

PDFs from Large Momentum expansion of current- current correlators

arXiv:2605.03234

第六届中国格点量子色动力学研讨会
香港中文大学（深圳）

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Content



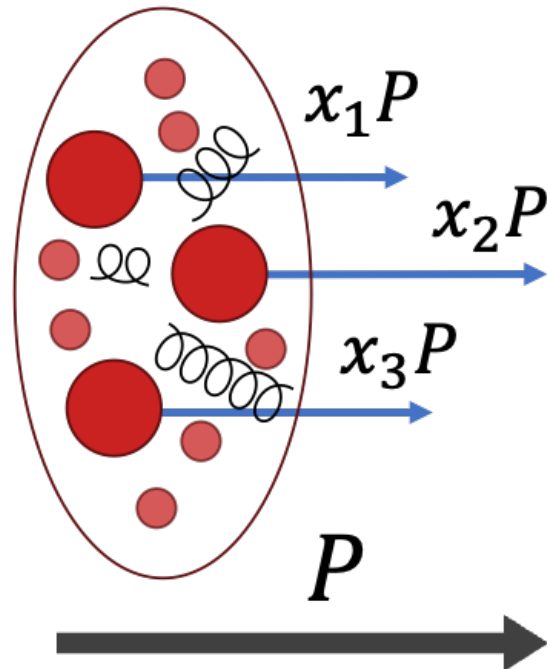
- 1 Background**
- 2 Theoretical foundation
- 3 Numerical Result
- 4 Summary and Outlook

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Hadron structure

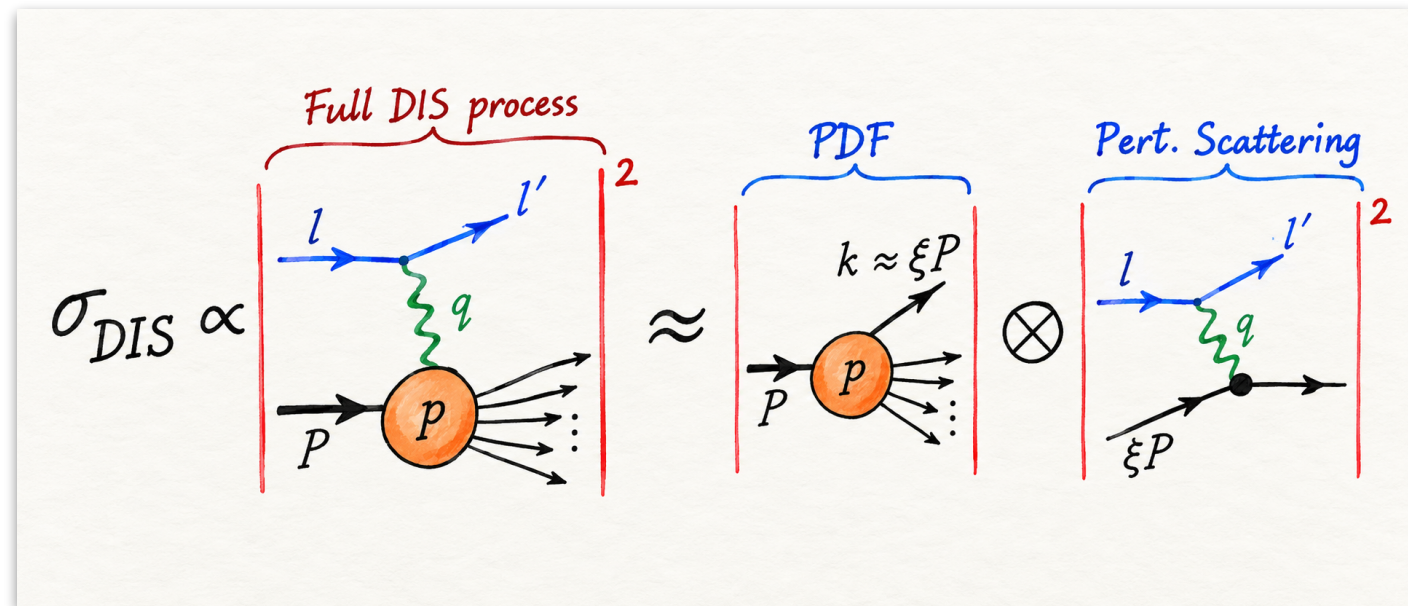
Collider: $\nu \rightarrow e$

Feynman's parton model



A proton moves at the speed of light.

Cross section factorized to PDF (hadron structure) and perturbative scattering



How to calculate PDF?



$$q(x, \mu) = \frac{1}{4\pi} \int d\xi^- e^{-ixP^+\xi^-} \left\langle P \left| \bar{\psi}(\xi^-) \gamma^+ W(\xi^-, 0) \psi(0) \right| P \right\rangle$$

ξ^- : on the lightcone

Mellin moments: $\langle x^{n-1} \rangle_q = \int_0^1 dx x^{n-1} [q(x) + (-1)^n \bar{q}(x)]$.

$\langle x^5 \rangle$ *A. Francis et al., PRL 136, 171903 (2026), ...*

- local operator
- short-distance factorization of Wilson-line operator *X. Gao et al., PRD 109, 054506 (2024), ...*
- Compton amplitude *A. J. Chambers, et al., PRL 118 (2017); M. Gockeler, et al., PRL 92 (2004)...*
- Heavy-quark Operator Product Expansion (HOPE) *W. Detmold, and C. J. David Lin, PRD 73 (2006); W. Detmold, et al., PRD 105 (2022)...*
- ...

Short-distance factorization (SDF):

- Pseudo PDF / Ioffe-time distribution *A. V. Radyushkin, PRD 96 (2017)...*
- Lattice cross sections... *Y.-Q. Ma and J.-W. Qiu, PRL 120, 022003 (2018)*

Large momentum effective theory (LaMET): Quasi-Distribution

Phys. Rev. Lett. 110, 262002 (2013)
Sci. China PMA 57, 1407 (2014)

Large Momentum effective theory (LaMET)



Define a quasi-distribution which is calculable on lattice:

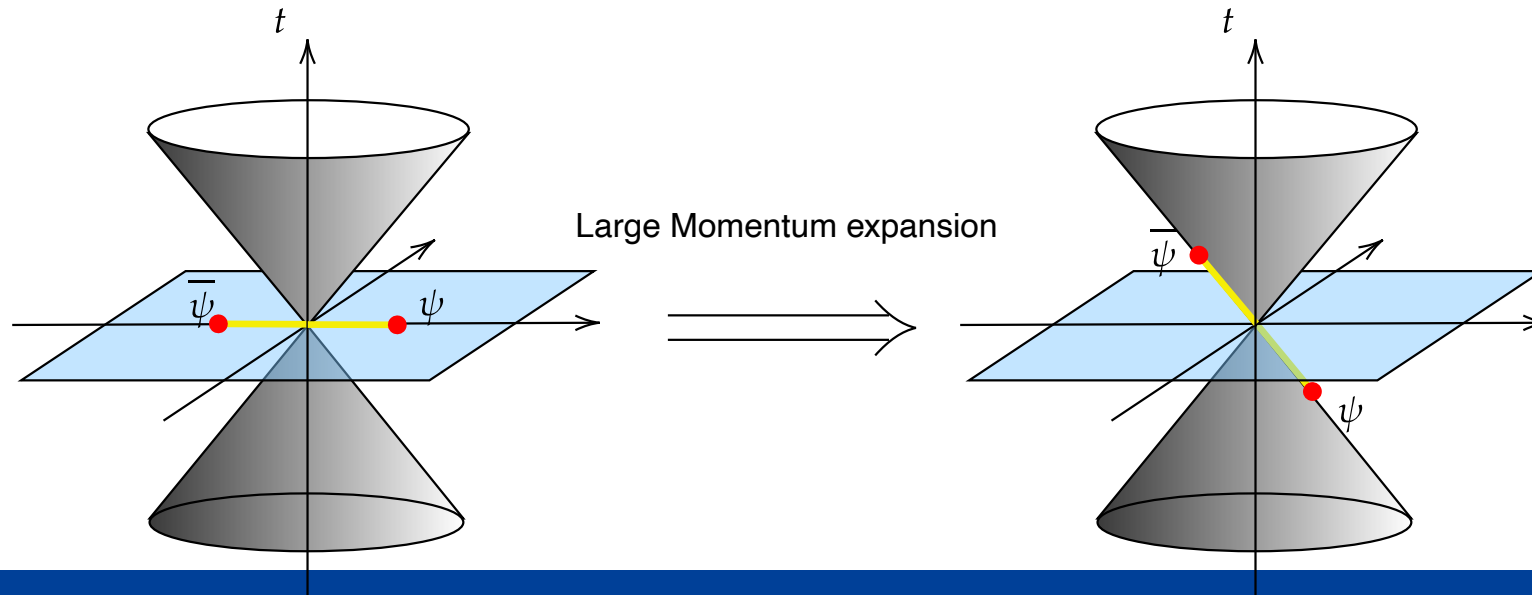
Phys. Rev. Lett. 110, 262002 (2013)

Sci. China PMA 57, 1407 (2014)

$$\tilde{q}(x, P^z, \mu) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dz e^{-ixP^z z} \left\langle P \left| \bar{\psi}(z) \Gamma W(z, 0) \psi(0) \right| P \right\rangle$$

Large momentum expansion:

$$q(x, \mu^2) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{|y|P_z}{\mu}\right) \tilde{q}(y, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^n}{(xP_z)^n}, \frac{\Lambda_{\text{QCD}}^n}{[(1-x)P_z]^n}, \frac{M^n}{P_z^n}\right)$$



LaMET: From Equal-Time Correlation to PDFs

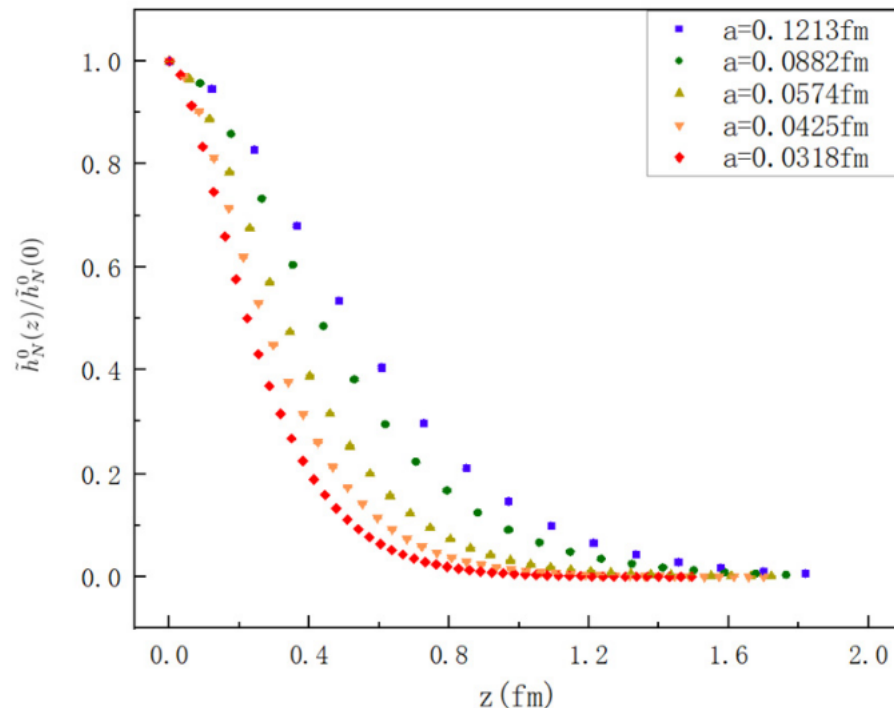


$$\tilde{q}(y, P_z) = \mathcal{F} \left[\langle P | \bar{\psi}(z) \color{red}{W(z,0)} \psi(0) | P \rangle \right]$$



To deal with Wilson line: Self-energy & Renormalon ambiguity

Matrix element change with lattice spacing a



Behaviour: $\exp\left(-\frac{\color{red}{\delta m}z}{a} - \color{red}{m_0}z\right)$

Statistic errors:

- Fit the behavior
- Eliminate the behavior

$$h^{bare}(z)e^{\frac{\delta m z}{a} + m_0 z}$$

Y.-K. Huo et al. (LPC), NPB 969, 115443 (2021)

LaMET: From Equal-Time Correlation to PDFs



$$\tilde{q}(y, P_z) = \mathcal{F} \left[\langle P | \bar{\psi}(z) \color{red}{W(z,0)} \psi(0) | P \rangle \right]$$



To deal with Wilson line: Self-energy & Renormalon ambiguity

- **Regularization-Independent Momentum-Subtraction scheme (RI/MOM)**

C. Alexandrou et al., NPB 923, 394 (2017); J.-W. Chen et al., PRD 97, 014505 (2018)...

- **Self renormalization**

Y.-K. Huo et al. (LPC), NPB 969, 115443 (2021)

- **Leading renormalon resummation (LRR)**

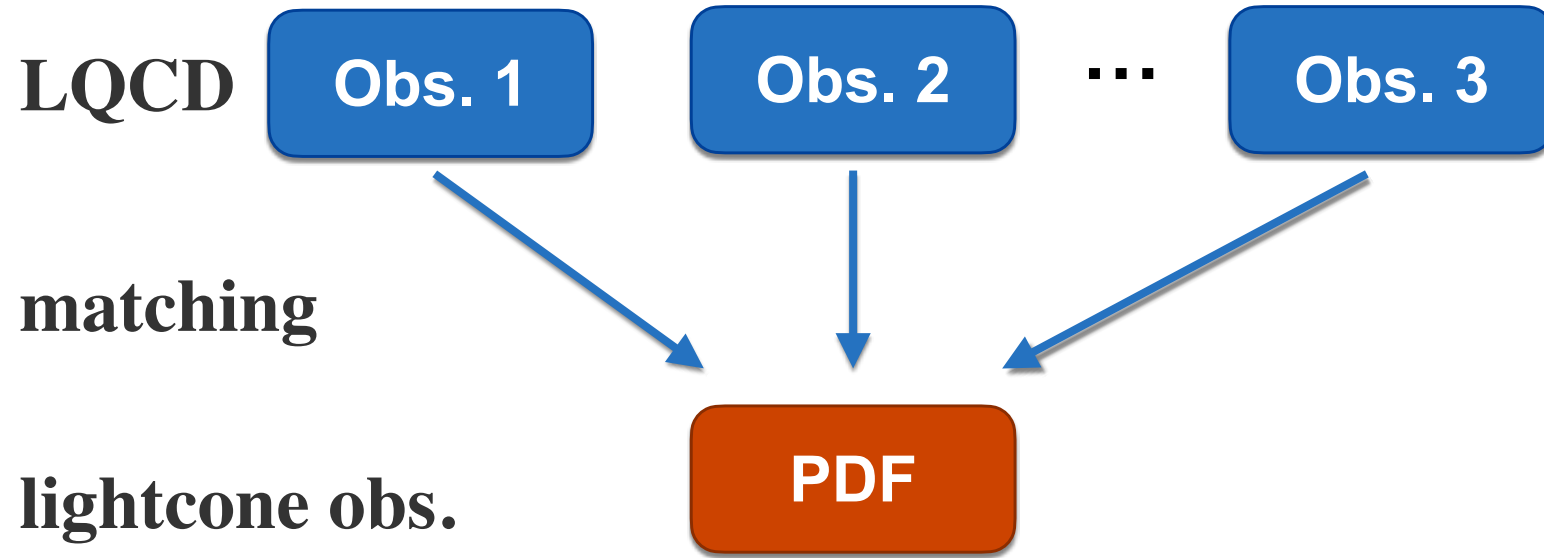
R. Zhang et al., PLB 844, 138081 (2023)

Can we get rid of the Wilson line from the beginning?

LaMET: A Universality Class of Euclidean Observables



LaMET is not limited to this kind of Wilson-line operators



The lattice observables form a universal class which can be matched to one light-cone operator

X. Ji et al., RMP 93, 035005 (2021)

Criteria

- Have the same IR physics
- Differ only by UV physics

Admit a factorization

Observables for Large Momentum Expansion



Quasi-PDF

$$\mathcal{F} \left[\langle P | \tilde{\psi}(z) \gamma^t W(z, 0) \psi(0) | P \rangle \right]$$

- Most studied
- Solid theoretical foundation
- All order factorization
- Complicated renormalization

Coulomb-gauge Quasi-PDF

$$\mathcal{F} \left[\langle P | \tilde{\psi}(z) \gamma^t \psi(0) \Big|_{\vec{\nabla} \cdot \vec{A} = 0} | P \rangle \right]$$

- No Wilson line, easier renormalization
- Better signal
- No all order factorization

Another operator sit in the same universal class

Current-Current Correlator



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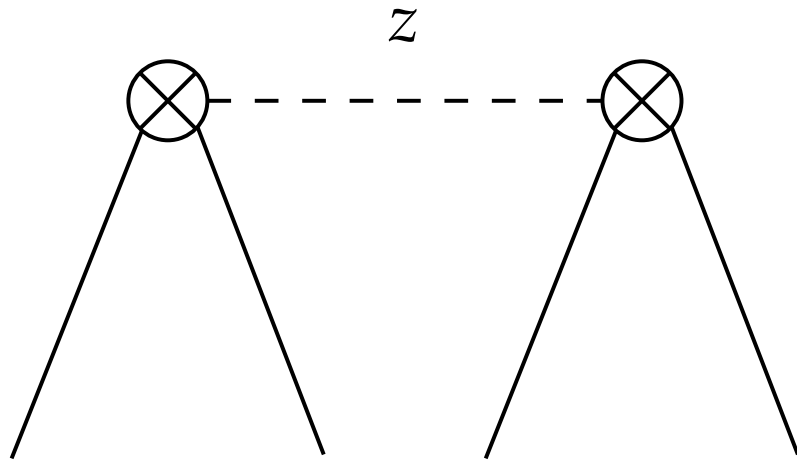
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Current-Current Correlators



Current-Current Correlator

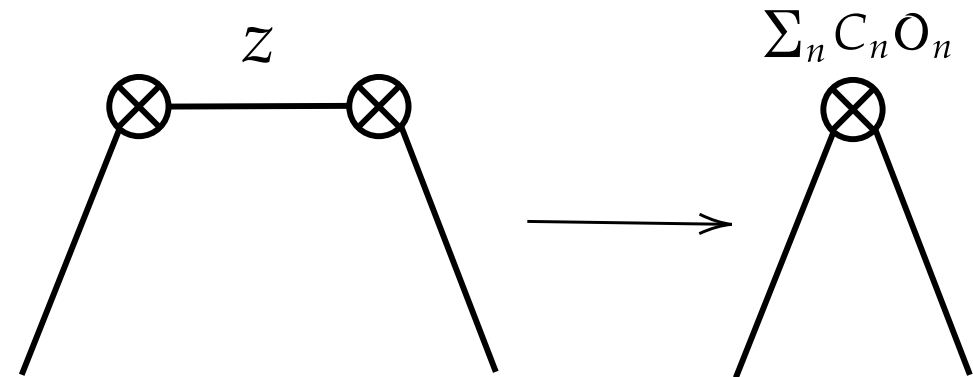
$$\langle P | J^\dagger(z) J(0) | P \rangle$$



Choices of J :

- $J_S := \bar{\psi}_q \psi_q$
- $J_V^\mu := \bar{\psi}_q \gamma^\mu \psi_q$
- $J_A^\mu := \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q$
- ...

The SDF of current-current correlators
has been established



Y.-Q. Ma and J.-W. Qiu, PRL 120, 022003 (2018)

R. S. Sufian et al., PRD 102, 054508 (2020)

...

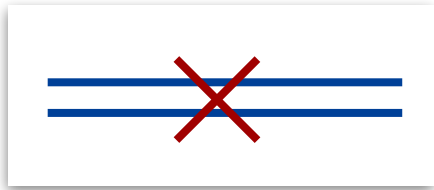
Why use current-current correlation



Advantages

No Wilson line

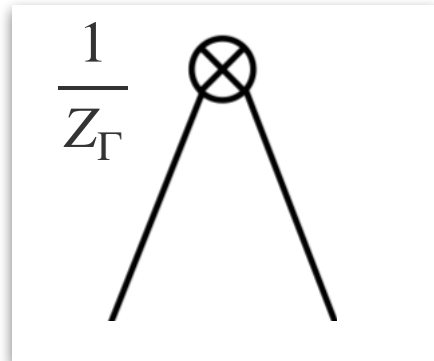
No Wilson-line linear divergence and associated renormalon ambiguity



$$\mathcal{P}\left[\exp\left(\int dt n \cdot A(nt)\right)\right]$$

Easier renormalization

Only need to do local composite operator renormalization

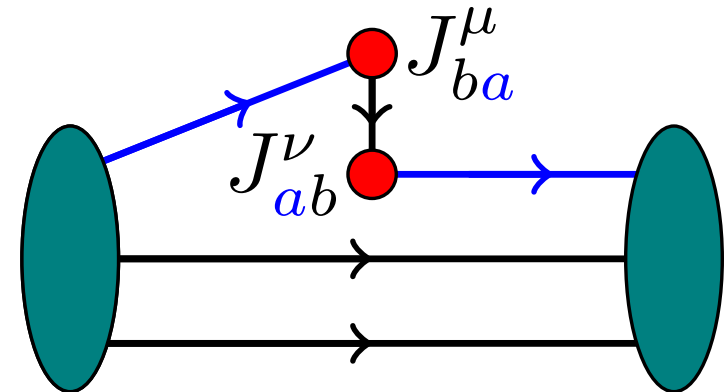


$$(\bar{\psi}\Gamma\psi)_R = \frac{1}{Z_\Gamma}(\bar{\psi}\Gamma\psi)_B$$

Drawback

It is a four point function

More expensive to get a competitive result



Current-current correlator



Current-current correlator used in this work

We consider the vector-vector (VV) and axial-axial (AA) correlators

$$M_{XY,ab}^{\mu\nu}(z, P) = \langle P | J_{X,ab}^{\mu}(z) J_{Y,ba}^{\nu}(0) | P \rangle, \quad X, Y = V, A$$

- **a: Flavour of PDF**

- **b: Auxiliary field**

$$J_{V,ab}^{\mu}(z) = \bar{q}_a(z) \gamma^{\mu} q_b(z), \quad J_{A,ab}^{\mu}(z) = \bar{q}_a(z) \gamma^{\mu} \gamma_5 q_b(z)$$

Leading contribution

For the unpolarized PDFs, the leading contribution is carried by transverse polarization

$$M_{AA}^{\perp\perp} \text{ \& } M_{VV}^{\perp\perp}$$

Momentum-space distribution

LaMET is for momentum-space distributions

$$W^{\perp\perp}(z, P) := -\frac{i\pi^2 z^4}{\lambda} M^{\perp\perp}(z, P)$$
$$\tilde{h}(y, P_z) = \int_{-\infty}^{\infty} d\lambda M^{\perp\perp}(z, P)$$

Large momentum expansion of VV and AA currents

Large momentum expansion

$$\underbrace{f(x, \mu^2)}_{\text{light-cone PDF}} = \int_{-\infty}^{\infty} \frac{dy}{|y|} \underbrace{C\left(\frac{x}{y}, \frac{|y|P_z}{\mu}\right)}_{\text{matching kernel}} \tilde{h}(y, P_z) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^n}{(xP_z)^n}, \frac{\Lambda_{\text{QCD}}^n}{[(1-x)P_z]^n}, \frac{M^n}{P_z^n}\right), \quad n \geq 2$$

light-cone PDF

matching kernel

- Iso-vector (u-d)

To avoid operator mixing

- No Wilson-line

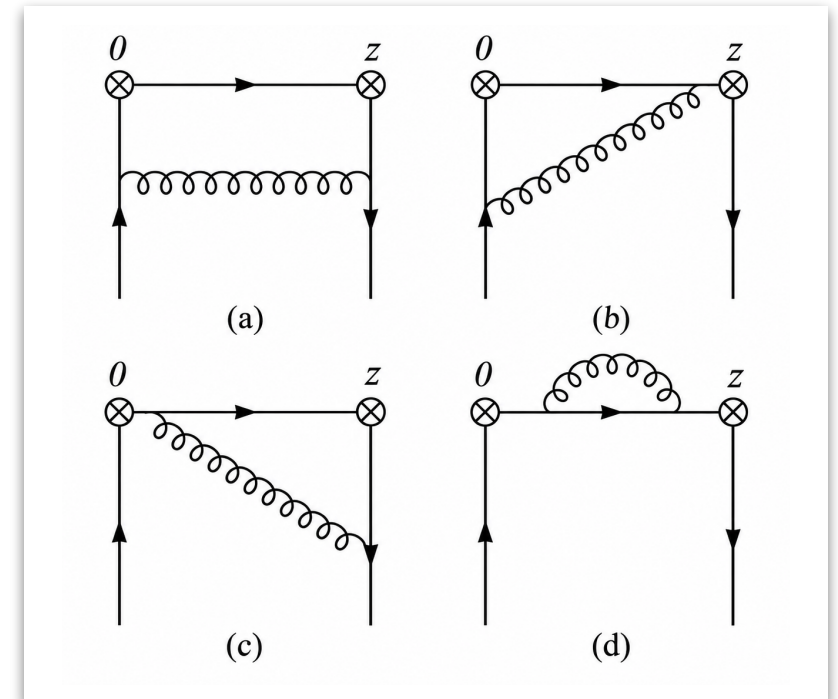
Power correction starts at n=2

Matching kernel at one-loop

At one-loop (**identical for VV and AA**)

$$C_{\text{ns}}^{\overline{\text{MS}}}\left(\xi, \frac{|y|P_z}{\mu}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{4\pi} \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi} c^{(1)}\left(\xi, \frac{|y|P_z}{\mu}\right), \quad \xi = \frac{x}{y}$$

$$c^{(1)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{(\xi^2+1) \log(\frac{\xi-1}{\xi}) + \xi + 1/2}{1-\xi} \right]_{+(1)}^{[1, \infty)} & \text{if } \xi > 1 \\ \left[\frac{2(\xi^2+3) \log(1-\xi) + 2(\xi^2+1) \log(\xi) + 2(\xi-4)\xi + 7}{2(\xi-1)} \right]_{+(1)}^{[0, 1]} + \left[\frac{1+\xi^2}{1-\xi} \right]_{+(1)}^{[0, 1]} \log\left(\frac{\mu^2}{4y^2 P_z^2}\right) & \text{if } 0 < \xi < 1 \\ \left[\frac{(\xi^2+1) \log(\frac{\xi-1}{\xi}) + \xi + 1/2}{(\xi-1)} \right]_{+(1)}^{(-\infty, 0]} & \text{if } \xi < 0 \end{cases}$$



Large momentum expansion of VA and AV currents

Alternative choice

$$W_{\text{VA}}^{\perp\perp} := \frac{\pi^2 z^3}{2E_P} \left[M_{\text{VA}}^{xy}(z, P) + M_{\text{AV}}^{xy}(z, P) \right]$$

OPE of VA correlation has been studied

R. S. Sufian et al., Phys. Rev. D 99, 074507 (2019)

R. S. Sufian et al., Phys. Rev. D 102, 054508 (2020)

**Alternative for
future study**

Matching kernel at one-loop

$$C_{\text{VA,ns}}^{\overline{\text{MS}}}(\xi) = C_{\text{VV,ns}}^{\overline{\text{MS}}}(\xi) + \frac{\alpha_s C_F}{\pi} [1 - \xi]_{+(1)}^{[0,1]} \theta(\xi) \theta(1 - \xi) .$$



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Lattice set-up

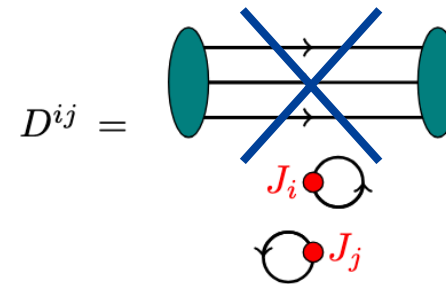
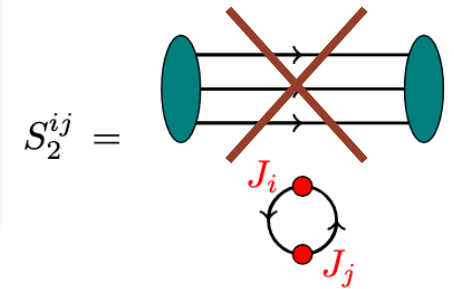
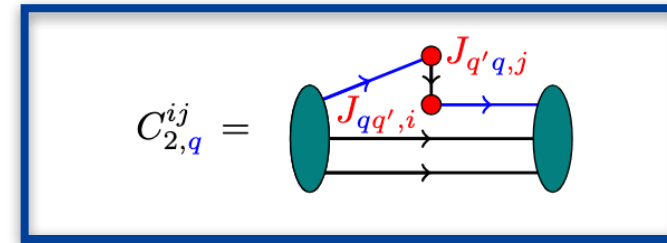
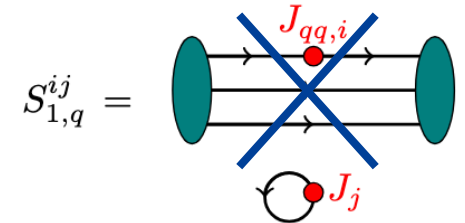
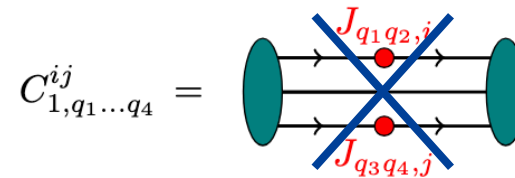


CLS H102 Ensemble

Nf=2+1 Wilson-clover

- Lattice size: $32^3 \times 96$
- Lattice spacing $a = 0.085 \text{ fm}$
- Pion mass $m_\pi = 355 \text{ MeV}$
- Proton momentum $\vec{P} = (\pm n, \pm n, \pm n)$
 $|\vec{P}|_{\max} = 1.57 \text{ GeV}$

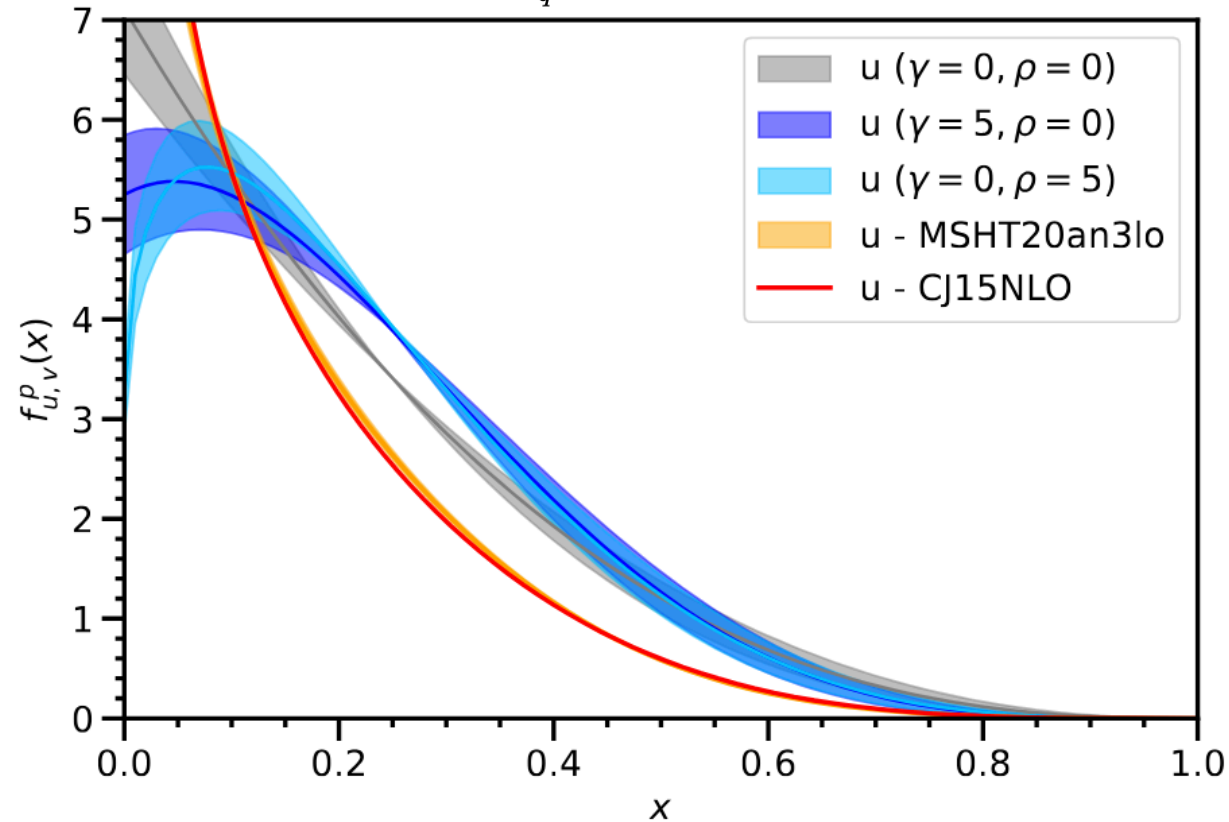
- **Flavour choice** $\bar{\psi}_a \gamma^\mu \psi_b$, $\bar{\psi}_b \gamma^\mu \psi_a$
- **Iso-vector** (u-d)



Same set of data, Different kinematic region

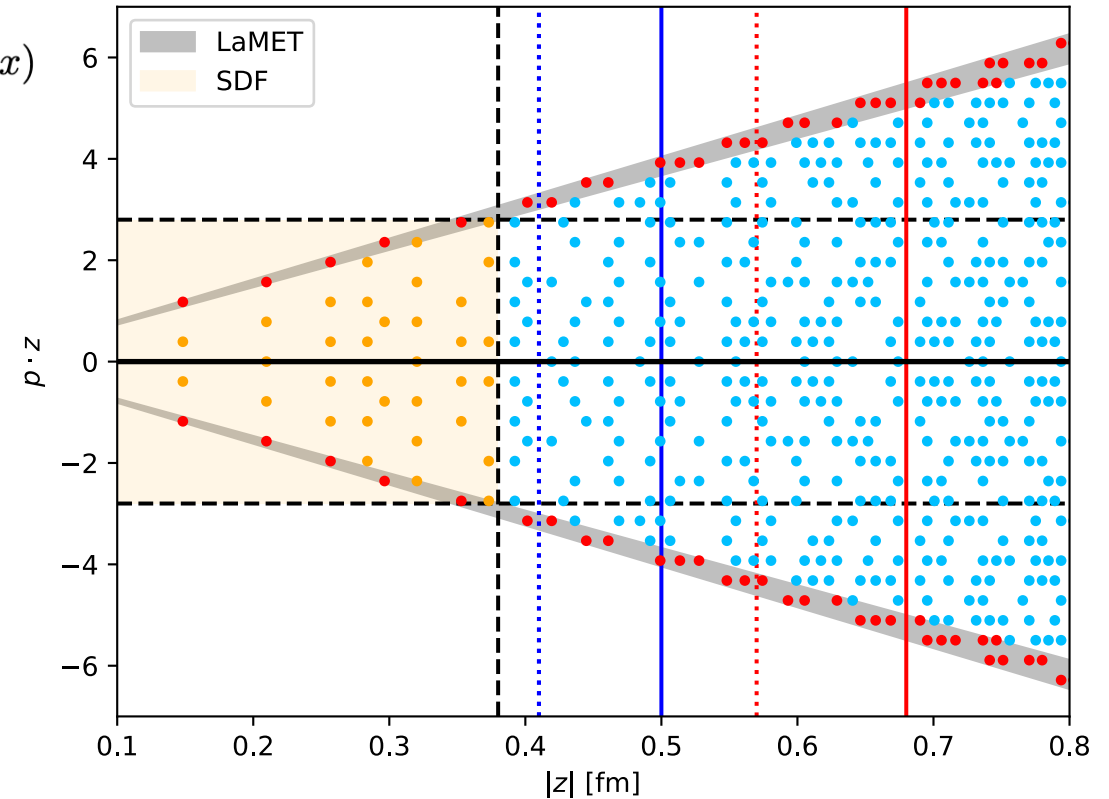


Choice of Model $\frac{f_q^p(x)}{N_q} = N(\alpha, \beta, \rho, \gamma) x^\alpha (1-x)^\beta (1 + \rho\sqrt{x} + \gamma x)$



Data: *G. S. Bali et al., JHEP 09 (2021) 106*

PDF: *C. Zimmermann and A. Schäfer, PRD 110, 074503 (2024)*



- LaMET uses a kinematic region different from short-distance factorization $\vec{z} \parallel \vec{P}$
- To get more data points:
 $|\vec{P}|(1 - 0.02) \leq \lambda/|\vec{z}| \leq |\vec{P}|(1 + 0.02)$

Lattice set-up



Quantity calculated

$$M^{\mu\nu}(z, P_z) = \frac{1}{2} (M_{VV}^{\mu\nu}(z, P_z) + M_{AA}^{\mu\nu}(z, P_z))$$

The combination is chosen to suppress chiral-odd Wilson-fermion artifacts.

G.S. Bali, et. Al., PRD 98, 094507 (2018)

Wilson-fermion propagator

$$S_W(p) = \frac{-i\gamma_\mu \bar{p}_\mu + \textcolor{red}{M}_W(p)}{\bar{p}^2 + M_W^2(p)}$$

$$\bar{p}_\mu = \frac{1}{a} \sin(ap_\mu), \quad M_W(p) = m_0 + \frac{r}{a} \sum_\mu [1 - \cos(ap_\mu)].$$

Isolating the transverse direction

Decomposition:

$$M^{\mu\nu}(z, P) = \frac{P^\mu P^\nu}{m^2} A(\lambda, z^2) - \textcolor{red}{g^{\mu\nu} B(\lambda, z^2)} + m^2 z^\mu z^\nu C(\lambda, z^2) + (P^\mu z^\nu + P^\nu z^\mu) D(\lambda, z^2),$$

$$\textcolor{brown}{M^{\perp\perp}(z, P_z) = B(\lambda^2, z) = P_{\mu\nu} M^{\mu\nu}(z, P)}$$

Projector:

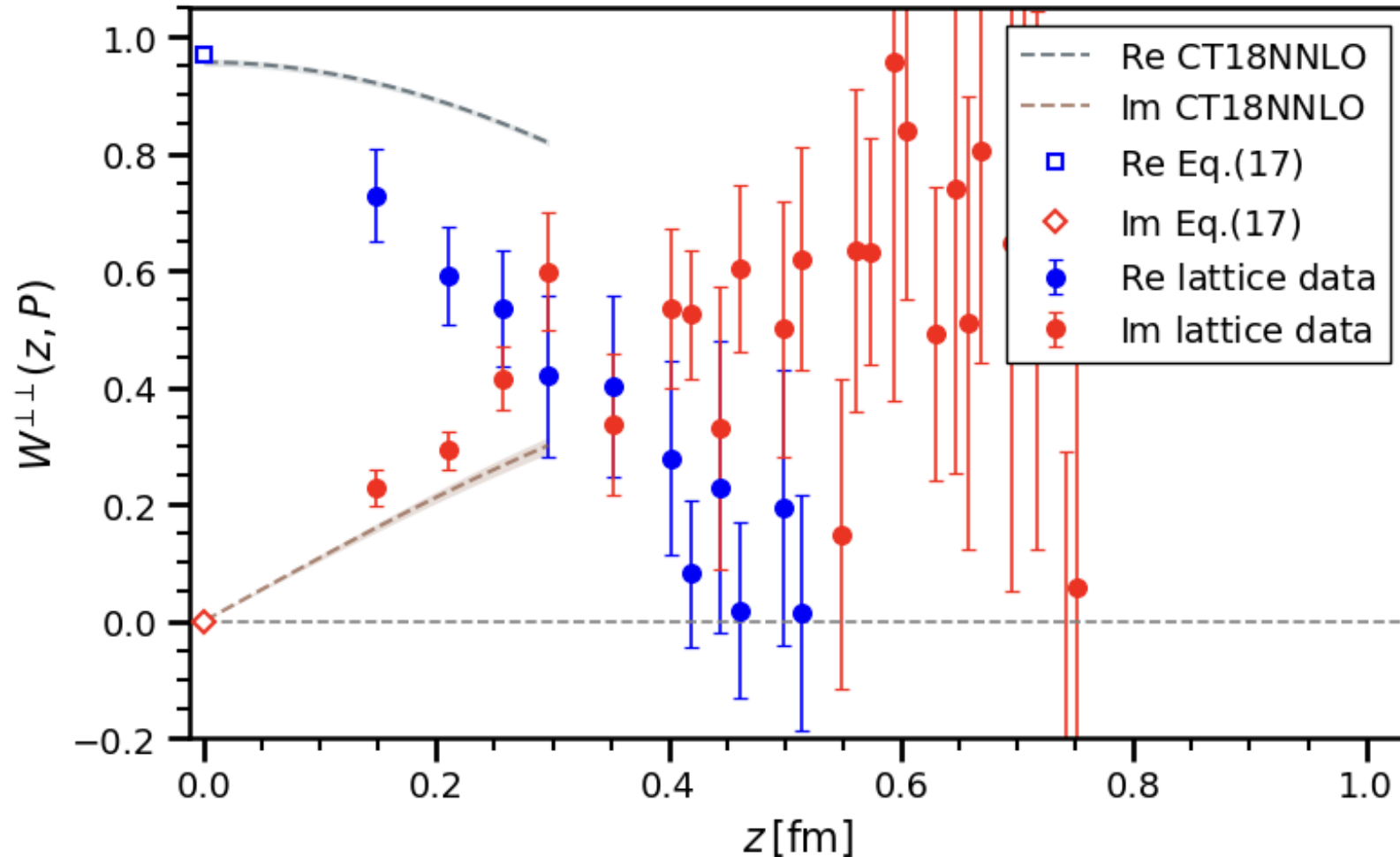
$$P_{00} = 0, \quad P_{0j} = P_{j0} = 0$$

$$P_{jk} = \frac{1}{2m^2} \left(\frac{p^j p^k}{\vec{p}^2} - \delta_{jk} \right)$$

Numerical results



Gray bands are obtained from forward LaMET matching followed by a Fourier transform



The moments are larger than CT18NNLO

Possible sources

- Pion mass
- Excited states
- Lattice spacing
- Momentum extrapolation



Large z Extrapolation

*X. Ji, Y. Liu and Y. Su,
arXiv:2601.12189 (2026).*

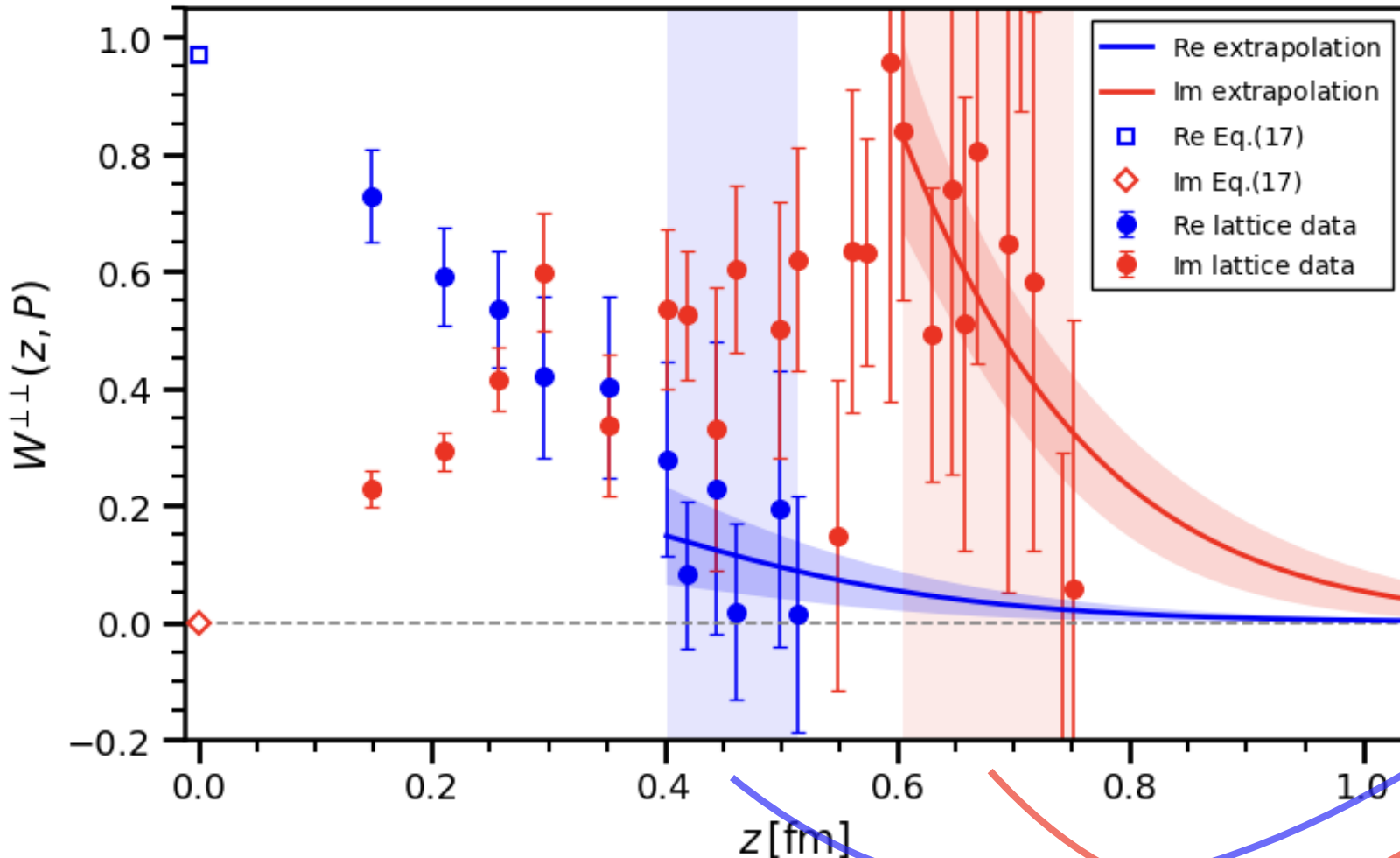
$$\sum_X \langle P | J^\mu(z) | X \rangle \langle X | J^\nu(0) | P \rangle$$

$$W^{\perp\perp}(z, P) = A |z|^{5/2} e^{i\phi \text{sign}(z)} e^{-m|z|}$$

Fit range

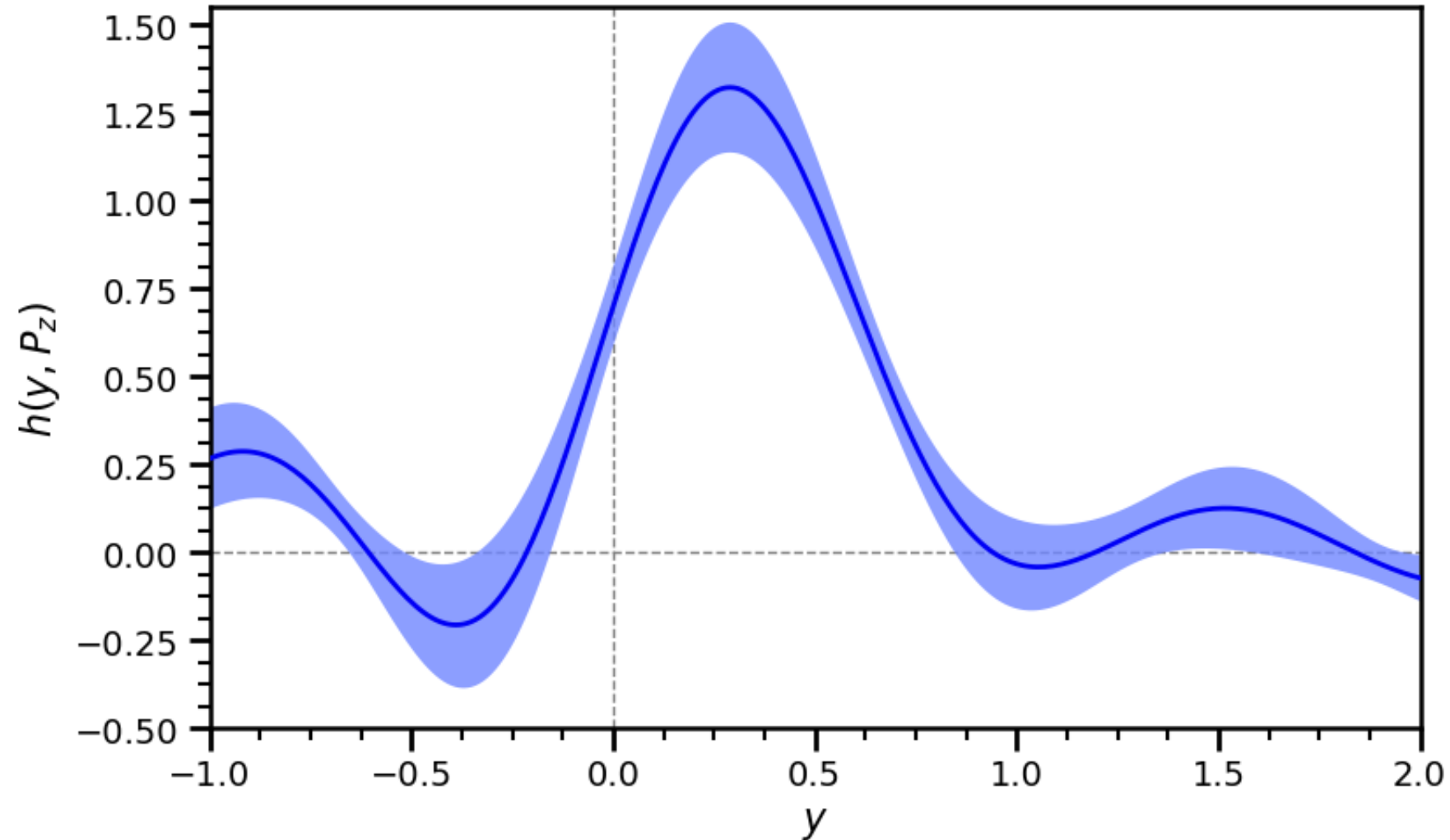
Re: 0.4-0.51 fm

Im: 0.61-0.75 fm





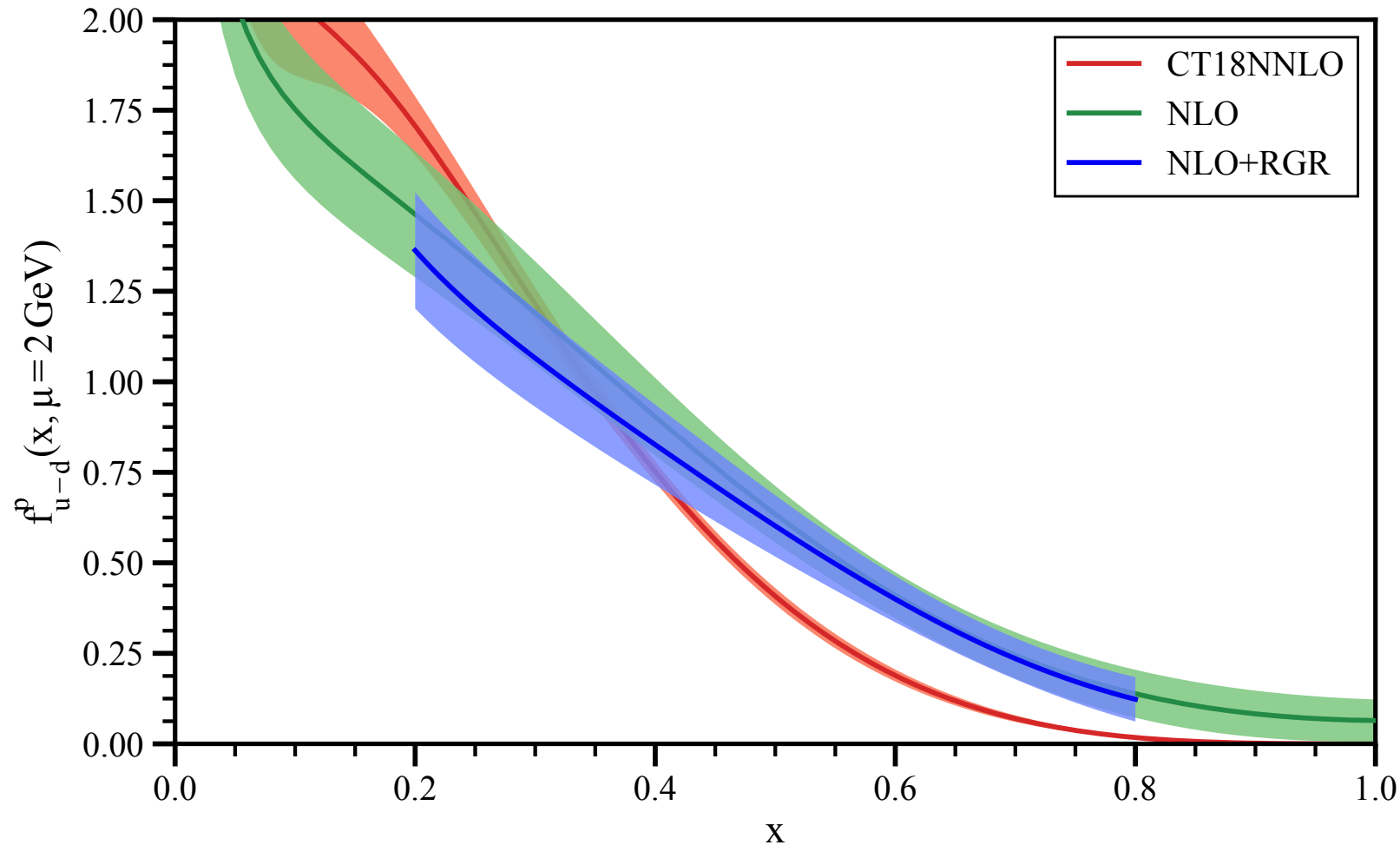
The quasi-distribution $h(y, P_z)$



Numerical results



$$f(x, \mu^2) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{|y|P_z}{\mu}\right) \tilde{h}(y, P_z) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^n}{(xP_z)^n}, \frac{\Lambda_{\text{QCD}}^n}{[(1-x)P_z]^n}, \frac{M^n}{P_z^n}\right), \quad n \geq 2$$



Red: CT18NNLO

Green: NLO

Blue: NLO+RGR

*Y. Su et al., Nucl. Phys. B
991, 116201 (2023)*



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Summary and Outlook



This work

- Established the factorization of VV (AA) correlator in LaMET framework
- Give the 1-loop kernel of Large momentum expansion for the first time
- Give the preliminary result for PDF from VV (AA) correlator

Outlook

- Improve precision: 4-point correlation on lattice
- Extend to other current-current correlator: VA, ...



Thank you for your listening!