

# Strong interaction origin of hadron masses

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Oct. 9th, 2026



CLQCD



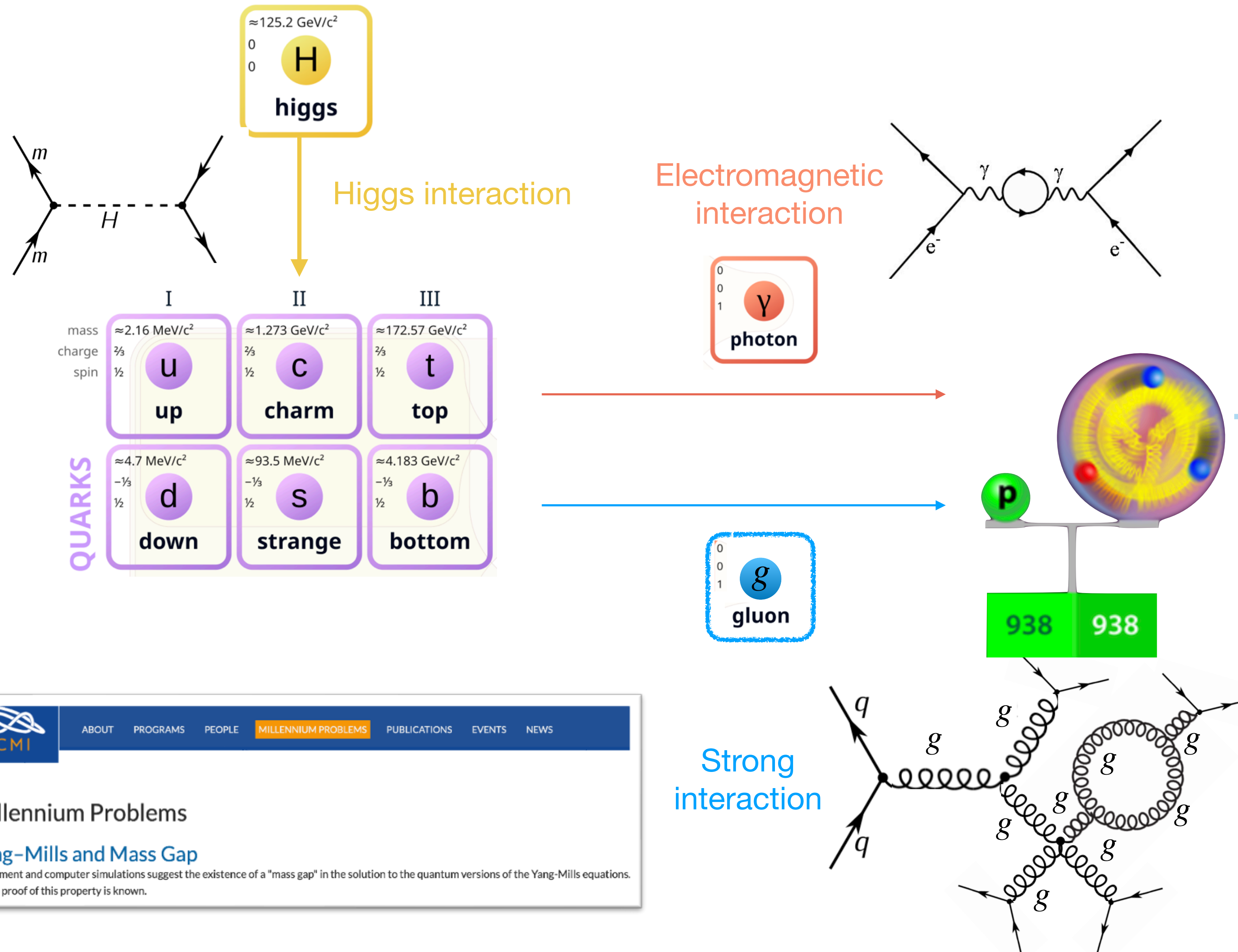
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# Mass in quantum field theory

## Multiple origins



- Quarks obtain mass through the **Higgs interaction**;
- **Electromagnetic interaction** can also be crucial in the cases like the proton-neutron mass difference;

- But **Strong interaction** is still the major origin of the mass of the hadronic matter and then the visible matter.

# Hadron masses

and their strong interaction origin



- Rescaled by  $\langle \bar{q}q \rangle_H$  which can be significantly larger than the valence quark number for the light quarks;

$$\bar{q}(x)\gamma_4 q(x) = \int \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} - a_{-,p}^\dagger a_{-,p})$$

$$\begin{aligned} \bar{q}(x)q(x) &= \int \frac{d^3p}{(2\pi^3)} \frac{m}{E} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \\ &\leq \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \end{aligned}$$

|        | I                        | II                        | III                        |
|--------|--------------------------|---------------------------|----------------------------|
| mass   | ≈2.16 MeV/c <sup>2</sup> | ≈1.273 GeV/c <sup>2</sup> | ≈172.57 GeV/c <sup>2</sup> |
| charge | 2/3                      | 2/3                       | 2/3                        |
| spin   | 1/2                      | 1/2                       | 1/2                        |
|        | <b>u</b><br>up           | <b>c</b><br>charm         | <b>t</b><br>top            |
| QUARKS | ≈4.7 MeV/c <sup>2</sup>  | ≈93.5 MeV/c <sup>2</sup>  | ≈4.183 GeV/c <sup>2</sup>  |
|        | -1/3                     | -1/3                      | -1/3                       |
|        | 1/2                      | 1/2                       | 1/2                        |
|        | <b>d</b><br>down         | <b>s</b><br>strange       | <b>b</b><br>bottom         |

$$\begin{aligned} m_H = & \sum_q m_q \langle \bar{q}q \rangle_H + \left\{ \sum_q \langle \bar{q} \vec{\gamma} \cdot \vec{D} q \rangle_H + \langle \text{Tr}[B^2 - E^2] \rangle_H + \frac{\epsilon}{4} \langle \text{Tr}[F_{\text{bare}}^2] \rangle_H \right\} \\ & + \left\{ \sum_q e_q^2 \Delta_{q,\text{self}}^{\text{QED}} m_H + \sum_{q,q' \neq q} e_q e_{q'} \Delta_{qq',\text{int}}^{\text{QED}} m_H + \mathcal{O}(\alpha^2) \right\} \end{aligned}$$

Electro-magnetic interaction



- Rescaling the hydrogen binding energy 13.6 eV by  $r_{\text{hydrogen}}/r_{\text{hadron}} \sim 10^5$  gives  $\mathcal{O}(\text{MeV})$  correction on the hadron mass;
- $\mathcal{O}(\alpha \Lambda_{\text{QCD}})$  gives a similar power counting.

- Direct contribution through the QCD anomalous dimension of  $m_q$  and  $\alpha_s$  (trace anomaly);
- Indirect contributions through  $\langle \bar{q}q \rangle_H$  and parton motions.



# QCD energy momentum tensor

## Sum rules

The decomposition of QCD energy momentum tensor (EMT) gives:

- Hamiltonian:

$$H \equiv T^{00} = \sum_q \overset{\text{Quark mass}}{m_q \bar{q}q} + \sum_{q,i} \overset{\text{Quark energy}}{\bar{q}\gamma^i D^i q} + \overset{\text{Gluon energy}}{\text{Tr}[B^2 - E^2]} + \overset{\text{Trace anomaly}}{\frac{\epsilon}{4} \text{Tr}[F_{\text{bare}}^2]};$$

X. Ji, PRL74(1995)1071

- “Pressure”:

$$\sum_i T^{ii} = \sum_{q,i} \bar{q}\gamma^i D^i q + \text{Tr}[B^2 - E^2] - \frac{3\epsilon}{4} \text{Tr}[F_{\text{bare}}^2];$$

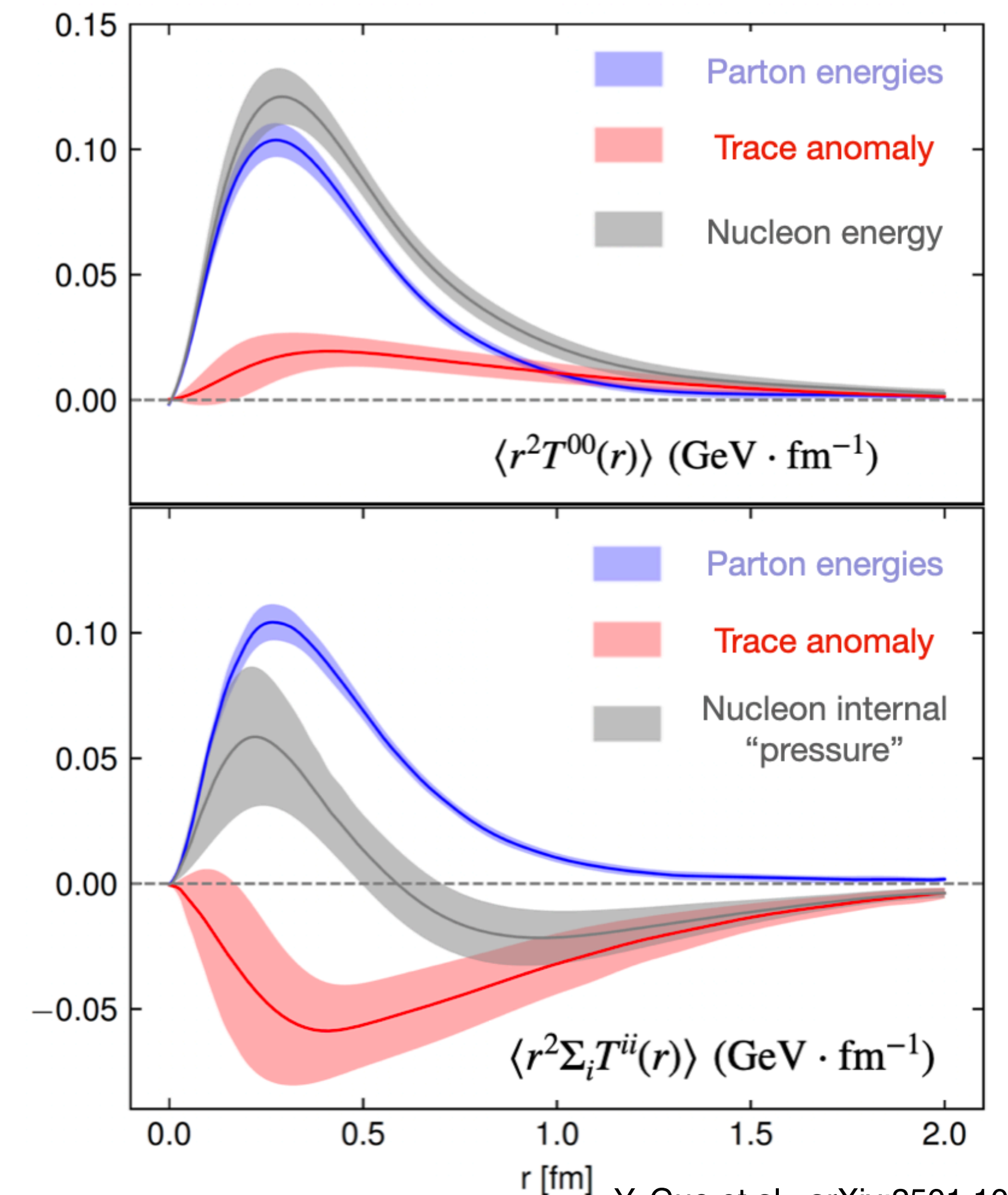
- And combining them gives the trace of EMT as:

$$T^\mu_\mu = T^{00} - \sum_i T^{ii} = \sum_q m_q \bar{q}q + \epsilon \text{Tr}[F_{\text{bare}}^2],$$

J. C. Collins et al., PRD16(1977)438  
N.K. Nielsen, NPB120(1977)212,  
M.A. Shifman et al, PLB78(1987)443

with

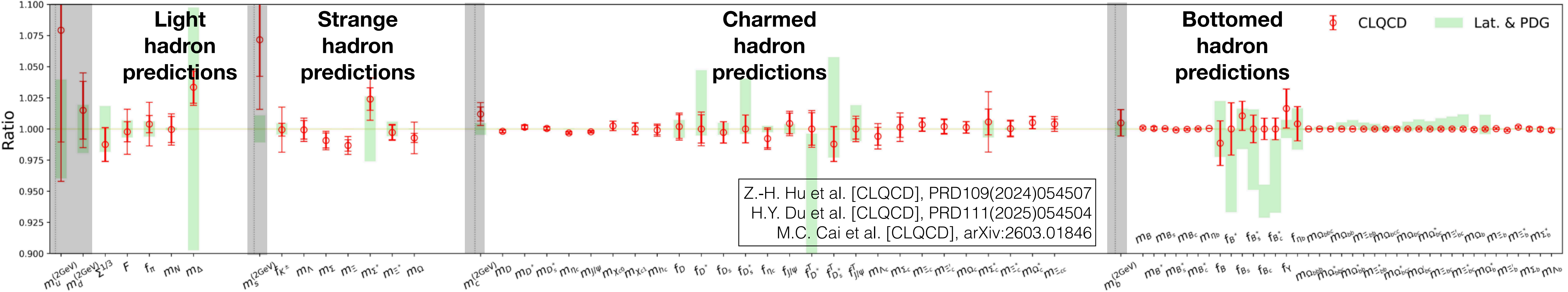
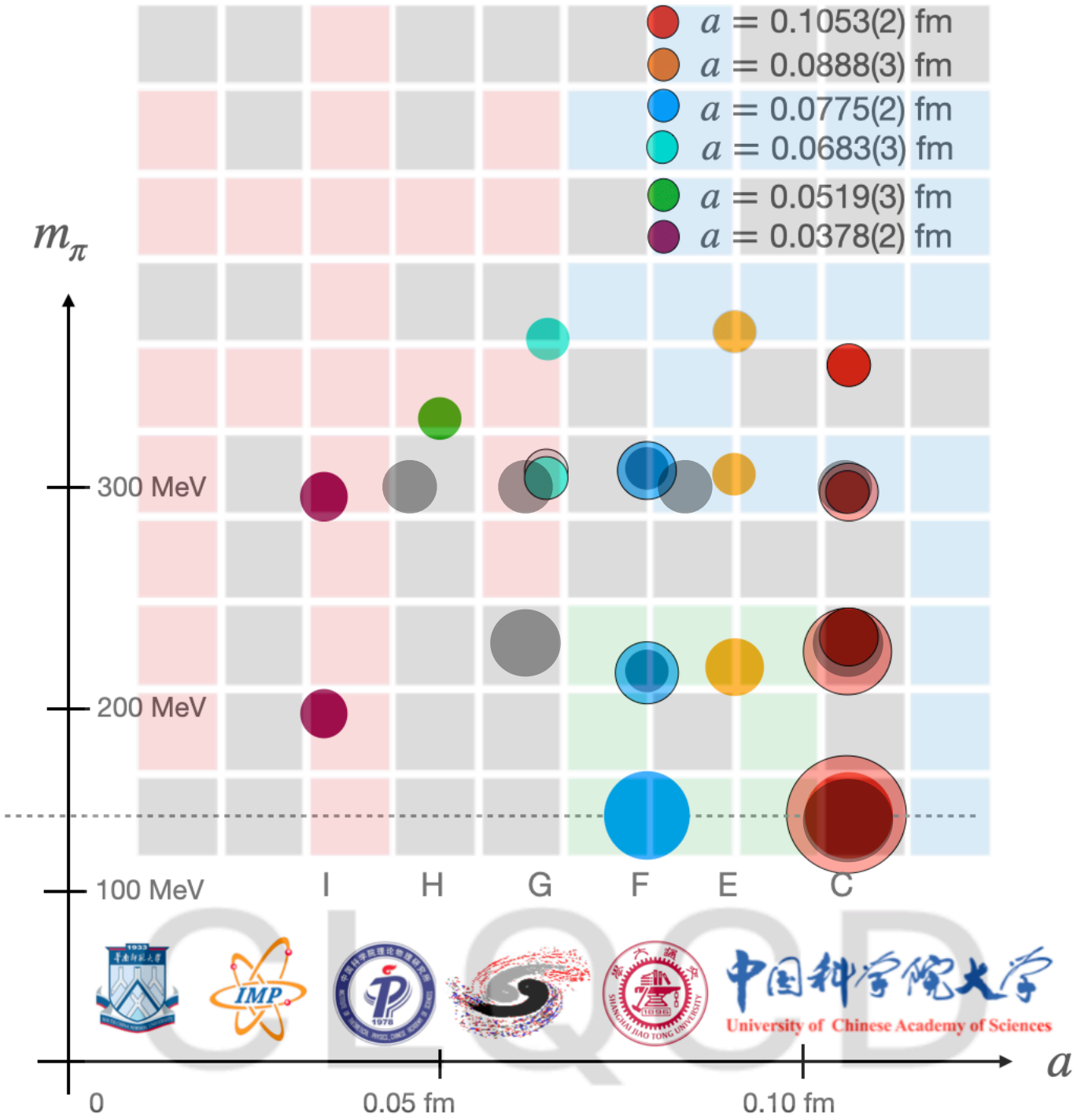
$$H_a \equiv \epsilon \text{Tr}[F_{\text{bare}}^2] = -\frac{2\mu^2}{m} \frac{\partial m}{\partial \mu^2} \sum_q m_q \bar{q}q + \frac{\mu^2}{\alpha_s} \frac{\partial \alpha_s}{\partial \mu^2} G^2 = \left[ \frac{2}{\pi} \alpha_s + \mathcal{O}(\alpha_s^2) \right] \sum_q m_q \bar{q}q + \left[ \left( -\frac{11}{8\pi} + \frac{N_f}{12\pi} \right) \alpha_s + \mathcal{O}(\alpha_s^2) \right] G^2$$



Y. Guo et al., arXiv:2501.10532  
X.Ji et al., arXiv:2503.01991

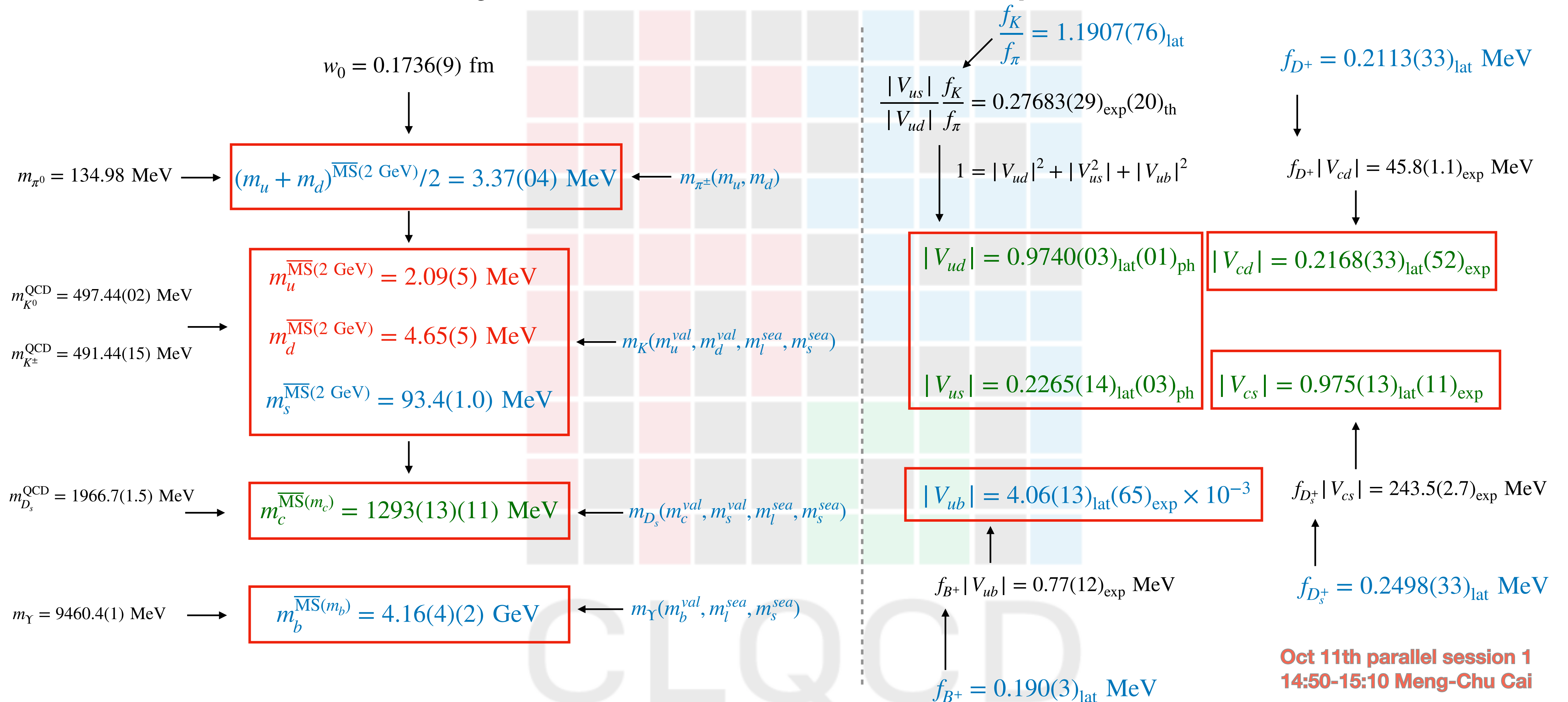
# CLQCD ensembles

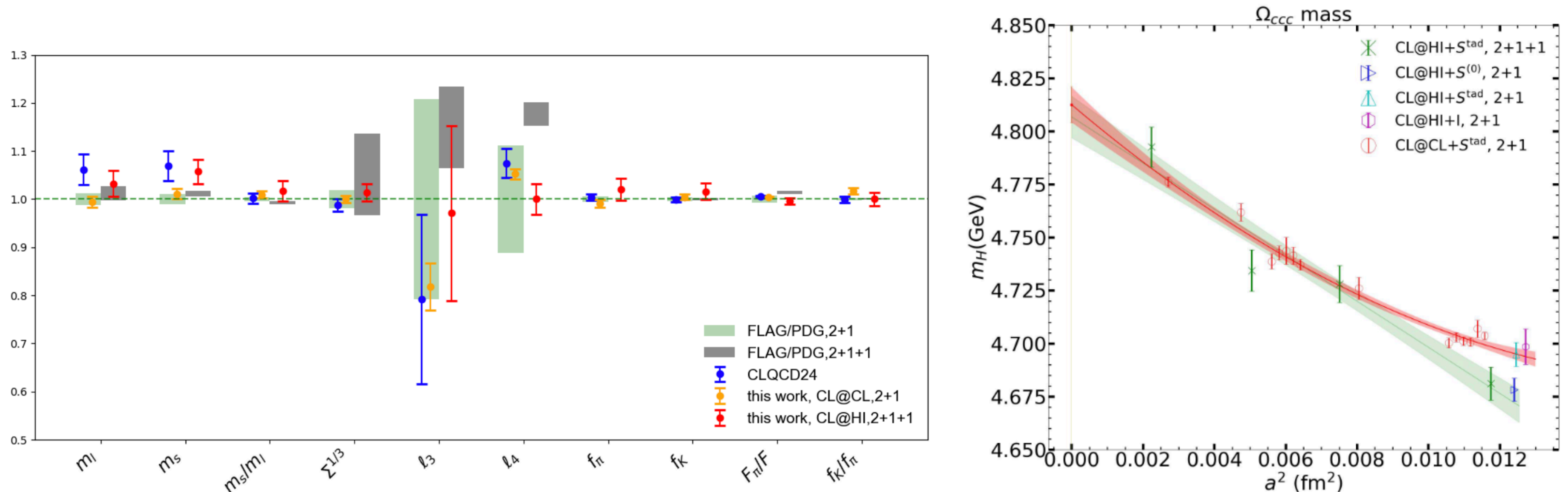
|       | Country/<br>Region | Smallest lattice<br>spacing | No. of<br>physical point<br>ensembles | Largest<br>spacial size | No. of<br>fermion<br>discretization |
|-------|--------------------|-----------------------------|---------------------------------------|-------------------------|-------------------------------------|
| MILC  | US                 | 0.03 fm                     | 5                                     | 5.8 fm                  | 1                                   |
| RBC   | US                 | 0.06 fm                     | 3                                     | 5.5 fm                  | 1                                   |
| BMW   | EN                 | 0.05 fm                     | 15                                    | 10 fm                   | 2                                   |
| CLS   | EN                 | 0.04 fm                     | 2                                     | 5.5 fm                  | 1                                   |
| ETM   | EN                 | 0.05 fm                     | 5                                     | 6.3 fm                  | 1                                   |
| PACS  | JP                 | 0.06 fm                     | 3                                     | 10 fm                   | 1                                   |
| CLQCD | CN                 | 0.04 fm                     | 4                                     | 6.7 fm                  | 2                                   |



# CLQCD ensembles

## Summary of our standard model parameter determinations

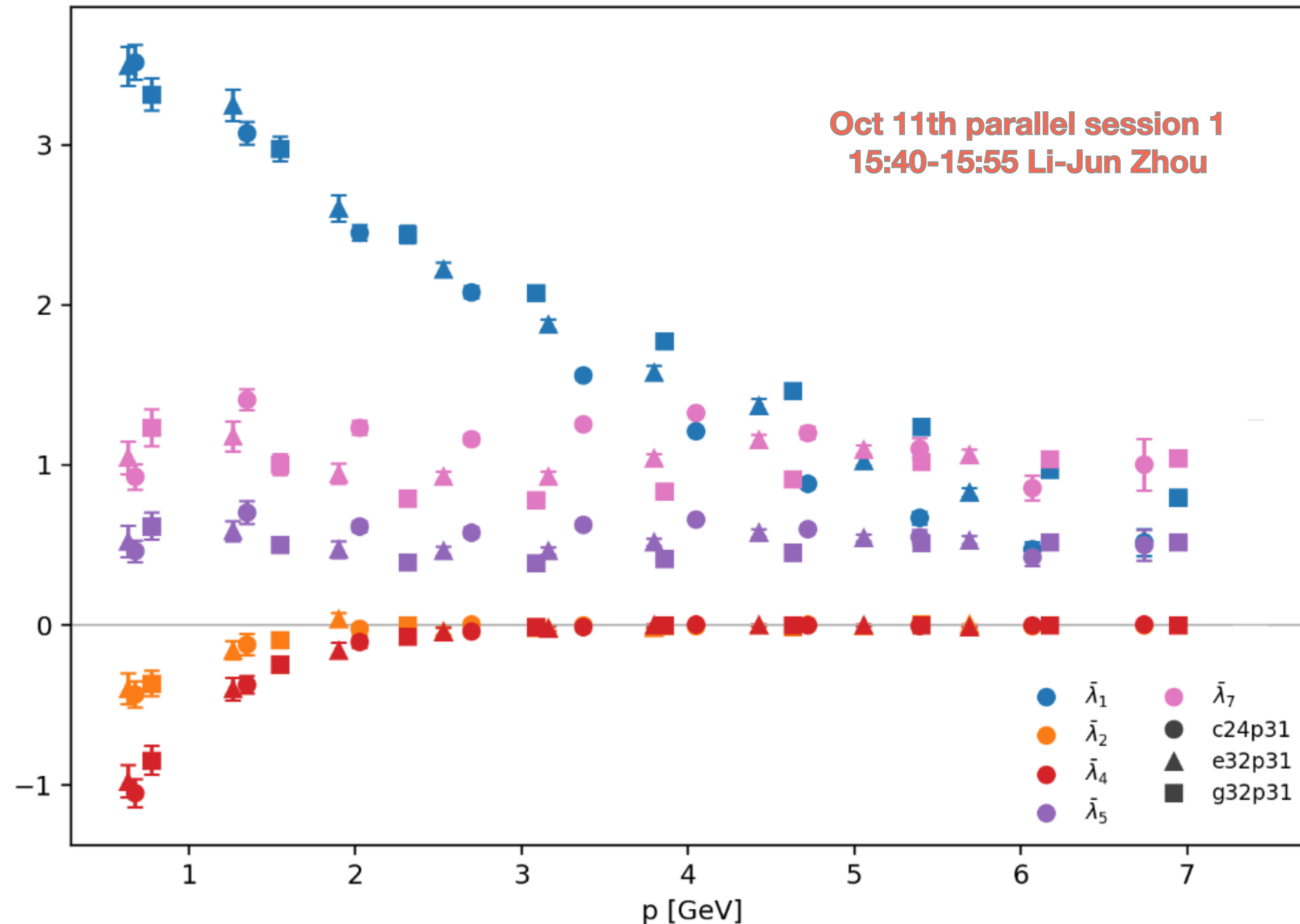




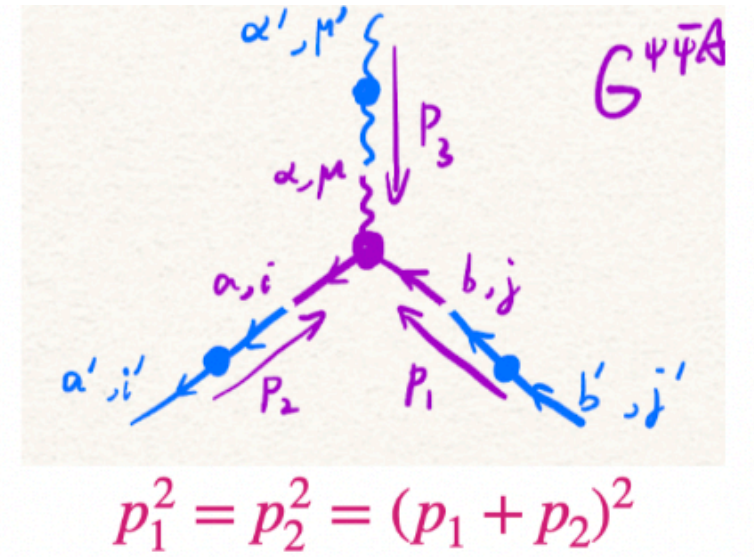
- The second generation of the CLQCD ensembles uses  $N_f=2(\text{light})+1(\text{strange})+1(\text{charm})$  setup and further suppresses the discretization errors;
- Good consistency between the 2+1 and 2+1+1 results for the light quark physics, using the same clover fermion action.
- Also good agreement for  $m_{\Omega_{ccc}}$  within 0.2% uncertainty.

# CLQCD ensembles

## Quark-gluon interactions



$$\begin{aligned}
 \bar{\psi} \not{D} \psi : & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(1)} \right]_{\mu} (p_2, p_1) = -i\gamma_{\mu}, \\
 \bar{\psi} \not{D}^2 \psi : & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(2)} \right]_{\mu} (p_2, p_1) = (p_1 - p_2)_{\mu} 1_s, \\
 & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(3)} \right]_{\mu} (p_2, p_1) = (\not{p}_1 - \not{p}_2) \gamma_{\mu}, \\
 & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(4)} \right]_{\mu} (p_2, p_1) = (\not{p}_1 + \not{p}_2) \gamma_{\mu}, \\
 \bar{\psi} \not{D}^3 \psi : & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(5)} \right]_{\mu} (p_2, p_1) = i(\not{p}_1 + \not{p}_2)(p_1 - p_2)_{\mu}, \\
 & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(6)} \right]_{\mu} (p_2, p_1) = i(\not{p}_1 - \not{p}_2)(p_1 - p_2)_{\mu}, \\
 & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(7)} \right]_{\mu} (p_2, p_1) = \frac{i}{2} [\not{p}_1, \not{p}_2] \gamma_{\mu}, \\
 \bar{\psi} \not{D}^4 \psi : & \quad \left[ \mathcal{T}_{\psi\bar{\psi}A}^{(8)} \right]_{\mu} (p_2, p_1) = -\frac{1}{2} [\not{p}_1, \not{p}_2] (p_1 - p_2)_{\mu}.
 \end{aligned}$$

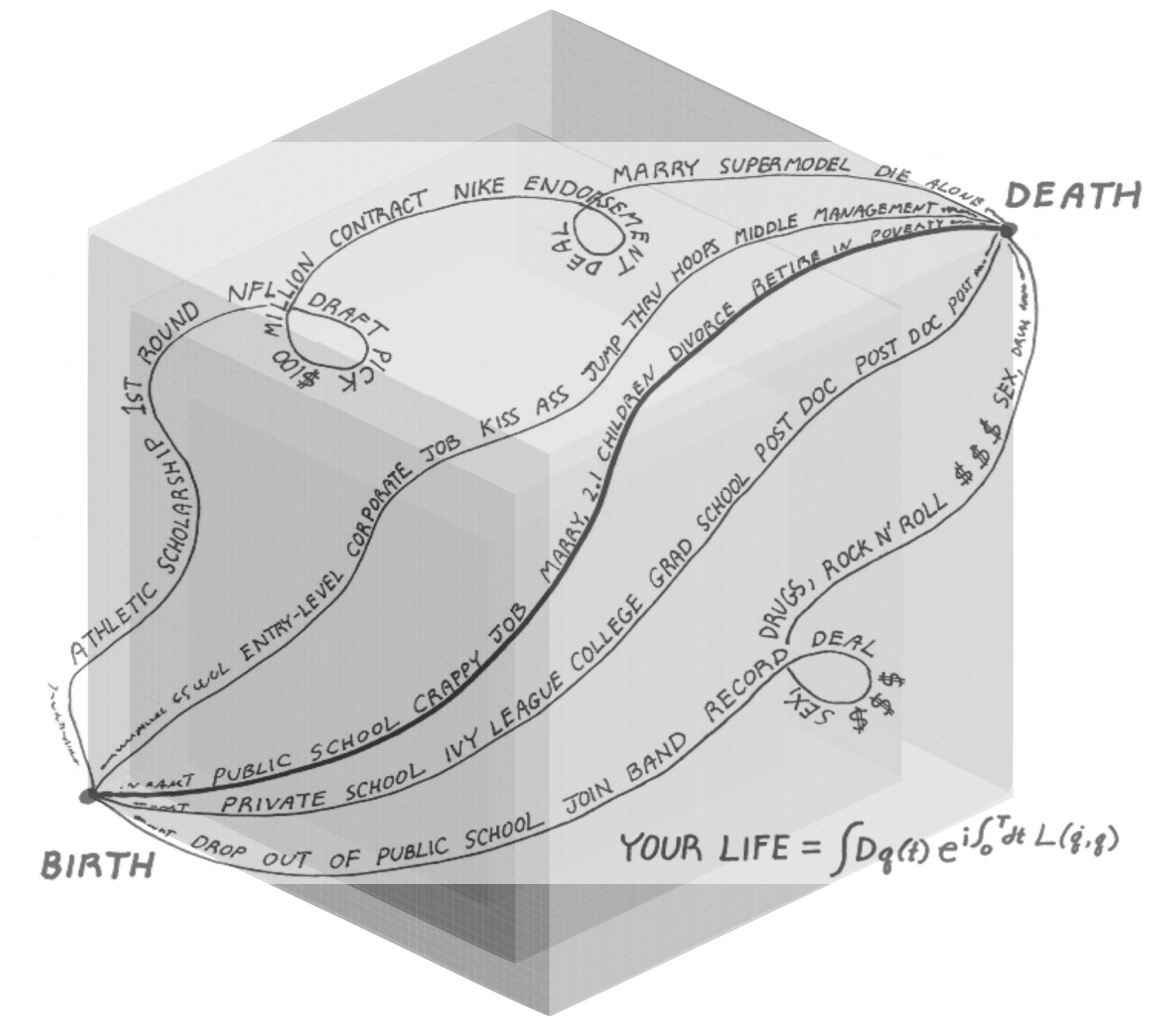


- For the first time, all the effective quark-gluon effective low energy interactions are calculated on the lattice;
- Using the chiral fermion on the 2+1+1 flavor ensembles and extrapolated to the continuum;
- High order term including the quark anomalous magnetic-moment term turn out to be important.

# Hadron mass decomposition

From Lattice QCD

$$m_H = \sum_q m_q \langle \bar{q}q \rangle_H + \left\{ \sum_q \langle \bar{q} \vec{\gamma} \cdot \vec{D} q \rangle_H + \langle \text{Tr}[B^2 - E^2] \rangle_H + \frac{\epsilon}{4} \langle \text{Tr}[F_{\text{bare}}^2] \rangle_H \right\} + \left\{ \sum_q e_q^2 \Delta_{q,\text{self}}^{\text{QED}} m_H + \sum_{q,q' \neq q} e_q e_{q'} \Delta_{qq',\text{int}}^{\text{QED}} m_H + \mathcal{O}(\alpha^2) \right\}$$



We extract:

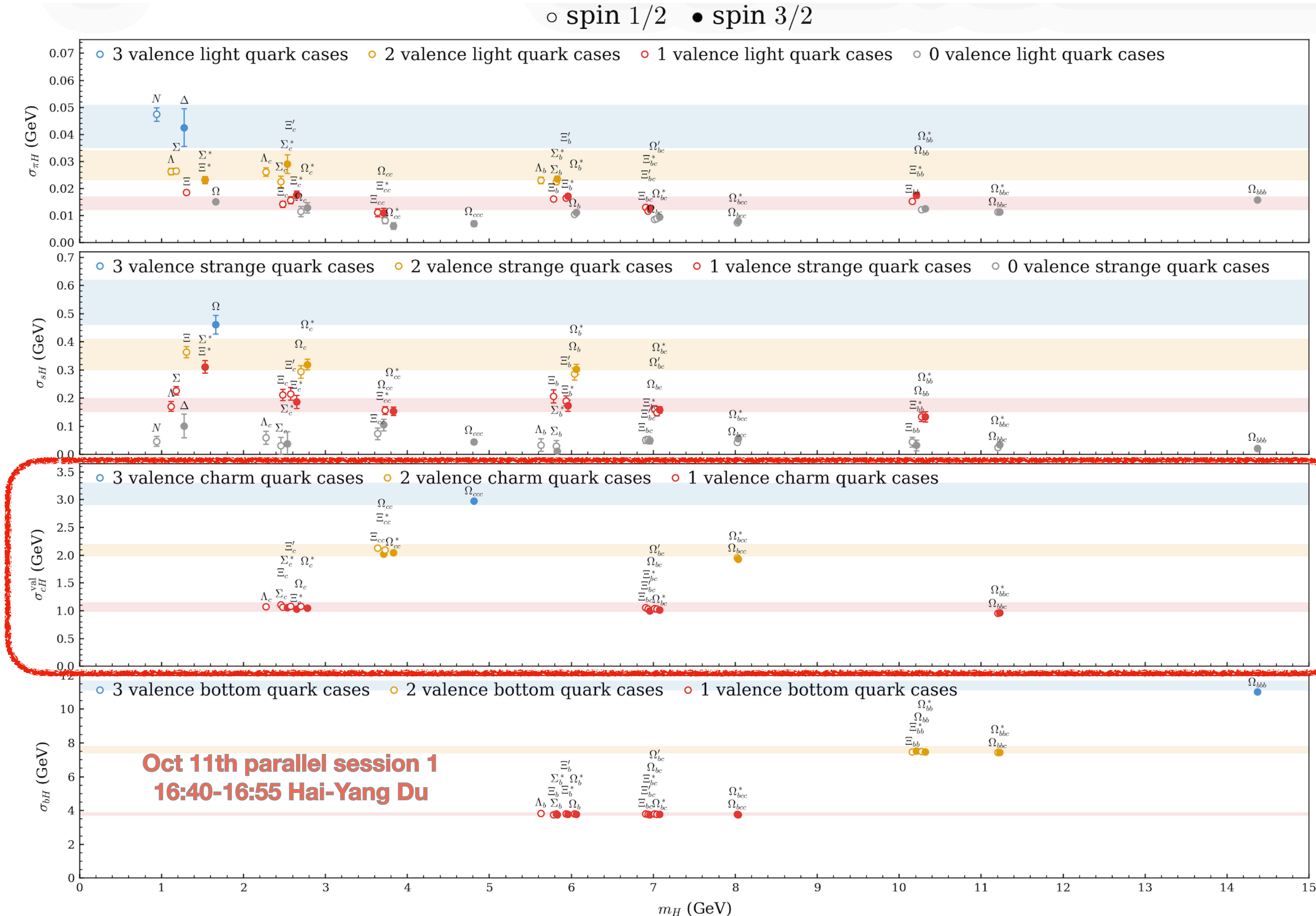
- Hadron mass  $m_H$  from the two point functions  $\sum_{\vec{x}, \vec{y}} \langle \mathcal{H}(\vec{x}, t) \mathcal{H}(\vec{y}, 0) \rangle_{t \rightarrow \infty} \rightarrow C e^{-m_H t}$  with different quark masses;
- Sigma term  $m_q \langle \bar{q}q \rangle$  from the Feynman-Hellman theorem  $m_q \langle \bar{q}q \rangle = m_q \frac{\partial m_H(m_q)}{\partial m_q}$ ;
- Trace anomaly from both the QCD EMT trace sum rule and also direct calculations;
- QED correction from the QCD + quenched  $\text{QED}_L$  with all the  $\alpha \alpha_s^n$  corrections except the QED vacuum polarization.

Oct 11th parallel session 1  
16:10-16:25 Chen-Yu Zhang  
16:25-16:40 Yi-Yang Liu  
16:40-16:55 Hai-Yang Du



# Quark mass contributions

## Charm quarks



$$\bar{q}(x)\gamma_4 q(x) = \int \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} - a_{-,p}^\dagger a_{-,p})$$

$$\begin{aligned} \bar{q}(x)q(x) &= \int \frac{d^3p}{(2\pi^3)} \frac{m}{E} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \\ &\leq \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \end{aligned}$$

- $m_c^{\overline{\text{MS}}(m_c)} = 1.29(2)$  GeV and  $m_c^{\overline{\text{MS}}(2 \text{ GeV})} = 1.09(1)$  GeV;
- Its scale independent components is 1.17(5) GeV; L. Chen, Z. Li, M. Niggetiedt, PLB 871(2025) 139935
- Contribution per valence flavor is also ~1.1 GeV;
- Slightly reduced with more valence charm.

# Quark mass contributions

## Bottom quarks

$$\bar{q}(x)\gamma_4 q(x) = \int \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} - a_{-,p}^\dagger a_{-,p})$$

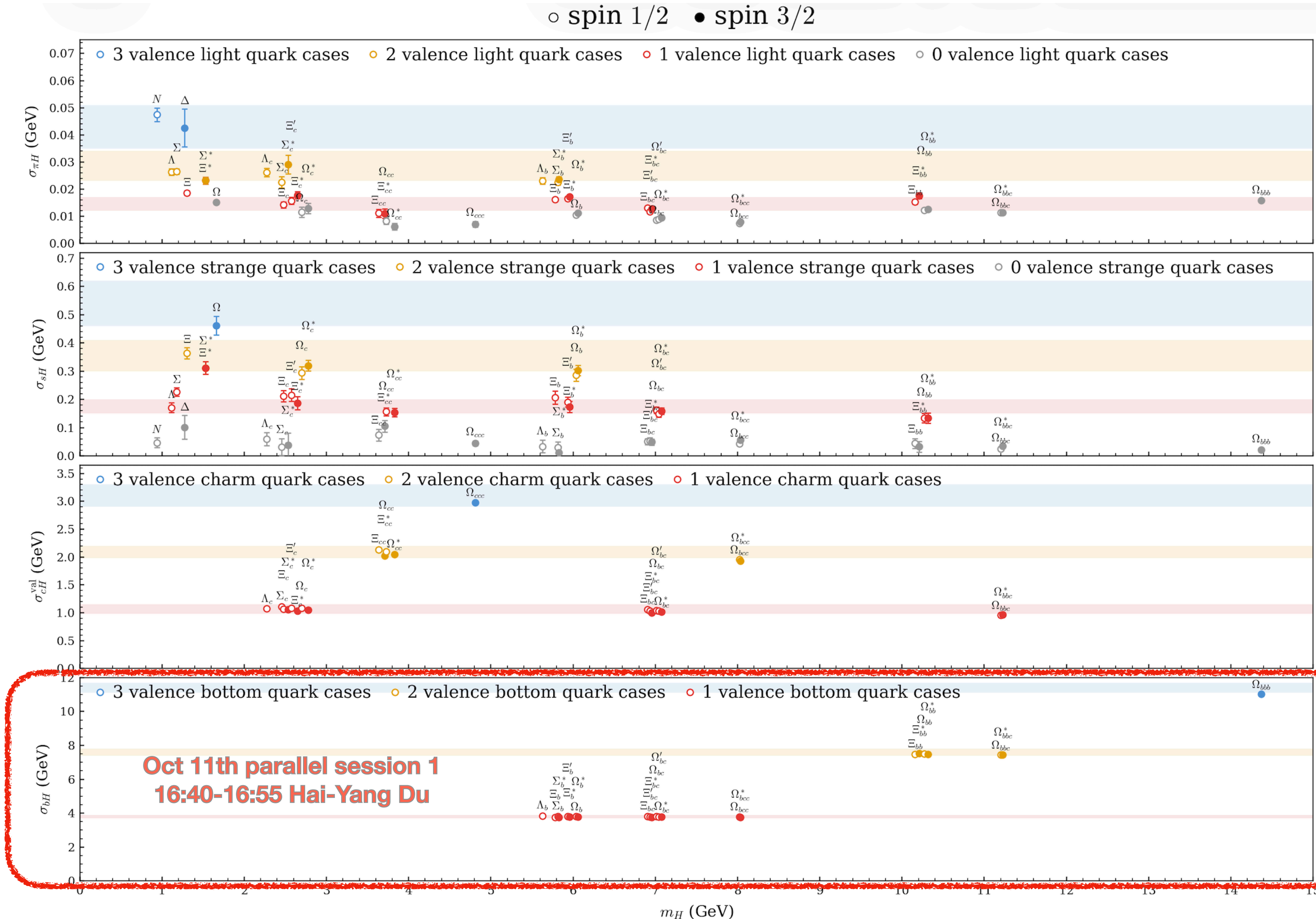
$$\begin{aligned} \bar{q}(x)q(x) &= \int \frac{d^3p}{(2\pi^3)} \frac{m}{E} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \\ &\leq \frac{d^3p}{(2\pi^3)} (a_{+,p}^\dagger a_{+,p} + a_{-,p}^\dagger a_{-,p}) \end{aligned}$$

- $m_b^{\overline{\text{MS}}(m_b)} = 4.16(4)$  GeV and  $m_b^{\overline{\text{MS}}(2 \text{ GeV})} = 4.94(5)$  GeV;

- Its scale independent components is  $3.97(8)$  GeV;

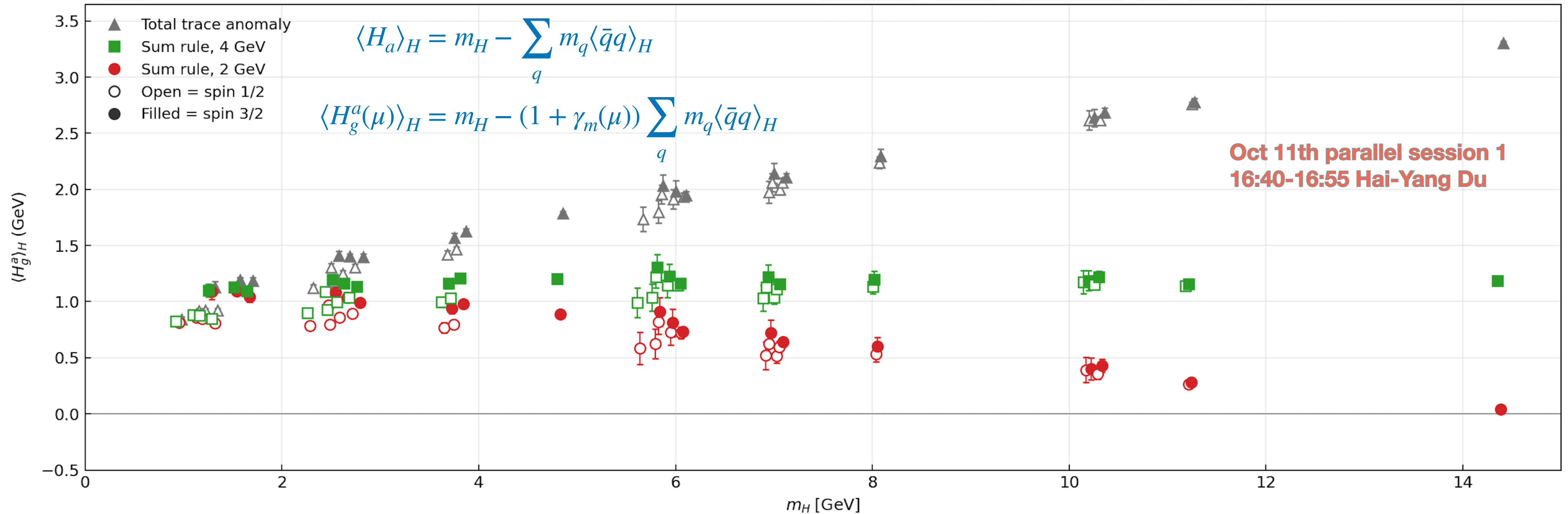
L. Chen, Z. Li, M. Niggetiedt, PLB 871(2025) 139935

- Contribution per valence flavor is also  $\sim 3.7$  GeV;
- Slightly reduced with more valence bottom.



B.-L. Hu, et. al., [CLQCD], arXiv: 2411.18402

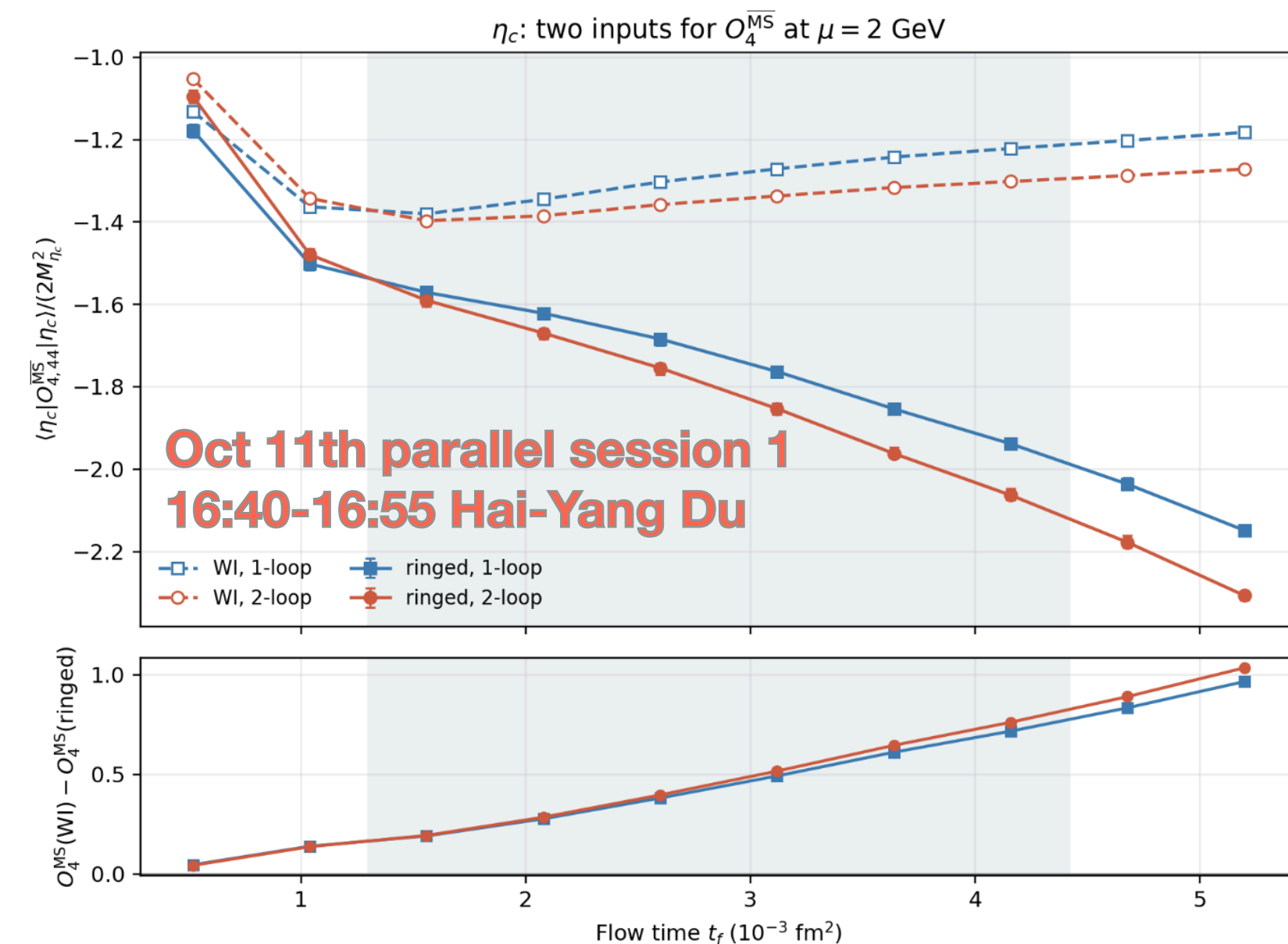
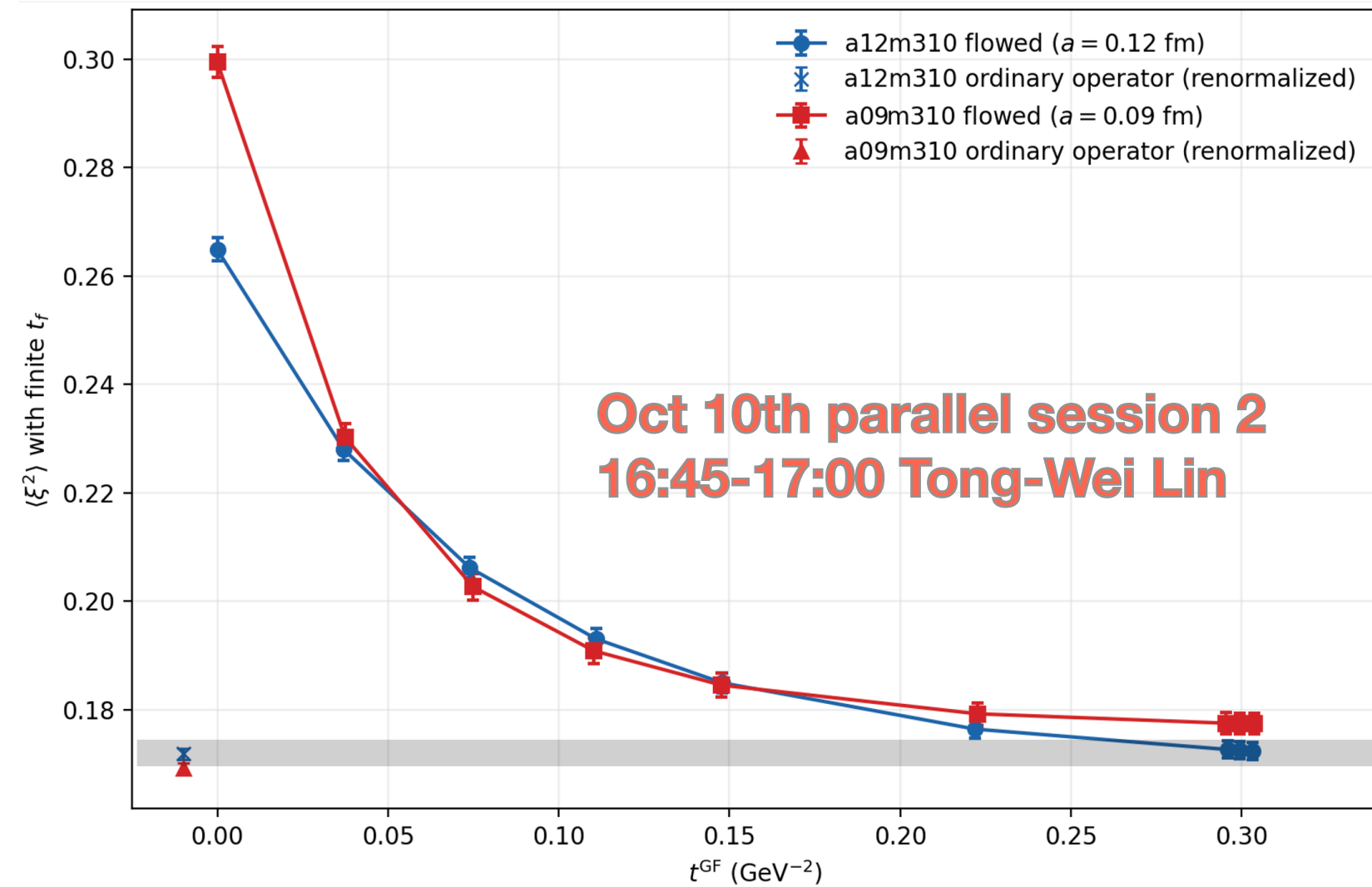
C.Y. Zhang, H.Y Du et al. [CLQCD], in preparation



- The total trace anomaly contribution increase linearly on the baryon mass based on the sum rule;
- $\langle H_g^a \rangle \sim 1$  GeV for all the ground state baryons from  $\sim 1$  GeV (nucleon) to 15 GeV  $\Omega_{bbb}$ , at  $\overline{\text{MS}}$  4 GeV;
- Scale dependence of the gluon trace anomaly becomes more obvious in the heavier baryon.

# Gradient flow

As an UV regulator



$$\int_{-\pi/a}^{\pi/a} \frac{d^4 p}{(2\pi)^4} \frac{1}{\left(\frac{4}{a^2} \sum_{\mu} \sin^2 \frac{p_{\mu} a}{2} + m^2\right)^n} \rightarrow \int_{-\pi/a}^{\pi/a} \frac{d^4 p}{(2\pi)^4} \frac{e^{-p^2 t^{\text{GF}}}}{\left(\frac{4}{a^2} \sum_{\mu} \sin^2 \frac{p_{\mu} a}{2} + m^2\right)^n}$$

$$= \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-p^2 t^{\text{GF}}}}{(p^2 + m^2)^n} \Big|_{t^{\text{GF}} \gg a^2/\pi^2}$$

- The gradient flow introduces a “soft” UV cutoff which can remove the impact of arbitrary regularization effect with large enough  $t^{\text{GF}}$ ;
- Taking the vector meson second moment as example, flowed ME become closer at  $t^{\text{GF}} \geq 0.05 \text{ GeV}^2$ , and similar to the Renormalized value;
- Using vector current conservation to define ringed  $Z_q(t^{\text{GF}})$  can suppress the  $\mathcal{O}((t^{\text{GF}})^n)$  correction of the charm sigma term at finite flow time  $t^{\text{GF}}$ .

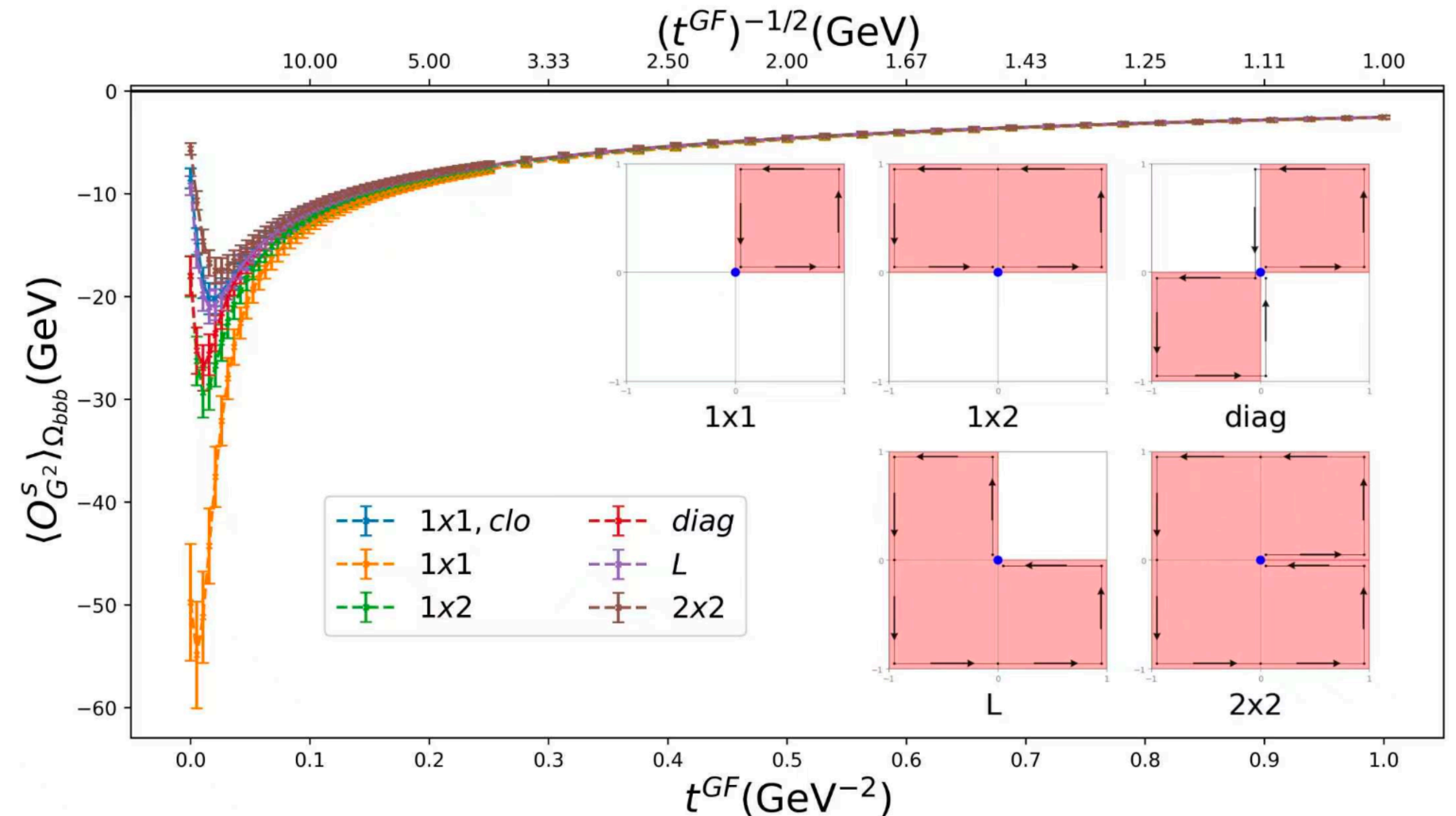
M. Q. Gao et al., arXiv:2609.23379  
D. Nollweg et al., PRL 137(2026)061901

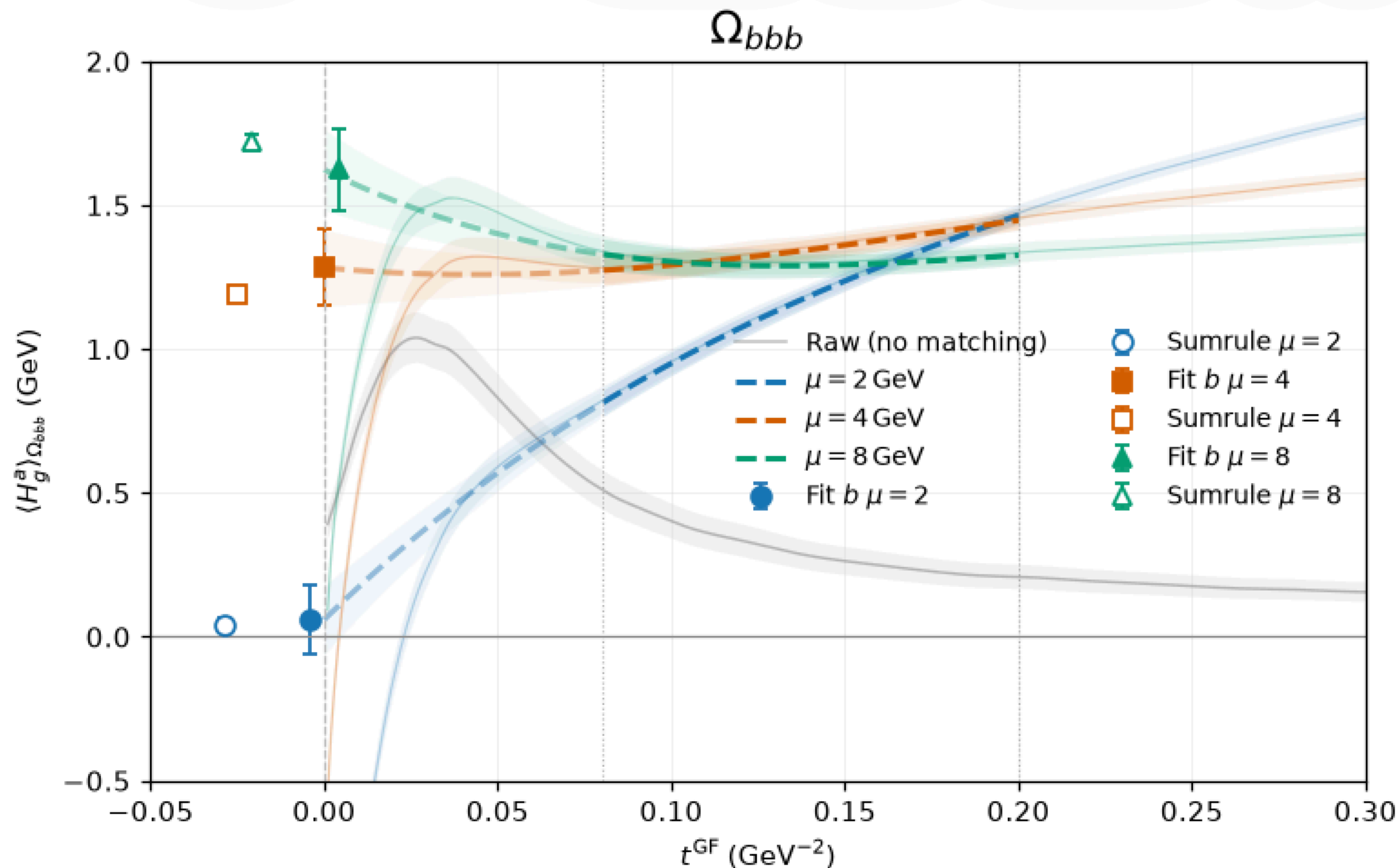
T. W. Lin et al., [CLQCD], in preparation  
C.Y. Zhang, H.Y. Du et al. [CLQCD], in preparation

- On the lattice, one can have many definitions of  $g^2 F^2$  which differ from each other at  $\mathcal{O}(a^2) + \mathcal{O}(\alpha_s)$ ;
- The gradient flow can also eliminate this ambiguity at large enough  $t^{\text{GF}}$ ;
- Taking  $\langle g^2 \text{Tr}[F^2] \rangle_{\Omega_{bbb}}$  at  $a = 0.052$  fm as example, the matrix element becomes independent of the definition of  $F^2$  at large enough  $t^{\text{GF}}$ .

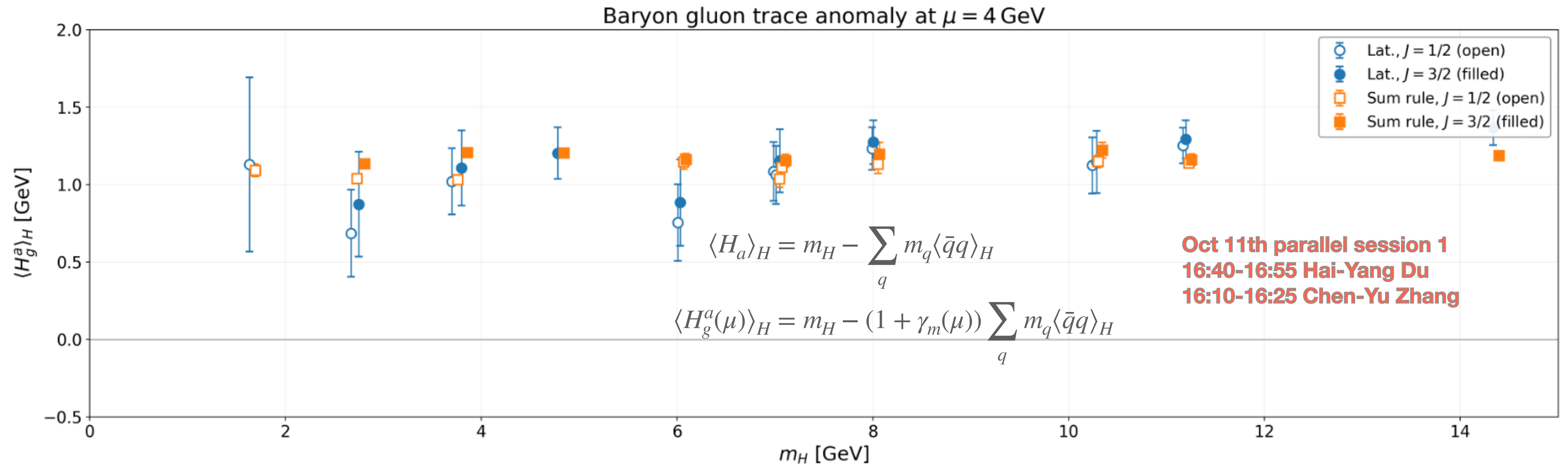
$$\int_{-\pi/a}^{\pi/a} \frac{d^4 p}{(2\pi)^4} \frac{1}{\left(\frac{4}{a^2} \sum_{\mu} \sin^2 \frac{p_{\mu} a}{2} + m^2\right)^n} \rightarrow \int_{-\pi/a}^{\pi/a} \frac{d^4 p}{(2\pi)^4} \frac{e^{-p^2 t^{\text{GF}}}}{\left(\frac{4}{a^2} \sum_{\mu} \sin^2 \frac{p_{\mu} a}{2} + m^2\right)^n}$$

$$= \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-p^2 t^{\text{GF}}}}{(p^2 + m^2)^n} \Big|_{t^{\text{GF}} \gg a^2/\pi^2}$$





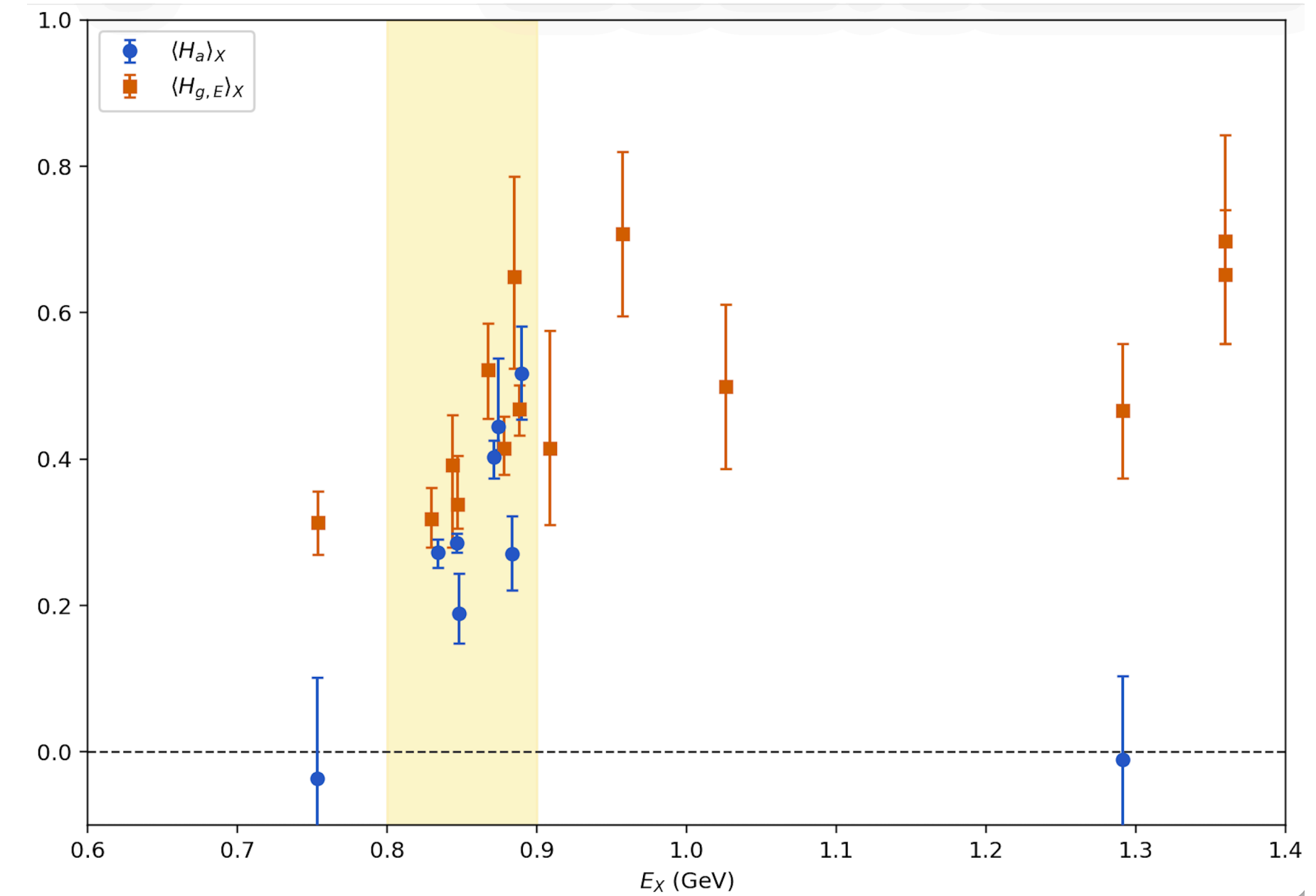
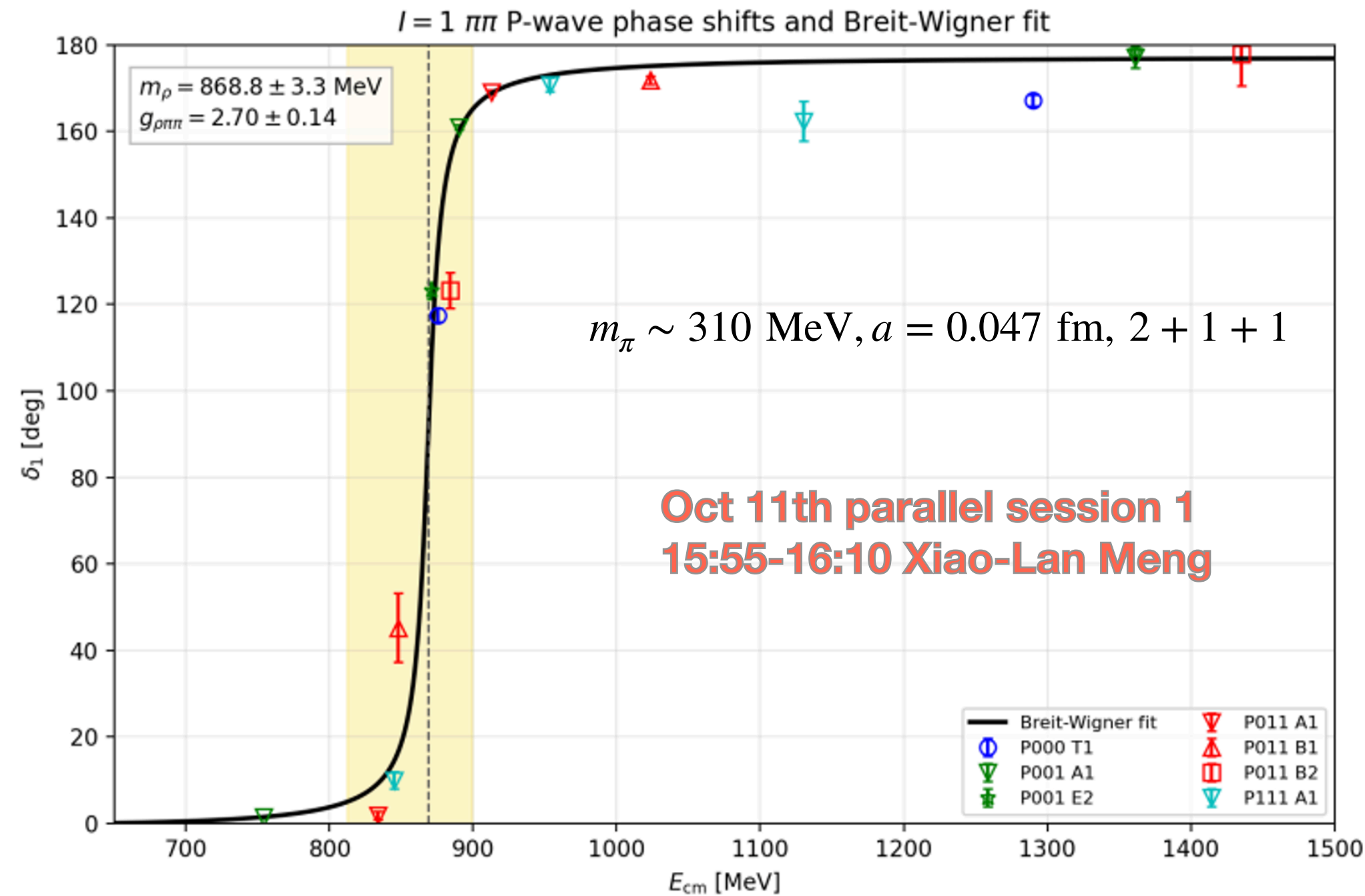
- Perturbative matching and also mixing with  $\sum_q m_q \langle \bar{q}q \rangle_H$  are required to convert the gluon trace anomaly at finite flow time  $t^{GF}$  into that in the specific  $\overline{\text{MS}}$  scale;
- $t^{GF} \rightarrow 0$  extrapolation is also required to eliminate the  $\mathcal{O}(t^n \Lambda_{\text{QCD}}^{2n})$  contaminations of the gradient flow scheme;
- Obvious scale dependence of the gluon trace anomaly  $\langle H_g^a \rangle_{\Omega_{bbb}}$  is reproduced after the extrapolations, and consistent with those from the sum rule.



- The direct calculation only covers kinds of hadrons without light quarks due to limited statistics;
- $\langle H_g^a \rangle^{\overline{\text{MS}}}(4 \text{ GeV}) \sim 1 \text{ GeV}$  for all the ground state baryons is verified by the direct calculation of the gluon trace anomaly.

# Trace anomaly

For unstable particles

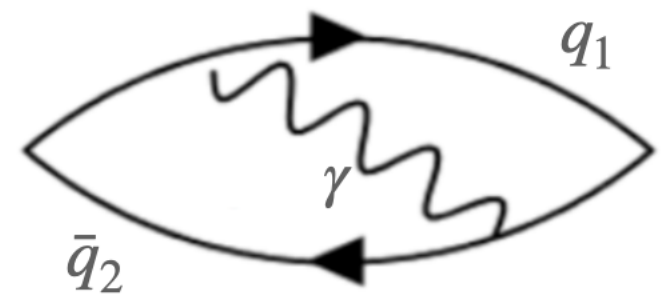


$$m_H = \langle \int d^3x T^{00}(x) \rangle_H = \sigma_H + \langle H_{q,E} \rangle_H + \langle H_{g,E} \rangle_H + \frac{1}{4} \langle H_a \rangle_H$$

$$\langle \sum_i \int d^3x T^{ii}(x) \rangle_H = \langle H_{q,E} \rangle_H + \langle H_{g,E} \rangle_H - \frac{3}{4} \langle H_a \rangle_H$$

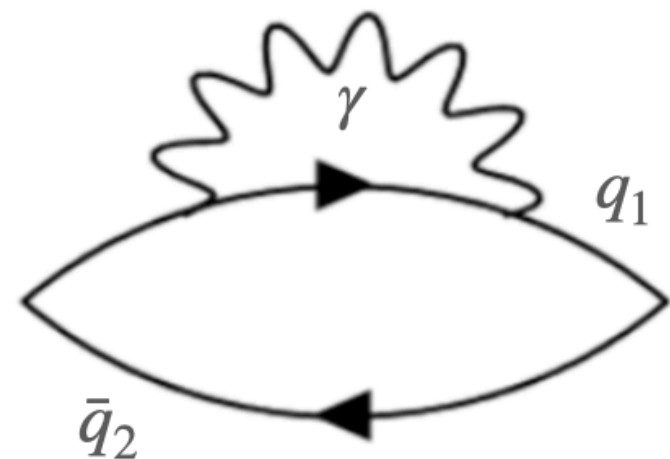
- The  $\rho - \pi\pi$  system has sizable  $\langle H_{g,E} \rangle_X$  for arbitrary energy level;
- On the other hand,  $\langle H_a \rangle_X$  has a peak around the  $\rho$  threshold while much smaller for  $\pi\pi$ .

$$\Delta^{\text{QED}} m_H = \sum_q e_q^2 \Delta_{q,\text{self}}^{\text{QED}} m_H + \frac{1}{2} \sum_{q,q' \neq q} e_q e_{q'} \Delta_{qq',\text{int}}^{\text{QED}} m_H + \mathcal{O}(\alpha^2)$$



QED UV finite (at 1-loop level)

$$-e_{q_1} e_{q_2} \Delta_{q_1 q_2, \text{int}}^{\text{QED}} m_H (m_q + \Delta^{\text{QED}} m_q) \\ \simeq -e_{q_1} e_{q_2} \Delta_{q_1 q_2, \text{int}}^{\text{QED}} m_H (m_q)$$

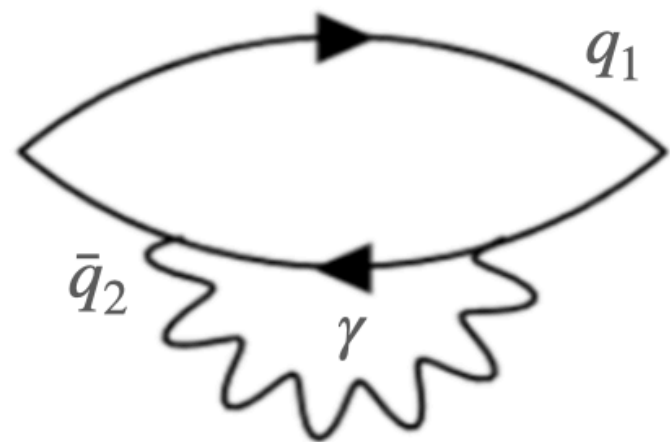


$$e_{q_1}^2 \delta_{\text{QED}}^{\text{self}} m_H$$

QED UV finite

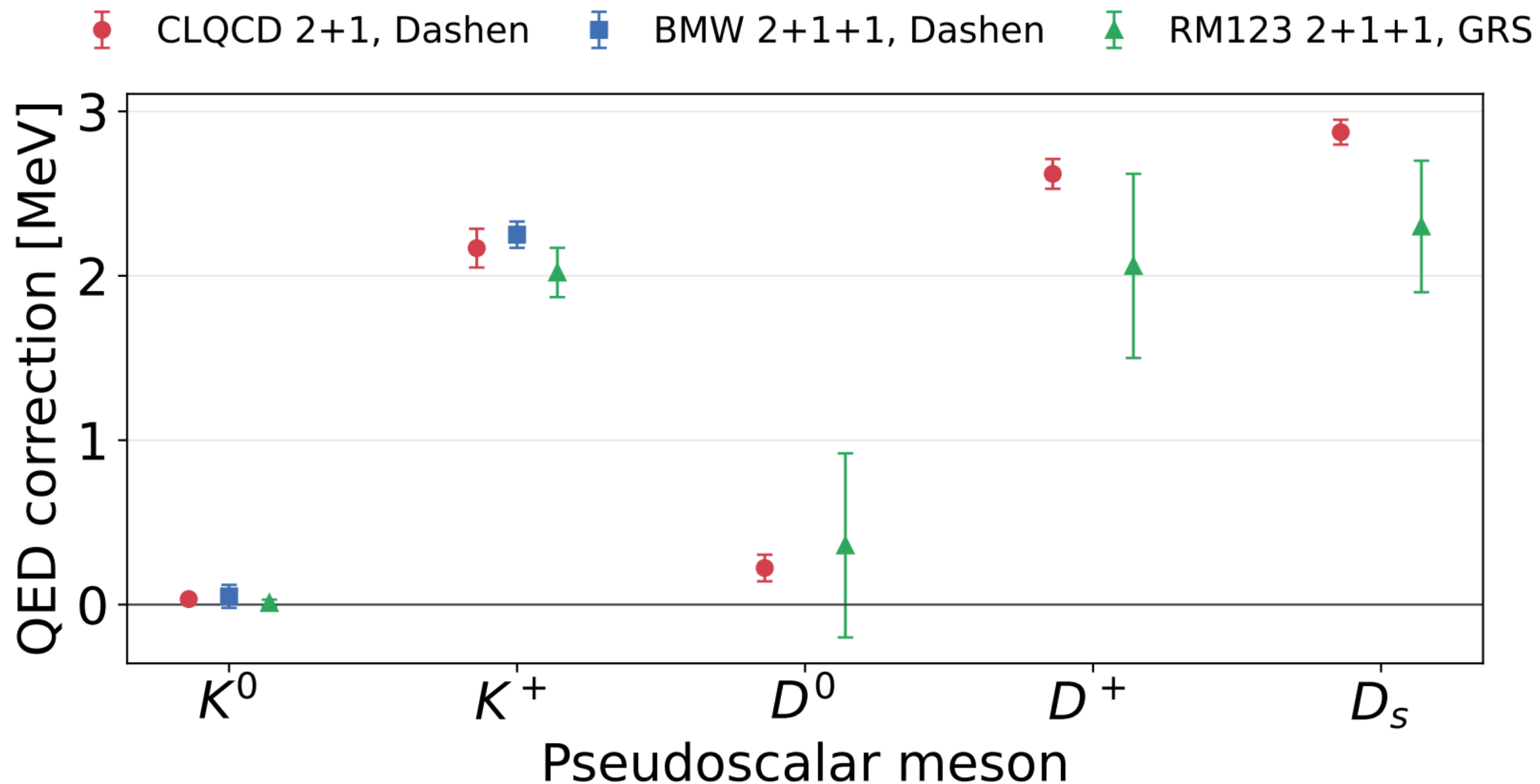
$$\Delta_{q,\text{self}}^{\text{QED}} m_H (m_q + \Delta^{\text{QED}} m_q) \\ \simeq \Delta_{q,\text{self}}^{\text{QED}} m_H (m_q) + \langle \bar{q} q \rangle_H \Delta^{\text{QED}} m_q$$

QED UV divergent    QED UV divergent

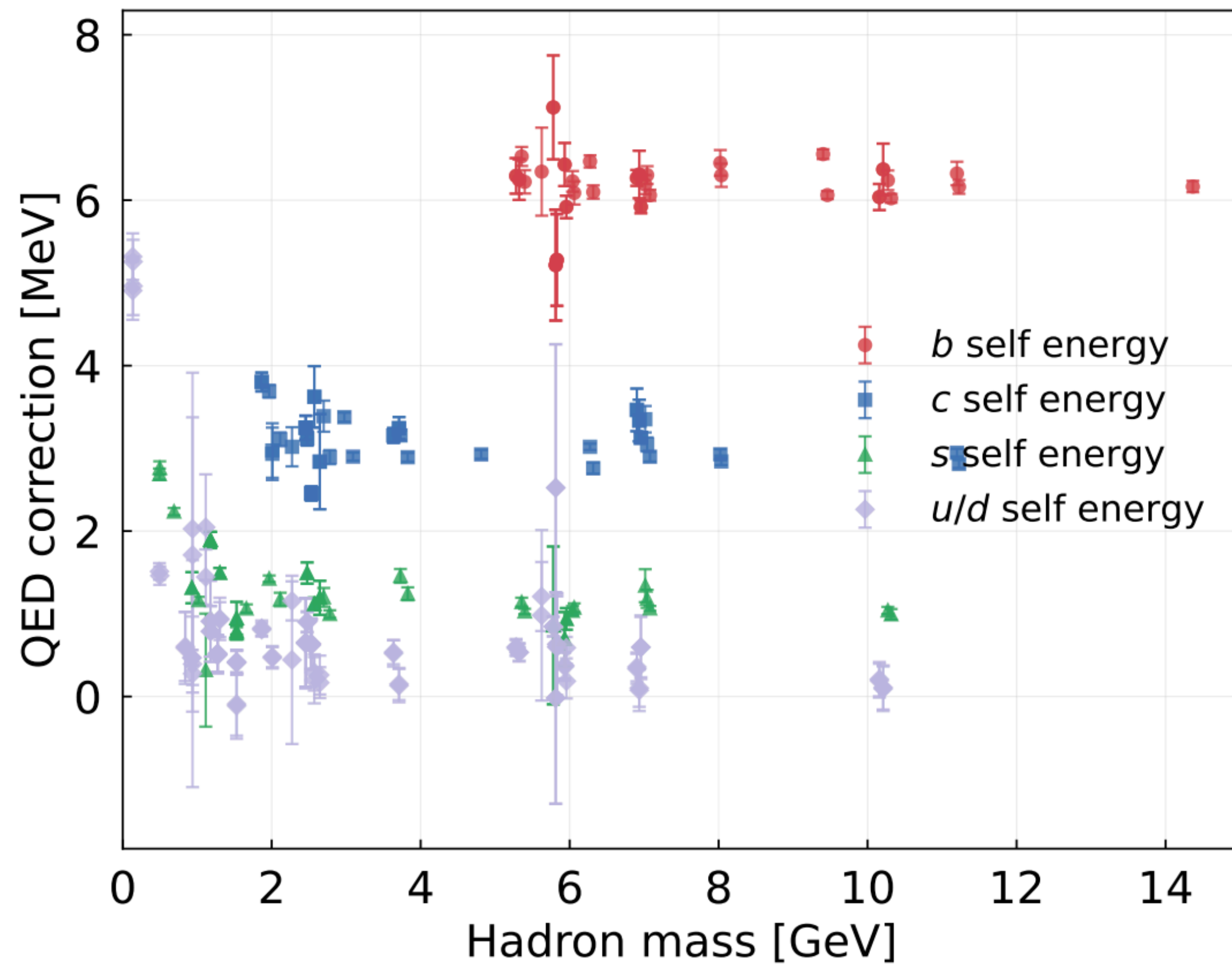


$$e_{q_2}^2 \delta_{\text{QED}}^{\text{self}} m_H$$

- QED correction of the hadron mass introduces additional UV divergence from the quark mass at  $\mathcal{O}(\alpha)$  level;
- Dashen scheme suggests matching condition  $m_{\eta_q}(m_q^{\text{QC+ED}}, e_q) = m_{\eta_q}(m_q^{\text{QCD}}, 0)$  to define the quark mass  $m_{q,\text{Das}}^{\text{QC+ED}}(\mu) = m_q^{\text{QCD}}(\mu)(1 + c_1(\mu)e_q^2\alpha + \mathcal{O}(\alpha^2))$ ;
- GRS scheme further matches the quark mass at  $\overline{\text{MS}}$  2 GeV,  $m_{q,\text{GRS}}^{\text{QC+ED}}(\mu) = m_q^{\text{QCD}}(\mu)(1 + (c_1(\mu) - c_1(2 \text{ GeV}))e_q^2\alpha + \mathcal{O}(\alpha^2))$ .



- Both the CLQCD and BMW  $\Delta^{\text{QED}} m_{K^{\pm/0}}$  results with Dashen scheme are consistent with that using the GRS scheme;
- $\Delta^{\text{QED}} m_{D_{(s)}^{\pm/0}}$  are also consistent while the Dashen scheme is free of the systematic uncertainty in the GRS scheme from the perturbative matching of the quark mass.
- QED corrections for the neutral particles are around zero, while those of the charged particles are much larger.



- $\Delta_{u/d/s,\text{self}}^{\text{QED}} m_H$  are around 1-2 MeV and suggest good SU(3) flavor symmetry, except the cases of light pseudo-scalar mesons;
- The QED self energy corrections for the heavy flavors are hadron insensitive:
- $\Delta_{b,\text{self}}^{\text{QED}} m_H \sim 6 \text{ MeV}$ ,  $e_b^2 \Delta_{b,\text{self}}^{\text{rmQED}} m_H \sim 0.6 \text{ MeV}$ ;
- $\Delta_{c,\text{self}}^{\text{QED}} m_H \sim 3 \text{ MeV}$ ,  $e_c^2 \Delta_{c,\text{self}}^{\text{rmQED}} m_H \sim 1.3 \text{ MeV}$ .
- This correction can not be eliminated through a quark mass independent renormalization, as  $\Delta_{b,\text{self}}^{\text{QED}} / \Delta_{c,\text{self}}^{\text{QED}} \neq m_b / m_c$  or  $m_b \langle \bar{b}b \rangle_H / (m_c \langle \bar{c}c \rangle_H)$ .
- $\Delta_{l,\text{self}}^{\text{QED}} m_\pi |_{m_l \rightarrow 0} \sim 33 \text{ MeV}$  based on the extrapolation.

# QED corrections

## Interaction contribution

$$\begin{aligned}\delta_{\text{QED}} m_{B_0} &= e_b^2 \delta_{\text{QED}}^{\text{self},b} m_{B_0} + e_d^2 \delta_{\text{QED}}^{\text{self},d} m_{B_0} + e_b e_d \delta_{\text{QED}}^{\text{int},bd} m_{B_0} \\ &= \frac{1}{9} 6.3 \text{ MeV} + \frac{1}{9} 0.6 \text{ MeV} - \frac{1}{9} 3.2 \text{ MeV} \\ &= 0.3 \text{ MeV}\end{aligned}$$

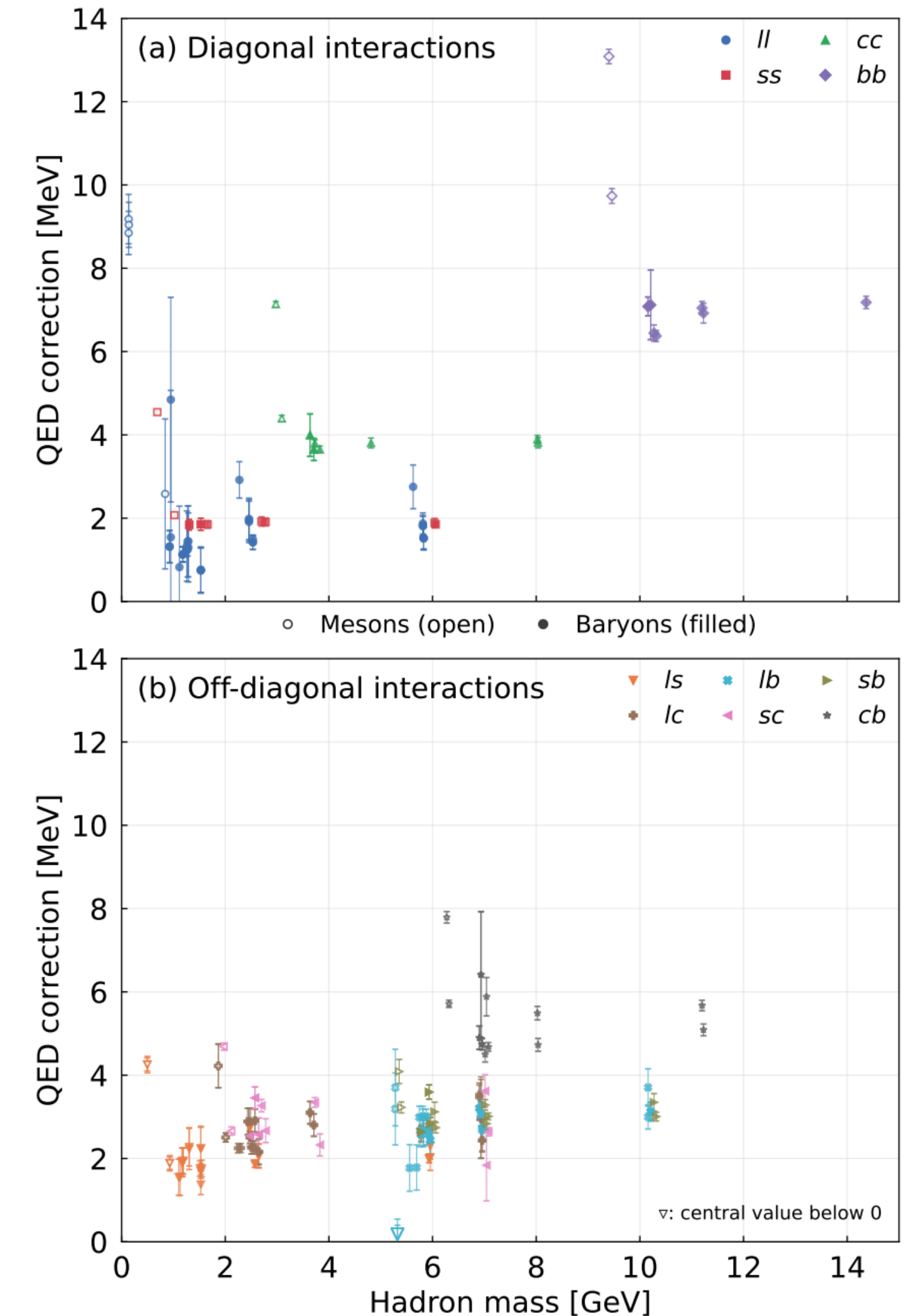
Self energy  
+  
Attractive interaction

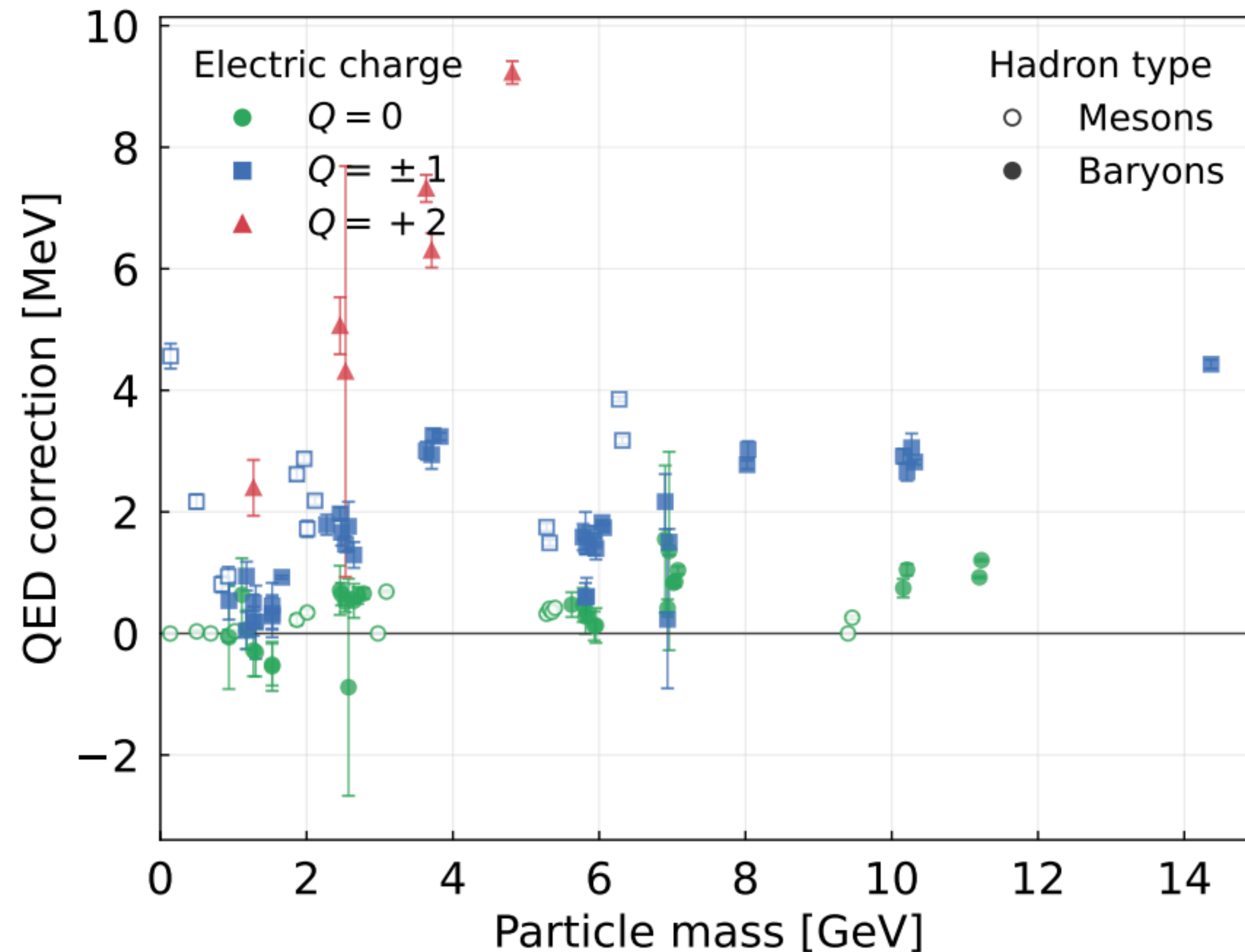
$$\begin{aligned}\delta_{\text{QED}} m_{B_+} &= e_b^2 \delta_{\text{QED}}^{\text{self},b} m_{B_+} + e_u^2 \delta_{\text{QED}}^{\text{self},u} m_{B_+} + e_b e_d \delta_{\text{QED}}^{\text{int},bd} m_{B_+} \\ &= \frac{1}{9} 6.3 \text{ MeV} + \frac{4}{9} 0.6 \text{ MeV} + \frac{2}{9} 3.2 \text{ MeV} \\ &= 1.7 \text{ MeV}\end{aligned}$$

Self energy  
+  
Repulsive interaction

$$\begin{aligned}\delta_{\text{QED}} m_{\Omega_{bbc}} &= 2e_b^2 \delta_{\text{QED}}^{\text{self},b} m_{\Omega_{bbc}} + e_c^2 \delta_{\text{QED}}^{\text{self},c} m_{\Omega_{bbc}} + e_b^2 \delta_{\text{QED}}^{\text{int},bb} m_{\Omega_{bbc}} + 2e_b e_c \delta_{\text{QED}}^{\text{int},bc} m_{\Omega_{bbc}} \\ &= 2\frac{1}{9} 6.3 \text{ MeV} + \frac{4}{9} 2.9 \text{ MeV} + \frac{1}{9} 7.0 \text{ MeV} - 2\frac{2}{9} 5.7 \text{ MeV} \\ &= 0.9 \text{ MeV}\end{aligned}$$

Self energy  
+  
Repulsive interaction  
+  
Attractive interaction

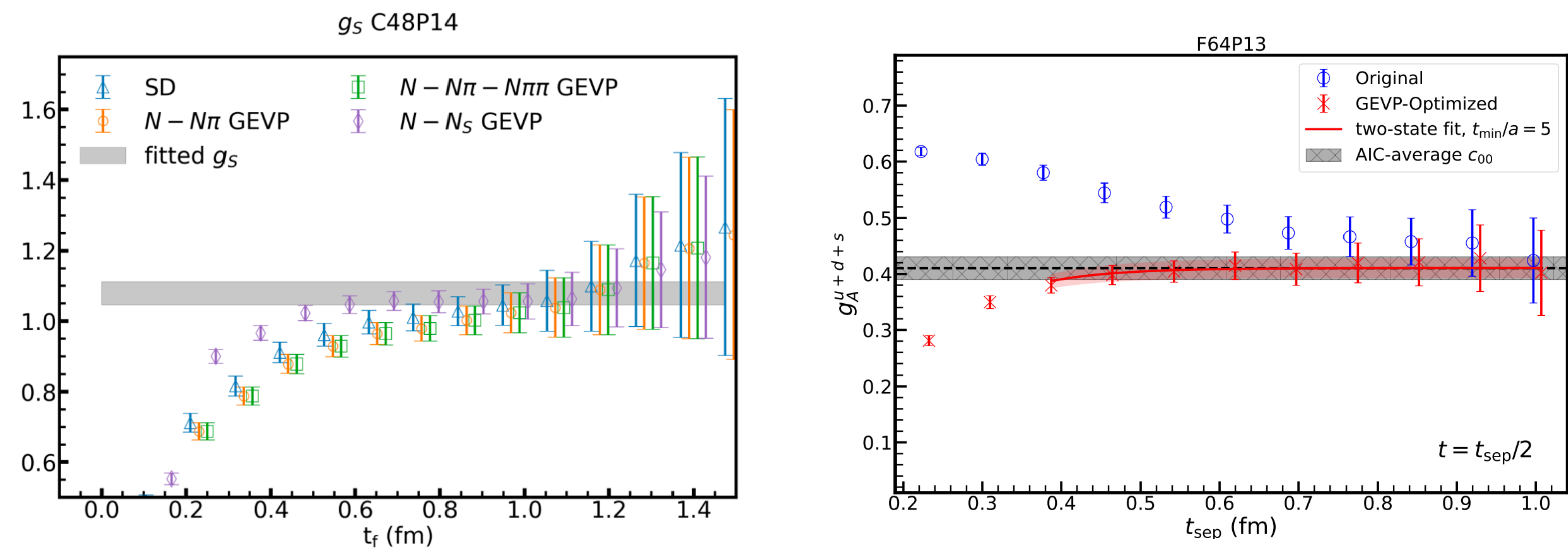




- Among the 45 neutral states shown, 40 have central values no larger than 1 MeV in magnitude;
- Singly charged states receive positive shifts below 5 MeV, with a median near 2 MeV;
- The doubly charged baryons have substantially larger shifts, ranging from approximately 3.7 to 8.4 MeV. The largest is  $\Delta^{\text{QED}} m_{\Omega_{ccc}} = 9.3(2)$  MeV.
- Those of the light baryons can be more precise in the near future with larger statistics.

# light baryons

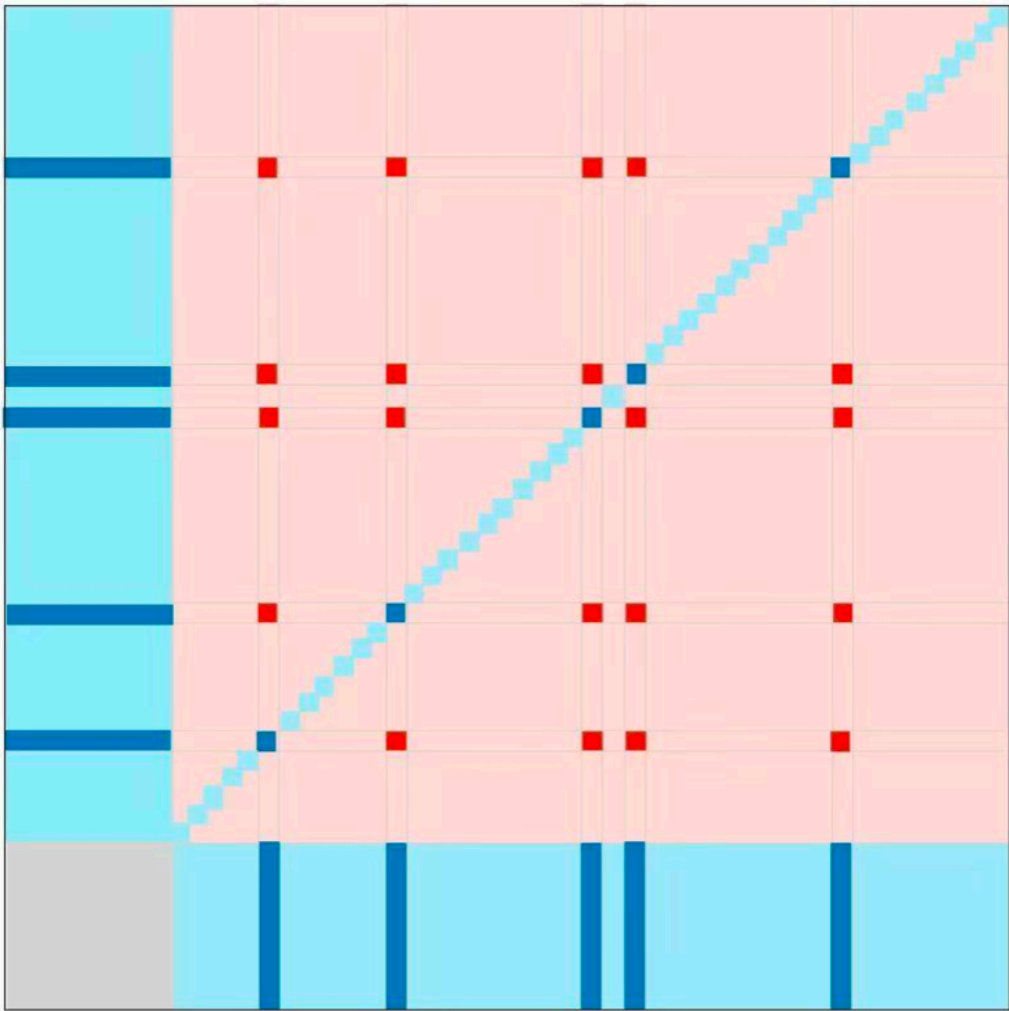
## Blending method



Counts of quark diagrams

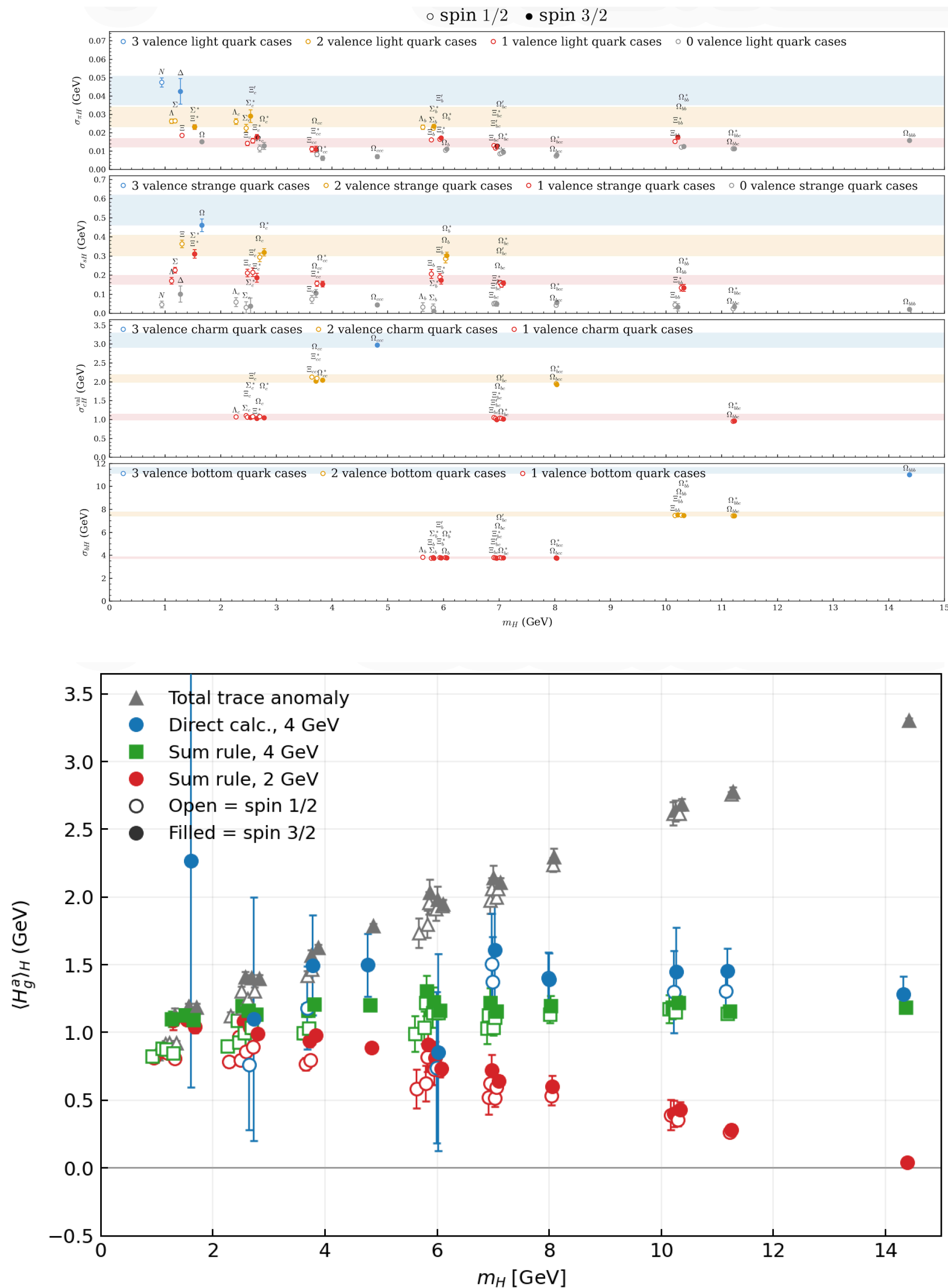
|                             | Unprojected | projected |
|-----------------------------|-------------|-----------|
| $C_{N_S, N_S}(t)$           | 26          | 54        |
| $C_{N_T, N_T}(t)$           | 26          | 486       |
| $C_{N\pi, N\pi}(t)$         | 102         | 1944      |
| $C_{N\pi\pi, N\pi\pi}(t)$   | 144         | 1178      |
| $C_{N, N_S}^J(t, \tau)$     | 26          | 42        |
| $C_{N, N_T}^J(t, \tau)$     | 26          | 126       |
| $C_{N, N\pi}^J(t, \tau)$    | 52          | 252       |
| $C_{N, N\pi\pi}^J(t, \tau)$ | 60          | 162       |

- Precise results for the light baryon likes nucleon requires good control of the excited state contaminations;
- Contaminations from  $N\pi$ ,  $N\pi\pi$  and also  $NJ$  states correspond to hundreds and thousands quark diagrams;
- The blending method can be so far the most efficient way to eliminate those contaminations.



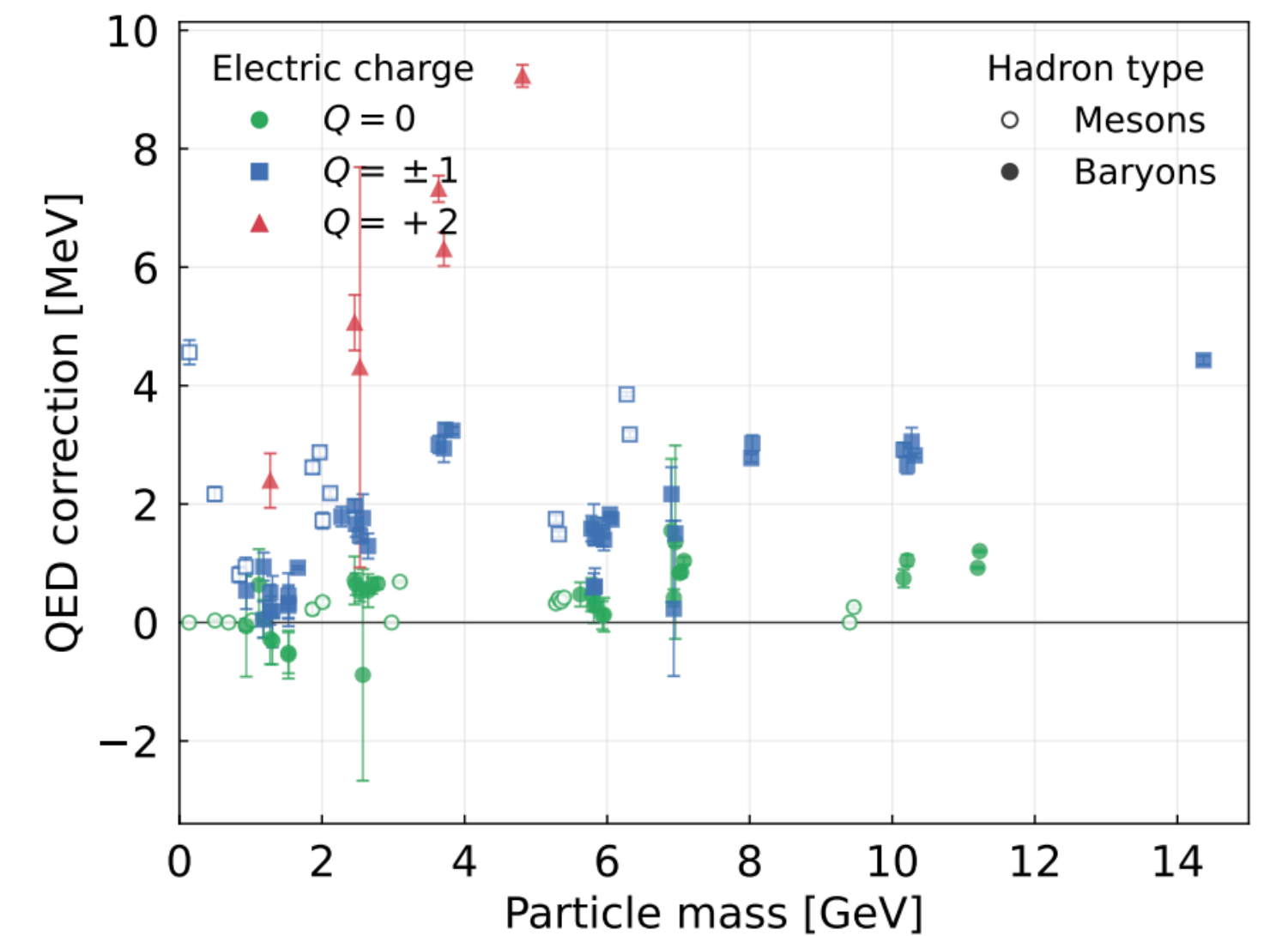
Oct 10th parallel session 2  
15:40-15:55 Ji-Hao Wang  
16:25-16:45 Zhi-Cheng Hu

# Summary



Strong interaction contributes to the hadron masses in three ways:

- Convert the scale dependent quark mass into scale independent contribution with flavor dependence enhancements;
- Flavor insensitive gluon trace anomaly at  $\overline{\text{MS}}$  4 GeV from the sum rule, with the coefficients verified with the direct gluon trace anomaly calculations.



- A few MeV QED corrections which include positive/negative correction for the repulsive/attractive interaction, and also flavor-dependent but hadron-insensitive quark self energy.