Electromagnetic form factors and radii of charmed light mesons from lattice QCD

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Background-Th, Ph

- The study of electromagnetic form factors and charge radii are important parts of understanding hadron structure.
- The study of charm-light mesons charge radius is challenging experimentally due to their large masses and short lifetimes.
- Various theoretical approaches have been used to predict these electromagnetic structure.

TABLE III. Charmed-meson electromagnetic observables from previous theoretical studies. Charge radii are quoted as r_C in fm, and magnetic moments as $G_M(0)$ in natural magnetons $e/(2M_V)$.

Observable	Meson	DSE	LFF	PM	CI	AM	CQM	ENJL	LQCD
r_C	D^+	0.435	0.429	0.510	–	0.680	0.505	0.46	0.45(24)/0.372(18)
	D^{*+}	0.473	–	–	–	–	–	–	0.430(28)
	D_s	0.352	0.352	0.465	0.26	0.372	0.377	0.39	0.465(57)
	D_s^*	0.381	–	–	0.369	–	–	–	–

Observable	Meson	DSE	ENJL	LCSR	EBM	BM	NR	ChPT
$G_M(0)$	D^{*+}	2.434	2.494	2.49(17)	2.279	2.515	2.83	$3.48^{+0.52}_{-0.17}$
	D_s^*	2.267	2.205	2.25(31)	1.958	2.3177	2.25	$1.553^{+0.49}_{-0.23}$

[1] Hwang, C.-W., Eur. Phys. J. C 23, 585 (2002).
[2] Luan, Y.-L. et al., Chin. Phys. C 39, 113103 (2015).
[3] Moita, R. M. et al., Phys. Rev. D 104, 096020 (2021).
[4] Almeida-Zamora, B. et al., Phys. Rev. D 109, 014016 (2024).
[5] Das, T. et al., Int. J. Theor. Phys. 63, 172 (2024).

Background-lat

	$r_{l/s}$/fm	r_c/fm	r /fm	a /fm	m_π/MeV	Action/Collaboration
D^+	0.585(57)	0.226(24)	0.372(18)	0.0907(13)	$300-700$	clover/PACS-CS
D^{*+}	0.689(68)	0.266(30)	0.430(28)	0.0907(13)	$300-700$	clover/PACS-CS
D^+	0.87(13)	0.50(57)	0.45(24)	0.085,0.067	300,315	twist mass/ETMC
D_s	0.826(27)	0.624(96)	0.465(57)	0.085,0.067	300,315	twist mass/ETMC

[1] Can, K. U et .al j.physletb.2012.12.050 (2013)
[2] Li, Ning et.al epja/i2017-12243-4 (2017)

For convenience to compare , The Ref1 results shown in this table are converted to charge radii. Its original results are are $\langle r^2{}_{D^+}\rangle = 0.138(13)\mathrm{fm}^2$ and $\langle r_{D^{*+}}\rangle = 0.185(24)\mathrm{fm}^2$ The uncertainties shown for the $\boldsymbol{r}$ are obtained by propagating, the uncertainties through the square root of $\boldsymbol{r^2}$.

Formula

The form factor can be obtained by decomposing the matrix elements.

For pseudoscalar meson

$$\langle D(p)|V^u(q)|D(p')\rangle = (p+p')^u G_C(Q^2)$$

For vector meson

$$\langle \lambda', \vec{p'}|j_\mu|\lambda, \vec{p}\rangle = \frac{1}{2\sqrt{E_i E_f}}\epsilon'^*_\alpha(\lambda', \vec{p'})\epsilon_\beta(\lambda, \vec{p})J^{\alpha\mu\beta}(p', p),$$

$$J^{\alpha\mu\beta}(p', p) = -\Big\{G_1(Q^2)g^{\alpha\beta}[p^\mu + p'^\mu] + G_2(Q^2)[g^{\mu\beta}q^\alpha - g^{\mu\alpha}q^\beta] + G_3(Q^2)q^\beta q^\alpha \frac{p^\mu + p'^\mu}{2mm'}\Big\},$$

$$G_C(Q^2) = G_1(Q^2) + \frac{2}{3}\eta G_Q(Q^2),$$

$$G_M(Q^2) = G_2(Q^2),$$

$$G_Q(Q^2) = G_1(Q^2) - G_2(Q^2) + (1+\eta)G_3(Q^2)$$

$$\eta = \frac{Q^2}{4m^2}$$

Define charge radius

$$r^2 = -6\frac{\partial G_C}{\partial Q^2}\Big|_{Q^2=0}$$

Numerical setup

TABLE I. Gauge ensembles and current-renormalization factors used in this calculation. The $N_f = 2+1$ clover ensembles are generated by the CLQCD Collaboration . The strange and charm quark masses are listed in lattice units. The local vector currents are renormalized by the factors Z_V obtained from Ward-identity normalization against the conserved current. The last two columns give the Gaussian-smearing parameters used for the quark sources and sinks.

Ensemble	a (fm)	m_π (MeV)	n_{cfg}	am_s	am_c	$L^3 \times T$	$(Z_V)_l$	$(Z_V)_s$	$(Z_V)_c$	n_{size}	n_{iter}
C24P29	0.10530(18)	292.7(1.2)	948	-0.2400	0.4479	$24^3 \times 72$	0.8209(3)	0.8618(3)	1.5613(7)	2	120
C32P23	0.10530(18)	228.0(1.2)	445	-0.2400	0.4500	$32^3 \times 64$	0.8173(4)	0.8608(4)	1.5623(11)	2	120
C48P14	0.10530(18)	135.5(1.4)	448	-0.2310	0.4502	$48^3 \times 96$	0.8120(4)	0.8688(5)	1.5623(15)	2	120
F32P30	0.07746(18)	303.2(1.3)	776	-0.2050	0.2079	$32^3 \times 64$	0.8499(2)	0.8766(2)	1.2983(3)	3	120
F32P21	0.07746(18)	210.9(2.2)	456	-0.2050	0.2104	$32^3 \times 64$	0.8470(2)	0.8766(2)	1.3010(4)	3	120
H48P32	0.05187(26)	317.2(0.9)	905	-0.1700	0.0581	$48^3 \times 144$	0.8778(1)	0.8938(2)	1.1273(2)	4	120

[1] Hu, Z.-C. et al. (CLQCD Collaboration), Phys. Rev. D 109, 054507 (2024)
[2] Y. Meng et al., Phys. Rev. D 109, 074511 (2024)..

Numerical details

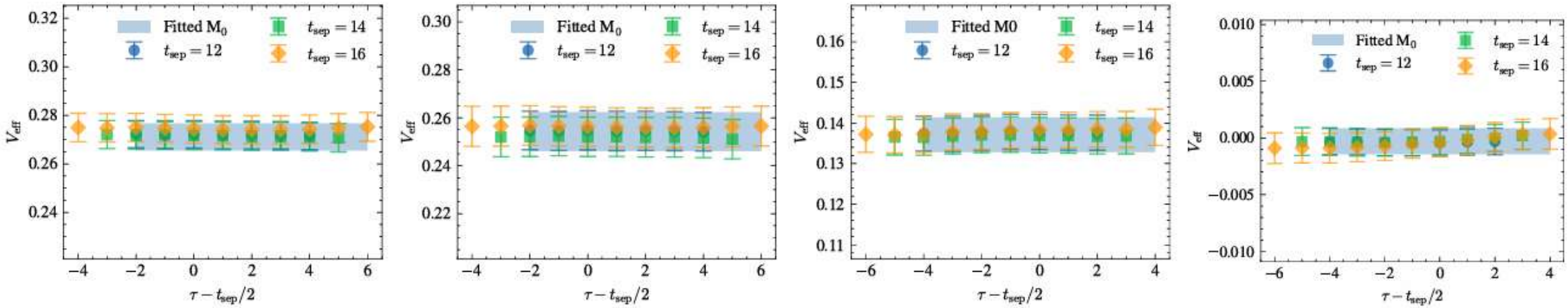
Matrix element fitting

Joint fit Results

$$C_{2i}(t) = |c_i|^2 e^{-E_i t'} + |c_i'|^2 e^{-E_i' t'},$$

$$C_{2f}(t) = |c_f|^2 e^{-E_f t'} + |c_f'|^2 e^{-E_f' t'},$$

$$C_3(t,\tau) = c_f c_i^* M_0 e^{-E_f(t-\tau)} e^{-E_i \tau} + c_f' c_i^* M_1 e^{-E_f'(t-\tau)} e^{-E_i \tau}$$
$$+ c_f c_i'^* M_2 e^{-E_f(t-\tau)} e^{-E_i' \tau} + c_f' c_i'^* M_3 e^{-E_f'(t-\tau)} e^{-E_i' \tau}$$



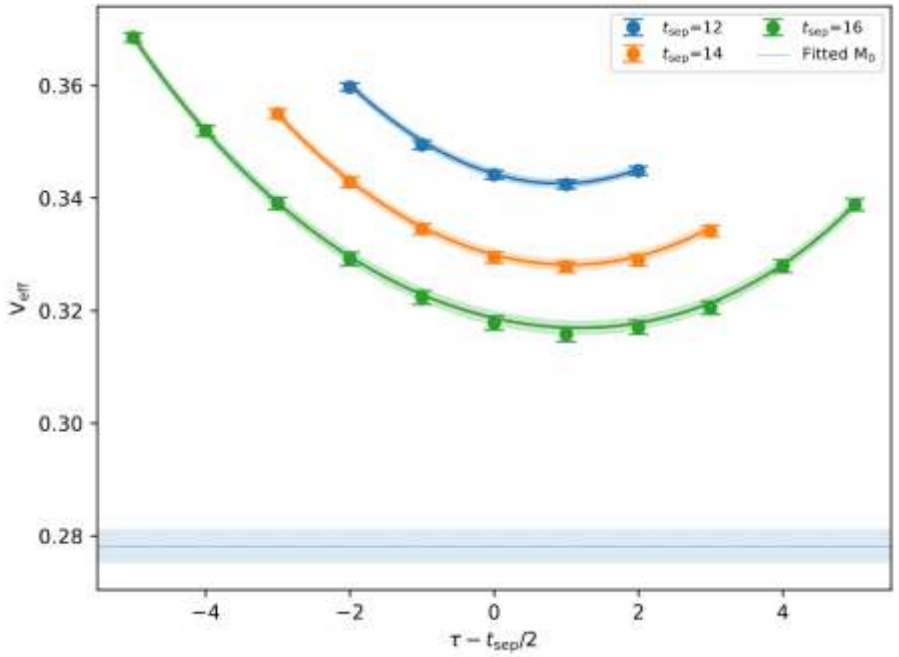
From left to right are results of $\langle P|\bar{l}\gamma_4 l|P\rangle$, and $\langle V_i|\bar{l}\gamma_4 l|V_i\rangle$ $\langle V_i|\bar{l}\gamma_i l|V_j\rangle$ $\langle V_i|\bar{l}\gamma_4 l|V_j\rangle$ for vector meson at F32P30 ensemble.

Numerical details

Matrix element fitting

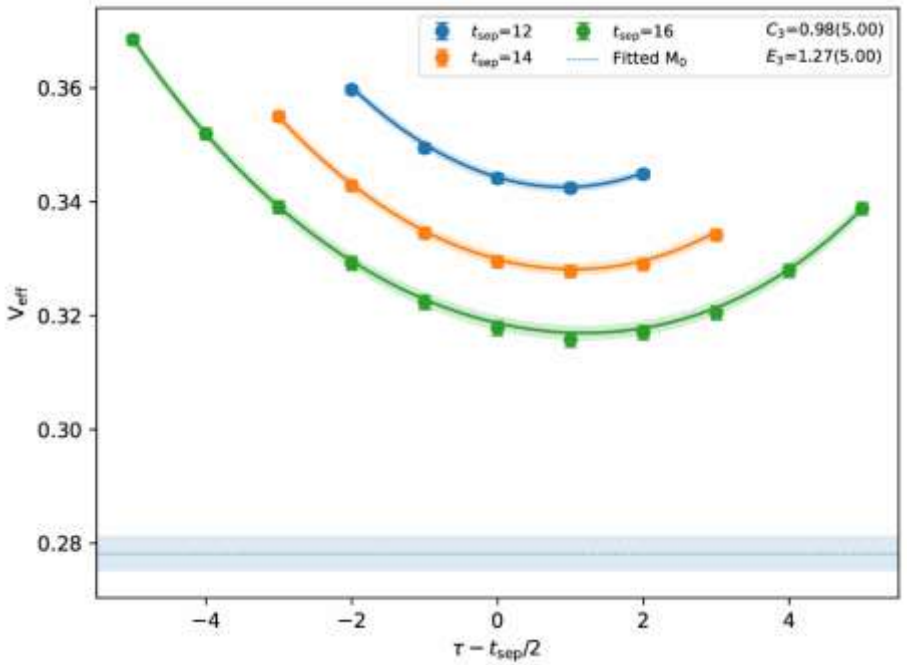
Take ratio

$$R_{\mu i} = C_{3\mu i}(t,\tau)\sqrt{\frac{2E_P C_{2P}(\tau)}{C_{2P}(t-\tau)C_{2P}(t)}}\sqrt{\frac{2E_V C_{2V}(t-\tau)}{C_{2V}(\tau)C_{2V}(t)}}$$
$$= M_0 + C_1 e^{-a\tau} + C_2 e^{-b(t-\tau)}.$$



Two-state

$$R_{\mu i} = C_{3\mu i}(t,\tau)\sqrt{\frac{2E_P C_{2P}(\tau)}{C_{2P}(t-\tau)C_{2P}(t)}}\sqrt{\frac{2E_V C_{2V}(t-\tau)}{C_{2V}(\tau)C_{2V}(t)}}$$
$$= M_0 + C_1 e^{-a\tau} + C_2 e^{-b(t-\tau)} + C_3 e^{-E_3 t}$$



Three-state

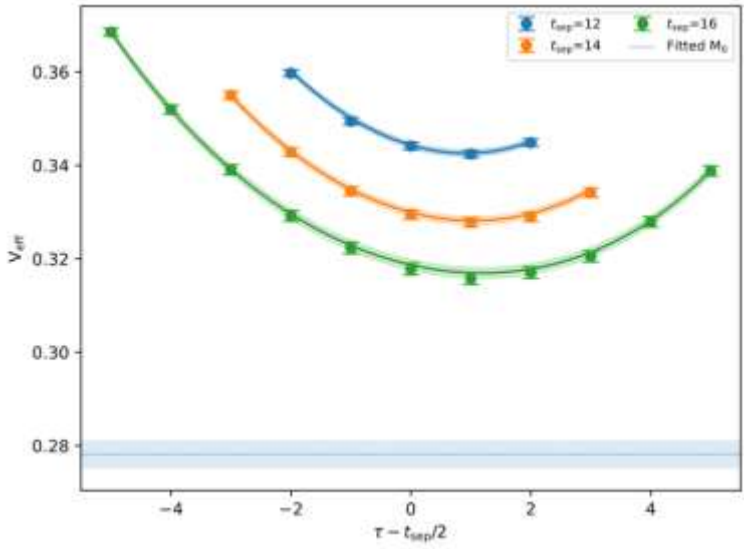
Numerical details

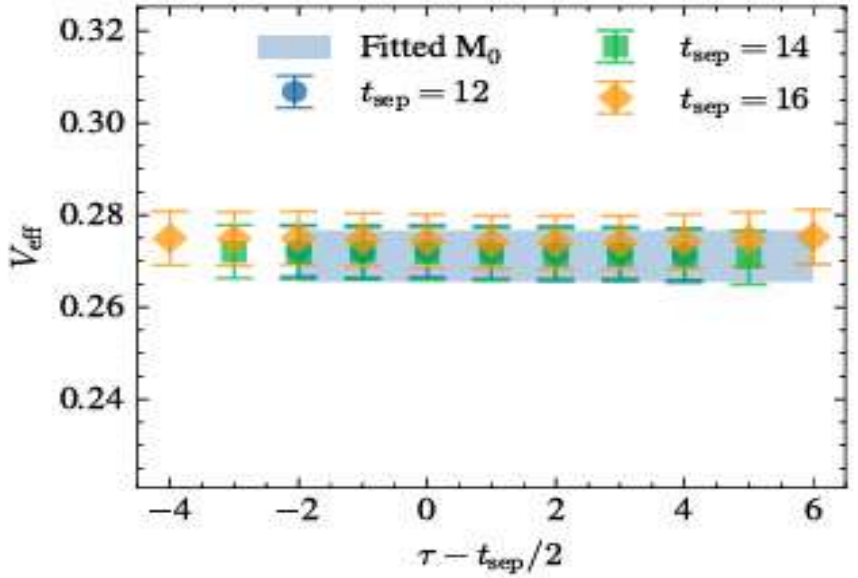
Matrix element fitting Systematic uncertainty

Fit range

Ensemble	2pt range t'	3pt range τ
F32P21	$(3 + \text{range}(0, 4), 19)$	$(3, t_{\text{sep}} - 3) + \text{range}(-1, 2)$
C32P23	$(2 + \text{range}(0, 4), 18)$	$(2, t_{\text{sep}} - 2) + \text{range}(-1, 2)$
H48P32	$(4 + \text{range}(0, 4), 20)$	$(4, t_{\text{sep}} - 4) + \text{range}(-1, 2)$
F32P30	$(3 + \text{range}(0, 4), 19)$	$(3, t_{\text{sep}} - 3) + \text{range}(-1, 2)$
C24P29	$(2 + \text{range}(0, 4), 18)$	$(2, t_{\text{sep}} - 2) + \text{range}(-1, 2)$
C48P14	$(2 + \text{range}(0, 4), 18)$	$(2, t_{\text{sep}} - 2) + \text{range}(-1, 2)$

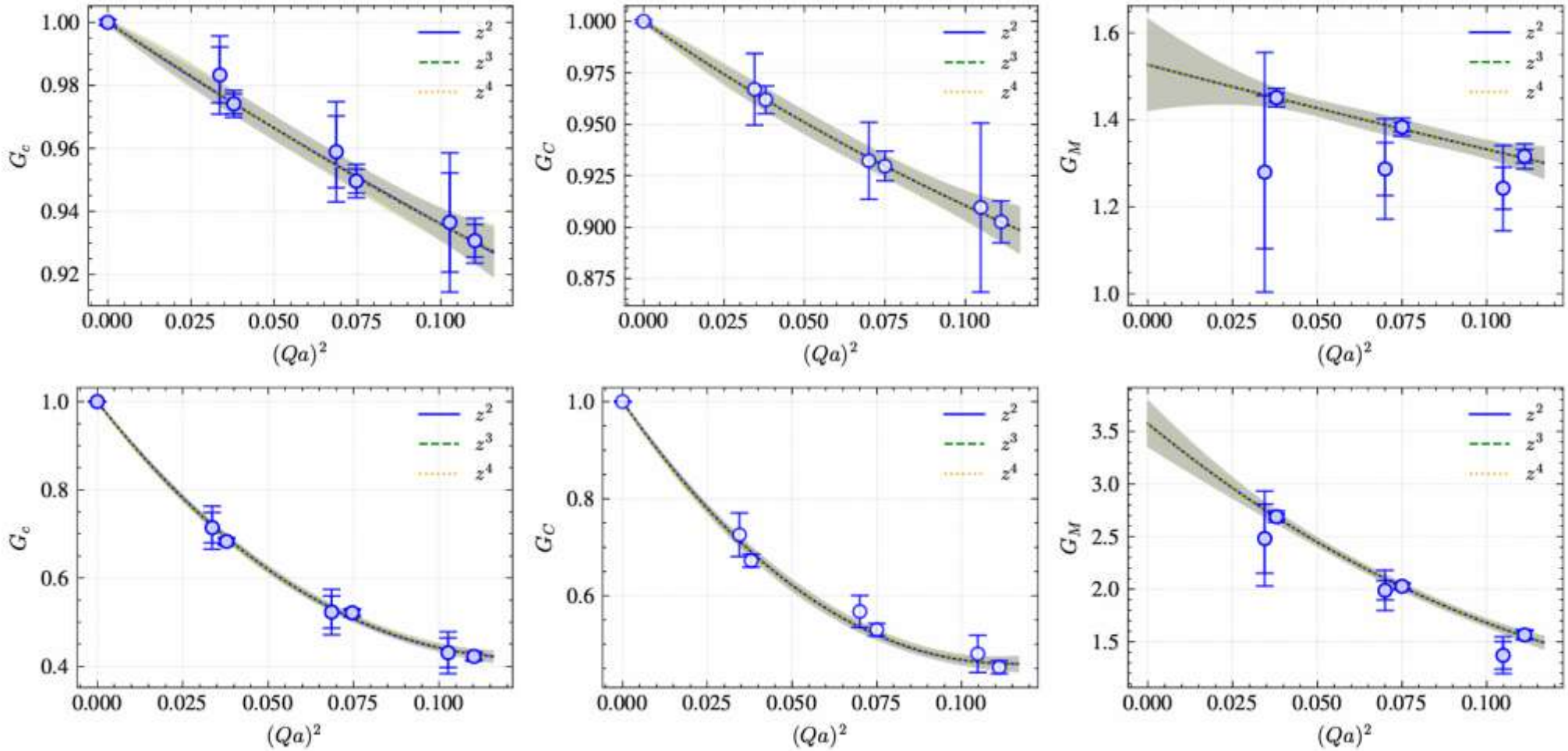
Fit method





Numerical details

Q^2 extrapolation



The Q^2 extrapolation result on the F32P30 ensemble. The upper part represents the contribution of charm quarks, and the lower part represents the contribution of light quarks. From left to right, the results are $r^2_{D^+}$, $r^2_{D^{*+}}$, $G_M^{D^{*+}}(0)$.

Numerical details

Chiral-continue extrapolation

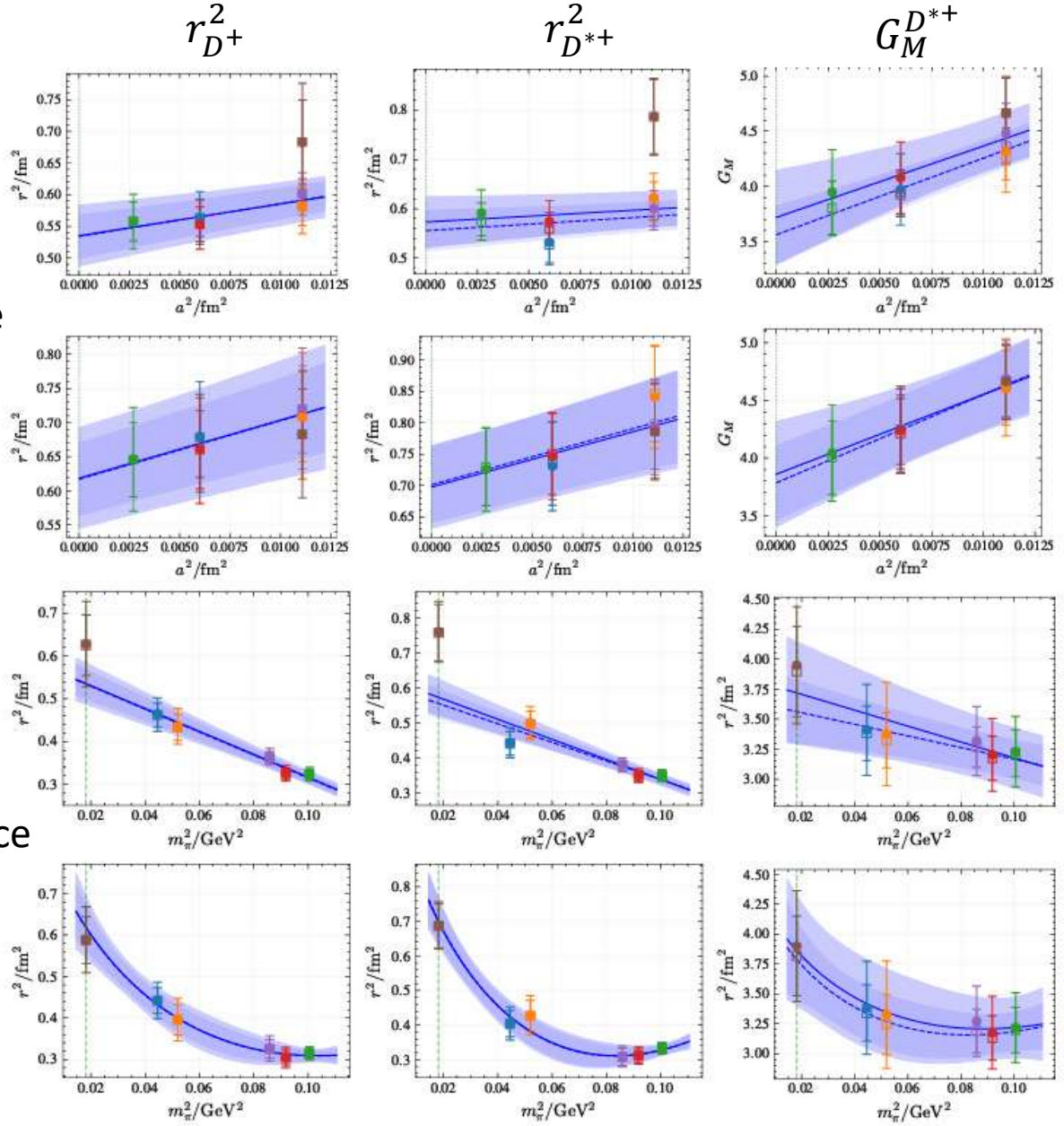
$$V\left(a^2, m_\pi^2\right) = V\left(0, m_{\pi,\mathrm{phy}}^2\right) + Aa^2 + B\left(m_\pi^2 - m_{\pi,\mathrm{phy}}^2\right) \quad (1)$$

a dependence

Light quark contribution

$$V\left(a^2, m_\pi^2\right) = V\left(0, m_{\pi,\mathrm{phy}}^2\right) + Aa^2 + B\left(m_\pi^2 - m_{\pi,\mathrm{phy}}^2\right) + C\left[m_\pi^2 \ln\left(m_\pi^2\right) - m_{\pi,\mathrm{phy}}^2 \ln\left(m_{\pi,\mathrm{phy}}^2\right)\right]. \quad (2)$$

m_π dependence



Numerical details

Chiral-continue extrapolation

$$V\left(a^2, m_\pi^2\right) = V\left(0, m_{\pi,\mathrm{phy}}^2\right) + Aa^2 + B\left(m_\pi^2 - m_{\pi,\mathrm{phy}}^2\right)$$

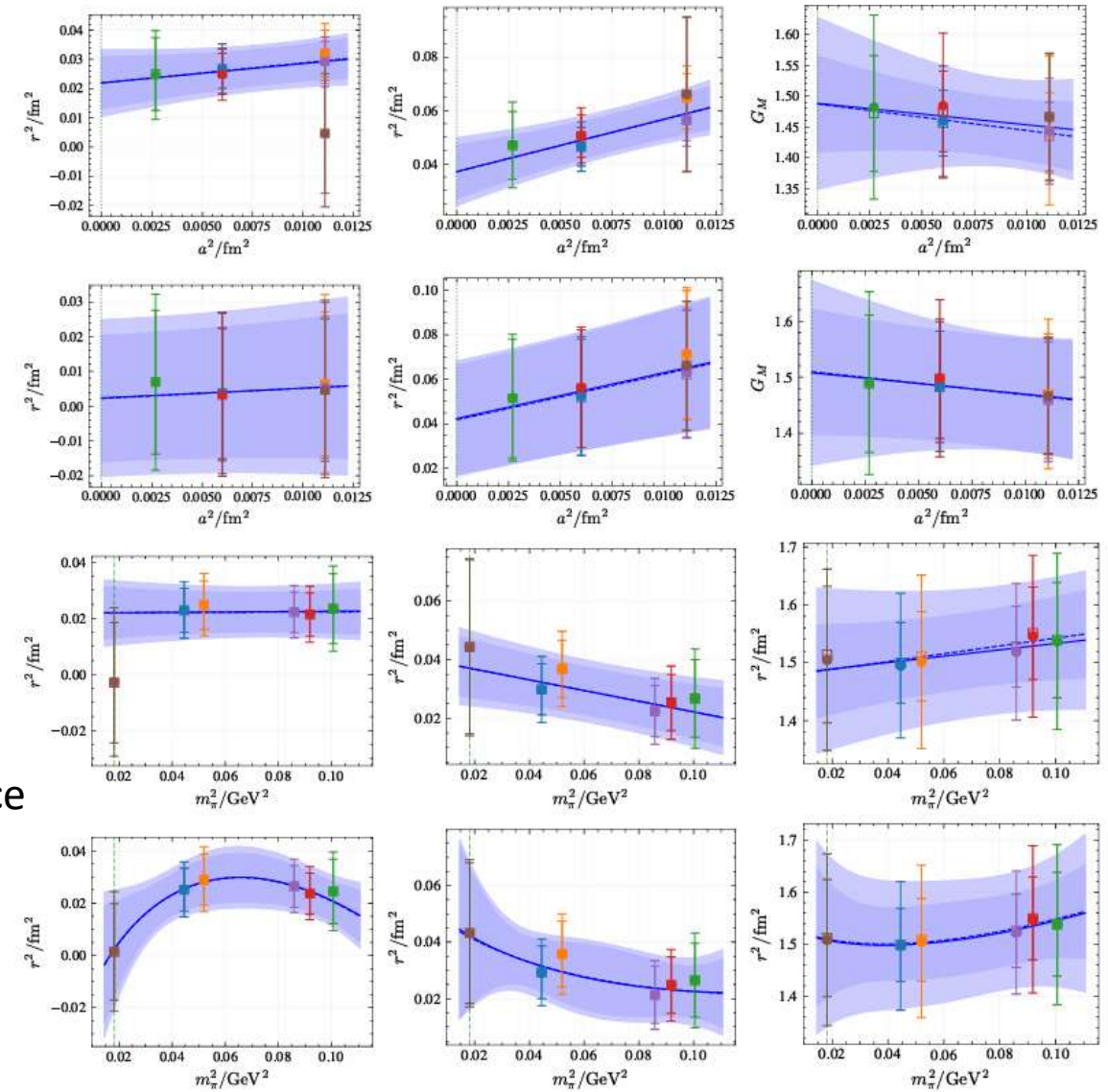
a dependence

Charm quark contribution

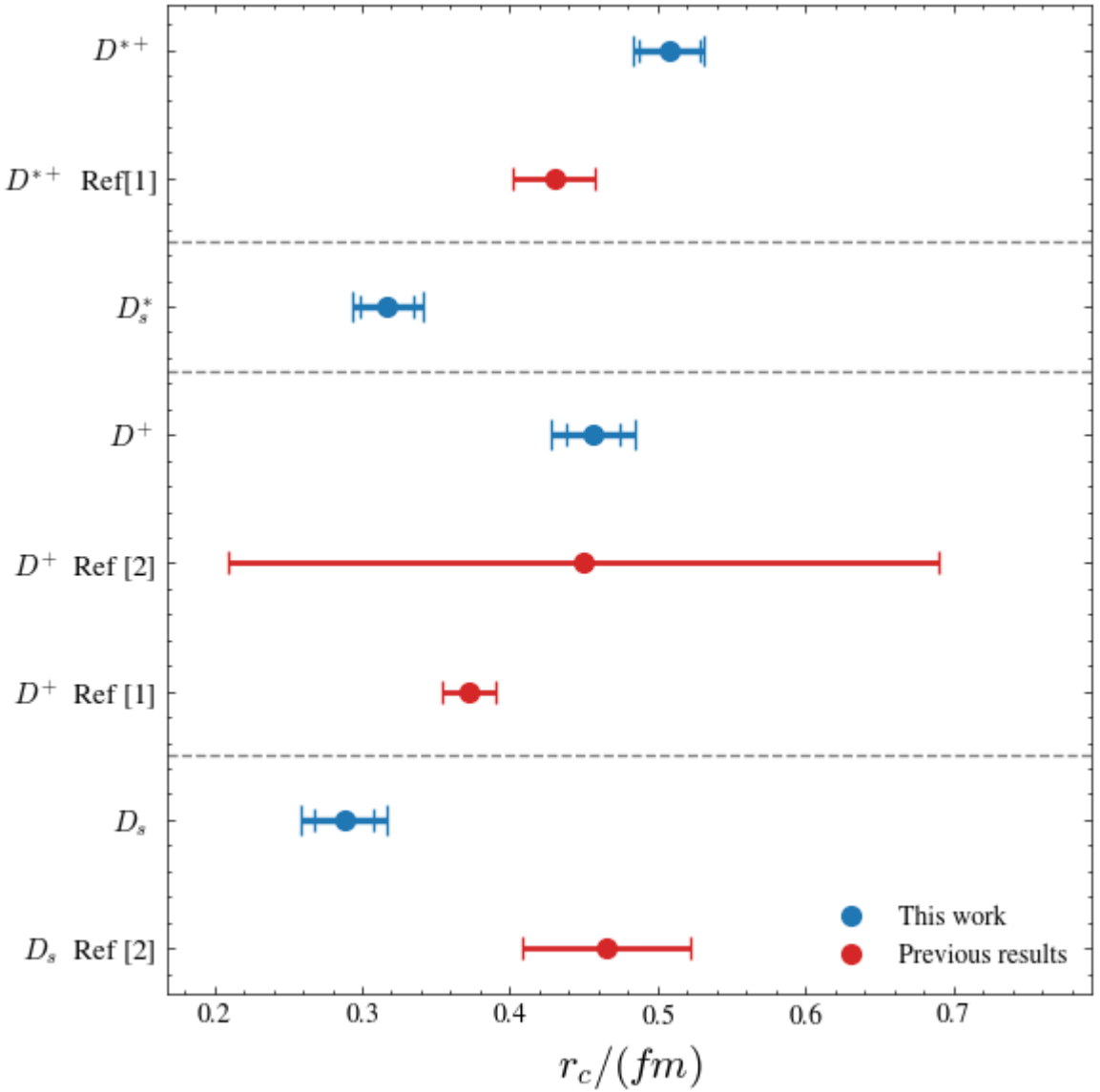
$$V\left(a^2, m_\pi^2\right) = V\left(0, m_{\pi,\mathrm{phy}}^2\right) + Aa^2 + B\left(m_\pi^2 - m_{\pi,\mathrm{phy}}^2\right) + C\left[m_\pi^2 \ln\left(m_\pi^2\right) - m_{\pi,\mathrm{phy}}^2 \ln\left(m_{\pi,\mathrm{phy}}^2\right)\right].$$

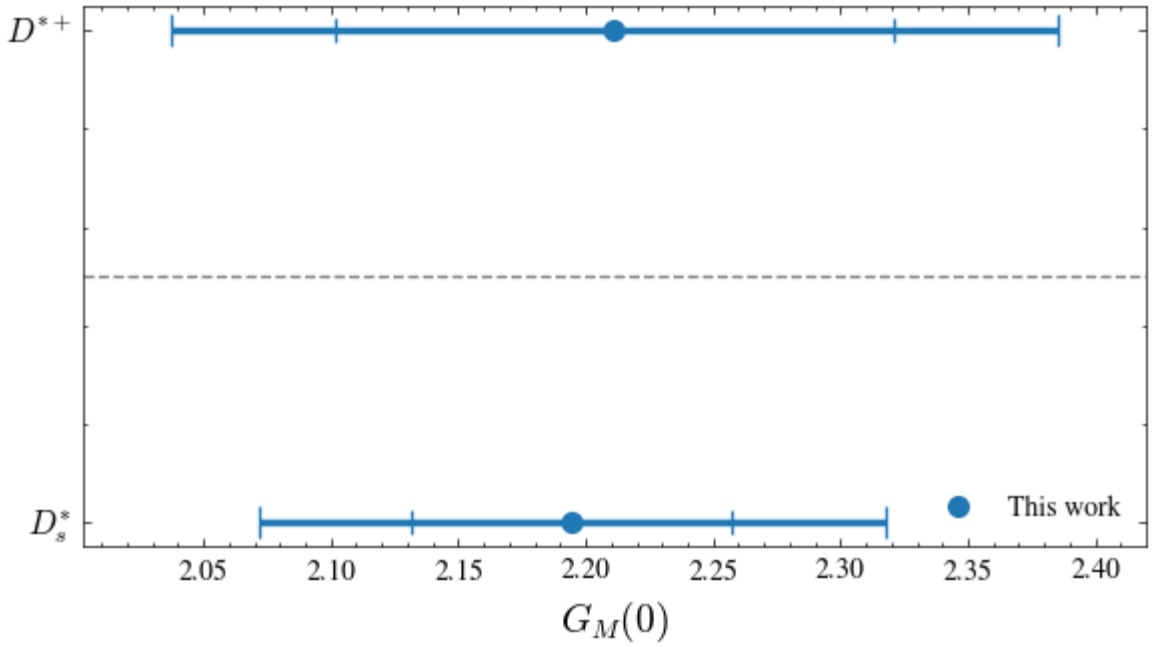
m_π dependence

$r_{D^+}^2$ $r_{D^{*+}}^2$ $G_M^{D^{*+}}$



Result Comparison





Ref 1 Can, K. U et .al j.physletb.2012.12.050 (2013)

300-700MeV a=0.0907(13)fm

Ref 2 Li, Ning et.al epja/i2017-12243-4 (2017)

300-315 MeV a=0.085fm a=0.067fm

Conculsion

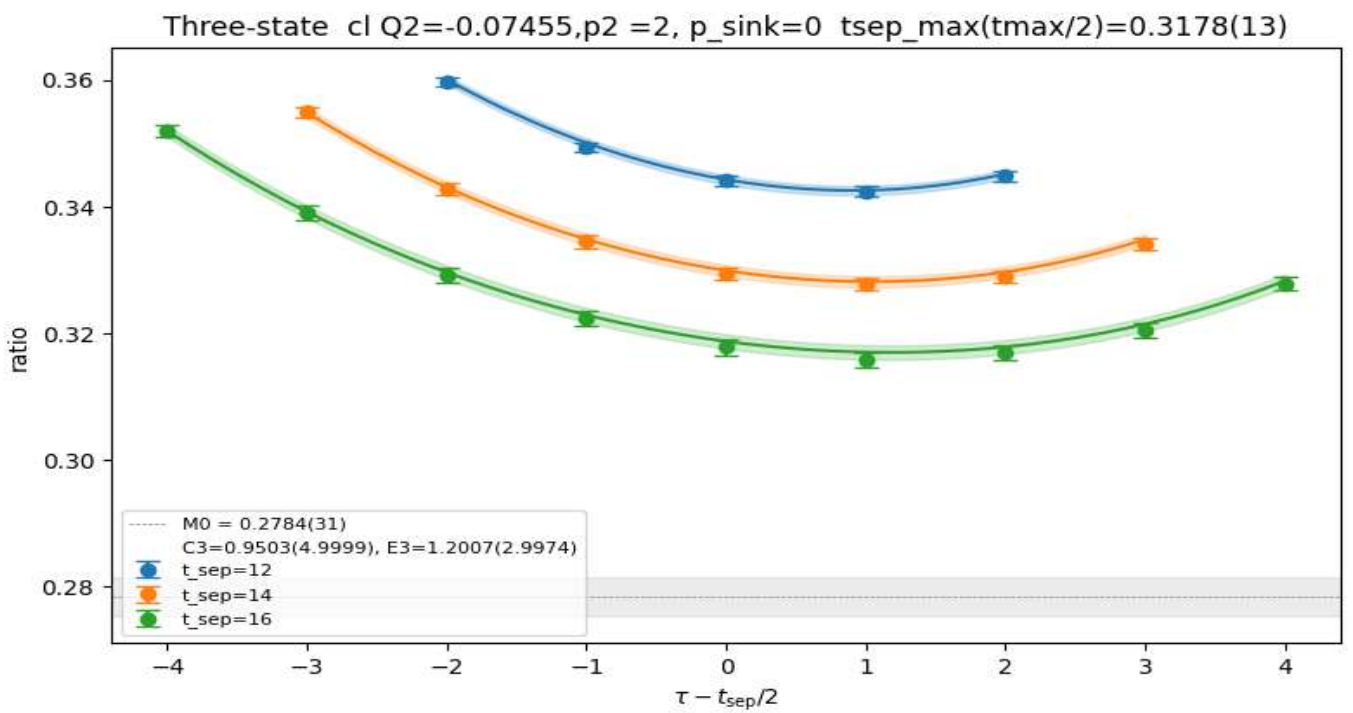
- We obtain the square of charge radii of $r^2_{D^{*+}} = 0.258(25)\ \mathrm{fm}^2$, $r^2_{D^+} = 0.208(26)\ \mathrm{fm}^2$, $r^2_{D_s^*} = 0.101(15)\ \mathrm{fm}^2$, $r^2_{D_s} = 0.083(17)\ \mathrm{fm}^2$

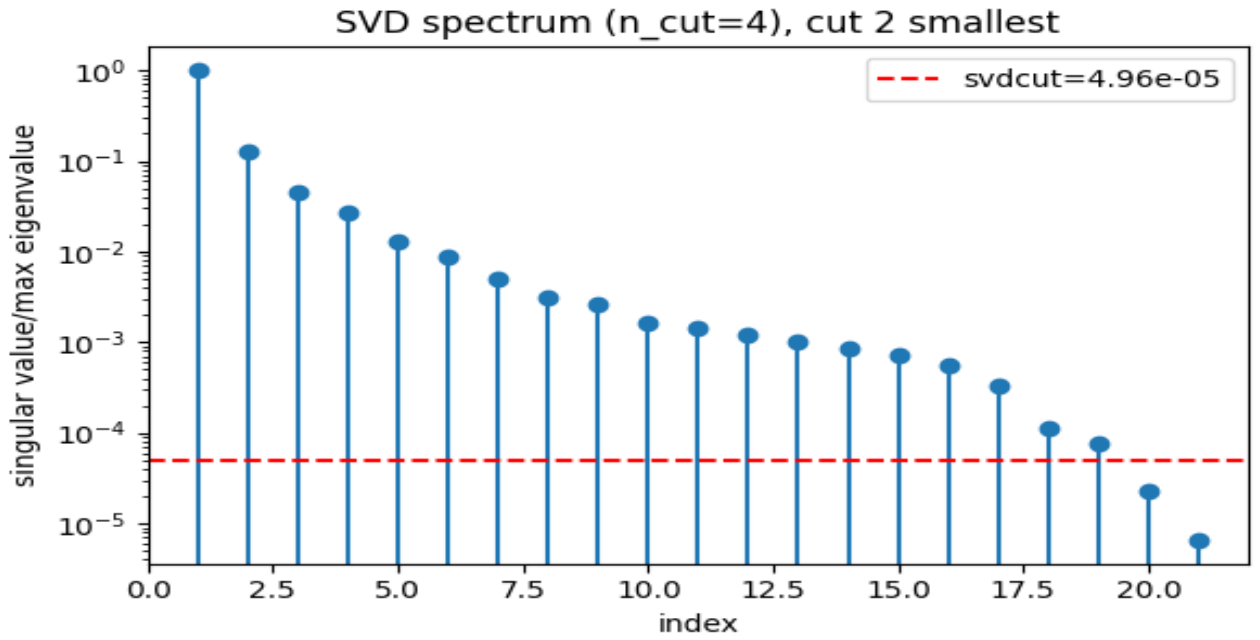
and the magnetic dipole moments of $G_M^{D^{*+}} = 2.21(17)$ and $G_M^{D_s^{*+}} = 2.19(12)$

- We also use the same technique to calculate the radiative decay coupling constants of D^* and D_s^*. $g_{D^{*+}D^+\gamma} = -0.204(22)\ \mathrm{GeV}^{-1}$, $g_{D^{*+}D^+\gamma} = 1.73(37)\ \mathrm{GeV}^{-1}$ and $g_{D_s^* D_s \gamma} = -0.120(14)\mathrm{GeV}^{-1}$

[1] W.-Z. Hou et al.Phys. Rev. D 113, 114513 (2026).

Thanks





light-front framework (LFF), QCD potential model (PM) , constituent quark model (CQM),
contact interaction model (CI), Algebraic model (AM), Extended Nambu–Jona-Lasinio model (ENJI
Dyson-Schwinger equations(DSE) lightconesum rules (LCSR) ,Extended-Bagmodel (EBM ,Bagmode
non-relativisticquark model (NR) chiral perturbationtheory (ChPT)