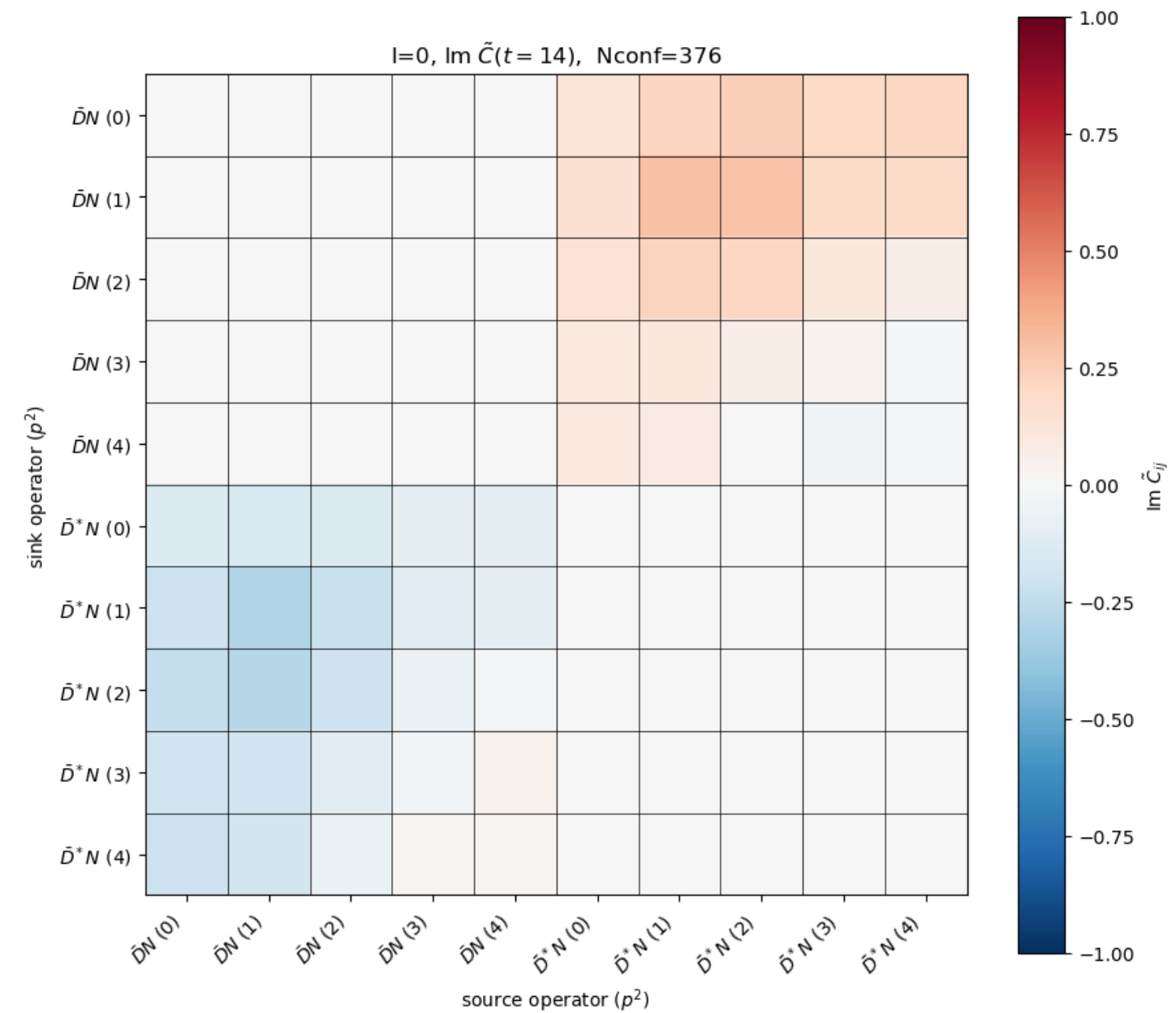
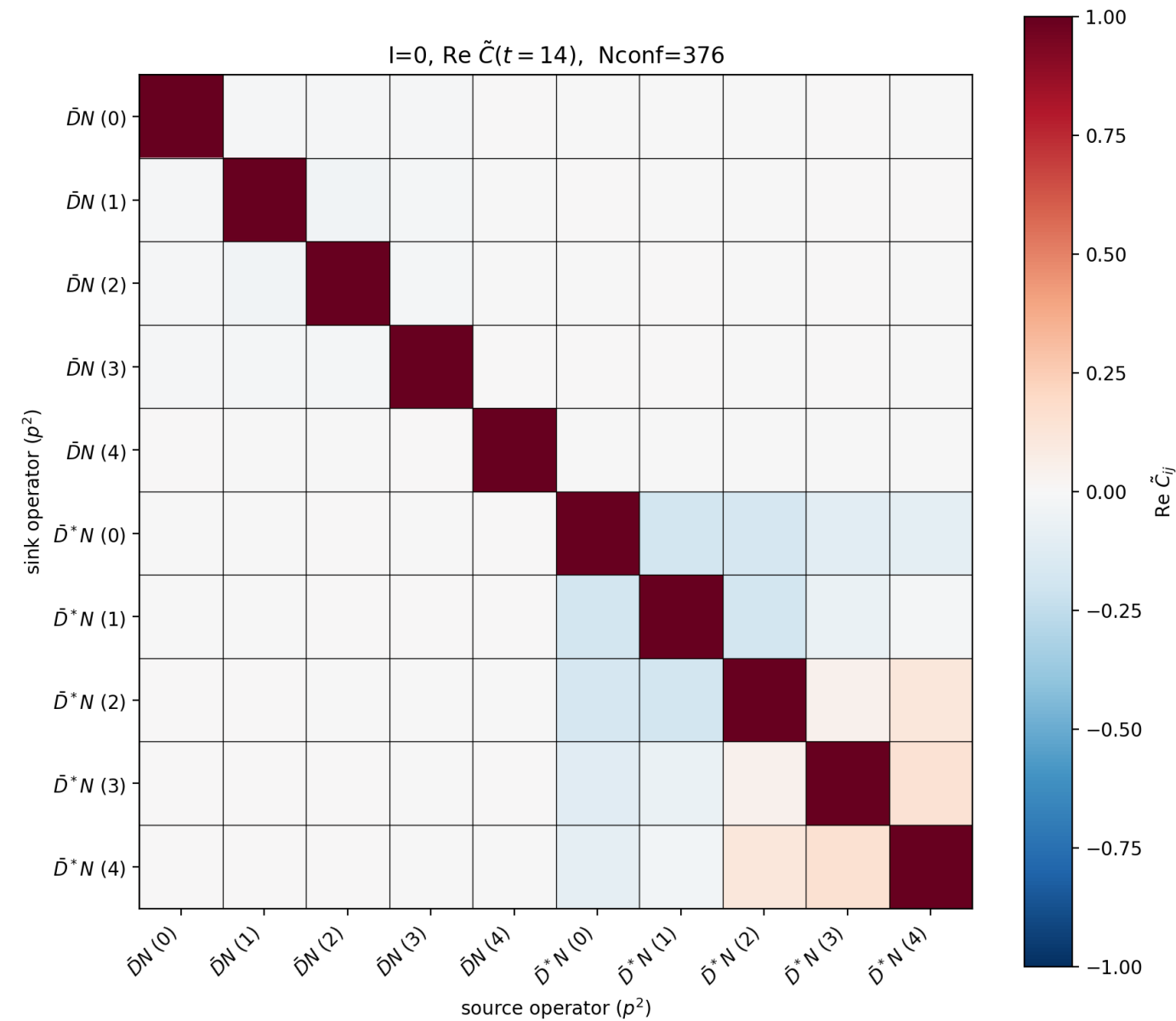


This spectral pattern suggests that a coupled-channel analysis of the $\bar{D}N - \bar{D}^*N$ system is needed.

two-channel correlation

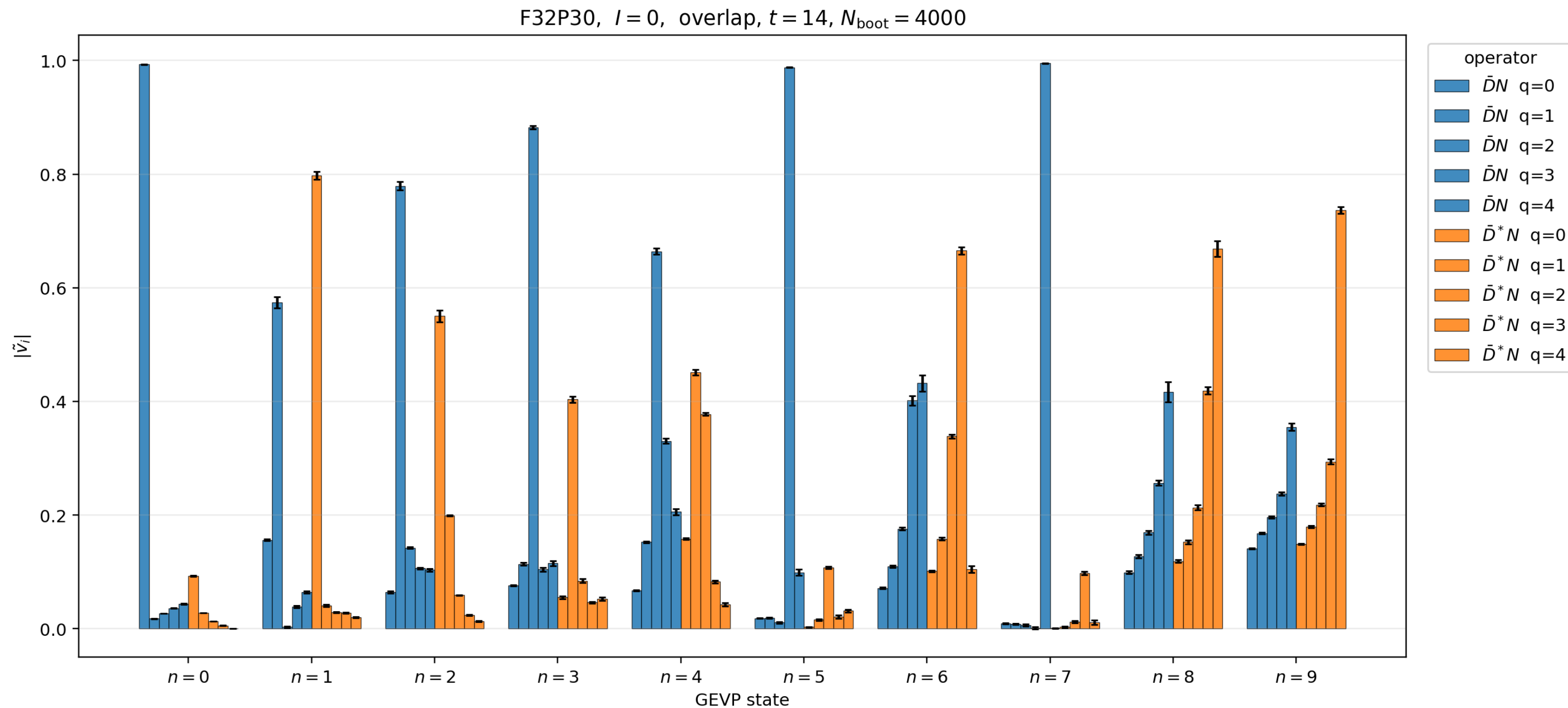
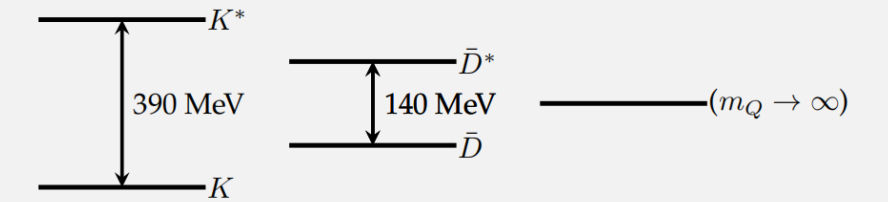
$$C(t) = \begin{pmatrix} C_{SS}(t) & C_{SV}(t) \\ C_{VS}(t) & C_{VV}(t) \end{pmatrix} \quad S = \bar{D}N, \quad V = \bar{D}^*N$$

10 × 10 correlation matrix

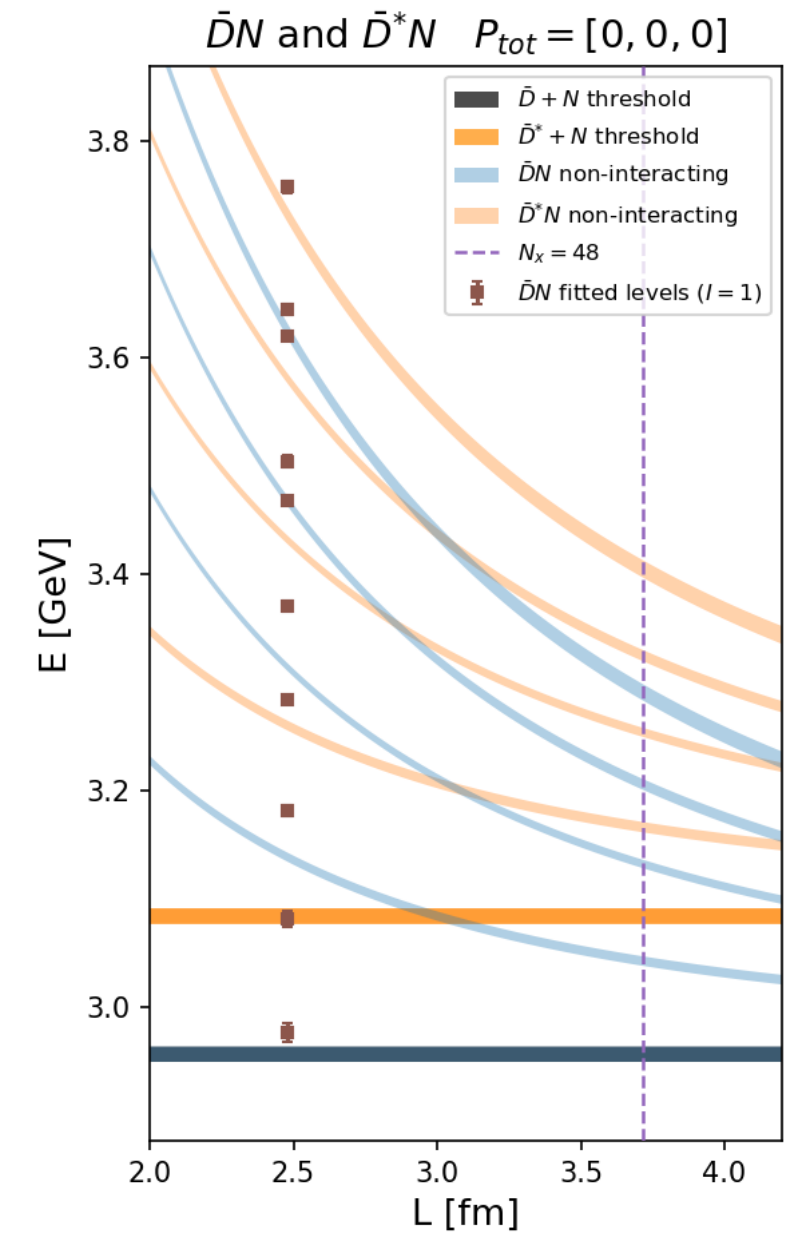
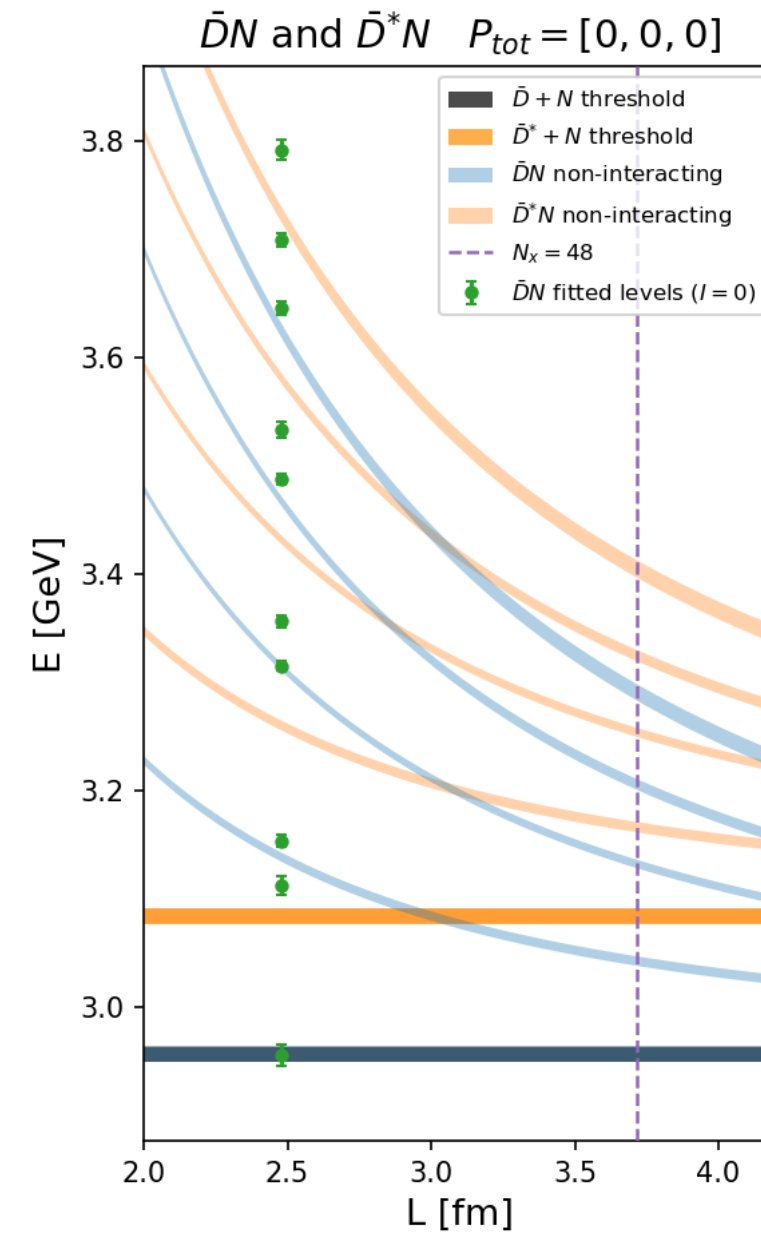
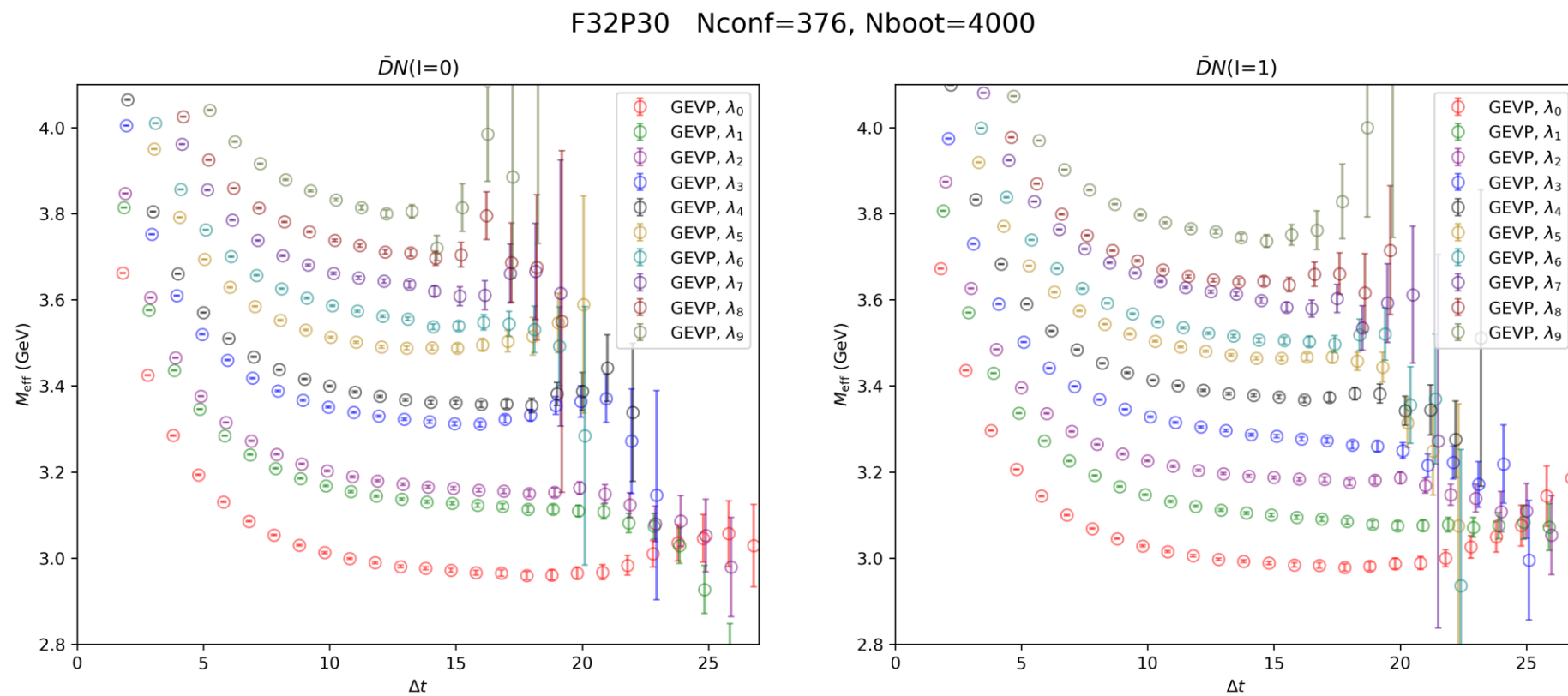


two-channel performance

Coupling to the $\bar{D}^*N$ channel (is thought to) enhance attraction ($\leftrightarrow KN$ system)



F32P30 Interaction energy level



F32P30 E_{int} from lsqfit

State n	0	1	2	3	4	5	6	7	8	9
I=0	2.9556(97)	3.1129(86)	3.1537(50)	3.3150(43)	3.3565(58)	3.4880(46)	3.5337(71)	3.6458(59)	3.7082(62)	3.7919(95)
I=1	2.9771(88)	3.0822(74)	3.1814(48)	3.2845(38)	3.3707(52)	3.4681(35)	3.5044(60)	3.6193(35)	3.6447(51)	3.7578(56)



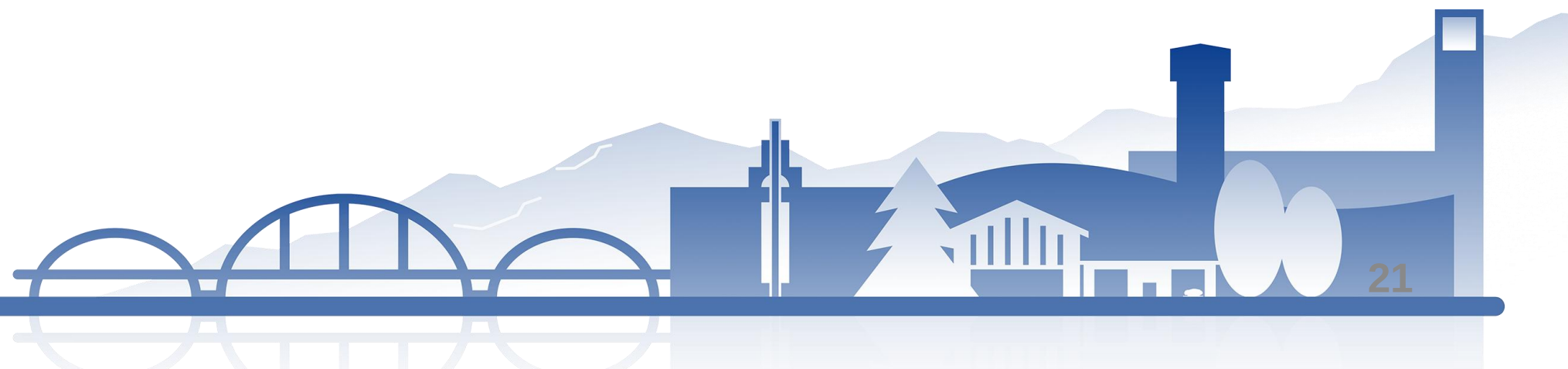
04

Summary and Outlook



Summary

- In the $I = 0, 1$, $\mathbf{P}_{\text{tot}} = \mathbf{0}$, and G_1^- sector, we constructed a 10×10 coupled-channel correlation matrix from five $\bar{D}N$ operators and five $\bar{D}^*N$ operators
- We extracted the finite-volume spectrum using the GEVP.
- We identified the F32P30 threshold and extracted several low-lying two-particle energy levels.
- The ten GEVP states belong to the same coupled-channel system, and significant channel mixing is observed within the current operator basis. A coupled-channel framework is therefore required for a more complete interpretation of the spectrum.



Outlook

01

Increase sample size

Increase the statistics and enlarge the set of lattice ensembles to test the stability of the energy levels and operator overlaps

02

higher partial waves

determine the locations of possible three-body thresholds, and assess the effects of higher partial waves and partial-wave mixing

03

Scattering analysis

Combine the finite-volume energy spectrum with Lüscher's method to extract scattering information, such as phase shifts, scattering lengths



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Thank you for watching

