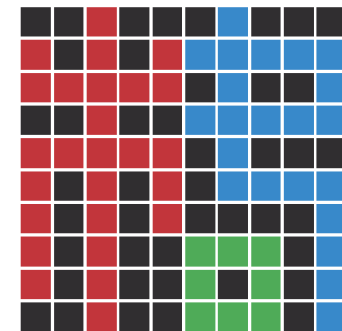


Precision determination of nucleon iso-vector scalar and tensor charges at the physical point

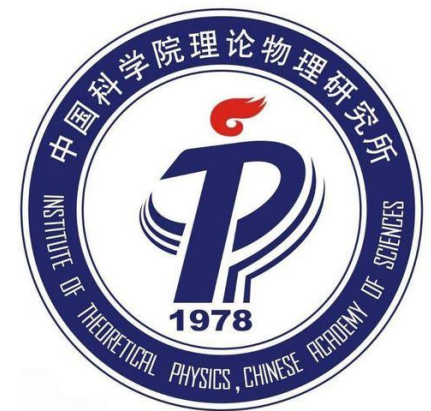
—Current enhance in scalar and tensor charge

Speaker: Ji-Hao Wang

Institute of Theoretical Physics, Chinese Academy of Sciences



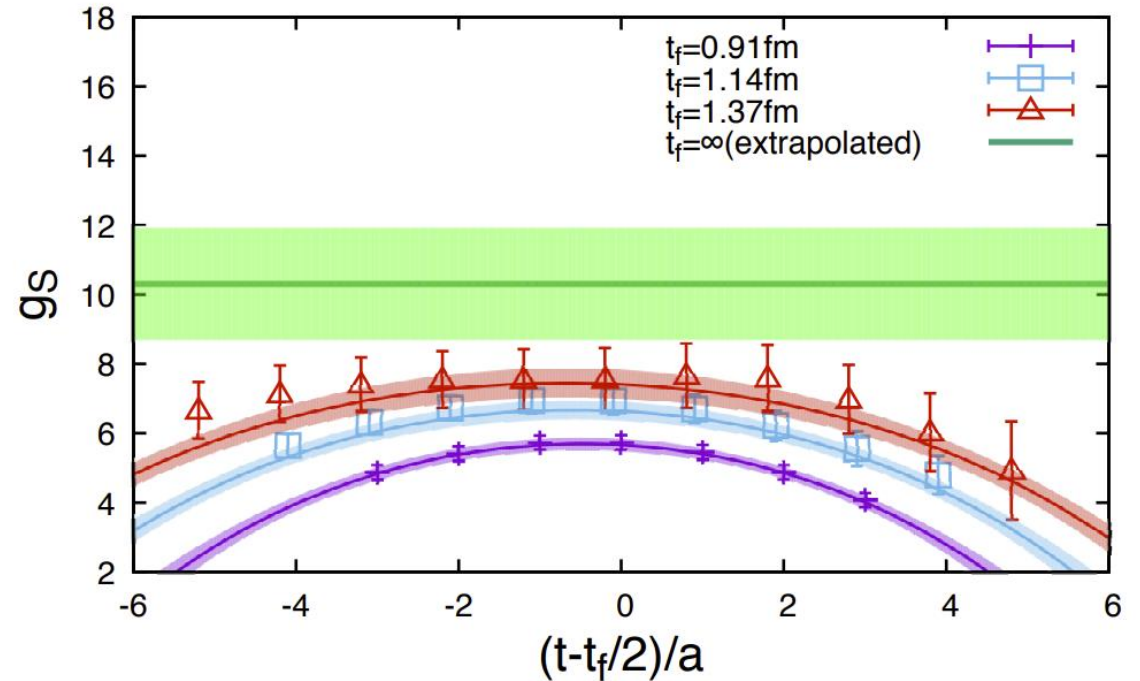
CLQCD



Introduction

The calculation of nucleon matrix elements is challenging because of severe excited-state contamination and increasing statistical noise at large time separations.

Using the nucleon iso-vector scalar and tensor charges, g_S and g_T as examples, I will show how **current-involved interpolating fields** [1,2] combined with the **blending method** [3] can be employed to control excited-state effects.



Y.-B. Yang et al., Phys. Rev. D 94, 054503 (2016), Fig. 2.

[1]. L. Barca, G. Bali, and S. Collins, Phys. Rev. D 111, L031505 (2025), arXiv:2412.13138 [hep-lat].

[2]. L. Barca, (2025), arXiv:2508.09006 [hep-lat].

[3]. Z.-C. Hu, J.-H. Wang, X. Jiang, L. Liu, S.-H. Su, P. Sun, and Y.-B. Yang, (2025), arXiv:2505.01719 [hep-lat].

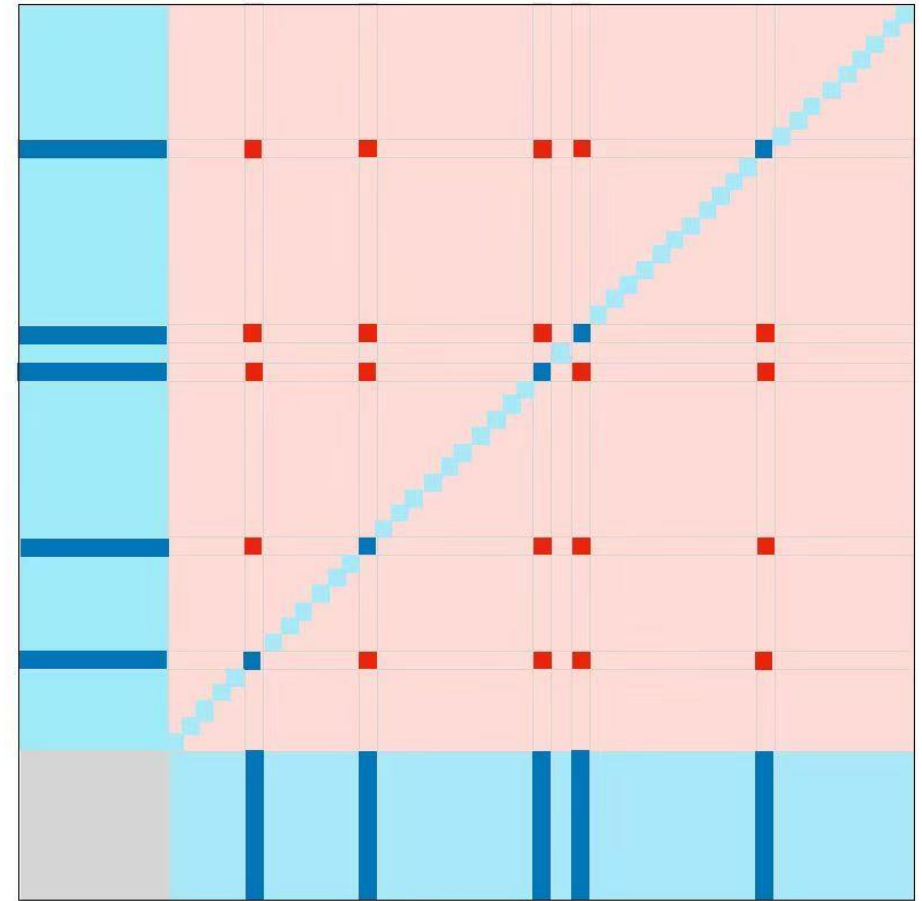
Blending method

The blending method provides an approach to calculate matrix elements within the distillation framework.

- In the distillation, only the information from $\mathcal{L}_1$ is used to approximate the all-to-all propagator.
- In the blending, full information from $\mathcal{L}$ is recovered through random sampling in $\mathcal{L}_2$.

definition :

- $\mathcal{L}$: the vector space associated with the lattice sites and color degrees of freedom
- $\mathcal{L}_1$: The span of N_e low-lying eigenvectors of the discrete Laplace operator
- $\mathcal{L}_2$: The orthogonal complement of $\mathcal{L}_1$ in $\mathcal{L}$

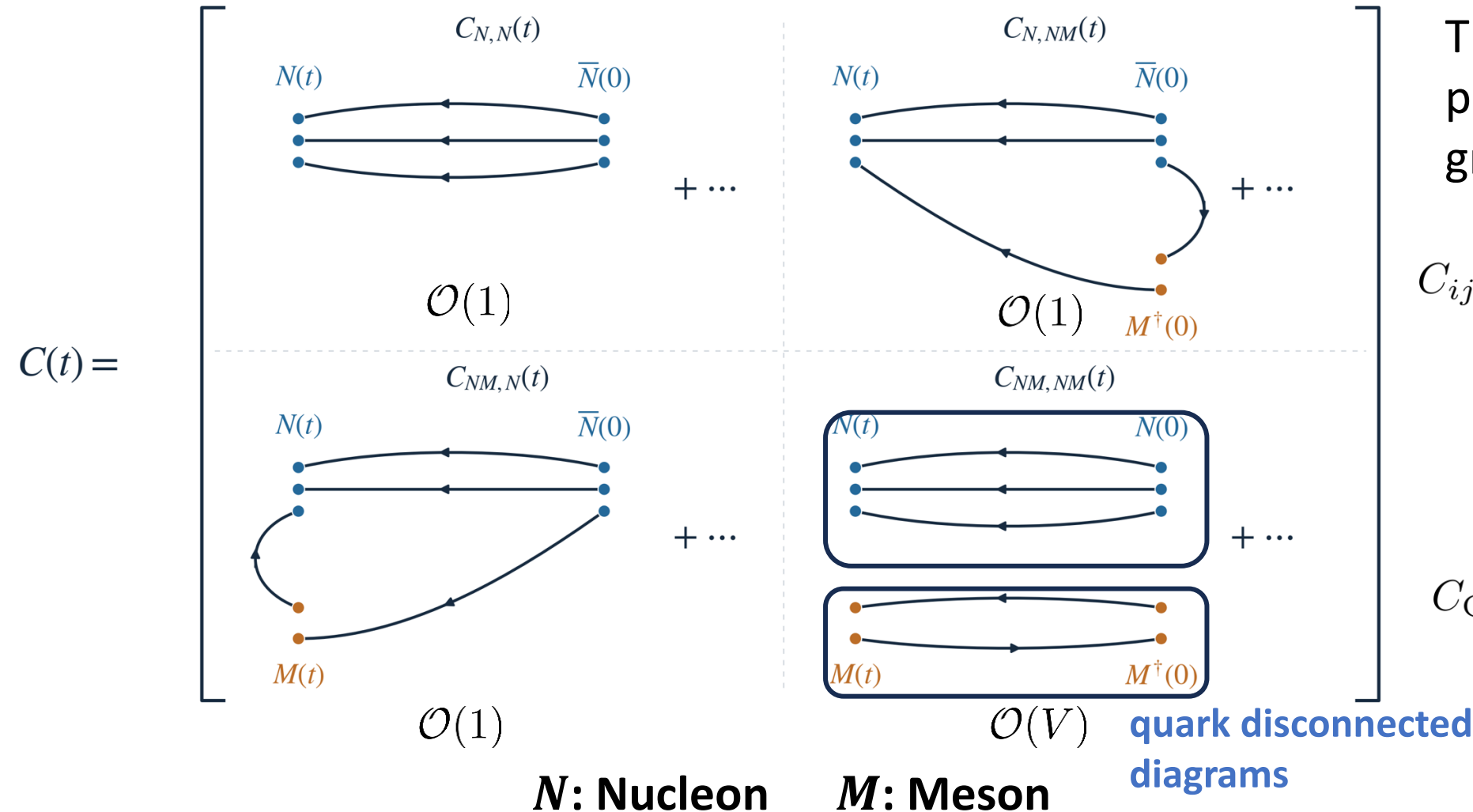


$$\Omega_{ij}^{(2)} = \begin{cases} 1 & \text{for } i, j \leq N_e, \\ \prod_{i=0}^1 \frac{V - N_e - i}{N_{st} - i} & \text{for } i, j > N_e, i \neq j \\ \frac{V - N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

The blending method inherits the benefit of flexible construction of complex hadron interpolation operators from distillation

Current enhance (2pt)

Consider GEVP for $\{\mathcal{O}_N, \mathcal{O}_{NM}\}$



This gives the volume power counting for the ground-state coefficient

$$C_{ij}(t)v_j^{(n)} = \lambda(t, t_0)C_{ij}(t_0)v_j^{(n)}$$

$$v_N^{(0)} \sim 1$$

$$v_{NM}^{(0)} \sim \mathcal{O}\left(\frac{1}{V}\right)$$

$$\begin{aligned} C_{\text{GEVP}}(t) &= v_i^{(0)*} C_{ij}(t) v_j^{(0)} \\ &= C_{N,N}(t) + \mathcal{O}\left(\frac{1}{V}\right) \end{aligned}$$

Current enhance (3pt)

Consider GEVP for $\{\mathcal{O}_N, \mathcal{O}_{NM}\}$

$$v_N^{(0)} \sim 1$$

$$v_{NM}^{(0)} \sim \mathcal{O}\left(\frac{1}{V}\right)$$

- If M and $\mathcal{J}$ have same quantum number (**current enhance**):

$$C_{N,NM}^{\mathcal{J}}(t, \tau) \sim \mathcal{O}(V)$$

$$C_{\text{GEVP}}^{\mathcal{J}}(t, \tau) = C_{N,N}^{\mathcal{J}}(t, \tau) + \mathcal{O}(1)$$

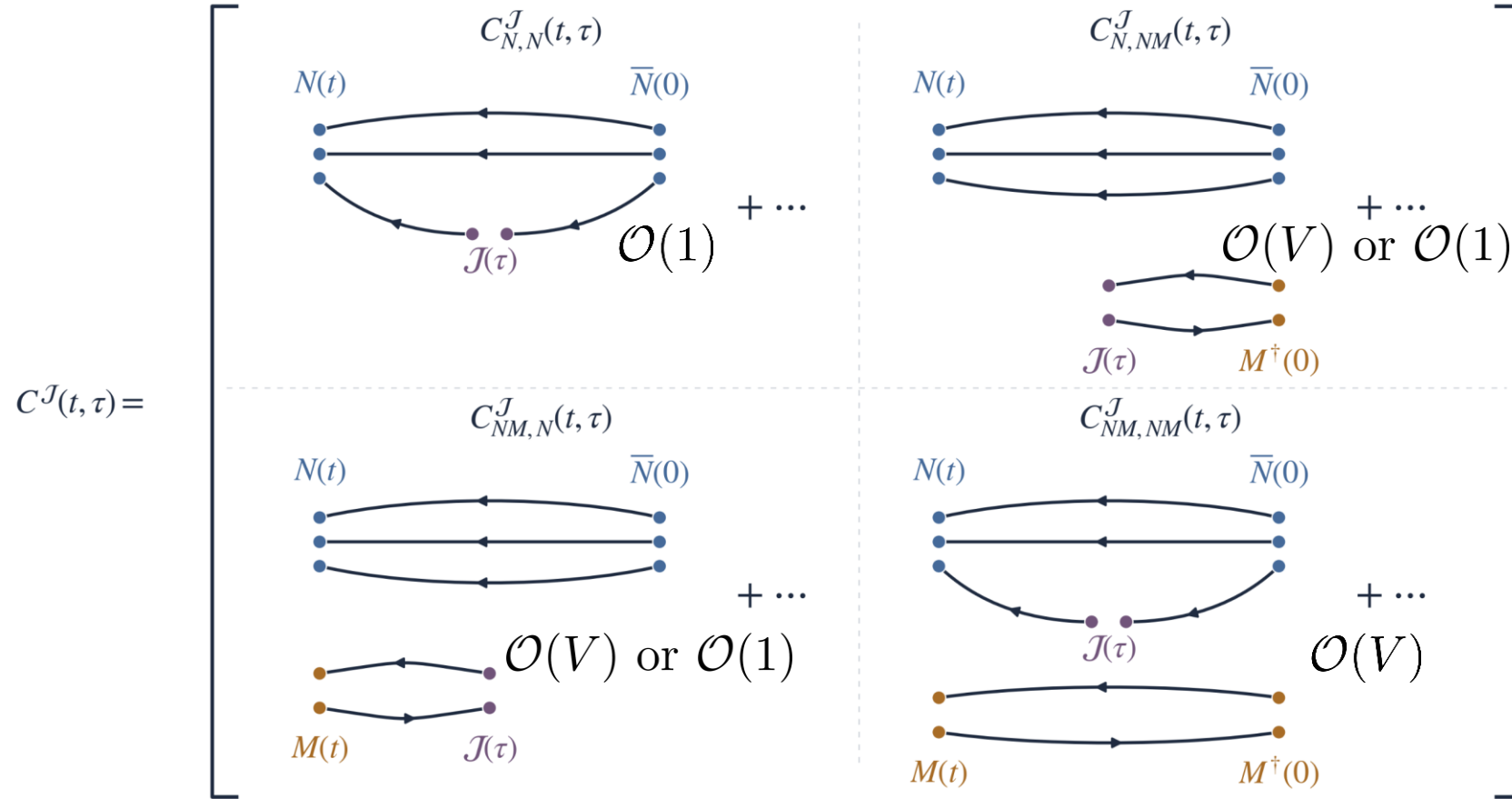
- If M and $\mathcal{J}$ don't have same quantum number (**quark disconnected diagrams do not contribute**):

$$C_{N,NM}^{\mathcal{J}}(t, \tau) \sim \mathcal{O}(1)$$

$$C_{\text{GEVP}}^{\mathcal{J}}(t, \tau) = C_{N,N}^{\mathcal{J}}(t, \tau) + \mathcal{O}\left(\frac{1}{V}\right)$$

When M and $\mathcal{J}$ have same quantum number, we refer to $\mathcal{O}_{NM}$ as the **current-involved interpolating fields**

Contributions from other interpolating fields are suppressed by a volume factor



N : Nucleon M : Meson $\mathcal{J}$: Current

Operator set

Iso-vector scalar charges g_S , $I^G(J^{PC}) = 1^-(0^{++})$

Meson:

$$\mathcal{O}_S^+ = \bar{u}d, \quad \mathcal{O}_S^- = \bar{d}u, \quad \mathcal{O}_S^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$$

project to iso-spin $\frac{1}{2}$:

$$\mathcal{N}_S = \sqrt{\frac{1}{3}} p \mathcal{O}_S^0 - \sqrt{\frac{2}{3}} n \mathcal{O}_S^+$$

Iso-vector tensor charges g_T , $I^G(J^{PC}) = 1^+(1^{+-})$

Meson:

$$\mathcal{O}_{T_i}^+ = \bar{u}T_id, \quad \mathcal{O}_{T_i}^- = \bar{d}T_iu, \quad \mathcal{O}_{T_i}^0 = \frac{1}{\sqrt{2}}(\bar{u}T_iu - \bar{d}T_id), \quad T_i = \epsilon_{ijk}\sigma_{jk} \text{ or } \gamma_5 D_i$$

project to iso-spin $\frac{1}{2}$ and spin $\frac{1}{2}$ (G_{1g} on lattice):

$$\mathcal{N}_T = \frac{1}{\sqrt{3}}(\mathcal{N}_\downarrow \mathcal{O}_{T_x} - i\mathcal{N}_\downarrow \mathcal{O}_{T_y} + \mathcal{N}_\uparrow \mathcal{O}_{T_z}) \quad \mathcal{N}_{\uparrow/\downarrow} \mathcal{O}_{T_i} = \sqrt{\frac{1}{3}} p_{\uparrow/\downarrow} \mathcal{O}_{T_i}^0 - \sqrt{\frac{2}{3}} n_{\uparrow/\downarrow} \mathcal{O}_{T_i}^+$$

We also consider the $(N\pi)_{\text{p-wave}}$ and $(N\pi\pi)_{\text{s-wave}}$, they are also project to iso-spin $\frac{1}{2}$ and spin $\frac{1}{2}$ (G_{1g})

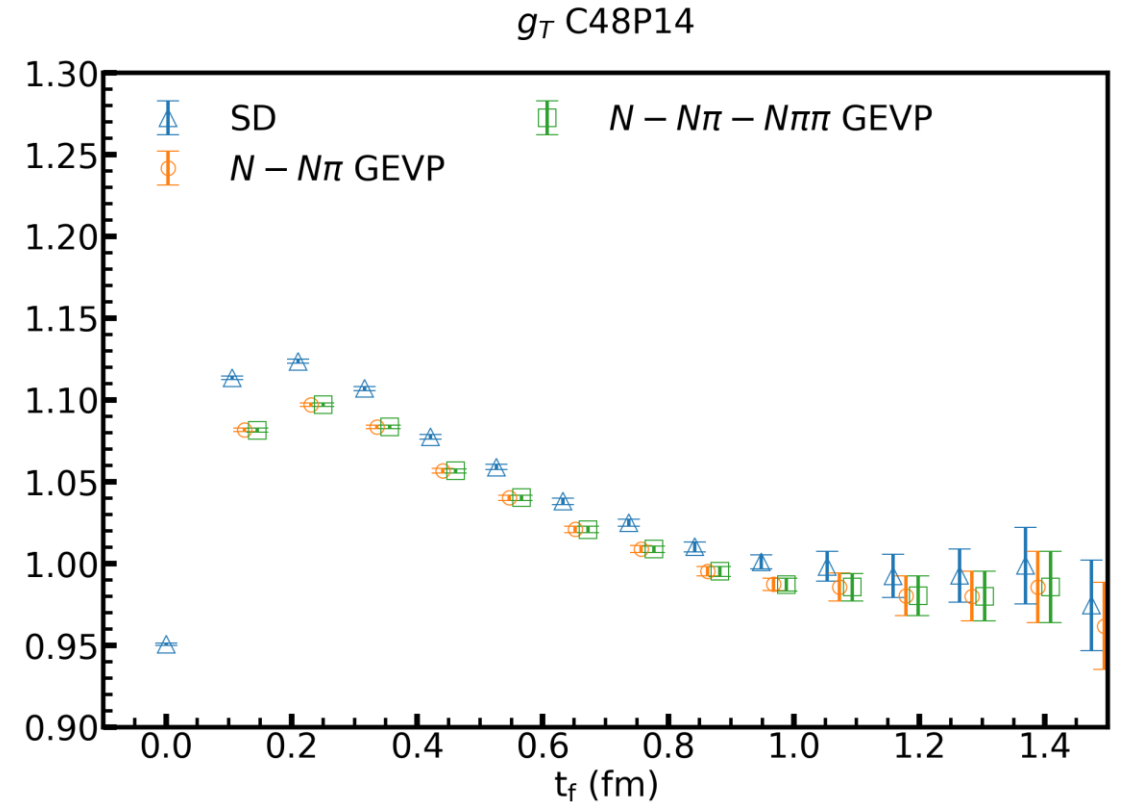
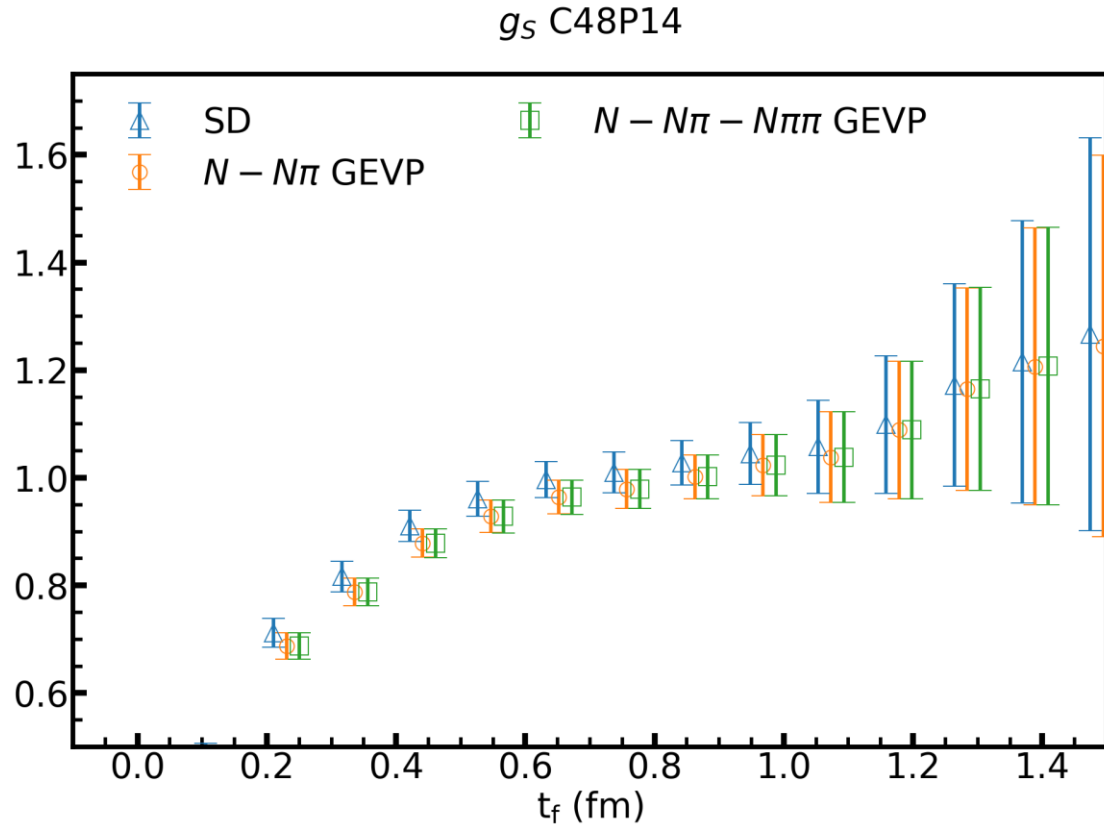
$$\begin{aligned} (\mathcal{N}\pi)_{\text{p-wave}} &= \frac{1}{\sqrt{3}}((\mathcal{N}_\downarrow \pi)_x - i(\mathcal{N}_\downarrow \pi)_y + (\mathcal{N}_\uparrow \pi)_z) & (\mathcal{N}_{\uparrow/\downarrow} \pi)_i &= \sqrt{\frac{1}{3}} p_{\uparrow/\downarrow}(p_i) \pi^0(-p_i) - \sqrt{\frac{2}{3}} n_{\uparrow/\downarrow}(p_i) \pi^+(-p_i) \\ (\mathcal{N}\pi\pi)_{\text{s-wave}} &= p(\sqrt{\frac{2}{5}} \pi^+ \pi^- - \sqrt{\frac{1}{5}} \pi^0 \pi^0) & & - \sqrt{\frac{1}{3}} p_{\uparrow/\downarrow}(-p_i) \pi^0(p_i) + \sqrt{\frac{2}{3}} n_{\uparrow/\downarrow}(-p_i) \pi^+(p_i) \end{aligned}$$

Counts of quark diagrams

	Unprojected	projected
$\mathcal{C}_{N_S, N_S}(t)$	26	54
$\mathcal{C}_{N_T, N_T}(t)$	26	486
$\mathcal{C}_{N\pi, N\pi}(t)$	102	1944
$\mathcal{C}_{N\pi\pi, N\pi\pi}(t)$	144	1178
$\mathcal{C}_{N, N_S}^J(t, \tau)$	26	42
$\mathcal{C}_{N, N_T}^J(t, \tau)$	26	126
$\mathcal{C}_{N, N\pi}^J(t, \tau)$	52	252
$\mathcal{C}_{N, N\pi\pi}^J(t, \tau)$	60	162

Influence of $N\pi$ and $N\pi\pi$

We calculate GEVP for $\{N, (N\pi)_{\text{p-wave}}, (N\pi\pi)_{\text{s-wave}}\}$ on C48P14 (**physical pion mass ensemble**) and apply the coefficients obtained from this GEVP to the 3pt functions



The contribution of $N\pi$ to the excited state contamination is **not significant**, and that of $N\pi\pi$ is almost **zero**

Further discussion of $N\pi\pi$

Consider GEVP for $\{\mathcal{O}_N, \mathcal{O}_{NM_1M_2}\}$

C_{N,NM_1M_2} is always $\mathcal{O}(1)$, since whenever $\langle M_1 M_2 \rangle$ have non-zero VEV, that VEV should be subtracted

$$v_N^{(0)} \sim 1$$

$$v_{NM_1M_2}^{(0)} \sim \mathcal{O}\left(\frac{1}{V^2}\right)$$

$$C_{\text{GEVP}}(t) = C_{N,N}(t) + \mathcal{O}\left(\frac{1}{V^2}\right)$$

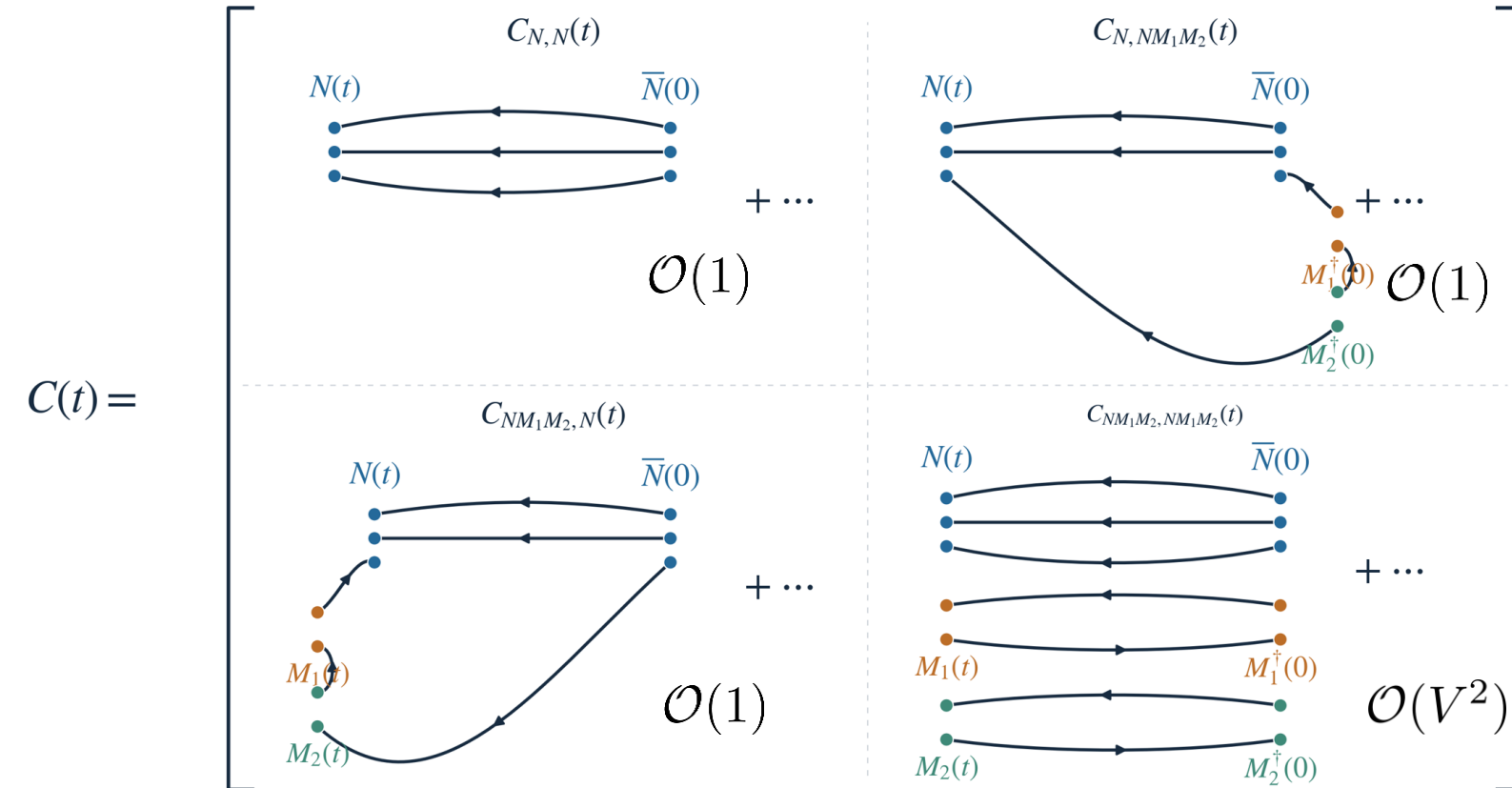
$$C_{\text{GEVP}}^{\mathcal{J}}(t, \tau) = C_{N,N}^{\mathcal{J}}(t, \tau) + \mathcal{O}\left(\frac{1}{V}\right) \quad \text{(current enhance)}$$

$$C_{\text{GEVP}}^{\mathcal{J}}(t, \tau) = C_{N,N}^{\mathcal{J}}(t, \tau) + \mathcal{O}\left(\frac{1}{V^2}\right) \quad \text{(no current enhance)}$$

The contributions from NM_1M_2 are less important than NM .

This explains why the contribution from $N\pi\pi$ is almost zero

Based on this, the current-involved interpolating fields are the first to be considered



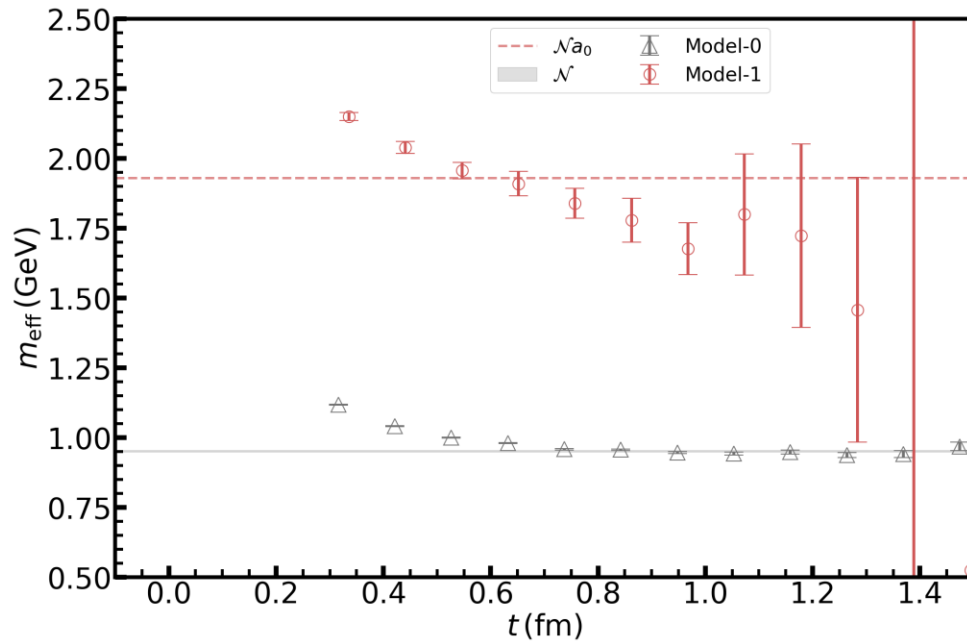
N : Nucleon

M_1, M_2 : Meson

current-involved interpolating fields

GEVP for $\{N, N_S\}$

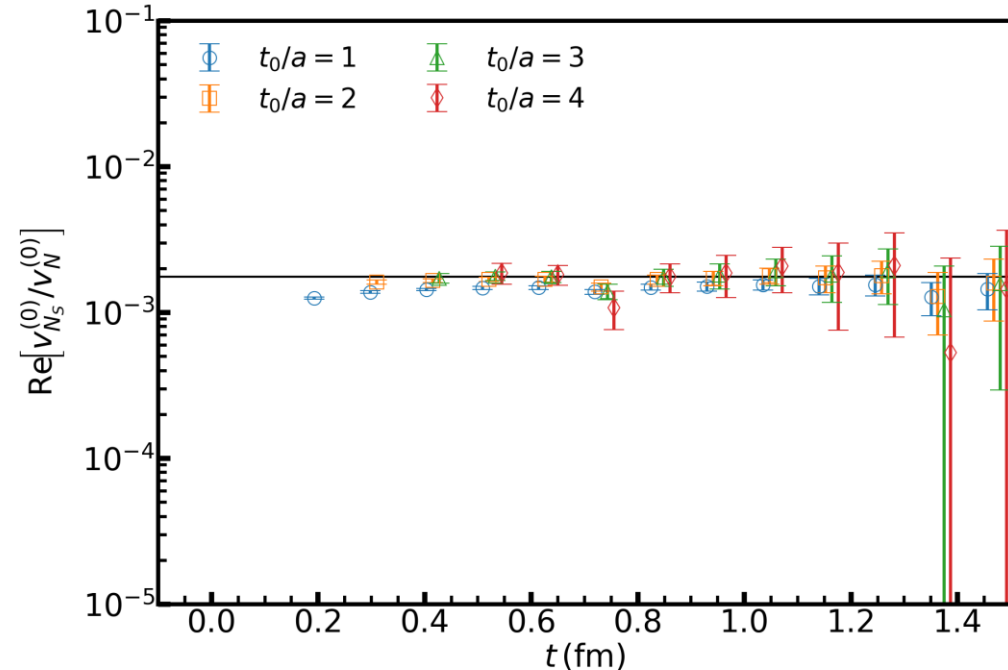
Effective mass for GEVP Eigenvalue $\lambda_i(t)$



The plateaus are close to, but slightly lower than, the corresponding free energy levels of $N a_0$

Here a_0 are the lightest mesons with the same $I^G(J^{PC})$ as g_S

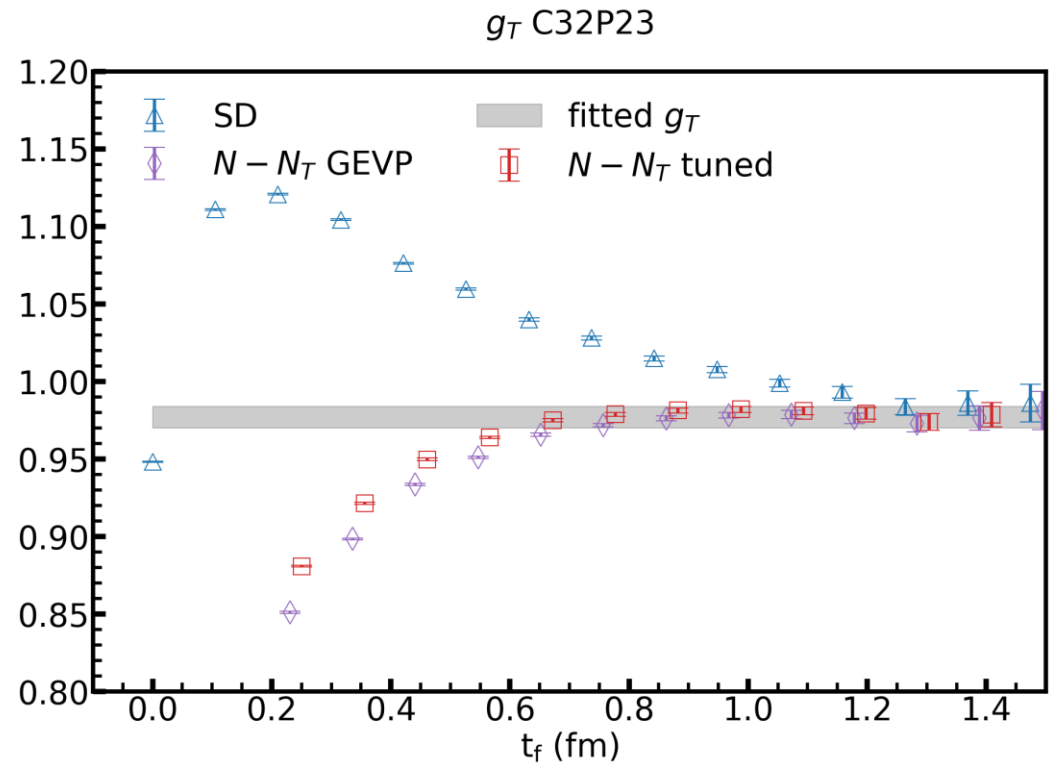
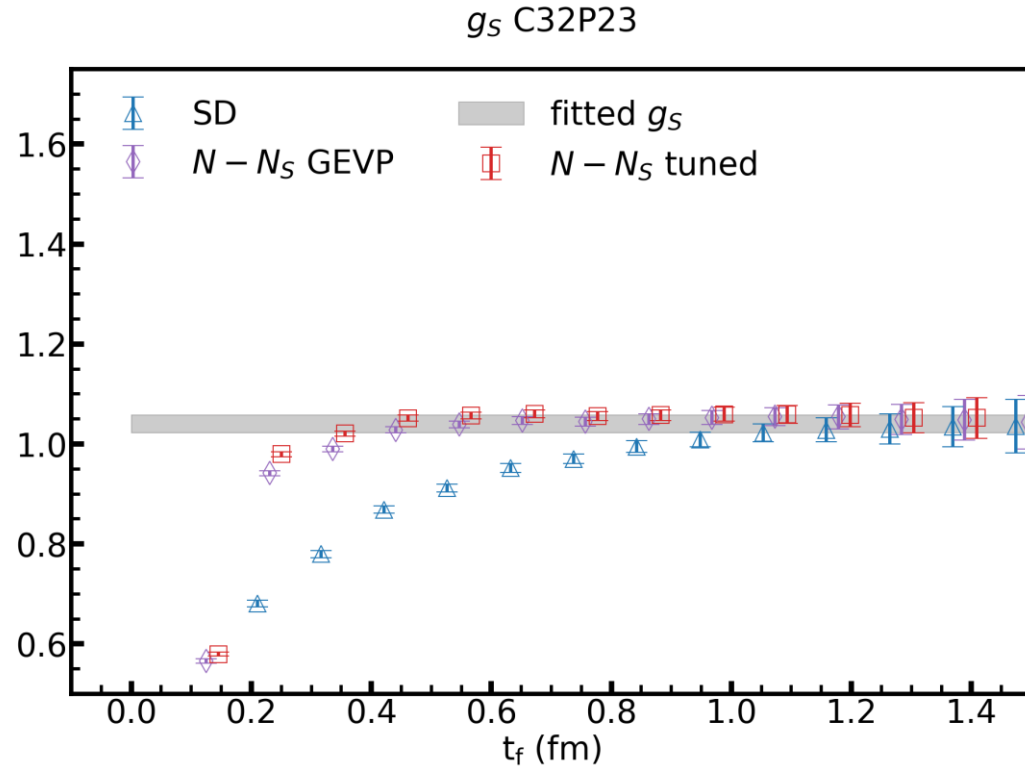
ground-state coefficient



Take $v(t_0, t)$ at $t_0 \approx 0.4$ fm and $t \approx 0.8$ fm

current-involved interpolating fields

Comparison of Standard and GEVP Results

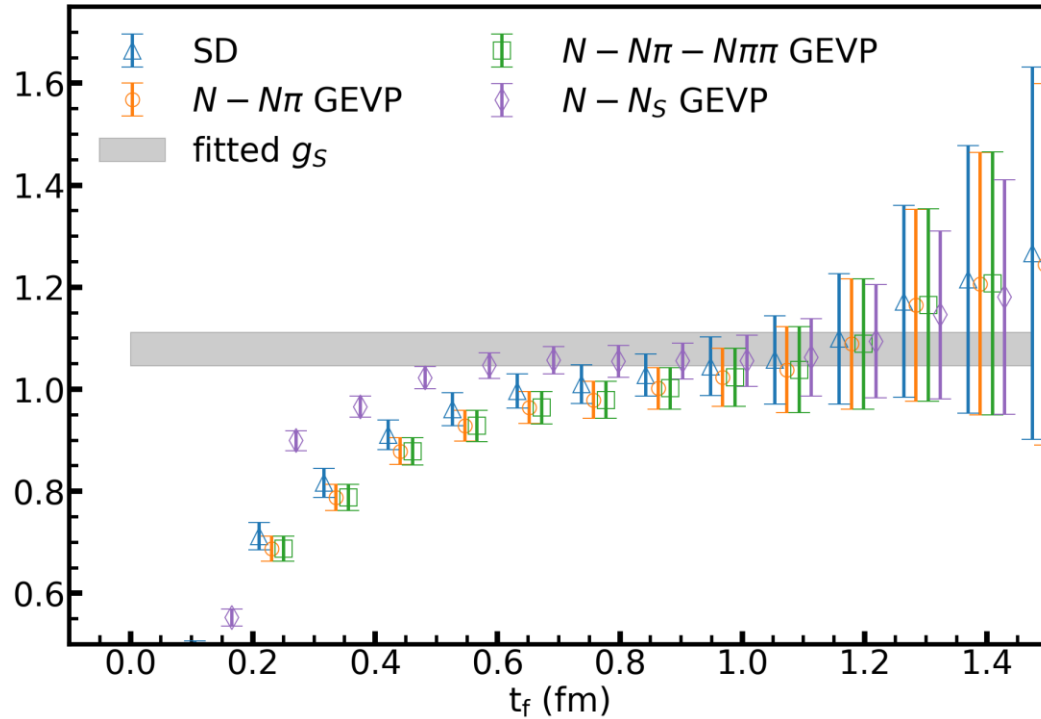


- The current-involved interpolating field can control excited-state effects very significant !
- The tuned results in [1] is very close the GEVP results

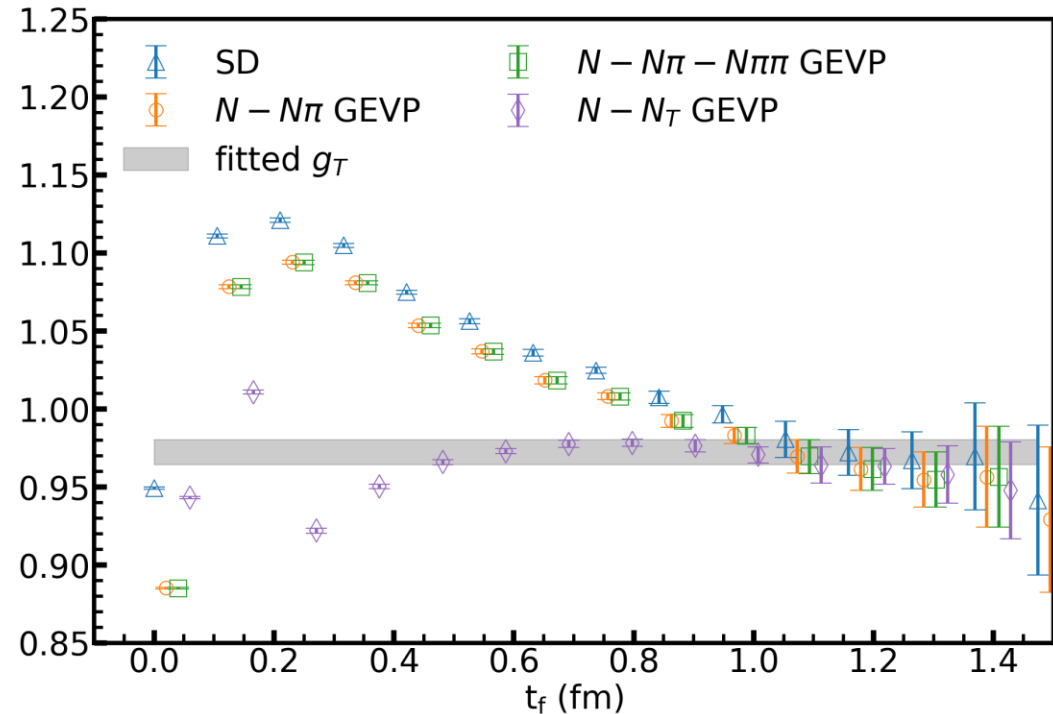
current-involved interpolating fields

Comparison on physical pion mass ensemble

g_S C48P14



g_T C48P14

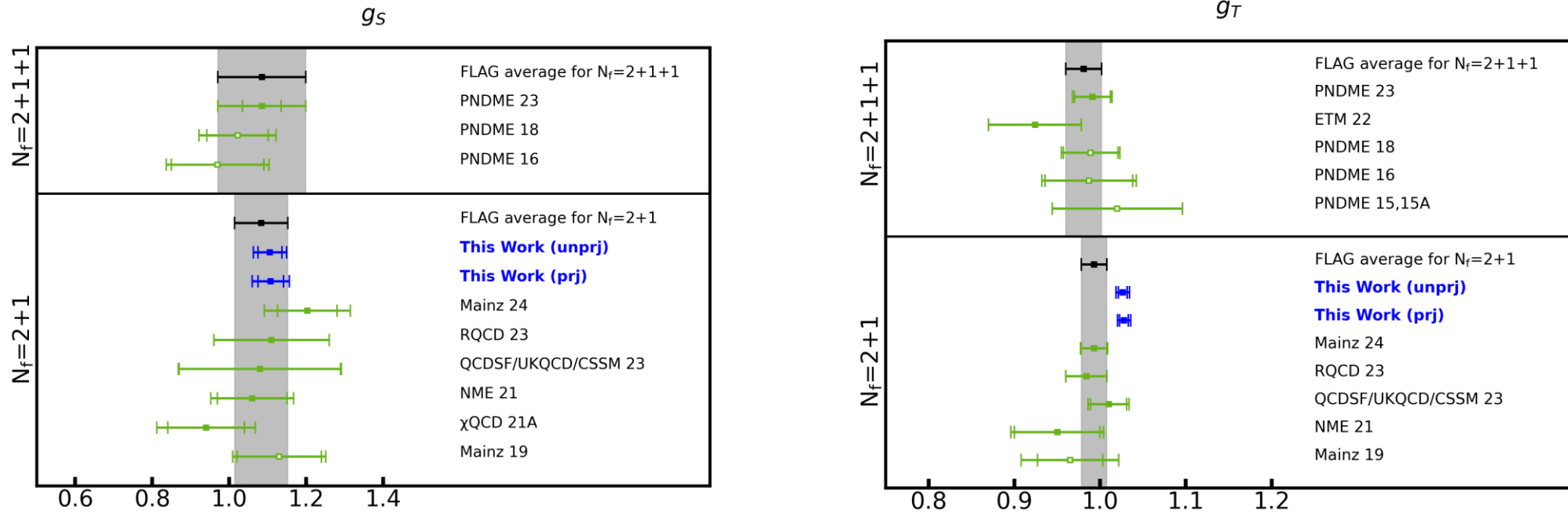


The current-involved interpolating field is very effective even on ensemble with physical pion mass

Results

J.H Wang, Z.C Hu et.al arxiv:2511.02326[hep-lat]

By using blending method and current-involved interpolating fields (tunning coefficient), we can obtain highly precise results for g_S and g_T



The results are consistent between unprojection and projection

Final Results

$$g_T^{\text{QCD}} = 1.0280[75]_{\text{tot}}(51)_{\text{stat}}(20)_a(48)_{\text{FV}}(01)_\chi(16)_{\text{ex}}(05)_{\text{re}},$$

$$g_S^{\text{QCD}} = 1.108[48]_{\text{tot}}(33)_{\text{stat}}(10)_a(29)_{\text{FV}}(01)_\chi(15)_{\text{ex}}(08)_{\text{re}},$$

Summary

- The contribution of $N\pi$ and $N\pi\pi$ to the excited state contamination is not significant.
- The current-involved interpolating field can control excited-state effects very significant, even on physical pion mass ensemble

