

S -wave scattering length of the $J/\psi J/\psi$ system from lattice QCD in coordinate space

JiaJun Xie¹

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第六届中国格点量子色动力学研讨会

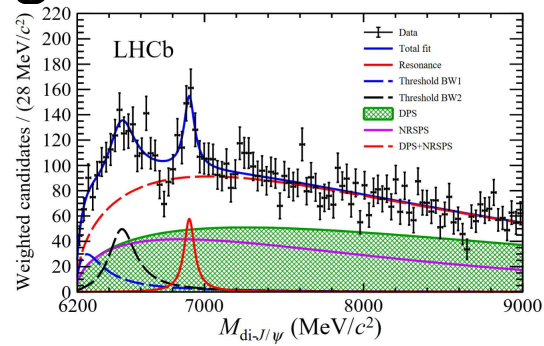
2025.10.10 深圳

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3. Department of Physics and Astronomy, University of Kentucky

Outline

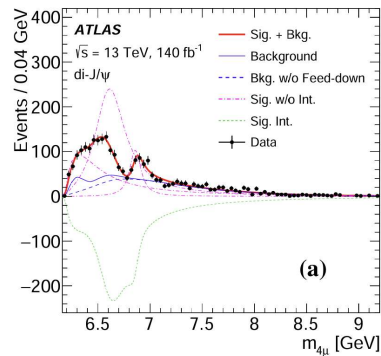
- Background
- Lattice calculation
- Lattice spectrum analysis
- Summary and outlook

Background



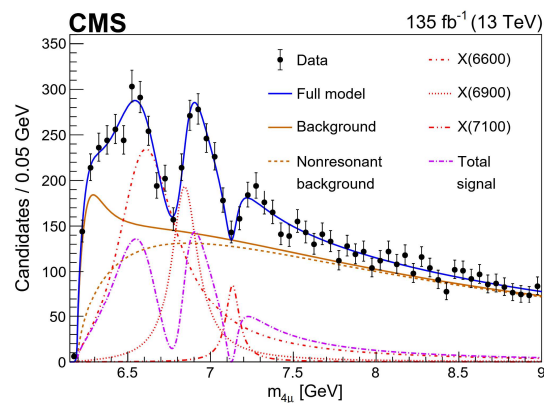
In 2020, LHCb collaboration reported an all-charm tetraquark candidate, X(6900).

[R. Aaij et al. \(LHCb Collaboration\), Sci. Bull. 65, 1983 \(2020\).](#)



In 2022, ATLAS collaboration also observed X(6900), together with a broad structure at lower mass.

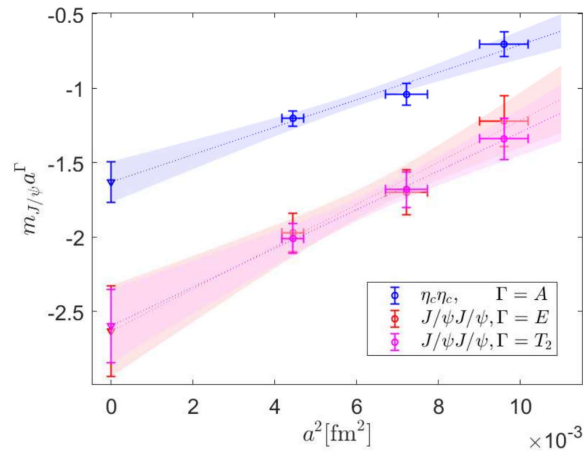
[G. Aad et al. \(ATLAS\), Phys. Rev. Lett. 131, 151902 \(2023\).](#)



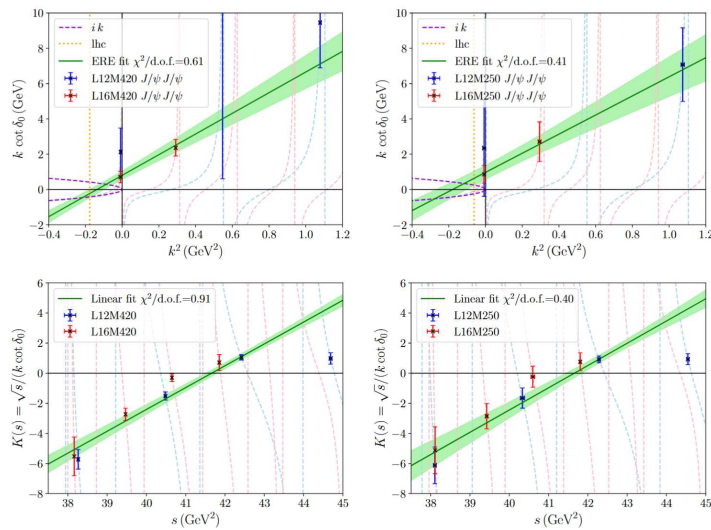
In 2023, CMS collaboration reported a tetraquark family consisting of X(6600), X(6900), and X(7300).

[A. Hayrapetyan et al. \(CMS\), Phys. Rev. Lett. 132, 111901 \(2024\).](#)

Background



CLQCD S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, Phys. Lett. B 833, 137391 (2022), Lattice calculation of the $\eta_c \eta_c$ and $J/\psi J/\psi$ s-wave scattering length.



G. Li, C. Shi, Y. Chen, and W. Sun, Phys. Rev. D 114, L031506 (2026), Scalar and tensor structures in $J/\psi J/\psi$ scattering from lattice QCD.

Lattice calculation

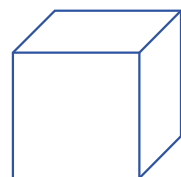
Symmetry

SO(3)



$$L^P = 0^+, 1^-, 2^+, \dots$$

O_h



$$A_1^\pm, A_2^\pm, E^\pm, T_1^\pm, T_2^\pm$$

SO(3)	O _h	O _h	SO(3)
0 ⁺	A ₁ ⁺	A ₁ ⁺	0 ⁺ , 4 ⁺ , 8 ⁺ , ...
1 ⁻	T ₁ ⁻	A ₁ ⁻	9 ⁻ , ...
2 ⁺	E ⁺ ⊕ T ₂ ⁺	A ₂ ⁺	6 ⁺ , ...
3 ⁻	A ₂ ⁻ ⊕ T ₁ ⁻ ⊕ T ₂ ⁻	A ₂ ⁻	3 ⁻ , 7 ⁻ , 9 ⁻ , ...
4 ⁺	A ₁ ⁺ ⊕ E ⁺ ⊕ T ₁ ⁺ ⊕ T ₂ ⁺	E ⁺	2 ⁺ , 4 ⁺ , 6 ⁺ , 8 ⁺ , ...
5 ⁻	E ⁻ ⊕ 2T ₁ ⁻ ⊕ T ₂ ⁻	E ⁻	5 ⁻ , 7 ⁻ , 9 ⁻ , ...
6 ⁺	A ₁ ⁺ ⊕ A ₂ ⁺ ⊕ E ⁺ ⊕ T ₁ ⁺ ⊕ 2T ₂ ⁺	T ₁ ⁺	4 ⁺ , 6 ⁺ , 8 ⁺ , ...
7 ⁻	A ₂ ⁻ ⊕ E ⁻ ⊕ 2T ₁ ⁻ ⊕ 2T ₂ ⁻	T ₁ ⁻	1 ⁻ , 3 ⁻ , 5 ⁻ , 7 ⁻ , 9 ⁻ , ...
8 ⁺	A ₁ ⁺ ⊕ 2E ⁺ ⊕ 2T ₁ ⁺ ⊕ 2T ₂ ⁺	T ₂ ⁺	2 ⁺ , 4 ⁺ , 6 ⁺ , 8 ⁺ , ...
9 ⁻	A ₁ ⁻ ⊕ A ₂ ⁻ ⊕ E ⁻ ⊕ 3T ₁ ⁻ ⊕ 2T ₂ ⁻	T ₂ ⁻	3 ⁻ , 5 ⁻ , 7 ⁻ , 9 ⁻ , ...

In the finite volume, an irreducible representation contains contributions from different partial waves.

How to project the correlation function into a specific irreducible representation ?

Momentum space
Coordinate space

Momentum space

Coordinate space

How to project the correlation function into a specific irreducible representation ?

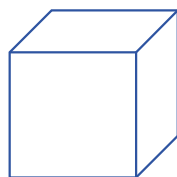
Symmetry

$SO(3)$



$$L^P = 0^+, 1^-, 2^+, \dots$$

O_h



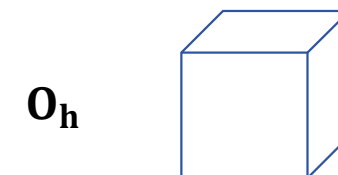
$$A_1^\pm, A_2^\pm, E^\pm, T_1^\pm, T_2^\pm$$

Momentum space: It requires substantial storage and computational resources, for instance, $\pi\pi$ system:

$$(\pi\pi)_{\vec{P}, \Lambda, \mu}^{[\vec{k}_1, \vec{k}_2]^\dagger} = \sum_{\substack{\vec{k}_1 \in \{\vec{k}_1\}^* \\ \vec{k}_2 \in \{\vec{k}_2\}^* \\ \vec{k}_1 + \vec{k}_2 = \vec{P}}} \mathcal{C}(\vec{P}, \Lambda, \mu; \vec{k}_1; \vec{k}_2) \pi^\dagger(\vec{k}_1) \pi^\dagger(\vec{k}_2)$$

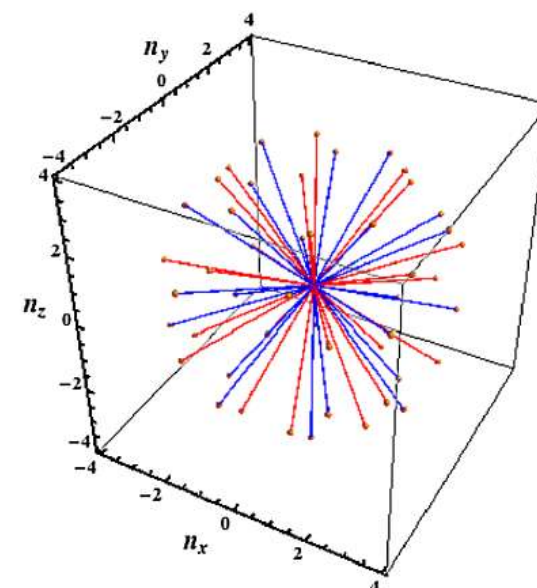
$$\pi^\dagger(\vec{k}, t) = \sum_{\vec{x}} e^{i\vec{k} \cdot \vec{x}} \left[\bar{\psi} \square_\sigma \mathbf{\Gamma}_t^\dagger \square_\sigma \psi \right] (\vec{x}, t)$$

Advantage: It allows for precise control over the scattering energy levels for each lattice momentum.



O_h

$A_1^\pm, A_2^\pm, E^\pm, T_1^\pm, T_2^\pm$



T. Luu and M. J. Savage, PRD 83 114508(2011)

Lattice calculation

How to project the correlation function into a specific irreducible representation ?

Momentum space

Coordinate space

Coordinate space: It significantly reduces storage and computational requirements.

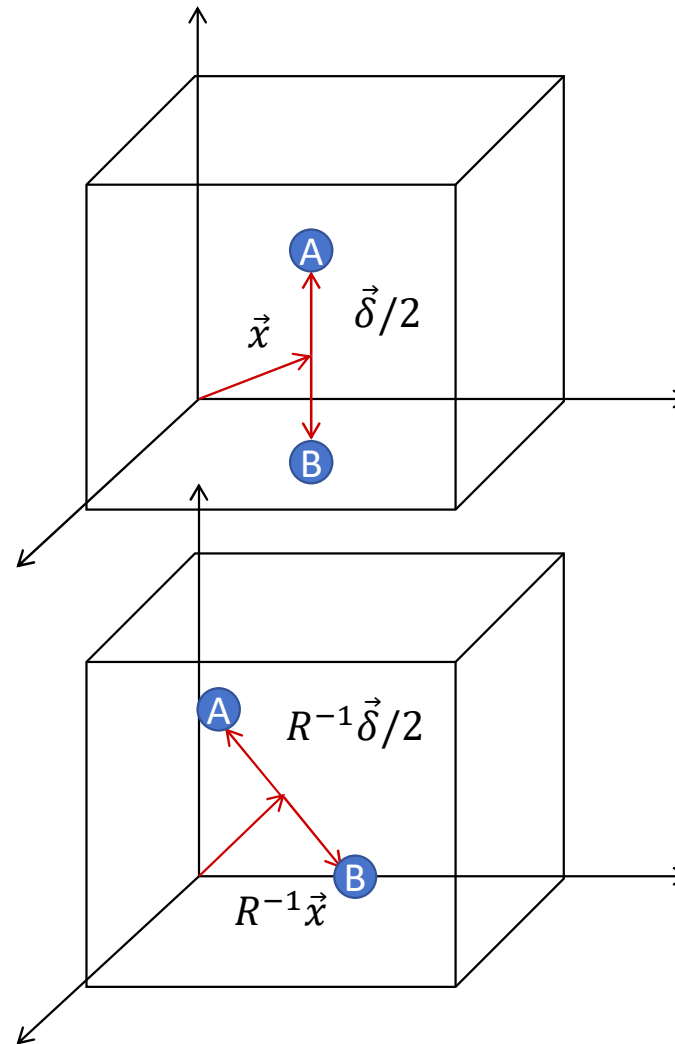
Disadvantage: We can only calculate the ground state for each irreducible representation.

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.

$\hat{R}$	Rotation Axis	Rotation Angle	(x_1, x_2, x_3)
$\hat{R}_1$	Any	0°	(x_1, x_2, x_3)
$\hat{R}_2$	(1,1,1)	-120°	(x_2, x_3, x_1)
...	...	...	...
$\hat{R}_{24}$	(0,0,1)	-180°	$(-x_1, -x_2, x_3)$
$\hat{R}_{25} = \hat{\pi} \hat{R}_1$	-	-	$(-x_1, -x_2, -x_3)$
$\hat{R}_{26} = \hat{\pi} \hat{R}_2$	-	-	$(-x_2, -x_3, -x_1)$
...	...	...	...
$\hat{R}_{48} = \hat{\pi} \hat{R}_{24}$	-	-	$(x_1, x_2, -x_3)$

Lattice calculation

Key method: Using rotations to generate the **regular representation space** of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.



$$\phi(\vec{x}, \vec{\delta}, t) = \chi_A(\vec{x} + \vec{\delta}/2, t) \chi_B(\vec{x} - \vec{\delta}/2, t)$$

Rotation Elements: $\hat{R} = \{\hat{R}_1, \hat{R}_2, \dots, \hat{R}_{48}\}$

$$\vec{\delta} = (\delta_x, \delta_y, \delta_z)$$

$$\delta_x \neq \delta_y \neq \delta_z$$

$$\vec{x} \pm \vec{\delta}/2 = \vec{n} \in \mathbb{Z}^3$$

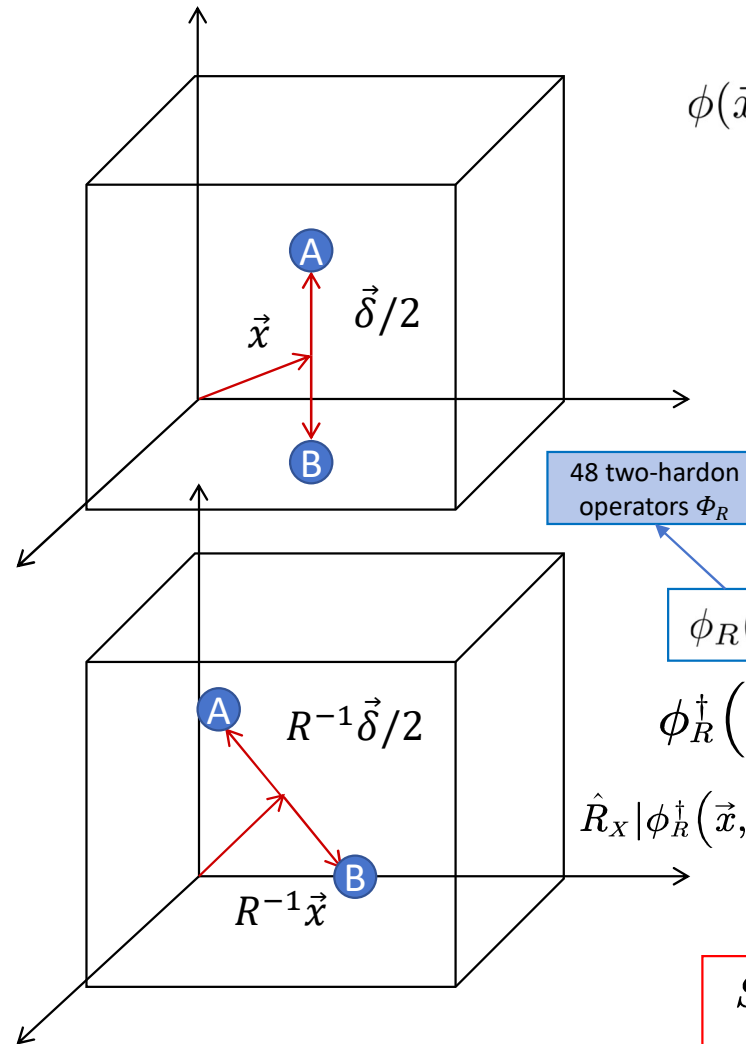
$$\vec{x} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \text{ and } \vec{\delta} = (1, 3, 5)$$

Simplifies continuum rotation symmetry

$$\phi_R(\vec{x}, \vec{\delta}, t) = \chi_A(\hat{R}^{-1}(\vec{x} + \vec{\delta}/2), t) \chi_B(\hat{R}^{-1}(\vec{x} - \vec{\delta}/2), t)$$

Lattice calculation

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$$\phi_R^\dagger(\vec{x}, \vec{\delta}, t) |\Omega\rangle = |\phi_R^\dagger(\vec{x}, \vec{\delta}, t)\rangle \sim |\phi_R^\dagger(\vec{x}, \vec{\delta})\rangle$$

$$\hat{R}_X |\phi_R^\dagger(\vec{x}, \vec{\delta}, t)\rangle = |\phi_{R_X R}^\dagger(\vec{x}, \vec{\delta}, t)\rangle = \sum_{R'} |\phi_{R'}^\dagger(\vec{x}, \vec{\delta}, t)\rangle [\bar{\Gamma}_{\text{reg}}(R_X)]_{R', R}$$

$$S^{-1} \bar{\Gamma}_{\text{reg}}(R_X) S = A_1^+ \oplus A_2^+ \oplus 2E^+ \oplus 3T_1^+ \oplus 3T_2^+ \\ \oplus A_1^- \oplus A_2^- \oplus 2E^- \oplus 3T_1^- \oplus 3T_2^-$$

Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then **performing a projection onto the regular representation** space to obtain the **operators** and correlation functions corresponding to specific irreducible representations.

$$\hat{R}_X |\phi_R^\dagger(\vec{x}, \vec{\delta}, t)\rangle = |\phi_{R_X R}^\dagger(\vec{x}, \vec{\delta}, t)\rangle = \sum_{R'} |\phi_{R'}^\dagger(\vec{x}, \vec{\delta}, t)\rangle [\bar{\Gamma}_{\text{reg}}(R_X)]_{R', R}$$

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$$= \bar{\Gamma}_{i, \Gamma, n; i', \Gamma', n'}$$

occurrence

Irreducible representation

n_{th} row

This block-diagonal matrices will lead to a new set of basis states: $|\Phi_{i, \Gamma, n}^\dagger(\vec{x}, \vec{\delta})\rangle$

$$\hat{R}_X |\Phi_{i, \Gamma, n}^\dagger(\vec{x}, \vec{\delta})\rangle = \sum_{i', \Gamma', n'} |\Phi_{i', \Gamma', n'}^\dagger(\vec{x}, \vec{\delta})\rangle [S^{-1} \bar{\Gamma}_{\text{reg}}(R_X) S]_{i', \Gamma', n'; i, \Gamma, n}$$

$$|\Phi_R^\dagger(\vec{x}, \vec{\delta})\rangle \xrightarrow{S} |\Phi_{i, \Gamma, n}^\dagger(\vec{x}, \vec{\delta})\rangle$$

$$|\Phi_{i, \Gamma, n}^\dagger(\vec{x}, \vec{\delta})\rangle = \sum_R |\Phi_R^\dagger(\vec{x}, \vec{\delta})\rangle [S]_{R; i, \Gamma, n}$$

Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the **operators** and **correlation functions** corresponding to specific irreducible representations.

$$\sum_{(\vec{y}-\vec{x}) \in \mathbb{Z}^3} e^{-i\vec{p} \cdot (\vec{x}-\vec{y})} \langle \Omega | \Phi_{i,\Gamma,n}(t, \vec{y}, \vec{\delta}) \Phi_{i',\Gamma',n'}^\dagger(0, \vec{x}, \vec{\delta}) | \Omega \rangle$$

$$= \delta_{i,i'} \delta_{\Gamma,\Gamma'} \delta_{n,n'} \sum_{(\vec{y}-\vec{x}) \in \mathbb{Z}^3} e^{-i\vec{p} \cdot (\vec{x}-\vec{y})} \frac{h}{l_\Gamma} \sum_R \chi_\Gamma(R) \langle \Omega | \phi_R(t, \vec{y}, \vec{\delta}) \phi^\dagger(0, \vec{x}, \vec{\delta}) | \Omega \rangle$$

Diagonal and Just rely on Γ

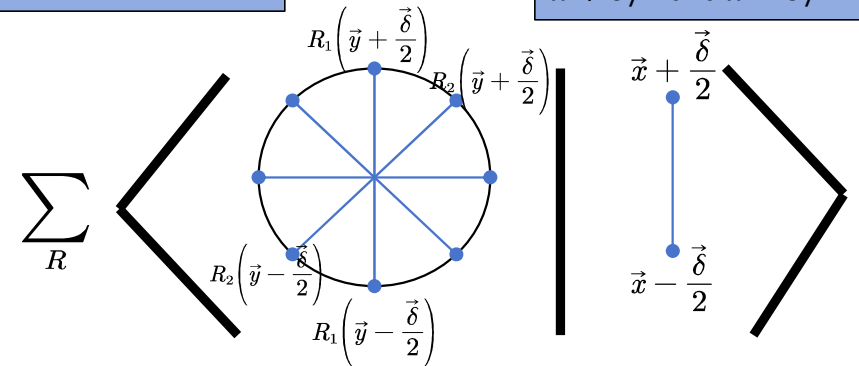
h and l_Γ are the orders of O_h group and the dimension of irreducible representation Γ .

Character Number of Γ

"Shell" Sink

Two source locations as $\vec{x} + \vec{\delta}/2$ and $\vec{x} - \vec{\delta}/2$

Coordinate space: It significantly reduces storage and computational requirements.



CSSM-QCDSF Collaboration Jia-Jun Wu et al. *PRD* 105 (2022) 7, 074506

Lattice calculation $J/\psi J/\psi$ system

$$J/\psi: \phi_\mu(\vec{x}) \equiv \sum_a \bar{c}^a(\vec{x}) \gamma_\mu c^a(\vec{x}), \quad \mu = x, y, z$$

$$\Phi_{\mu\nu\hat{R}}(\vec{x}, \vec{\delta}) \equiv \hat{P}_{\hat{R}} \phi_\mu(\vec{x} + \vec{\delta}/2) \phi_\nu(\vec{x} - \vec{\delta}/2) \hat{P}_{\hat{R}^{-1}}$$

$$= \sum_{\rho, \sigma} \bar{D}_{\sigma\mu}(\hat{R}_{\text{in}}) \bar{D}_{\rho\nu}(\hat{R}_{\text{in}}) \phi_\sigma(\hat{R}^{-1}(\vec{x} + \vec{\delta}/2)) \phi_\rho(\hat{R}^{-1}(\vec{x} - \vec{\delta}/2))$$

$|\vec{P}|^2 = 0$

$T_1^- \otimes T_1^-$

$\left\{ \begin{array}{l} A_1^+ \longleftrightarrow S=0 \\ T_1^+ \longleftrightarrow S=1 \\ E^+, T_2^+ \longleftrightarrow S=2 \end{array} \right\}$

$s_1 = 1 \otimes s_2 = 1$

The irreps for S-wave and P-wave

Partial wave	$ P ^2=0$	$ P ^2=1$	$ P ^2=2$	$ P ^2=3$
$^1S_0$	A_1^+	A_1	A_1	A_1
$^3P_1$	$T_1^+(\times)$	A_2, E	A_2, B_1, B_2	A_2, E
$^5S_2$	E^+	A_1, B_1	A_1, A_2	E
	T_2^+	B_2, E	A_1, A_2, B_1	A_1, E

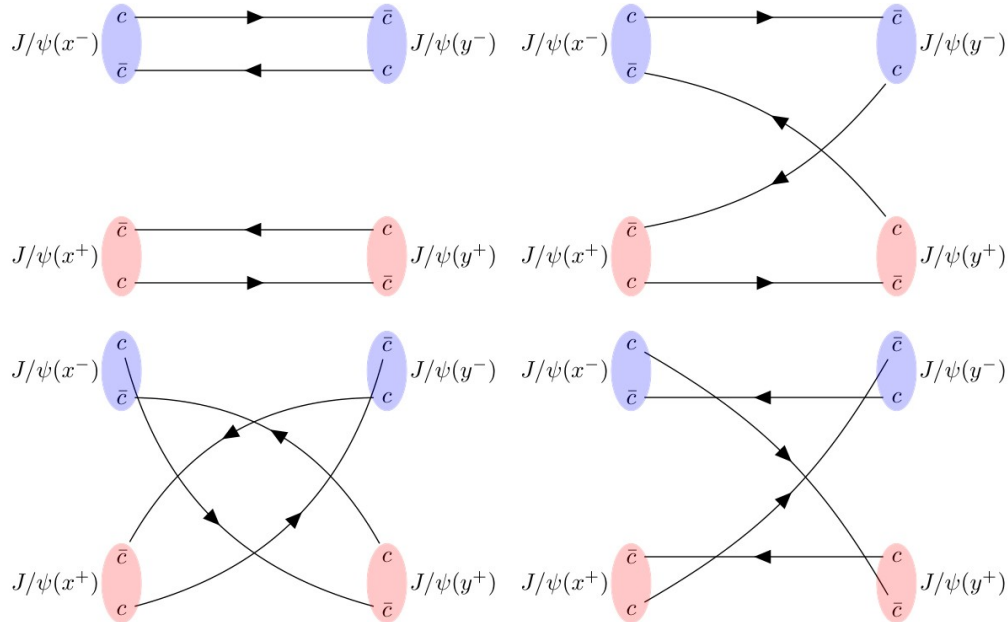
Lattice calculation $J/\psi J/\psi$ system

Lattice setup

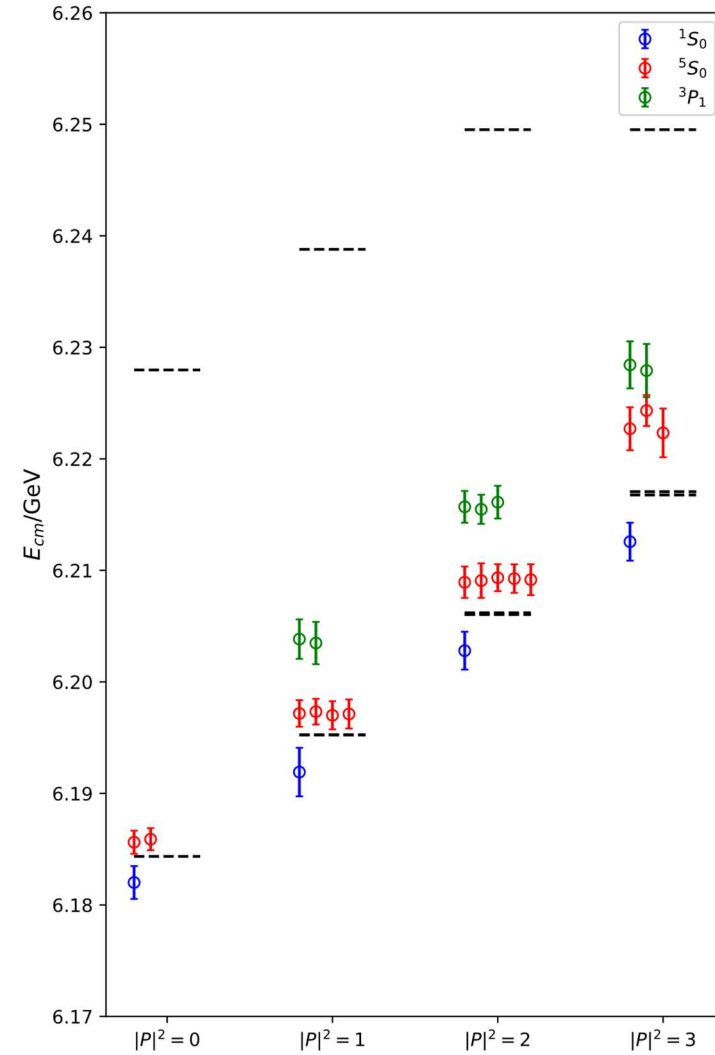
Config	$L^3 \times T$	$a[fm]$	$m_\pi [MeV]$	$m_{J/\psi}[MeV]$	N_{cfg}
C32P29	$32^3 \times 64$	0.10530	292.4	3092.2	1048



Wick contraction diagrams

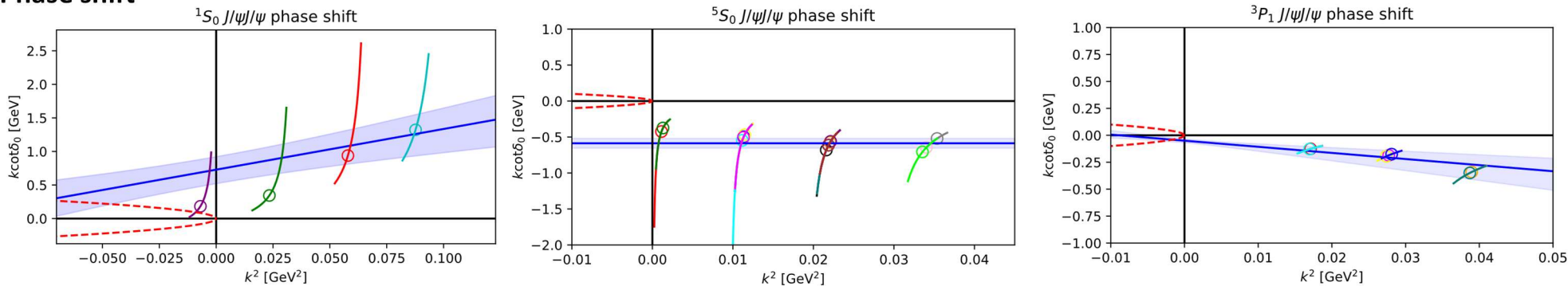


Lattice spectrum



Lattice spectrum analysis

Phase shift



Scattering length

	Partial wave	$a_0[fm]$	$r_0[fm]$	Pole/GeV
Geng Li	$^1S_0$	0.25(7) 0.20(8)	2.31(33) 2.14(41)	-0.028(10) -0.038(20)
	$^5S_2$	-0.163(16) -0.171(29)	/	$6.543(10) - \frac{i}{2}0.548(34)$ $6.543(10) - \frac{i}{2}0.548(34)$
Yu Meng	$^5S_2$	-0.165(16)	/	/
Our result	$^1S_0$	0.29(14)	2.35(1.04)	-0.049(26)
	$^5S_2$	-0.180(20)	/	/
	$^3P_1$	-2.25(1.72)	-1.30(56)	/

CLQCD S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, Phys. Lett. B 833, 137391 (2022)

G. Li, C. Shi, Y. Chen, and W. Sun, Phys. Rev. D 114,L031506 (2026)

Summary and outlook

- Utilizing a new method which is based on the projection of rotation in coordinate space, we construct the $J/\psi J/\psi$ operator referring to different irreps and calculate corresponding lattice energy spectrum.
- In $^1S_0$ channel, the interaction is attractive, for $^5S_0$ channel and $^3P_1$ channel, the interaction is repulsive and we extract corresponding phase shift and scattering length.
- Introducing operators with higher momentum.