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Chiral anomaly neutral pion lifetime and axion–photon coupling

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➤ Bell & Jackiw and, independently, Adler found:

Bell & Jackiw, 1969; Adler, 1969

For massless fermions: $\mathcal{L}_{\text{fermion}} = \bar{\psi} \gamma^\mu D_\mu \psi$

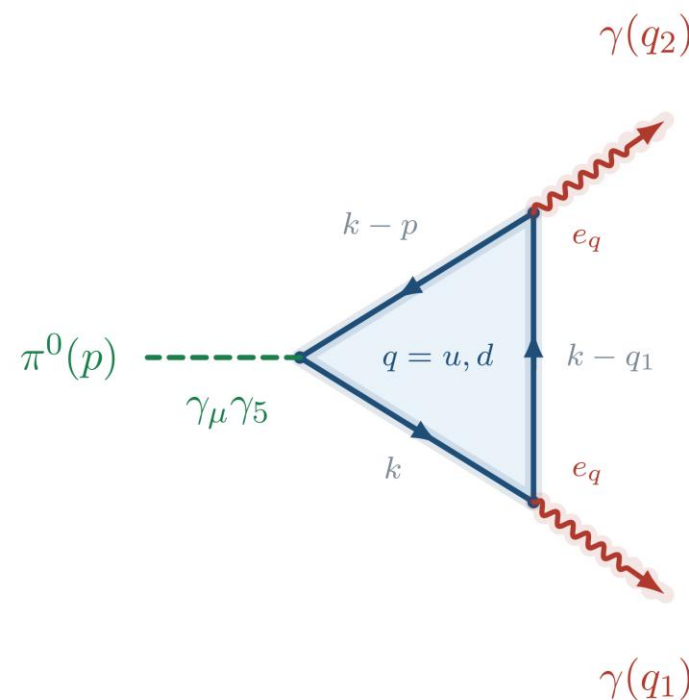
$$\begin{array}{ccc} \partial_\mu A_\mu^3 = 0 & \xrightarrow{\text{quantization}} & \partial_\mu A_\mu^3 = \frac{e^2}{16\pi^2} F \tilde{F} \\ \text{classical conserved} & & U(1)_A \text{ anomaly} \end{array}$$

➤ A physical consequence: $\pi^0 \rightarrow \gamma\gamma$

- Chiral anomaly induces $\pi^0 \rightarrow \gamma\gamma$
and fixes $\pi^0 \rightarrow \gamma\gamma$ amplitude in chiral limit:

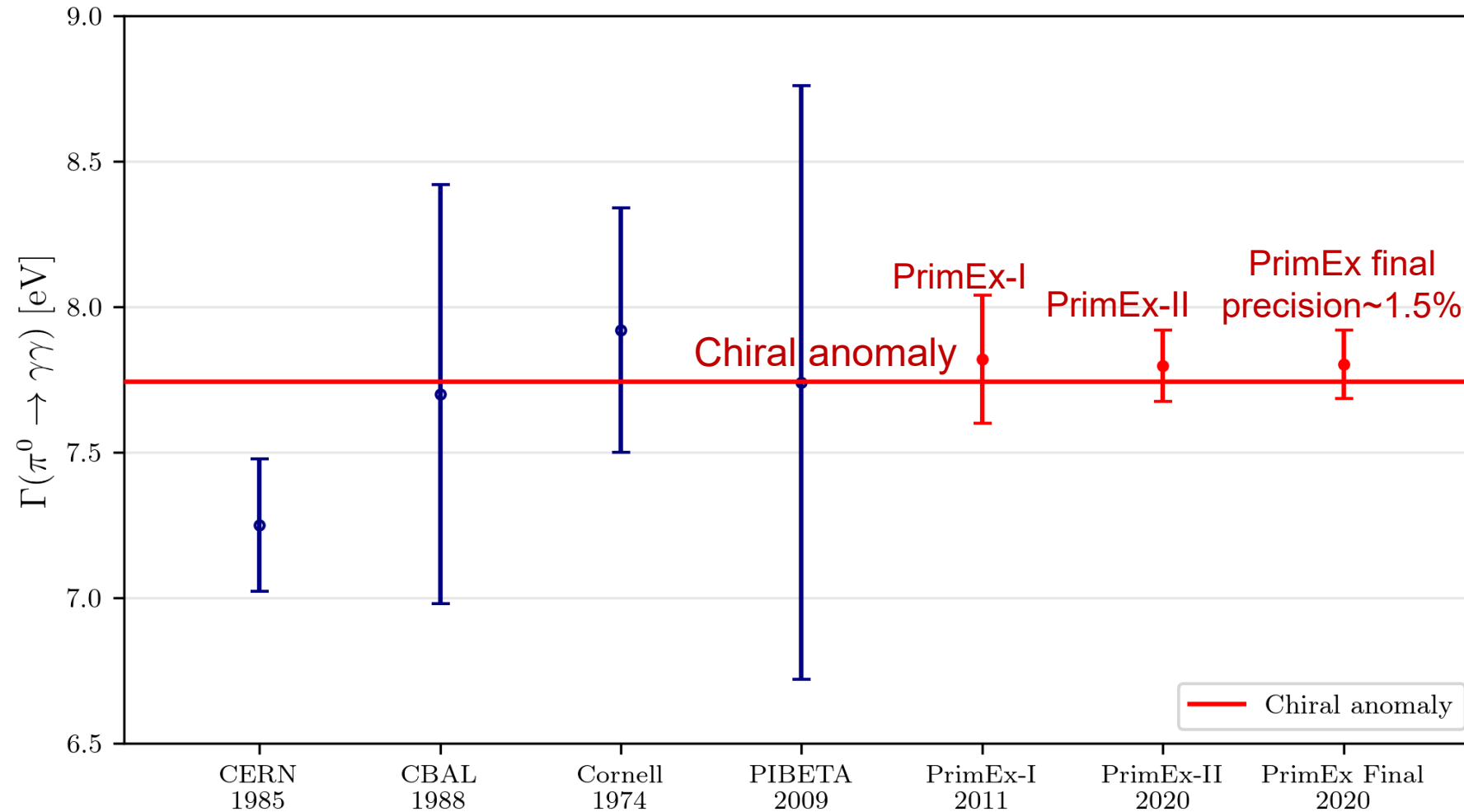
$$\Gamma_{\text{anomaly}} = \frac{\alpha^2 m_\pi^3}{64\pi^3 F_\pi^2} \approx 7.74 \text{eV}$$

- $\pi^0 \rightarrow \gamma\gamma$ can be measured precisely, probing the anomaly and low energy QCD dynamics



$\pi^0 \rightarrow \gamma\gamma$ experiment

$\pi^0 \rightarrow \gamma\gamma$ decay width

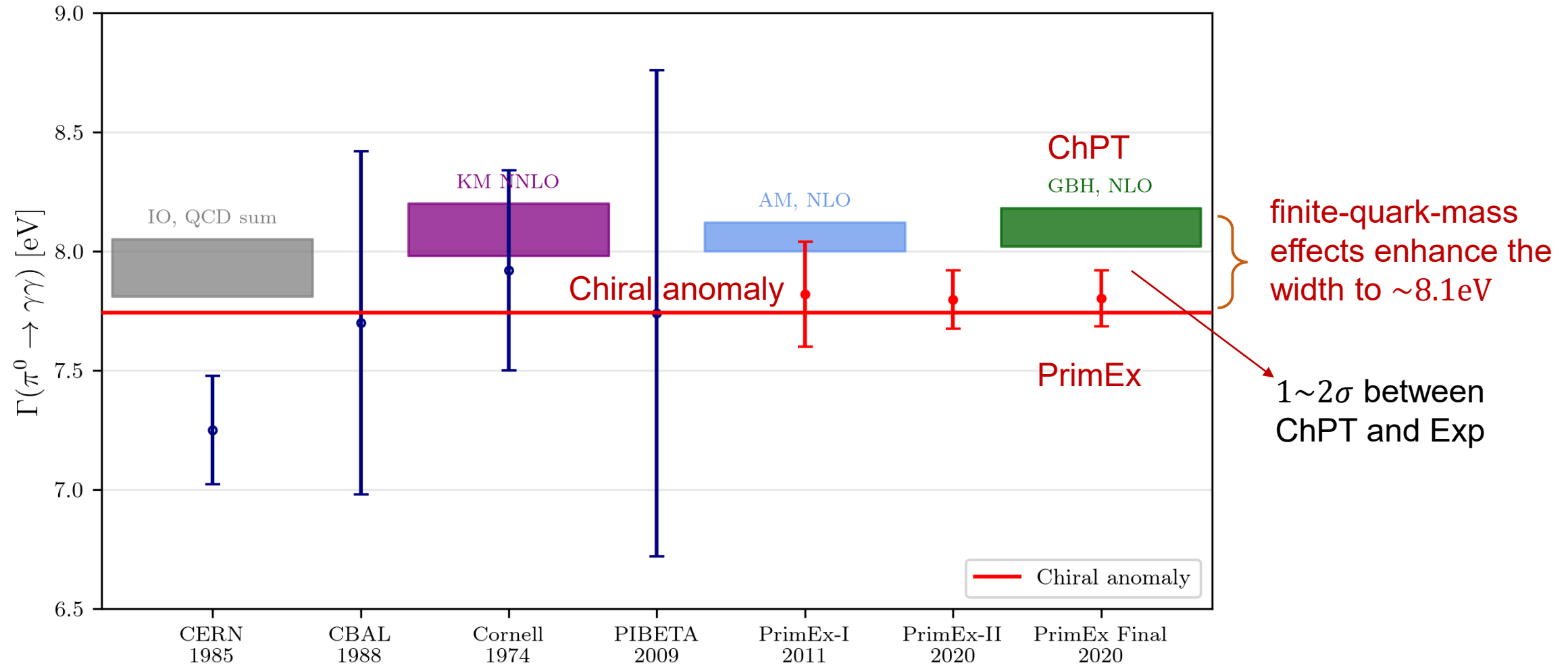


Future program:
JLab 22GeV upgrade
target precision ~1%

Atherton et al. 1985; Williams et al. 1988; Browman et al. 1974; Bychkov et al. 2009; Larin et al. 2011, 2020

$\pi^0 \rightarrow \gamma\gamma$ theory vs experiment

$\pi^0 \rightarrow \gamma\gamma$ decay width

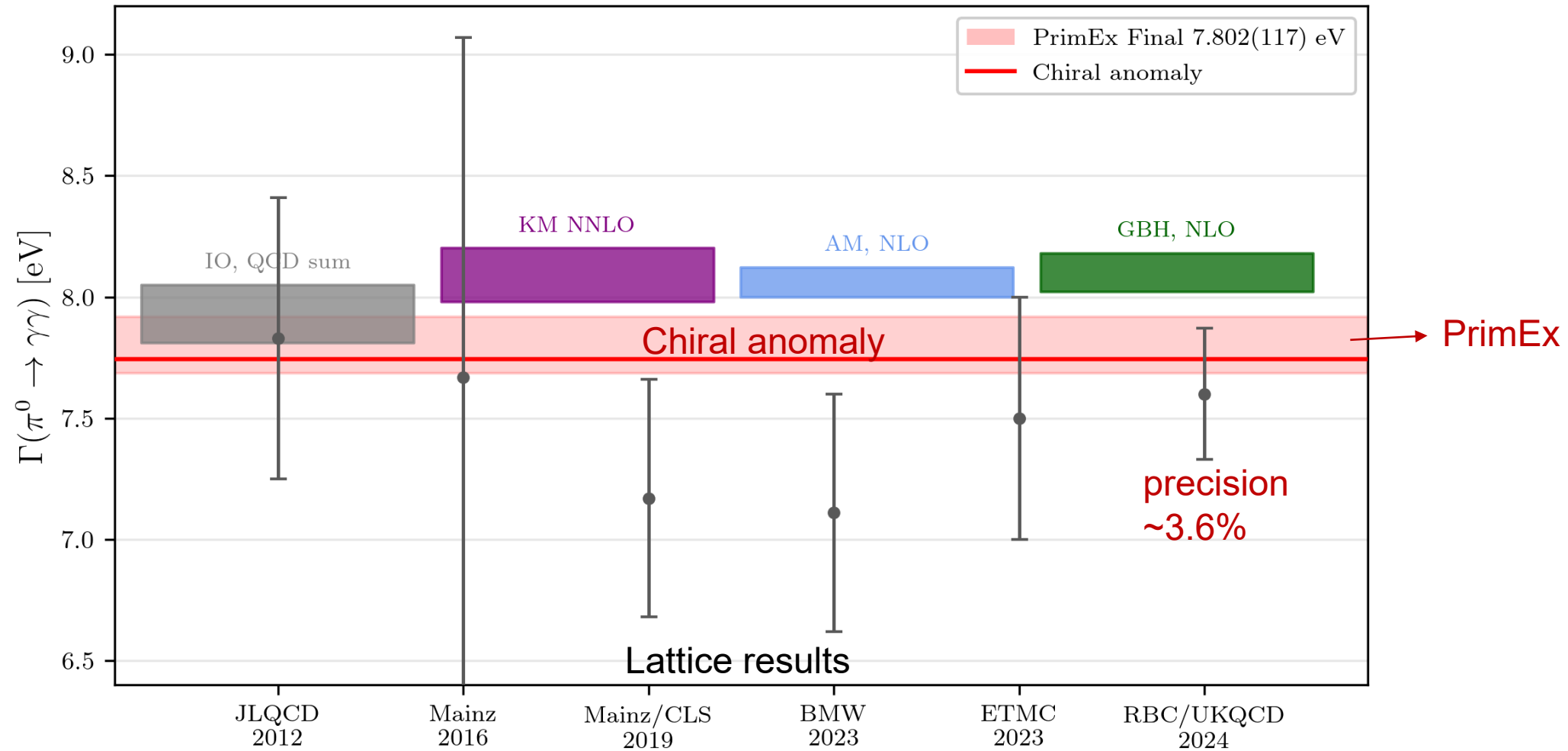


Can lattice QCD get results with comparable precision?

Atherton et al. 1985; Williams et al. 1988; Browman et al. 1974; Bychkov et al. 2009; Larin et al. 2011, 2020

Ioffe & Oganesian 2007; Kampf & Moussallam 2009; Ananthanarayan & Moussallam 2002; Goity et al. 2002

$\pi^0 \rightarrow \gamma\gamma$ decay width



Earlier lattice studies:

- larger uncertainty compared vs experiment
- No IB effects

Feng et al. 2012; Gérardin et al. 2016, 2019, 2023; Alexandrou et al. 2023; Lin et al. 2024

➤ A natural thought:

Can we calculate mass correction directly?

➤ Treat mass correction perturbatively yields a computation of 4-point function

$$\frac{\partial}{\partial m_q} \langle J_\mu J_\nu P \rangle \sim - \int d^4 z \langle J_\mu J_\nu P \bar{q} q(z) \rangle.$$

➤ However ...

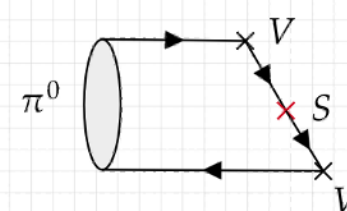
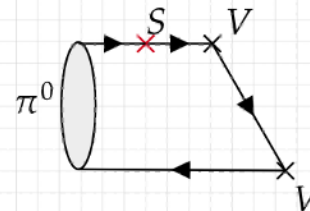
- The operator $O_{IS} = \frac{1}{2}(\bar{u}u + \bar{d}d)$ yields **noisy disconnected diagrams**.
- Both $\frac{df_\pi}{dm_\pi^2}$, $\frac{dF_{\pi\gamma\gamma}}{dm_\pi^2}$ contain chiral log, the **cancellation** of the two terms leads to large uncertainties
- Finally, the results carry a **$\sim 4\%$** error

We need to seek for a better method!

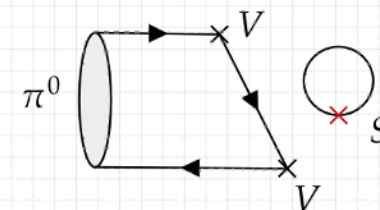
➔ **ABJ anomaly is exactly known. Can we make use of it?**

Quark mass correction of $\pi \rightarrow \gamma\gamma$ (Isospin symmetry)

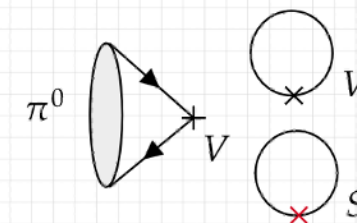
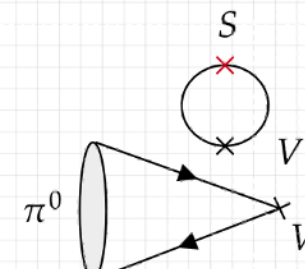
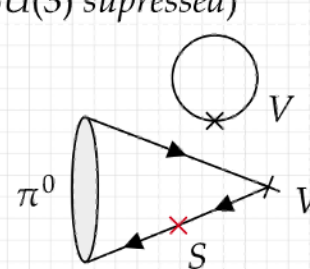
connected diagram



S – type disconnected diagram



V – type disconnected diagram
(SU(3) suppressed)



3pt method: application of anomalous ward identity

- The anomaly contribution remains exact at non-zero quark mass

$$\partial_\mu A_\mu^3 = (m_u + m_d)P^3 + (m_u - m_d)P^0 - \pi K_0 \alpha F_{\mu\nu}^{em} \tilde{F}_{\mu\nu}^{em}$$

↓total divergence
 ↓non-zero m_π
 ↓IB effects
 ↓ABJ anomaly

- evaluate amplitude: $\langle 0 | \partial_\mu A_\mu^3 | \gamma\gamma \rangle$ leading to relation between form factors

- $q^2 = 0$

$$K_0 = \hat{\mathcal{F}}^{(3)}(0) + \underbrace{\hat{\mathcal{F}}^{(0)}(0)}_{\text{off shell } \pi^0, \eta, \eta' \dots}$$

exactly known

- $q^2 = -m_\pi^2$

$$K = \lim_{q^2 + m_\pi^2 \rightarrow 0} \frac{q^2 + m_\pi^2}{m_\pi^2} \underbrace{(\hat{\mathcal{F}}^{(3)}(-q^2) + \hat{\mathcal{F}}^{(0)}(-q^2))}_{\text{on shell } \pi^0}$$

on shell π^0

- $K = (K - K_0) + K_0$

↓
mass
correction

↓
anomaly
 $K_0 = 1/4\pi^2$

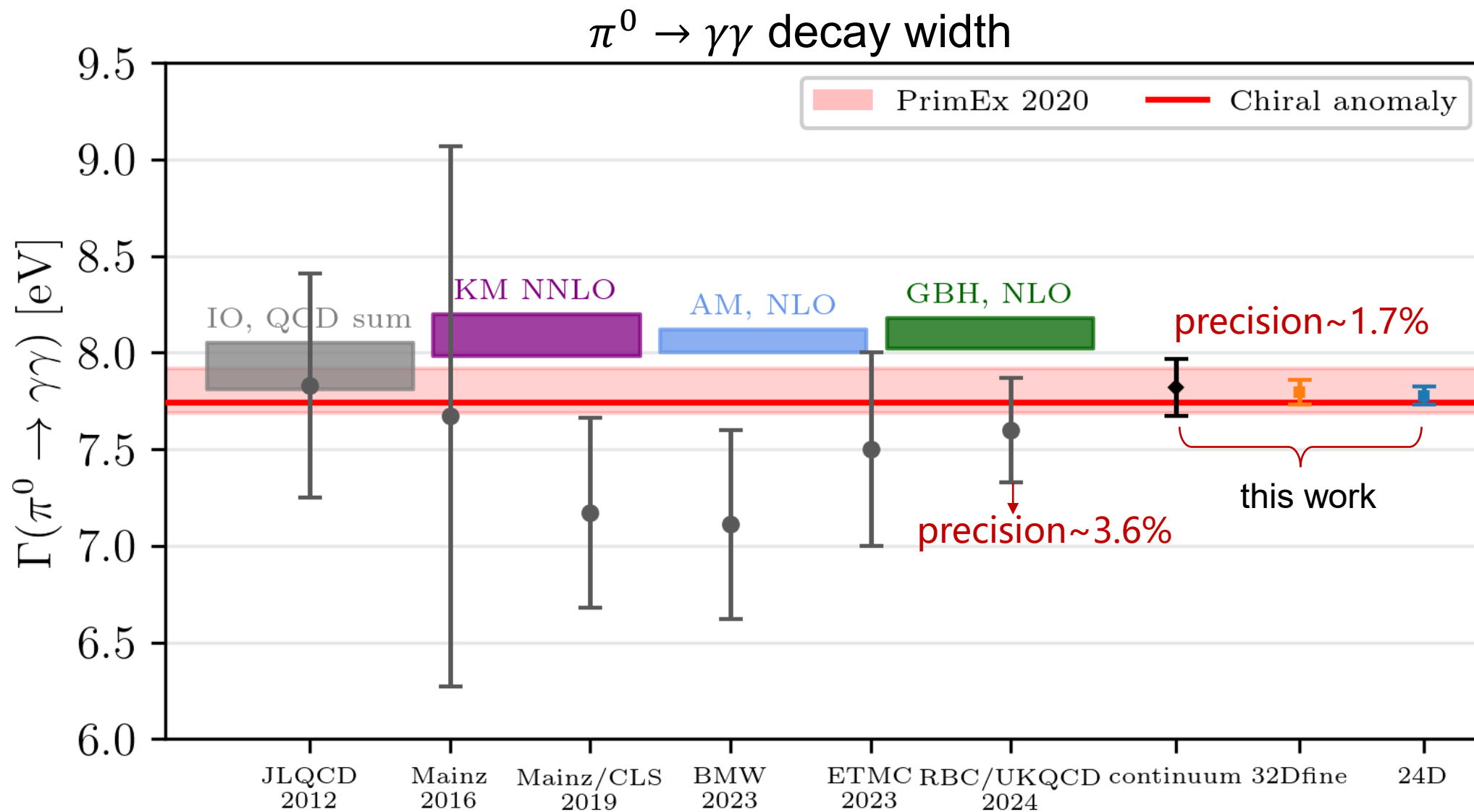
- Lattice can calculate mass correction via $\langle PVV \rangle$ 3pt correlator

Ensembles	$a[\text{GeV}^{-1}]$	L/a	T/a	L/fm	$m_\pi[\text{MeV}]$	$m_\pi L$	# of confs
24D	1.023	24	64	4.7	142.8(2)	3.3	92
32Dfine	1.378	32	64	4.7	142.9(4)	3.3	49

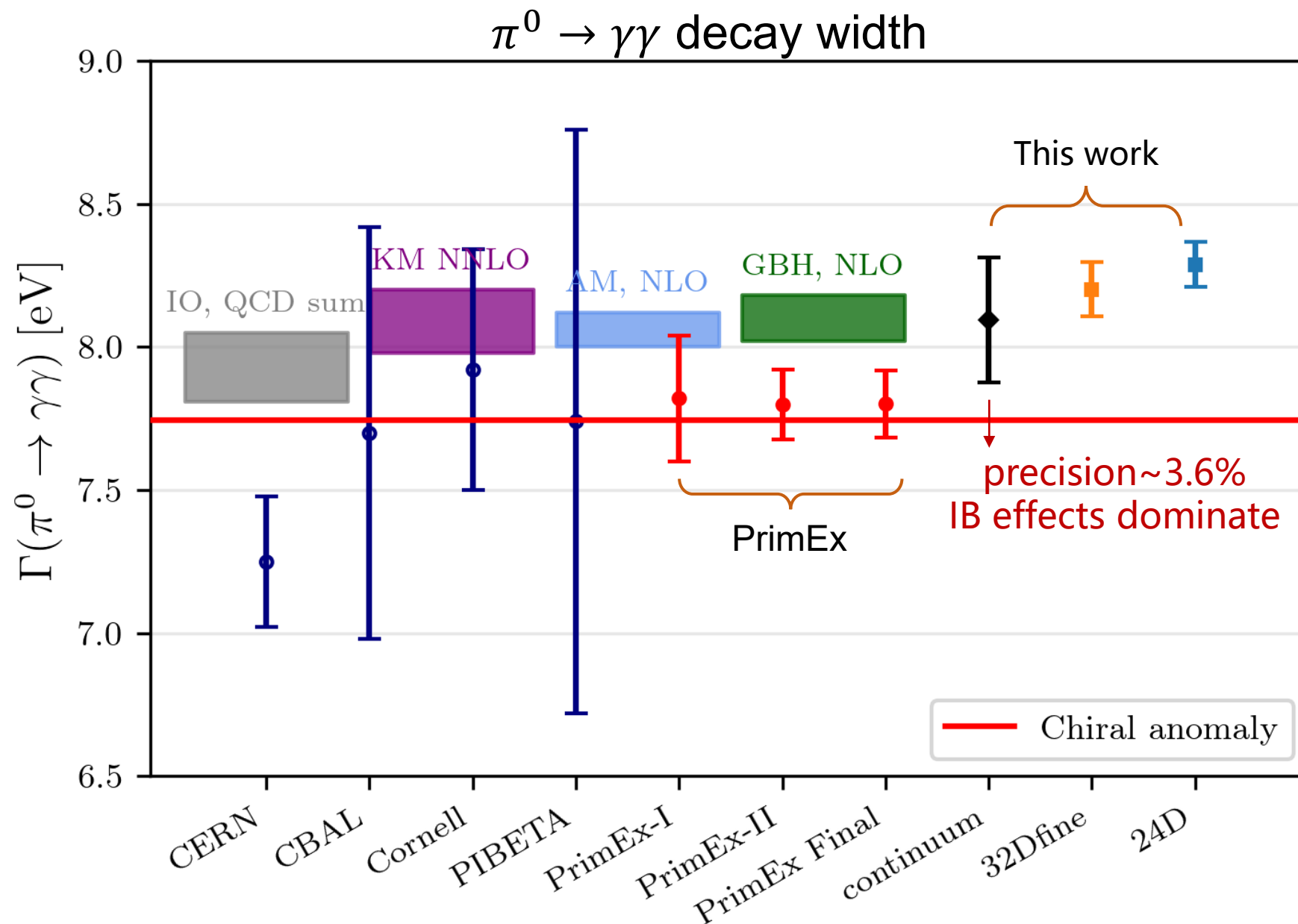
Generated by RBC/UKQCD(RBC et al., 2016)

- near physical pion mass
- Domain wall fermions + Iwasaki gauge action(+DSDR)
- Two different lattice spacings for extrapolation

π^0 decay width in isospin limit



π^0 decay width with IB effects



QCD axion and axion-photon coupling

➤ QCD axion

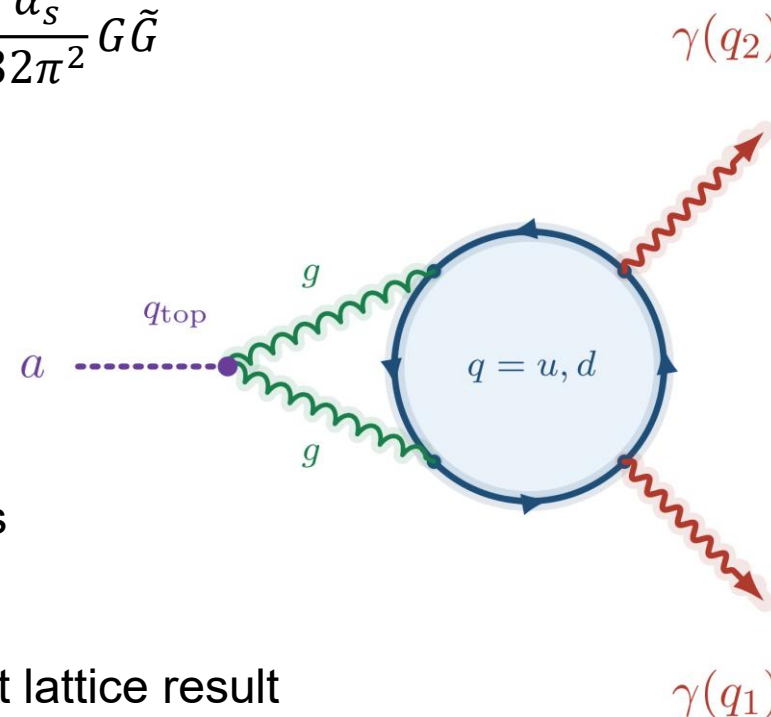
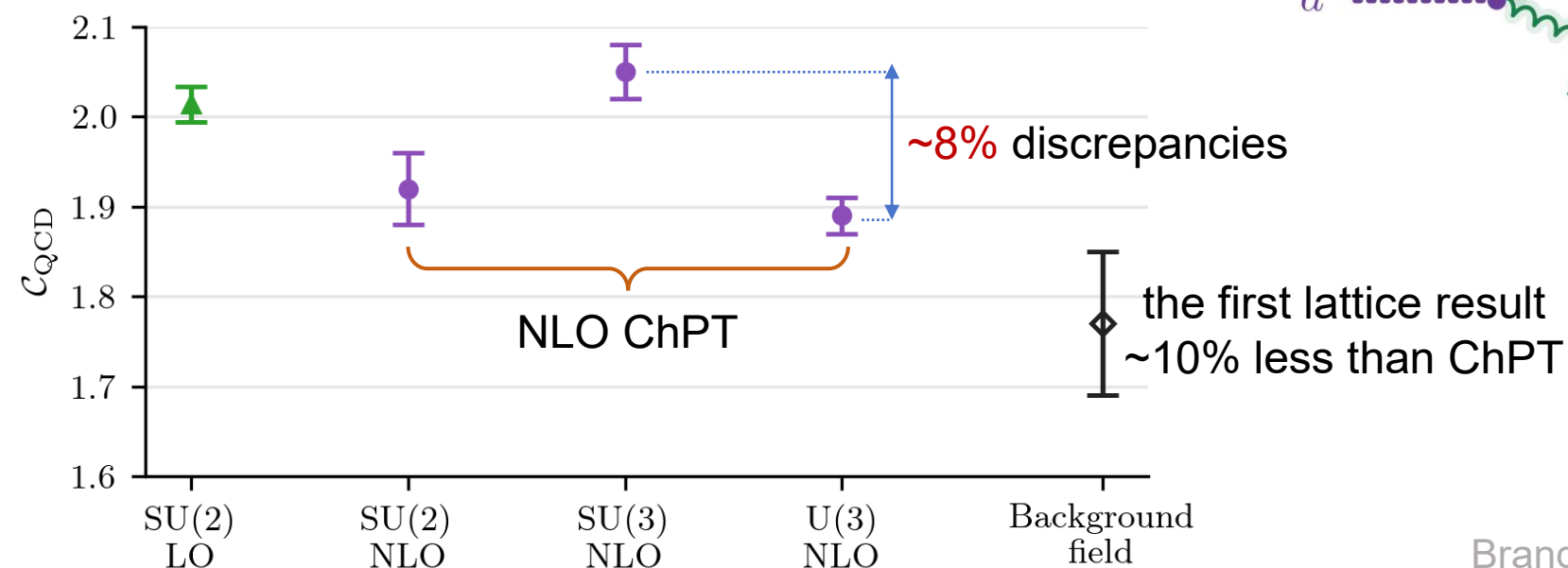
- solves strong CP problem

$$\mathcal{L}_a \supset \frac{a}{f_a} q_{top} \quad \text{with} \quad q_{top} = \frac{\alpha_s^2}{32\pi^2} G\tilde{G}$$

- is a dark-matter candidate.

➤ axion-photon coupling

- Its photon coupling enables experimental searches
- QCD contribution $\langle 0 | q_{top} | \gamma\gamma \rangle \Rightarrow C_{QCD}$



(Hint: The ChPT prediction has been updated and we will return to this later)

Brandt et al., 2026; Cortona et al., 2016; Gao et al., 2026; Lu et al., 2020

Method: Anomalous ward identity

➤ make use of anomalous WI

$$\partial_\mu A_\mu^3 = (m_u + m_d)P^3 + (m_u - m_d)P^0 - \pi K_0 \alpha F_{\mu\nu}^{em} \tilde{F}_{\mu\nu}^{em} \quad (\text{iso-vector channel})$$

$$\partial_\mu A_\mu^0 = (m_u + m_d)P^0 + (m_u - m_d)P^3 - \frac{5}{3} \pi K_0 \alpha F_{\mu\nu}^{em} \tilde{F}_{\mu\nu}^{em} - 2q_{top} \quad (\text{iso-singlet channel})$$

- evaluate amplitude $\langle 0 | \partial_\mu A_\mu^a | \gamma \gamma \rangle$ leading to relation between form factors

- $q^2 = 0$

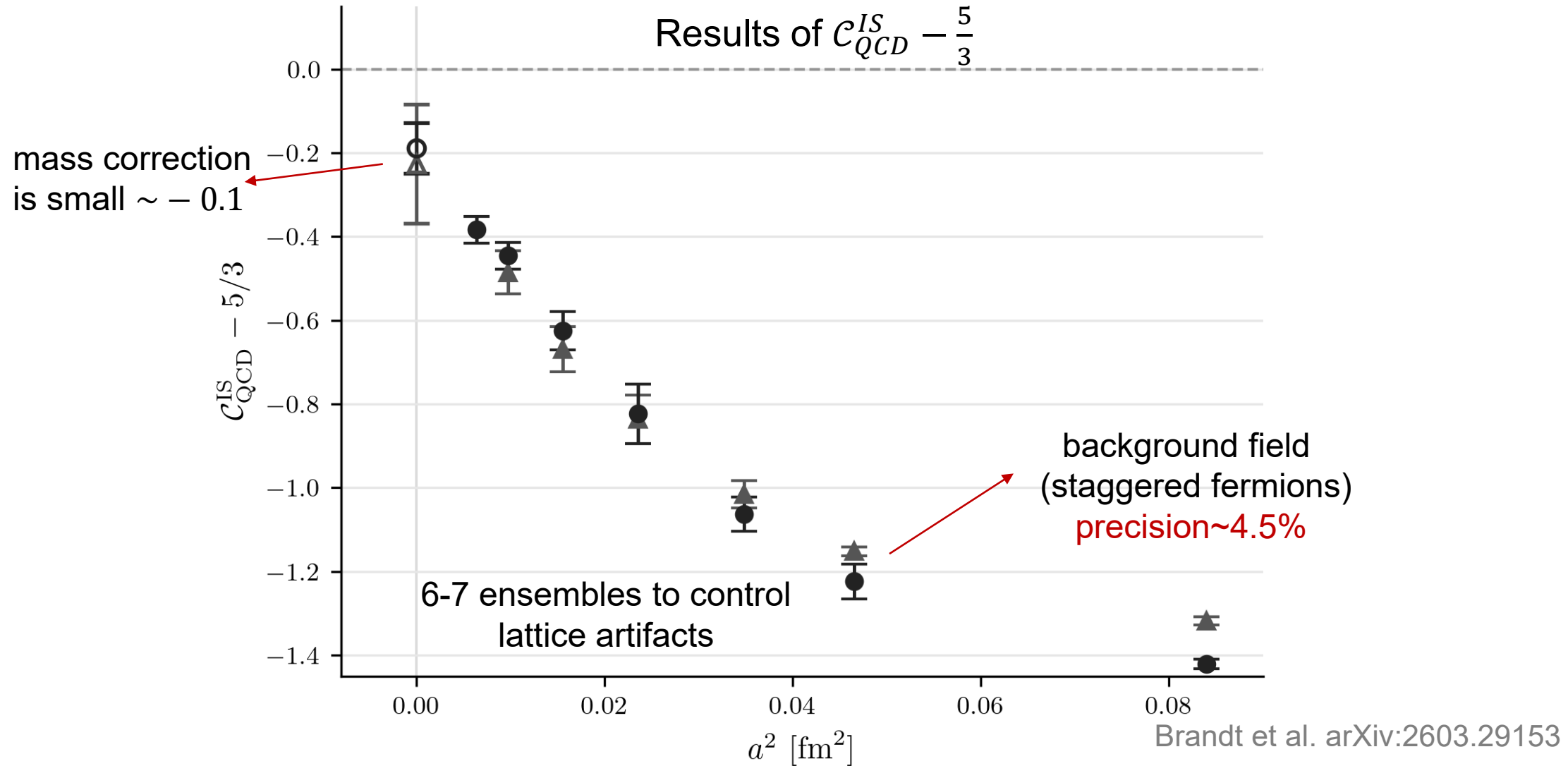
$$K_0 = 2m_l \mathcal{F}^{(3)}(0) + \delta m \mathcal{F}^{(0)}(0)$$

$$\frac{5}{3} K_0 = 2m_l \mathcal{F}^{(0)}(0) + \delta m \mathcal{F}^{(3)}(0) - 2\mathcal{F}_q(0) \longrightarrow \text{axion-photon coupling} \quad \mathcal{C}_{QCD} = -8\pi^2 \mathcal{F}_q(0)$$

$$\mathcal{C}_{QCD} = \underbrace{\frac{5}{3}}_{\text{anomaly}} - \underbrace{\xi}_{\text{IB effects} \sim 20\%} - \underbrace{8\pi^2 m_l (1 - \xi^2) \mathcal{F}^{(0)}(0)}_{\text{mass correction}} \quad \text{with } \xi = \frac{\delta m}{2m_l} = -0.347(17) \text{ (FLAG 26)}$$

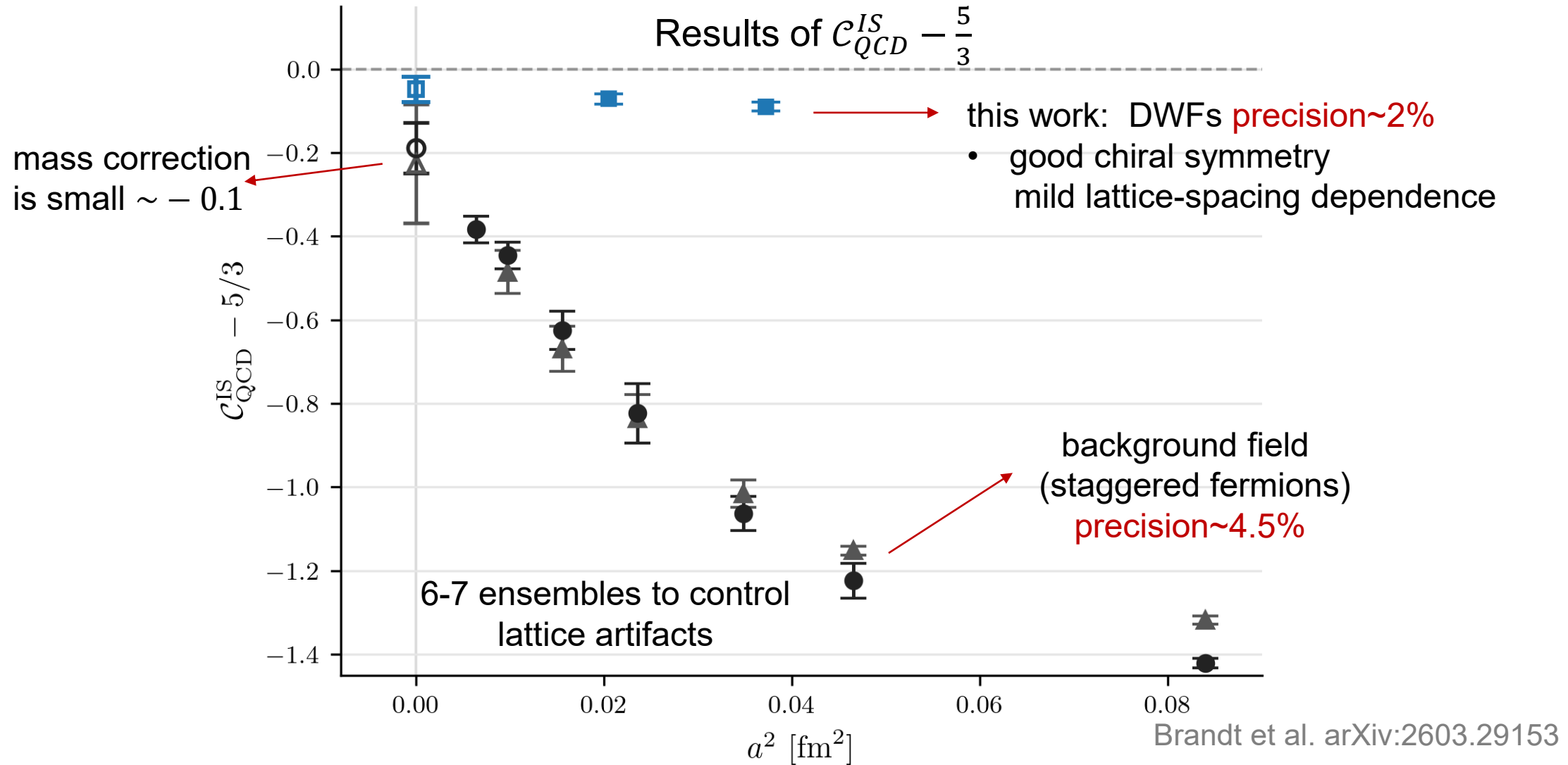
Axion-photon coupling in isospin limit

- $\mathcal{C}_{QCD}^{IS} = \frac{5}{3} - 8\pi^2 m_l \mathcal{F}^{(0)}(0)|_{\delta m=0}$



Axion-photon coupling in isospin limit

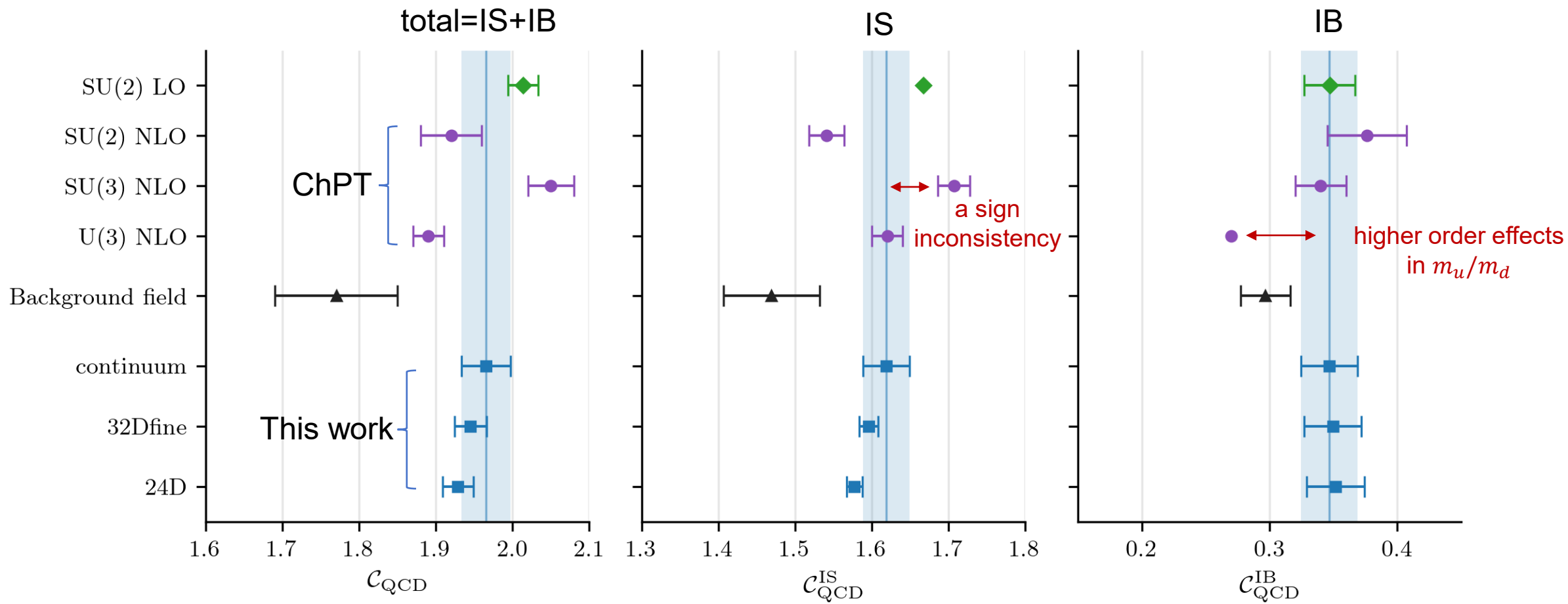
- $\mathcal{C}_{QCD}^{IS} = \frac{5}{3} - 8\pi^2 m_l \mathcal{F}^{(0)}(0)|_{\delta m=0}$



Axion-photon coupling

QCD contribution to $C_{a\gamma}^{\text{eff}} = E/N - \mathcal{C}_{\text{QCD}}$

◆ LO χPT
● NLO χPT
▲ Background field
 ■ This work



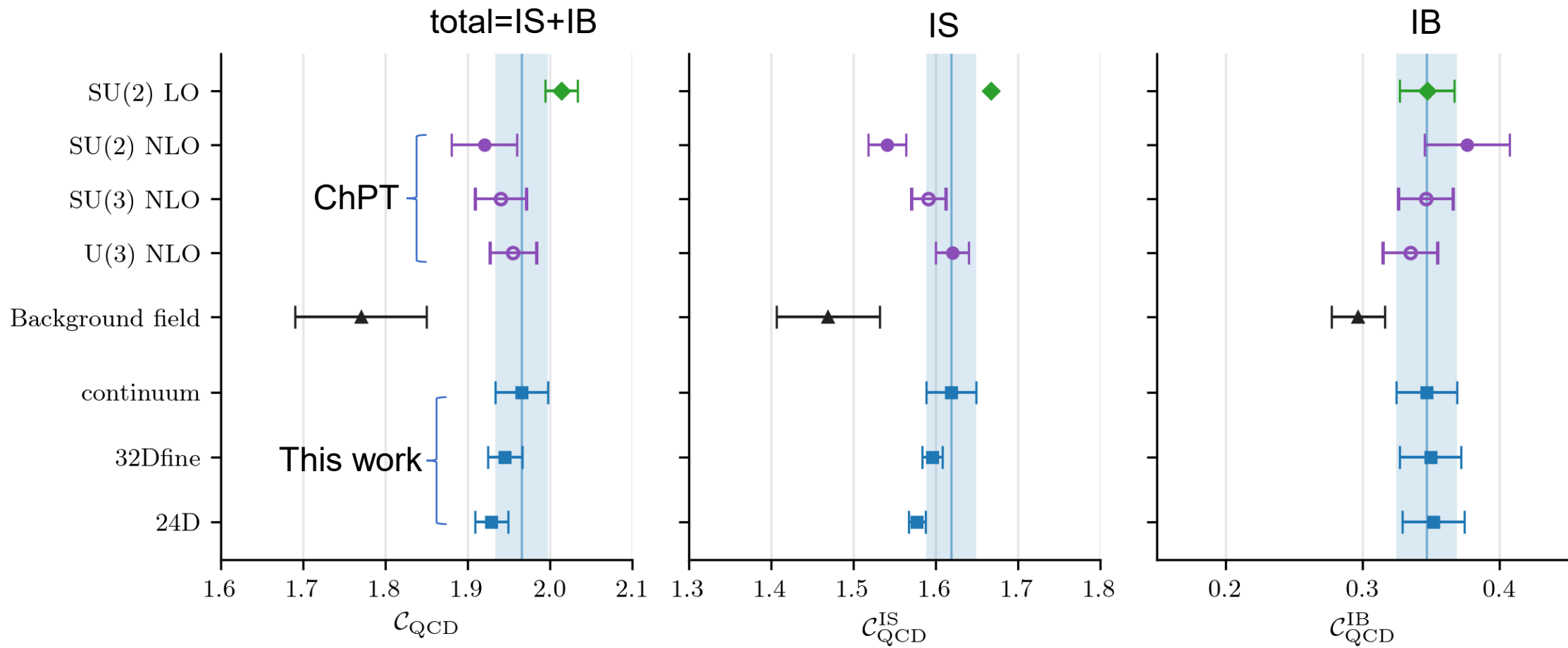
- $C_{\text{QCD}} = 1.965(35) \sim 2\%$ precision

Brandt et al., 2026; Cortona et al., 2016; Gao et al., 2026; Lu et al., 2020

Axion-photon coupling (updated)

QCD contribution to $C_{a\gamma}^{\text{eff}} = E/N - \mathcal{C}_{\text{QCD}}$

◆ LO χPT ● NLO χPT ▲ Background field ■ This work



- Our results agree well with ChPT
- Lattice provides an important benchmark!

Brandt et al., 2026; Cortona et al., 2016; Gao et al., 2026; Lu et al., 2020

Framework

- Apply the anomalous Ward identities (AWI) to separate the exact anomaly contribution from the finite-quark-mass corrections.

Results

- First lattice determination of the neutral pion decay width with isospin-breaking effects:

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 8.09(22)\text{eV}$$

arXiv: 2608.31024

- Model-independent QCD contribution to the axion–photon coupling:

$$C_{QCD} = 1.965(35)$$

arXiv: 2609.22698

Outlook

- Reduce the uncertainty and full error budget analysis.
- Extend the formalism to $\eta, \eta' \rightarrow \gamma\gamma$ and to other anomalous processes $\gamma \rightarrow 3\pi, K \bar{K} \rightarrow 3\pi$