Determination of Standard Model Parameters with CLQCD ensembles

CLQCD 2026

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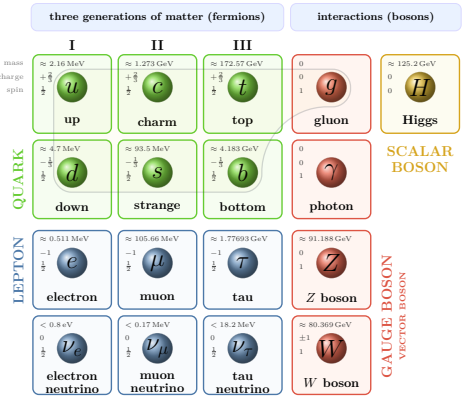
CLQCD

Outline

1. What lattice QCD can determine
2. Renormalized quark masses
3. CKM matrix elements
4. Summary and outlook

Standard Model parameters

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}_f \left[\gamma_\mu \left(\partial_\mu + i g A_\mu \right) + m_f \right] \psi_f$$



- renormalized quark masses $m_f(\mu)$

$$m_u,\ m_d,\ m_s,\ m_c,\ m_b$$

- the strong coupling $\alpha_s(\mu) = g^2(\mu)/4\pi$

$$\mathcal{L}^{\text{int}}_{\text{QCD}} = i g \overline{\psi}_f A_\mu \psi_f$$

- CKM elements $V_{q_1 q_2}$ through decay amplitudes

$$\mathcal{H}_{\text{eff}} \propto G_F\, V_{q_1 q_2} \left[\bar{q}_1 \gamma_\mu \left(1 - \gamma_5 \right) q_2 \right] \left[\bar{\ell} \gamma_\mu \left(1 - \gamma_5 \right) \nu_\ell \right]$$

Renormalized quark masses

Chiral Fermion

$$m_q^R = Z_m m_q^{\text{bare}} \qquad Z_m = 1/Z_S$$

Clover Fermion

$$m_q^R = Z_m(m_q^{\text{bare}} - m_{\text{res}})$$

m_{res}: residual quark mass

$$\langle\Omega|\partial_\mu A^\mu(x)|H\rangle = 2m_q^{\text{PCAC}}\langle\Omega|P(x)|H\rangle$$

m_q^{PCAC}: PCAC quark mass

$$m_q^R = Z_A/Z_P\, m_q^{\text{PCAC}}$$

Nonperturbative renormalization

$$\begin{aligned} G^B_{\mathcal{O}_\Gamma}(p_1,p_2) &= \sum_{x,y} e^{-i(p_1\cdot x - p_2\cdot y)}\langle\psi(x)\,[\bar\psi(0)\Gamma\psi(0)]\,\bar\psi(y)\rangle \\ &= S(p_1)\Lambda^B_{\mathcal{O}_\Gamma}(p_1,p_2)S(p_2) \\ S(p) &= \sum_x \mathrm{e}^{-\mathrm{i}p\cdot x}\left\langle\psi_f(x)\bar\psi_f(0)\right\rangle \end{aligned}$$

RI/SMOM: $p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$

RI/MOM: $p_1 = p_2$, $p_1^2 = \mu^2$

$$\frac{Z_q^{\text{SMOM}}}{Z_{\mathcal{O}}^{\text{SMOM}}} = \frac{\mathrm{Tr}[\Lambda^B_{\mathcal{O}}\Lambda^{\text{tree}}_{\mathcal{O}}]}{\mathrm{Tr}[\Lambda^{\text{tree}}_{\mathcal{O}}\Lambda^{\text{tree}}_{\mathcal{O}}]}$$

$C^{\overline{\text{MS}}}_{\text{SMOM}}(\mu)$ converts to $\overline{\text{MS}}$, $R(\mu_0,\mu)$ runs within $\overline{\text{MS}}$

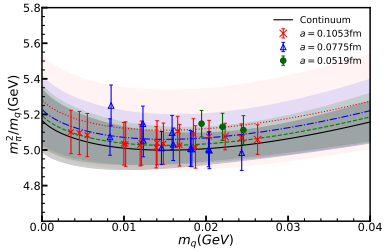
$$Z^{\overline{\text{MS}}}(\mu_0) = R^{\overline{\text{MS}}}(\mu_0,\mu)C^{\overline{\text{MS}}}_{\text{SMOM}}(\mu)Z^{\text{SMOM}}(\mu)$$

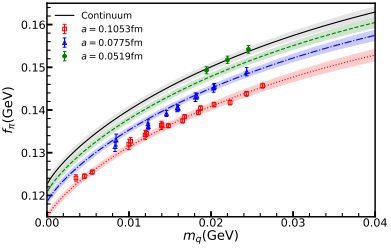
Light quark

NLO partially quenched χPT

$$
\begin{aligned}
m^2_{\pi,\mathrm{vv}} &= \Lambda_\chi^2 2y_\mathrm{v}\Big\{1+\frac{2}{N_f}\big[(2y_\mathrm{v}-y_\mathrm{s})\ln(2y_\mathrm{v})+(y_\mathrm{v}-y_\mathrm{s})\big] \\
&\quad + 2y_\mathrm{v}(2\alpha_8-\alpha_5)+2y_\mathrm{s}N_f(2\alpha_6-\alpha_4)\Big\} \\
&\quad \times\Big[1+c^\pi_L e^{-m_\pi L}+c^\pi_s(m^2_{\eta_s}-m^2_{\eta_s,\mathrm{phys}})\Big] \\
&\quad \times(1+c^\pi_{a^2}a^2), \\
F_{\pi,\mathrm{vv}} &= F(1-\frac{N_f}{2}(y_\mathrm{v}+y_\mathrm{s})\ln(y_\mathrm{v}+y_\mathrm{s})+y_\mathrm{v}\alpha_5+y_\mathrm{s}N_f\alpha_4) \\
&\quad \times\Big[1+d^\pi_L e^{-m_\pi L}+d^\pi_s(m^2_{\eta_s}-m^2_{\eta_s,\mathrm{phys}})\Big] \\
&\quad \times(1+d^\pi_{a^2}a^2) \\
y_{\mathrm{v/s}} &= \Sigma\, m_l^{\mathrm{v/s}}/F^2\Lambda_\chi^2 \quad \Lambda_\chi = 4\pi F
\end{aligned}
$$

log function behavior at the small quark mass





Hu et al., 2024, *Phys. Rev. D*

Light quark

Alexandrou et al., 2021, *Phys. Rev. D*

m_q^s: sea quark mass, m_q^v: valence quark mass

$$
\begin{aligned}
&m_K^2(m_l^{\mathrm{v}}, m_l^{\mathrm{s}}, m_s^{\mathrm{v}}, m_s^{\mathrm{s}}, a, 1/L) \\
&\quad = (b_s^{\mathrm{v}} m_s^{\mathrm{v}} + b_s^{\mathrm{s}} m_s^{\mathrm{s}} + b_l^{\mathrm{v}} m_l^{\mathrm{v}} + b_l^{\mathrm{s}} m_l^{\mathrm{s}}) \\
&\quad \times \left[1 + c_l^K m_l^{\mathrm{v}} + c_{a^2}^K a^2 + c_L^K e^{-m_\pi L}\right]
\end{aligned}
$$

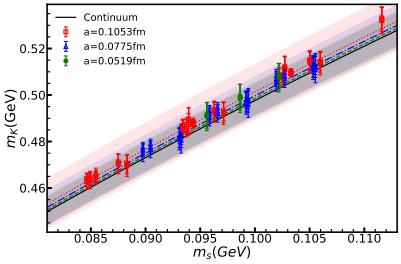
determine m_u and m_d with

$$
\begin{aligned}
m_K\left(m_d^{\mathrm{phys}}, m_l^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_s^{\mathrm{phys}}\right) &= m_{K^0,\mathrm{QCD}}, \\
m_K\left(m_u^{\mathrm{phys}}, m_l^{\mathrm{phys}}, m_s^{\mathrm{phys}}, m_s^{\mathrm{phys}}\right) &= m_{K^\pm,\mathrm{QCD}}, \\
m_u^{\mathrm{phys}} + m_d^{\mathrm{phys}} &= 2\, m_l^{\mathrm{phys}}.
\end{aligned}
$$

Kaon mass in pure QCD

Giusti et al., 2017, *Phys. Rev. D*

$$
\begin{aligned}
m_{K^0,\mathrm{QCD}} &= m_{K^0}^{\mathrm{phys}} - \Delta_{\mathrm{QED}} m_{K^0} = 497.44(02)\,\mathrm{MeV}, \\
m_{K^\pm,\mathrm{QCD}} &= m_{K^\pm}^{\mathrm{phys}} - \Delta_{\mathrm{QED}} m_{K^\pm} = 491.44(15)\,\mathrm{MeV},
\end{aligned}
$$



	CLQCD 23	CLQCD 26
m_{ud} [MeV]	3.60(11)(15)	3.51(08)(14)
m_u [MeV]	2.45(22)(20)	2.320(86)*
m_d [MeV]	4.74(11)(09)	4.700(86)*
m_s [MeV]	98.8(2.9)(4.7)	97.5(1.7)(4.1)

* CLQCD 26 m_u and m_d are preliminary

Hu et al., 2024, *Phys. Rev. D* *Lin et al.*, 2026, *Commun. Theor. Phys.*

Strong isospin breaking

$$m_{K^\pm}^{\text{phys}} - m_K^{\text{isoQCD}} = \Delta_{\text{QED}}\, m_{K^\pm} + \Delta_{\text{IB}}\, m_{K^\pm}$$

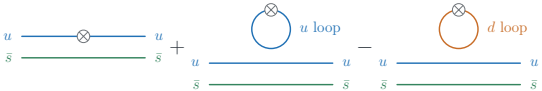
Feynman-Hellmann theorem

$$m_q \frac{\partial m_H}{\partial m_q} = \langle H | m_q \bar{q} q | H \rangle$$

Isospin symmetry breaking (IB)

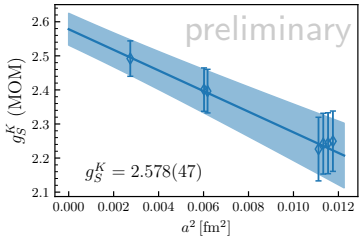
$$\Delta m_{K^+}^{\text{IB}} = (m_u - m_{ud}) \langle K^+ | \bar{u} u | K^+ \rangle_{\text{isoQCD}} + (m_d - m_{ud}) \langle K^+ | \bar{d} d | K^+ \rangle_{\text{isoQCD}}$$

only connected diagram is required



blending method allows computation of local current matrix element in the distillation framework

$$m_d - m_u = [3.934(20) + (2.25(8) - 0.05(7))]/g_S^K \text{ MeV} = 2.379(60) \text{ MeV}$$



$$-\nabla^2_{\boldsymbol{xy}}(t) = 6\delta_{\boldsymbol{xy}} - \sum_{j=1}^{3} \tilde{U}_j(\boldsymbol{x},t)\delta_{\boldsymbol{x}+\hat{j},\boldsymbol{y}} + \tilde{U}_j^\dagger(\boldsymbol{x}-\hat{j},t)\delta_{\boldsymbol{x}-\hat{j},\boldsymbol{y}}$$

$$\Box_{\boldsymbol{xy}}(t) = V_{\boldsymbol{x}}(t)V_{\boldsymbol{y}}^\dagger(t) = \sum_{k=1}^{N} v_{\boldsymbol{x}}^{(k)}(t) v_{\boldsymbol{y}}^{(k)\dagger}(t)$$

$$C_{3pt} = \langle \bar{s}(x)\gamma^5 u(x)\ \bar{u}(z)\Gamma u(z)\ \bar{u}(y)\gamma^5 s(y)\rangle = \sum_{k,j=1}^{N_{ex}} \sum_{i=1}^{N_{ex}+N_b} \langle \tau_{jk}^s(t_y,t_x)\gamma^5 \tau_{ki}^u(t_x,t_z)\Omega_i \tau_{ij}^u(t_z,t_y))\gamma^5\rangle$$

$$\tau_{ij}(t,t') = v_{\boldsymbol{x}}^{(i)\dagger}(t) S(\boldsymbol{x},t;\boldsymbol{x}',t') v_{\boldsymbol{x}'}^{(j)}(t')$$

Boccaletti et al., 2026, *Nature*

Peardon et al., 2009, *Phys. Rev. D*

Hu et al., 2025, *arXiv:2505.01719*

Heavy quark

Du et al., 2025, *Phys. Rev. D*

Cai et al., 2026, *arXiv:2603.01846*

tune valence m_c and m_b on each ensemble

$$m_{D_s}(m_s^{\text{phys}}, m_c^{\text{val}}, a) = m_{D_s}^{\text{QCD}} \qquad m_\Upsilon(m_b^{\text{val}}, a) = m_\Upsilon^{\text{phys}}$$

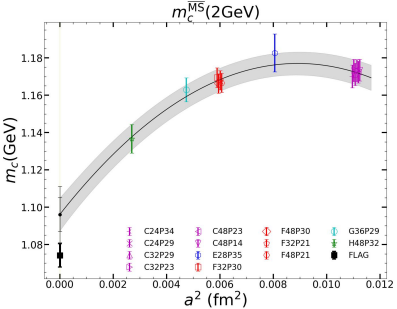
extrapolation is parametrized as

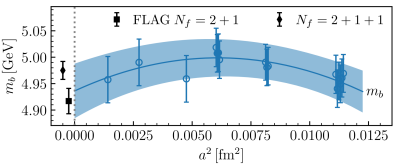
$$\begin{aligned} &X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0) \\ &+ c_{m_l}\left(m_\pi^2 - (m_\pi^{\text{phys}})^2\right) + c_{m_s}\left(m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2\right) \\ &+ d_{a^2} a^2 + d_{a^4} a^4 + c_L e^{-m_\pi L} \end{aligned}$$

anisotropic clover action for heavy quark

$$\begin{aligned} S = a^4 \sum_x \bar{\psi}(x) \Bigg[& m_q + \left(\gamma_0 D_0 - \frac{a}{2} D_0^2\right) + \nu \sum_i \left(\gamma_i D_i - \frac{a}{2} D_i^2\right) \\ & - \frac{a}{2} c_B \left(\sum_{i<j} \sigma_{ij} F_{ij}\right) - \frac{a}{2} c_E \left(\sum_i \sigma_{0i} F_{0i}\right) \Bigg] \psi(x) \end{aligned}$$

Chen, 2001, *Phys. Rev. D*

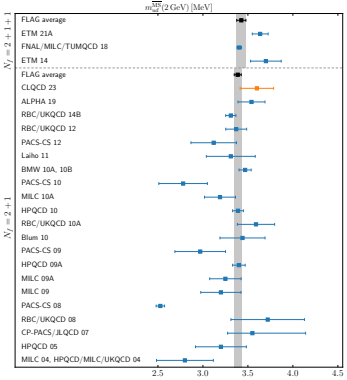


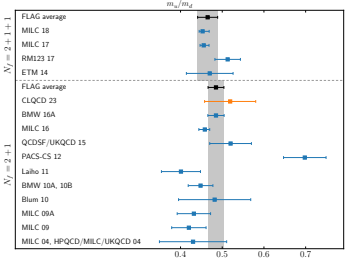


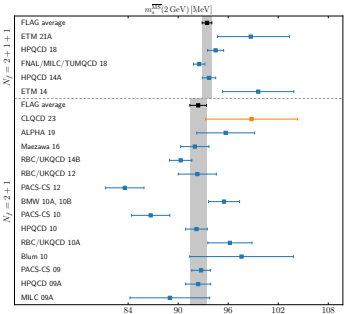
$$m_c(m_c) = 1.2933(72)(95)\ \text{GeV} \qquad m_b(m_b) = 4.185(37)\ \text{GeV}$$

Comparison of quark mass

Aoki et al., 2026, FLAG 24, *Phys. Rev. D*



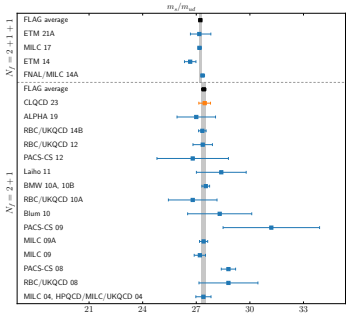


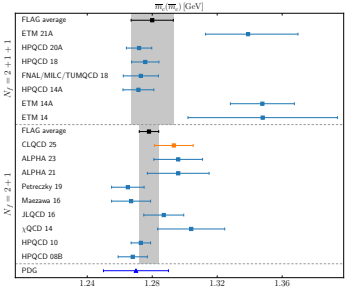


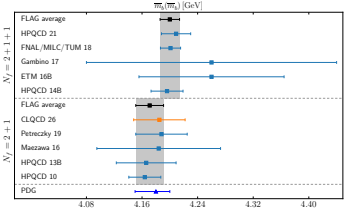
Results from CLQCD are well consistent with FLAG average

Comparison of quark mass

Aoki et al., 2026, FLAG 24, *Phys. Rev. D*





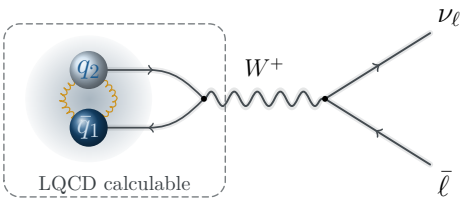


Results from CLQCD are well consistent with FLAG average

It gives us confidence to pursue high precision calculations

Leptonic decay

$$\mathcal{H}_{\text{eff}} \propto G_F\, V_{q_1 q_2} \left[\bar{q}_1 \gamma_\mu \left(1-\gamma_5\right) q_2\right] \left[\bar{\ell} \gamma_\mu \left(1-\gamma_5\right) \nu_\ell\right]$$



leptonic decay of a pseudoscalar meson H

$$\Gamma(H \to \bar{\ell}\nu_\ell) = \frac{G_F^2}{8\pi} f_H^2 m_\ell^2 M_H \left(1 - \frac{m_\ell^2}{M_H^2}\right)^2 |V_{q_1 q_2}|^2$$

$$i f_H p^\mu = Z_A \langle 0 | \bar{q}_1 \gamma^\mu \gamma_5 q_2 | H(p) \rangle$$

$|V_{us}|/|V_{ud}|\, f_K/f_\pi = 0.27599(41)$

$|V_{ub}| = 3.82(24) \times 10^{-3}$

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

$f_D |V_{cd}| = 45.82(1.10)$ MeV

$f_{D_s} |V_{cs}| = 243.5(2.7)$ MeV

$f_B\, |V_{ub}| = 0.77 \pm 0.12$ MeV

Aoki et al., 2026, FLAG 24, *Phys. Rev. D*

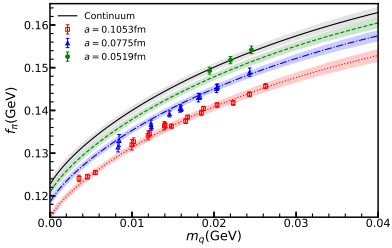
f_π and f_K

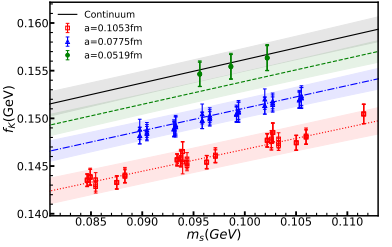
$$F_{\pi,\mathrm{vv}} = F(1 - \frac{N_f}{2}(y_\mathrm{v} + y_\mathrm{s})\ln(y_\mathrm{v} + y_\mathrm{s}) + y_\mathrm{v}\alpha_5 + y_\mathrm{s}N_f\alpha_4)$$
$$\times\left[1 + d_L^\pi e^{-m_\pi L} + d_s^\pi(m_{\eta_s}^2 - m_{\eta_s,\mathrm{phys}}^2)\right]$$
$$\times(1 + d_{a^2}^\pi a^2)$$
$$y_{\mathrm{v/s}} = \Sigma\, m_l^{\mathrm{v/s}}/F^2\Lambda_\chi^2 \quad \Lambda_\chi = 4\pi F$$

m_q^s: sea quark mass, m_q^v: valence quark mass

$$f_K(m_l^\mathrm{v}, m_l^\mathrm{s}, m_s^\mathrm{v}, m_s^\mathrm{s}, a, 1/L)$$
$$= (d_f + d_s^\mathrm{v}m_s^\mathrm{v} + d_s^\mathrm{s}m_s^\mathrm{s} + d_l^\mathrm{v}m_l^\mathrm{v} + d_l^\mathrm{s}m_l^\mathrm{s})$$
$$\times\left[1 + d_{a^2}^K a^2 + d_L^K e^{-m_\pi L}\right]$$

$$f_K/f_\pi$$
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \Rightarrow \begin{matrix}|V_{ud}| = 0.9740(03)_\mathrm{lat}(01)_\mathrm{exp},\\ |V_{us}| = 0.2264(15)_\mathrm{lat}(03)_\mathrm{exp}\end{matrix}$$





Hu et al., 2024, *Phys. Rev. D*

Heavy meson

$$f_{\rm PS} m_{\rm PS} = Z_A \langle 0|\bar{q}_1 \gamma^\mu \gamma_5 q_2|H\rangle$$
$$Z_A = (Z_A/Z_V)^{\rm RI}\ Z_V$$

extrapolation is parameterized as

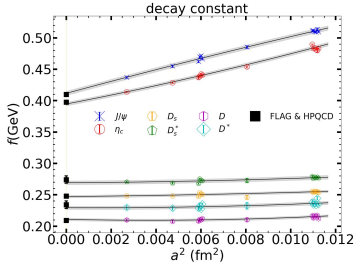
$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\rm phys}, m_{\eta_s}^{\rm phys}, 0)$$
$$+ c_{m_l}\left(m_\pi^2 - (m_\pi^{\rm phys})^2\right)$$
$$+ c_{m_s}\left(m_{\eta_s}^2 - (m_{\eta_s}^{\rm phys})^2\right)$$
$$+ d_{a^2} a^2 + d_{a^4} a^4 + c_L e^{-m_\pi L}$$

to improve discretization errors

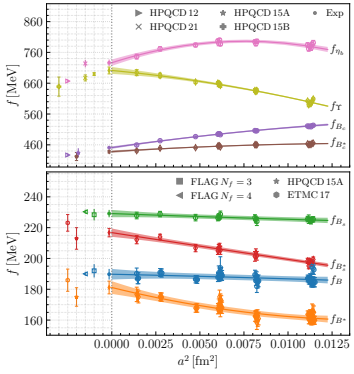
$$Z_V(m_c)\langle \eta_c|V^t|\eta_c\rangle = 2P^0$$

anisotropic action

$$Z_V^i(m_b)\langle \eta_b|V^i|\eta_b\rangle = 2P^i$$
$$Z_V^t(m_b)\langle \eta_b|V^t|\eta_b\rangle = 2P^0$$



$$f_D \Rightarrow |V_{cs}| = 0.986(08)_{\rm lat}(11)_{\rm exp},$$
$$f_{D_s} \Rightarrow |V_{cd}| = 0.2173(21)_{\rm lat}(52)_{\rm exp}$$

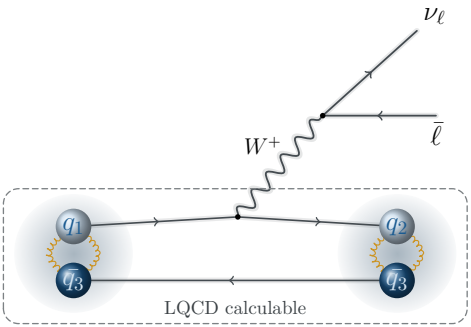


$$f_B \Rightarrow |V_{ub}| = 4.06(08)_{\rm lat}(63)_{\rm exp} \times 10^{-3}$$

Du et al., 2025, *Phys. Rev. D* *Cai et al.*, 2026, *arXiv:2603.01846*

Semileptonic form factors

$$\mathcal{H}_{\text{eff}} \propto G_F\, V_{q_1 q_2} \left[\bar{q}_1 \gamma_\mu \left(1-\gamma_5\right) q_2\right] \left[\bar{\ell} \gamma_\mu \left(1-\gamma_5\right) \nu_\ell\right]$$



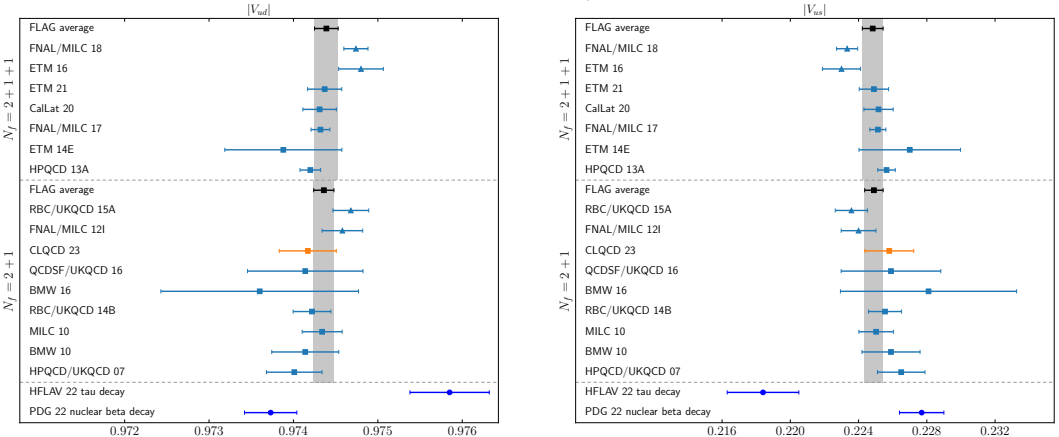
For $H \to P\ell\nu$, the vector-current matrix element is decomposed as

$$\langle P(p')|V^\mu|H(p)\rangle = f_+(q^2)\left[p^\mu + p'^\mu - \frac{M_H^2 - M_P^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M_H^2 - M_P^2}{q^2} q^\mu, \quad q = p - p'.$$

- arXiv:2510.14478: $D_s \to \phi\ell\nu_\ell$ gives $V_{cs} = 0.952(12)_{\text{stat}}(23)_{\text{PDG}}$ and $V_{cs} = 0.945(12)_{\text{stat}}(24)_{\text{PDG}}$ for μ and e channels *Fan et al.*, 2025, e-Print: 2510.14478
- semi-leptonic decays of bottom meson, e.g. $B \to \pi\nu\ell$, give more precise determination of V_{ub}. $B \to D\nu\ell$ gives access to V_{cb}

Comparison of CKM matrix elements

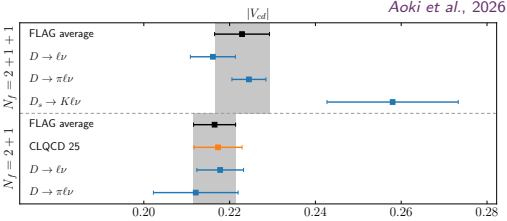
Aoki et al., 2026, FLAG, *Phys. Rev. D*

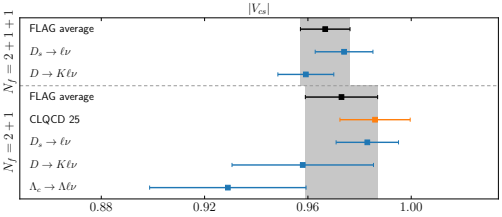


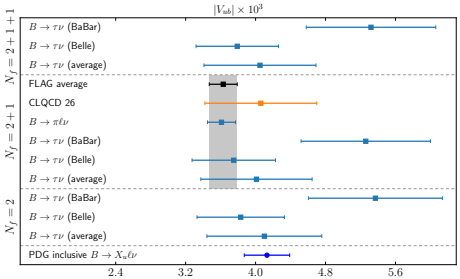
Results from CLQCD are well consistent with FLAG average

Comparison of CKM matrix elements

Aoki et al., 2026, FLAG, *Phys. Rev. D*





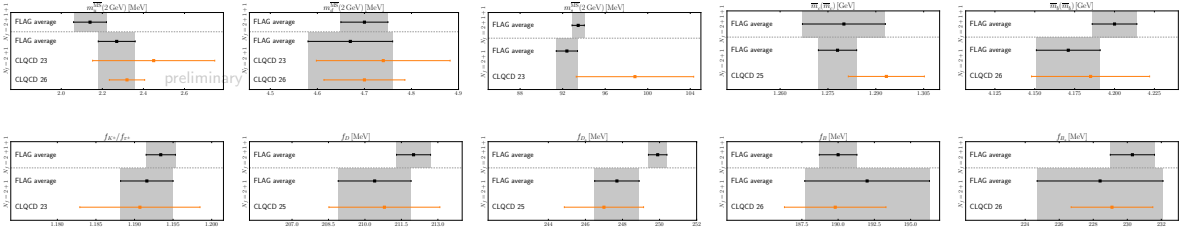


Results from CLQCD are well consistent with FLAG average

Summary

Aoki et al., 2026, Phys. Rev. D

Lin et al., 2026, Commun. Theor. Phys. Hu et al., 2024, Phys. Rev. D Du et al., 2025, Phys. Rev. D Cai et al., 2026, arXiv:2603.01846



conclusion:

- Calculations from CLQCD ensembles are well consistent with FLAG average
- g_S^K computed via the blending method enables a high-precision determination of $m_d - m_u$
- anisotropic clove action makes the calculation of bottom physics on CLQCD ensembles possible

outlook:

- QED corrections from CLQCD ensembles
- bottom meson form factors from anisotropic clover action

THANKS

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