

Lattice-QCD Study of the



$\Lambda \rightarrow p \ell \bar{\nu}_\ell$ Semileptonic Decay

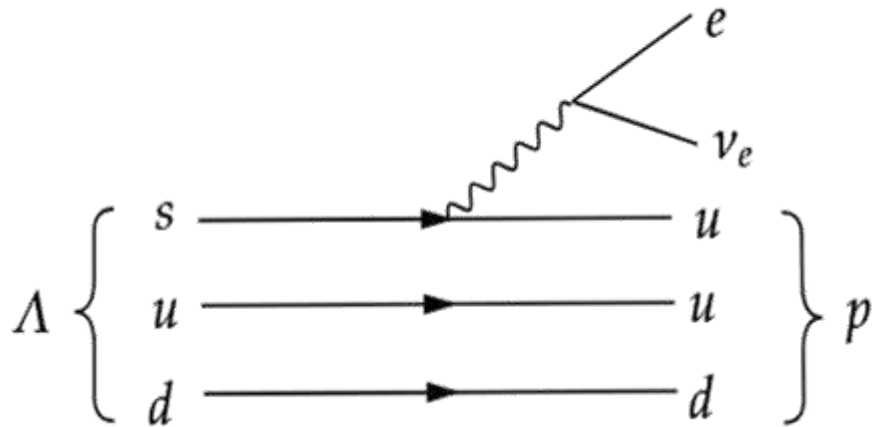
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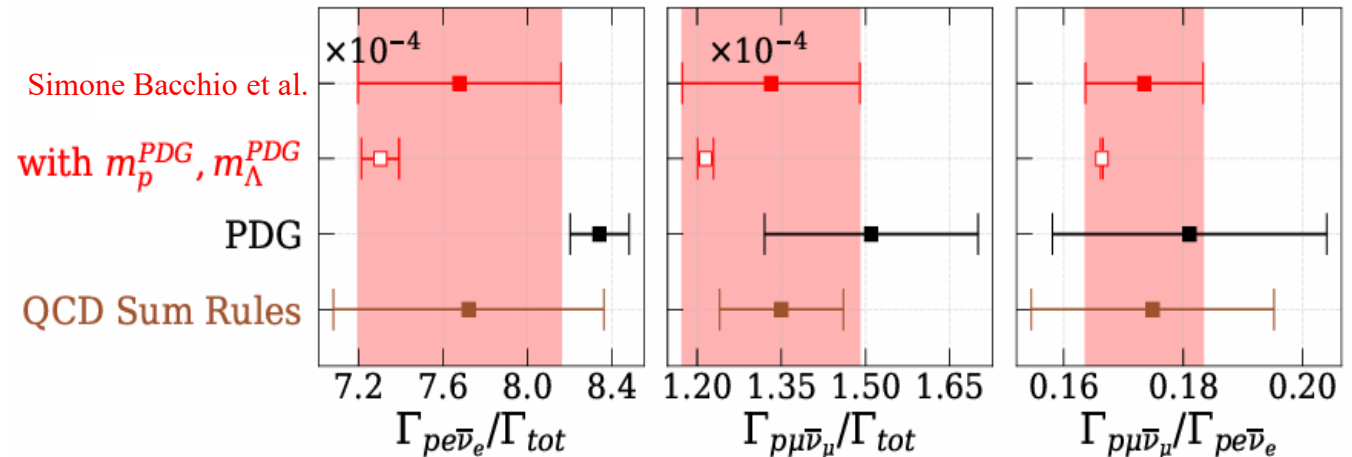
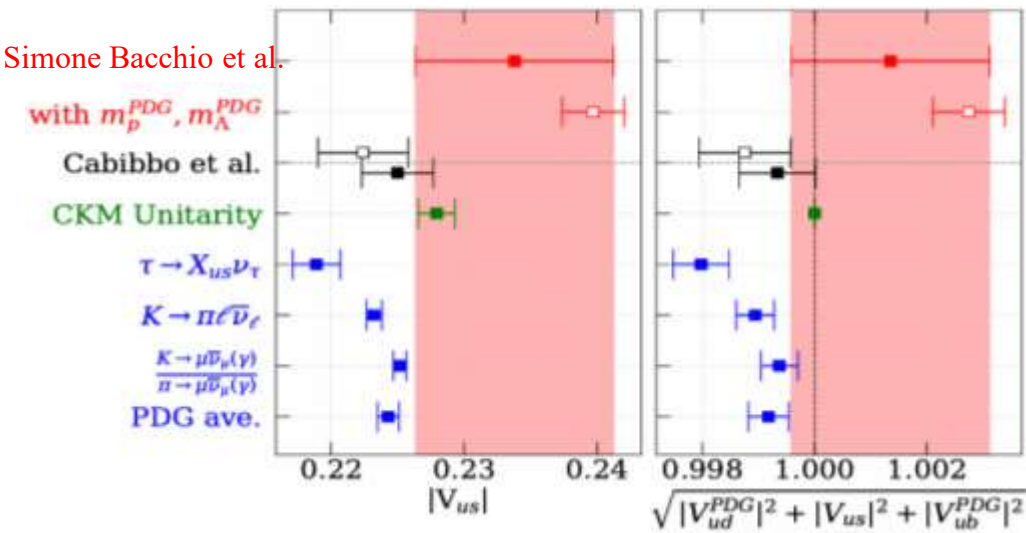
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Introduction



Hyperon semileptonic decays provide an independent baryon-sector determination of V_{us} , as well as a way to study form factors and test lepton universality. They may also shed light on the Cabibbo-angle anomaly and constrain new physics. BESIII can use $e^+e^- \rightarrow \Lambda\bar{\Lambda}$ pair production to analyze the production and decay angular distributions jointly.

In 2025, Simone Bacchio et al. performed a lattice calculation using an ETMC physical-point ensemble.



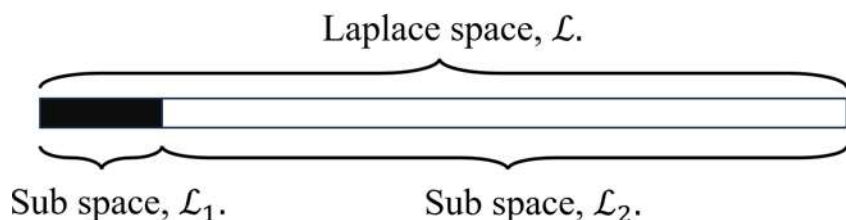
Source: Bacchio and Konstantinou, arXiv:2507.09970v3 (2025).



distillation

The quark field is approximated using only the lowest 100 eigenvectors of the Laplacian operator. The spatial and color information is stored in the eigenvector matrix V , reducing the storage cost of the perambulator.

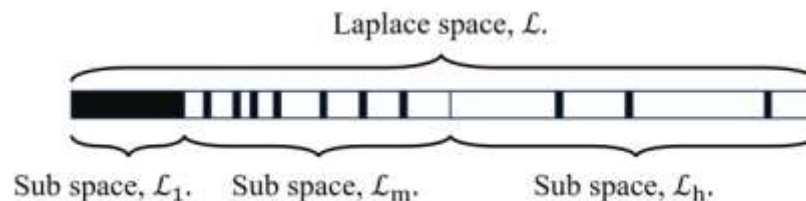
$$\begin{aligned} & \square (\vec{x}, \vec{y}, t) \\ &= \sum_{k=1}^{R_D} \xi^{(k)}(\vec{x}, t) \xi^{(k)\dagger}(\vec{y}, t) \\ \psi^{\text{dist.}}(x) &= \sum_{i=1}^{N_{\text{vec}}} v_i \langle v_i | \psi(x) \rangle \\ P_{\alpha\beta}(t, t_0) &= V^\dagger(t) G_{\alpha\beta} V(t_0) \end{aligned}$$

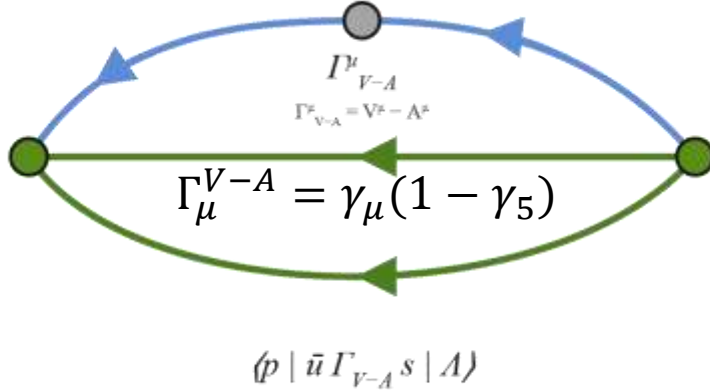


blending method

Building on distillation, N_{st} stochastic vectors are generated and orthogonalized using the Gram–Schmidt procedure. These vectors are combined with the exact Laplacian eigenvectors to form a weighted hybrid subspace, which approximates the full space spanned by the Laplacian eigenvectors.

$$\begin{aligned} \hat{I} &= \sum_{i=1}^{[\mathcal{L}]} |V_i\rangle \langle V_i| = \lim_{N_{st} \rightarrow [\mathcal{L}_2]} \sum_{k=1}^{N_e + N_{st}} \Omega_k^{(1)} |\phi_k\rangle \langle \phi_k| \\ &\begin{cases} 1 & \text{for } k \leq N_e \\ \omega_0 & \text{for } k > N_e \end{cases} \quad \omega_n = ([\mathcal{L}_2 - n]) / (N_{st} - n) \end{aligned}$$





The hyperon semileptonic decay amplitude is written as

$$M = \frac{G_F}{\sqrt{2}} V_{us} \langle p | \bar{u} \Gamma_\mu^{V-A} s | \Lambda \rangle \cdot \langle \ell \bar{\nu}_\ell | \bar{\ell} \Gamma_\mu^{V-A} \nu_\ell | 0 \rangle$$

The strong-interaction effects are encoded in the matrix element $\langle p | \bar{u} \Gamma_\mu^{V-A} s | \Lambda \rangle$. By computing this matrix element on the lattice, we extract the form factors and then determine physical observables such as the decay rate.

In our calculation, the Λ at the source is kept at rest. The proton at the sink is assigned momenta with $p^2 = 0, 1, 2, 3, 4$. We use 14 source-sink separations, $t_{sep} = 7..20..1$

$$\chi_P(x) = \epsilon^{abc} (u^a(x)^T C \gamma_5 d^b(x)) u^c(x)$$

$$\begin{aligned} \chi_\Lambda(x) = & \frac{1}{\sqrt{6}} \epsilon^{abc} [2(u_a^T(x) C \gamma_5 d^b(x)) s^c(x) \\ & + (u^a(x)^T C \gamma_5 s^b(x)) d^c(x) \\ & - (d^a(x)^T C \gamma_5 s^b(x)) u^c(x)] \end{aligned}$$

symbol	lattice size	a [fm]	m_π (Mev)	N_{cfg}
C24P29	$24^3 \times 72$	0.10524	292.7(1.2)	200
E32P29	$32^3 \times 64$	0.0897	286.7(1.8)	200
F32P30	$32^3 \times 96$	0.07753	303.2(1.3)	192
C48P14	$48^3 \times 96$	0.10524	135.5(1.6)	116

correlation function and matrix-element



Construction of the two-point correlation function: $C_B(\vec{p}_B, t_s) = \sum_{\vec{x}_s} \text{Tr} [P_0 \langle \chi_B(x_s) \bar{\chi}_B(0) \rangle e^{-i\vec{x}_s \cdot \vec{p}_B}]$ $P_0 = \frac{(1 + \gamma_t)}{2}$

Construction of the three-point correlation function:

$$C_{\mu\nu}^{V-A}(\vec{p}_P, \vec{p}_\Lambda, t_s, t_{ins}) = \sum_{\vec{x}_{ins}, \vec{x}_s} e^{i\vec{x}_{ins} \cdot (\vec{p}_P - \vec{p}_\Lambda) - i\vec{x}_s \cdot \vec{p}_P} \times \text{Tr} [P_\nu \langle \chi_P(x_s) \bar{u}(x_{ins}) \Gamma_\mu^{V-A} s(x_{ins}) \bar{\chi}_\Lambda(0) \rangle] \quad P_\nu = i \frac{(1 + \gamma_t)}{2} \gamma_5 \gamma_k (k=x,y,z)$$

The ratio of the three-point and two-point functions is used to remove the dominant exponential decay.

$$R_{\mu\nu}^{V-A}(t_s, t_{ins}) = \frac{C_{\mu\nu}^{V-A}(t_s, t_{ins})}{C_P(t_s)} \sqrt{\frac{C_\Lambda(t_s - t_{ins}) C_P(t_{ins}) C_P(t_s)}{C_P(t_s - t_{ins}) C_\Lambda(t_{ins}) C_\Lambda(t_s)}}$$

$$M_\mu^{V-A} \equiv \langle p | \bar{u} \Gamma_\mu^{V-A} s | \Lambda \rangle = \bar{u}_p [\gamma_\mu (F_1(q^2) - G_1(q^2) \gamma_5) - i \sigma_{\mu\nu} q^\nu \frac{F_2(q^2) - G_2(q^2) \gamma_5}{m_\Lambda} + q_\mu \frac{F_3(q^2) - G_3(q^2) \gamma_5}{m_\Lambda}] u_\Lambda \quad \text{Weinberg form factors}$$



$$M_\mu^V = \bar{u}_p \left[\frac{(m_\Lambda - m_p) q^\mu}{q^2} F_0(q^2) + \frac{m_\Lambda + m_p}{s_+} \left(p_\Lambda^\mu + p_p^\mu - \frac{m_\Lambda^2 - m_p^2}{q^2} q^\mu \right) F_+(q^2) + \left(\gamma^\mu - \frac{2m_N p_\Lambda^\mu}{s_+} - \frac{2m_\Lambda p_p^\mu}{s_+} \right) F_\perp(q^2) \right] u_\Lambda$$

helicity form factors

$$M_\mu^A = -\bar{u}_p \gamma_5 \left[\frac{(m_\Lambda + m_N) q^\mu}{q^2} G_0(q^2) + \frac{m_\Lambda - m_N}{s_-} \left(p_\Lambda^\mu + p_p^\mu - \frac{m_\Lambda^2 - m_N^2}{q^2} q^\mu \right) G_+(q^2) + \left(\gamma^\mu + \frac{2m_N p_\Lambda^\mu}{s_-} - \frac{2m_\Lambda p_p^\mu}{s_-} \right) G_\perp(q^2) \right] u_\Lambda$$



We renormalize the matrix elements using the RI/MOM scheme [3].

symbol	Z_V	Z_A/Z_V
C24P29	0.79814(23)	1.07244(70)
E32P29	0.8221(12)	1.0516(14)
F32P30	0.83548(12)	1.05549(54)
C48P14	0.79957(06)	1.07320(68)

Du H Y, Hu B, Cai M, et al. Charmed meson masses and decay constants in the continuum limit from the tadpole improved clover ensembles[J]. Physical Review D, 2025, 111(5): 054504.

The Z-expansion is used to describe the momentum-transfer dependence of the form factors.

$$H(q^2) = \sum_{k=0}^{k_{max}} a_k z(q^2)^k \quad z(q^2) = \frac{\sqrt{A - q^2} - \sqrt{A}}{\sqrt{A - q^2} + \sqrt{A}} \quad A = (m_\pi + m_K)^2$$

At low- q^2 expansion, each form factor can be expanded in a Taylor series. This allows us to define the corresponding charge, which characterizes the strength of the transition matrix element at zero momentum transfer, and the corresponding radius, which describes the momentum-transfer dependence of the form factor.

$$H(q) = \int e^{iqr} H(r) dr = H(0) \left(1 + \frac{\langle r_H^2 \rangle}{6} q^2 \right) + O(q^4) \quad \longrightarrow \quad H(0) = f, \quad \langle r_H^2 \rangle = \frac{6}{H(0)} \frac{\partial H(q^2)}{\partial q^2} \Big|_{q^2=0}.$$



The derivative of the decay rate with respect to the momentum transfer is given by [4]:

[S.Meinel,Phys.Rev.D97,034511\(2018\),arXiv:1712.05783 \[hep-lat\].](#)

$$\begin{aligned} \frac{d\Gamma}{dq^2} = & \frac{G_F^2 |V_{us}|^2 \sqrt{s_+ s_-}}{192 \pi^3 m_\Lambda^3} \left(1 - \frac{m_\ell^2}{q^2} \right)^2 \\ & \times \left\{ \frac{3m_\ell^2}{2q^2} [s_- ((m_\Lambda + m_N) G_0)^2 + s_+ ((m_\Lambda - m_N) F_0)^2] + \frac{m_\ell^2 + 2q^2}{2q^2} [s_+ ((m_\Lambda - m_N) G_+)^2 + s_- ((m_\Lambda \right. \\ & \left. + (m_\ell^2 + 2q^2) [s_+ G_\perp^2 + s_- F_\perp^2]) \right\}. \end{aligned}$$

$$\Gamma_\ell = \int_{m_\ell^2}^{(m_\Lambda - m_p)^2} dq^2 \frac{d\Gamma_\ell}{dq^2}.$$

Integrating the differential decay rate yields $\frac{\Gamma_\ell}{|V_{us}|^2}$ which is independent of $|V_{us}|^2$. Using the experimental decay rate as input, V_{us}^{lat} can then be determined. Conversely, using the PDG value of V_{us}^{PDG} , we can obtain the decay rate Γ_ℓ .

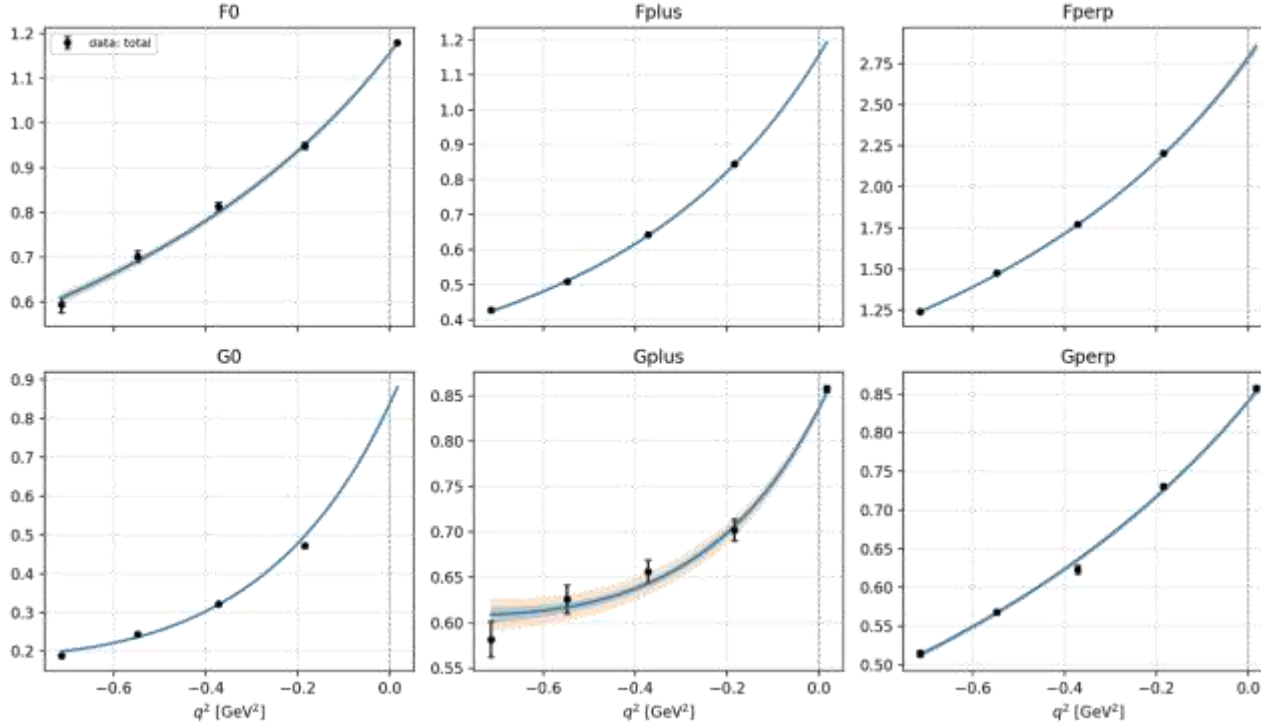
Combining this result with the PDG values of V_{ud}^{PDG} and V_{ub}^{PDG} , we can test the first-row CKM unitarity relation:

$$\Delta_{\text{CKM}} = \sqrt{|V_{ud}^{PDG}|^2 + |V_{us}^{lat}|^2 + |V_{ub}^{PDG}|^2}.$$

Lattice calculation results



E32P29



f_0	f_+	$f_\perp$
1.1553(39)	1.1553(39)	2.787(31)
$\langle r_{F_0}^2 \rangle m_\Lambda^2$	$\langle r_{F_+}^2 \rangle m_\Lambda^2$	$\langle r_{F_\perp}^2 \rangle m_\Lambda^2$
8.48(36)	13.86(21)	10.31(45)

g_0	g_+	$g_\perp$
0.8359(43)	0.8359(43)	0.8410(36)
$\langle r_{G_0}^2 \rangle m_\Lambda^2$	$\langle r_{G_+}^2 \rangle m_\Lambda^2$	$\langle r_{G_\perp}^2 \rangle m_\Lambda^2$
22.58(25)	8.86(69)	6.35(19)

$$\frac{\Gamma_{pe\bar{\nu}_e}}{|V_{us}|^2} = 5.560(41) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{|V_{us}|^2} = 0.9178(65) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{\Gamma_{pe\bar{\nu}_e}} = 0.16507(14).$$

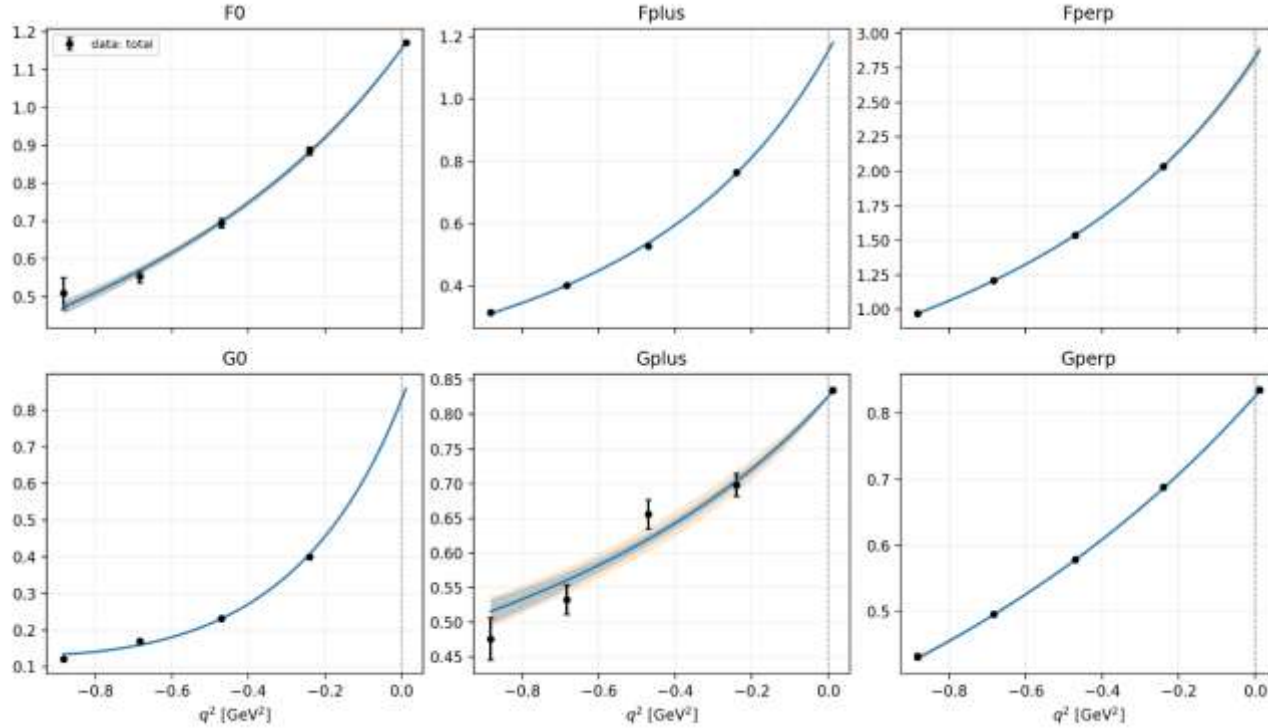
$$|V_{us}|_{lat}^{(3264)} = 0.2394(22),$$

$$\Delta_{CKM}^{(3264)} = \sqrt{|V_{ud}^{PDG}|^2 + \left(|V_{us}|_{lat}^{(2472)}\right)^2 + |V_{ub}^{PDG}|^2} = 1.00268(62).$$

Lattice calculation results



C24P29



f_0	f_+	$f_\perp$
1.1538(23)	1.1538(23)	2.830(54)
$\langle r_{F_0}^2 \rangle m_\Lambda^2$	$\langle r_{F_+}^2 \rangle m_\Lambda^2$	$\langle r_{F_\perp}^2 \rangle m_\Lambda^2$
8.95(53)	14.14(15)	11.12(55)
g_0	g_+	$g_\perp$
0.8259(35)	0.8259(35)	0.8256(30)
$\langle r_{G_0}^2 \rangle m_\Lambda^2$	$\langle r_{G_+}^2 \rangle m_\Lambda^2$	$\langle r_{G_\perp}^2 \rangle m_\Lambda^2$
23.61(22)	5.60(11)	5.84(18)

$$\frac{\Gamma_{pe\bar{\nu}_e}}{|V_{us}|^2} = 5.434(28) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{|V_{us}|^2} = 0.8957(46) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{\Gamma_{pe\bar{\nu}_e}} = 0.16483(20).$$

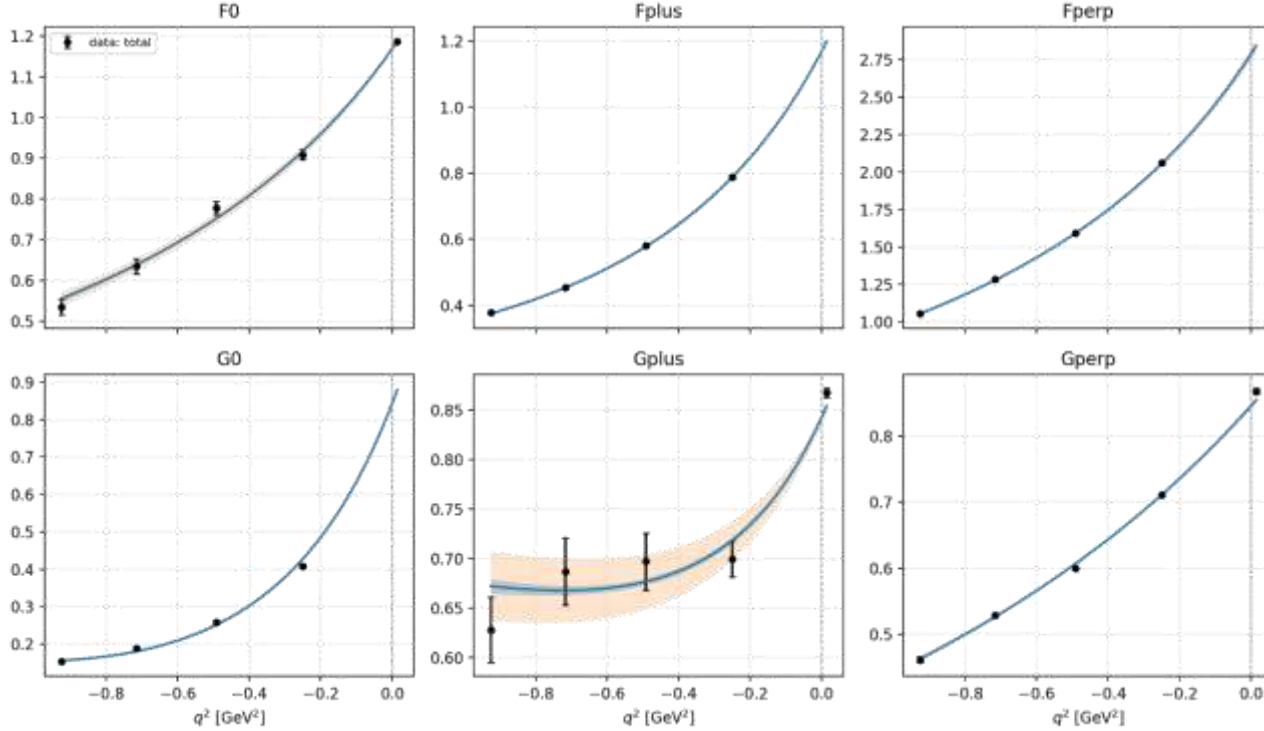
$$|V_{us}|_{lat}^{(2472)} = 0.2422(22),$$

$$\Delta_{CKM}^{(2472)} = \sqrt{|V_{ud}^{PDG}|^2 + \left(|V_{us}|_{lat}^{(2472)}\right)^2 + |V_{ub}^{PDG}|^2} = 1.00334(61).$$

Lattice calculation results



F32P30



f_0	f_+	$f_\perp$
1.1689(30)	1.1689(30)	2.787(34)
$\langle r_{F_0}^2 \rangle m_\Lambda^2$	$\langle r_{F_+}^2 \rangle m_\Lambda^2$	$\langle r_{F_\perp}^2 \rangle m_\Lambda^2$
8.00(48)	13.21(21)	9.93(39)

g_0	g_+	$g_\perp$
0.8427(43)	0.8427(43)	0.8410(36)
$\langle r_{G_0}^2 \rangle m_\Lambda^2$	$\langle r_{G_+}^2 \rangle m_\Lambda^2$	$\langle r_{G_\perp}^2 \rangle m_\Lambda^2$
21.97(25)	7.00(90)	5.33(20)

$$\frac{\Gamma_{pe\bar{\nu}_e}}{|V_{us}|^2} = 5.634(37) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{|V_{us}|^2} = 0.9278(65) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{\Gamma_{pe\bar{\nu}_e}} = 0.16467(20).$$

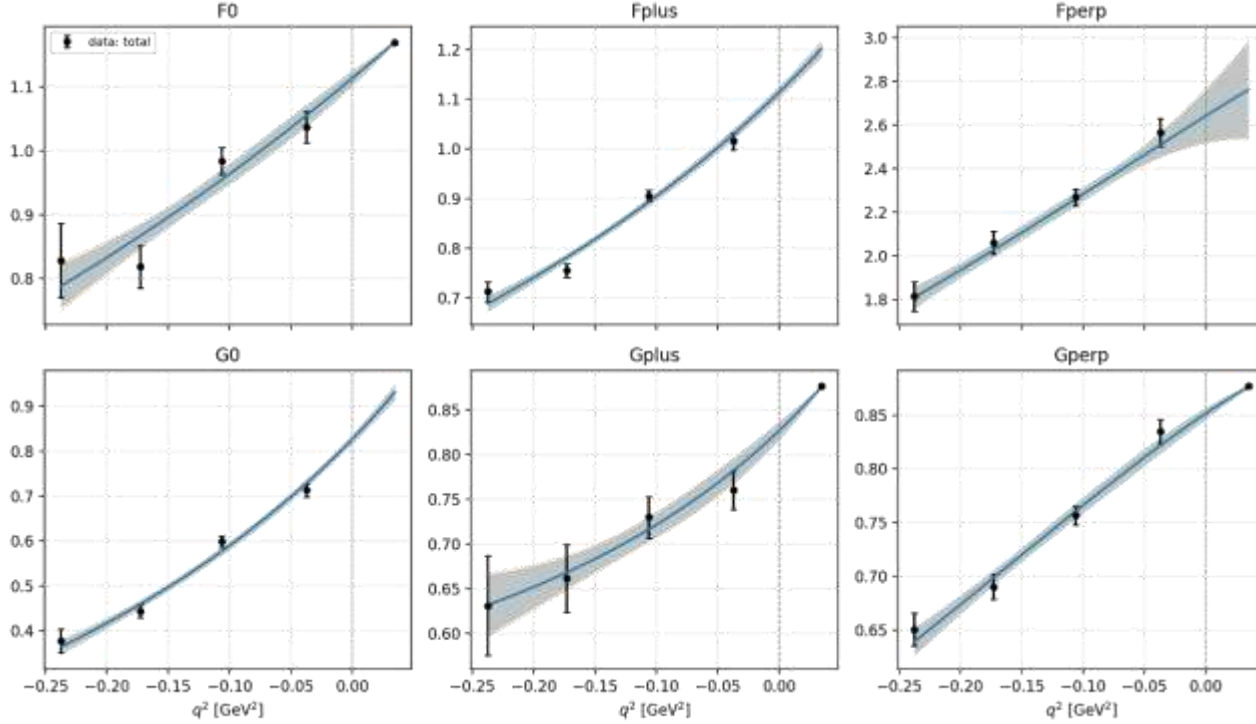
$$|V_{us}|_{lat}^{(3296)} = 0.2378(22),$$

$$\Delta_{CKM}^{(3296)} = \sqrt{|V_{ud}^{PDG}|^2 + \left(|V_{us}|_{lat}^{(2472)}\right)^2 + |V_{ub}^{PDG}|^2} = 1.00230(61).$$

Lattice calculation results



C48P14



f_0	f_+	$f_\perp$
1.114(12)	1.114(12)	2.64(13)
$\langle r_{F_0}^2 \rangle m_\Lambda^2$	$\langle r_{F_+}^2 \rangle m_\Lambda^2$	$\langle r_{F_\perp}^2 \rangle m_\Lambda^2$
10.70(17)	16.00(14)	10.00(67)

g_0	g_+	$g_\perp$
0.827(10)	0.827(10)	0.8515(55)
$\langle r_{G_0}^2 \rangle m_\Lambda^2$	$\langle r_{G_+}^2 \rangle m_\Lambda^2$	$\langle r_{G_\perp}^2 \rangle m_\Lambda^2$
25.4(23)	11.8(19)	6.71(96)

$$\frac{\Gamma_{pe\bar{\nu}_e}}{|V_{us}|^2} = 5.429(63) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{|V_{us}|^2} = 0.91026(76) \times 10^7 \text{ s}^{-1},$$

$$\frac{\Gamma_{p\mu\bar{\nu}_\mu}}{\Gamma_{pe\bar{\nu}_e}} = 0.16625(66).$$

$$|V_{us}|_{lat}^{(4896)} = 0.2423(25),$$

$$\Delta_{CKM}^{(4896)} = \sqrt{|V_{ud}^{PDG}|^2 + \left(|V_{us}|_{lat}^{(2472)}\right)^2 + |V_{ub}^{PDG}|^2} = 1.00337(68).$$



Summary

1. We established a complete lattice-QCD workflow from two- and three-point correlation functions to the form factors of the decay $\Lambda \rightarrow p \ell \bar{\nu}_\ell$.
2. At four lattice spacings, we extracted six helicity form factors, their corresponding charges and radii, and the decay rates.
3. Our current results provide an independent determination of V_{us} and an independent test of first-row CKM unitarity.

outlook

Extrapolate the results from the four lattice spacings to the continuum limit.

Thanks



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- [2] T. Feldmann and M.W.Y. Yip, Phys. Rev. D 85, 014035(2012); 86, 079901(E) (2012).
- [3] Du H Y, Hu B, Cai M, et al. Charmed meson masses and decay constants in the continuum limit from the tadpole improved clover ensembles[J]. Physical Review D, 2025, 111(5): 054504.
- [4] S. Meinel, Phys. Rev. D 97, 034511(2018), arXiv:1712.05783 [hep-lat].
- [5] R. J. Hill and G. Paz, Phys. Rev. D 82, 113005 (2010), arXiv:1008.4619 [hep-ph].
- [6] S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024).
- [7] Bacchio S, Konstantinou A. Study of the $\Lambda \rightarrow p \ell \bar{\nu}_\ell$ semileptonic decay in lattice QCD[EB/OL]. arXiv:2507.09970v3 [hep-lat], 2025.



Some input quantities used in the calculation are taken from the PDG.

$$\begin{aligned}
 \mathcal{B}(\Lambda \rightarrow pe\bar{\nu}_e) &= 8.34(14) \times 10^{-4} & \tau_{\Lambda}^{PDG} &= 2.617(10) \times 10^{-10} \text{ s} \\
 |V_{ud}^{PDG}| &= 0.97367(32) & |V_{ub}^{PDG}| &= 3.82(20) \times 10^{-3} & |V_{us}^{PDG}| &= 0.2243(8). \\
 m_e &= 0.511 \text{ MeV} & m_{\mu} &= 0.10566 \text{ GeV} \\
 m_p^{PRD} &= 0.93827 \text{ GeV} & m_{\Lambda}^{PRD} &= 1.11568 \text{ GeV} \\
 G_F &= 1.16638 \cdot (1 + 0.0105) \cdot 10^{-5} \text{ GeV}^{-2}
 \end{aligned}$$

Conversion relations between the Weinberg form factors and the helicity form factors

$$\begin{aligned}
 F_+(q^2) &= F_1(q^2) + \frac{q^2}{m_{\Lambda}(m_{\Lambda} + m_N)} F_2(q^2), & G_+(q^2) &= G_1(q^2) - \frac{q^2}{m_{\Lambda}(m_{\Lambda} - m_N)} G_2(q^2), \\
 F_{\perp}(q^2) &= F_1(q^2) + \frac{m_{\Lambda} + m_N}{m_{\Lambda}} F_2(q^2), & G_{\perp}(q^2) &= G_1(q^2) - \frac{m_{\Lambda} - m_N}{m_{\Lambda}} G_2(q^2), \\
 F_0(q^2) &= F_1(q^2) + \frac{q^2}{m_{\Lambda}(m_{\Lambda} - m_N)} F_3(q^2), & G_0(q^2) &= G_1(q^2) - \frac{q^2}{m_{\Lambda}(m_{\Lambda} + m_N)} G_3(q^2).
 \end{aligned}$$

Form-factor extraction.

$$\Pi_{ij}^V(\vec{p}_P) = \frac{\epsilon_{ijk} p_P^k \mathcal{K}}{2m_p} F_{\perp}(q^2),$$

$$\Pi_{00}^V(\vec{p}_P) = \frac{(E_P + m_p) \mathcal{K}}{2m_p} \left\{ F_{\perp}(q^2) - \frac{2m_{\Lambda}(E_P + m_p)}{s_+} (F_{\perp}(q^2) - F_+(q^2)) - \frac{(E_P - m_{\Lambda})(m_{\Lambda} - m_p)}{q^2} (F_0(q^2) - F_+(q^2)) \right\},$$

$$\Pi_{i0}^V(\vec{p}_P) = \frac{ip_P^i \mathcal{K}}{2m_p} \left\{ F_{\perp}(q^2) - \frac{2m_{\Lambda}(E_P + m_p)}{s_+} (F_{\perp}(q^2) - F_+(q^2)) - \frac{(E_P + m_p)(m_{\Lambda} - m_p)}{q^2} (F_0(q^2) - F_+(q^2)) \right\},$$

$$\Pi_{0j}^A(\vec{p}_P) = \frac{p_P^j \mathcal{K}}{2m_p} \left\{ \frac{(m_p + m_{\Lambda})(E_P - m_{\Lambda})}{q^2} (G_+(q^2) - G_0(q^2)) + G_{\perp}(q^2) - \frac{2m_{\Lambda}(E_P - m_p)}{s_-} (G_{\perp}(q^2) - G_+(q^2)) \right\},$$

$$\Pi_{ij}^A(\vec{p}_P) = \frac{-i(m_p + m_{\Lambda}) \mathcal{K}}{2m_p} \left\{ \delta_{ij} \frac{E_P + m_p}{m_p + m_{\Lambda}} G_{\perp}(q^2) + \frac{p_P^i p_P^j}{q^2} \left(G_+(q^2) - G_0(q^2) + \frac{2m_{\Lambda} q^2 (G_+(q^2) - G_{\perp}(q^2))}{s_-(m_p + m_{\Lambda})} \right) \right\}.$$

$$s_{\pm} = (m_{\Lambda} \pm m_p)^2 - q^2 \quad \mathcal{K} = \sqrt{\frac{2m_p^2}{E_p(E_p + m_p)}} \quad q^2 = (p_{\Lambda} - p_p)^2$$



Z-fit

Conventionally, the Z-expansion is constrained by $a_{k_{max}} = -\sum_{k=0}^{k_{max}-1} a_k$ which ensures convergence in the large- q^2 limit [R. J. Hill and G. Paz, Phys. Rev. D 82, 113005 \(2010\), arXiv:1008.4619 \[hep-ph\]](#).

By evaluating the form factors at $q^2=0$, $q^2=q_{max}^2$, two additional constraint conditions can also be obtained.

$$f_0 = f_+, g_0 = g_+ \qquad G_+(q_{max}^2) = G_\perp(q_{max}^2)$$



Renormalization-coefficient table

TABLE IV. Z_V, Z_V^c and ratios of different renormalization constants at different lattice spacing

		Z_V	Z_V^c	Z_A/Z_V	Z_S/Z_V	Z_P/Z_V	Z_T/Z_V
a (fm)	0.10524(05)(62)	0.8021(10)	1.5790(21)	1.0718(07)	1.2019(33)	0.9209(45)	1.0811(09)
	0.08973(20)(53)	0.8221(12)	1.4107(22)	1.0516(14)	1.0958(63)	0.8583(70)	1.0963(18)
	0.07753(03)(45)	0.8387(10)	1.3104(17)	1.0527(08)	1.0492(33)	0.8399(36)	1.1017(10)
	0.06887(12)(41)	0.8502(12)	1.2445(19)	1.0418(09)	1.0053(71)	0.8133(42)	1.1097(15)
	0.05199(08)(31)	0.8727(12)	1.1334(17)	1.0344(10)	0.9175(86)	0.7801(50)	1.1236(16)
Para	c_1	-0.0207(18)	-0.0119(08)	0.0346(68)	-0.008(34)	0.036(44)	0.021(12)
	c_2	-0.0044(11)	-0.0126(06)				
	c_3	-0.0013(05)	-0.0070(03)				

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TABLE V. The renormalization constants on each ensemble.

a (fm)	Symbol	Z_V	Z_V^* (optional)	Z_V^c	Z_A/Z_V	Z_P/Z_V	Z_S/Z_V	Z_T/Z_V
0.10524(05)(62)	C24P34	0.79676(32)	0.85034(10)	1.57130(20)	1.07677(72)	0.921(12)	1.205(05)	1.0856(20)
	C24P29	0.79814(23)	0.85184(06)	1.57353(18)	1.07244(70)	0.922(17)	1.194(06)	1.0825(10)
	C32P29	0.79810(13)	0.85167(04)	1.57163(14)	1.07648(63)	0.923(07)	1.199(08)	1.0826(10)
	C32P23	0.79957(13)	0.85350(04)	1.57644(12)	1.07375(40)	0.929(08)	1.197(06)	1.0818(10)
	C48P23	0.79954(05)	0.85339(02)	1.57326(08)	1.07317(70)	0.923(09)	1.208(10)	1.0819(10)
	C48P14	0.79957(06)	0.85359(02)	1.57415(08)	1.07320(68)	0.915(11)	1.204(04)	1.0831(10)
0.08973(20)(53)	E28P35	0.81768(04)	0.85877(02)	1.40361(05)	1.06029(53)	0.862(06)	1.095(05)	1.0991(10)
0.07753(03)(45)	F32P30	0.83548(12)	0.86900(03)	1.30566(07)	1.05549(54)	0.841(10)	1.042(09)	1.1036(10)
	F48P30	0.83511(04)	0.86880(02)	1.30451(06)	1.05687(66)	0.841(04)	1.053(09)	1.1052(10)
	F32P21	0.83579(09)	0.87031(03)	1.30782(08)	1.05416(50)	0.845(05)	1.042(06)	1.1008(10)
	F48P21	0.83567(05)	0.86880(02)	1.30673(04)	1.05434(88)	0.840(07)	1.051(04)	1.1033(10)
0.06887(12)(41)	G36P29	0.84636(09)	0.87473(05)	1.23990(13)	1.04500(22)	0.814(03)	1.006(07)	1.1118(10)
0.05199(08)(31)	H48P32	0.86855(04)	0.88780(01)	1.12882(11)	1.03802(28)	0.783(04)	0.916(09)	1.1260(10)