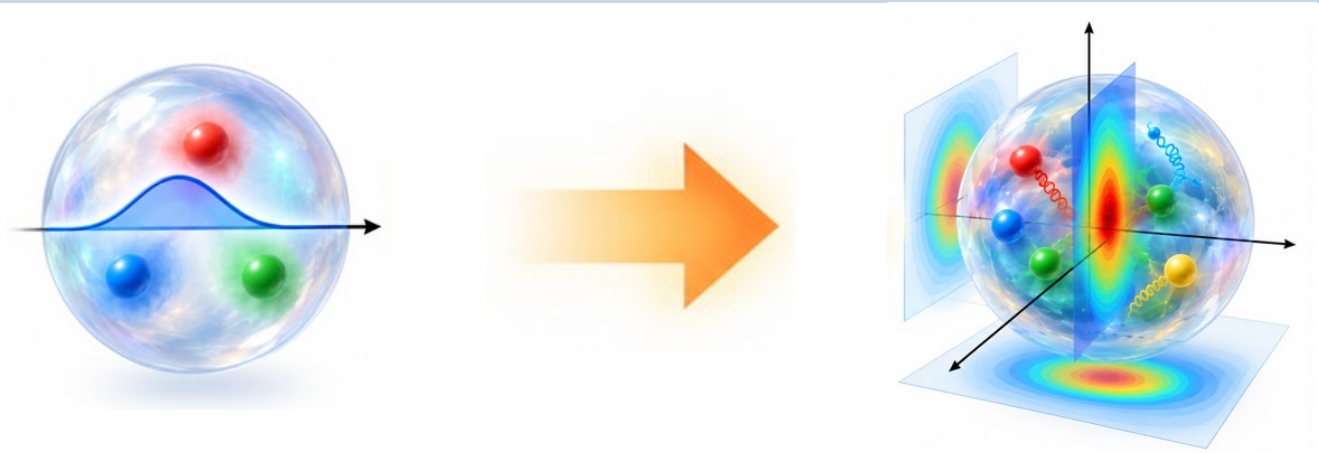
华南师範大學
SOUTH CHINA NORMAL UNIVERSITY

Lattice Parton Collaboration

From Longitudinal to Transverse Structure:

Hadron Tomography from Lattice QCD



华俊 华南师范大学

On behalf of Lattice Parton Collaboration（LPC）

2026 10/10 @ 第6届中国格点量子色动力学研讨会，深圳

OUTLINE

- Motivation
- Renormalization
- Exclusive 1-D structure LCDA
- 3-D TMDs
- Summary and Outlook

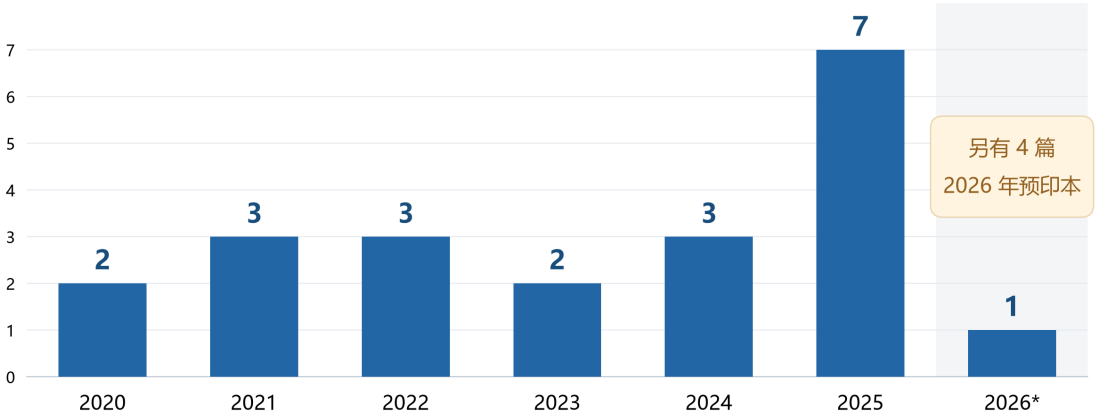
Lattice Parton Collaboration (LPC)

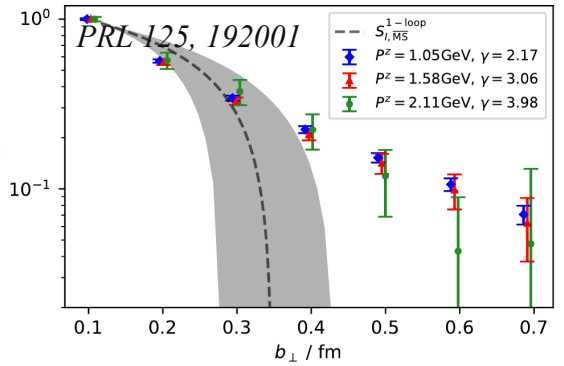
Lattice Parton Collaboration

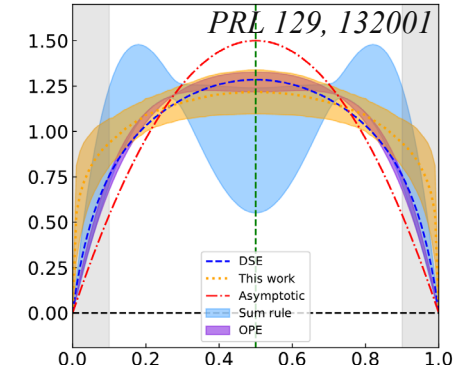
2019

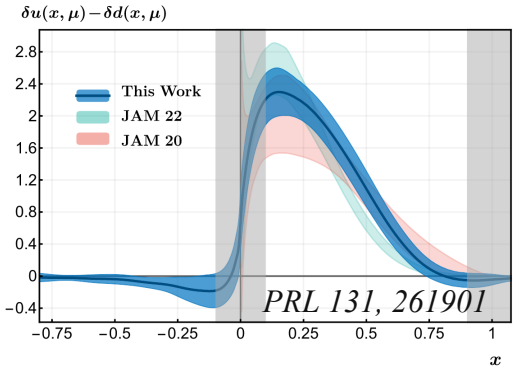
2019 → 2026 Hadronic papers 26

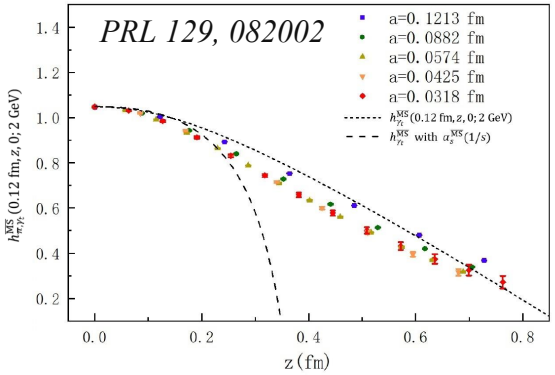


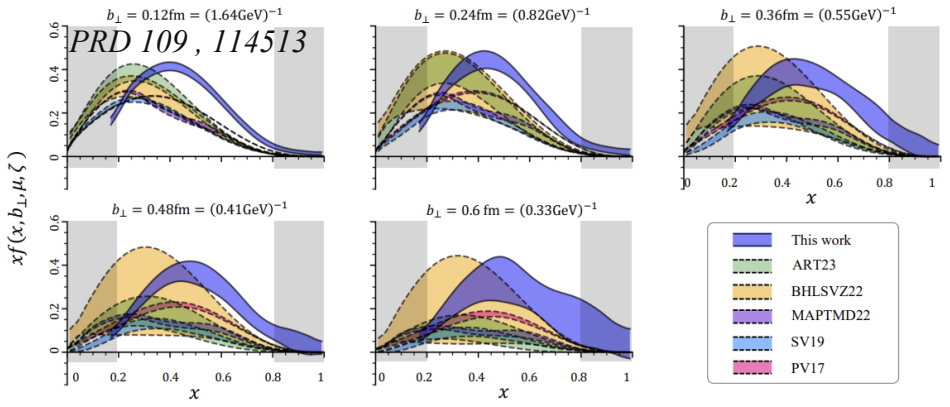






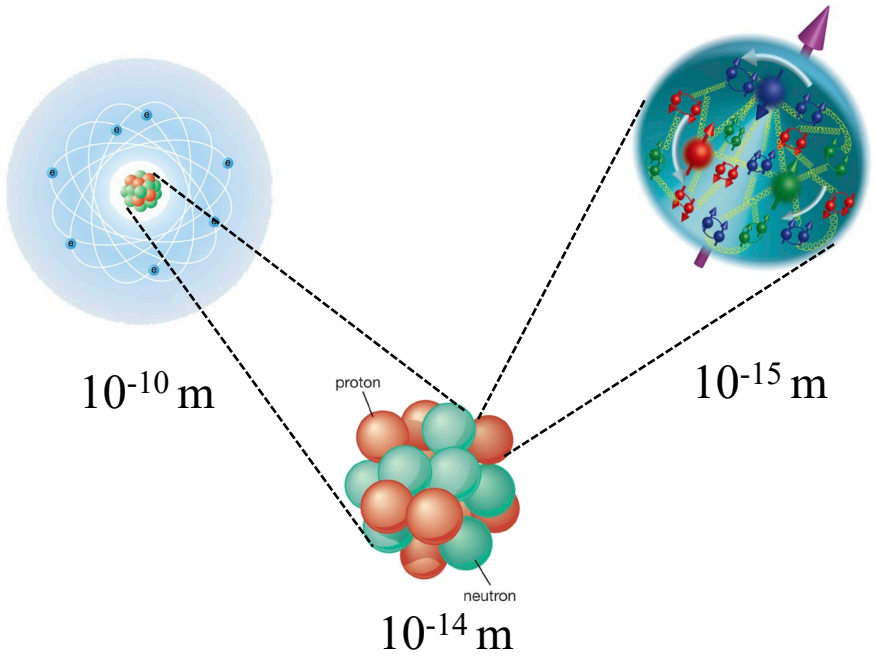






Nucleons: Building Blocks of the Visible Universe

Despite more than half a century of exploration, the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe

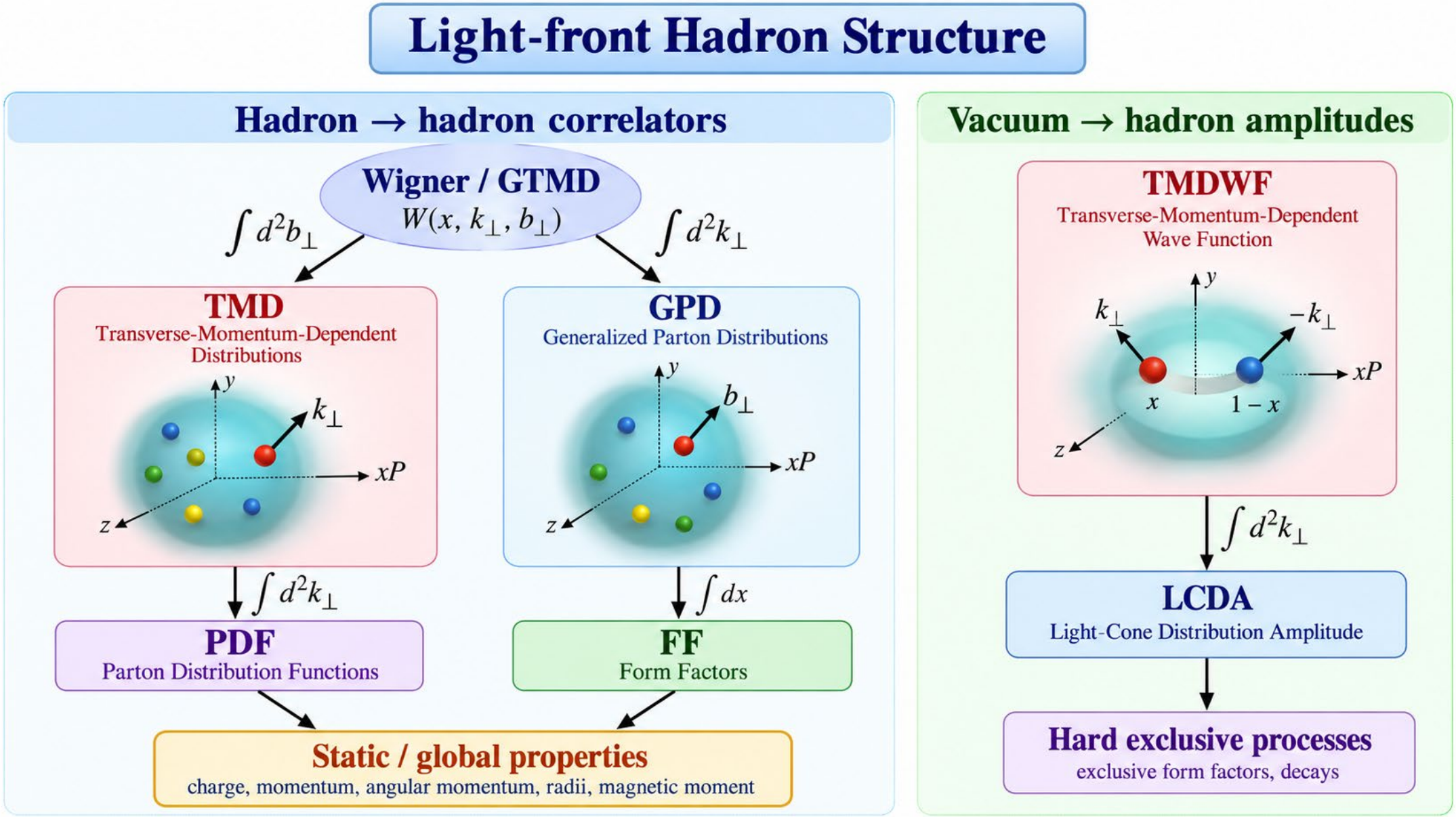


- How does quarks & gluons form the nucleon, how do they distribute?
- How do these distributions and partonic orbital motion generate the nucleon spin?

— 1st of the 3 central science questions driving the EIC program.

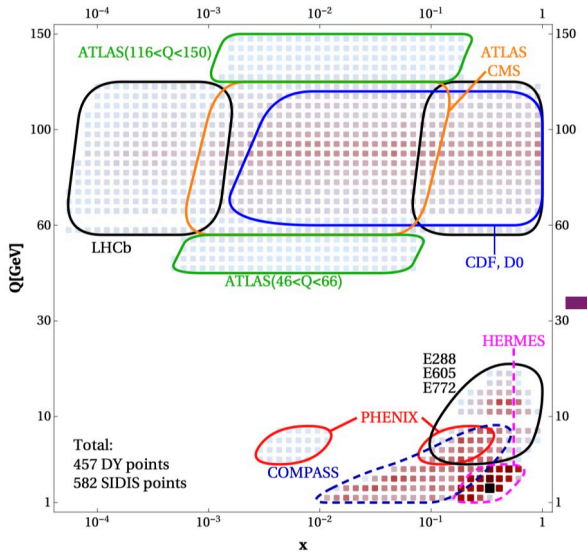
Adapted from the White Paper of EIC, EPJA52, 268 (2016).

The Landscape of Hadron Structure



How do we access the Hadron Structures?

Exp.

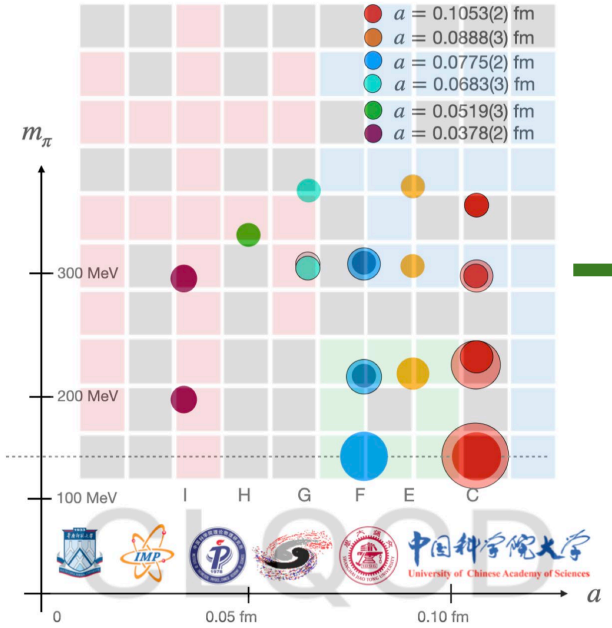


→ **Cross Sections, Asymmetries, …** → Global Analysis / Fitting

$$\chi^2 = \sum_i^{N_{\text{meas}}} \frac{(O_i - E_i)^2}{E_i}$$

→ Parameters

Lat.



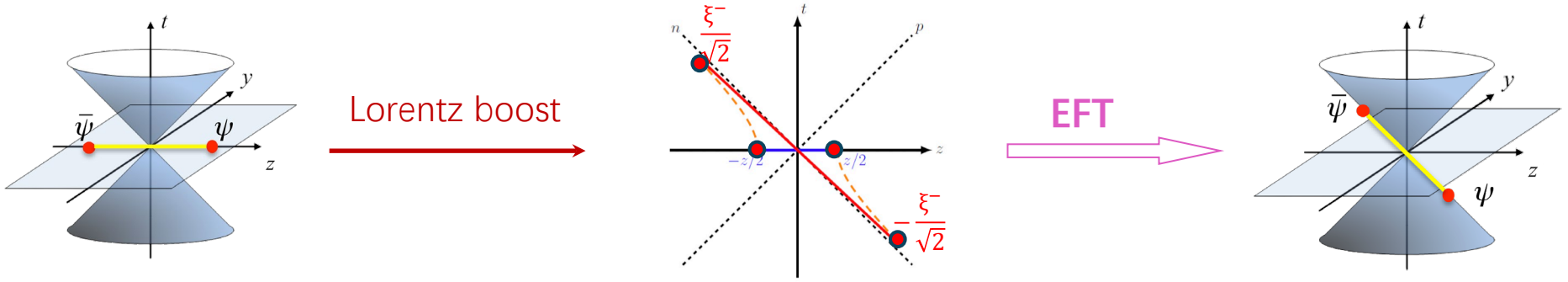
→ Correlation Function → Effective Theory

$$\langle \tilde{O} \rangle_{\text{lat.}} = C \otimes \langle O \rangle_{l.c.}$$

→ Distributions

LaMET: A New Horizon for Nucleon Structure from Lattice

Large-momentum effective theory (LaMET): connecting Euclidean lattice and physical observables



Ji, PRL110, 262002 (2013), see Rev.Mod.Phys.93, 035005 (2021) for review

- **Equal-time distributions can be directly computed in lattice QCD:**

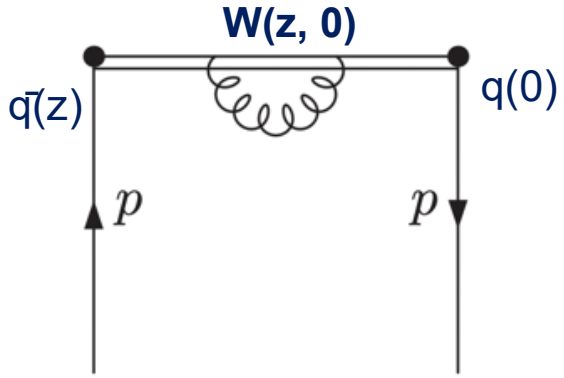
$$\tilde{q}(x, P^z) = \int \frac{dz}{2\pi} e^{-izP^z} \langle P^z | \bar{\psi}(z) \Gamma \mathcal{U}(z,0) \psi(0) | P^z \rangle$$

- **Recovering light-cone distributions in the large-momentum limit:**

$$q(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{q}(x, P^z) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(yP^z, (1-y)P^z)^2}\right)$$

Linear Divergence: The Roadblock to Nonlocal Renormalization

Wilson-line self-energy

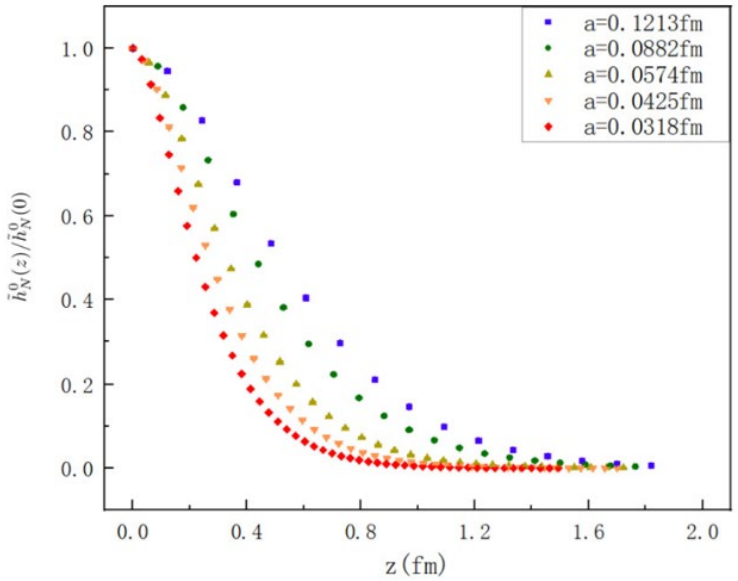


$O\Gamma(z) = \bar{q}(z)\,\Gamma\,W(z,0)\,q(0)$

$hB(z,a) \propto \exp[-\delta m(a)\,|z|]$

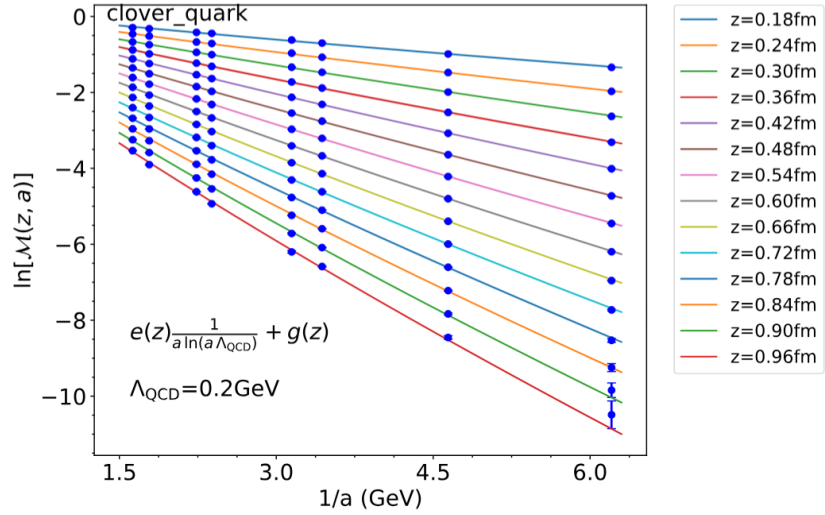
$\delta m(a) \sim \alpha_s(1/a)/a$

Cutoff dependence of the bare correlator



Smaller a gives stronger exponential suppression

Zero momentum matrix element to extract linear div



Self renormalization

Huo et al. (LPC), NPB 969, 115443 (2021)

Hybrid renormalization

Ji et al. (LPC), NPB 964, 115311 (2021)

Diagnose residual divergences in RI/MOM

K. Zhang et al. PRD 104, 074501 (2021)

Subtract staple-operator UV divergences

K. Zhang et al. (LPC), PRL 129, 082002 (2022)

Control gauge-fixing errors

K. Zhang et al. (LPC), PRD 110, 074505 (2024)

Extend hybrid subtraction to baryon DAs

Bai et al. (LPC), PRD 112, 114515 (2025)

Hybrid & Self renormalization: The Reliable Path Forward

Mass subtraction and renormalons

Consistent mass subtraction (LRR)

R. Zhang, Holligan, Ji & Su

PLB 844, 138081 (2023) [2305.05212]

LRR + two-scale RGR for pion DA

Holligan, Ji, Lin, Su & R. Zhang

NPB 993, 116282 (2023) [2301.10372]

Renormalon cancellation at threshold

Y. Liu & Su

JHEP 02 (2024) 204 [2311.06907]

Hybrid subtraction for three-quark operators

Han, Su, Wang & J.-L. Zhang, JHEP 12 (2023) 044 [2308.16793]

Momentum and threshold logarithms

Resum parton-momentum logarithms (RGR)

Su, Holligan, Ji, Yao, J.-H. Zhang & R. Zhang

NPB 991, 116201 (2023) [2209.01236]

Factorize and resum threshold logarithms

Ji, Y. Liu & Su

JHEP 08 (2023) 037 [2305.04416]

Quantify threshold resummation + LRR

Ji, Y. Liu, Su & R. Zhang

JHEP 03 (2025) 045 [2410.12910]

Gradient-Flow Renormalization of Nonlocal Quasi Operators

A tunable UV scale for Wilson-line self-energy subtraction

Flow radius and self-energy

$r = \sqrt{(8\tau)}$ $\quad \delta m(\tau) \sim 1/r$

Finite flow regulates UV modes
Small-flow-time self-energy still needs subtraction

Gao et al., arXiv:2609.23379 — Talk by 高美琪

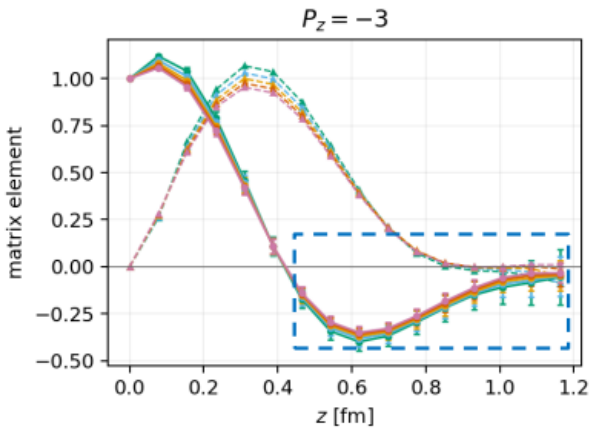
Ward-identity normalization of flowed quarks
Validation across local bilinear channels

Huang et al., in preparation — Talk by 黄俊元

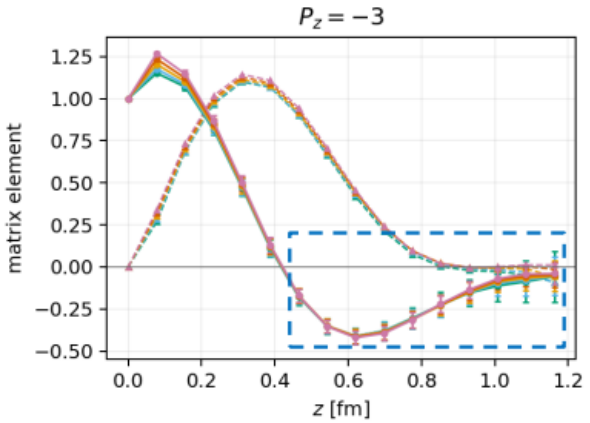
Extract self-energy from gauge-fixed Wilson lines
Apply a common subtraction to quasi matrix elements
Match the flowed operators to MS-bar

Preliminary pion quasi-DA test

Before matching



After matching



Flow radii: 0.12, 0.14, 0.16, 0.18, 0.20 fm

Reduced flow-time dependence after self-energy subtraction and matching

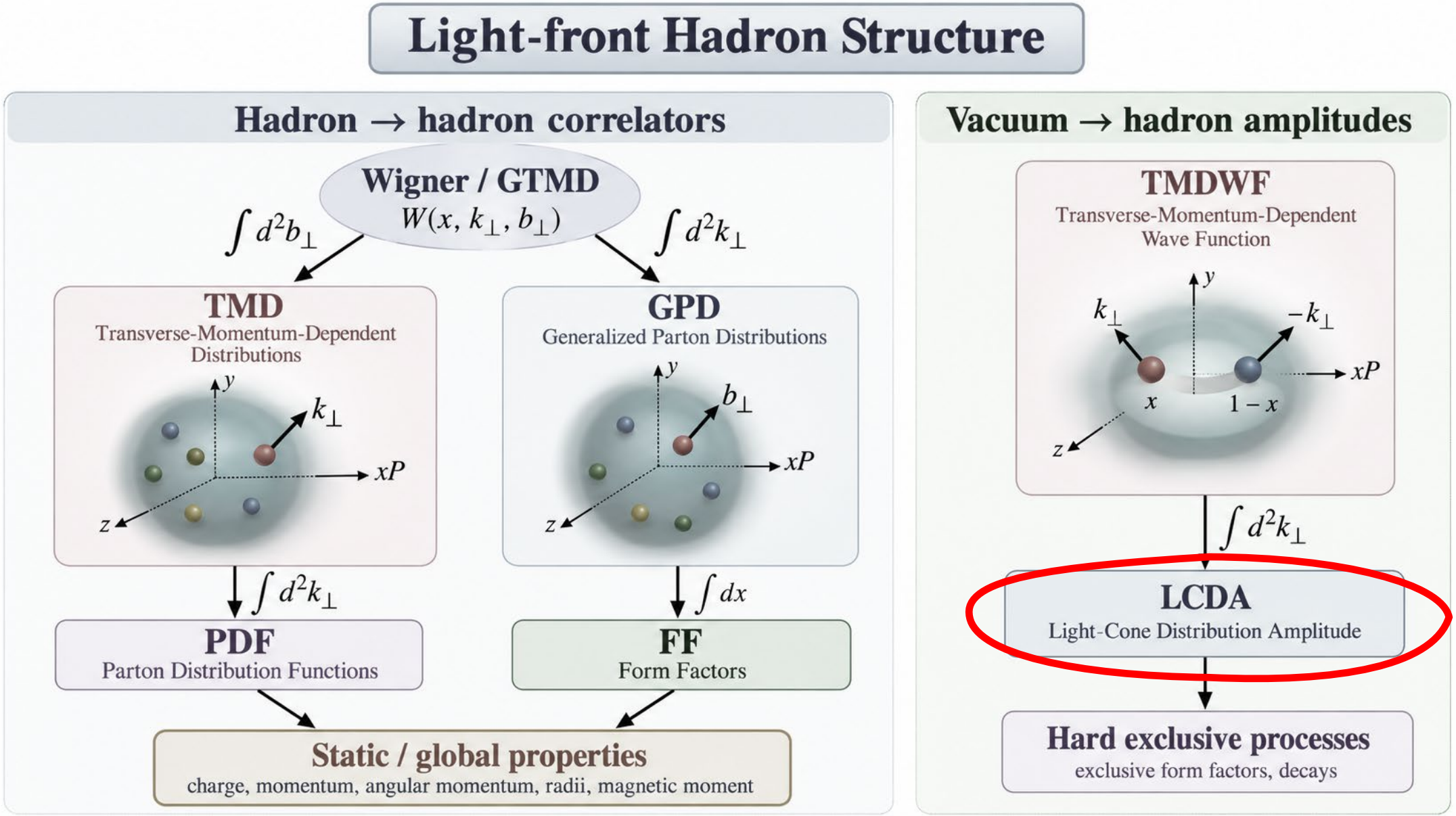
Compatible self-energy coefficients across pion DA, pion PDF and proton PDF

Two CLQCD ensembles: a ≈ 0.105 and 0.077 fm

Next: continuum and small-flow-time control for nonlocal matrix elements

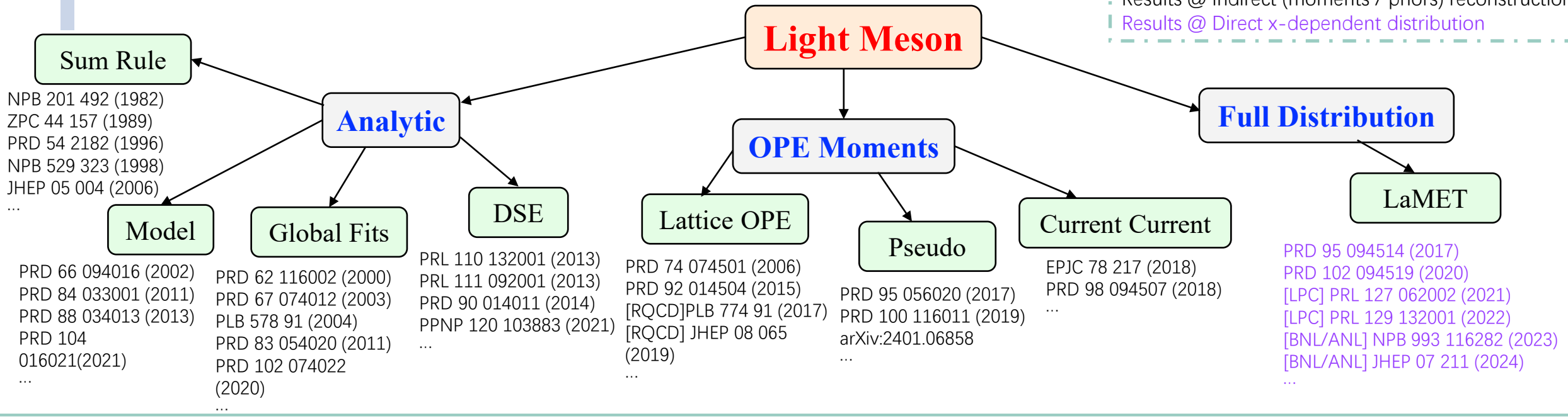
Lüscher, JHEP 08 (2010) 071 *Brambilla & Wang, JHEP 06 (2024) 210*

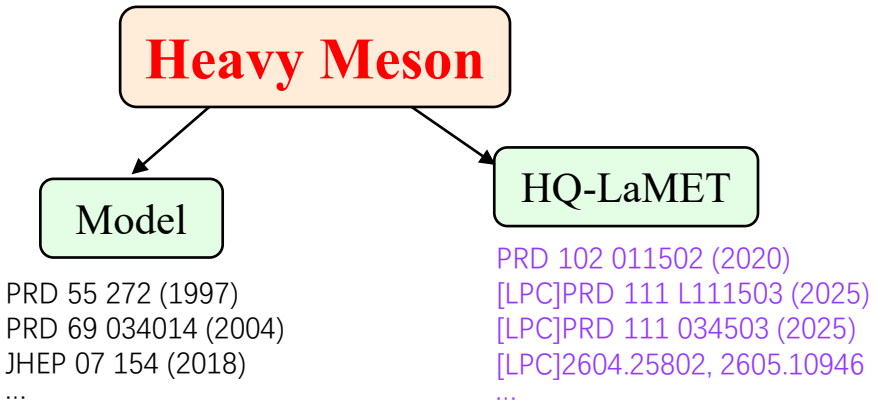
Exclusive 1-D structure LCDA

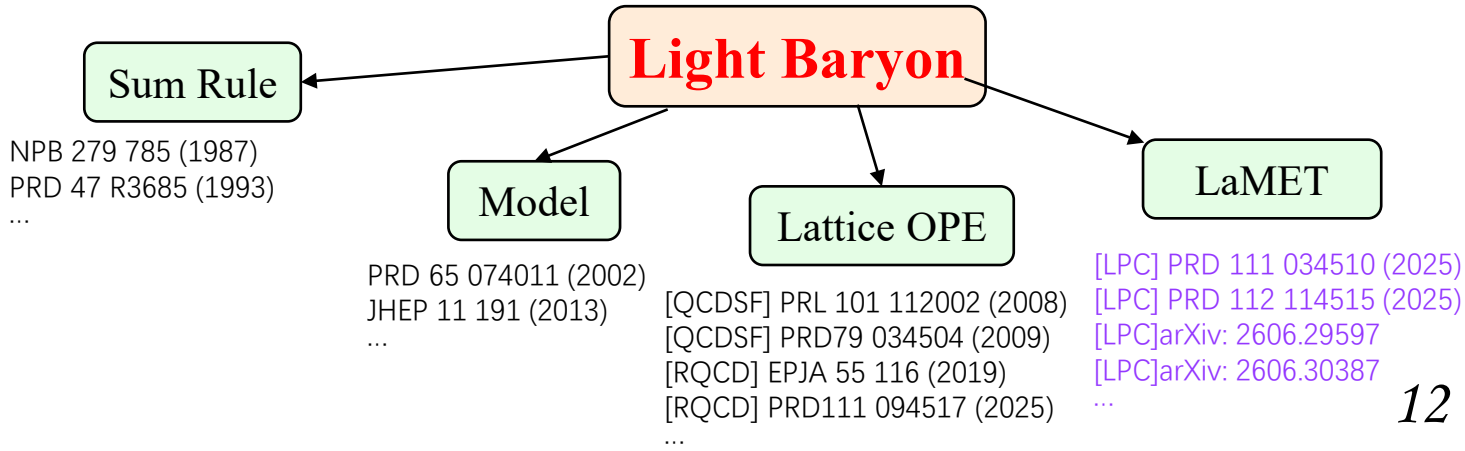


Exclusive 1-D structure LCDA

Results @ Indirect (moments / priors) reconstruction
Results @ Direct x-dependent distribution







Light Meson LCDA

Vector mesons: K* and φ

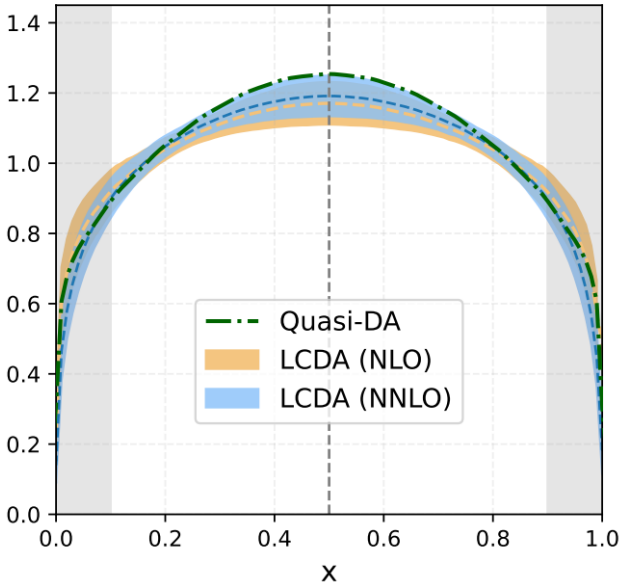
(LPC) PRL 127, 062002 (2021)

Pseudoscalars: π and K

(LPC) PRL 129, 132001 (2022)

NNLO meson-DA matching

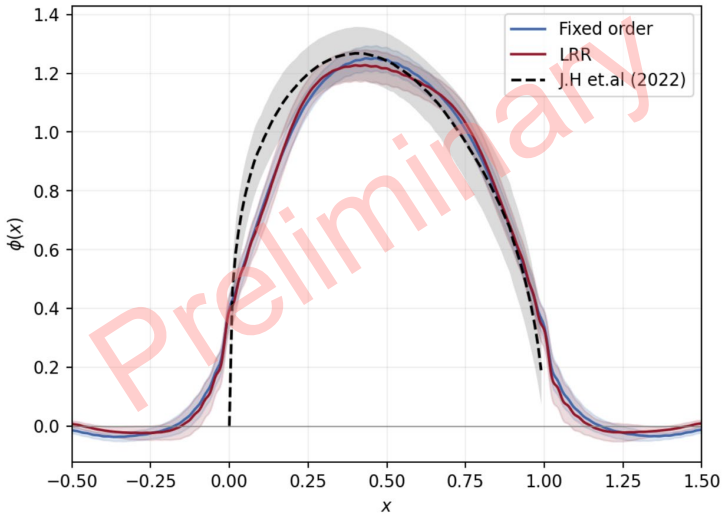
Y. Ji, F. Yao & J.-H. Zhang
arXiv:2504.09367 (2025)



Pion at Pz = 2.15 GeV, μ = 2 GeV

New kaon LCDA with an improved operator

- Excited-state contamination and Pz dependence controlled.
- Moments agree with independent OPE benchmarks.
- Our most precise LaMET hadron-structure result to date.



Moments from OPE:

$$a_{1,K} = -0.0477(23) \quad a_{2,K} = 0.158(14)$$

Benchmark from LaMET:

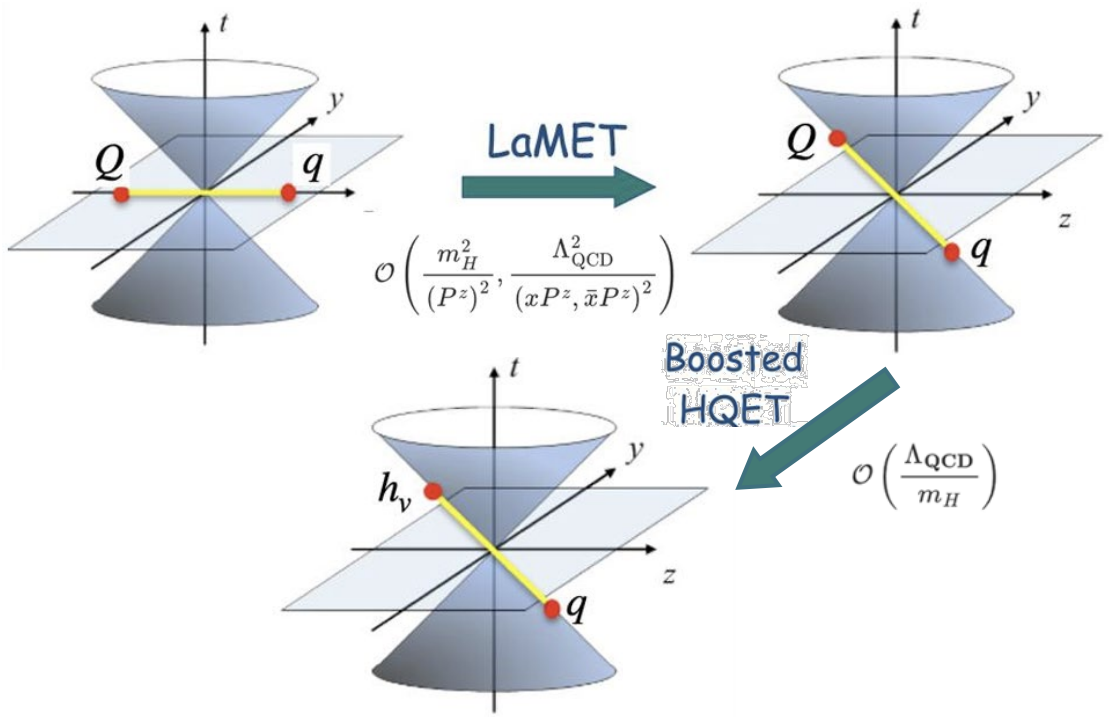
$$a_{1,K} = -0.032(18) \quad a_{2,K} = 0.143(23)$$

Heavy Meson LCDA

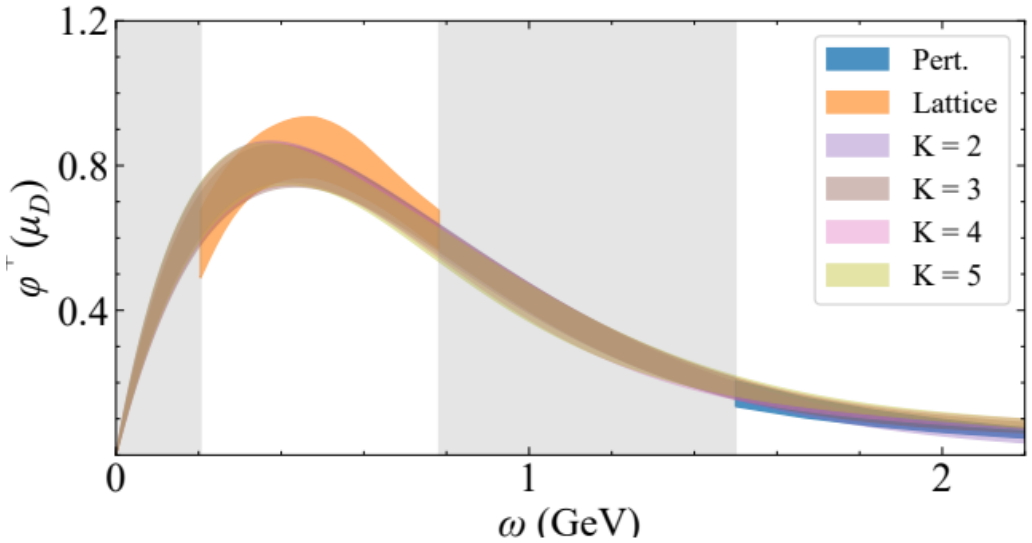
HQET LCDAs for exclusive B decays

Han et al. (LPC), PRD 111, 034503 (2025): exploratory lattice determination

Han et al., arXiv:2605.10946 Gao et al., arXiv:2604.25802 (LPC, 2026) — Talk by 高浩飞



HQET LCDA at μ = mD



Lattice peak and perturbative tail constrain a parametrized interpolation between them

Continuum and physical-mass extrapolations, with independent OPE-moment cross-checks

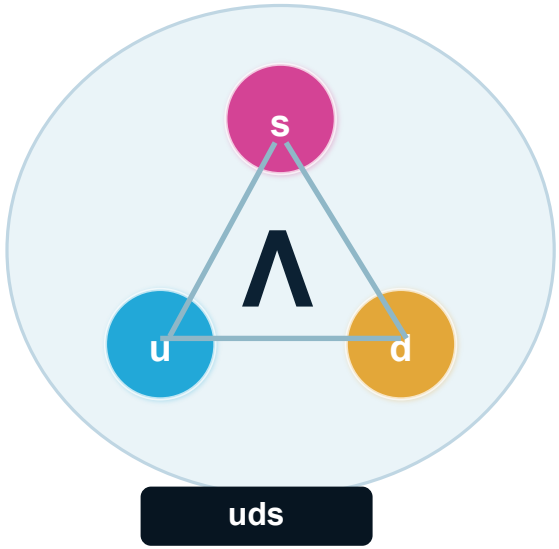
Light baryon LCDA

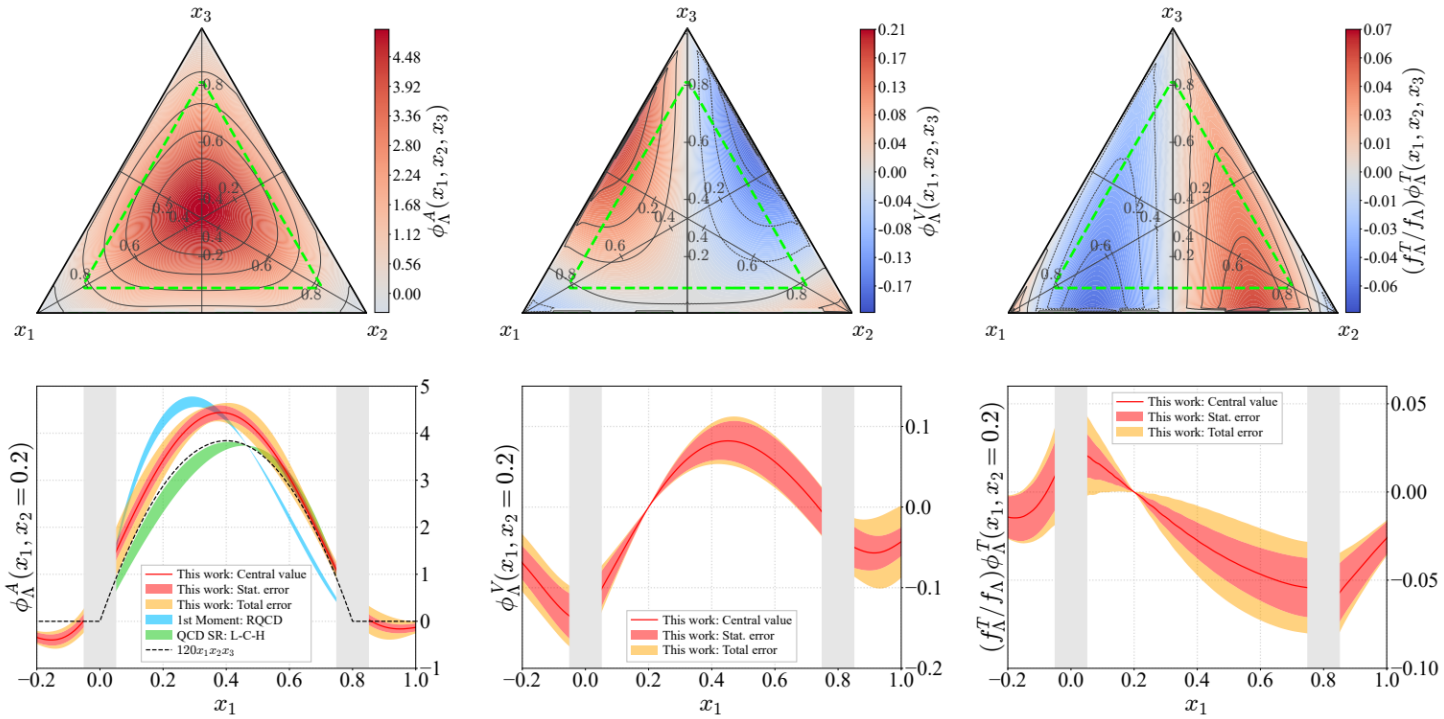
(LPC), PRD 111, 034510 (2025): first study (LPC), PRD 112, 114515 (2026):Hybrid renormalization

Zhang et al. (LPC), arXiv:2606.29597 and 2606.30387 (2026)

Complete leading-twist Λ LCDAs: A, V and T — Talk by 张沐华

Two independent longitudinal fractions, $x_1 + x_2 + x_3 = 1$





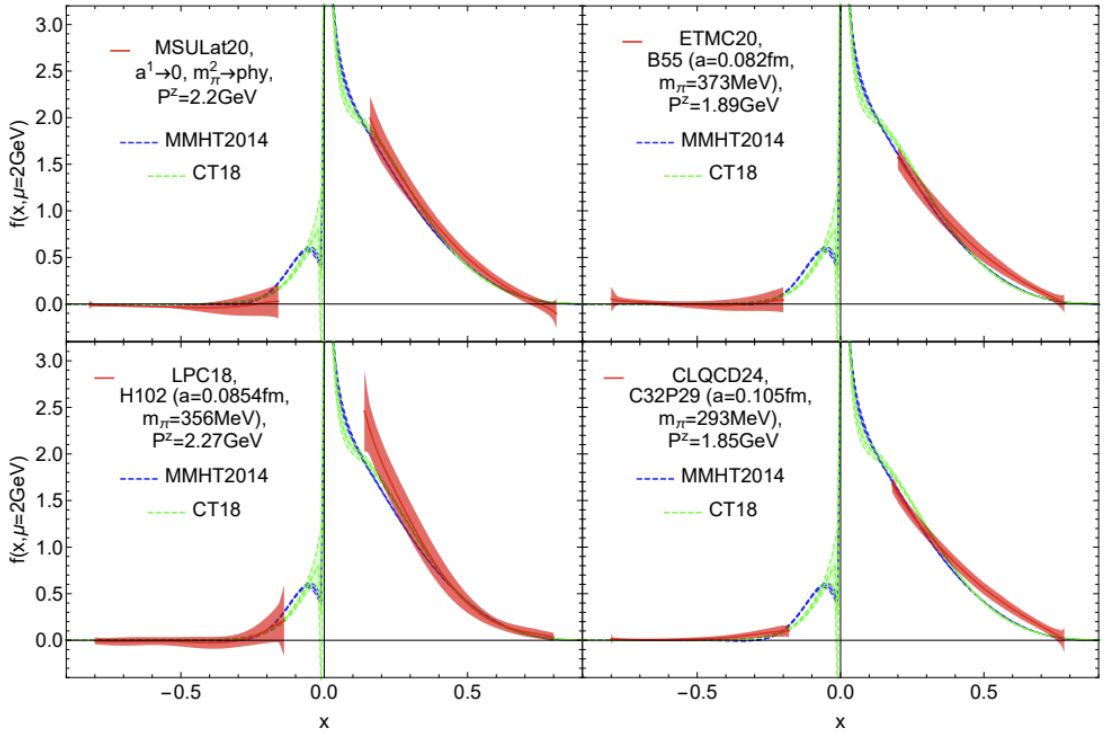
A is symmetric under $x_1 \leftrightarrow x_2$, while V and T change sign

Example: the lattice DA shifts a Λ electromagnetic form factor by about 10% versus the asymptotic DA

Precision PDF & exploratory PDFs

Nucleon u − d PDF: toward precision

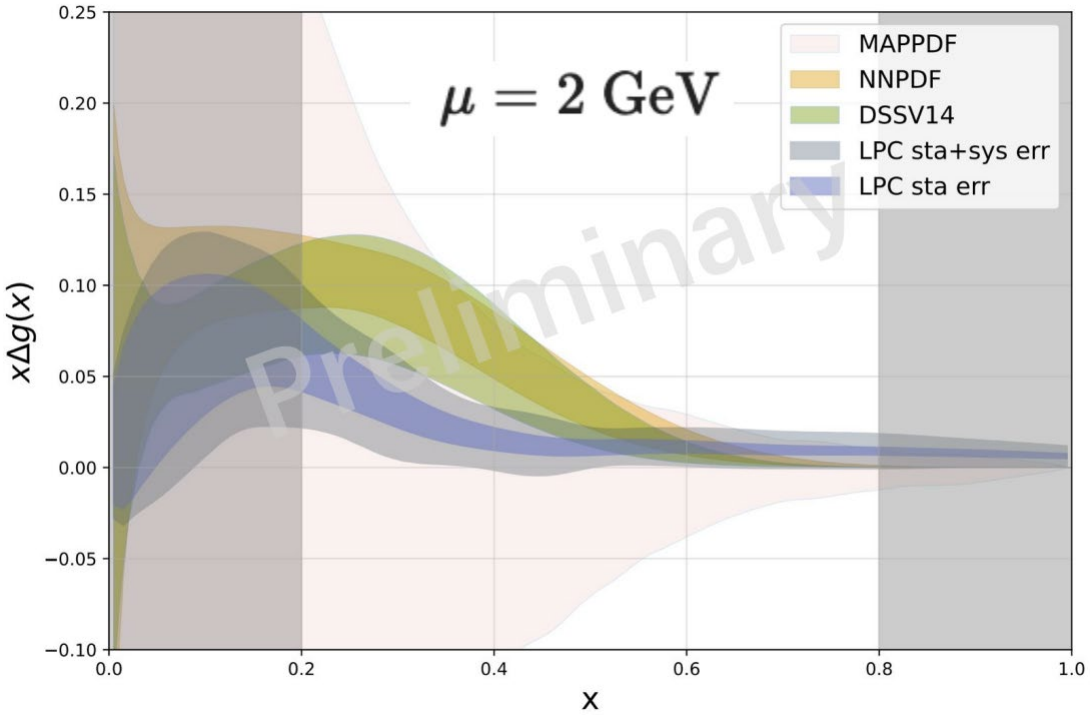
Ji & Su arXiv:2606.22224 (2026)



First-moment-constrained empirical phase rotation.
Excited-state contamination may be the main bottleneck for precision nucleon PDFs.

Gluon helicity: exploring Δg(x) — Talk by 钟诗怡

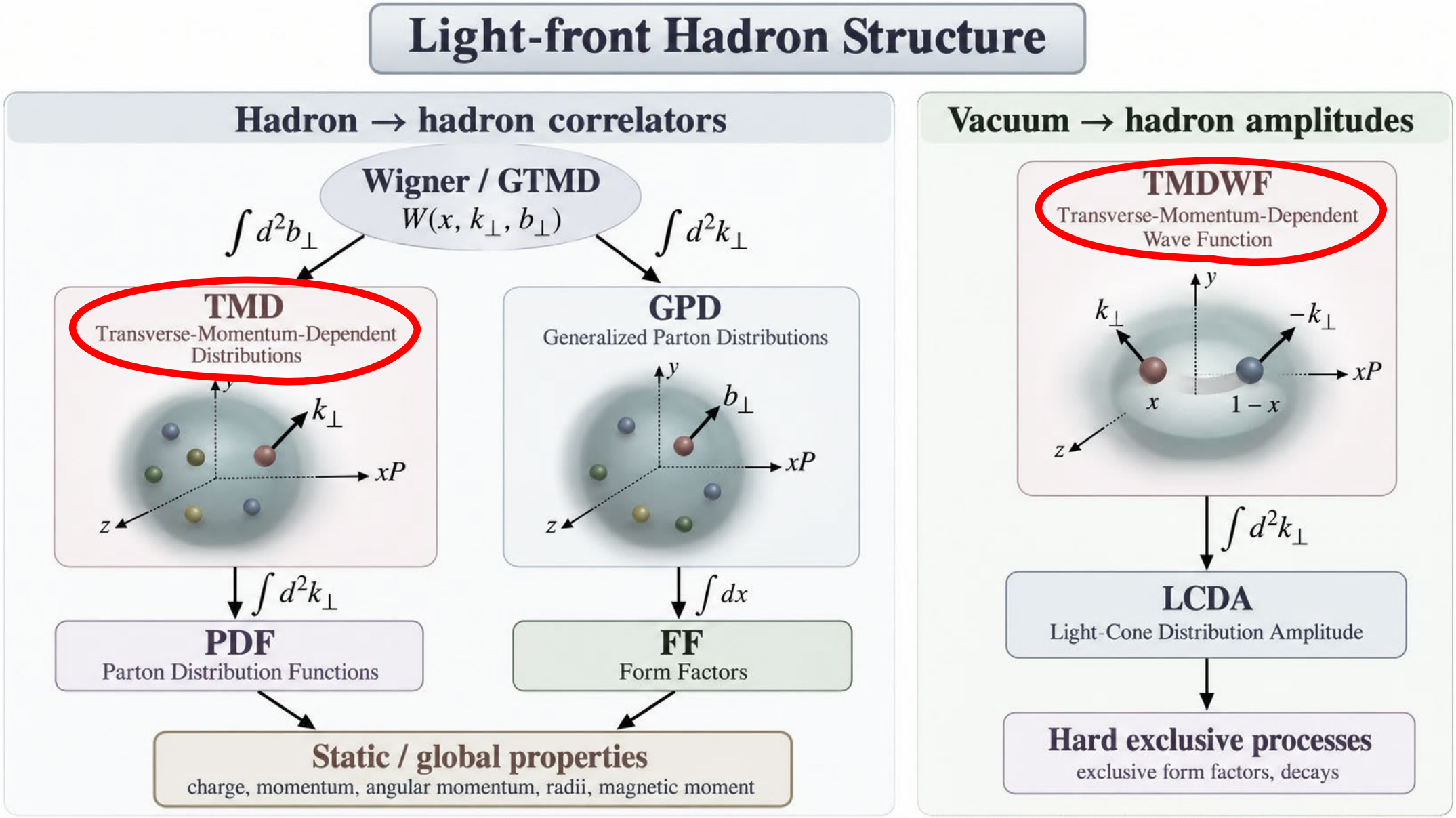
Shi-Yi Zhong et al., LaMET 2026 (preliminary)



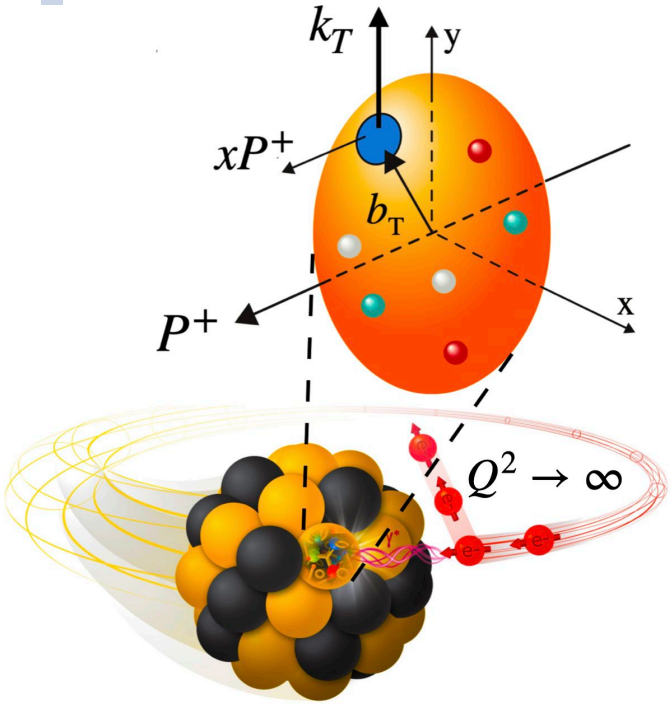
Continuum and large-momentum limits at mπ ≈ 300 MeV.
Exploratory result with sizable uncertainties;
physical-mass dependence remains to be studied.

Earlier LPC: continuum/physical-mass transversity PDF — Yao et al., PRL 131, 261901 (2023).
Accurate lower moments of PDF — PRD 109, 074507 (2024).

TMDs: 3D Momentum Tomography of Nucleon



TMDs: 3D Momentum Tomography of Nucleon



TMDs: Spin-dependent 3D momentum space images from semi-inclusive scattering

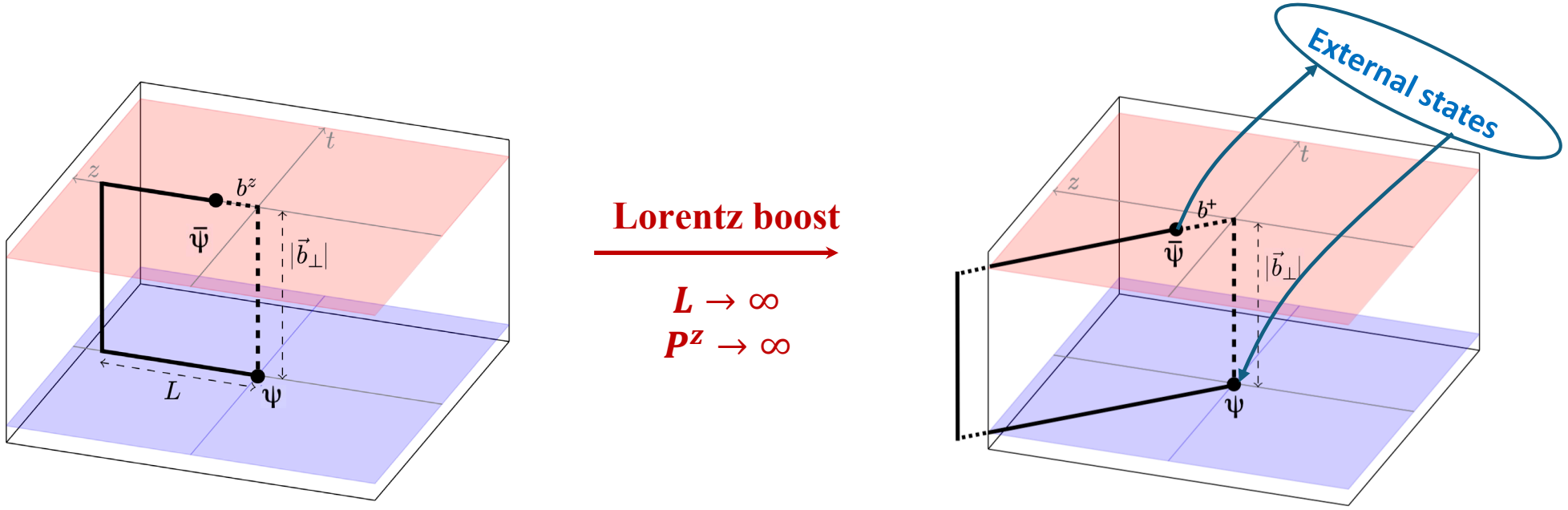
Leading Quark TMDPDFs

Nucleon Spin Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity; $h_{1T}^\perp$ = Pretzelosity

TMDs from Lattice QCD within LaMET

Quasi-TMDs: Equal-time correlators, Lattice calculable



Factorization in LaMET:

PLB811, 135946 (2020); JHEP04, 178, (2022)

Perturbative matching kernel

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma(\zeta^z, \mu)\exp\left[\frac{1}{2}K(b_\perp, \mu)\ln\frac{\zeta^z}{\zeta}\right]f(x, b_\perp, \mu, \zeta)+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta^z}\right)$$

Quasi TMDs | Intrinsic Soft Function | Collins-Soper kernel | Light-cone TMDs

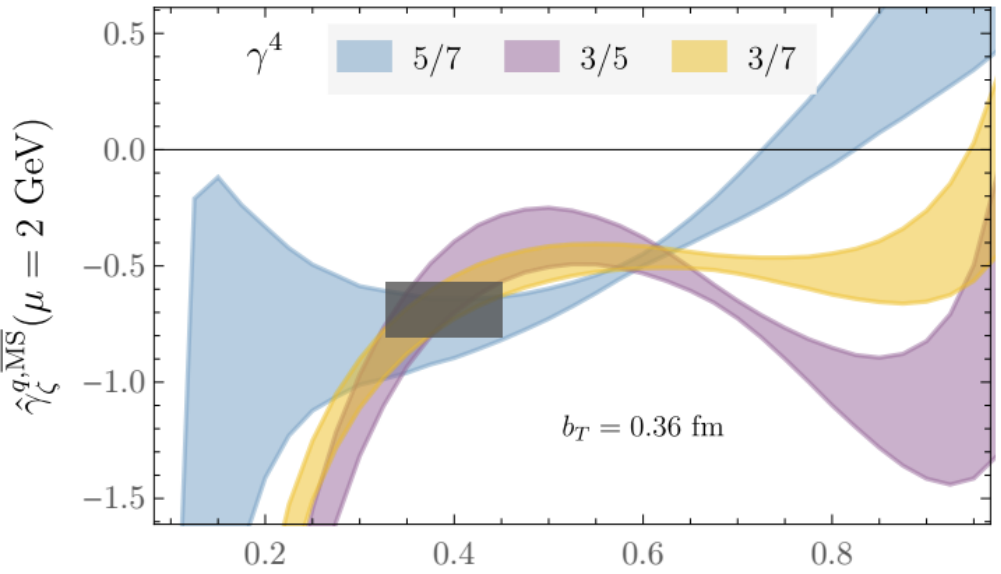
By quasi-TMDWFs

CS Kernel from Quasi-TMDWF VS. Quasi-TMDPDF

From collinear LCDAs to transverse amplitudes: TMD wave functions.

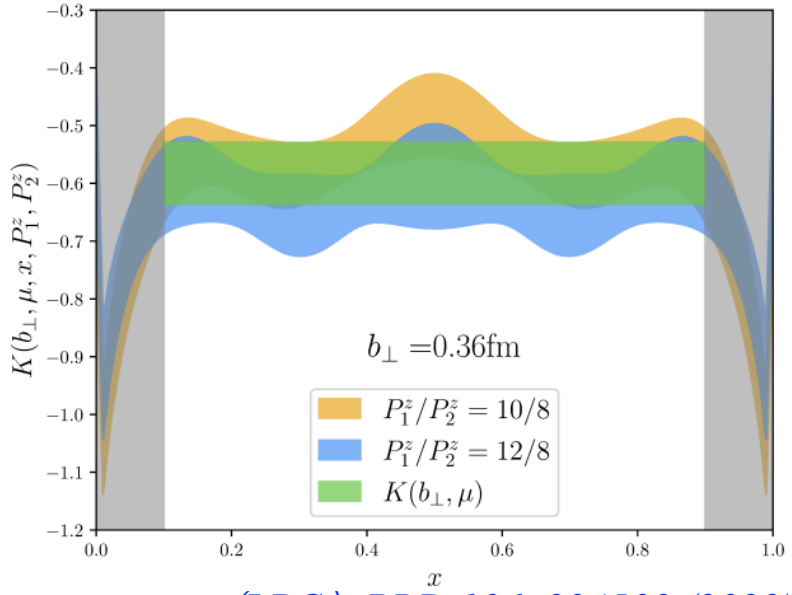
Quasi-TMDWFs provide the soft function and CS kernel needed to extract TMDs.

Quasi-TMDPDF



(BNL/ANL) PRD 104, 114502 (2021)

Quasi-TMDWF



(LPC) PRD 106, 034509 (2022)

2021-2022: quasi-TMDWFs yield a much more stable x-independent CS-kernel plateau.
This advantage motivated later TMDWF-based extractions

BNL/ANL: Gao et al., PLB 852, 138617 (2024), arXiv:2403.00664

Progress in Lattice TMDs (2020–2026)

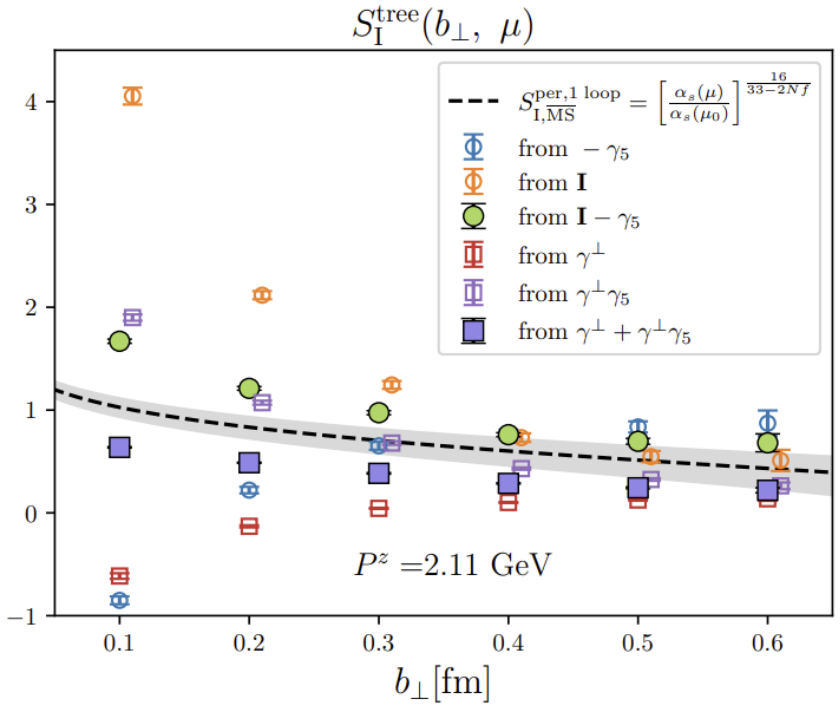
Physical development	LPC collaboration papers	Related work
Soft function and CS-kernel 2020–2026	*Zhang et al., PRL 125, 192001 (2020)* *Chu et al., PRD 106, 034509 (2022)* *Chu et al., JHEP 08 (2023) 172* *Tan et al., PRD 113, 054505 (2026)* Continuum and physical-mass limits	*Shu et al., PRD 108, 074519 (2023)* *Avkhadiev et al., PRL 132, 231901 (2024)* *Bollweg et al., PLB 852, 138617 (2024)* *Francis et al., arXiv:2606.19221 (2026)* .....
Hadron structure in (x, b⊥) 2023–2024	*Chu et al., PRD 109, L091503 (2024)* Pion TMD wave function *He et al., PRD 109, 114513 (2024)* Nucleon unpolarized TMDPDF	*Deng et al., JHEP 09 (2022) 046* One-loop TMDWF / soft matching *del Río & Vladimirov, PRD 108, 114009* (2023): NNLO quasi-TMDPDF matching
Spin–momentum correlations 2025	*Walter et al., PRD 111, 094507 (2025)* Pion Boer–Mulders function *Ma et al., JHEP 08 (2025) 086* Nucleon Boer–Mulders function	*Liu & Su, JHEP 02 (2024) 204* Related Sudakov / renormalon theory NNLO and RG improvements support the published Boer–Mulders analyses
Precision TMDPDFs 2026	LPC 2026, preliminary / in preparation Unpolarized and helicity TMDPDFs Continuum and physical-mass limits	Global-fit benchmarks *MAP24, JHEP 08 (2024) 232* *MAP24pol, PRL 134, 121901 (2025)*

Operator foundation: K. Zhang et al. (LPC), PRL 129, 082002 (2022), staple-operator renormalization

CS Kernel and Intrinsic Soft Function

Current choice controls higher-twist contamination

Chu et al.(LPC), JHEP 08 172 (2023)



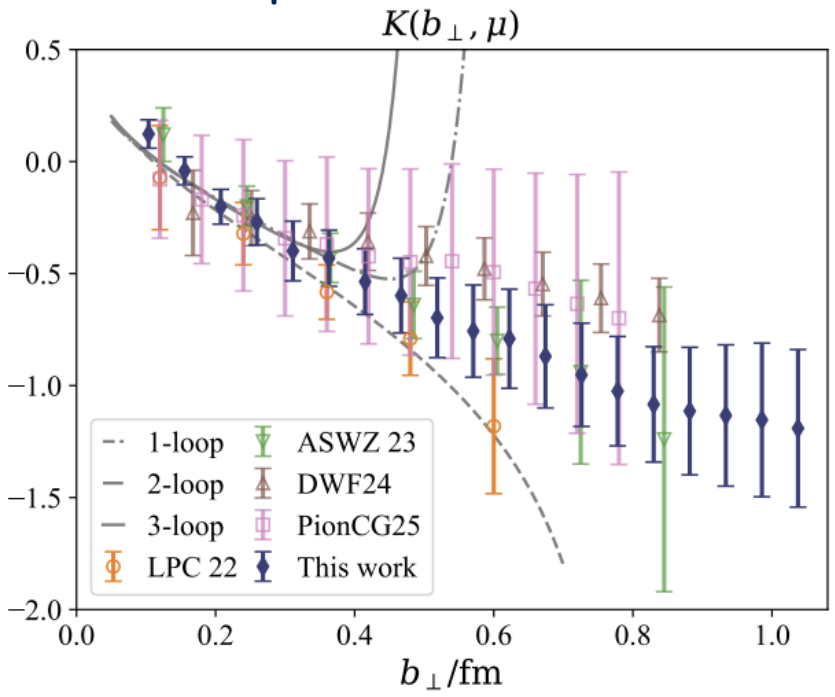
Fierz-selected current combinations suppress higher-twist effects in the soft-function extraction.

CS kernel: rapidity evolution

Tan et al. (LPC), PRD 113, 054505 (2026)

Signal-enhanced quasi-TMDWF operator

Our most precise CS kernel to date

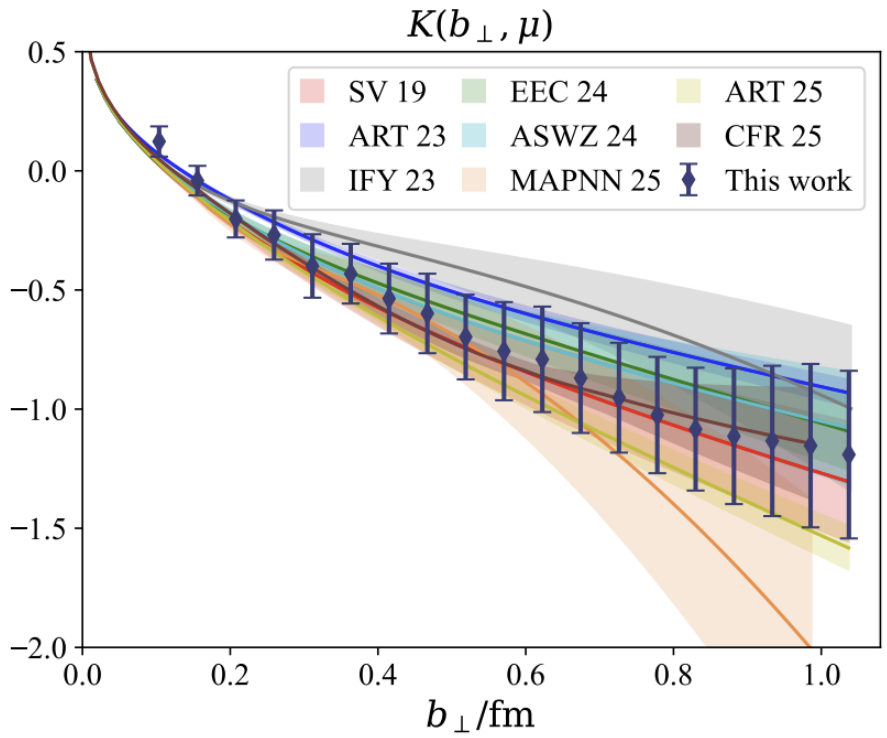


Nonperturbative evolution up to b⊥ ≈ 1 fm

CS Kernel and Intrinsic Soft Function

CS kernel: rapidity evolution

Tan et al. (LPC), PRD 113, 054505 (2026)



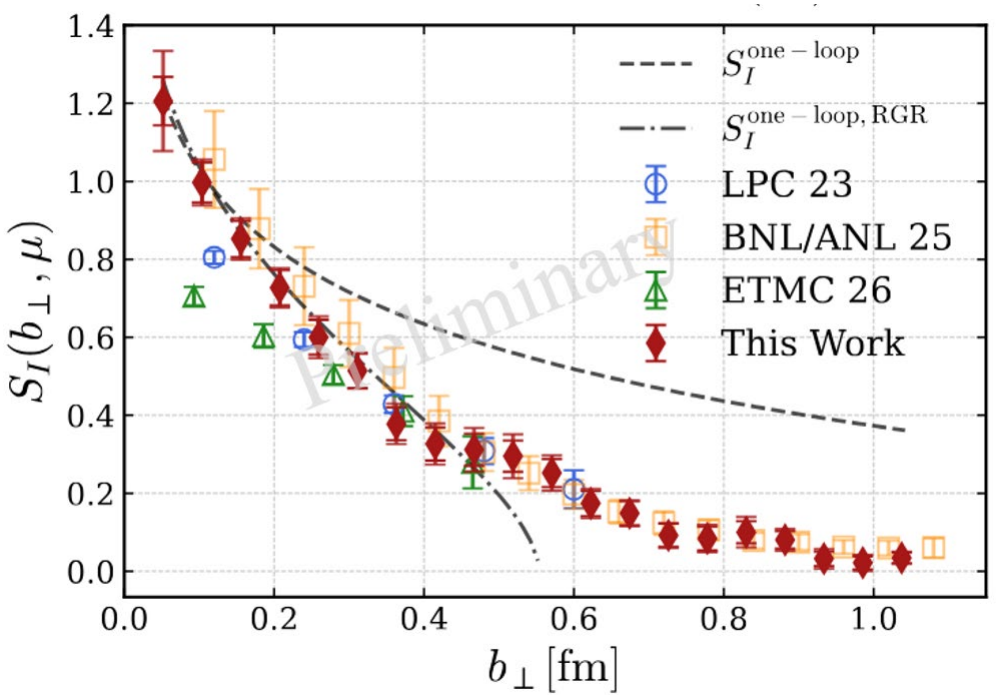
Nonperturbative evolution up to b⊥ ≈ 1 fm

Intrinsic soft function

— Talk by 龚智超

LPC 2026, preliminary / in preparation

Continuum, physical-mass and infinite-momentum limits



Soft input extends to b⊥ ≈ 1 fm

Published TMD Results

Pion TMD wave function

Chu et al. (LPC), PRD 109, L091503 (2024)

Transverse dependence beyond the collinear LCDA

Nucleon unpolarized TMDPDF

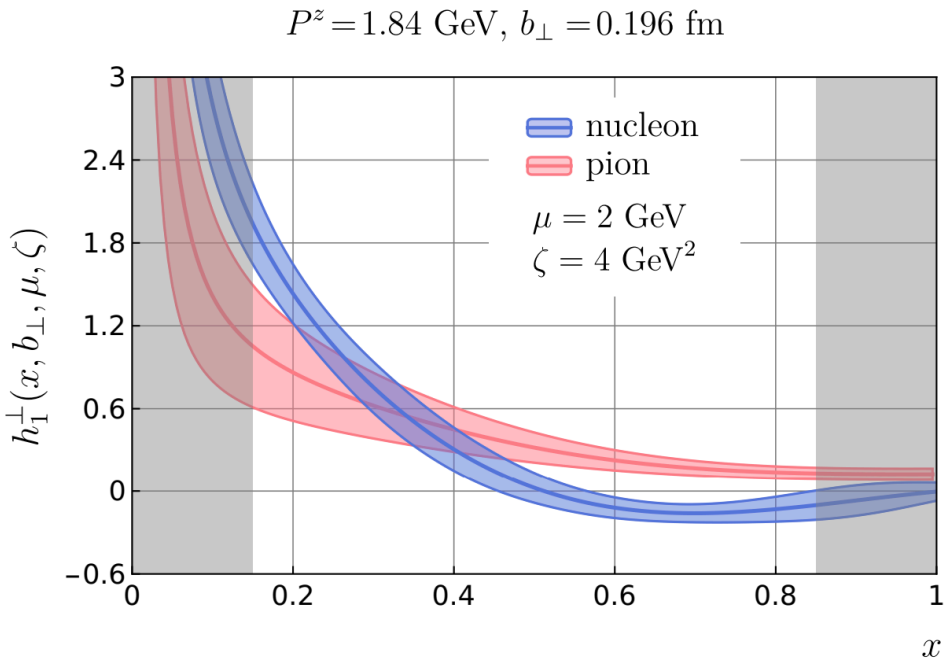
He et al. (LPC), PRD 109, 114513 (2024)

Exploratory isovector $f_1(x, b_\perp)$ determination

Pion and nucleon Quark Spin-transverse-momentum Correlation (Boer-Mulders) functions

Walter et al (LPC)., PRD 111, 094507 (2025)
Ma et al (LPC)., JHEP 08 (2025) 086

T-odd quark transverse spin-momentum correlation



Published Boer-Mulders comparison

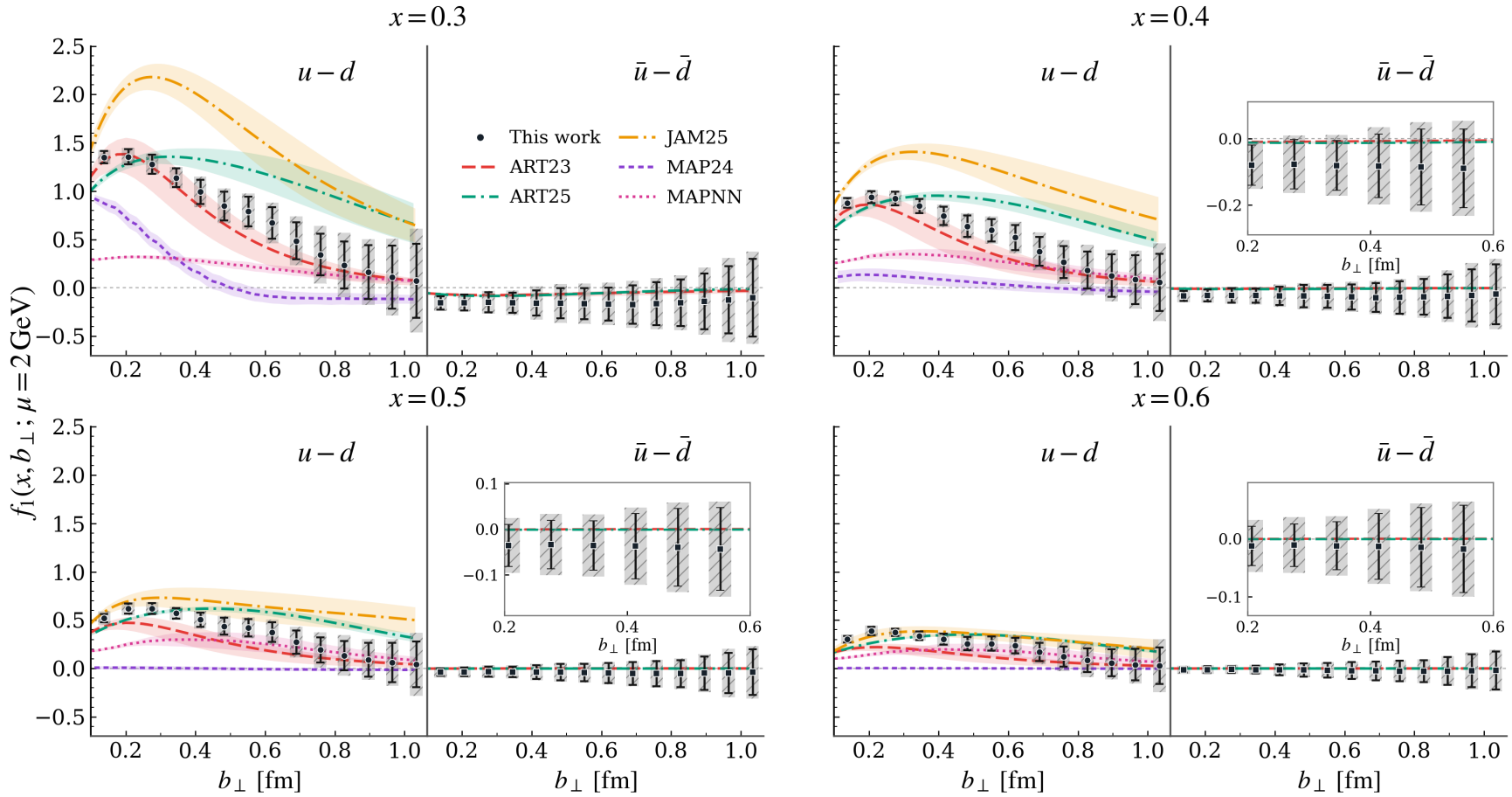
A finite-boost slice, not a common physical-limit comparison

Published studies established access to TMD shapes. The 2026 calculations address precision.

Precision Unpolarized TMDPDFs

LPC 2026, preliminary / in preparation — Talk by 谭金鑫

Continuum and physical-mass limits, with b⊥ reaching about 1 fm



Moderate-x lattice results constrain transverse shapes and distinguish global-fit predictions

Summary and Outlook

Renormalization: a foundation for precision

Self/hybrid renormalization and LRR/RGR improve systematic control;
gradient flow opens a new route for nonlocal operators.

LCDAs: resolving longitudinal parton sharing

Light mesons, heavy mesons and baryons connect lattice amplitudes to exclusive processes.
The new kaon result provides a precision benchmark against OPE moments.

TMDs: transverse motion, evolution and spin

TMD wave functions connect LCDAs to transverse structure and supply soft/evolution inputs.
Precision CS kernels and nucleon TMDPDFs sharpen comparisons with phenomenology.

Outlook: turn precision hadron structure into quantitative tests of QCD

Validate the latest preliminary results, control remaining systematics,
and propagate lattice uncertainties to exclusive processes and TMD observables.