



Direct determination of gluon trace anomaly contributions to baryon masses from LQCD using gradient flow

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Research Background

Origin of mass::

- SM elementary particles -> visible matter
- Higgs mechanism -> masses of elementary particles
- QCD quantum corrections (dominant) -> mass of visible matter

Trace of EMT -> hadron mass decomposition:

$$\begin{aligned} T_{\mu}^{\mu} &= H_m + (\gamma_m H_m + \frac{\beta}{2g} F^2) \\ m_H &= \langle T_{\mu}^{\mu} \rangle_H = (1 + \gamma_m) \sigma_H + \langle H_g^a \rangle_H \\ &= (1 + \gamma_m) \langle m_q \bar{q}q \rangle_H + \hat{\beta} \langle g^2 \text{Tr}[G_{\rho\sigma} G_{\rho\sigma}] \rangle_H \end{aligned} \quad (1)$$

Current Status

Status & Motivation

Sum rules: trace anomaly significant in nucleon and charmed-baryon masses [1][2].

First direct calc.: trace-anomaly contributions differ between pion and nucleon [3].

Gradient-flow calc.: all EMT components and mass-decomposition sum rule verified for η_c and J/ψ [7].

Problems:

Previous direct calculations lacked nonperturbative renormalization, so sum rule (1) could not be tested quantitatively [3].

Systematic direct calculations, especially for bottomed hadrons, are still lacking.

This work:

Direct gradient-flow calc. matched to $\overline{\text{MS}}$.

Test flavor universality.

[1]Y.-B. Yang, J. Liang, Y.-J. Bi, Y. Chen, T. Draper, K.-F. Liu, and Z. Liu, Phys. Rev. Lett. 121, 212001 (2018), arXiv:1808.08677 [hep-lat].

[2]B. Hu, H. Du, X. Jiang, K.-F. Liu, P. Sun, and Y.-B. Yang, (2024), arXiv:2411.18402 [hep-lat].

[3]F. He, P. Sun, and Y.-B. Yang (χ QCD), Phys. Rev. D104, 074507 (2021), arXiv:2101.04942 [hep-lat].

[7]D. Bollweg, H.-T. Ding, X. Gao, R. Luo, and S. Mukherjee, Phys. Rev. Lett. 137, 061901 (2026), arXiv:2601.13070 [hep-lat].

Numerical Calculation: Lattice Matrix Element Extraction and Operator Definition

·2pt/3pt and $Tr[\langle g^2 F^2 \rangle]_H$ matrix element extraction [3]:

$$\begin{aligned}
 C_2(t_f; \mathcal{H}) &= \langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle \\
 SC_3^{q_s, g}(t_f; \mathcal{H}) &= \sum_{0 < t < t_f} \sum_{\vec{y}} \sum_{\vec{x}} \langle [C_2(t_f, \vec{y}; \mathcal{M}) - \langle C_2(t_f, \vec{y}; \mathcal{M}) \rangle] [O^g(t, \vec{x}) - \langle O^{q_s, g}(t, \vec{x}) \rangle] \rangle \\
 \Delta R^g(t_f) &\equiv \frac{SC_3^g(t_f; \mathcal{H})}{C_2(t_f; \mathcal{H})} - \frac{SC_3^g(t_f - 1; \mathcal{H})}{C_2(t_f - 1; \mathcal{H})}
 \end{aligned} \tag{2}$$

$$Tr[\langle g^2 F^2 \rangle]_H = \Delta R^g(t_f; H) + \mathcal{O}(e^{-\delta m t_f})$$

where:

$$\begin{aligned}
 &O_s^{loop} \\
 &\propto Tr[1 - U_s^{loop}] \xrightarrow{\text{continuum limit}} Tr[\langle g^2 F^2 \rangle]_H \\
 &U(x, y) = \text{Exp}[-ig \int_x^y dz A(z)]
 \end{aligned} \tag{3}$$

Gradient Flow

$$O_s^{loop} \propto Tr[1 - U_s^{loop}] \xrightarrow{\text{continuum limit}} Tr[\langle g^2 F^2 \rangle]_H$$

s denotes a closed Wilson loop on the lattice; different loops produce different $O(a^2)$ corrections. Therefore, we introduce gradient flow (Wilson flow) [4][5], and take η_c as an example to compare different gradient-flow schemes.

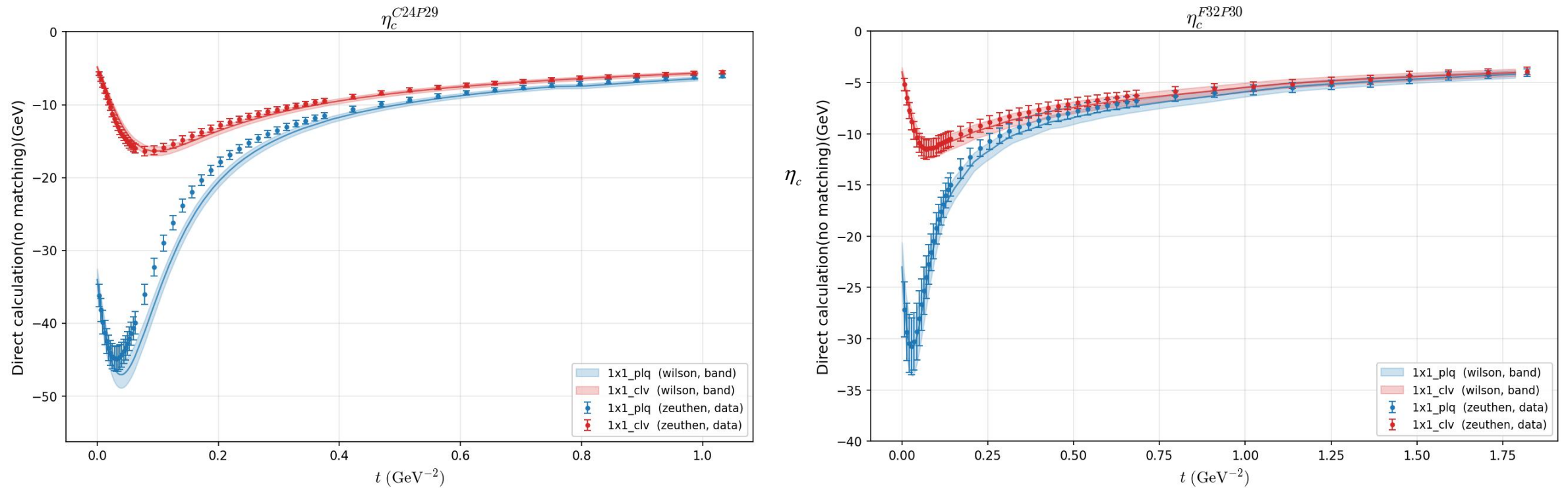


Fig. 1. Comparison of results from different gradient-flow schemes.

Gradient Flow

$$\partial_{t^{GF}} \tilde{A}_\mu(t^{GF}) = D_\nu \tilde{G}_{\nu\mu}(t^{GF}), \quad \tilde{A}_\mu(0) = A_\mu \quad (4)$$

Introduces a soft UV cutoff and suppresses high-momentum modes. As the flow time increases, the gauge field is smoothed.

Different Wilson-loop constructions tend to the same physical trajectory under gradient flow. At $t^{GF} \sim 0.1 \text{ GeV}^{-2}$, we can use 1×1 clv as the main calculation object.

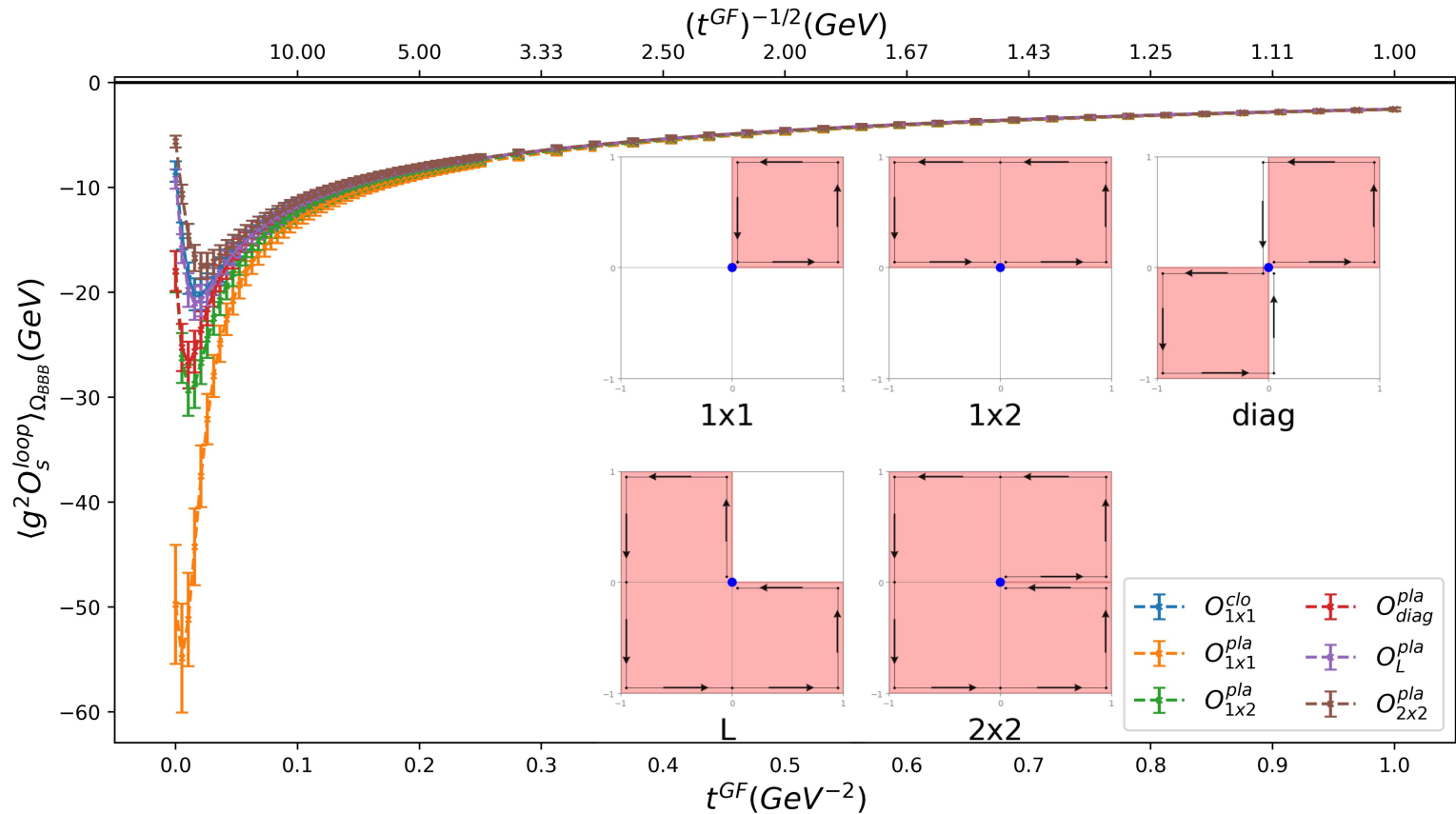


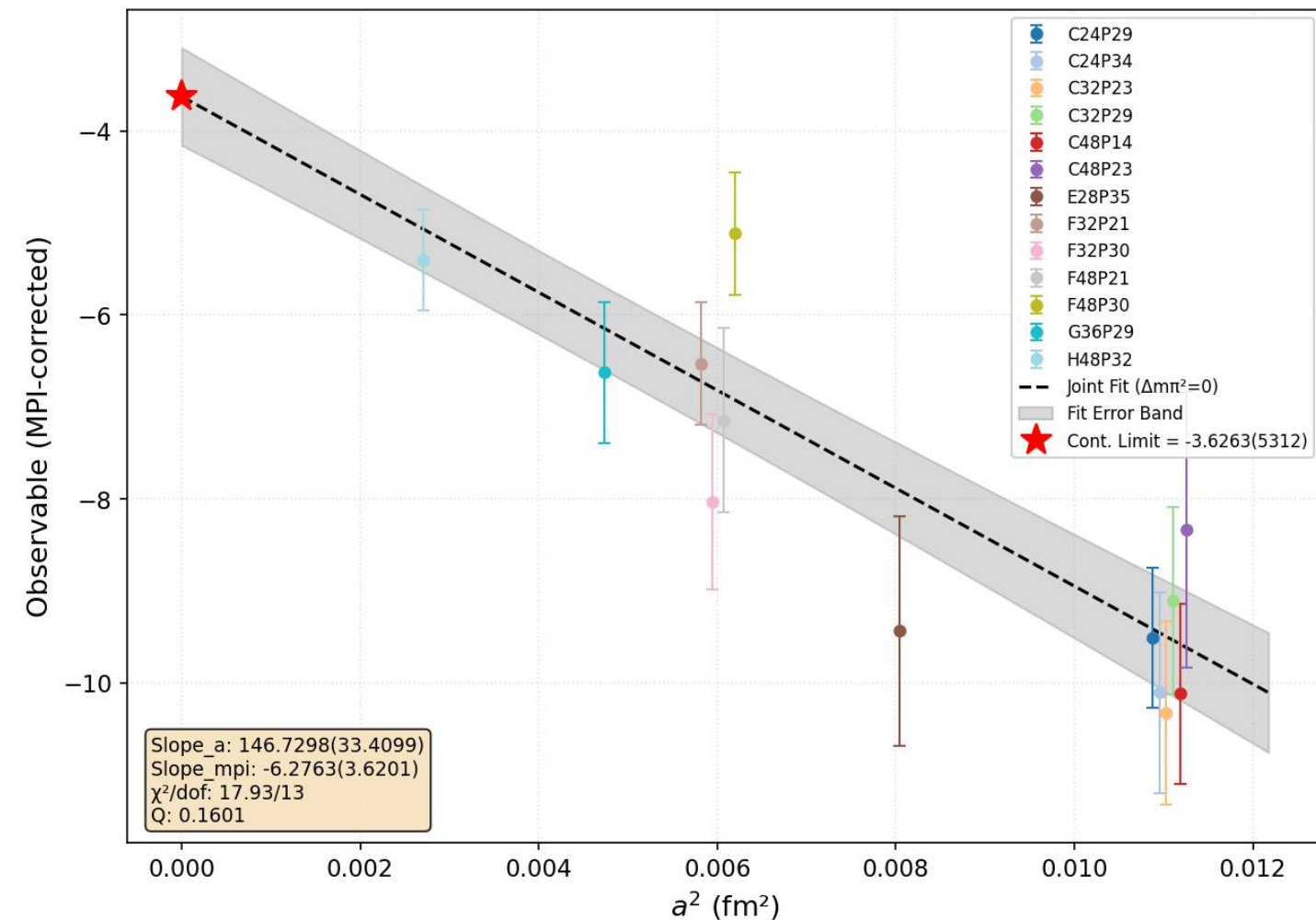
Fig.2. Behavior of $Tr[\langle g^2 F^2 \rangle]_{\Omega_{BBB}}$ of different s at $a = 0.052 \text{ fm}$ under gradient flow,

ensemble	a (fm)	m_π (MeV)	m_{η_s} (MeV)	$\tilde{L}^3 \times \tilde{T}$
C24P34	0.10542(17)(62)	340.6(1.7)	749.2(0.7)	$24^3 \times 64$
C24P29		292.4(1.0)	658.5(0.6)	$24^3 \times 72$
C32P29		293.4(0.8)	659.5(0.4)	$32^3 \times 64$
C32P23		228.1(1.2)	644.6(0.5)	$32^3 \times 64$
C48P23		224.3(1.2)	644.8(0.6)	$48^3 \times 96$
C48P14		136.5(1.7)	707.3(0.4)	$48^3 \times 96$
E28P35	0.09013(25)(53)	345.4(1.1)	710.7(1.0)	$28^3 \times 64$
F32P30	0.07760(07)(46)	301.0(1.2)	677.3(1.0)	$32^3 \times 96$
F48P30		302.7(0.9)	674.8(0.6)	$48^3 \times 96$
F32P21		210.5(2.2)	660.1(1.0)	$32^3 \times 64$
F48P21		207.9(1.1)	663.7(0.6)	$48^3 \times 96$
G36P29	0.06895(17)(41)	295.7(1.1)	692.6(0.5)	$36^3 \times 108$
H48P32	0.05235(11)(31)	316.1(1.0)	690.6(0.7)	$48^3 \times 144$

A total of 13 ensembles, covering different lattice spacings and pion masses.

Table 1. Lattice ensembles from the CLQCD Collaboration

Continuum Extrapolation



·The following formula is used for extrapolation:

$$\begin{aligned} &\langle \tilde{O}(t^{GF}; m_\pi, a) \rangle \\ &= \{ \langle \tilde{O} \rangle_H^{phys}(t^{GF}) + c_1(m_\pi^2 - (m_H^{phys})^2) \} (1 + c_2 a^2) \end{aligned} \quad (5)$$

·Discretization errors are visible, but the overall extrapolation describes the data.

Fig. 3. Continuum extrapolation of $\langle H_g^a \rangle_{\Omega_{bbb}}$ at 0.1 GeV^{-2} .

Matching and $t^{GF} \rightarrow 0$ Extrapolation

· Match the results from gradient flow to $\overline{MS}$ [6], then extrapolate $t^{GF} \rightarrow 0$. Here the one-loop coefficient is shown; the actual calculation uses two loops.

$$\langle H_g^a(\mu_{\overline{MS}}) \rangle_H = \hat{\beta}(\mu_{\overline{MS}}) \left\{ \left(1 - \frac{21}{8\pi} \alpha_s(\mu_{\overline{MS}}) \right) \langle \tilde{O}(t^{GF}) \rangle_H - \left[\frac{20}{3} + 8 \ln(2\mu_{\overline{MS}}^2 t^{GF}) + 8\gamma_E \right] \alpha_s^2(\mu_{\overline{MS}}) \sigma_H \right\} + \mathcal{O}(t^{GF}) \quad (6)$$

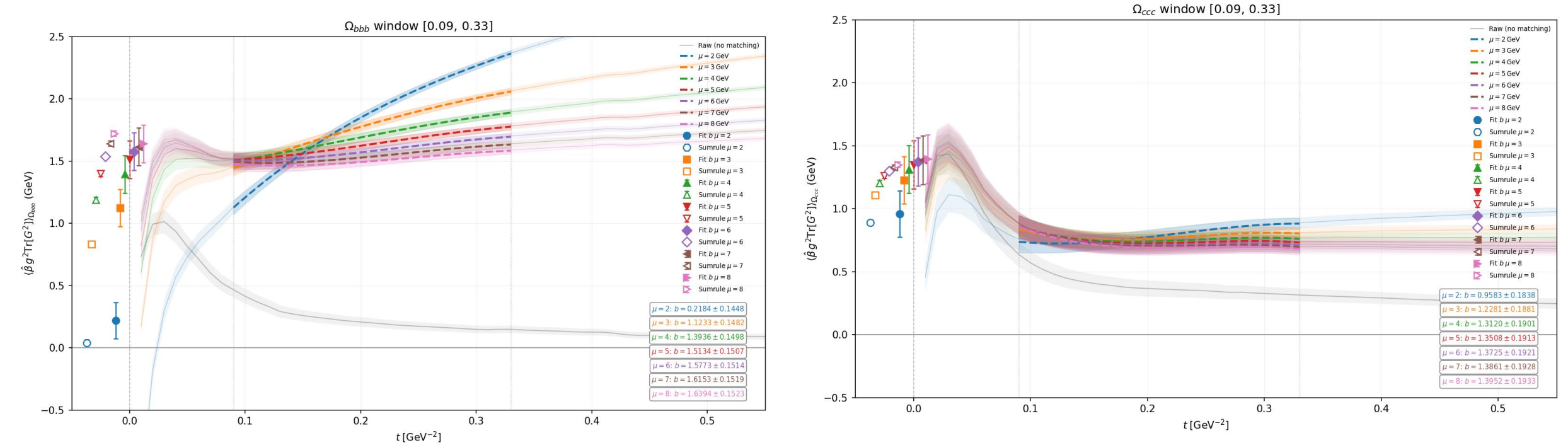


Fig. 4. Matching and extrapolation($t^{GF} \rightarrow 0$) of $\langle H_g^a \rangle_{\Omega_{ccc}}$ $\langle H_g^a \rangle_{\Omega_{bbb}}$ from 2-8 GeV (filled symbols near 0), compared with sum-rule results (open symbols near 0).

The window where the extrapolation is close to the sum-rule result is used and then applied to other particles.

Coefficient running

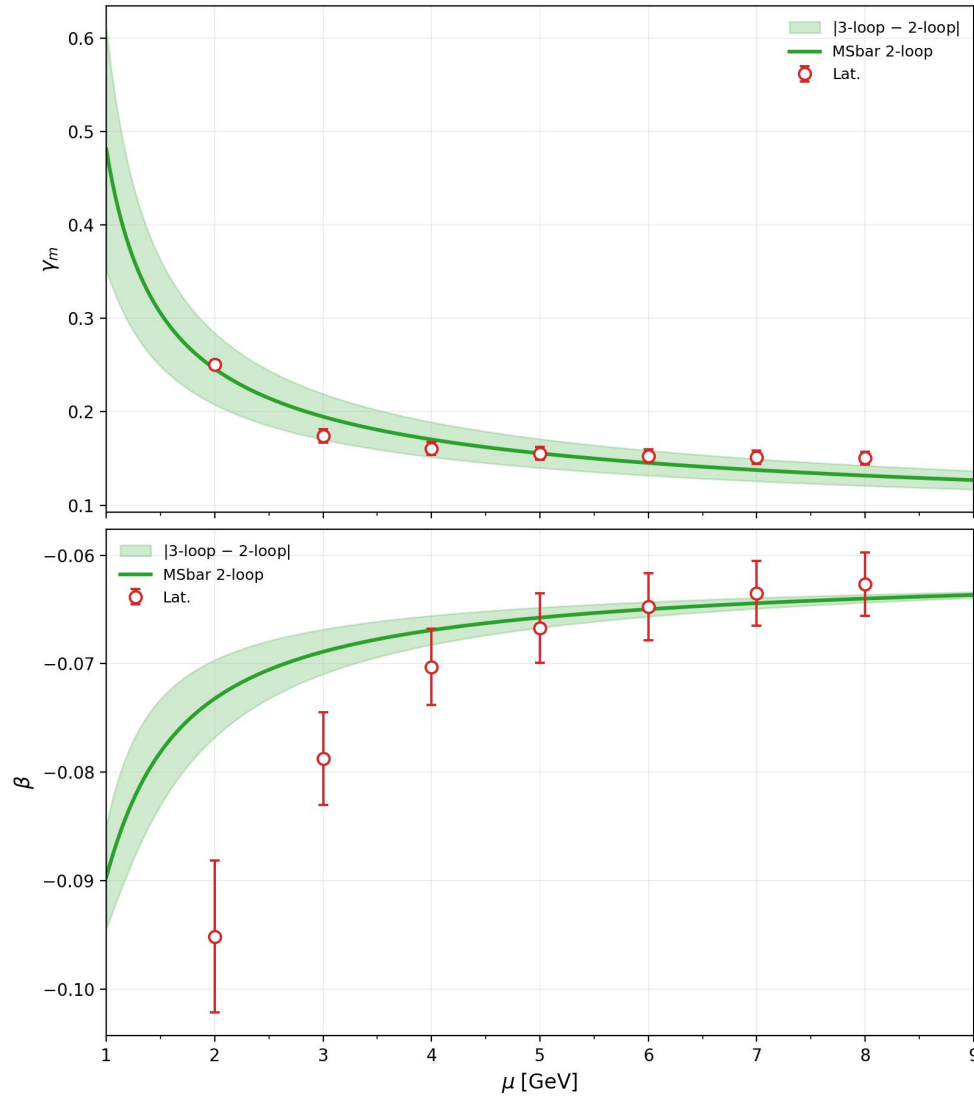


Fig. 5. Running of anomalous coefficients: Lattice results (red) compared with $\overline{MS}$ (green) results

·Coefficients in Eq. (1):

$$\begin{cases} \gamma_m = -\frac{\mu}{m} \frac{\partial m}{\partial \mu} = \frac{2}{\pi} \alpha_s + O(\alpha_s^2) \\ \hat{\beta} = -\frac{1}{4\pi} \frac{\mu^2}{\alpha_s^2} \frac{\partial \alpha_s}{\partial \mu^2} = -\left(\frac{1}{4\pi}\right)^2 (11 - \frac{2n_f}{3}) + O(\alpha_s) \end{cases}$$

·Calculated the trace anomaly of 28 hadrons containing l, s, c, b quarks, and performed a joint fit:

$$m_H = (1 + \gamma_m) \langle m_q \bar{q}q \rangle_H + \hat{\beta} \langle g^2 \text{Tr}[G_{\rho\sigma} G_{\rho\sigma}] \rangle_H$$

·Overall, the coefficients obtained from gradient flow converge in trend and are self-consistent with each other at certain energy scales.

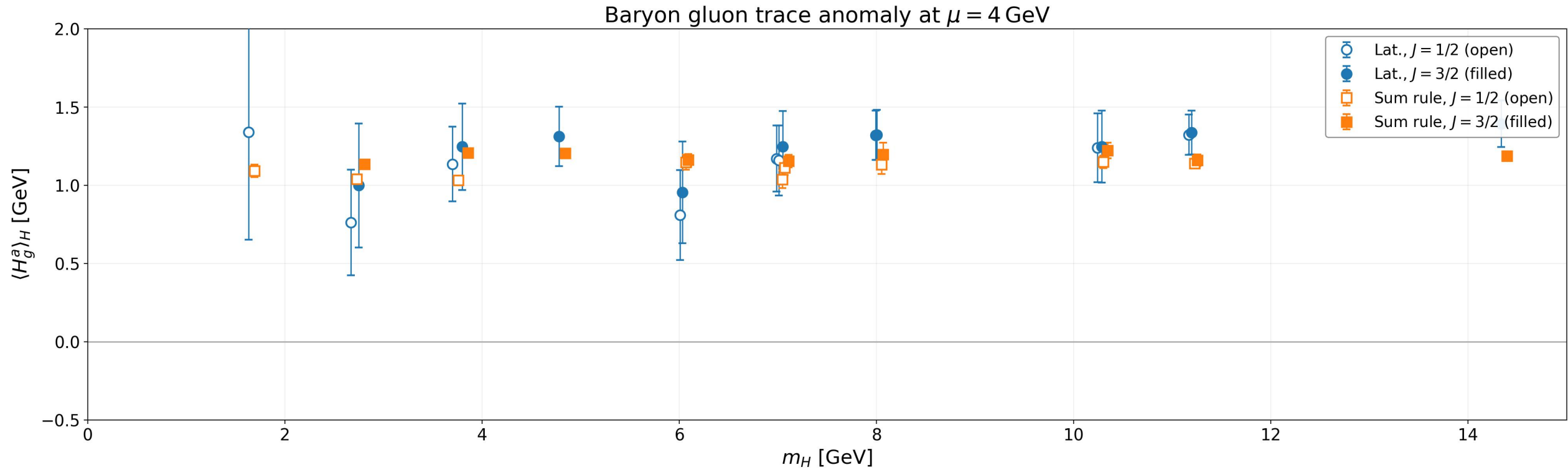


Fig. 6. At 4 GeV, comparison between direct calculation results (blue) and sum-rule results (red).

- Light quarks require more statistics, not shown here.
- At 4 GeV, the direct calculation results and sum-rule results are basically consistent within 2 sigma, and the contributions are similar in magnitude.

Summary:

- Systematic and broad calculation: 13 CLQCD ensembles, 28 hadrons containing l, s, c, b quarks, direct gradient-flow calculation matched to $\overline{\text{MS}}$.
- Wilson flow convergence: different s paths converge to the same physical trajectory at $t_{\text{GF}} \approx 0.1 \text{ GeV}^{-2}$, reducing operator ambiguity.
- Coefficient running: joint fit of γ_m and β_{hat} shows trends consistent with $\overline{\text{MS}}$ and mutual self-consistency at certain energy scales.
- Sum rule test: at $\mu = 4 \text{ GeV}$, direct calculation results and sum-rule results are basically consistent within 2 sigma.

Outlook:

Increase statistics, improve light-quark signals, and test ensembles with different actions, e.g., HISQ configurations.



Thanks