



内蒙古大学
INNER MONGOLIA UNIVERSITY

Form factors for $\Xi^- \rightarrow \Lambda \ell^- \bar{\nu}_\ell$ from lattice QCD

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第六届中国格点量子色动力学研讨会

CUHK-Shenzhen, Shenzhen • 8–11 October 2026

Outline

1. Motivation

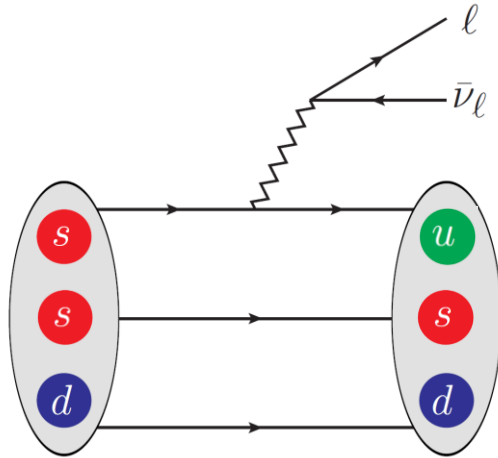
2. Form factor formalism

3. Numerical results

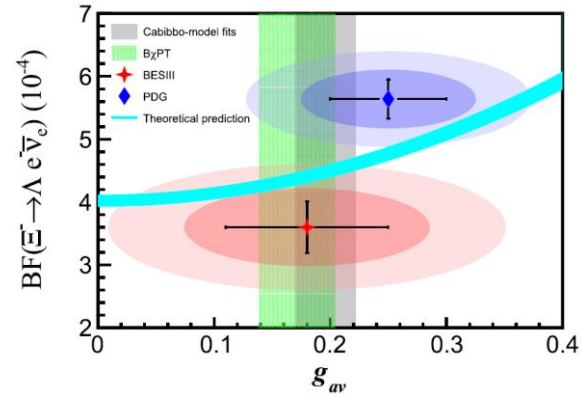
4. Summary and outlook

$\Xi \rightarrow \Lambda$ semileptonic decay

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Branch ratio of $\Xi \rightarrow \Lambda e \bar{\nu}_e$ in experiment measurements



PDG: 2024 & Z.Phys.C 21 (1983) 27

Phys.Rev.Lett. 137 (2026) 9, 091801

	BESIII	PDG	
$\text{Br}(\Xi \rightarrow \Lambda e^- \bar{\nu}_e)$	$(3.60 \pm 0.40_{\text{stat}} \pm 0.10_{\text{syst}}) \times 10^{-4}$	$(5.63 \pm 0.31_{\text{total}}) \times 10^{-4}$	3.9σ
$\text{Br}(\Xi \rightarrow \Lambda \mu^- \bar{\nu}_\mu)$	-	$3.5^{+3.5}_{-2.2} \times 10^{-4}$	

1. Electron mode: the 3.9 σ tension between experiments
2. Determination of $g_{av} = g_1(0)/f_1(0)$
3. Test lepton-flavor universality
4. Provide theoretical input for precision measurements of the μ mode

$|V_{us}|$ and CKM unitarity

-4-

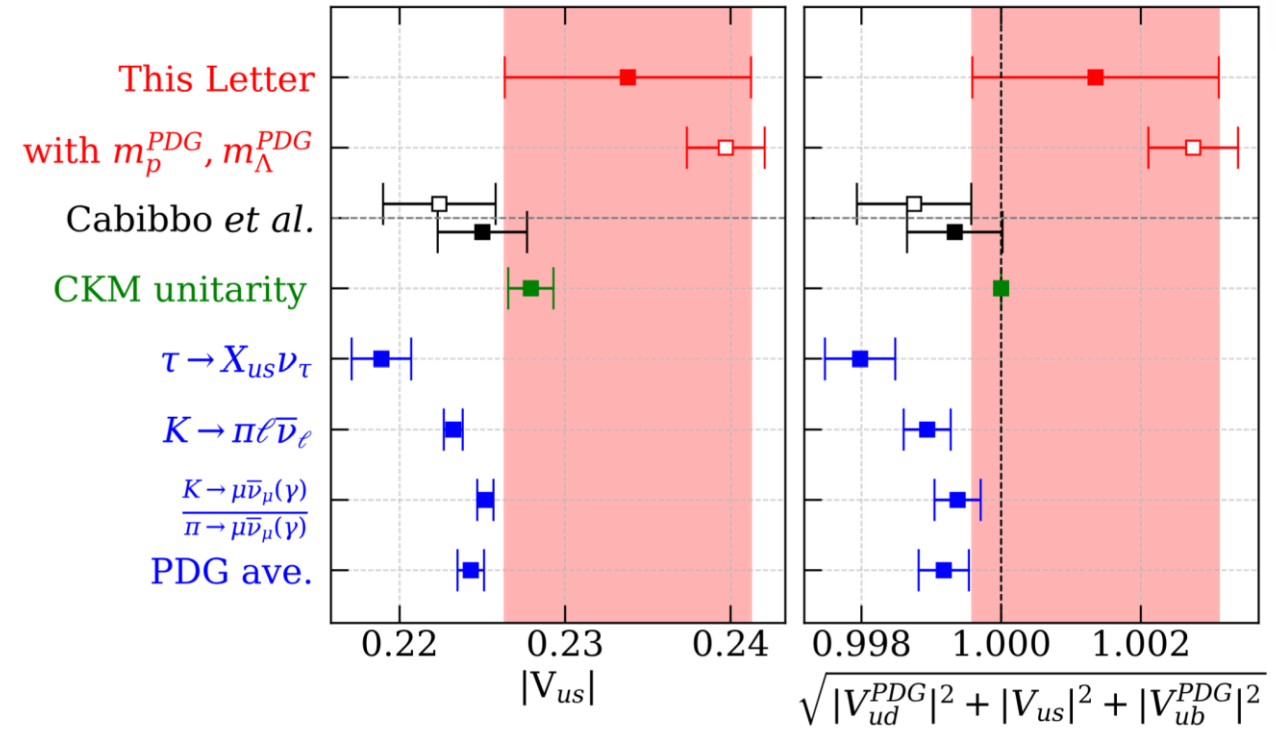
PHYSICAL REVIEW LETTERS 135, 231901 (2025)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

PDG2026

$$\begin{aligned} |V_{ud}| &= 0.97367 \pm 0.00032 & 0.03\% \\ |V_{us}| &= 0.22431 \pm 0.00085 & 0.4\% \\ |V_{ub}| &= 0.00382 \pm 0.00020 & 5\% \end{aligned}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0007$$



τ and K decay determinations of $|V_{us}|$ differ by more than 2σ . Hyperon decays provide an independent test.

5. CKM matrix element unitarity test from baryon decay

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What does lattice QCD calculate?

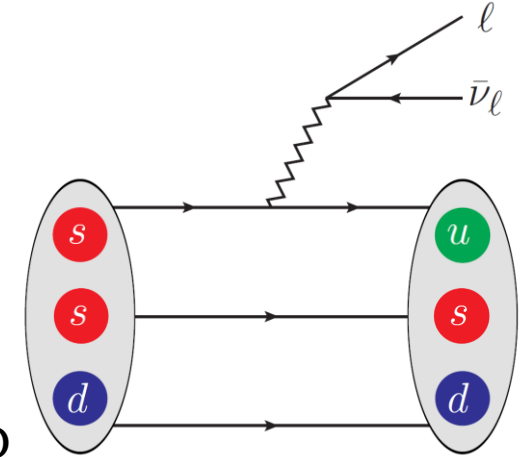
-6-

Weak decay amplitude

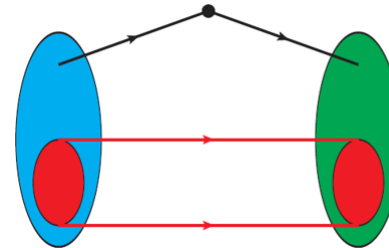
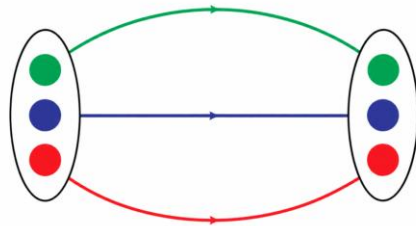
$$\mathcal{M} = \frac{G_F V_{us}}{\sqrt{2}} L^\mu H_\mu$$

$$L^\mu = \bar{u}_\ell \gamma^\mu (1 - \gamma_5) v_{\bar{\nu}_\ell} \quad \text{Leptonic current: known}$$

$$H_\mu = \langle \Lambda | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Xi^- \rangle \quad \text{Hadronic matrix element: calculated using lattice QCD}$$



On the lattice, we calculate **2pt** for the initial and final baryons and **3pt** with a weak-current insertion.



3pt in lattice QCD

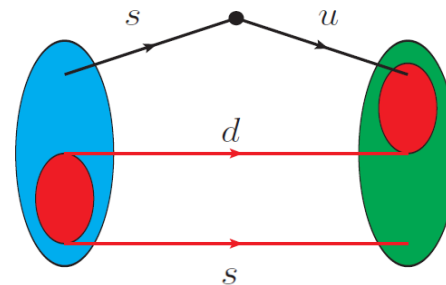
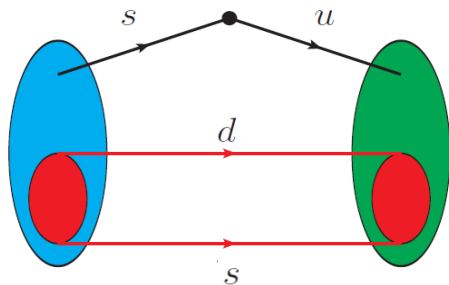
-7-

Interpolator of initial and final baryon

$$O_{\Lambda} = \epsilon_{ijk}(u_i^T C \gamma_5 d_j) s_k,$$

$$O_{\Xi^-} = \epsilon_{ijk}(d_i^T C \gamma_5 s_j) P_+ s_k,$$

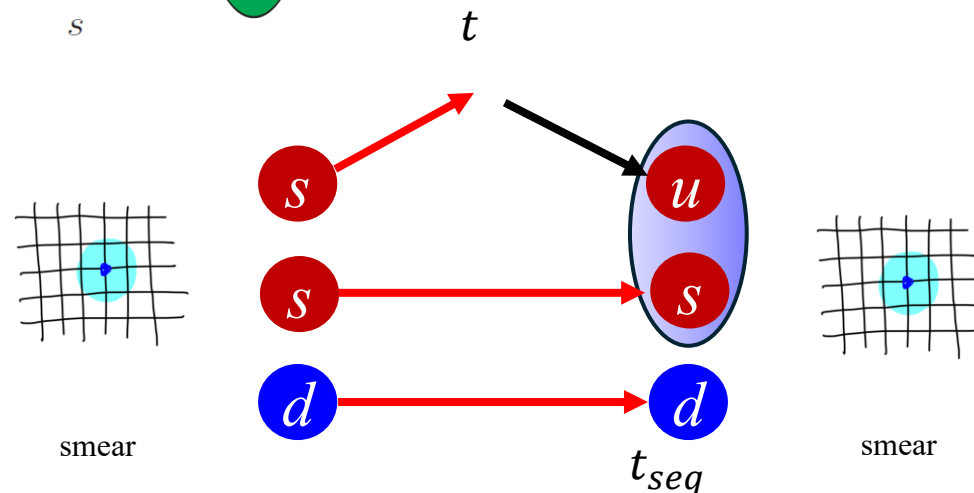
Two Wick contractions



Sequential source method

$$C_{3\text{pt}}(t, t_{\text{seq}}, \vec{p}, \vec{q}) T = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \text{Tr}\{G^{\text{seq}}(x, 0) \Gamma G(x, 0)\}$$

Sequential propagator



Form factors

We use the **helicity based** definition of the form factors

$$\begin{aligned}
 \langle \Lambda(p', s') | \bar{u} \gamma_\mu s | \Xi(p, s) \rangle &= \bar{u}_\Lambda(p', s') \left[f_0(q^2) (m_i - m_f) \frac{q^\mu}{q^2} \right. \\
 &\quad + f_+(q^2) \frac{(m_i + m_f)}{s_+} \left(p^\mu + p'^\mu - (m_i^2 - m_f^2) \frac{q^\mu}{q^2} \right) \\
 &\quad \left. + f_\perp(q^2) \left(\gamma^\mu - \frac{2m_f}{s_+} p^\mu - \frac{2m_i}{s_+} p'^\mu \right) \right] u_\Xi(p, s) \\
 \langle \Lambda(p', s') | \bar{u} \gamma_\mu \gamma_5 s | \Xi(p, s) \rangle &= -\bar{u}_\Lambda(p', s') \gamma_5 \left[g_0(q^2) (m_i + m_f) \frac{q^\mu}{q^2} \right. \\
 &\quad + g_+(q^2) \frac{(m_i - m_f)}{s_-} \left(p^\mu + p'^\mu - (m_i^2 - m_f^2) \frac{q^\mu}{q^2} \right) \\
 &\quad \left. + g_\perp(q^2) \left(\gamma^\mu + \frac{2m_f}{s_-} p^\mu - \frac{2m_i}{s_-} p'^\mu \right) \right] u_\Xi(p, s)
 \end{aligned}$$

PRD85, 014035 (2012)

Weinberg form factors for g_{av}

$$\begin{aligned}
 \langle \Lambda(p_f, s_f) | \bar{u} \gamma_\mu (1 - \gamma_5) s | \Xi(p_i, s_i) \rangle &= \bar{u}_\Lambda(p_f) \left[\gamma_\mu f_1(q^2) + i \sigma_{\mu\nu} q^\nu \frac{f_2(q^2)}{m_i} + q_\mu \frac{f_3(q^2)}{m_i} \right] u_\Xi(p_i) \\
 &\quad - \bar{u}_\Lambda(p_f) \left[\gamma_\mu g_1(q^2) + i \sigma_{\mu\nu} q^\nu \frac{g_2(q^2)}{m_i} + q_\mu \frac{g_3(q^2)}{m_i} \right] \gamma_5 u_\Xi(p_i)
 \end{aligned}$$

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Axial charge g_A^H : $g_A^H = f_1^{SU(3)} * g_{av}$

Axial-vector to vector coupling ratio:

$$g_{av} = g_1(0)/f_1(0)$$

Vector form factor: $f_1^{SU(3)} = -\sqrt{3/2}$

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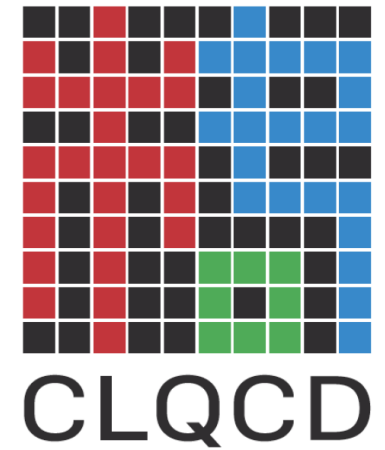
Lattice setup

-10-

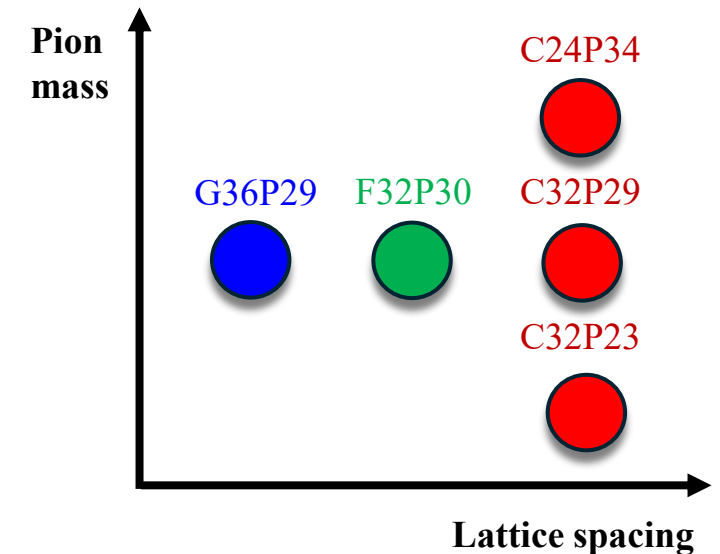
Ensembles from CLQCD:

Lattice spacing: $[0.06887, 0.10524]$ fm $\rightarrow$ **continuum limit extrapolation**

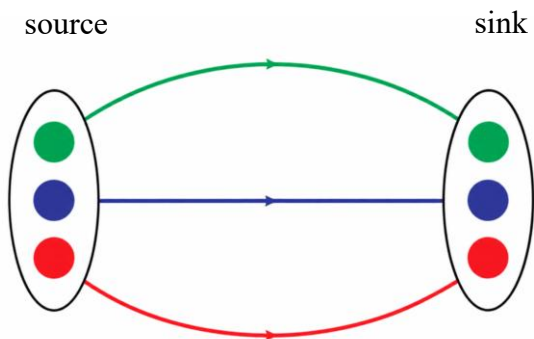
Pion : $[227.9, 340.2]$ MeV $\rightarrow$ **Pion mass extrapolation**



Ensemble	$L^3 \times T$	a (fm)	smear size	m_π (MeV)	Nconf	Nsrc
C24P34	$24^3 \times 64$	0.10524	4 30	340.2	300	30
C32P29	$32^3 \times 64$	0.10524	4 30	293.1	984	2
C32P23	$32^3 \times 64$	0.10524	4 30	227.9	954	5
F32P30	$32^3 \times 96$	0.07753	5 50	300.4	772	4
G36P29	$36^3 \times 108$	0.06887	5 50	297.2	656	4

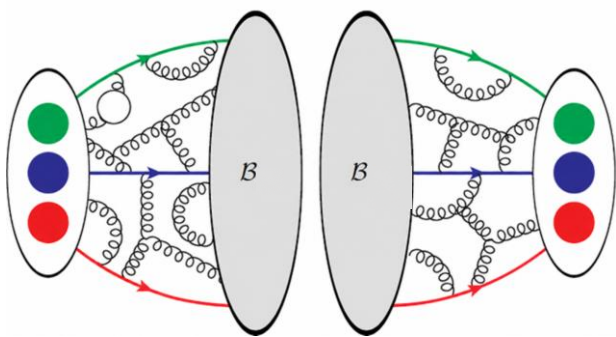


2pt



Parameterize in terms of decay constant

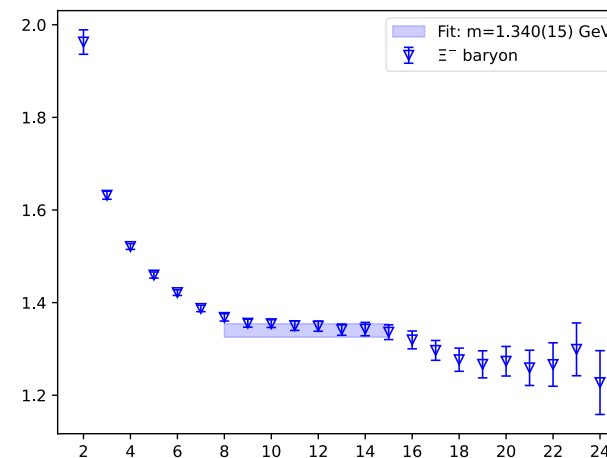
$$\langle 0 | O(0) | \mathcal{B}(p) \rangle = m_{\mathcal{B}} f_{\mathcal{B}} u_{\mathcal{B}}(p)$$



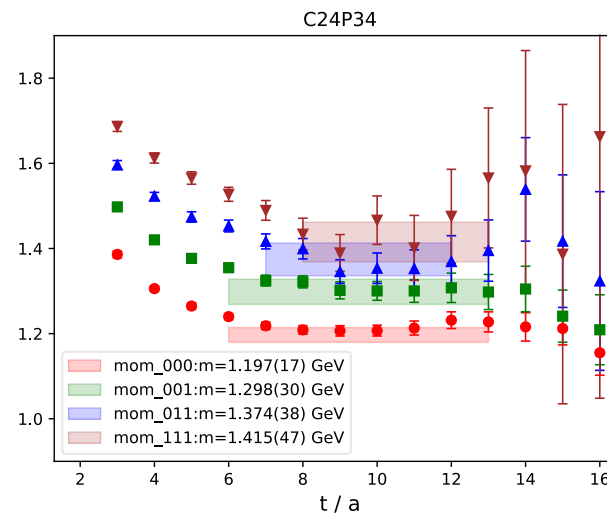
$$C_{2pt}^+(t) = 2m_{\mathcal{B}}^2 f_{\mathcal{B}}^{(0)2} e^{-m_{\mathcal{B}} t} (1 + \Delta C \cdot e^{-\Delta m \cdot t})$$

C24P34

Effective mass of Ξ^-



Effective energies of Λ

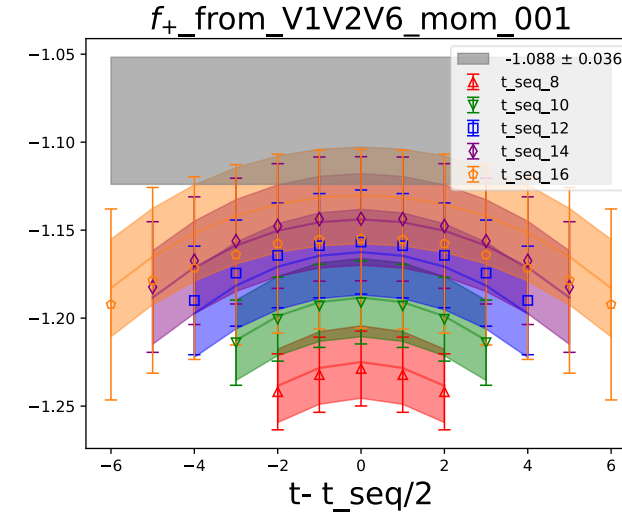


Reduction formula

$$C_{3pt}(t, t_{seq})T \sim \sum_{m,n} e^{(E_m(\vec{p}_f) - E_n(\vec{p}_i))t} e^{-E_m(\vec{p}_f)t_{seq}} \\ \times \langle 0|O(x)|m(\vec{p}_f)\rangle \langle m(\vec{p}_f)|J_\mu(y)|n(\vec{p}_i)\rangle \langle n(\vec{p}_i)|\bar{O}(x)|0\rangle T$$

$$\sqrt{\frac{C_{3pt}(t, t_{seq})T * C_{3pt}(t_{seq} - t, t_{seq})T}{C_{Bf,2pt}(t_{seq})T' * C_{Bi,2pt}(t_{seq})T'}} \propto \underbrace{\langle m(\vec{p}_f)|J_\mu(x)|n(\vec{p}_i)\rangle}_{\text{current}} \underbrace{T}_{\text{projector}} \sim V_i(A_i)$$

mom=(0, 0, 1)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$
$T = 1$	0	0	V_1	V_2
$T = \gamma^x$	V_3	0	0	0
$T = \gamma^y$	0	V_3	0	0
$T = \gamma^x \gamma^y$	0	0	0	0
$T = \gamma^z$	0	0	V_4	V_5
$T = \gamma^x \gamma^z$	V_6	0	0	0
$T = \gamma^y \gamma^z$	0	V_6	0	0
$T = \gamma^5 \gamma^t$	0	0	0	0



tseq extrapolation

Form Factors:

$$V_i = af_+(q^2) + bf_\perp(q^2) + cf_0(q^2), \quad i = 1 \sim 6$$

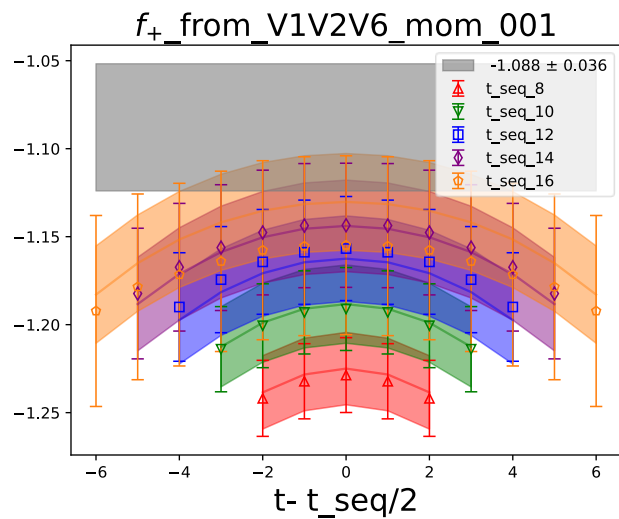
$$A_i = ag_+(q^2) + bg_\perp(q^2) + cg_0(q^2), \quad i = 1 \sim 6$$

Extrapolation to $t_{seq} \rightarrow \infty$

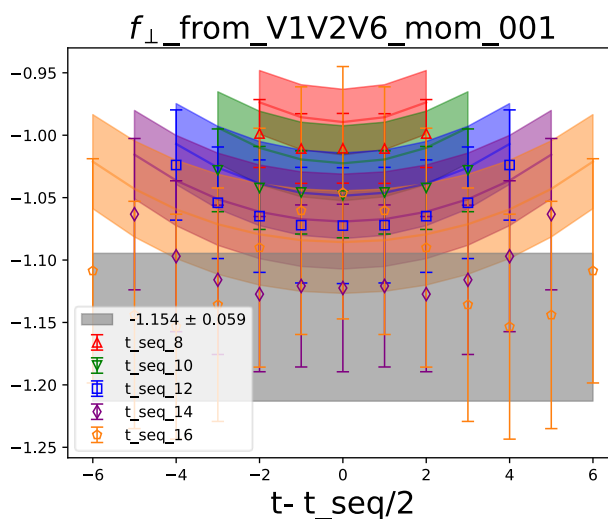
C32P23: (0, 0, 1)

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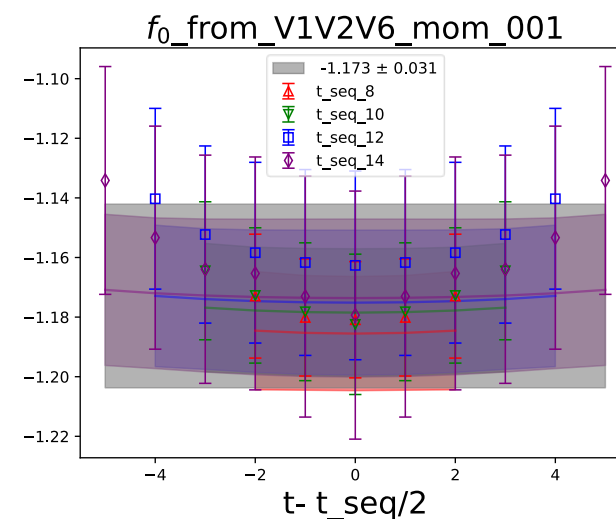
f_+



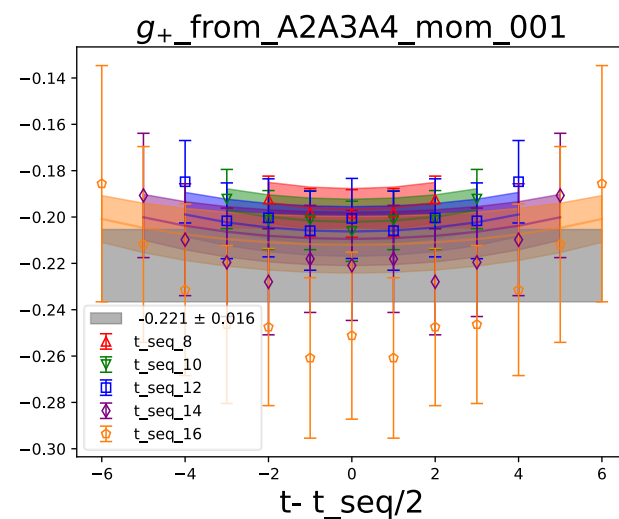
$f_\perp$



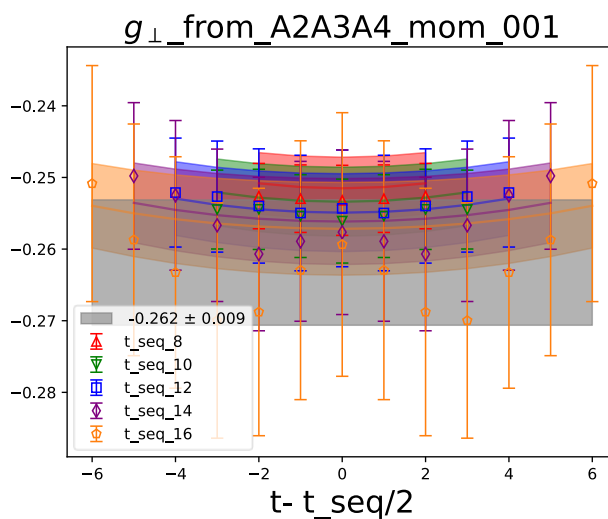
f_0



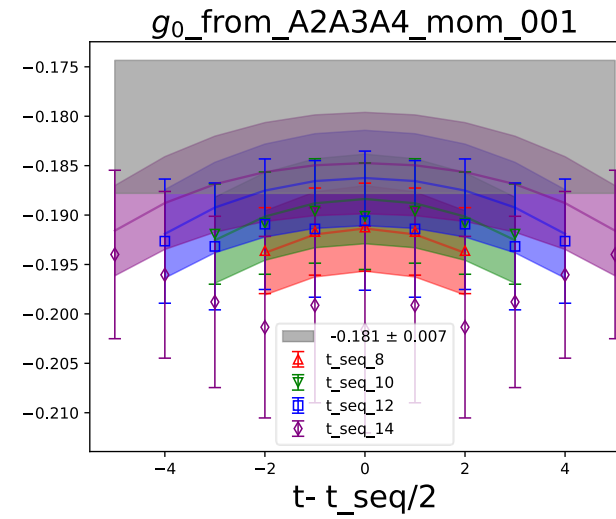
g_+



$g_\perp$



g_0



Renormalization

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Bare lattice currents

$$V_\mu^{\text{lat}} = \bar{u}\gamma_\mu s, \quad A_\mu^{\text{lat}} = \bar{u}\gamma_\mu\gamma_5 s$$

$$V_\mu^R = Z_V^{us} V_\mu^{\text{lat}}, \quad A_\mu^R = Z_A^{us} A_\mu^{\text{lat}}$$

Renormalization of form factors

$$f_i^R(q^2) = Z_V^{us} f_i^{\text{bare}}(q^2), \quad g_i^R(q^2) = Z_A^{us} g_i^{\text{bare}}(q^2)$$

For the flavor-changing current $J = \bar{u}\Gamma_{V/A}s$, we use

$$Z_V^{us} = \sqrt{Z_V Z_V^s}, \quad Z_A^{us} = \sqrt{Z_V Z_V^s} \frac{Z_A}{Z_V}$$

Phys. Rev. D 111, no.5, 054504 (2025)

TABLE VI. The renormalization constants on each ensemble.

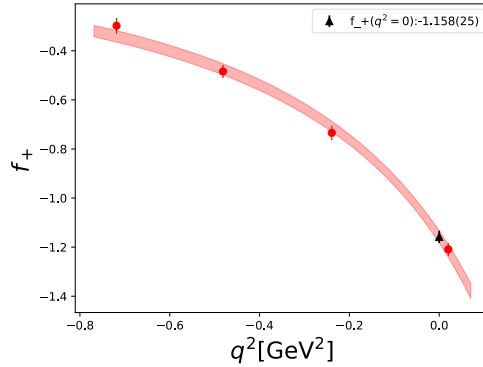
a (fm)	Symbol	Z_V	Z_V^s (optional)	Z_V^c	Z_A/Z_V
0.10524(05)(62)	C24P34	0.79676(32)	0.85034(10)	1.57130(20)	1.07677(72)
	C24P29	0.79814(23)	0.85184(06)	1.57353(18)	1.07244(70)
	C32P29	0.79810(13)	0.85167(04)	1.57163(14)	1.07648(63)
	C32P23	0.79957(13)	0.85350(04)	1.57644(12)	1.07375(40)
	C48P23	0.79954(05)	0.85339(02)	1.57326(08)	1.07317(70)
	C48P14	0.79957(06)	0.85359(02)	1.57415(08)	1.07320(68)
0.08973(20)(53)	E28P35	0.81768(04)	0.85877(02)	1.40361(05)	1.06029(53)
0.07753(03)(45)	F32P30	0.83548(12)	0.86900(03)	1.30566(07)	1.05549(54)
	F48P30	0.83511(04)	0.86880(02)	1.30451(06)	1.05687(66)
	F32P21	0.83579(09)	0.87031(03)	1.30782(08)	1.05416(50)
	F48P21	0.83567(05)	0.86880(02)	1.30673(04)	1.05434(88)
0.06887(12)(41)	G36P29	0.84636(09)	0.87473(05)	1.23990(13)	1.04500(22)
0.05199(08)(31)	H48P32	0.86855(04)	0.88780(01)	1.12882(11)	1.03802(28)

q^2 dependence from the z-expansion

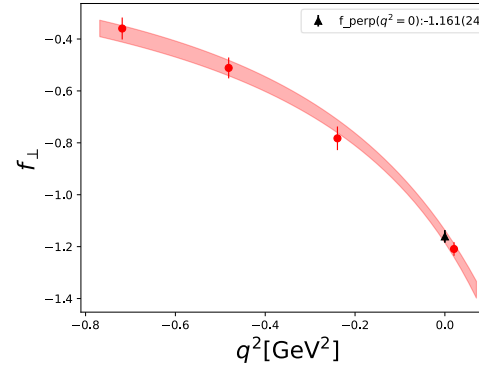
C24P34

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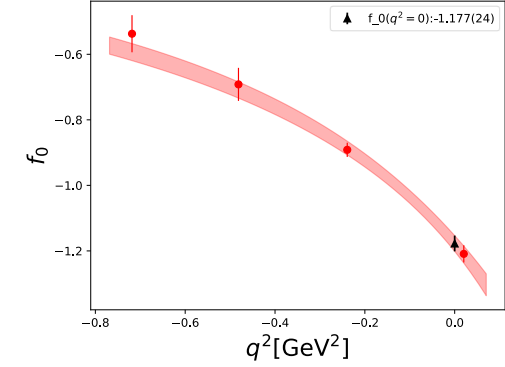
f_+ $m_{pole} = m_{K^*} = 0.892$ GeV



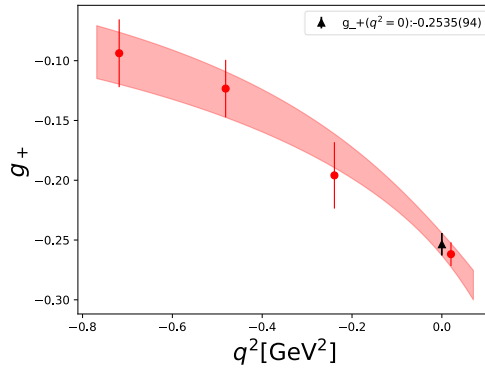
$f_\perp$ $m_{pole} = m_{K^*} = 0.892$ GeV



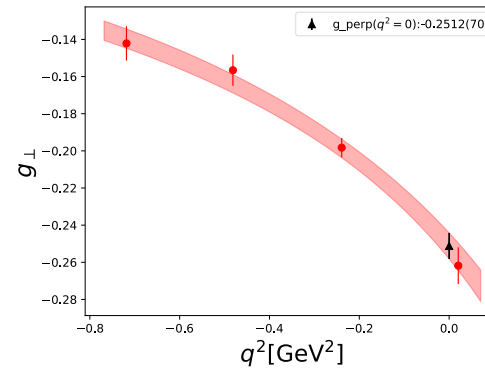
f_0 $m_{pole} = m_{K_0^*} = 0.838$ GeV



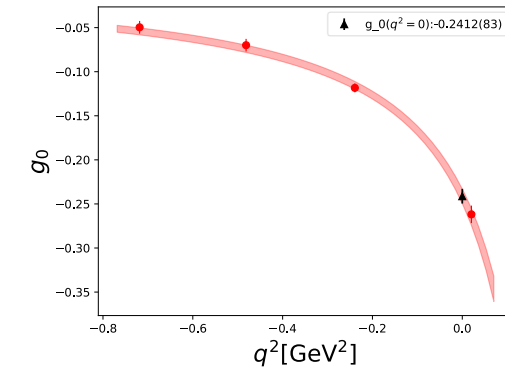
g_+ $m_{pole} = m_{K_1} = 1.27$ GeV



$g_\perp$ $m_{pole} = m_{K_1} = 1.27$ GeV



g_0 $m_{pole} = m_{K^\pm} = 0.494$ GeV



z-expansion:

$$f(q^2) = \frac{a_0 + a_1 z(q^2)}{1 - q^2 / (m_{pole})^2}$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

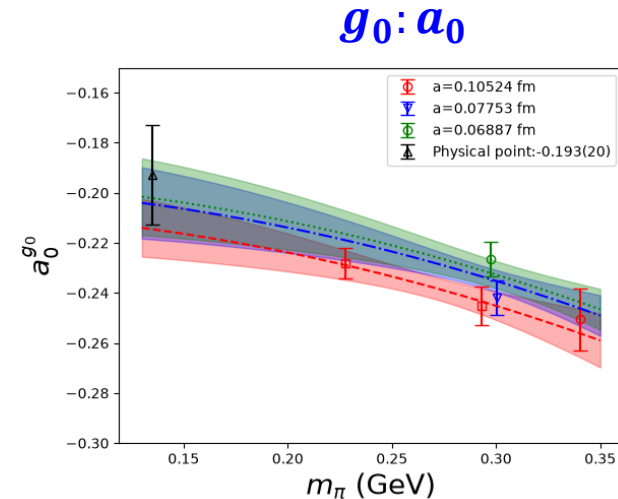
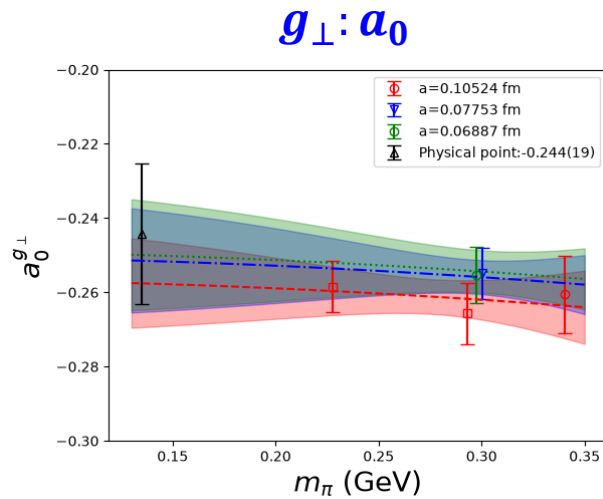
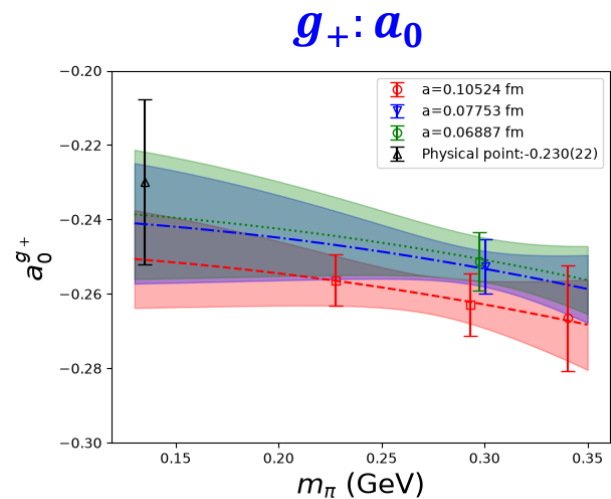
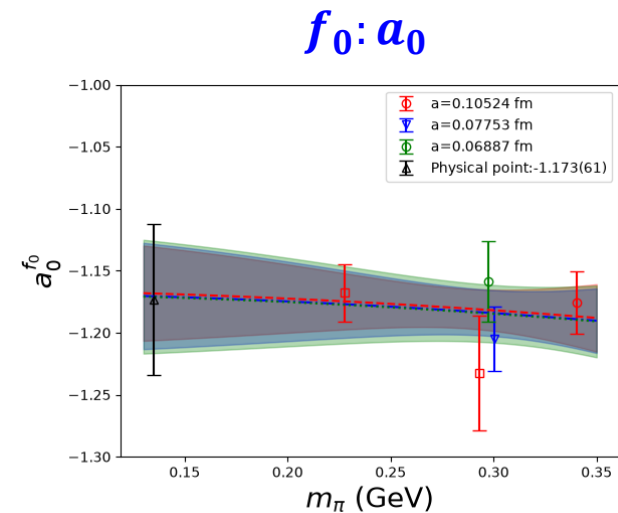
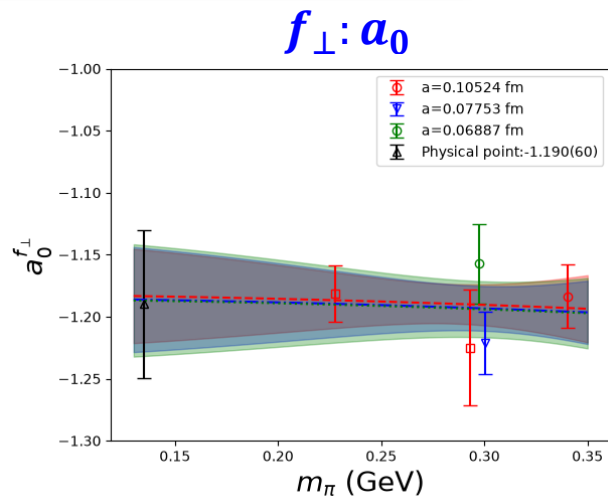
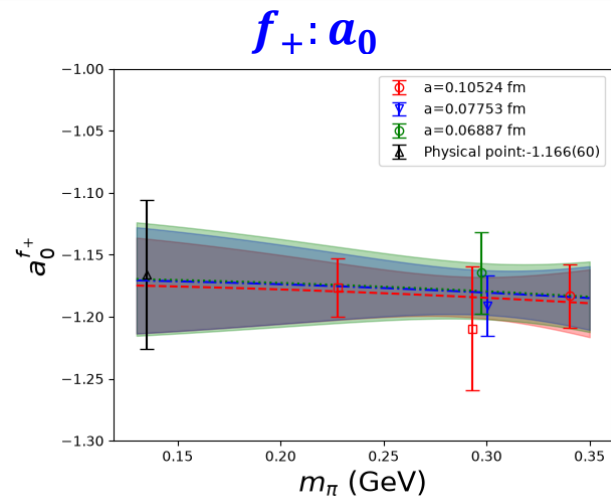
$$t_+ = (m_K + m_\pi)^2$$

$$t_0 = q_{max}^2 = (m_{\Xi^-} - m_\Lambda)^2$$

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Chiral and continuum extrapolation

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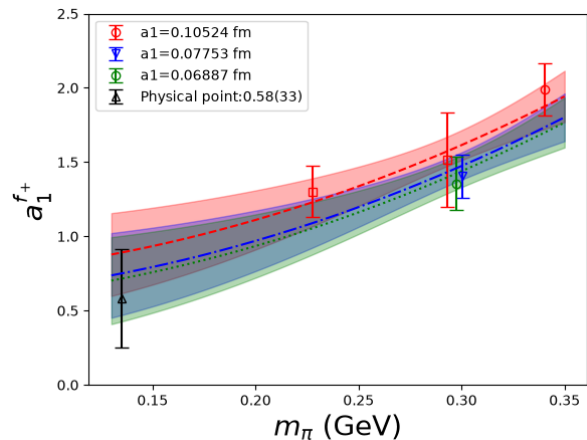


$$a_i = a_{i,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2, \quad i = 0, 1$$

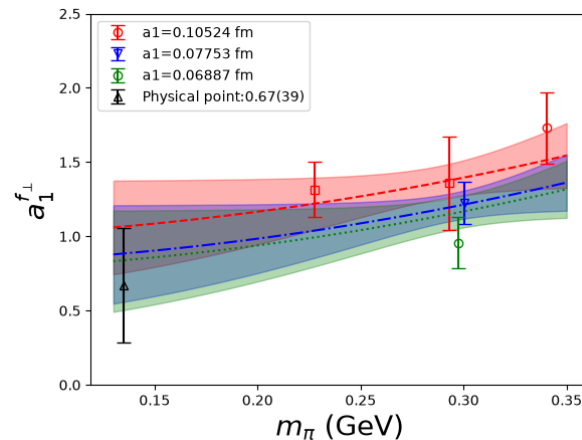
Chiral and continuum extrapolation

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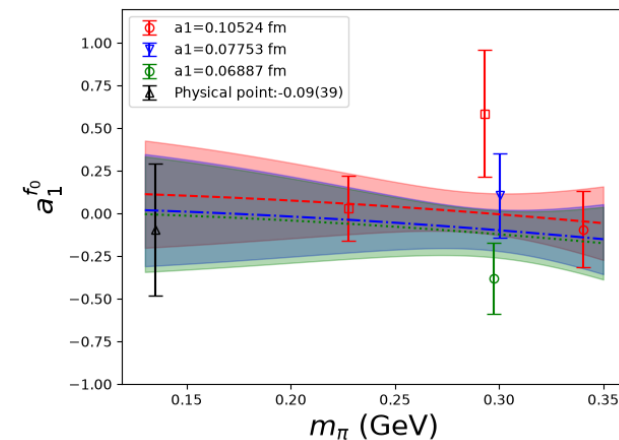
$f_+ : a_1$



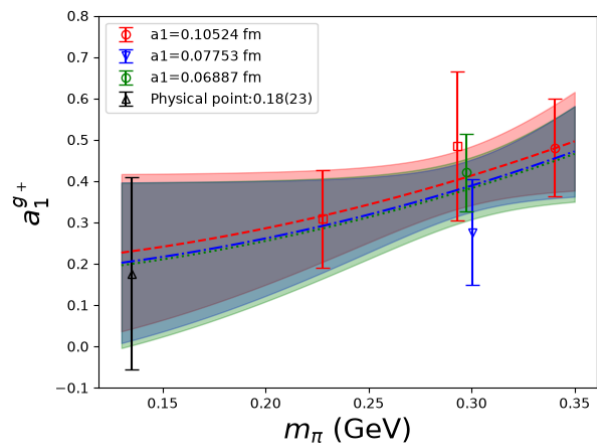
$f_\perp : a_1$



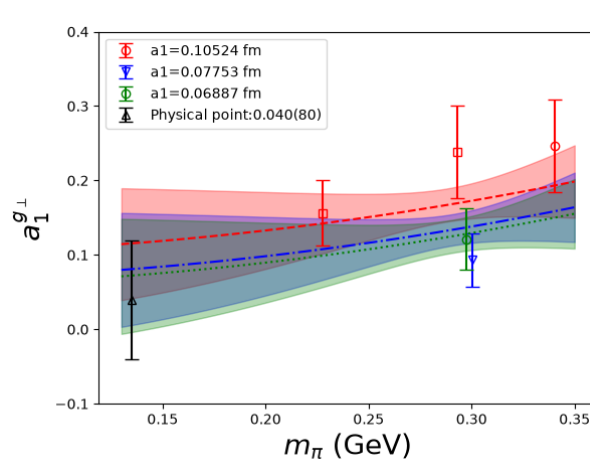
$f_0 : a_1$



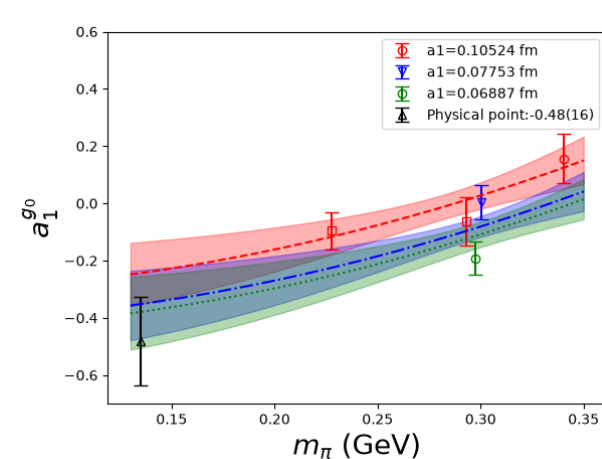
$g_+ : a_1$



$g_\perp : a_1$



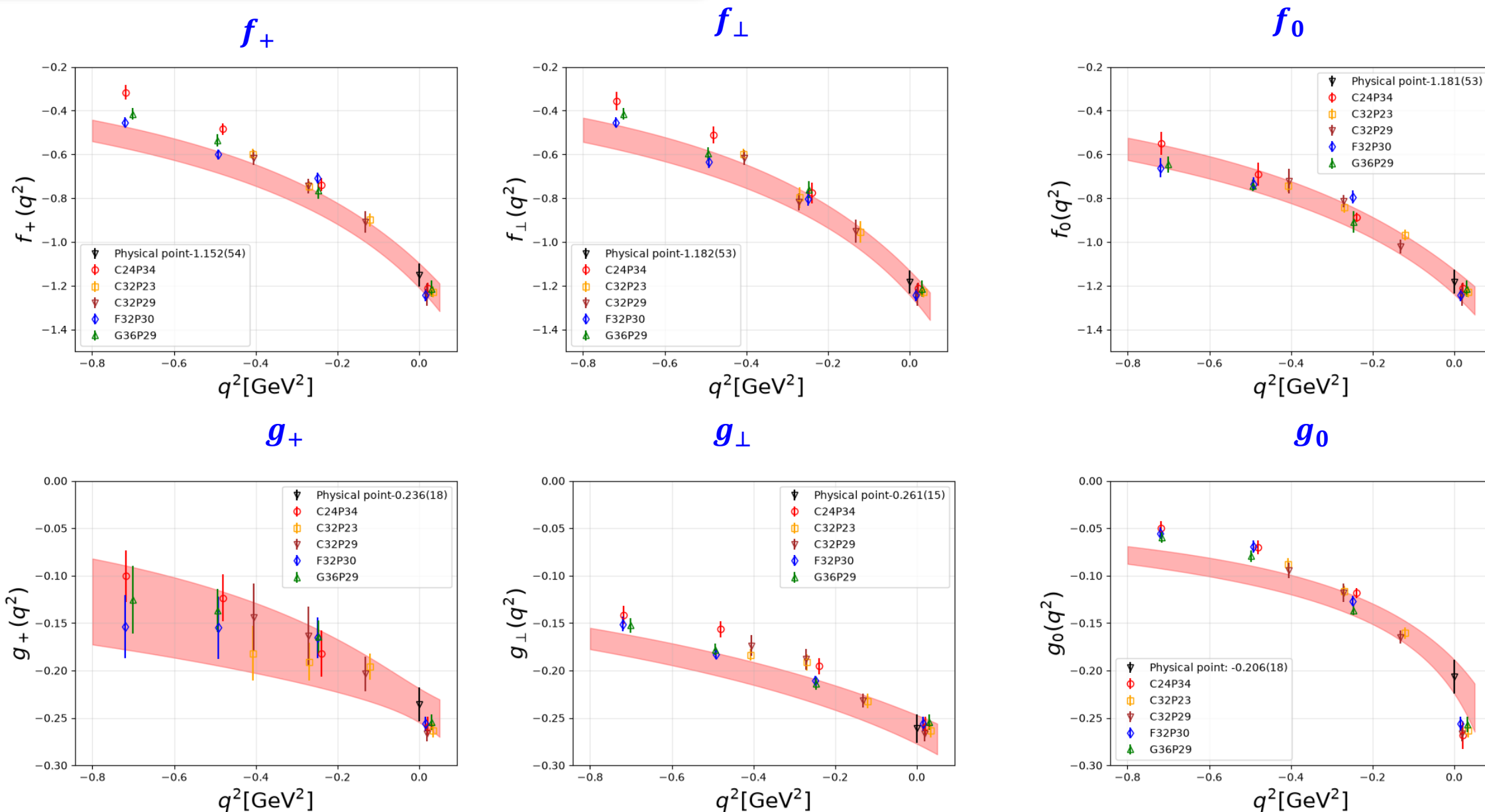
$g_0 : a_1$



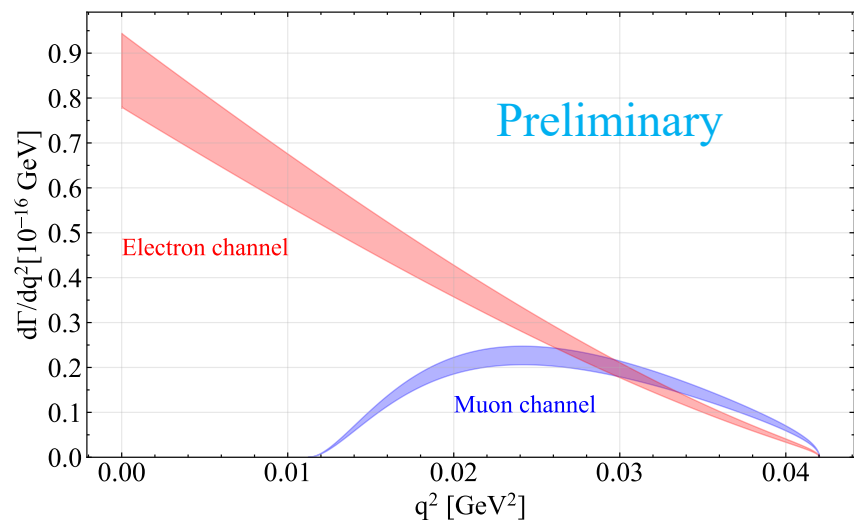
$$a_i = a_{i,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2, \quad i = 0, 1$$

Form factors at the physical point

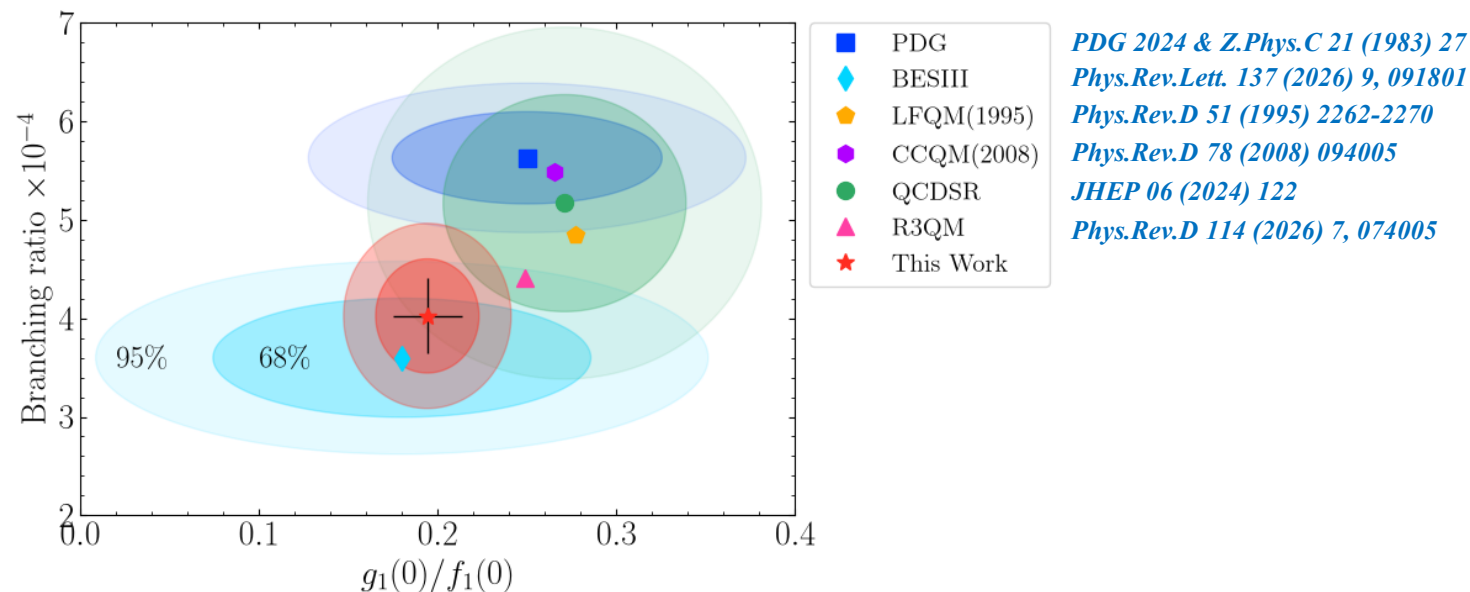
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Differential decay width



Branching ratio



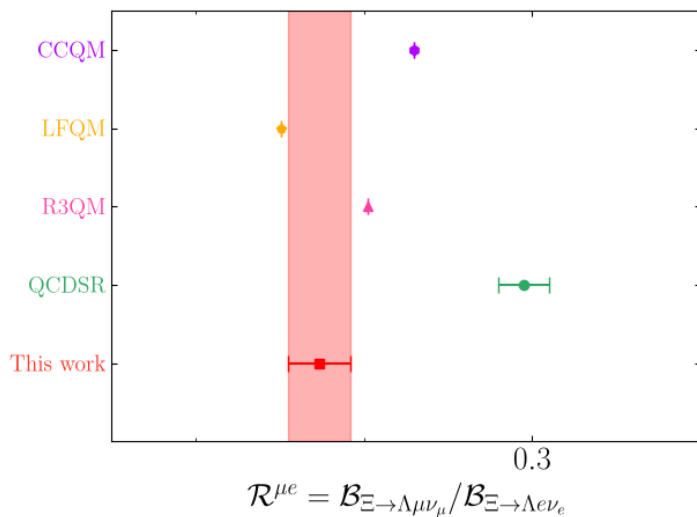
	This work	BESIII	PDG
$\text{Br}(\Xi \rightarrow \Lambda e^- \bar{\nu}_e) \times 10^4$	$4.06(39)_{\text{stat}}$	$3.60(40)_{\text{stat}}(10)_{\text{syst}}$	$5.63(31)_{\text{total}}$
g_{av}	$0.194(19)_{\text{stat}}$	$0.18(7)_{\text{stat}}(1)_{\text{syst}}$	$0.25(5)_{\text{total}}$

Electron-mode branching fraction: within 1σ of BESIII, but more than 2σ from PDG.

Numerical results

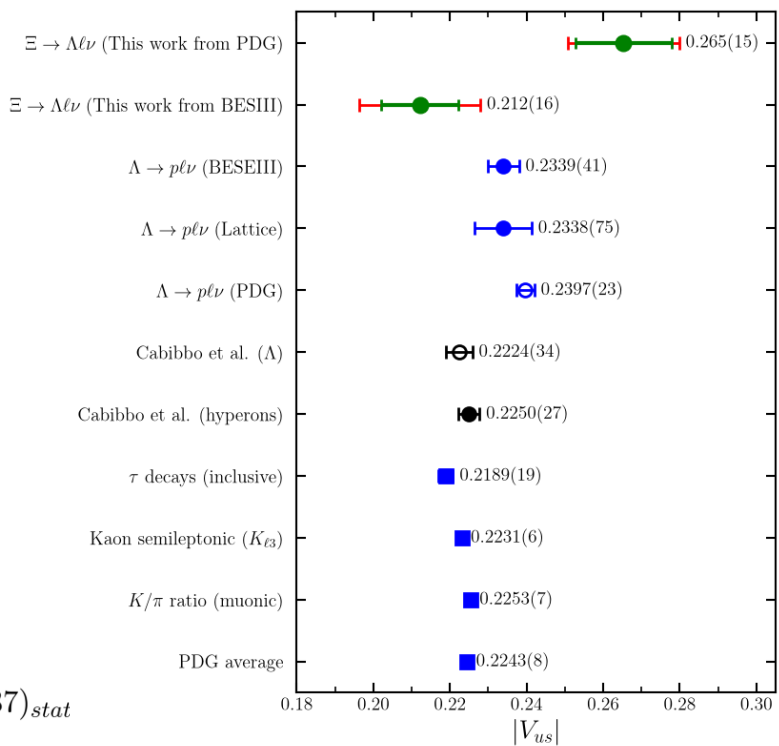
-20-

Lepton-flavor ratio



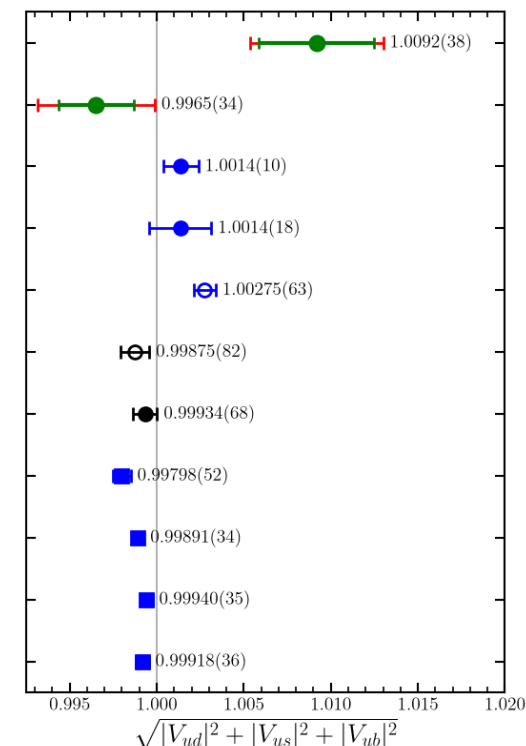
$$\mathcal{R}^{\mu e} \equiv \mathcal{B}(\Xi^- \rightarrow \Lambda \mu^- \bar{\nu}_\mu) / \mathcal{B}(\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e) = 0.2747(37)_{stat}$$

CKM matrix element $|V_{us}|$



$$|V_{us}| \equiv \left(\frac{\Gamma_{\Lambda e \bar{\nu}_e}}{|V_{us}|^2 \Gamma_{\Lambda e \bar{\nu}_e}^{\text{exp}}} \right)^{-1/2}$$

Unitarity



$$\sqrt{|V_{ud}^{\text{PDG}}|^2 + |V_{us}|^2 + |V_{ub}^{\text{PDG}}|^2} :$$

Nature 657 (2026) 8130, 92-97

Phys.Rev.Lett. 135 (2025) 23, 231901

Summary and outlook

-21-

Summary:

1. We calculated vector and axial helicity form factors of $\Xi \rightarrow \Lambda \ell \bar{\nu}_\ell$
2. Performed chiral and continuum extrapolations
3. Obtained the preliminary results of branching ratio and g_{av}

Outlook:

Extend the calculation to other baryon semileptonic decays,
like $\Xi_c \rightarrow \Xi \ell \bar{\nu}_\ell$

Apply 3pt to other baryon observables

...

**Thanks for
your attention!**

Appendix

约化方程： $\mathbf{p}=(0,0,0)$

-23-

mom=(0,0,0)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$	$\Gamma = \gamma_x \gamma_5$	$\Gamma = \gamma_y \gamma_5$	$\Gamma = \gamma_z \gamma_5$	$\Gamma = \gamma_t \gamma_5$
$T = \text{gamma}(0) = 1$	0	0	0	V_1	0	0	0	0
$T = \text{gamma}(1) = \gamma^x$	0	0	0	0	0	0	0	0
$T = \text{gamma}(2) = \gamma^y$	0	0	0	0	0	0	0	0
$T = \text{gamma}(3) = \gamma^x \gamma^y$	0	0	0	0	0	0	A_1	0
$T = \text{gamma}(4) = \gamma^z$	0	0	0	0	0	0	0	0
$T = \text{gamma}(5) = \gamma^x \gamma^z$	0	0	0	0	0	$-A_1$	0	0
$T = \text{gamma}(6) = \gamma^y \gamma^z$	0	0	0	0	A_1	0	0	0
$T = \text{gamma}(7) = \gamma^5 \gamma^t$	0	0	0	0	0	0	0	0
$T = \text{gamma}(8) = \gamma^t$	0	0	0	V_1	0	0	0	0
$T = \text{gamma}(9) = \gamma^x \gamma^t$	0	0	0	0	0	0	0	0
$T = \text{gamma}(10) = \gamma^y \gamma^t$	0	0	0	0	0	0	0	0
$T = \text{gamma}(11) = \gamma^z \gamma^5$	0	0	0	0	0	0	iA_1	0
$T = \text{gamma}(12) = \gamma^z \gamma^t$	0	0	0	0	0	0	0	0
$T = \text{gamma}(13) = \gamma^5 \gamma^y$	0	0	0	0	0	$-iA_1$	0	0
$T = \text{gamma}(14) = \gamma^x \gamma^5$	0	0	0	0	iA_1	0	0	0
$T = \text{gamma}(15) = \gamma^5$	0	0	0	0	0	0	0	0

$$V1 = 8m_f m_i f_0$$

$$A1 = 8im_i m_f g_{\perp}$$

表 3: Γ 代表流算符的洛伦兹结构 $\bar{\psi}\Gamma\psi$ ， T 代表投影矩阵，动量 $\vec{p} = (0,0,0)$ 情况

约化方程: mom=(0,0,1)

$$V_i = A f_+(q^2) + B f_\perp(q^2) + C f_0(q^2),$$

$$i = 1 \sim 6$$

-24-

mom=(0,0,p)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$	$\Gamma = \gamma_x \gamma_5$	$\Gamma = \gamma_y \gamma_5$	$\Gamma = \gamma_z \gamma_5$	$\Gamma = \gamma_t \gamma_5$
T = gamma(0) = 1	0	0	V_1	V_2	0	0	0	0
T = gamma(1) = γ^x	V_3	0	0	0	0	A_1	0	0
T = gamma(2) = γ^y	0	V_3	0	0	$-A_1$	0	0	0
T = gamma(3) = $\gamma^x \gamma^y$	0	0	0	0	0	0	A_2	A_3
T = gamma(4) = γ^z	0	0	V_4	V_5	0	0	0	0
T = gamma(5) = $\gamma^x \gamma^z$	V_6	0	0	0	0	A_4	0	0
T = gamma(6) = $\gamma^y \gamma^z$	0	V_6	0	0	$-A_4$	0	0	0
T = gamma(7) = $\gamma^5 \gamma^t$	0	0	0	0	0	0	A_5	A_6
T = gamma(8) = γ^t	0	0	V_1	V_2	0	0	0	0
T = gamma(9) = $\gamma^x \gamma^t$	$-V_3$	0	0	0	0	$-A_1$	0	0
T = gamma(10) = $\gamma^y \gamma^t$	0	$-V_3$	0	0	A_1	0	0	0
T = gamma(11) = $\gamma^z \gamma^5$	0	0	0	0	0	0	iA_2	iA_3
T = gamma(12) = $\gamma^z \gamma^t$	0	0	V_4	$-V_5$	0	0	0	0
T = gamma(13) = $\gamma^5 \gamma^y$	iV_6	0	0	0	0	iA_4	0	0
T = gamma(14) = $\gamma^x \gamma^5$	0	iV_6	0	0	$-iA_4$	0	0	0
T = gamma(15) = γ^5	0	0	0	0	0	0	$-A_5$	$-A_6$

表 6: Γ 代表流算符的洛伦兹结构 $\bar{\psi}\Gamma\psi$, T代表投影矩阵, 动量 $\vec{p} = \frac{2\pi}{aL}(0,0,p)$ 情况

$$A_1 = 4i m_i p g_\perp \quad A_2 = -\frac{4i m_i (m_f + m_i) p^2}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p^2 (m_f^2 - m_i^2 - q^2)}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ + \frac{4i m_i [m_f^3 - 2m_f^2 m_i - 2m_i p^2 + m_f (m_i^2 - q^2) + E_f (m_f^2 - 2m_f m_i + m_i^2 - q^2)]}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_\perp$$

$$A_3 = \frac{4i (E_f - m_i) m_i (m_f + m_i) p}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p [m_i (m_f^2 - m_i^2 + q^2) + E_f (-m_f^2 + m_i^2 + q^2)]}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ - \frac{4i m_i p (m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_\perp$$

$$A_4 = -\frac{(E_f + m_f) A_1}{p} \quad A_5 = -i \frac{(E_f - m_f) A_2}{p} \quad A_6 = -i \frac{(E_f - m_f) A_3}{p}$$

$$V_1 = -\frac{4(E_f + m_f)(m_f - m_i)m_i p}{q^2} f_0 + \frac{4(E_f + m_f)m_i(m_f + m_i)p(m_f^2 - m_i^2 - q^2)}{(m_f^2 + 2m_f m_i + m_i^2 - q^2)q^2} f_+ - \frac{4m_i p(m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_\perp$$

$$V_2 = \frac{4(E_f + m_f)(E_f - m_i)(m_f - m_i)m_i}{q^2} f_0 + \frac{4(E_f + m_f)m_i(m_f + m_i)[m_i(m_f^2 - m_i^2 + q^2) + E_f(-m_f^2 + m_i^2 + q^2)]}{(m_f^2 + 2m_f m_i + m_i^2 - q^2)q^2} f_+ - \frac{4(E_f + m_f)m_i(-m_f^2 + 2E_f m_i - m_i^2 + q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_\perp$$

$$V_3 = 4(-E_f + m_f)m_i f_\perp$$

$$V_4 = \frac{pV_1}{(E_f + m_f)}$$

$$V_5 = \frac{pV_2}{(E_f + m_f)}$$

$$V_6 = -\frac{(E_f + m_f)}{pV_3}$$

约化方程: $\mathbf{p}=(0,1,1)$

-25-

mom=(0, p, p)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$	$\Gamma = \gamma_x \gamma_5$	$\Gamma = \gamma_y \gamma_5$	$\Gamma = \gamma_z \gamma_5$	$\Gamma = \gamma_t \gamma_5$
T = gamma(0) = 1	0	V_1	V_1	V_2	0	0	0	0
T = gamma(1) = γ^x	V_3	0	0	0	0	A_1	$-A_1$	0
T = gamma(2) = γ^y	0	V_4	V_5	V_6	$-A_1$	0	0	0
T = gamma(3) = $\gamma^x \gamma^y$	V_7	0	0	0	0	A_2	A_3	A_4
T = gamma(4) = γ^z	0	V_5	V_4	V_6	A_1	0	0	0
T = gamma(5) = $\gamma^x \gamma^z$	V_7	0	0	0	0	$-A_3$	$-A_2$	$-A_4$
T = gamma(6) = $\gamma^y \gamma^z$	0	V_7	$-V_7$	0	A_5	0	0	0
T = gamma(7) = $\gamma^5 \gamma^t$	0	0	0	0	0	A_6	A_6	A_7
T = gamma(8) = γ^t	0	V_1	V_1	V_2	0	0	0	0
T = gamma(9) = $\gamma^x \gamma^t$	$-V_3$	0	0	0	0	$-A_1$	A_1	0
T = gamma(10) = $\gamma^y \gamma^t$	0	$-V_4$	$-V_5$	$-V_6$	A_1	0	0	0
T = gamma(11) = $\gamma^z \gamma^5$	iV_7	0	0	0	0	iA_2	iA_3	iA_4
T = gamma(12) = $\gamma^z \gamma^t$	0	$-V_5$	$-V_4$	$-V_6$	$-A_1$	0	0	0
T = gamma(13) = $\gamma^5 \gamma^y$	iV_7	0	0	0	0	iA_4	$-iA_2$	$-iA_4$
T = gamma(14) = $\gamma^x \gamma^5$	0	iV_7	$-iV_7$	0	iA_5	0	0	0
T = gamma(15) = γ^5	0	0	0	0	0	$-A_6$	$-A_6$	$-A_7$

表 9: Γ 代表流算符的洛伦兹结构 $\bar{\psi}\Gamma\psi$, T代表投影矩阵, 动量 $\vec{p} = \frac{2\pi}{aL}(0, p, p)$ 情况

$$A_1 = 4i m_i p g_{\perp} \quad A_2 = -\frac{4i m_i (m_f + m_i) p^2}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p^2 (m_f^2 - m_i^2 - q^2)}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ - \frac{8i m_i^2 p^2}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_{\perp}$$

$$A_4 = \frac{4i (E_f - m_i) m_i (m_f + m_i) p}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p [m_i (m_f^2 - m_i^2 + q^2) + E_f (-m_f^2 + m_i^2 + q^2)]}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ - \frac{4i m_i p (m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_{\perp}$$

$$A_3 = A_2 + \frac{A_1 (E_f + m_f)}{p} \quad A_5 = \frac{A_1 (E_f + m_f)}{p} \quad A_6 = -i \frac{A_2 (E_f - m_f) + A_1 p}{p} \quad A_7 = -i \frac{A_4 (E_f - m_f)}{p}$$

$$V_1 = -\frac{4 (E_f + m_f) (m_f - m_i) m_i p}{q^2} f_0 + \frac{4 (E_f + m_f) m_i (m_f + m_i) p (m_f^2 - m_i^2 - q^2)}{(m_f^2 + 2m_f m_i + m_i^2 - q^2) q^2} f_+ - \frac{4 m_i p (m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_{\perp}$$

$$V_2 = \frac{4 (E_f + m_f) (E_f - m_i) (m_f - m_i) m_i}{q^2} f_0 + \frac{4 (E_f + m_f) m_i (m_f + m_i) [m_i (m_f^2 - m_i^2 + q^2) + E_f (-m_f^2 + m_i^2 + q^2)]}{(m_f^2 + 2m_f m_i + m_i^2 - q^2) q^2} f_+ - \frac{4 (E_f + m_f) m_i (-m_f^2 + 2E_f m_i - m_i^2 + q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_{\perp}$$

$$V_3 = 4 (-E_f + m_f) m_i f_{\perp}$$

$$V_4 = \frac{p V_1}{(E_f + m_f)} + \frac{V_3}{2}$$

$$V_5 = \frac{p V_1}{(E_f + m_f)} - \frac{V_3}{2}$$

$$V_6 = \frac{p V_2}{(E_f + m_f)}$$

$$V_7 = -\frac{(E_f + m_f) V_3}{2p}$$

约化方程: $\mathbf{p}=(1,1,1)$

mom= (p, p, p)	$\Gamma = \gamma_x$	$\Gamma = \gamma_y$	$\Gamma = \gamma_z$	$\Gamma = \gamma_t$	$\Gamma = \gamma_x \gamma_5$	$\Gamma = \gamma_y \gamma_5$	$\Gamma = \gamma_z \gamma_5$	$\Gamma = \gamma_t \gamma_5$
T = gamma(0)	V_1	V_1	V_1	V_2	0	0	0	0
T = gamma(1)	V_3	V_4	V_4	V_5	0	A_1	$-A_1$	0
T = gamma(2)	V_4	V_3	V_4	V_5	$-A_1$	0	A_1	0
T = gamma(3)	V_6	$-V_6$	0	0	A_2	A_2	A_3	A_4
T = gamma(4)	V_4	V_4	V_3	V_5	A_1	$-A_1$	0	0
T = gamma(5)	V_6	0	$-V_6$	0	$-A_2$	$-A_3$	$-A_2$	$-A_4$
T = gamma(6)	0	V_6	$-V_6$	0	A_3	A_2	A_2	A_4
T = gamma(7)	0	0	0	0	A_5	A_5	A_5	A_6
T = gamma(8)	V_1	V_1	V_1	V_2	0	0	0	0
T = gamma(9)	$-V_3$	$-V_4$	$-V_4$	$-V_5$	0	$-A_1$	A_1	0
T = gamma(10)	$-V_4$	$-V_3$	$-V_4$	$-V_5$	$-A_1$	0	$-A_1$	0
T = gamma(11)	iV_6	$-iV_6$	0	0	iA_2	iA_2	$-A_3$	iA_4
T = gamma(12)	$-V_4$	$-V_4$	$-V_3$	$-V_5$	$-A_1$	A_1	0	0
T = gamma(13)	iV_6	0	$-iV_6$	0	$-iA_2$	$-iA_3$	$-iA_2$	$-iA_4$
T = gamma(14)	0	iV_6	$-iV_6$	0	iA_3	iA_2	iA_2	iA_4
T = gamma(15)	0	0	0	0	$-A_5$	$-A_5$	$-A_5$	$-A_6$

表 12: Γ 代表流算符的洛伦兹结构 $\bar{\psi}\Gamma\psi$, T代表投影矩阵, 动量 $\vec{p} = \frac{2\pi}{La}(p, p, p)$ 情况

$$A_1 = 4i m_i p g_{\perp}$$

$$A_2 = -\frac{4i m_i (m_f + m_i) p^2}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p^2 (m_f^2 - m_i^2 - q^2)}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ - \frac{8i m_i^2 p^2}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_{\perp}$$

$$A_4 = \frac{4i (E_f - m_i) m_i (m_f + m_i) p}{q^2} g_0 + \frac{4i (m_f - m_i) m_i p [m_i (m_f^2 - m_i^2 + q^2) + E_f (-m_f^2 + m_i^2 + q^2)]}{(m_f^2 - 2m_f m_i + m_i^2 - q^2) q^2} g_+ - \frac{4i m_i p (m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 - 2m_f m_i + m_i^2 - q^2} g_{\perp}$$

$$A_3 = A_2 + \frac{A_1 (E_f + m_f)}{p}$$

$$A_5 = -i \frac{A_2 (E_f - m_f) + p A_1}{p}$$

$$A_6 = -i \frac{A_4 (E_f - m_f)}{p}$$

$$V_1 = -\frac{4 (E_f + m_f) (m_f - m_i) m_i p}{q^2} f_0 + \frac{4 (E_f + m_f) m_i (m_f + m_i) p (m_f^2 - m_i^2 - q^2)}{(m_f^2 + 2m_f m_i + m_i^2 - q^2) q^2} f_+ - \frac{4 m_i p (m_f^2 - 2E_f m_i + m_i^2 - q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_{\perp}$$

$$V_2 = \frac{4 (E_f + m_f) (E_f - m_i) (m_f - m_i) m_i}{q^2} f_0 + \frac{4 (E_f + m_f) m_i (m_f + m_i) [m_i (m_f^2 - m_i^2 + q^2) + E_f (-m_f^2 + m_i^2 + q^2)]}{(m_f^2 + 2m_f m_i + m_i^2 - q^2) q^2} - \frac{4 (E_f + m_f) m_i (-m_f^2 + 2E_f m_i - m_i^2 + q^2)}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_{\perp}$$

$$V_4 = -\frac{4 (m_f - m_i) m_i p^2}{q^2} f_0 - \frac{4 m_i (m_f + m_i) p^2 (-m_f^2 + m_i^2 + q^2)}{(m_f^2 + 2m_f m_i + m_i^2 - q^2) q^2} f_+ + \frac{8 m_i^2 p^2}{m_f^2 + 2m_f m_i + m_i^2 - q^2} f_{\perp}$$

$$V_3 = \frac{3pV_1}{(E_f + m_f)} - 2V_4$$

$$V_5 = \frac{pV_2}{(E_f + m_f)}$$

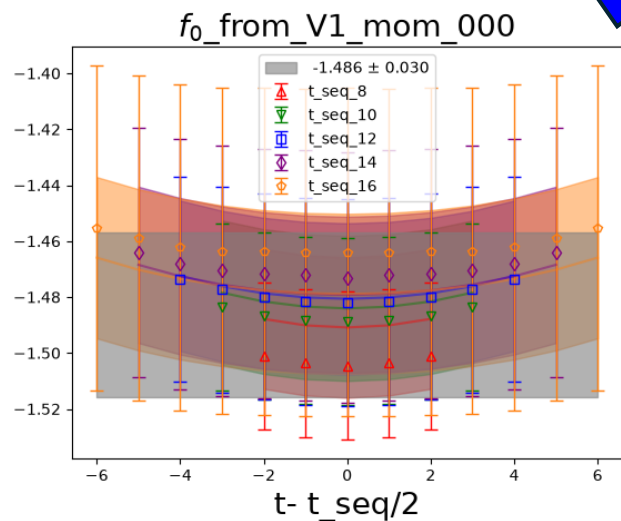
$$V_6 = -V_1 + \frac{(E_f + m_f)V_4}{p}$$

Extrapolation to $t_{seq} \rightarrow \infty$

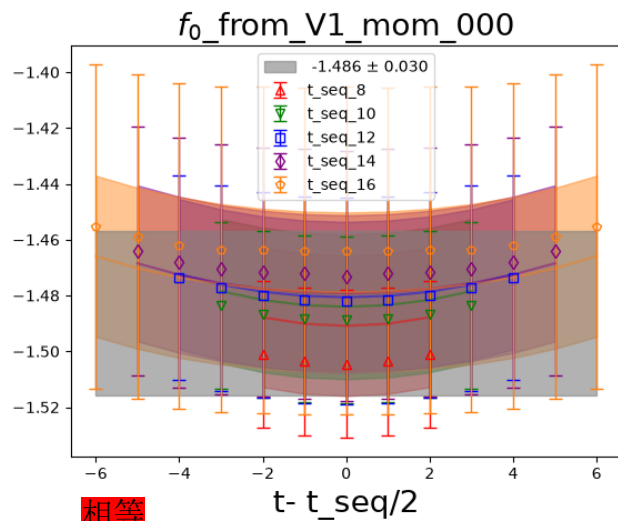
C32P23: (0, 0, 0)

-27-

f_+

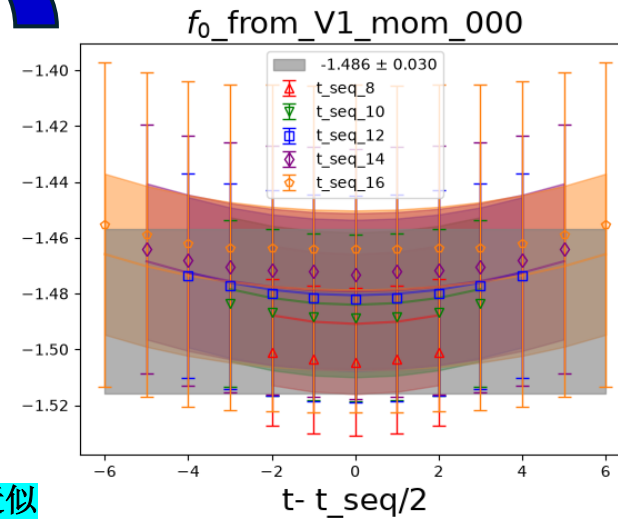


$f_{\perp}$



近似

f_0



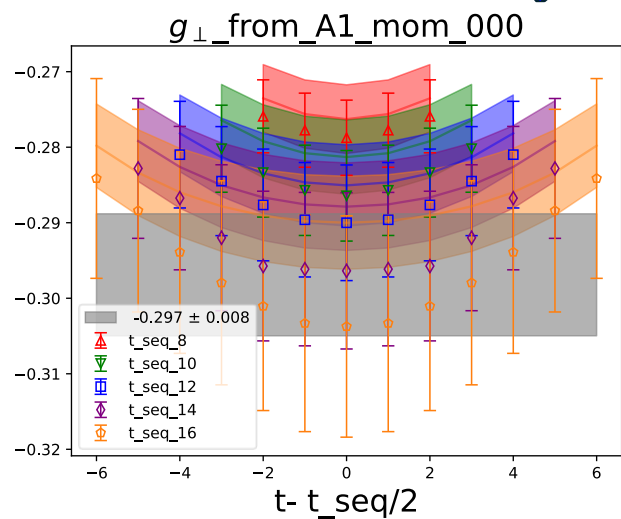
$$V1 = 8m_fm_if_0$$

$$A1 = 8im_im_fg_{\perp}$$

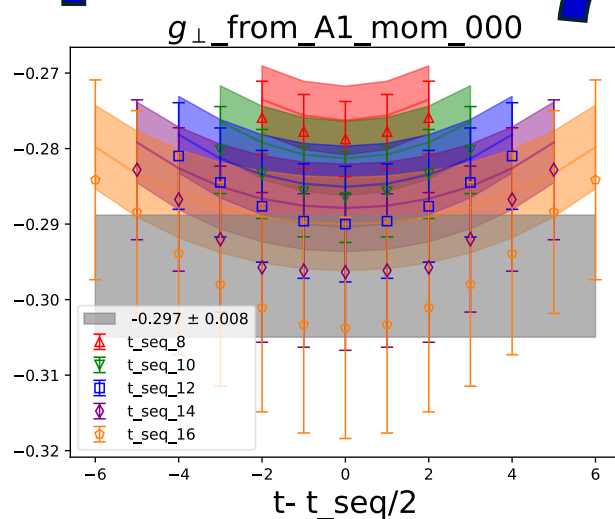
相等

近似

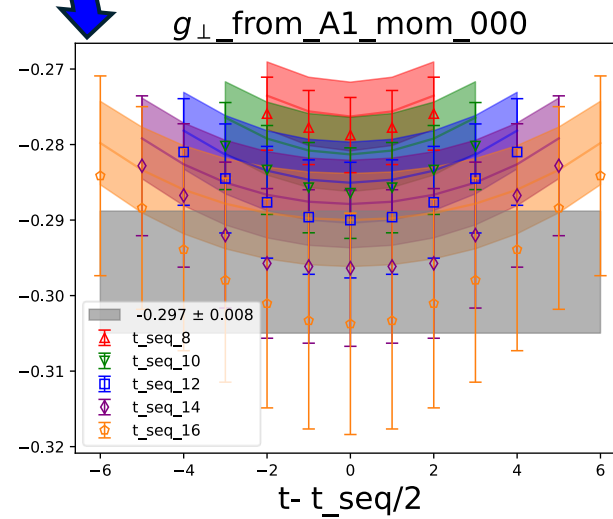
g_+



$g_{\perp}$



g_0

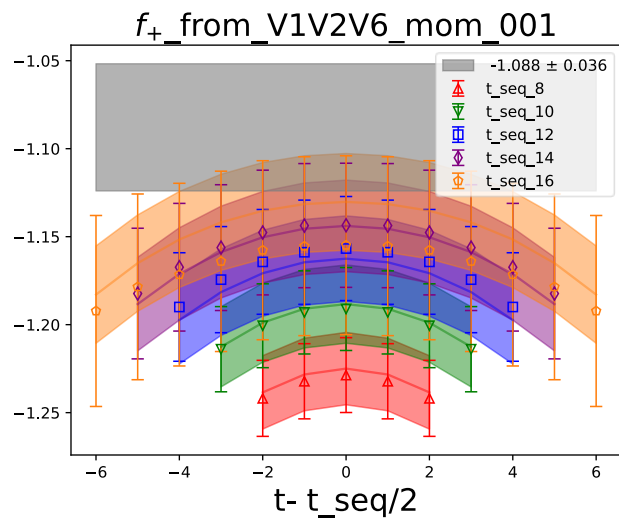


Extrapolation to $t_{seq} \rightarrow \infty$

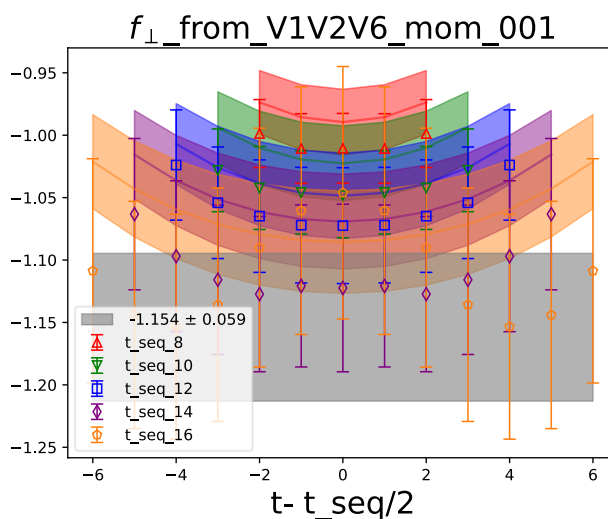
C32P23: (0, 0, 1)

-28-

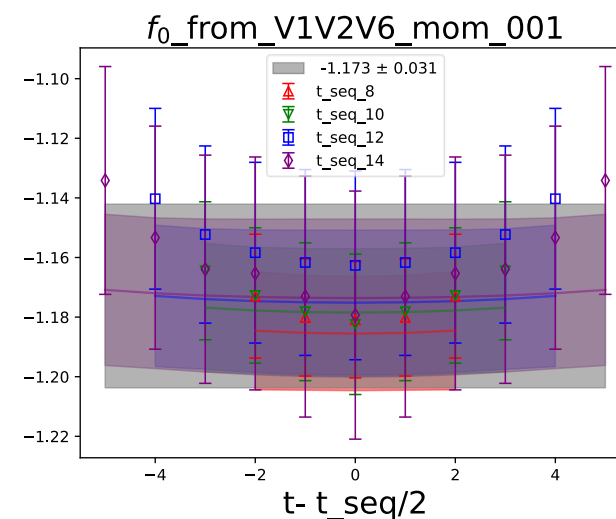
f_+



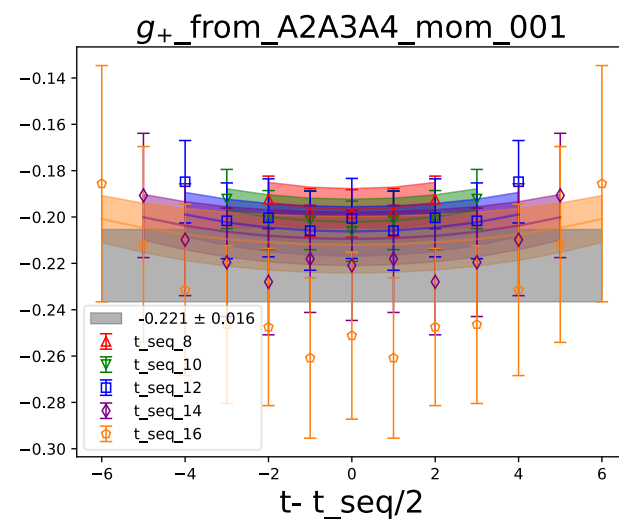
$f_\perp$



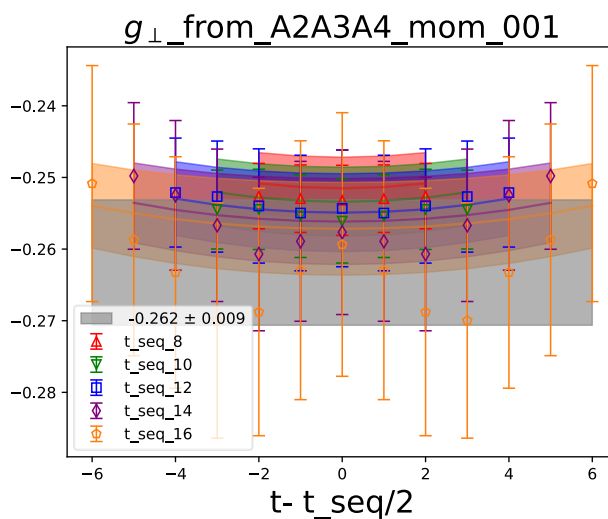
f_0



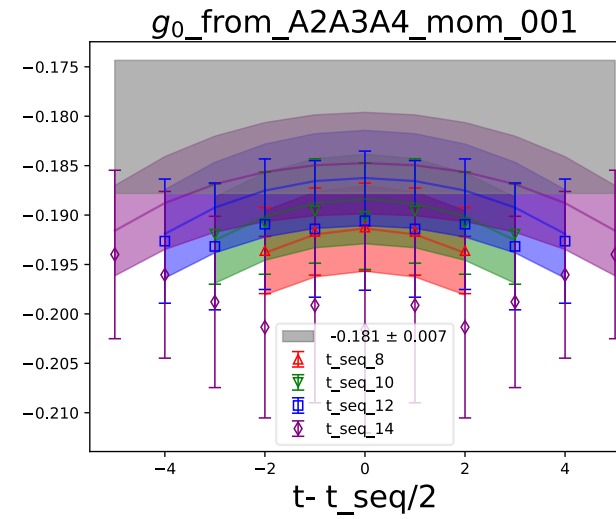
g_+



$g_\perp$



g_0

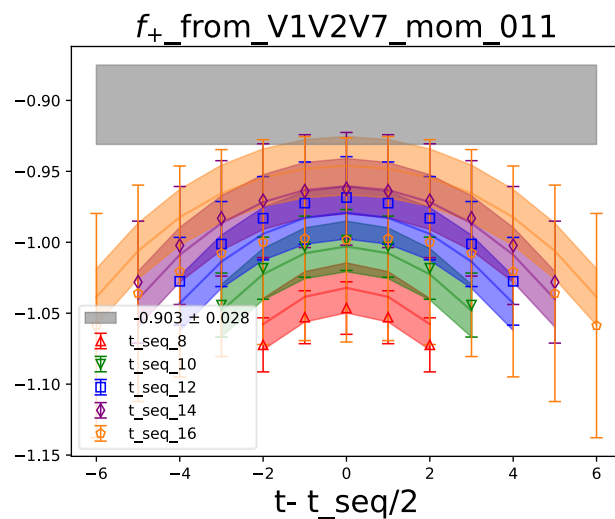


Extrapolation to $t_{seq} \rightarrow \infty$

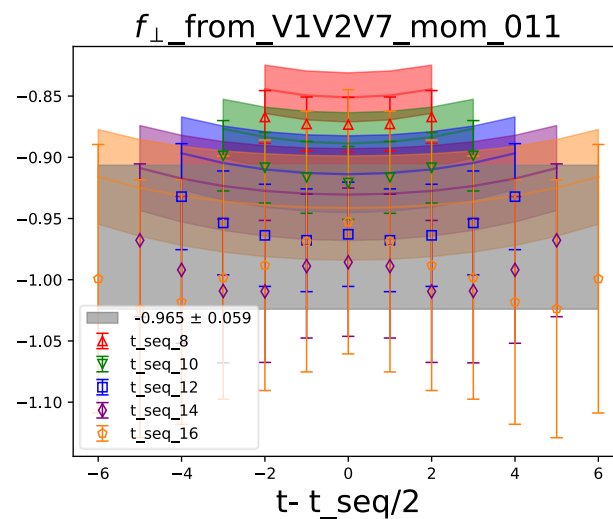
C32P23: (0, 1, 1)

-29-

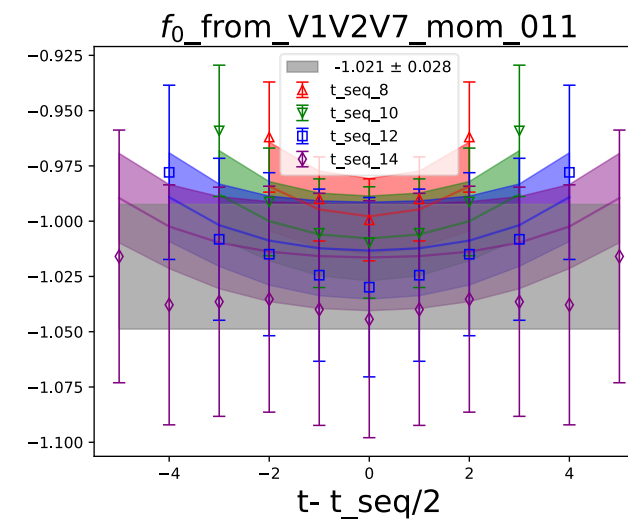
f_+



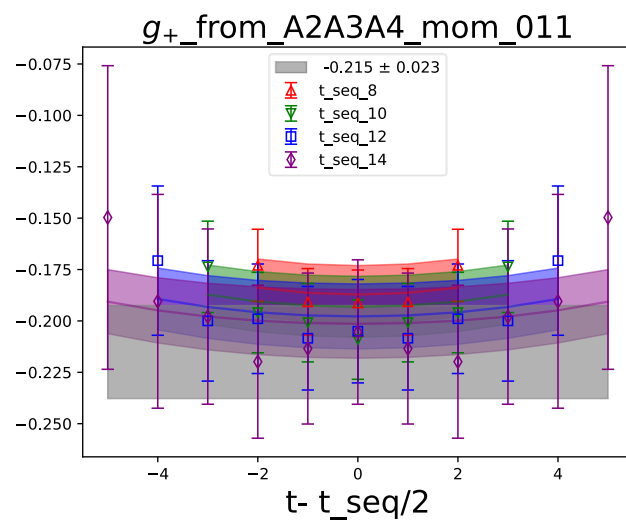
$f_{\perp}$



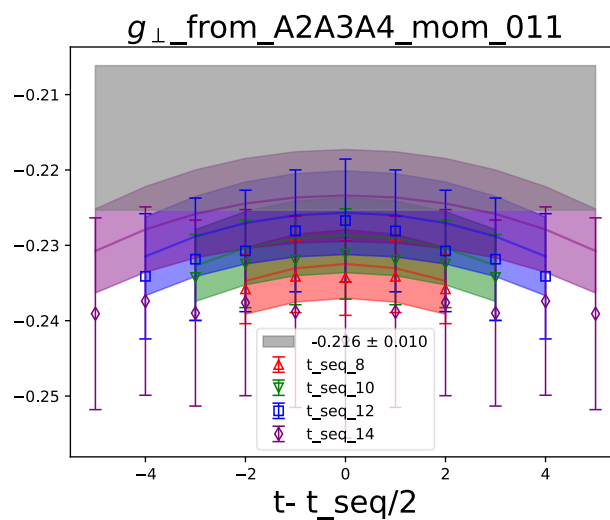
f_0



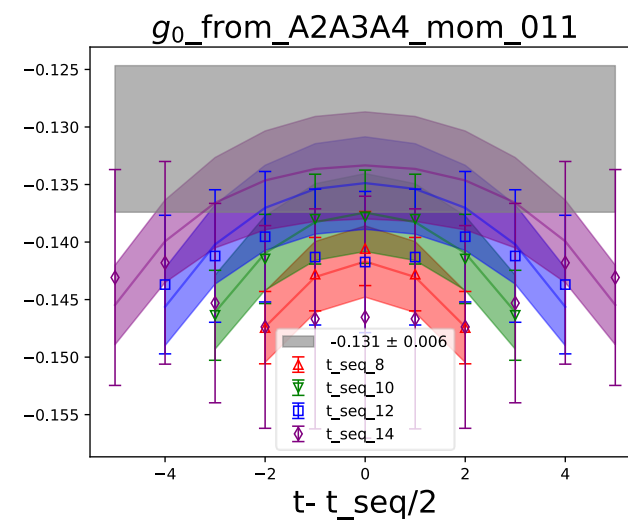
g_+



$g_{\perp}$



g_0

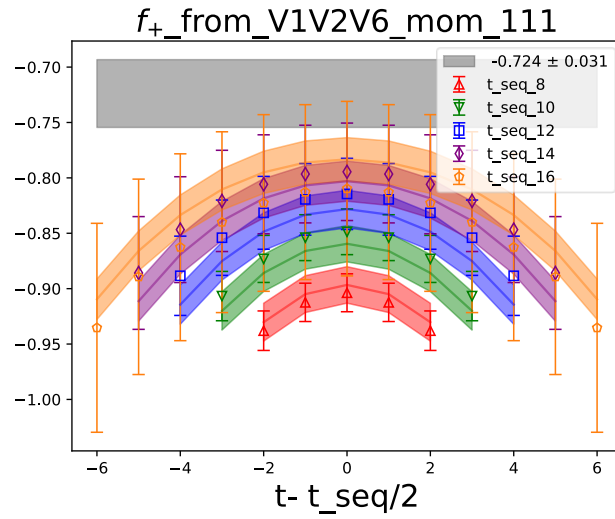


Extrapolation to $t_{seq} \rightarrow \infty$

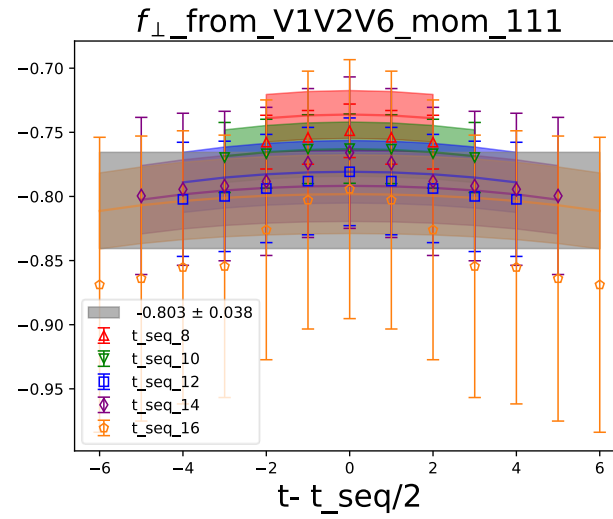
C32P23: (1, 1, 1)

-30-

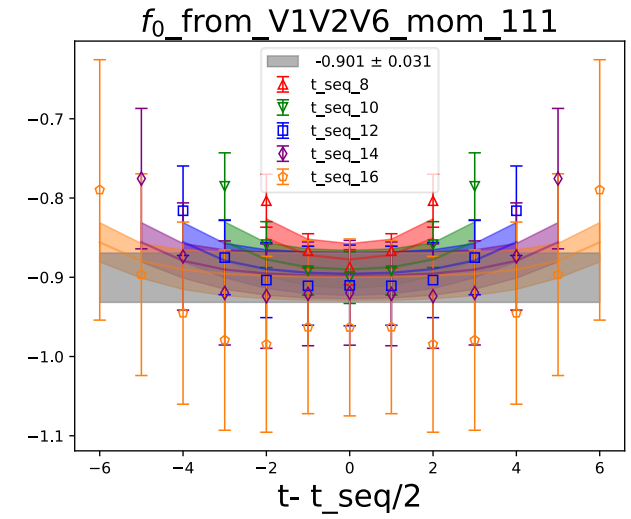
f_+



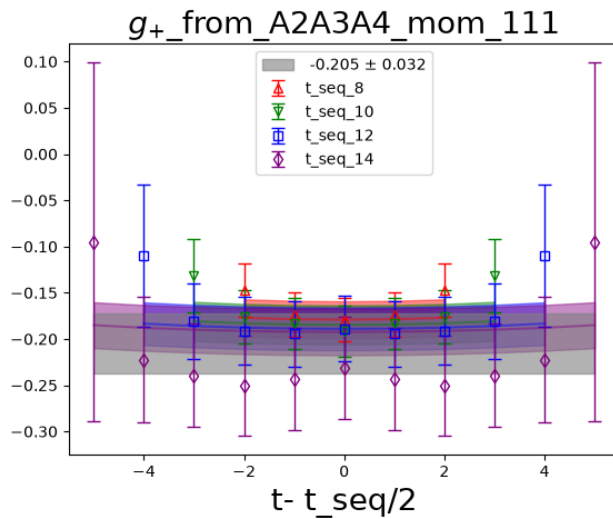
$f_{\perp}$



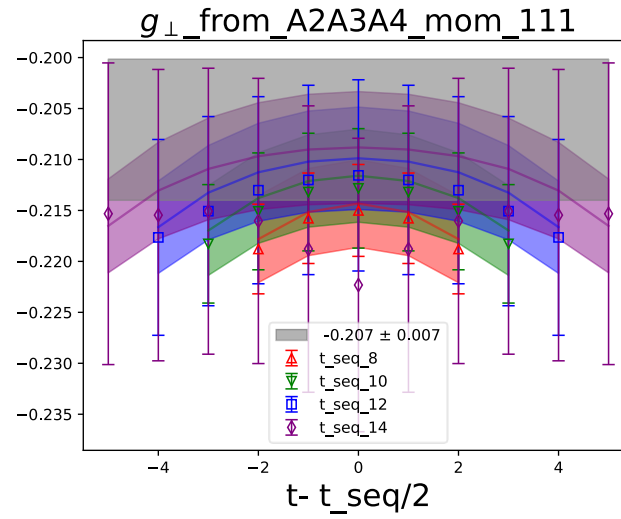
f_0



g_+



$g_{\perp}$



g_0

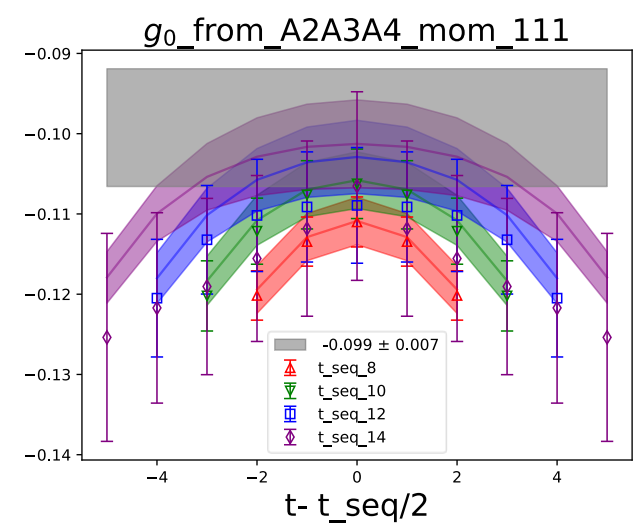


TABLE VII. Helicity Form factors for $\Xi^- \rightarrow \Lambda l^- \bar{\nu}_\ell$ at $q^2 = 0$ with statistical errors

Method	$f_+(0)$	$f_\perp(0)$	$f_0(0)$	$g_+(0)$	$g_\perp(0)$	$g_0(0)$	
LFQM(1995)	-1.190	-1.360	-1.190	-0.330	-0.328	-0.330	
χ QM(1999)	-1.220	-1.150	-1.220	-0.270	-0.269	-0.270	
CCQM(2008)	-1.252	-1.551	-1.252	-0.332	-0.320	-0.332	
χ QSM(2008)	-1.225	-1.151	-1.225	-0.260	-0.260	-0.260	
χ CQM(2009)	-1.225	-0.775	-1.225	-0.262	-0.258	-0.262	
QCDSR(2024)	-1.202(72)	-1.420(90)	-1.202(72)	-0.309(34)	-0.309(34)	-0.309(34)	<i>JHEP 06 (2024) 122</i>
R3QM(2026)	-1.160	-0.686	-1.160	-0.284	-0.292	-0.284	<i>Phys.Rev.D 114 (2026) 7, 074005</i>
This work	-1.148(57)	-1.171(57)	-1.176(57)	-0.233(18)	-0.261(14)	-0.219(16)	

$$\frac{d\Gamma(\mathcal{B}_1 \rightarrow \mathcal{B}_2 \ell \nu)}{dq^2} = \frac{G_F^2 |V_{us}|^2 q^2 \sqrt{Q_+ Q_-}}{384 \pi^3 M_1^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 H_{tot} \quad H_{tot} = \left(1 + \frac{m_\ell^2}{2q^2}\right) \left(H_{\frac{1}{2},1}^2 + H_{-\frac{1}{2},-1}^2 + H_{\frac{1}{2},0}^2 + H_{-\frac{1}{2},0}^2\right) + \frac{3m_\ell^2}{2q^2} \left(H_{\frac{1}{2},t}^2 + H_{-\frac{1}{2},t}^2\right)$$

$$\begin{aligned} H_{\frac{1}{2},0}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} (M_1 + M_2) f_+, & H_{\frac{1}{2},1}^V &= -i \sqrt{2Q_-} f_\perp, & H_{\frac{1}{2},t}^V &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} (M_1 - M_2) f_0, & H_{\lambda_2, \lambda_W} &\equiv H_{\lambda_2, \lambda_W}^V - H_{\lambda_2, \lambda_W}^A \\ H_{\frac{1}{2},0}^A &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} (M_1 - M_2) g_+, & H_{\frac{1}{2},1}^A &= -i \sqrt{2Q_+} g_\perp, & H_{\frac{1}{2},t}^A &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} (M_1 + M_2) g_0, & Q_\pm &= (M_1 \pm M_2)^2 - q^2 \end{aligned}$$

Wienberg form factors and helicity form factors

$$\begin{aligned}
 f_+ &= f_1 - \frac{q^2}{m_i(m_i + m_f)} f_2 & f_\perp &= f_1 - \frac{m_i + m_f}{m_i} f_2 & f_0 &= f_1 + \frac{q^2}{m_i(m_i - m_f)} f_3 \\
 g_+ &= g_1 + \frac{q^2}{m_i(m_i - m_f)} g_2 & g_\perp &= g_1 + \frac{m_i - m_f}{m_i} g_2 & g_0 &= g_1 - \frac{q^2}{m_i(m_i + m_f)} g_3
 \end{aligned}$$

Obtain decay width from formfactors

$$\frac{d\Gamma(\mathcal{B}_1 \rightarrow \mathcal{B}_2 \ell \nu)}{dq^2} = \frac{G_F^2 |V_{us}|^2 q^2 \sqrt{Q_+ Q_-}}{384 \pi^3 M_1^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 H_{tot} \quad H_{tot} = \left(1 + \frac{m_\ell^2}{2q^2}\right) \left(H_{\frac{1}{2},1}^2 + H_{-\frac{1}{2},-1}^2 + H_{\frac{1}{2},0}^2 + H_{-\frac{1}{2},0}^2\right) + \frac{3m_\ell^2}{2q^2} \left(H_{\frac{1}{2},t}^2 + H_{-\frac{1}{2},t}^2\right)$$

$$\begin{aligned}
 H_{\frac{1}{2},0}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} (M_1 + M_2) f_+, & H_{\frac{1}{2},1}^V &= -i \sqrt{2Q_-} f_\perp, & H_{\frac{1}{2},t}^V &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} (M_1 - M_2) f_0, & H_{\lambda_2, \lambda_W} &\equiv H_{\lambda_2, \lambda_W}^V - H_{\lambda_2, \lambda_W}^A \\
 H_{\frac{1}{2},0}^A &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} (M_1 - M_2) g_+, & H_{\frac{1}{2},1}^A &= -i \sqrt{2Q_+} g_\perp, & H_{\frac{1}{2},t}^A &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} (M_1 + M_2) g_0. & Q_\pm &= (M_1 \pm M_2)^2 - q^2
 \end{aligned}$$