



華中師範大學
CENTRAL CHINA NORMAL UNIVERSITY



Nuclear Science
Computing Center at CCNU

Chiral and $U(1)_A$ Symmetries in Background Magnetic Fields from Lattice QCD

Dan Zhang (张丹)

Central China Normal University

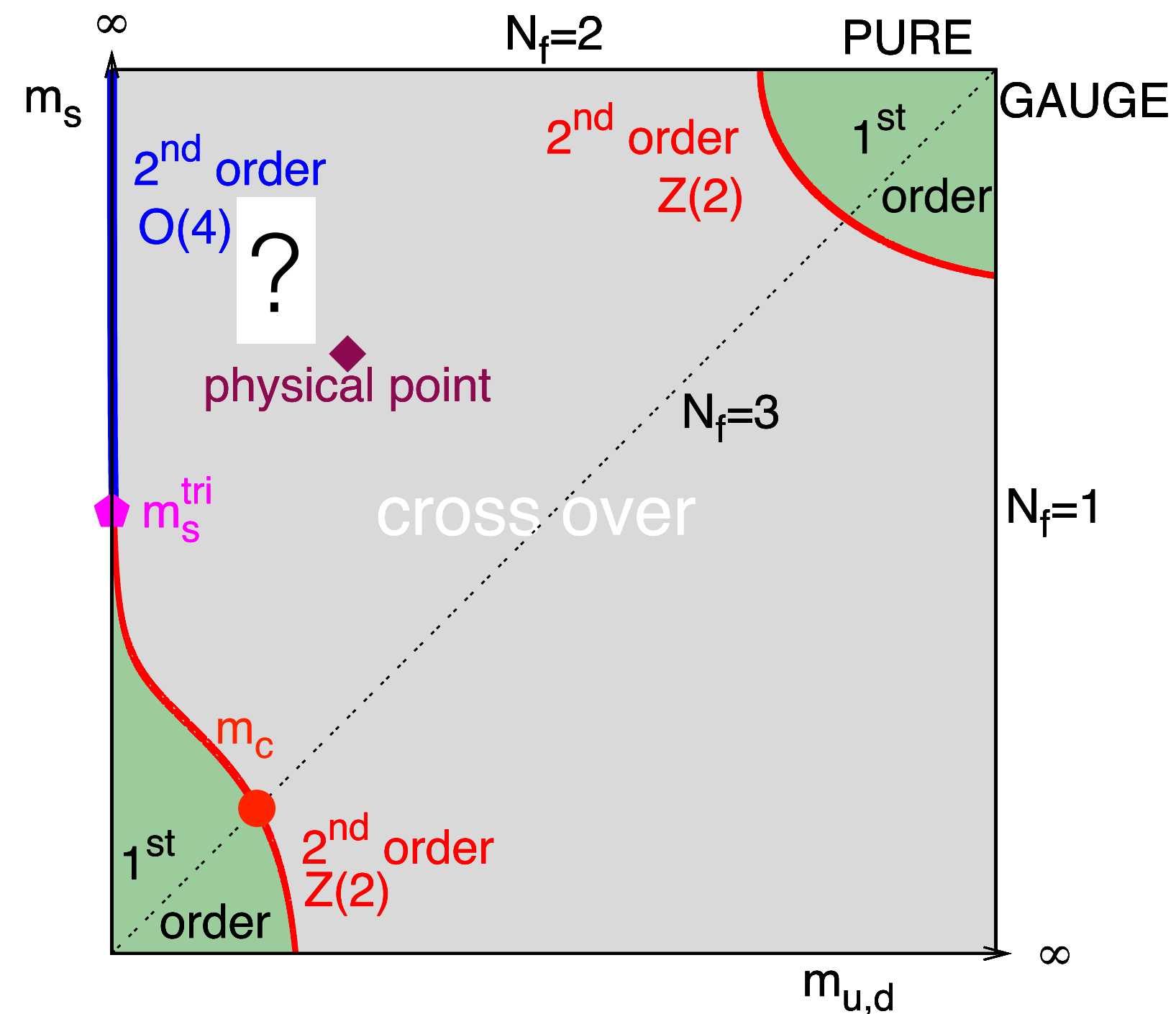
In collaboration with Heng-Tong Ding and José Javier Hernández Hernández

Based on: arXiv:2607.11625

第六届中国格点量子色动力学研讨会
二〇二六年十月 8-11日 @ 香港中文大学 (深圳)

Why does the axial $U(1)$ anomaly matter?

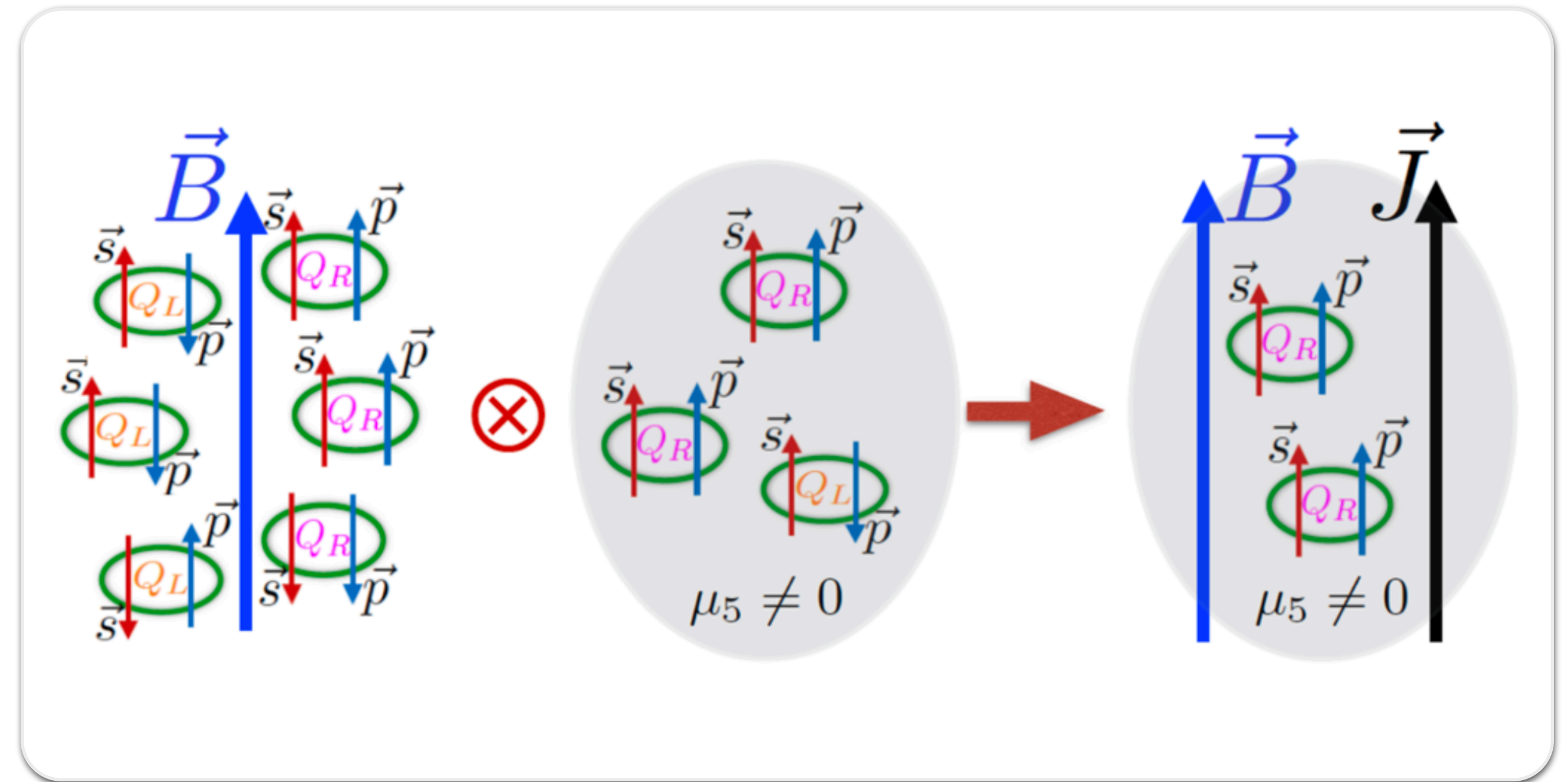
Columbia plot



HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274

Near the chiral limit, the fate of $U(1)_A$ symmetry can affect the nature of the QCD thermal transition.

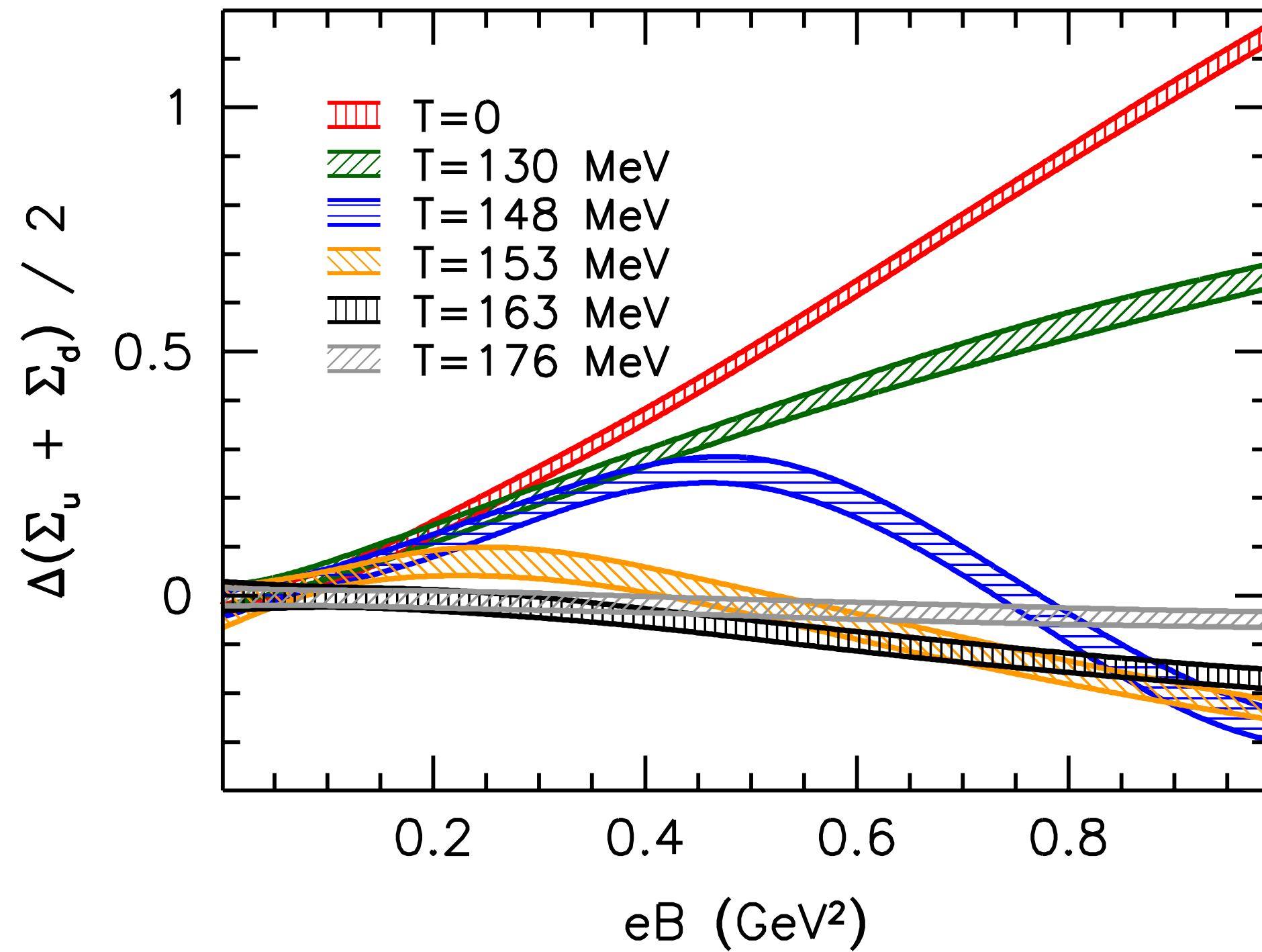
The chiral magnetic effect



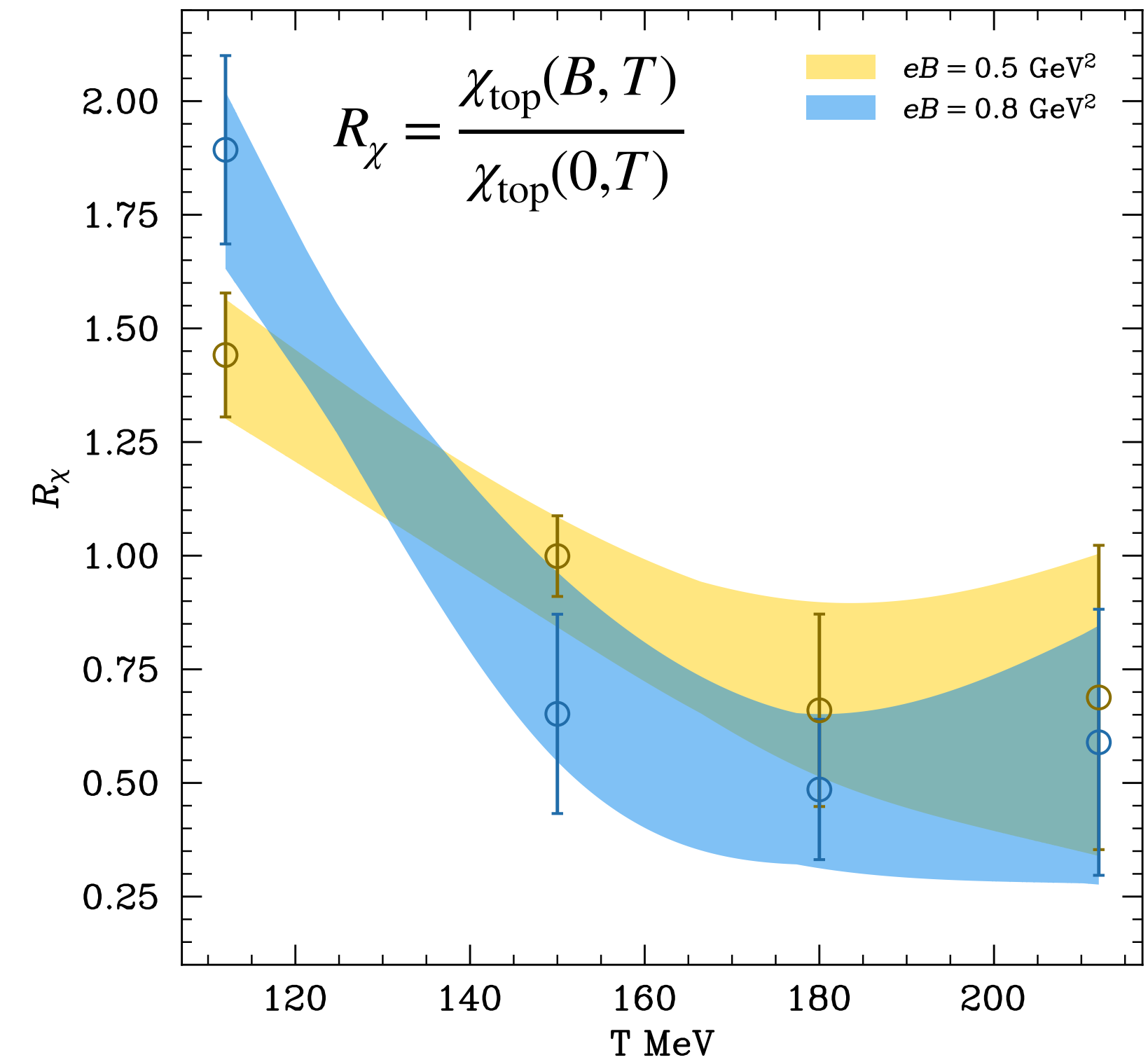
Kharzeev et al., arXiv:1511.04050

Topology, axial-charge fluctuations, and magnetic fields are connected, as illustrated by the chiral magnetic effect.

Magnetic fields reshape chiral symmetry and topology



Bali et al., arXiv:1206.4205



Brandt et al., arXiv:2409.00796

Low T : magnetic catalysis (MC)

Near T_{pc} : inverse magnetic catalysis (IMC)

Low T : χ_{top} is enhanced by B

Near T_{pc} : IMC-like response

At $B = 0$: classical symmetries and their realization

Two-flavor chiral limit

► Classical light-quark symmetry: $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$

► Decompose the non-singlet part:

Vector direction

$$U_L = U_R \implies SU(2)_V$$

preserved by the vacuum

Axial direction

$$U_L = U_R^{-1} \implies SU(2)_A$$

spontaneously broken

► Singlet axial symmetry $U(1)_A$: anomalously broken

► Remaining unbroken symmetry: $SU(2)_V \times U(1)_V$

At $B \neq 0$: the magnetic field selects a neutral flavor direction

- ▶ Magnetic field couples through electric charge

$$Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1}{6}\mathbf{1} + \frac{1}{2}\tau^3,$$

- ▶ Symmetry condition: $[\mathcal{T}, Q] = 0$

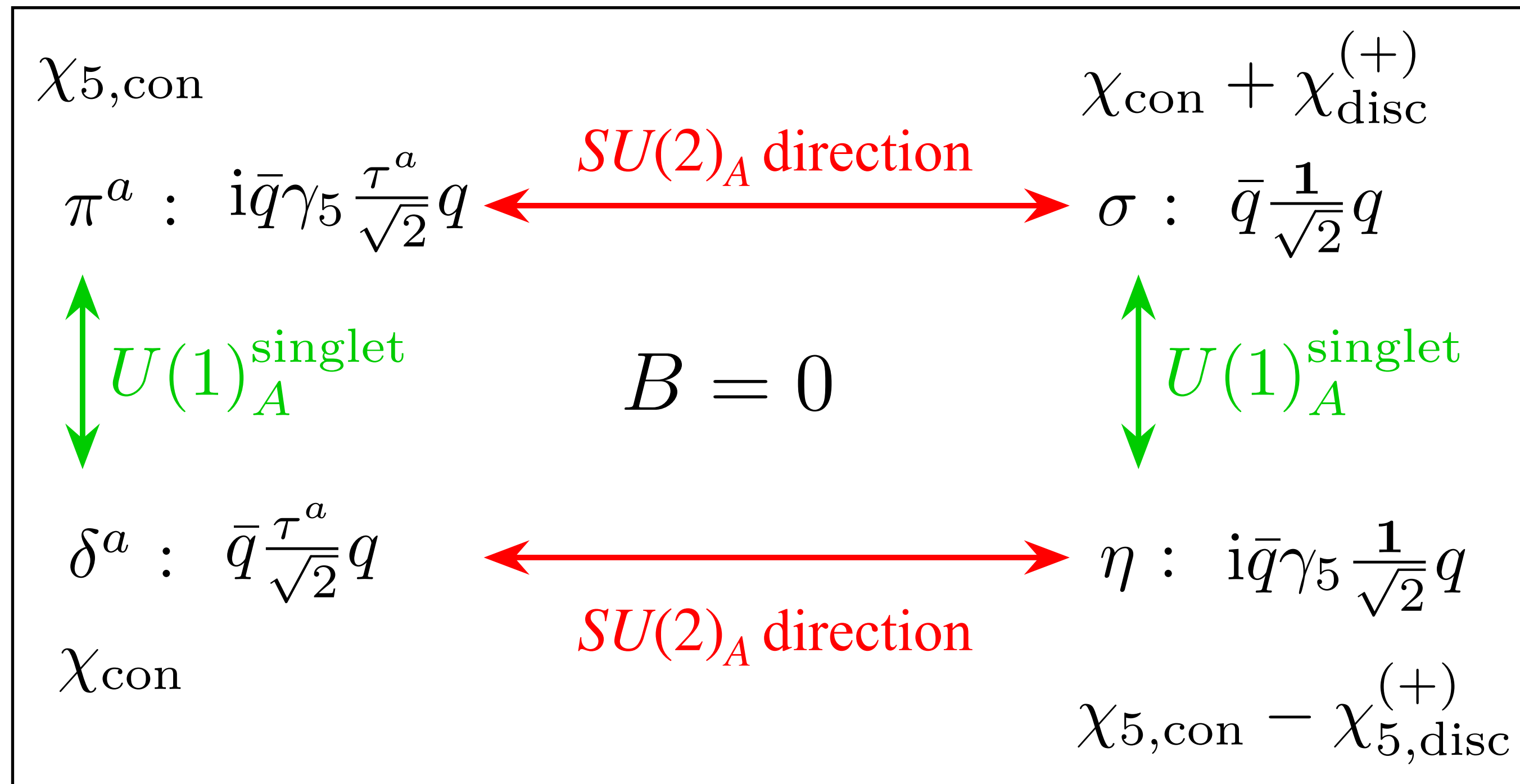
- 📌 $[\tau^3, Q] = 0$: neutral flavor direction survives

- 📌 $[\tau^{1,2}, Q] \neq 0$: charged directions are explicitly broken by B

- ▶ Surviving neutral directions:
$$\begin{cases} SU(2)_V \longrightarrow U(1)_V^{(3)} \\ SU(2)_A \longrightarrow U(1)_A^{(3)} \end{cases}$$

The superscript $^{(3)}$ denotes the neutral flavor direction.

Chiral and $U(1)_A$ patterns at $B = 0$



$SU(2)_A: \quad \pi \leftrightarrow \sigma, \quad \delta \leftrightarrow \eta$

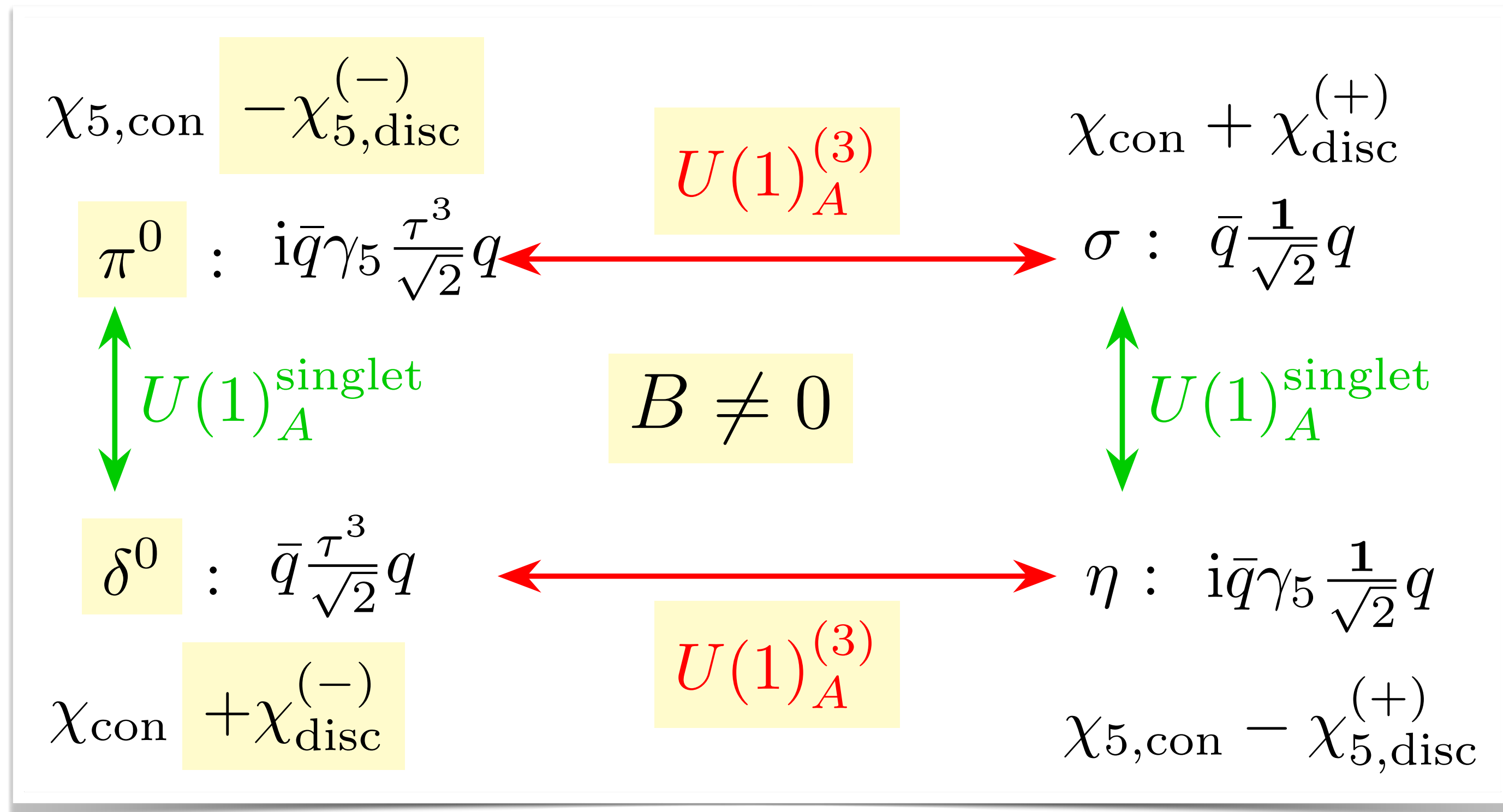
$U(1)_A^{\text{singlet}}: \quad \pi \leftrightarrow \delta, \quad \sigma \leftrightarrow \eta$

HotQCD Collaboration, PRD 86, 094503 (2012).

► At $B = 0$, exact isospin makes the three isovector components equivalent

Neutral sector partners at $B \neq 0$

- At $B \neq 0$, charged channels are no longer symmetry-equivalent to the neutral direction.



- $U(1)_A^{(3)} : \quad \pi^0 \leftrightarrow \sigma, \quad \delta^0 \leftrightarrow \eta$

Probe: $O_\chi \propto \chi_{\pi^0} - \chi_\sigma$

- $U(1)_A^{\text{singlet}} : \quad \pi^0 \leftrightarrow \delta^0, \quad \sigma \leftrightarrow \eta$

Probe: $O_A \propto \chi_{\pi^0} - \chi_{\delta^0}$

$\Rightarrow O_A, O_\chi \simeq 0 \Rightarrow$ approximate partner degeneracy

- ◆ **Flavors:** $N_f = 2 + 1$
- ◆ **Action:** Highly Improved Staggered Quarks (HISQ) and Tree-level Improved Symanzik action
- ◆ **Quark masses:** $m_l = m_s^{\text{phys}}/10$ ($am_s = 0.0506$); $M_\pi \approx 220$ MeV at $B = 0$
- ◆ **Fixed scale:** $32^3 \times N_\tau$, $a \simeq 0.117$ fm; $\beta = 6.680$
- ◆ **eB range:** $0 \leq eB \leq 2.51$ GeV² ($0 \sim 50m_\pi^2$)
- ❖ **Quantized:** $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$
 $N_b = 0, 1, 2, 3, 4, 6, 8, 10, 12, 16, 20, 24, 32, 40, 48$ (magnetic fluxes)

The results use two primary neutral-sector splittings

• Neutral non-singlet chiral-partner splitting

$$O_{\chi}(B, T) \equiv \frac{m_s^2}{f_K^4} [\chi_{\pi^0}(B, T) - \chi_{\sigma}(B, T)]$$

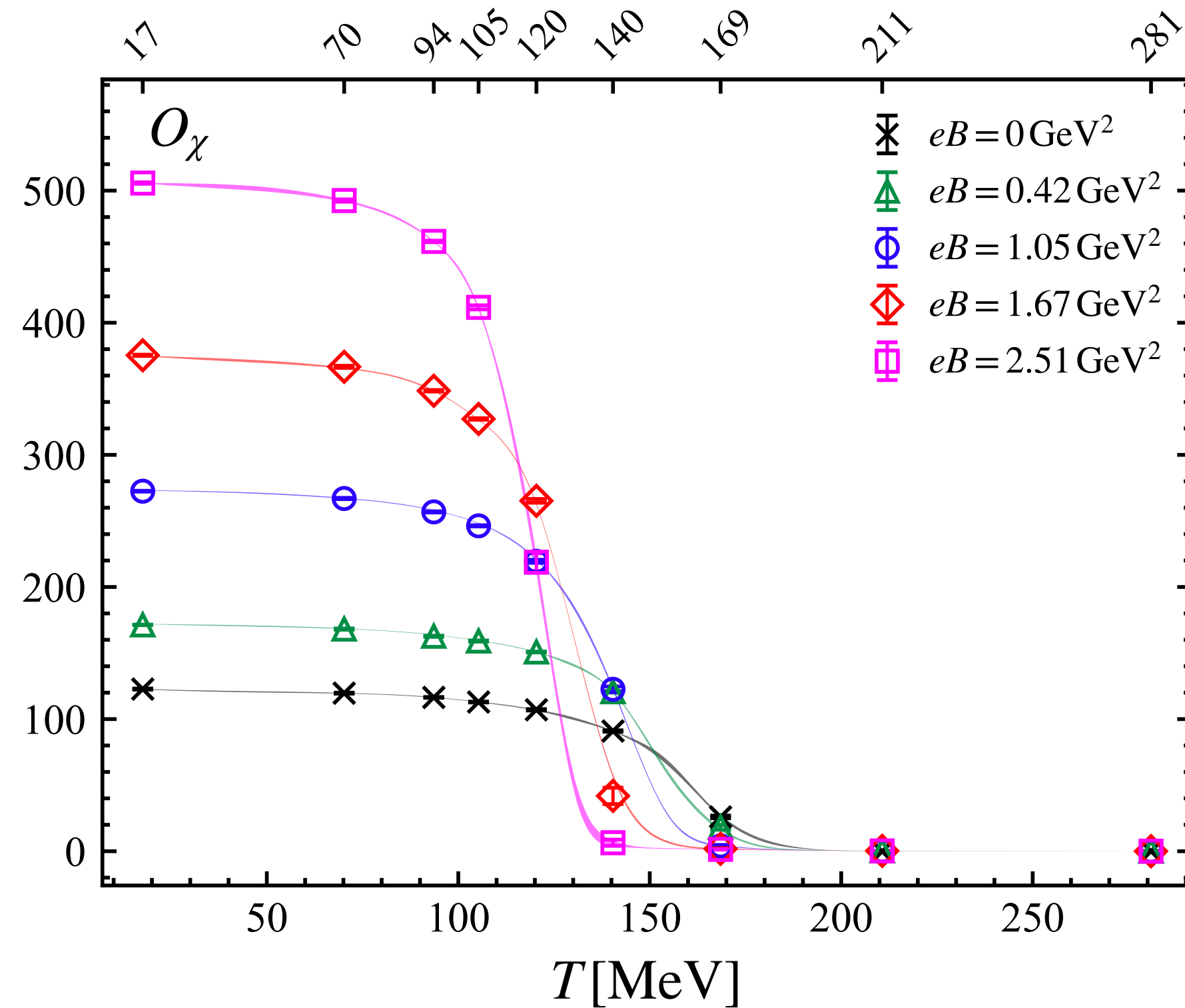
• Singlet $U(1)_A$ partner splitting

$$O_A(B, T) \equiv \frac{m_s^2}{f_K^4} [\chi_{\pi^0}(B, T) - \chi_{\delta^0}(B, T)]$$

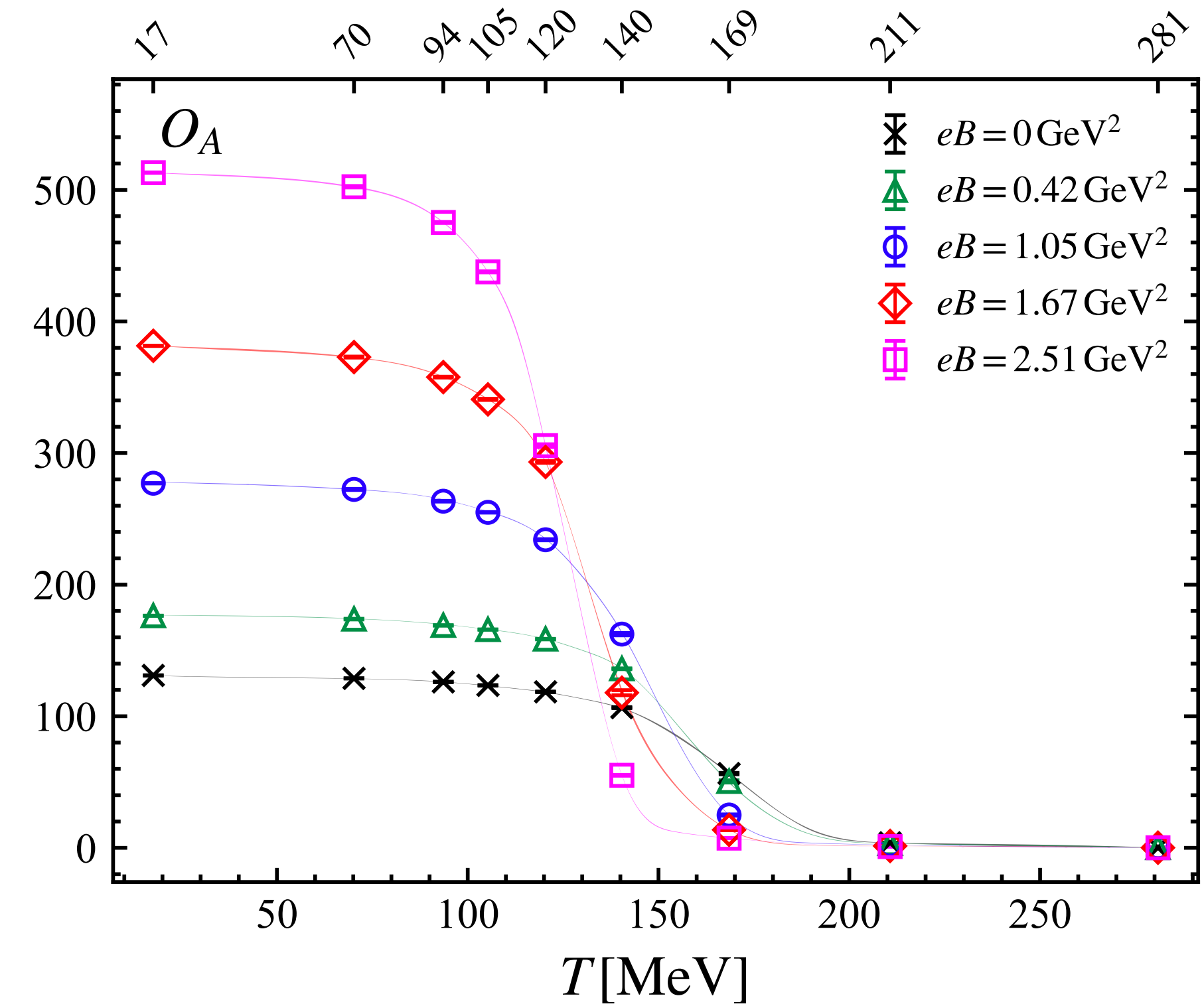
- non-singlet axial Ward identity, integrated over spacetime: $2m_l \chi_{\pi^0} = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle$

Thermal response: both splittings are suppressed

Measure of chiral symmetry breaking



Measure of singlet $U(1)_A$ breaking

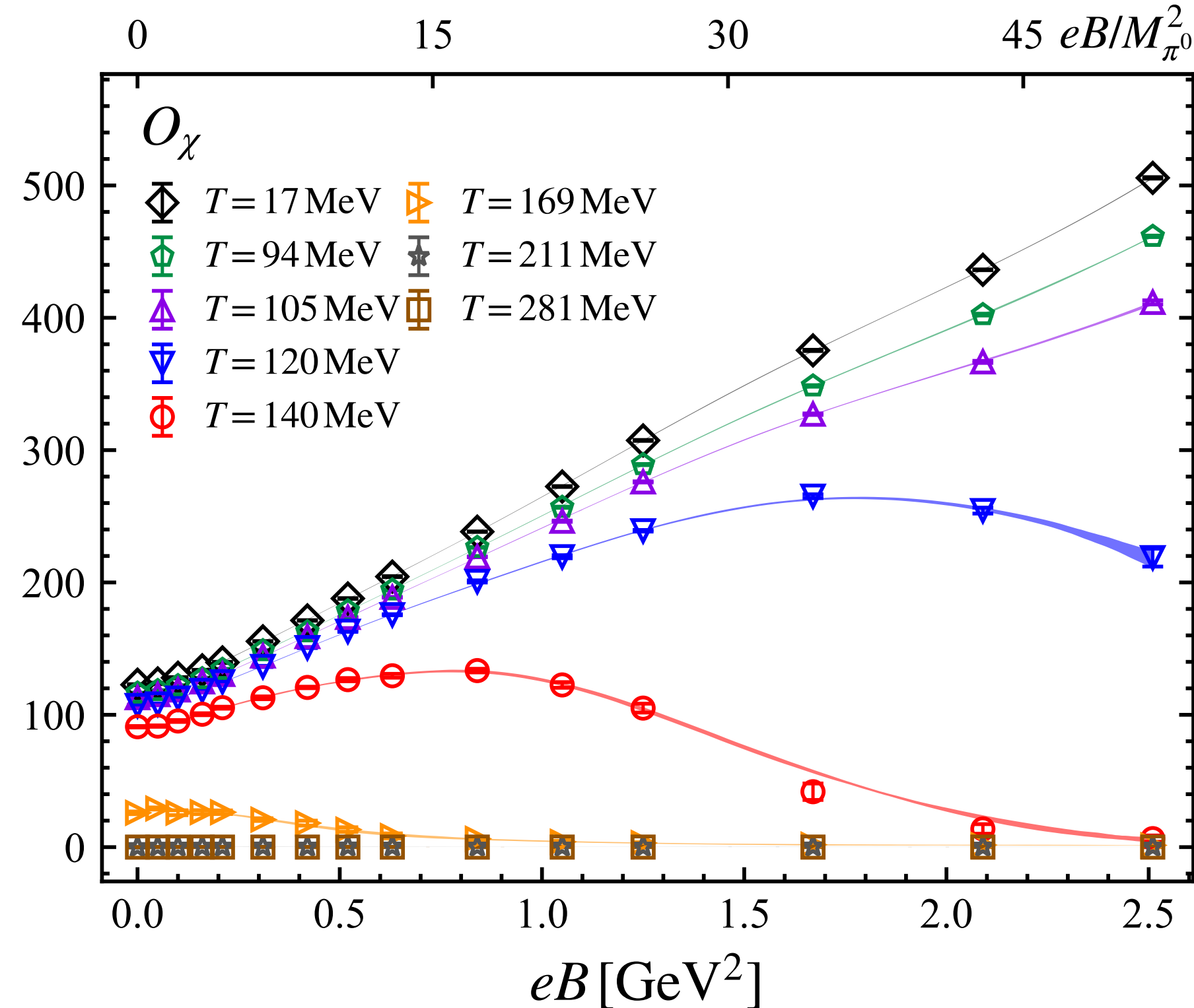


- ☑ O_χ is rapidly reduced between $T = 120$ and 169 MeV, and is nearly zero at $T = 211, 281$ MeV.
- ☑ O_A follows the same trend, but is more persistent.
- ☑ Neutral-chiral degeneracy is approached before singlet- $U(1)_A$ breaking effects become negligible.

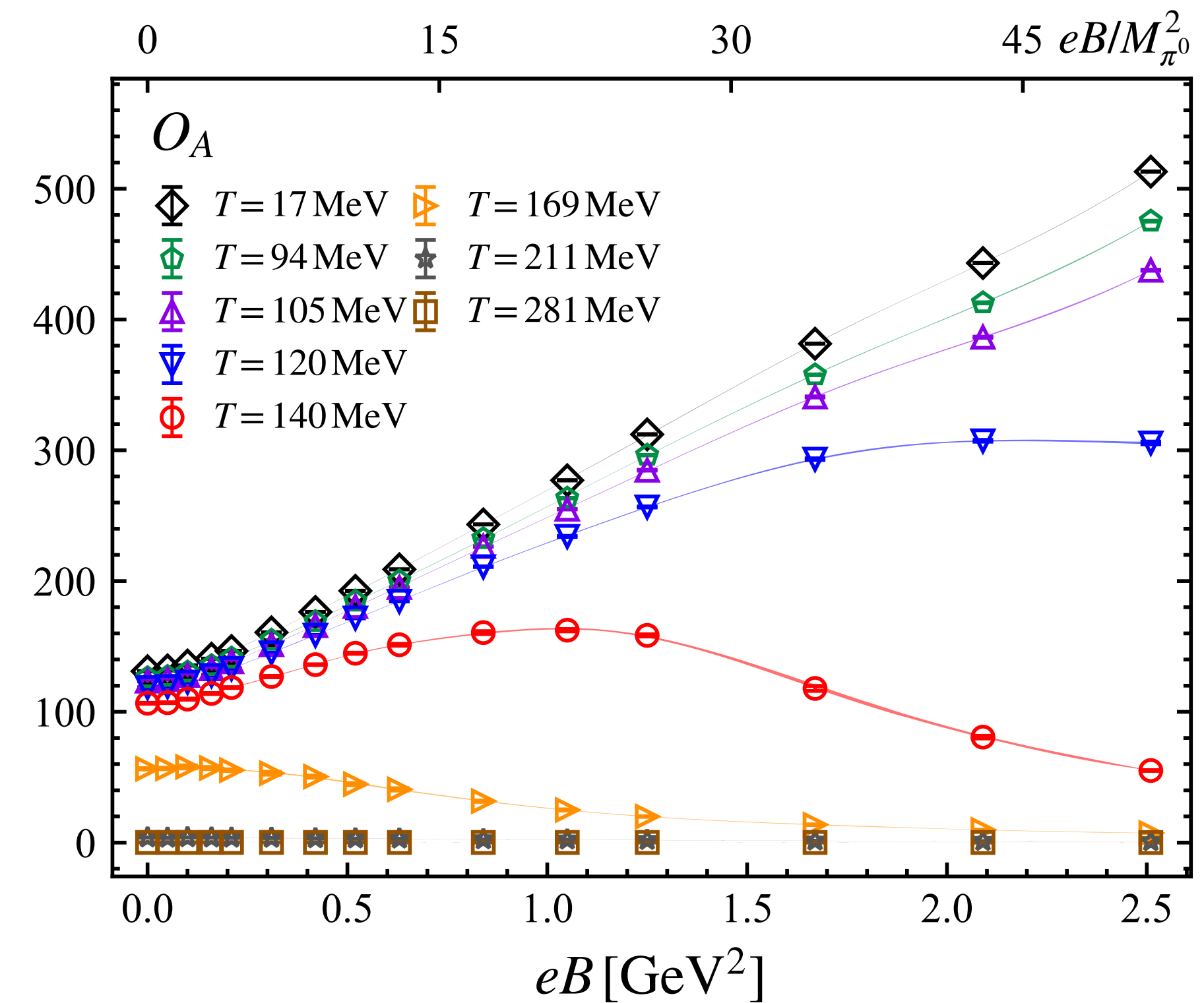
Magnetic response: MC-like enhancement and IMC-like reduction



Measure of chiral symmetry breaking



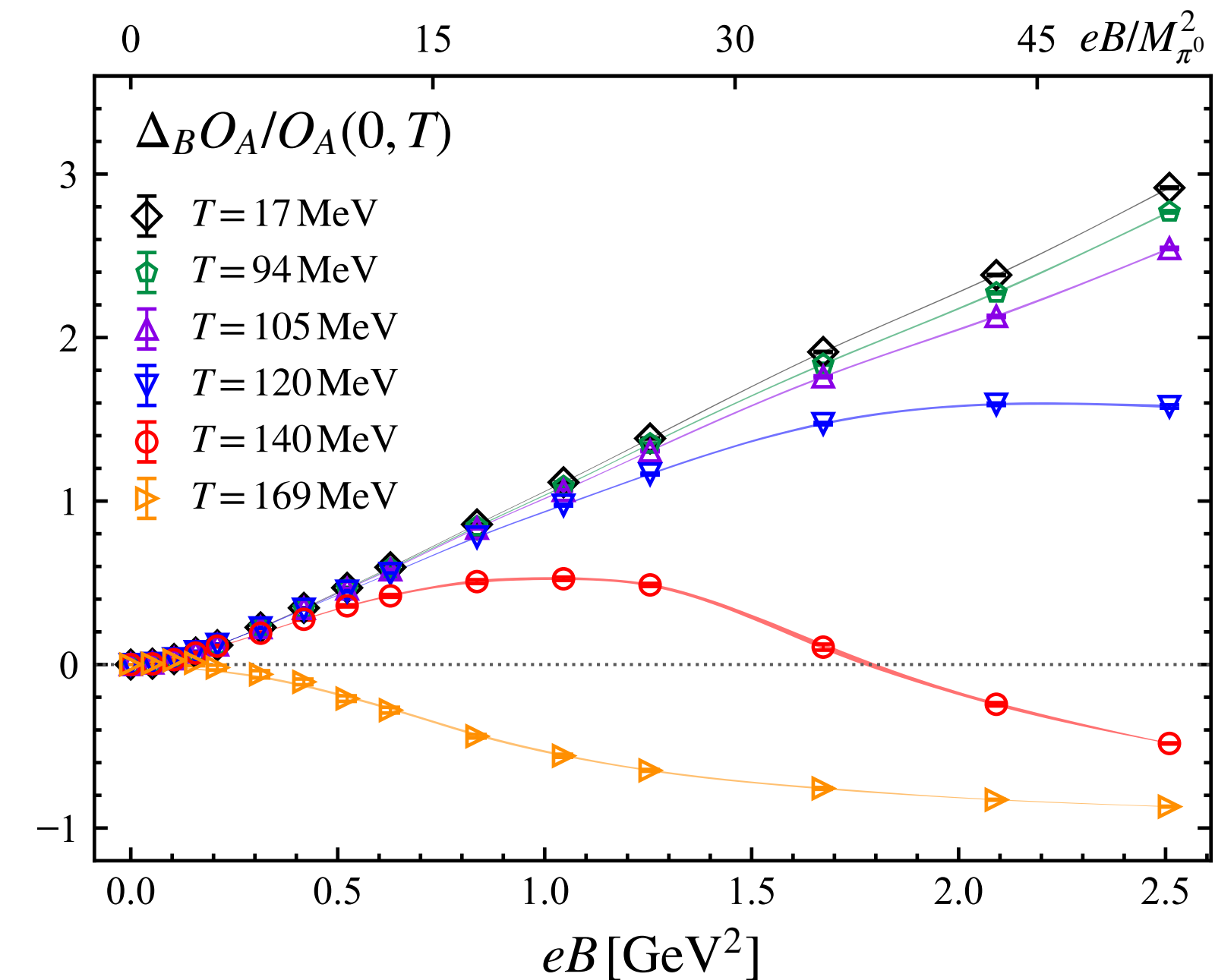
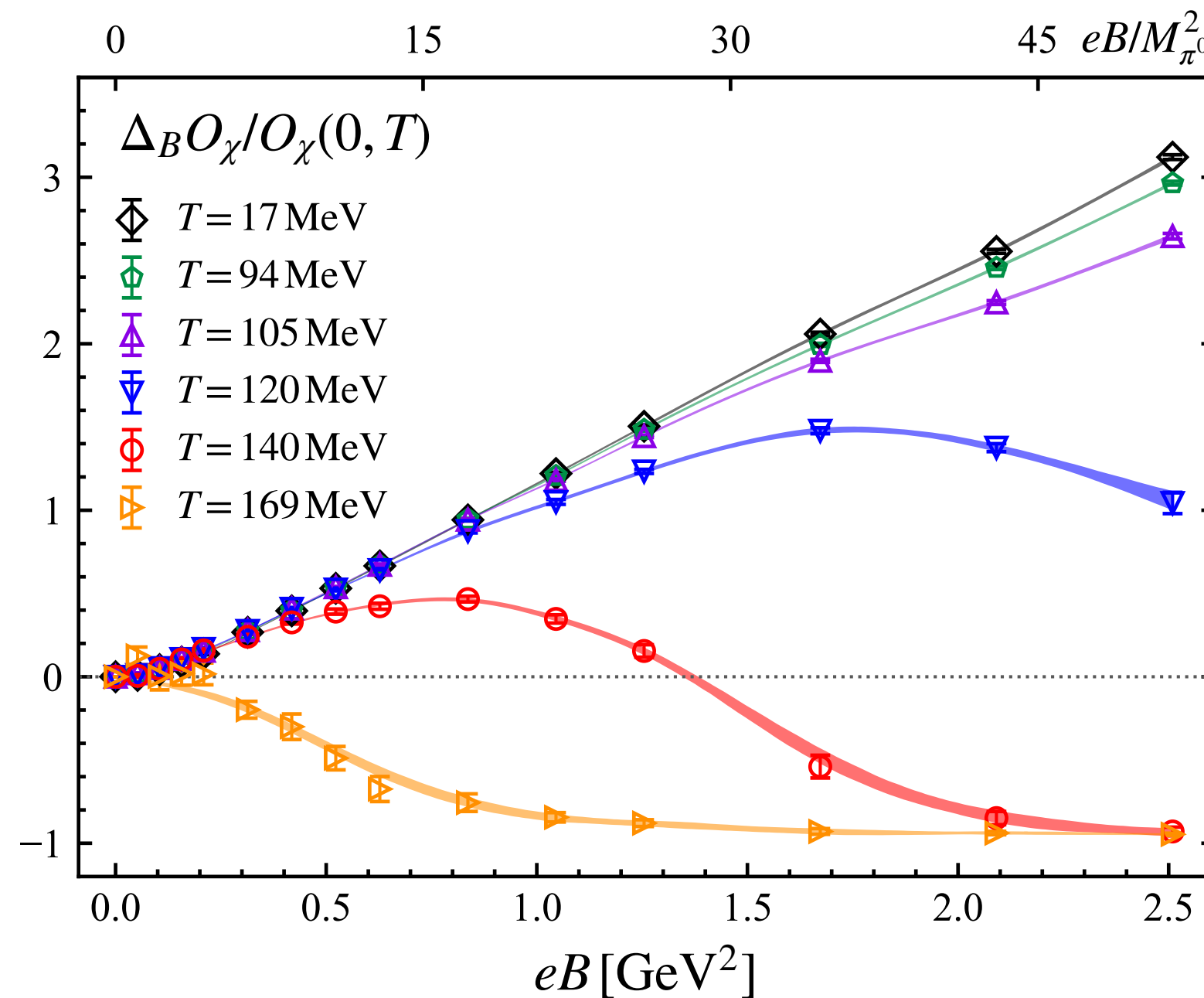
Measure of singlet $U(1)_A$ breaking



- ☑ Low T : both increase monotonically with eB ; MC counterpart in O_χ ; MC-like enhancement in O_A .
- ☑ $T = 120$ MeV: O_χ turns over at large eB , while O_A continues to increase and tends to saturate.
- ☑ $T = 140$ MeV: both become nonmonotonic; the decrease of O_A sets in at larger eB and is milder.
- ☑ $T = 169$ MeV: both splittings are already small and are further reduced by eB .

Relative magnetic response of O_χ and O_A

$$R_{O_i}(eB, T) \equiv \frac{\Delta_B O_i(eB, T)}{O_i(0, T)} = \frac{O_i(eB, T) - O_i(0, T)}{O_i(0, T)}.$$



- ☑ $T = 17, 94, 105$ MeV: both responses are positive and comparable.
- ☑ $T = 120$ MeV: R_{O_A} remains positive at large eB , while R_{O_χ} decreases from its maximum.
- ☑ $T = 140$ MeV: both responses change sign at larger eB ; milder reduction in R_{O_A} .

Ratios are used mainly up to $T = 169$ MeV; at higher T , the $eB = 0$ splittings are already very small.

- A magnetic field selects the neutral flavor direction; the clean probes are

$$O_\chi \propto \chi_{\pi^0} - \chi_\sigma, \quad O_A \propto \chi_{\pi^0} - \chi_{\delta^0}.$$

- With increasing temperature, both splittings are suppressed: O_χ is reduced earlier, while O_A is more persistent.
- Low T : O_χ gives the susceptibility counterpart of MC; O_A shows an analogous MC-like enhancement.
- Near the crossover: sufficiently strong eB gives an IMC counterpart in O_χ and an IMC-like reduction in O_A .
- Relative response: the reduction of O_A sets in at larger eB and is milder than that of O_χ .

Thank You for your attention!



Backup

Based on: arXiv:2607.11625

► Flavor-diagonal scalar and pseudoscalar bilinears:

$$S_f = \bar{f}f, \quad P_f = i\bar{f}\gamma_5 f, \quad f = u, d$$

► The neutral pseudoscalar and scalar operators:

$$\pi^0 = \frac{P_u - P_d}{\sqrt{2}}, \quad \eta = \frac{P_u + P_d}{\sqrt{2}}$$

$$\delta^0 = \frac{S_u - S_d}{\sqrt{2}}, \quad \sigma = \frac{S_u + S_d}{\sqrt{2}}$$

► Susceptibility of $\mathcal{O} = \pi^0, \eta, \delta^0, \sigma$:

$$\chi_{\mathcal{O}} = \sum_x [\langle \mathcal{O}(x)\mathcal{O}(0) \rangle - \langle \mathcal{O} \rangle^2].$$

- ♦ The Wick contractions of $\chi_{\mathcal{O}}$ naturally separate into connected and disconnected contributions.

► The neutral susceptibilities decompose as:

$$\chi_{\pi^0} = \chi_{5,\text{con}} - \chi_{5,\text{disc}}^{(-)}, \quad \chi_{\eta} = \chi_{5,\text{con}} - \chi_{5,\text{disc}}^{(+)}$$

$$\chi_{\delta^0} = \chi_{\text{con}} + \chi_{\text{disc}}^{(-)}, \quad \chi_{\sigma} = \chi_{\text{con}} + \chi_{\text{disc}}^{(+)}$$

► The connected part:

$$\chi_{\text{con}} = -\frac{1}{2V_4} \langle \text{Tr } \mathcal{M}_u^{-2} + \text{Tr } \mathcal{M}_d^{-2} \rangle,$$

$$\chi_{5,\text{con}} = \frac{1}{2V_4} \langle \text{Tr } (\gamma_5 \mathcal{M}_u^{-1})^2 + \text{Tr } (\gamma_5 \mathcal{M}_d^{-1})^2 \rangle,$$

► The disconnected part:

$$\chi_{\text{disc}}^{(\pm)} = \frac{1}{2V_4} \left[\langle (X_u \pm X_d)^2 \rangle - \langle X_u \pm X_d \rangle^2 \right],$$

$$\chi_{5,\text{disc}}^{(\pm)} = \frac{1}{2V_4} \left[\langle (\mathcal{P}_u \pm \mathcal{P}_d)^2 \rangle - \langle \mathcal{P}_u \pm \mathcal{P}_d \rangle^2 \right],$$

- ♦ where $X_f = \text{Tr } \mathcal{M}_f^{-1}$, $\mathcal{P}_f = \text{Tr } \gamma_5 \mathcal{M}_f^{-1}$.

► At $B \neq 0$, $q_u \neq q_d$, the neutral-pion channel can contain a local disconnected contribution.

► The integrated pseudoscalar loop,

$$\mathcal{P}_f = \text{Tr } \gamma_5 \mathcal{M}_f^{-1},$$

is an index-type quantity in the continuum: $\mathcal{P}_f = \frac{\text{index}(D_f)}{m_l}$.

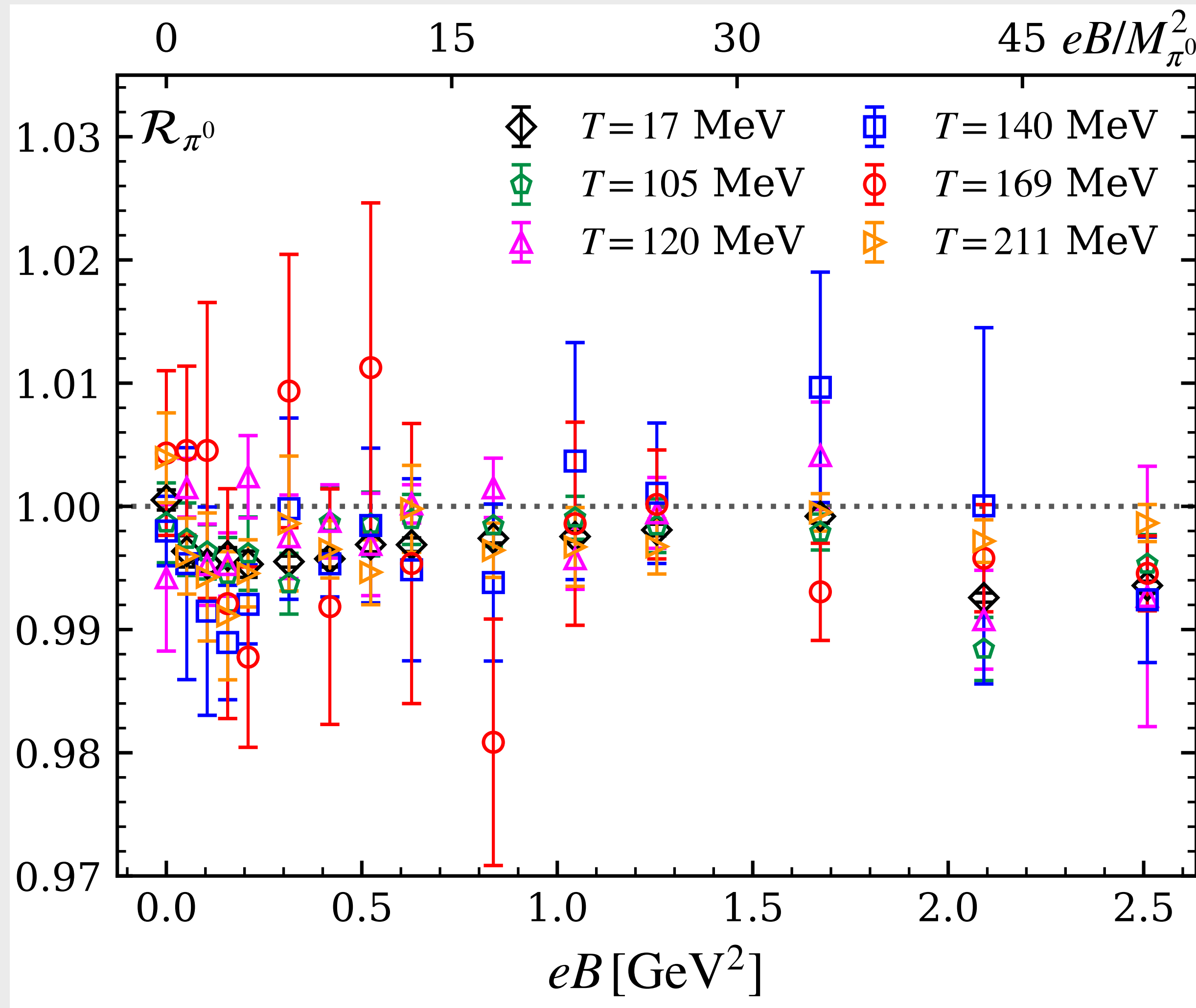
► For a pure magnetic background: $F^{\text{em}} \widetilde{F}^{\text{em}} \propto \mathbf{E} \cdot \mathbf{B} = 0$.

► Therefore the remaining index is controlled by the flavor-blind gluonic topological charge: $\mathcal{P}_u = \mathcal{P}_d$.

⇒ Integrated consequence: $\chi_{5,\text{disc}}^{(-)} = 0$.

- This is an integrated continuum statement, not a local equality.

► Direct $\chi_{5,\text{disc}}^{(\pm)}$ measurements are noisy and sensitive to topology, zero modes, and staggered-taste effects.



- ◆ Use the non-singlet axial Ward identity

$$2m_l \chi_{\pi^0} = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle.$$

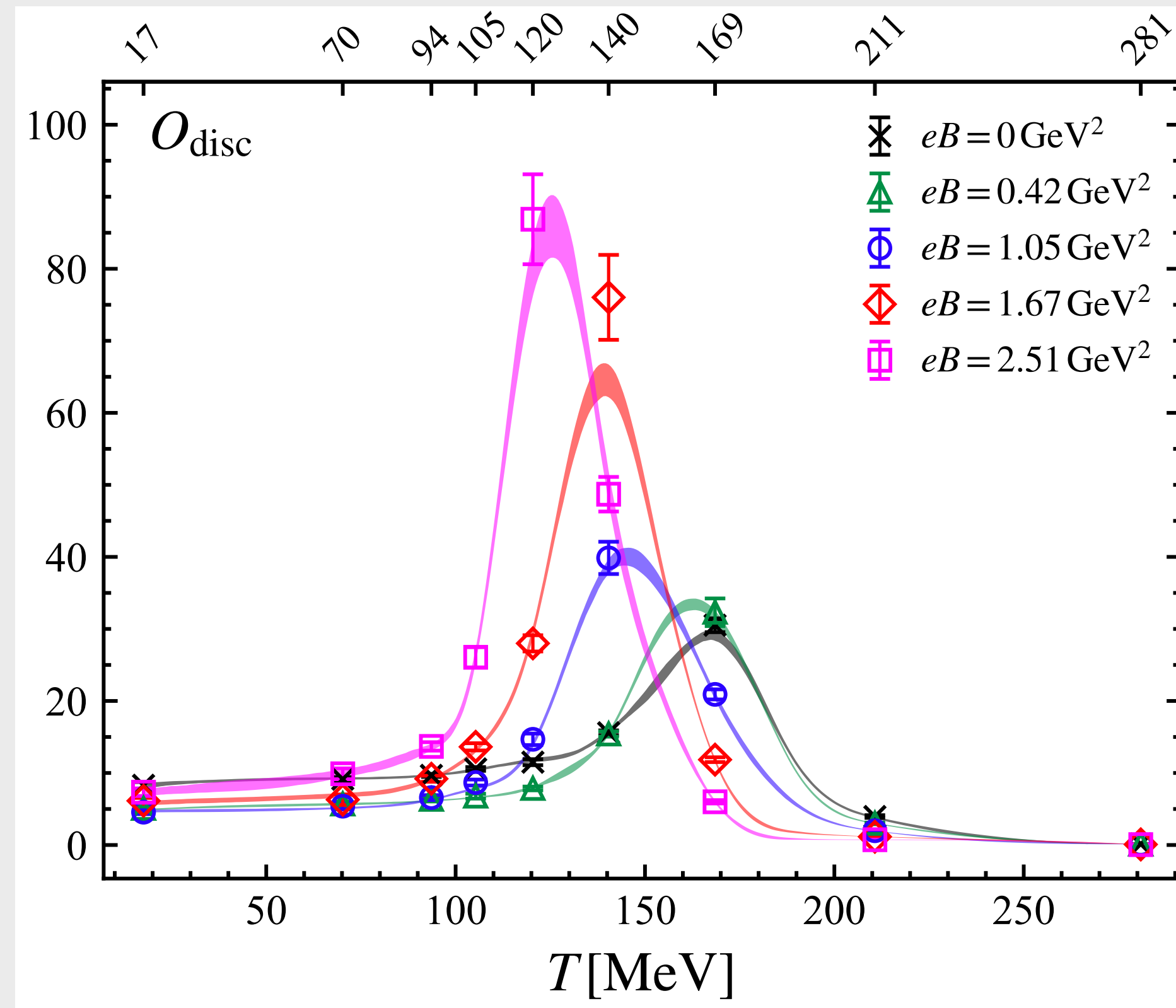
- ◆ Check against the connected pion susceptibility:

$$\mathcal{R}_{\pi^0} = \frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{2m_l \chi_{5,\text{con}}}.$$

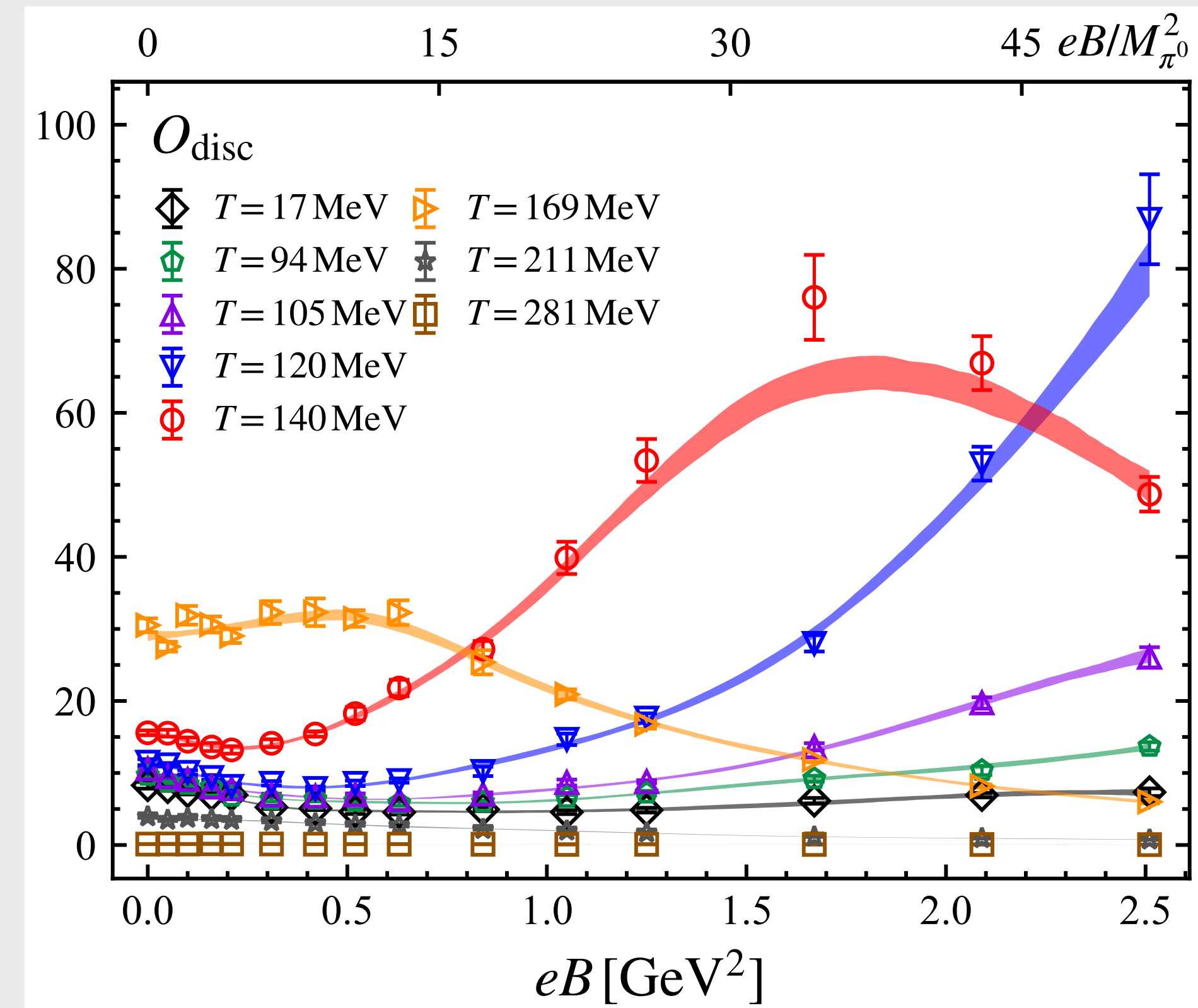
- ◆ $\mathcal{R}_{\pi^0} \approx 1$ within 2% for all studied T and eB .
- ◆ This supports $\chi_{5,\text{disc}}^{(-)} \approx 0$ in the present lattice setup.

Difference between two splittings: $O_{\text{disc}} = O_A - O_\chi$

$$O_{\text{disc}}(B, T) \equiv O_A(B, T) - O_\chi(B, T) = \frac{m_s^2}{f_K^4} \left[\chi_{\text{disc}}^{(+)}(B, T) - \chi_{\text{disc}}^{(-)}(B, T) \right]$$



☑ Small at low and high T ; peaked around T_{pc} .



☑ Most visible at $T = 120$ – 140 MeV.

► Zero-field reference

For these $M_\pi \simeq 220$ MeV ensembles,

$$T_{pc}(eB = 0) \simeq 170 \text{ MeV}$$

H.-T. Ding et al., Phys. Rev. D **105**, 034514 (2022)

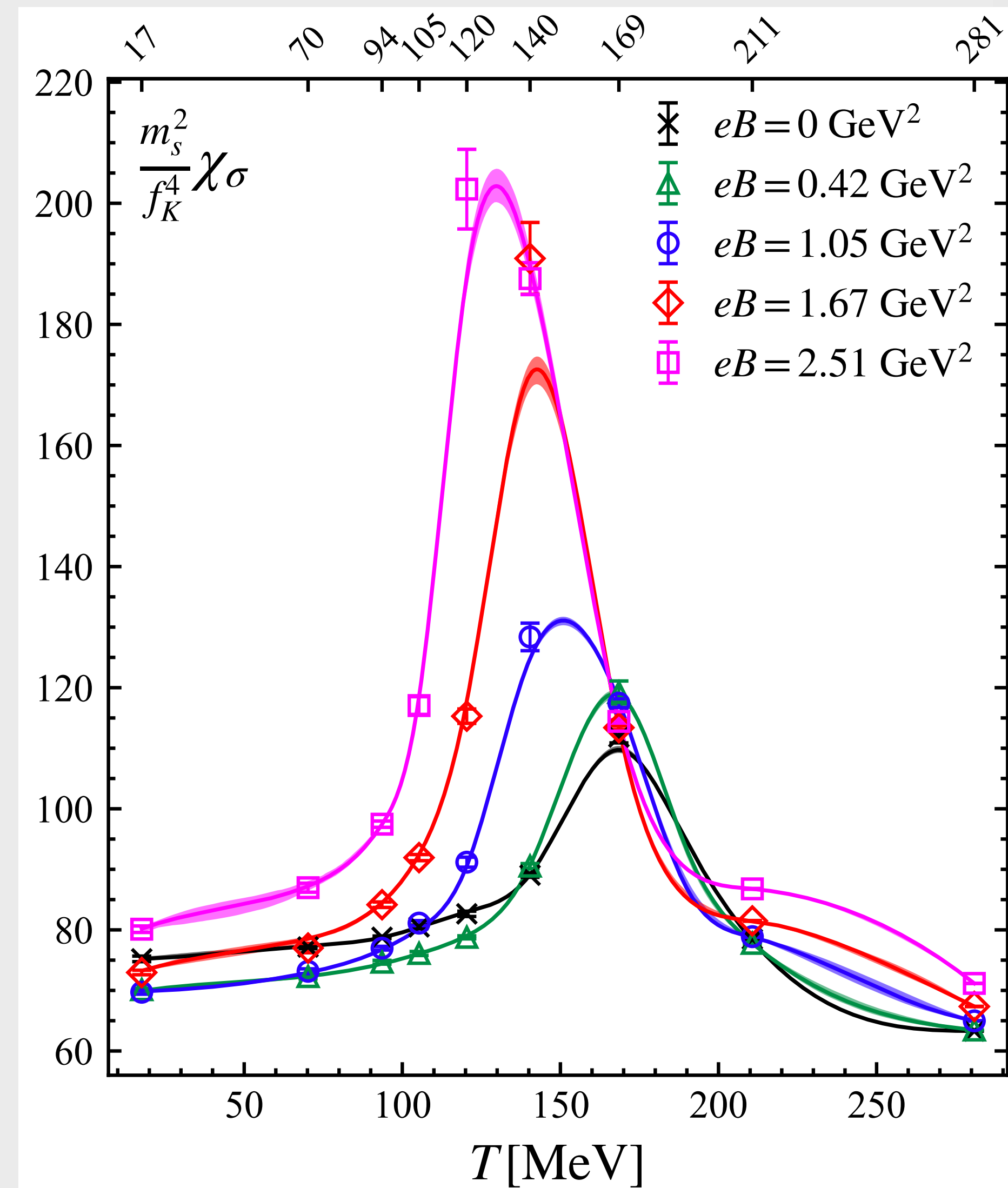
H.-T. Ding et al., Eur. Phys. J. A **57**, 202 (2021)

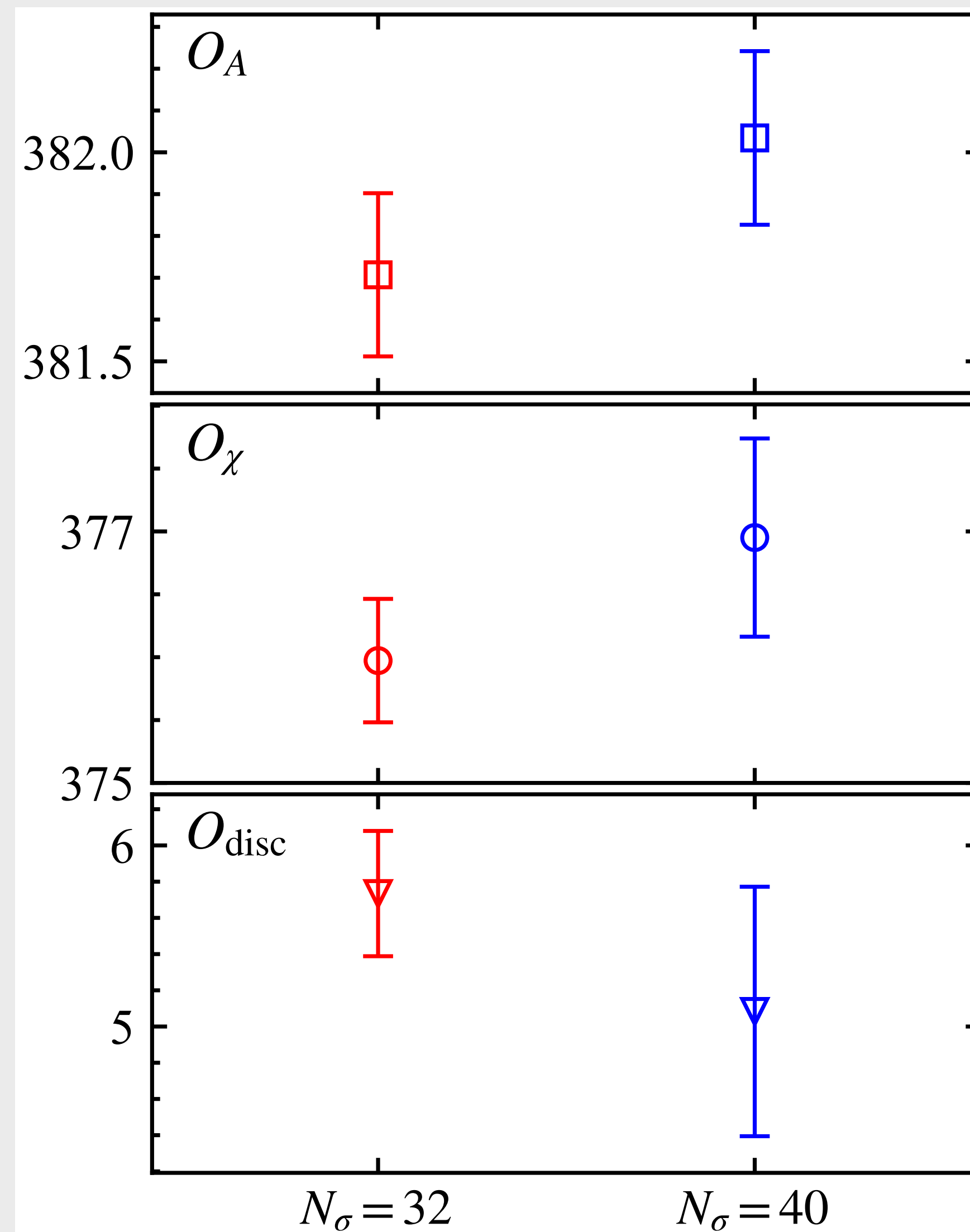
► At nonzero magnetic field

$$\chi_\sigma = \chi_{\text{con}} + \chi_{\text{disc}}^{(+)}$$

shows a broad enhancement in the crossover region.

🔊 *The temperature spacing does not allow a precise extraction of $T_{pc}(eB)$.*





- ◆ $T \simeq 17 \text{ MeV}$, $eB \simeq 1.67 \text{ GeV}^2$
- ◆ Same lattice spacing and quark masses.

	$32^3 \times 96$	$40^3 \times 96$
N_b	32	50
N_{conf}	1100	1100
Measurement spacing	10 trajectories	5 trajectories

- ◆ Equal N_b/N_s^2 ensures the same magnetic field.

- ☑ Central-value shifts below 0.3 % for O_A and O_χ .
- ☑ O_{disc} agrees within statistical uncertainties.

- ◆ Number of configurations used for the main $32^3 \times N_\tau$ ensembles.
- ◆ Measurement spacing: 10 trajectories for $N_\tau = 96, 18$; predominantly 5 for the other ensembles.

N_τ	T [MeV]	N_{conf} at fixed N_b														
		0	1	2	3	4	6	8	10	12	16	20	24	32	40	48
6	281	4800	4990	4620	4490	2610	4740	7110	2390	4390	5800	3000	5060	5220	3000	5460
8	211	5690	7110	6620	5660	6660	6490	5610	7790	7540	6520	5930	5410	4470	3000	4530
10	169	8450	8270	6890	6810	8030	6600	6550	8970	6360	6070	6160	6710	6170	10570	5750
12	140	8890	12480	9100	8430	8890	5340	6960	7600	5420	6540	6720	6820	5820	7780	8510
14	120	4480	9420	6750	7390	6440	6330	7550	6450	9280	6130	8050	6360	7630	8180	5820
16	105	6390	7500	7500	7500	7500	7500	7490	7500	6720	6590	6830	7800	7110	6990	7650
18	94	4020	4160	3680	2270	3930	4230	3690	3630	3660	3890	2370	2680	2690	2000	2610
24	70	6600	3720	2240	1380	1230	1170	1930	1170	1230	1730	1630	1010	1960	1390	2000
96	17	1480	1630	1830	1850	1510	1860	1690	2170	1390	2880	1220	1390	2000	1230	1880